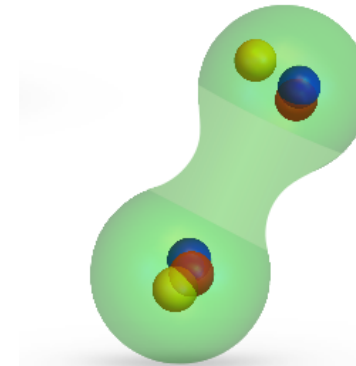
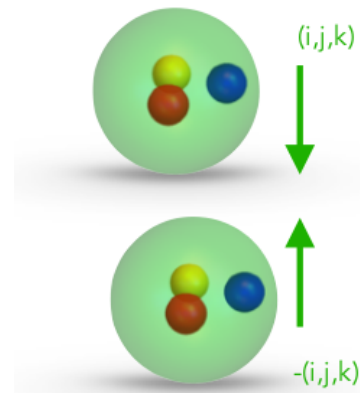
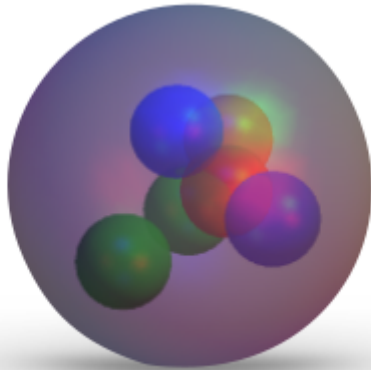




# NUCLEAR PHYSICS FROM THE STANDARD MODEL



Robert J. Perry

University of Barcelona, Spain

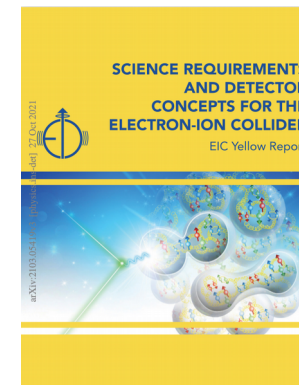
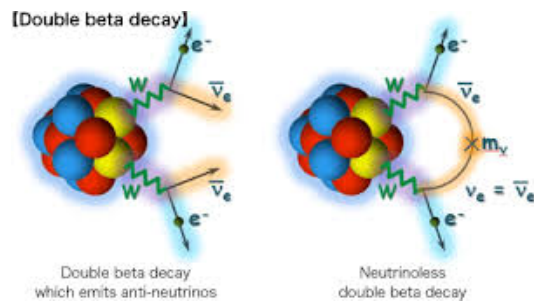
perryrobertjames@gmail.com

On Behalf Of:

Zohreh Davoudi, William Detmold, Anthony V. Grebe, Marc Illa, William I. Jay, Assumpta Parreño, Phiala E. Shanahan, Michael L. Wagman  
(NPLQCD)

# WHY NUCLEAR PHYSICS USING LQCD?

- **Understand** emergence of nuclear complexity from the quarks and gluons.
- Particle content of SM has been observed, but SM cannot explain:
  - Dark matter,
  - Neutrino masses,
  - etc.
- Nuclear objects often employed as targets in BSM physics searches: often large source of systematic uncertainty.
  - LQCD has potential to be relevant to a broad set of experimental programs.



# VIA MULTI-HADRON SPECTROSCOPY

- (Hermitian) matrix of correlators:

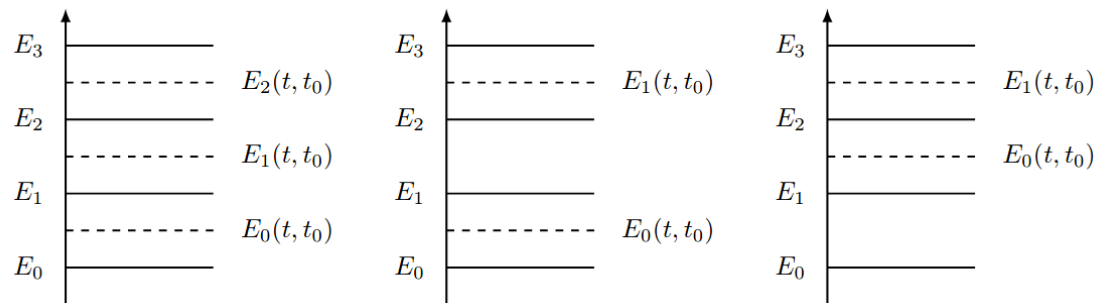
$$\mathbf{C}(t) = \begin{bmatrix} \text{Diagram 1} & \text{Diagram 2} \\ \text{Diagram 3} & \text{Diagram 4} \end{bmatrix}$$

Most recent work: up to 46x46 correlation matrix!

- Solve Generalized Eigenvalue Problem (GEVP):

$$\mathbf{C}(t)\vec{v}_n(t, t_0) = \lambda_n(t, t_0)\mathbf{C}(t_0)\vec{v}_n(t, t_0)$$

- Interlacing theorem: GEVP Eigenvalues provide **rigorous (stochastic) variational upper bounds on energy levels**



- Require multiple L. Generally work with **one** lattice spacing...

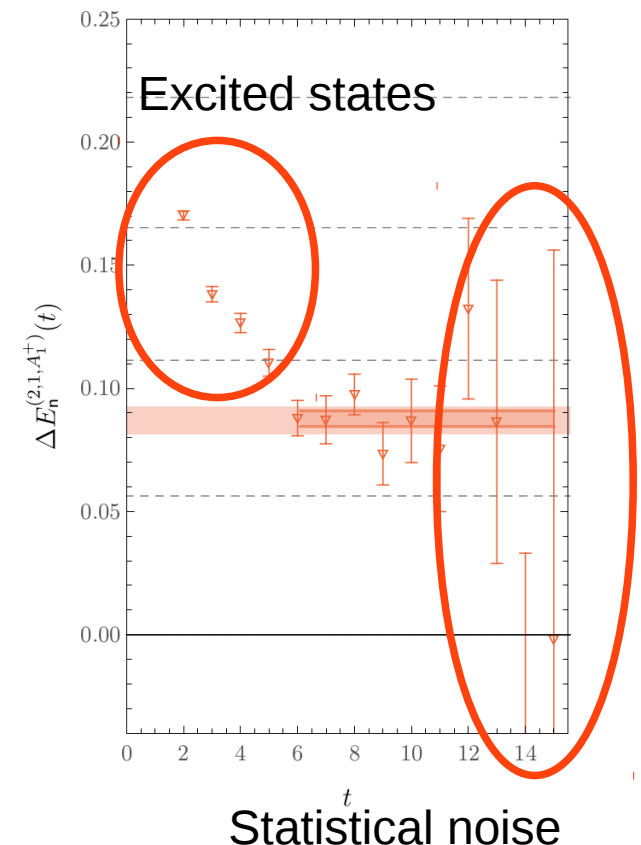
# WHY ARE NUCLEAR SYSTEMS CHALLENGING?

- Common to all lattice calculations:
  - Light quarks are expensive
  - Continuum limit requires multiple lattice ensembles
- BB: 6 quark system → large cost computing quark contractions
- StN problem at large Euclidean time
- Small energy gaps:

$$C_{\chi\chi}(t) = |Z_{0\chi}|^2 e^{-tE_0} + |Z_{1\chi}|^2 e^{-tE_1} + \dots$$

$$\Delta E = E_1 - E_0 \sim \frac{1}{L^2}$$

$$E_{\text{eff}} = \ln \frac{C_{\chi\chi'}(t)}{C_{\chi\chi'}(t+1)}$$



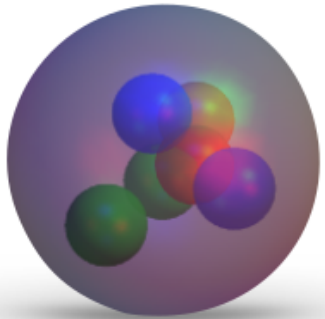
# RECENT WORK



*Variational Studies*

- NN @  $m_\pi = 806$  MeV:
  - **NN32**: “Variational study of two-nucleon systems with lattice QCD”, Phys.Rev.D 107 (2023) 9, 094508.
  - **NN24**: “Constraints on the finite volume two-nucleon spectrum at  $m_\pi = 806$  MeV”, arXiv:2404.12039 [hep-lat].
- $0\nu 2\beta$ :
  - “Long-Distance Nuclear Matrix Elements for Neutrinoless Double-Beta Decay from Lattice QCD”, e-Print: 2402.09362 [hep-lat].
- Many (6144) body physics
  - “Lattice quantum chromodynamics at large isospin density”, Phys.Rev.D 108 (2023) 11, 114506.
- Formal Developments
  - “Multi-particle interpolating operators in quantum field theories with cubic symmetry”, e-Print: 2403.00672 [hep-lat].

# TYPES OF OPERATORS

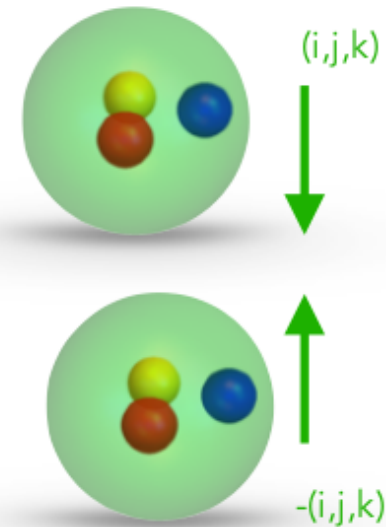


## Local hexaquark operators

- Six Gaussian smeared quarks at a point

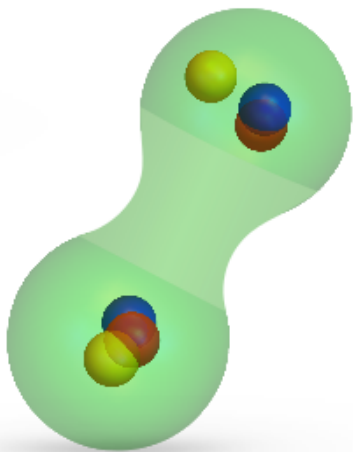
## Dibaryon Operators

- Two spatially-separated plane-wave baryons with relative momenta
- Relative momentum: up to four units  $\rightarrow$  5 operators



## Quasi-local Operators

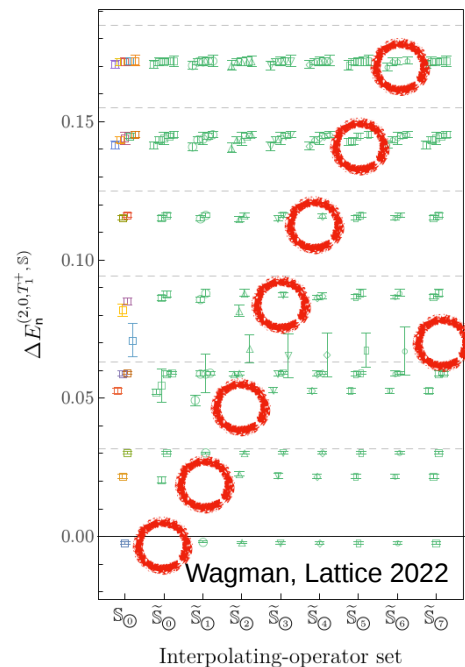
- Two exponentially localized baryons
- NN -EFT motivated deuteron-like structure



# NUCLEON-NUCLEON SPECTROSCOPY AT 800 MEV

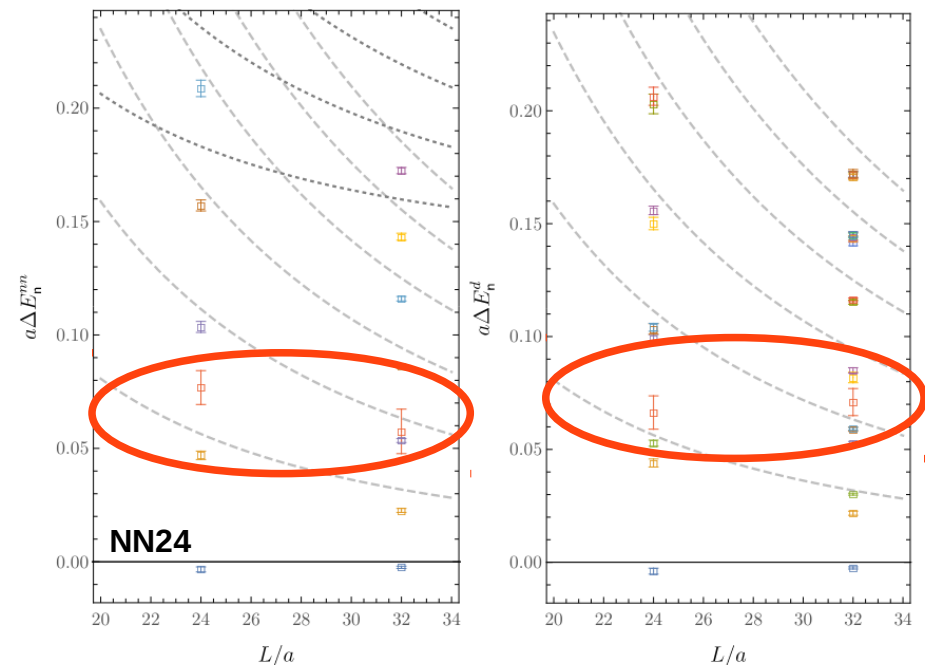
## NN32

- Variational upper bounds. No evidence for (or against) bound states
- Operator dependence on variational bounds
- Additional bound: large overlap to Hex



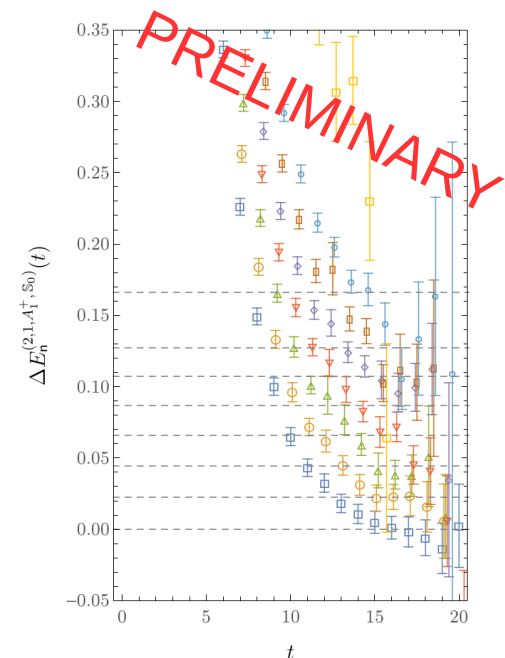
## NN24

- No evidence for (or against) bound states
- Complete basis of local NN hexaquark operators
- Additional bound observed at two lattice volumes



# APPROACHING THE PHYSICAL POINT

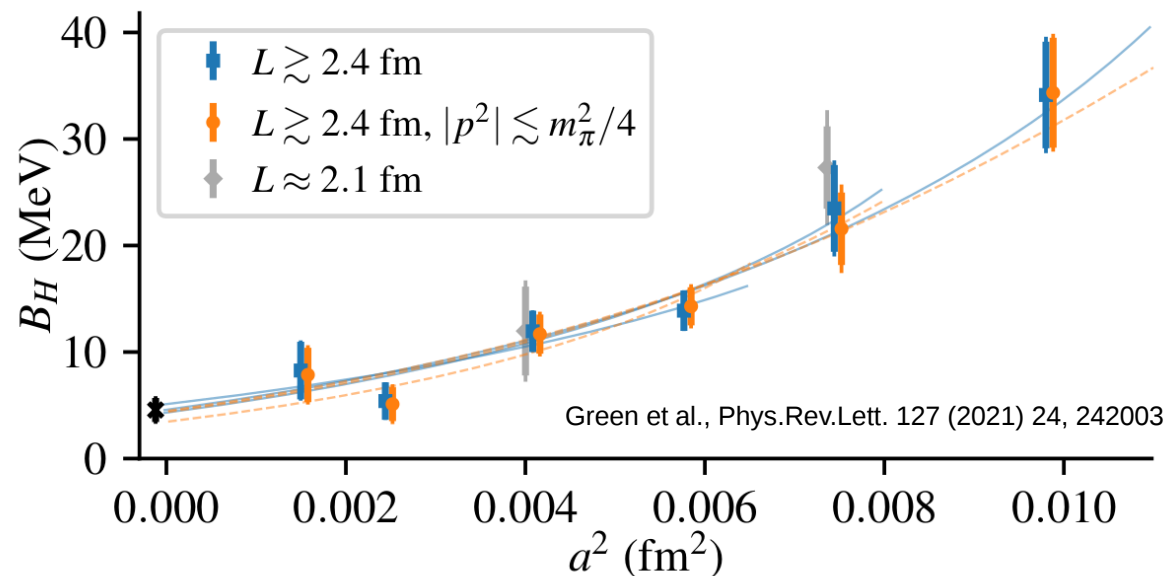
- Ongoing calculation: Octet baryon-Octet baryon @  $m_\pi = 170$  MeV
- Employed previous year's allocation
- Now running more gauge fields and propagators on **INCITE** + other (US+European) resources
- Enable nucleon-nucleon (NN), hyperon-nucleon (YN), and hyperon-hyperon (YY) interactions at the close-to-physical quark masses.
- Multiple lattice spacings.
- Multi-year project!
- Will be important to understand systematics of calculation
  - Including lattice artifacts...





# LATTICE ARTIFACTS: GREEN ET AL, 2021

- **SU(3)-symmetric point with  $m_\pi = m_K \approx 420$  MeV**
- **Only** singlet irrep of flavor SU(3): Ground state is the H-dibaryon
- Compute matrix of correlators – Employ GEVP to define optimized interpolating operators



**Potentially large lattice artifacts! ... statistically challenging analysis**

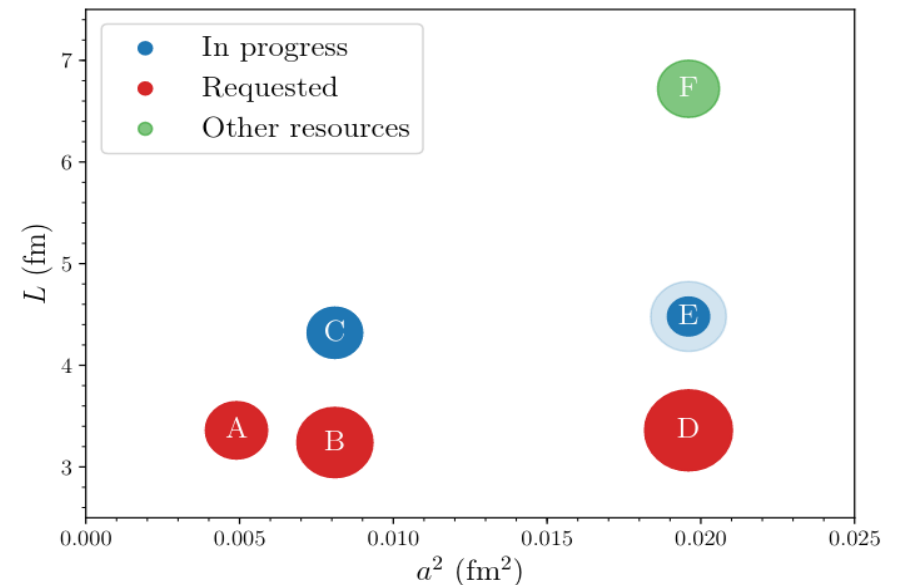
# PROPOSAL: STUDY OCTET BARYON-OCTET BARYON SPECTRA AT 800 MEV

Ensemble	$\beta$	$L/a$	$T/a$	$am_q$	$c_{sw}$	$a$ (fm)	$N_{sparse}$	$L$ (fm)	$T$ (fm)
<b>A</b>	<b>6.5</b>	<b>48</b>	<b>96</b>	<b>-0.1788</b>	<b>1.1701</b>	<b>0.072</b>	<b>8</b>	<b>3.46</b>	<b>6.91</b>
<b>B</b>	<b>6.3</b>	<b>36</b>	<b>64</b>	<b>-0.2050</b>	<b>1.2054</b>	<b>0.086</b>	<b>6</b>	<b>3.10</b>	<b>5.50</b>
C	6.3	48	64	-0.205	1.2054	0.086	6	4.13	5.50
D	6.1	24	48	-0.245	1.2493	0.145	4	3.48	6.96
E	6.1	32	48	-0.245	1.2493	0.145	4	4.64	6.96
F	6.1	48	64	-0.245	1.2493	0.145	4	6.96	9.28

## Tasks:

- Gauge field generation on **A & B**
- Propagator calculation on **A & B**
- Contractions on **A, B & D**

Perform global fit in  $a$  and  $L$  to variational bounds.



# FLAVOR IRREPS

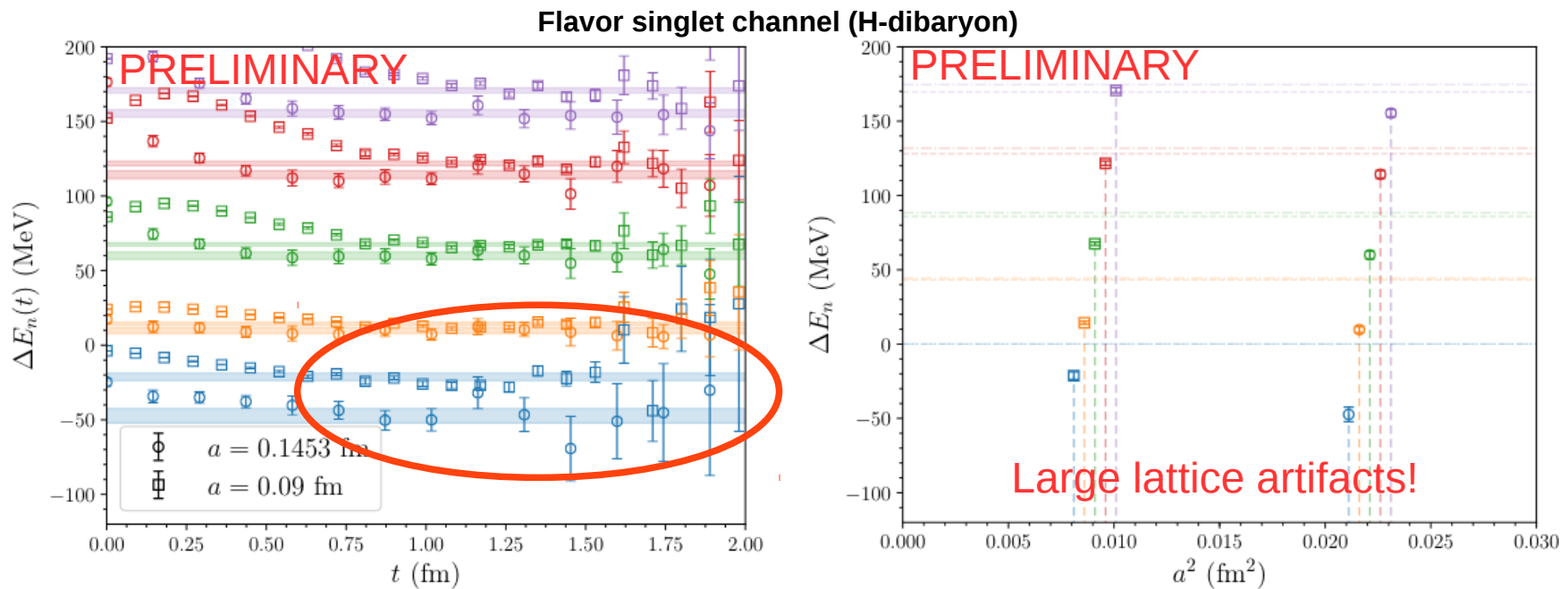
- Efficient contraction code allows us to compute **all SU(3) irreps**.
- Makes sense to compute all SU(3) flavor irreps (recycle baryon blocks).

Flavor States	$I$	$J$	$-S$	$N_u : N_d : N_s$	Cost	Storage
$\Xi^- p, \Xi^0 n, \Lambda\Lambda, \Sigma^+\Sigma^-, \Sigma^0\Sigma^0$	0	0	2	2:2:2	400	25
$\Xi^- p, \Xi^0 n$	0	1	2	2:2:2	96	12
$\Lambda\Sigma^+, \Xi^0 p$	1	0	2	3:1:2	48	4
$\Lambda\Sigma^+, \Sigma^0\Sigma^+, \Xi^0 p$	1	1	2	3:1:2	324	27
$\Sigma^+\Sigma^+$	2	0	2	4:0:2	48	1
				Total	916	69

# LATTICE ARTIFACTS IN TWO-BARYON VARIATIONAL BOUNDS

- 2023: Commenced study of all SU(3) irreps at two different lattice spacings
- Subset of results presented @ Lattice 2023

Ensemble	$\beta$	$L/a$	$T/a$	$am_q$	$c_{sw}$	$a$ (fm)	$N_{\text{sparse}}$	$L$ (fm)	$T$ (fm)
C	6.3	48	64	-0.205	1.2054	0.086	6	4.13	5.50
E	6.1	32	48	-0.245	1.2493	0.145	4	4.64	6.96



**Assuming** variational bounds have saturated, observe large lattice artifacts in ground state

# EXPECTED OUTCOMES

- 2 **new** ensembles @  $m_\pi = 800$  MeV.
- SU(3) flavor symmetry: provide variational bounds on all octet baryon-octet baryon channels.
- Provide important cross check of claimed large lattice artifacts.
- Inform computational strategy of ongoing  $m_\pi = 170$  MeV study.

# SPC QUESTIONS & ANSWERS

- 1) *Variational analysis is clearly a superior methodology (albeit at a cost), and GEVP analysis of the correlators instead of  $\Delta E_{\text{eff}}$  will yield a more robust upper bound on the energy gap. Is the lower bound from GEVP also demonstrably more robust (e.g., based on some analytic argument or lattice data)?*

As you state, GEVP has advantages over analysis of combinations of asymmetric correlation functions, but is more expensive to use. The positive definiteness of principal correlators is usefully constraining. At this point in time it possible to use GEVP even for large correlator matrices that are built from  $O(50)$  interpolating operators as we have in our recent works. Nevertheless the GEVP approach provides rigorous (stochastic) upper bounds on the energy eigenvalues. Consequently, GEVP analysis can not rigorously give lower (or indeed upper) bounds on the energy difference between a bound state and threshold ( $E_{\text{BB}} - 2E_{\text{B}}$ ). The GEVP can provide only rigorous statements on the minimum number of energy eigenvalues that appear below specific values in the one- and multi-body spectra. Specifically, one can state rigorously that at least  $n$  true energy eigenstate exists below the lowest  $n$  variational bounds in a tower, but the variational analysis can not exclude additional energy eigenstates onto which the chosen interpolating operators have small overlap. A lower bound on the binding energy of a multi-body spectrum can be achieved under the assumption that the one-body spectra have been extracted reliably.

- 2) *Since the precision of propagators is crucial in multi-baryon contractions, and your current tolerance approaches the double-precision roundoff, do you expect that at some point you will need to calculate propagators using quad-precision, especially with more roundoff accumulation on larger-size, finer lattices?*

For the ensembles we propose to use here, we do not expect that quad-precision will be necessary. We have experience with this question (including in nuclear systems in studies dating back to ~2010 where precision of propagator solves was extensively investigated. More recently in our calculations of many pion systems in Phys.Rev.D 108 (2023) 11, 114506 [2307.15014]), we explicitly investigated the effect of adding noise to propagators at the level of solver tolerance and for even systems involving products of very large numbers,  $O(1000\text{s})$ , of propagators the effects were very small (what was seen to be important in these and earlier studies was the precision used in the contractions themselves). We do not expect precision to become a problem for our current generation of studies of two-baryon systems, but plan to revisit the question as we return to studying larger systems in the future.

# CHALLENGING DATA ANALYSIS

Analysis strategy (Green et al):

1) Fits to logarithm of ratio (**non-convex**)

$$R_n(t) = \frac{\tilde{C}_{nn}(t)}{C_{\Lambda}^{\vec{p}_1}(t)C_{\Lambda}^{\vec{p}_2}(t)}$$

2) Large statistical uncertainties in region where single-state dominates nucleon correlator.

