QCD + QED studies

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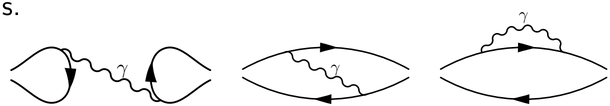
Apr 19, 2024

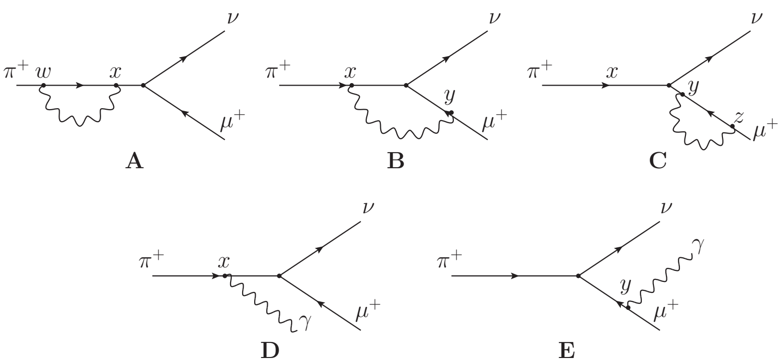
USQCD All Hands' Meeting 2024

Online

Outline

- Precision for some important observables, $m_{\pi/K}$, $f_{\pi/K}$, HVP, have been improving steadily.
- QCD + QED is needed for sub-percent accuracy. Already very relevant at present.
- Our overall strategy to include QED in lattice QCD calculations:
  Calculate the pure QCD matrix elements of local vector currents, which couple to the QED gauge fields.





Master formula for QED correction to hadron masses

- The final expression for QED correction to hadron mass is split into two parts:

$$\Delta M = \mathcal{I} = \mathcal{I}^{(s)} + \mathcal{I}^{(l)}$$

- For the short distance part: $\mathcal{I}^{(s)} \approx \mathcal{I}^{(s,L)} = \frac{1}{2}\int_{-t_s}^{t_s} dt \int_{L/2}^{L/2} d^3x\, \mathcal{H}^L_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x)$

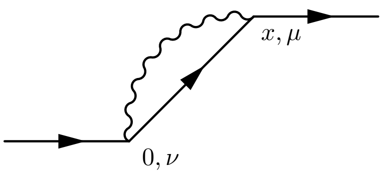
- For the long distance part: $\mathcal{I}^{(l)} \approx \mathcal{I}^{(l,L)} = \int_{-L/2}^{L/2} d^3x\, \mathcal{H}^L_{\mu,\nu}(t_s,\vec{x}) L_{\mu,\nu}(t_s,\vec{x})$

- For Feynman gauge:

$$S^{\gamma}_{\mu,\nu}(x) = \frac{\delta_{\mu,\nu}}{4\pi^2 x^2} \qquad L_{\mu,\nu}(t_s,\vec{x}) = \frac{\delta_{\mu,\nu}}{2\pi^2}\int_0^{\infty} dp \frac{\sin(p|\vec{x}|)}{2(p+E_p-M)|\vec{x}|} e^{-pt_s}$$

- Only use $\mathcal{H}^L_{\mu,\nu}(t,\vec{x})$ within $-t_s \le t \le t_s$.

- Choose $t_s = L/2$, **finite volume errors and the ignored excited states contribution to $\mathcal{I}^{(l)}$ are both exponentially suppressed by the spatial lattice size L.**

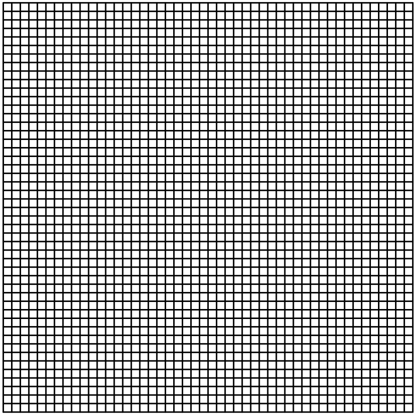


X. Feng, L. Jin. PRD [arXiv:1812.09817]

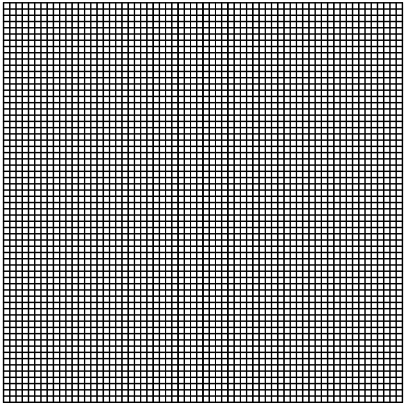
N. H. Christ, et al. PRD [arXiv:2304.08026]

Lattice QCD Ensembles from RBC/UKQCD

48I



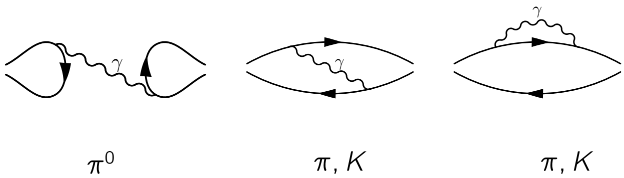
64I



- Domain wall fermion action (preserves Chiral symmetry, no $\mathcal{O}(a)$ lattice artifacts).
- Iwasaki gauge action.
- $M_\pi = 139$ MeV, $L = 5.5$ fm box, $1/a_{48I} = 1.73$ GeV, $1/a_{64I} = 2.359$ GeV.

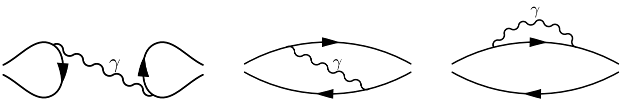
RBC/UKQCD, PRD [arXiv:1411.7017]

Hadronic matrix elements - meson mass



$$H^{(s)}_{\rho,\sigma}(x_t, \vec{x}) = H^{(s)}_{\rho,\sigma}(x) = \frac{1}{2m_\pi}\langle\pi(\vec{0})|T\{J^{\text{EM}}_\rho(x)J^{\text{EM}}_\sigma(0)\}|\pi(\vec{0})\rangle \quad (1)$$

- Calculated with the 48I ensemble.
- Coulomb gauge fixed wall sources propagator at all time slices and point source propagators at randomly selected 2048 locations. We save these propagators after sparsening ($1/16$ ratio).
- Wall sources to interpolate the meson state.
- Keep the time separation between the wall sources and its closest J^{EM} operator fixed at a large enough distance (~ 1.5 fm) to control the excited state effects.
- Use point sources at one J^{EM} location, perform contraction at the other J^{EM} location after sparsening.



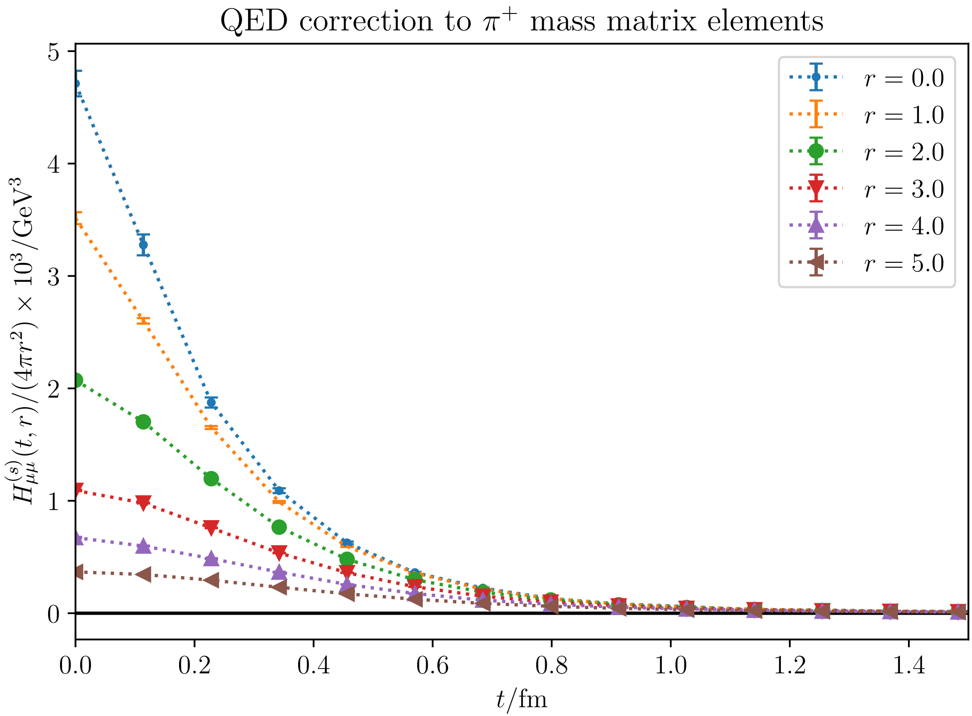
π^0 $\qquad$ π, K $\qquad$ π, K

$$H^{(s)}_{\rho,\sigma}(x_t,\vec{x}) = H^{(s)}_{\rho,\sigma}(x) = \frac{1}{2m_\pi}\langle\pi(\vec{0})|T\{J^{\mathrm{EM}}_\rho(x)J^{\mathrm{EM}}_\sigma(0)\}|\pi(\vec{0})\rangle \tag{2}$$

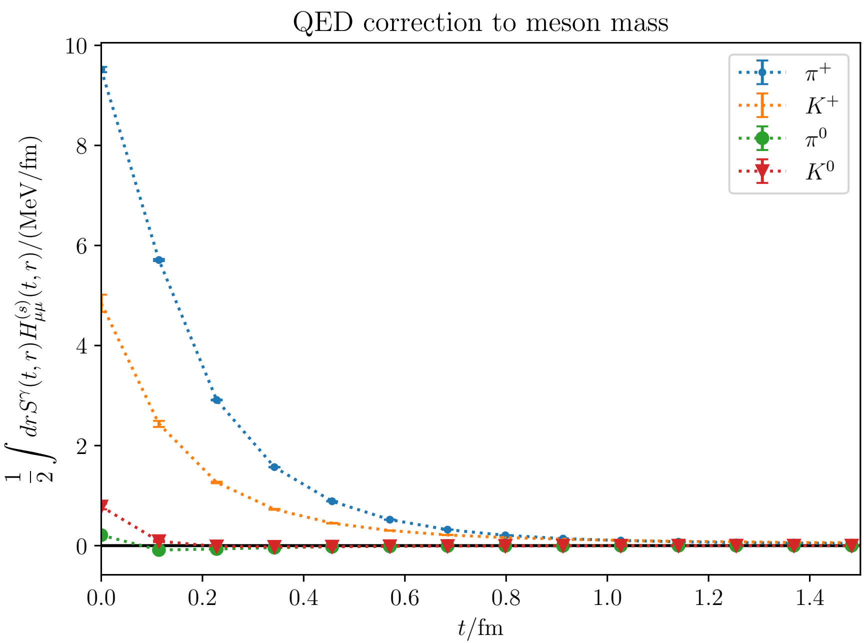
$$H^{(s)}_{\mu\mu}(t,r) = \int d^3\vec{x}\,\delta(|\vec{x}|-r)H^{(s)}_{\mu,\mu}(t,\vec{x}) \to \sum_{\vec{x},|\vec{x}|=r} H^{(s)}_{\mu,\mu}(t,\vec{x}) \tag{3}$$

$$\frac{1}{4\pi|\vec{x}|^2}H^{(s)}_{\mu\mu}(t,r) = \frac{\int d^3\vec{x}\,\delta(|\vec{x}|-r)H^{(s)}_{\mu\mu}(t,\vec{x})}{\int d^3\vec{x}\,\delta(|\vec{x}|-r)} \tag{4}$$

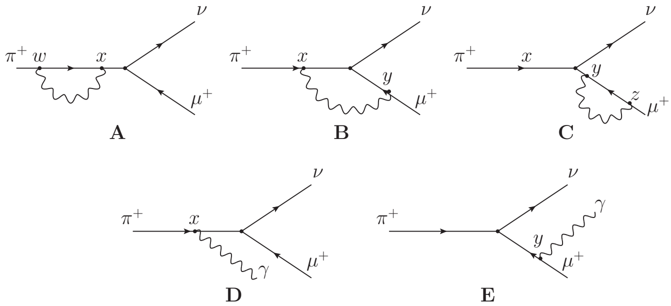
$$\to \frac{\sum_{\vec{x},|\vec{x}|=r} H^{(s)}_{\mu,\mu}(t,\vec{x})}{\sum_{\vec{x},|\vec{x}|=r} 1} = \frac{1}{\sum_{\vec{x},|\vec{x}|=r} 1}H^{(s)}_{\mu\mu}(t,r) \tag{5}$$


QED correction to π^+ mass matrix elements
$H^{(s)}_{\mu\mu}(t,r)/(4\pi r^2) \times 10^3/\mathrm{GeV}^3$
t/fm
$r = 0.0$
$r = 1.0$
$r = 2.0$
$r = 3.0$
$r = 4.0$
$r = 5.0$
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Hadronic matrix elements - meson mass



- Integrate over t (from $-\infty$ to $+\infty$) give the meson mass shift.



- χPT: V. Cirigliano and H. Neufeld. PLB [arXiv:1102.0563]

$$\delta R_K = 0.0064(24) \tag{6}$$

$$\delta R_\pi = 0.0176(21) \tag{7}$$

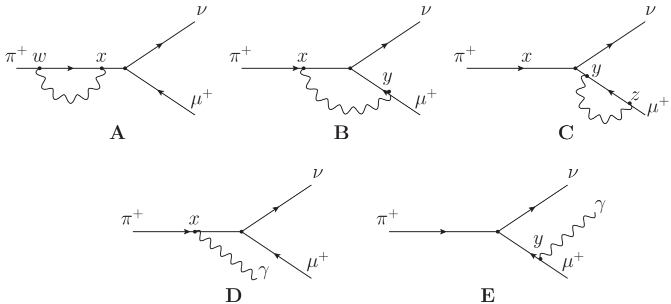
$$\delta R_{K\pi} = \delta R_K - \delta R_\pi = -0.0112(21) \tag{8}$$

- Lattice ETMC: M. Di Carlo, et al. PRD [arXiv:1904.08731]

$$\delta R_K = 0.0024(10) \tag{9}$$

$$\delta R_\pi = 0.0153(19) \tag{10}$$

$$\delta R_{K\pi} = \delta R_K - \delta R_\pi = -0.0126(14) \tag{11}$$



- Lattice ETMC: M. Di Carlo, et al. PRD [arXiv:1904.08731]

$$\delta R_K = 0.0024(10) \tag{12}$$

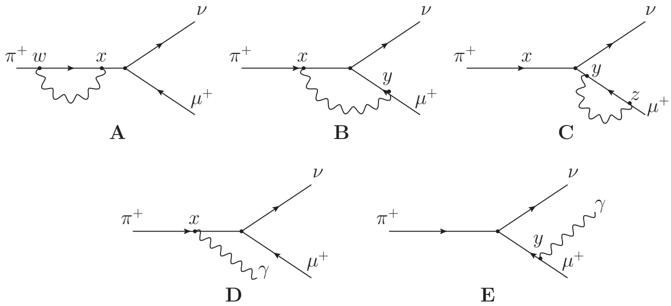
$$\delta R_\pi = 0.0153(19) \tag{13}$$

$$\delta R_{K\pi} = \delta R_K - \delta R_\pi = -0.0126(14) \tag{14}$$

- Lattice RBC-UKQCD: P. Boyle, et al. JHEP [arXiv:2211.12865]

$$\delta R_{K\pi} = -0.0086(3)_{\text{stat}}\left(\,^{+11}_{-4}\,\right)_{\text{fit}}(5)_{\text{disc.}}(5)_{\text{quench.}}(39)_{\text{vol.}} \tag{15}$$

Also use 48I ensemble (physical pion mass). Major systematic error from finite volume effects.



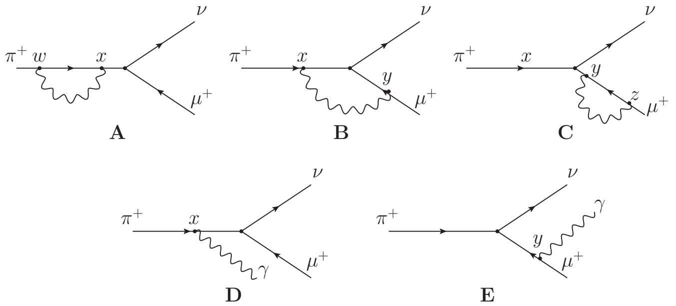
$$H^{(0)}_{\mu} = \langle 0|J^{W}_{\mu}(0)|\pi(\vec{0})\rangle \quad (16)$$

$$H^{(1)}_{\mu,\rho}(x) = H^{(1)}_{\mu,\rho}(x_t,\vec{x}) = \langle 0|T\{J^{W}_{\mu}(0)J^{\mathrm{EM}}_{\rho}(x)\}|\pi(\vec{0})\rangle \quad (17)$$

$$H^{(2)}_{\mu,\rho,\sigma}(t_1,t_2,\vec{x}) = \int d^3\vec{w}\langle 0|T\{J^{W}_{\mu}(0)J^{\mathrm{EM}}_{\rho}(t_1,\vec{w}+\vec{x})J^{\mathrm{EM}}_{\sigma}(t_2,\vec{w})\}|\pi(\vec{0})\rangle \quad (18)$$

- Calculated with the 48I ensemble.
- Coulomb gauge fixed wall sources propagator at all time slices and point source propagators at randomly selected 2048 locations. We save these propagators after sparsening (1/16 ratio).
- This is the same set of propagators as the meson mass calculation.

Hadronic matrix elements - leptonic decay



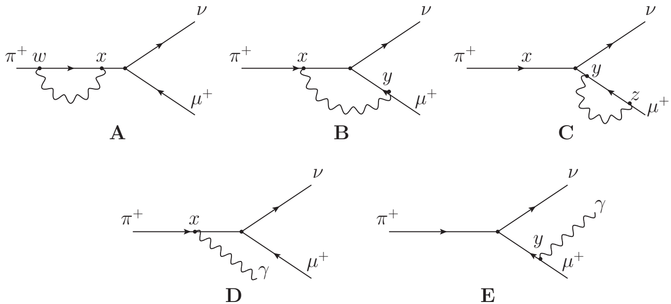
$$H^{(0)}_{\mu} = \langle 0 | J^{W}_{\mu}(0) | \pi(\vec{0}) \rangle \tag{19}$$

$$H^{(1)}_{\mu,\rho}(x) = H^{(1)}_{\mu,\rho}(x_t, \vec{x}) = \langle 0 | T\{J^{W}_{\mu}(0) J^{\text{EM}}_{\rho}(x)\} | \pi(\vec{0}) \rangle \tag{20}$$

$$H^{(2)}_{\mu,\rho,\sigma}(t_1, t_2, \vec{x}) = \int d^3\vec{w} \langle 0 | T\{J^{W}_{\mu}(0) J^{\text{EM}}_{\rho}(t_1, \vec{w}+\vec{x}) J^{\text{EM}}_{\sigma}(t_2, \vec{w})\} | \pi(\vec{0}) \rangle \tag{21}$$

- Wall sources to interpolate the meson state.
- Keep the time separation between the wall sources and its closest J^{EM} or J^{W} operator fixed at a large enough distance (~ 1.5 fm) to control the excited state effects.
- $J^{W}_{\mu} = J^{W,V}_{\mu} - J^{W,A}_{\mu} = \bar{d}\gamma_{\mu}u + \bar{s}\gamma_{\mu}u - \bar{d}\gamma_{\mu}\gamma_5 u - \bar{s}\gamma_{\mu}\gamma_5 u.$

Hadronic matrix elements - leptonic decay

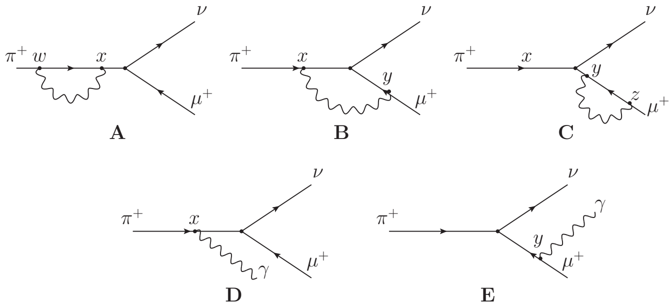


$$H^{(0)}_{\mu} = \langle 0|J^{W}_{\mu}(0)|\pi(\vec{0})\rangle \tag{22}$$

$$H^{(1)}_{\mu,\rho}(x) = H^{(1)}_{\mu,\rho}(x_t,\vec{x}) = \langle 0|T\{J^{W}_{\mu}(0)J^{\mathrm{EM}}_{\rho}(x)\}|\pi(\vec{0})\rangle \tag{23}$$

$$H^{(2)}_{\mu,\rho,\sigma}(t_1,t_2,\vec{x}) = \int d^3\vec{w}\langle 0|T\{J^{W}_{\mu}(0)J^{\mathrm{EM}}_{\rho}(t_1,\vec{w}+\vec{x})J^{\mathrm{EM}}_{\sigma}(t_2,\vec{w})\}|\pi(\vec{0})\rangle \tag{24}$$

- For diagram B and D, use point sources at one J^{EM} location, perform contraction at the other J^{EM} location after sparsening.
- For diagram A, use point sources at one J^{EM} location and the J^{W} location, perform contract at the other J^{EM} location. Use aggresive sparsening to reduce the contraction cost.



$$H_{\mu}^{(0)} = \langle 0 | J_{\mu}^{W}(0) | \pi(\vec{0}) \rangle \tag{25}$$

$$H_{\mu,\rho}^{(1)}(x) = H_{\mu,\rho}^{(1)}(x_t, \vec{x}) = \langle 0 | T\{J_{\mu}^{W}(0) J_{\rho}^{\mathrm{EM}}(x)\} | \pi(\vec{0}) \rangle \tag{26}$$

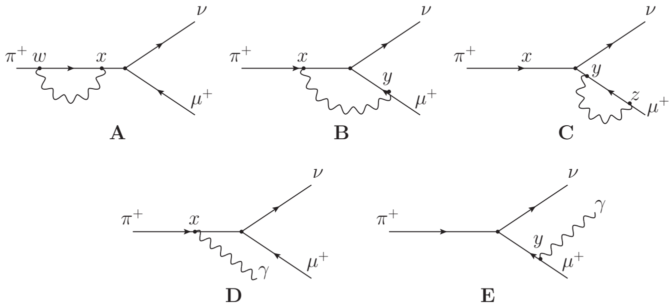
$$H_{\mu,\rho,\sigma}^{(2)}(t_1, t_2, \vec{x}) = \int d^3\vec{w} \langle 0 | T\{J_{\mu}^{W}(0) J_{\rho}^{\mathrm{EM}}(t_1, \vec{w}+\vec{x}) J_{\sigma}^{\mathrm{EM}}(t_2, \vec{w})\} | \pi(\vec{0}) \rangle \tag{27}$$

- For $H^{(0)}$:

$$H_{\mu}^{(0)} = -\delta_{\mu,t} \langle 0 | J_{t}^{W,A}(0) | \pi(\vec{0}) \rangle = -\delta_{\mu,t} i m_{\pi} f_{\pi} \tag{28}$$

where $f_{\pi} \approx 130$ MeV

Hadronic matrix elements - leptonic decay



$$H_\mu^{(0)} = \langle 0 | J_\mu^W(0) | \pi(\vec{0}) \rangle \quad (29)$$

$$H_{\mu,\rho}^{(1)}(x) = H_{\mu,\rho}^{(1)}(x_t, \vec{x}) = \langle 0 | T\{J_\mu^W(0) J_\rho^{\mathrm{EM}}(x)\} | \pi(\vec{0}) \rangle \quad (30)$$

$$H_{\mu,\rho,\sigma}^{(2)}(t_1, t_2, \vec{x}) = \int d^3\vec{w} \langle 0 | T\{J_\mu^W(0) J_\rho^{\mathrm{EM}}(t_1, \vec{w}+\vec{x}) J_\sigma^{\mathrm{EM}}(t_2, \vec{w})\} | \pi(\vec{0}) \rangle \quad (31)$$

- For $H^{(2)}$:

$$H_{\mu,\rho,\sigma}^{(2)}(t_1, t_2, \vec{x}) = -\delta_{\mu,t} \int d^3\vec{w} \langle 0 | T\{J_t^{W,A}(0) J_\rho^{\mathrm{EM}}(t_1, \vec{w}+\vec{x}) J_\sigma^{\mathrm{EM}}(t_2, \vec{w})\} | \pi(\vec{0}) \rangle \quad (32)$$

$$H_{t\mu\mu}^{(2)}(t_1, t_2, r) = \int d^3\vec{x}\, \delta(|\vec{x}| - r) H_{t,\mu,\mu}^{(2)}(t_1, t_2, \vec{x}) \quad (33)$$

$$H_{\mu}^{(0)} = \langle 0| J_{\mu}^{W}(0) |\pi(\vec{0})\rangle \tag{34}$$

$$H_{\mu,\rho}^{(1)}(x) = H_{\mu,\rho}^{(1)}(x_t, \vec{x}) = \langle 0| T\{J_{\mu}^{W}(0) J_{\rho}^{\text{EM}}(x)\} |\pi(\vec{0})\rangle \tag{35}$$

$$H_{\mu,\rho,\sigma}^{(2)}(t_1, t_2, \vec{x}) = \int d^3\vec{w} \langle 0| T\{J_{\mu}^{W}(0) J_{\rho}^{\text{EM}}(t_1, \vec{w}+\vec{x}) J_{\sigma}^{\text{EM}}(t_2, \vec{w})\} |\pi(\vec{0})\rangle \tag{36}$$

- For $H^{(1)}$:

$$H_{\mu,\rho}^{(1,V)}(x) = \langle 0| T\{J_{\mu}^{W,V}(0) J_{\rho}^{\text{EM}}(x)\} |\pi(\vec{0})\rangle \tag{37}$$

$$H_{\mu,\rho}^{(1,A)}(x) = \langle 0| T\{J_{\mu}^{W,A}(0) J_{\rho}^{\text{EM}}(x)\} |\pi(\vec{0})\rangle \tag{38}$$

$$H_{V}^{(1)}(t, r) = \frac{1}{2} \int d^3\vec{x}\, \delta(|\vec{x}| - r) \epsilon_{i,j,k} \frac{x_i}{|\vec{x}|} H_{j,k}^{(1,V)}(t, \vec{x}) \tag{39}$$

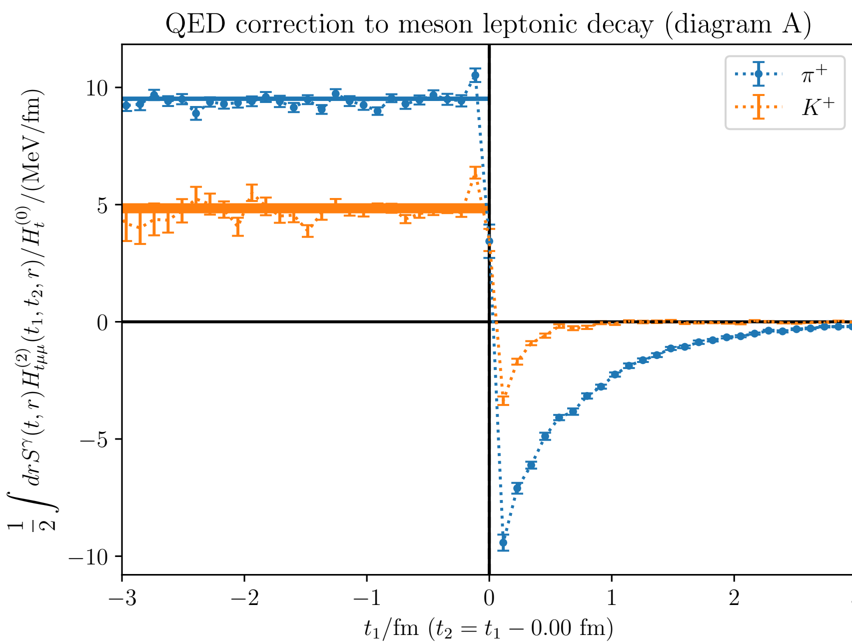
$$H_{Att}^{(1)}(t, r) = \int d^3\vec{x}\, \delta(|\vec{x}| - r) H_{t,t}^{(1,A)}(t, \vec{x}) \tag{40}$$

$$H_{Atx}^{(1)}(t, r) = \int d^3\vec{x}\, \delta(|\vec{x}| - r) \frac{x_i}{|\vec{x}|} H_{t,i}^{(1,A)}(t, \vec{x}) \tag{41}$$

$$H_{Axt}^{(1)}(t, r) = \int d^3\vec{x}\, \delta(|\vec{x}| - r) \frac{x_i}{|\vec{x}|} H_{i,t}^{(1,A)}(t, \vec{x}) \tag{42}$$

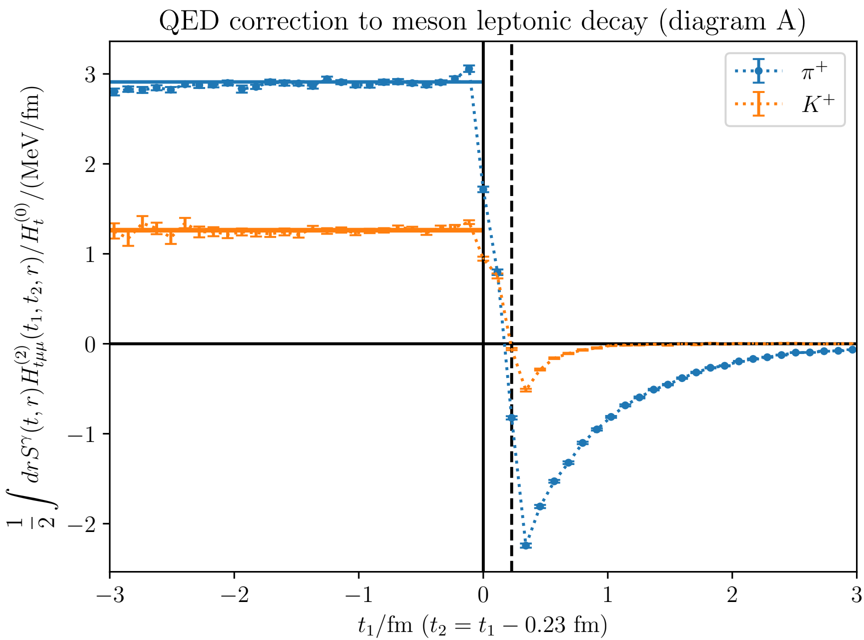
$$H_{Aii}^{(1)}(t, r) = \int d^3\vec{x}\, \delta(|\vec{x}| - r) \frac{1}{3} H_{i,i}^{(1,A)}(t, \vec{x}) \tag{43}$$

$$H_{Axx}^{(1)}(t, r) = \int d^3\vec{x}\, \delta(|\vec{x}| - r) \frac{3}{2} \left(\frac{x_i x_j}{|\vec{x}|^2} - \frac{1}{3}\delta_{i,j} \right) H_{i,j}^{(1,A)}(t, \vec{x}) \tag{44}$$



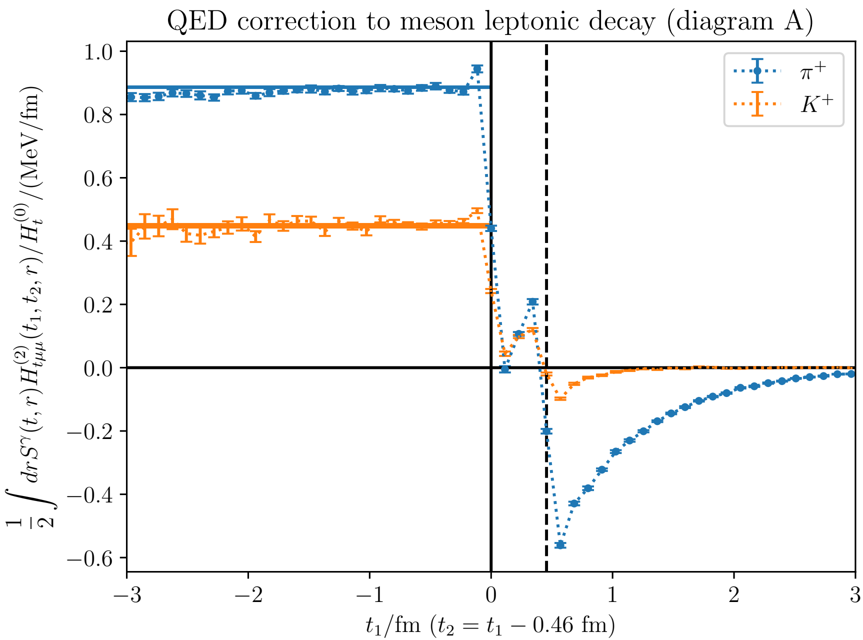
- Horizontal band represent the corresponding value from the mass shift calculation.

$$H^{(2)}_{t\mu\mu}(t_1, t_2, r)/H^{(0)}_t \approx H^{(s)}_{\mu\mu}(t_1 - t_2, r) \qquad (t_1, t_2 \ll 0) \tag{45}$$



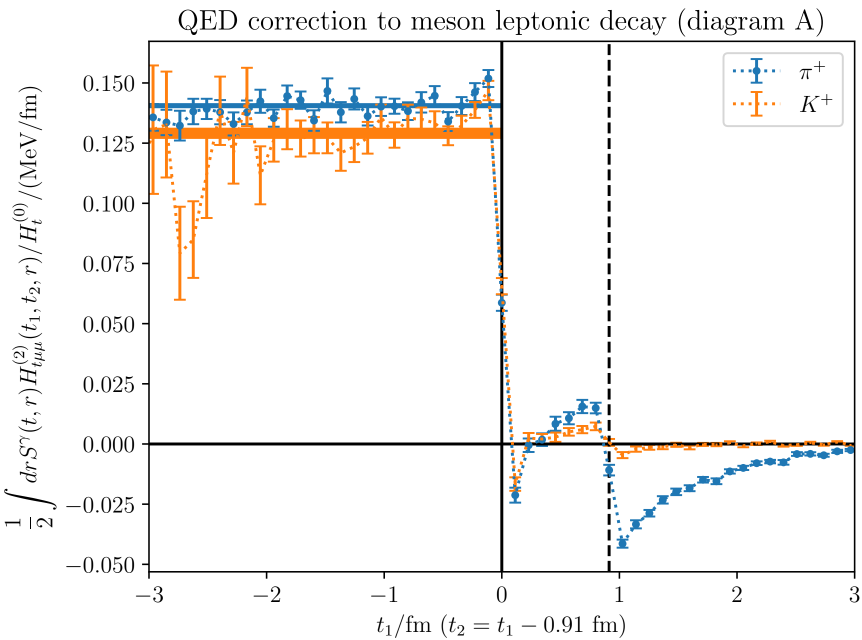
- Horizontal band represent the corresponding value from the mass shift calculation.

$$H^{(2)}_{t\mu\mu}(t_1,t_2,r)/H^{(0)}_t \approx H^{(s)}_{\mu\mu}(t_1-t_2,r) \qquad (t_1,t_2 \ll 0) \qquad (46)$$



- Horizontal band represent the corresponding value from the mass shift calculation.

$$H^{(2)}_{t\mu\mu}(t_1, t_2, r)/H^{(0)}_t \approx H^{(s)}_{\mu\mu}(t_1 - t_2, r) \qquad (t_1, t_2 \ll 0) \qquad (47)$$



- Horizontal band represent the corresponding value from the mass shift calculation.

$$H^{(2)}_{t\mu\mu}(t_1, t_2, r)/H^{(0)}_t \approx H^{(s)}_{\mu\mu}(t_1 - t_2, r) \qquad (t_1, t_2 \ll 0) \qquad (48)$$

Summary

- Calculations of QED corrections to kaon mass and leptonic decay are in progress.
- Other recent works supported by this proposal:
  - "Nucleon electric polarizabilities and nucleon-pion scattering at physical pion mass," [arXiv:2310.01168 [hep-lat]].
  - "Lattice QCD Calculation of Electroweak Box Contributions to Superallowed Nuclear and Neutron Beta Decays," [arXiv:2308.16755 [hep-lat]].
  - "Lattice QCD Calculation of $\pi 0 \rightarrow e+e-$ Decay," Phys. Rev. Lett. **130**, no.19, 191901 (2023) [arXiv:2208.03834 [hep-lat]].
  - "Lattice QCD calculation of the light sterile neutrino contribution in $0\nu 2\beta$ decay," Phys. Rev. D **106**, no.7, 074510 (2022) [arXiv:2206.00879 [hep-lat]].
  - "Lattice QCD Calculation of the Two-Photon Exchange Contribution to the Muonic-Hydrogen Lamb Shift," Phys. Rev. Lett. **128**, no.17, 172002 (2022) [arXiv:2202.01472 [hep-lat]].
  - "Lattice QCD Calculation of the Pion Mass Splitting," Phys. Rev. Lett. **128**, no.5, 052003 (2022) [arXiv:2108.05311 [hep-lat]].

Thank You!