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Radiative Correction to β-decays

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Introduction

- CKM matrix describes the weak interactions in the mass-eigenstate basis:

$$H_W \supset G_F (\bar{u} \quad \bar{c} \quad \bar{t}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \Gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix} (\bar{e} \quad \bar{\mu} \quad \bar{\tau}) \Gamma^\mu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \text{ (Semileptonic)}$$

$$+ G_F (\overline{\nu_e} \quad \overline{\nu_\mu} \quad \overline{\nu_\tau}) \Gamma_\mu \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} (\bar{e} \quad \bar{\mu} \quad \bar{\tau}) \Gamma^\mu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \text{ (Leptonic)}$$

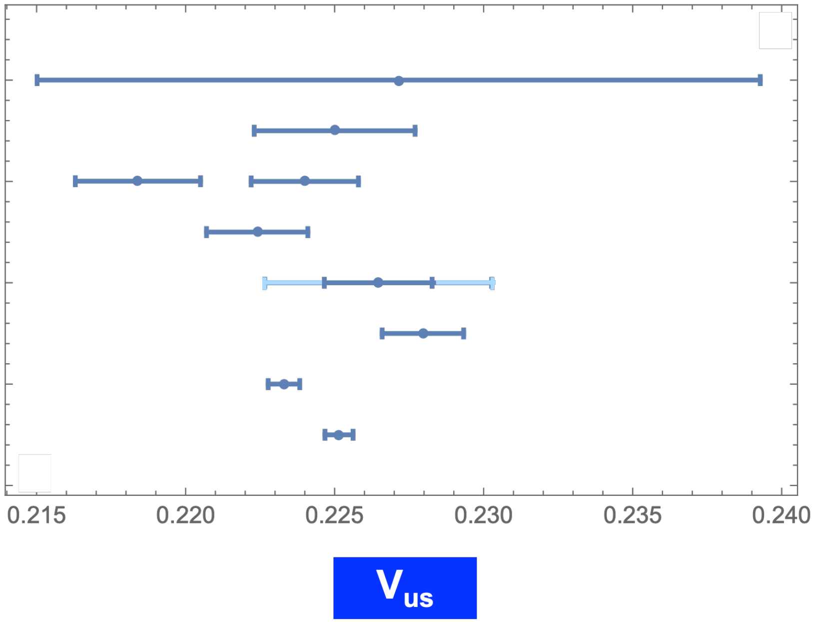
- CKM matrix is unitary in the standard model, in particular,

$$|V_{ud}|^2 \quad + \quad |V_{us}|^2 \quad + \quad |V_{ub}|^2 \quad = 1$$

$$0.95\ (6\times10^{-4}) + 0.05\ (2\times10^{-4}) + 1.5\ (0.1)\times10^{-5} = 1$$

- G_F determined from Muon decays ($\mu \to e\nu_\mu\overline{\nu_e}$)
- Pion, Kaon, Tau decays give V_{ud} and V_{us} (e.g., $\pi \to e\,\overline{\nu_e}$, $K \to \pi\, e\,\overline{\nu_e}$, $\tau \to \pi\,\nu_\tau$)

V_{us} assuming unitarity



	Fractional uncertainty	Largest uncertainty
$\pi^{\pm} \to \pi^{0} e \nu$	5.3%	EXP
Hyperons	1.2% +?	EXP + TH
τ inclusive	0.8% + ?	EXP + TH
τ exclusive	0.8%	EXP + TH
$n \to p e \nu$	0.8% (1.7%) PDG	EXP
$0^{+} \to 0^{+}$	0.6%	TH
K → π l ν	0.24%	EXP + TH
K → μν / π → μν	0.21%	TH

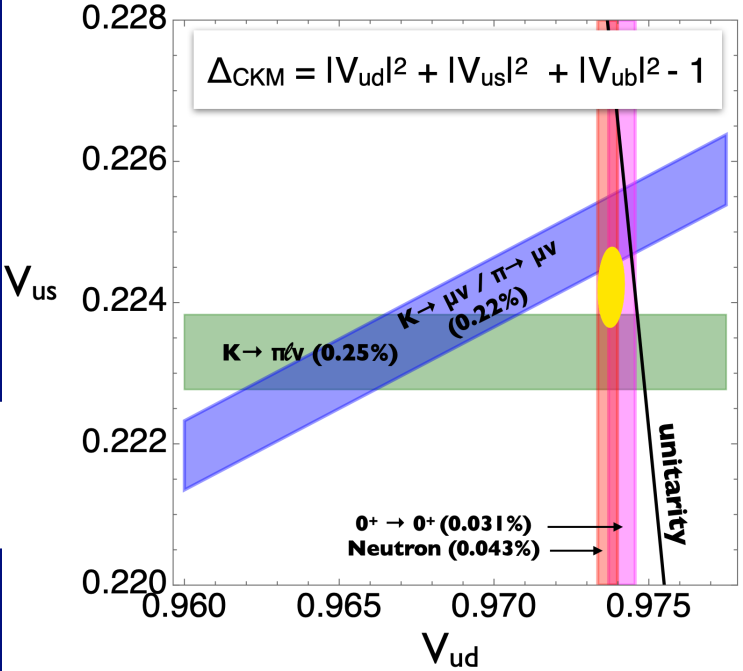
From Vincenzo Cirigliano

Phenomenology: Status on the V_{ud}- V_{us} plane

- Leptonic and semileptonic meson decays inconsistent with unitarity
- Baryonic β-decays much closer to unitarity.
- Global fit has has about 3σ tension

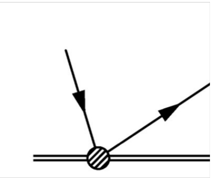
$$\Delta_{CKM} = -1.48(53)\times 10^{-3}$$

- Problem with hard photon corrections?



Cirigliano et al., 2208.11707

Effective field theory calculation of radiative corrections



$$g_V(\mu_e) = U(\mu_e, \Lambda_\chi) \left[1 + \overline{\Box}^V_{\rm Had}(\mu_0) + \frac{\alpha(\Lambda_\chi)}{\pi}\,\kappa(\mu_0)\right] U(\Lambda_\chi, \mu_W)\; C_\beta(\mu_W)$$

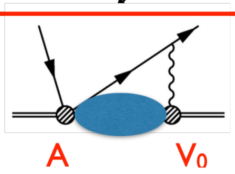
NLL RGE in ChPT and pion-less EFT

Non-perturbative 'triangle' diagram: the usual 'box' minus the DIS contributions for $Q^2 > (\mu_0)^2$, which in the EFT approach belongs to the Wilson coefficient

NLL RGE in Fermi Theory

Wilson Coefficient at $\mu_W \sim m_W$

$\overline{\Box}^V_{\rm Had} \sim$



+0.026% shift in total radiative correction to neutron decay compared to previous literature

- Cirigliano et al., 2306.03138

The running from weak scale to the chiral-matching scale done perturbatively with resummation of leading logarithms.

Nonperturbative input needed for the IR-finite matching coefficient at the chiral scale.

$V - A$ Interactions and lattice input

Γ_μ is $V - A$, but different processes pick up different components

- Semileptonic pseudoscalar mesons (and $0^+ \to 0^+$ nuclear) decays purely V.
  - Need lattice input like f_+ (and nuclear theory).
- Leptonic decays of pseudoscalar mesons are purely A.
  - Need lattice input like f_K.
- Nucleon (and tau decays) need both V and A.
  - Need g_V and g_A.
  - These appear in the coefficients of the 2-quark+2-lepton four-fermi terms.
  - Differs from lattice determined coefficients of the quark bilinear vector and axial currents by hard electromagnetic ‘vertex corrections’.
  - $\lambda \equiv g_A/g_V$ well determined by A-asymmetry of $n \to p\, e\, \bar{\nu}_e$ β-decays.

Neutron decay phenomenology

$$|V_{ud}|^2 = \frac{5283.321(5)s}{\tau_n(1+3\lambda^2)(1+\Delta_f)(1+\Delta_R)}$$

$\lambda = g_A/g_V$

Long distance

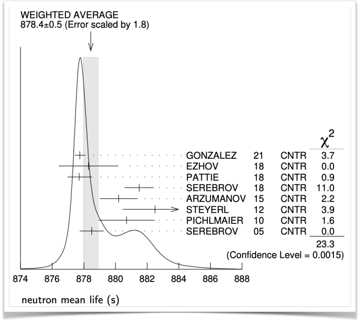
$$\Gamma_n = \frac{G_F^2 |V_{ud}|^2 m_e^5}{2\pi^3}(1+3\lambda^2)\cdot f_0 \cdot (1+\Delta_f)\cdot(1+\Delta_R),$$

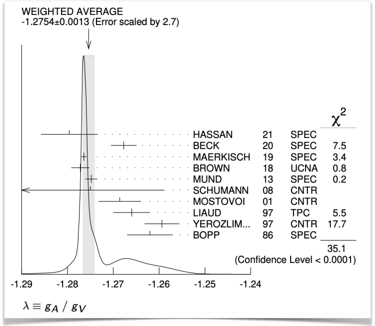
Short distance

$\Delta_R = 4.044(27)\%$
$\Delta_f = 3.573(5)\%$

VC, W. Dekens, E. Mereghetti, O. Tomalak, 2306. 03138

- Radiative corrections: NLL setup + LECs in terms of 'γ-W box' (dispersive & Lattice QCD)
- Experimental input: PDG averages include large scale factor, particularly for g_A/g_V





x

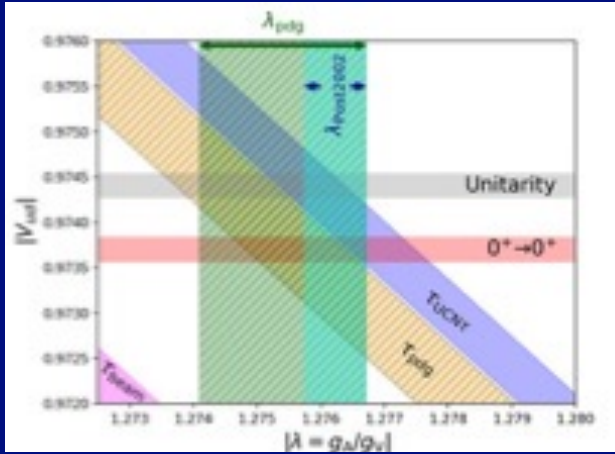
Single most precise measurements of lifetime and λ imply very competitive V_{ud}!

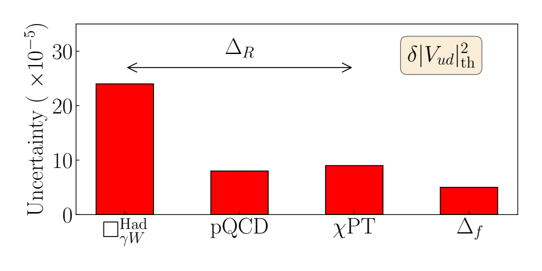
Maerkish et al, 1812.04666 Gonzalez et al, 2106.10375

$$V_{ud}^{n,\mathrm{PDG}} = 0.97430(2)_{\Delta_f}(13)_{\Delta_R}(82)_{\lambda}(28)_{\tau_n}[88]_{\mathrm{total}}$$

$$V_{ud}^{n,\mathrm{best}} = 0.97402(2)_{\Delta_f}(13)_{\Delta_R}(35)_{\lambda}(20)_{\tau_n}[42]_{\mathrm{total}}$$

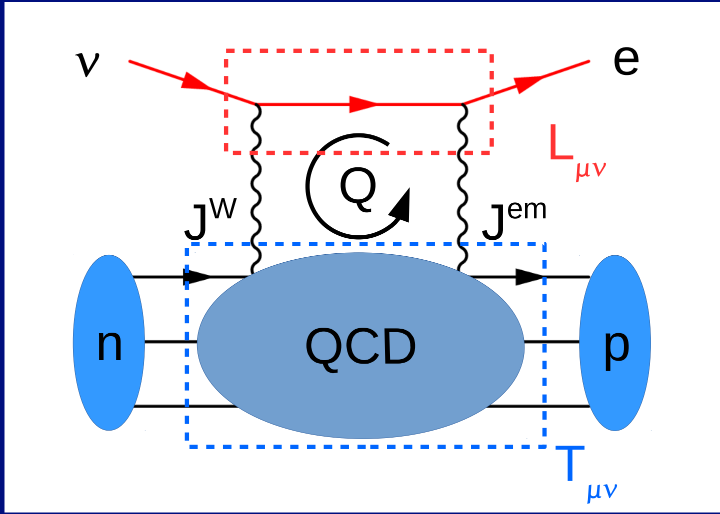
Need improvements in lifetime and g_A/g_V.
Within reach in next 5 years

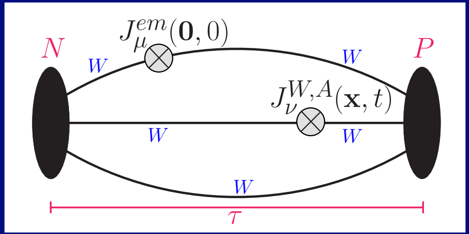


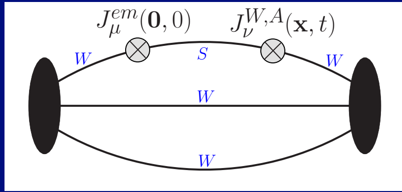


From Vincenzo Cirigliano

Lattice Methodology





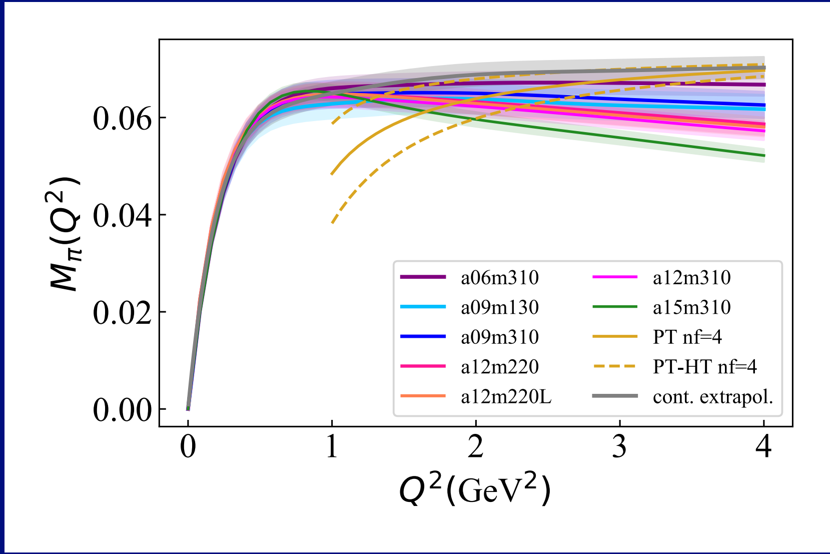


+ Disconnected diagrams

From Vincenzo Cirigliano

Lattice Systematics

- Four-point functions are difficult on the lattice.
- Nucleon correlators dominated by noise beyond about 1.5 fm
- Excited states significant for about 0.5 fm.
- Physical pion masses may have low-lying multihadron excited states.
- Mesonic analogue calculation shows consistency with perturbative calculations.



Lattice Status

	LANL PRD 108, 034508 (2023)	P. Ma, Xu Feng, et al
$\square_{\gamma W}^{VA}\big\vert_{\pi}$	$2.810(26)\times10^{-3}$	$2.830(11)(26)\times10^{-3}$ PRL 124, 192002 (2020) PRD 103 114503 (2021)
$\square_{\gamma W}^{VA}\big\vert_{K}$	$2.389(17)\times10^{-3}$	$2.437(44)\times10^{-3}$
$\square_{\gamma W}^{VA}\big\vert_{n}$		$3.65(8)_{\text{lat}}(44)_{\text{PT}}\times10^{-3}$ arXiv:2308.16755

- Results for mesons agree within stated errors.
- LANL and Ma et al. calculations differ in systematics.
  - LANL uses physical strange quark and varying light quark mass.
  - Full accounting of systematics will need further study.
- Excited state uncertainties are unclear.
- Important for multiple calculations to agree.

Proposed Calculation

- Use two HISQ ensembles
  - a=0.1207(11) fm, M_π=305.3(4) MeV (sea), M_π=310(3) MeV (sea), 24^3 x 64
  - a=0.1202(12) fm, M_π=218.1(4) MeV (sea), M_π=225(2) MeV (sea), 24^3 x 64
- Same volume and lattice spacing, differs in pion mass.
- Do calculation at multiple source-sink separations.
- Look for
  - Statistical signal
  - Systematics associated with source-sink separation
  - Systematics associated with pion mass