

# Semileptonic $B$ -decays with a vector final state

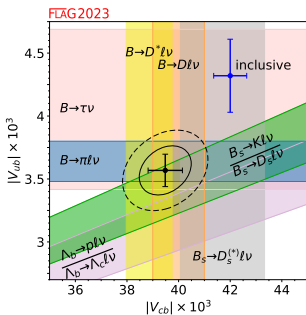
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04.19.24  
USQCD All Hands Meeting  
Zoom@MIT

# Intro & Motivation

- Semileptonic decays are a rich source of information for determining CKM matrix elements.
- Lattice data a critical source of input for testing the CKM paradigm.



- Overall aim: Precision ( $\sim 1\%$ ) determination of a range of  $B_{(s)}$  (and  $D_{(s)}$ ) semileptonic form factors, of direct relevance for current and upcoming experimental programs.
- Here we discuss our work extending this program to decays with vector final states (specifically the processes  $B_{(s)} \rightarrow D_{(s)}^*$ ).
- Support/enhance physics of  $B \rightarrow D^*$  with FNAL heavy quarks on asqtad (and hisq) sea. 2105.14019

# Outline

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- Intro & Motivation
- FNAL/MILC all-HISQ semileptonic decays
  - ▶ Calculation framework
  - ▶ Update on results w/ pseudoscalar final state
- Progress with vector final states
- Summary

# FNAL-MILC allhisq working group

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## Heavy quarks

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Treatment of  $c$  and especially  $b$  quarks challenging in lattice simulation due to lattice artifacts which grow as  $(am_h)^n$

- May use an effective theory framework to handle the  $b$  quark.
  - ▶ Fermilab method, RHQ, OK, NRQCD
  - ▶ Pros: Solves problem w/  $am_h$  artifacts.
  - ▶ Cons: Requires matching, can still have  $ap$  artifacts.
- Also possible to use relativistic fermion provided  $a$  is sufficiently small  $am_c \ll 1$ ,  $am_b < 1$ .
  - ▶ Use improved actions e.g.  $\mathcal{O}(a^2) \rightarrow \mathcal{O}(\alpha_s a^2)$
  - ▶ Pros: Absolutely normalised current, straightforward continuum extrap.
  - ▶ Cons: Numerically expensive, extrapolate  $m_h \rightarrow m_b$ .

## allhisq simulations

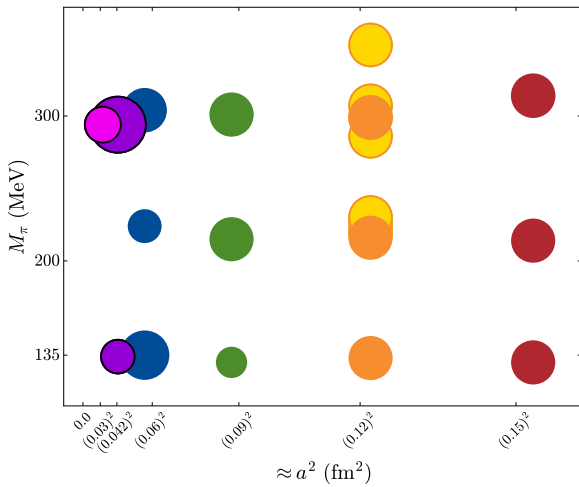
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- Here we simulate *all* quarks with the HISQ action.
- Unified treatment for wide range of  $B_{(s)}$  (and  $D_{(s)}$ ) to pseudoscalar transitions
  - ▶  $B_{(s)} \rightarrow D_{(s)}^{(*)}$
  - ▶  $B_{(s)} \rightarrow K$
  - ▶  $B \rightarrow \pi$
- Ensembles with (HISQ) sea quarks down to physical at each lattice spacing.
- Enables correlated studies of ff *ratios*.

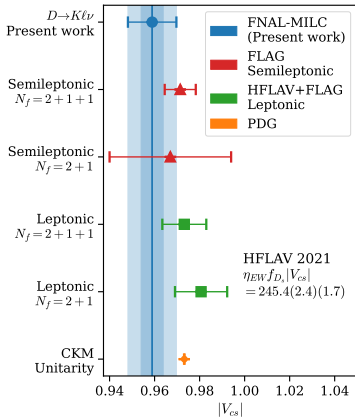
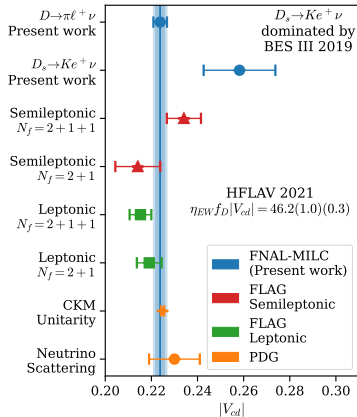
See Lattice 2023 proceeding for more details. 2403.03959

- HISQ fermion action.
  - ▶ Discretization errors begin at  $\mathcal{O}(\alpha_s a^2)$ .
  - ▶ Designed for simulating heavy quarks ( $m_c$  and higher at current lattice spacings).
- Symanzik-improved gauge action, takes into account  $\mathcal{O}(N_f \alpha_s a^2)$  effects of HISQ quarks in sea. [0812.0503]
- Multiple lattice spacings down to  $\sim 0.042$  (now 0.03) fm.
- Effects of  $u/d$ ,  $s$ , and  $c$  quarks in the sea.
- Multiple light-quark input parameters down to physical pion mass.
  - ▶ Chiral fits.
  - ▶ Reduce statistical errors.





allHISQ  $b$  updates

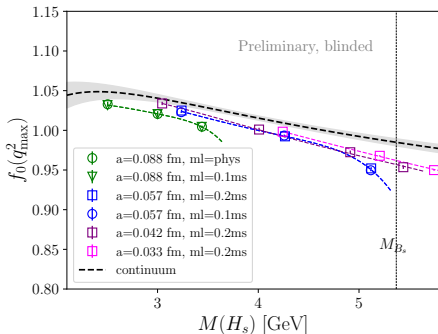


- Use a heavy valence mass  $h$  as a proxy for the  $b$  quark.
- Work at a range of  $m_h$ , with  $am_c < am_h \lesssim 1$  on each ensemble. On sufficiently fine ensembles,  $m_h$  is near to  $m_b$  (e.g.  $m_b$  at  $am_h \approx 0.65$  on  $a = 0.03$  fm).
- Map out physical dependence on  $m_h$ , remove discretisation effects  $\sim (am_h)^{2n}$  using information from several ensembles. Extrapolate results  $a^2 \rightarrow 0, m_h \rightarrow m_b$ .

## $B_s \rightarrow D_s$ - a simple $f_0(q_{\max}^2)$ fit

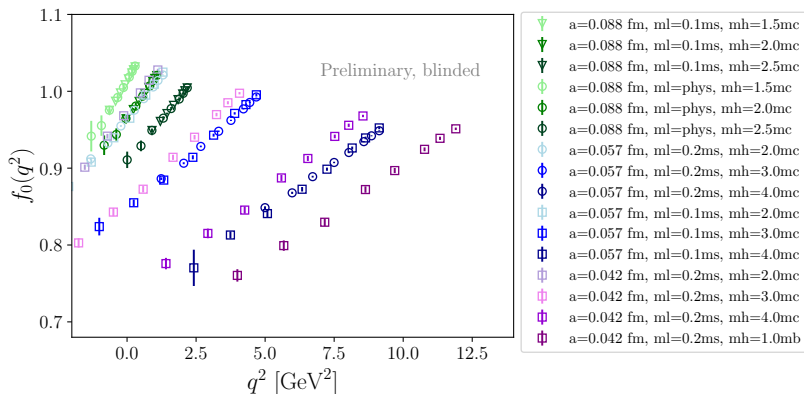
Basic fit parameterizing  $M_H$  dependence and heavy quark discretization.

$$f_0(q_{\max}^2)[M_H, am_h] = \sum_{ij} c_{ij} \left(\frac{1}{M_H}\right)^i (am_h)^{2j}$$



Good precision obtained ( $\sim 0.5\%$ ) at  $M_{B_s}$ .

# $B_s \rightarrow D_s: f_0(q^2)$



- Good precision out to  $p = 400$
- Rightmost points on figure have  $m_h = m_b$

## Chiral/cont. extrapolations

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Build from chiral forms used in  $D$  analysis.

$$f_{0,\parallel,\perp}(E) = \frac{c_0}{E + \Delta} (1 + \cdots + c_H \chi_{H_s} + \cdots)$$

$$\Delta = \frac{M_{D^*}^2 - M_{D_s}^2 - M_K^2}{2M_{D_s}}, \quad \chi_{H_s} = \frac{\Lambda_{\text{HQET}}}{M_{H_s}} - \frac{\Lambda_{\text{HQET}}}{M_{D_s}^{\text{PDG}}}$$

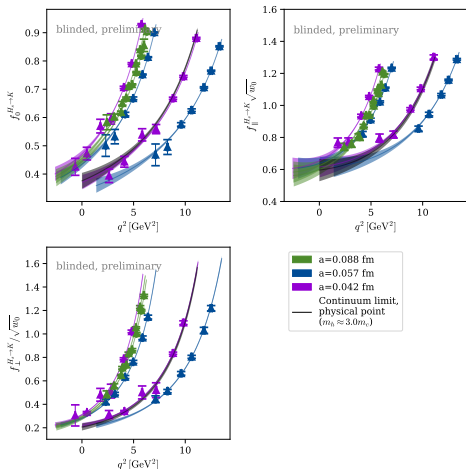
Generalize to incorporate HQET expansion:

$$c_0 \rightarrow c_0 + c_1 \frac{\Lambda_{\text{HQET}}}{M_{H_s}} + \cdots, \quad \Delta \rightarrow \frac{M_{D^*}^2 - M_{D_s}^2 - M_K^2}{2M_{H_s}} \quad (\text{1st order})$$

$$\chi_{H_s} = \frac{\Lambda_{\text{HQET}}}{M_{H_s}} - \frac{\Lambda_{\text{HQET}}}{M_{H_s}^{\text{phys}}}$$

# $H_s \rightarrow K$

Here building off  $D_s$  chiral analysis, working out towards  $B_s$ .



- Data at  $2-3m_c$ , 3 lattice spacings, 3  $m_{l,sea}$  values
- Note 0.057 fm has  $m_h \approx 2.2, 3.3m_c$
- Reasonable  $\chi^2/\text{dof} = 0.92, 1.79, 0.75$  for  $f_0, f_{\parallel}, f_{\perp}$



# Tensor current renormalization

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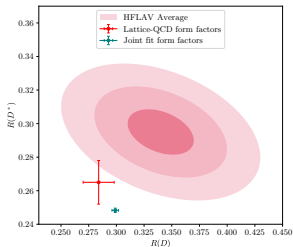
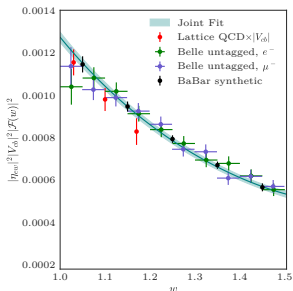
Started computing tensor  $Z$ -factors for (one-link) tensor currents. (Also relevant for  $P \rightarrow V$  tensor currents.)

- Use tensor decay constant  $f_{J/\psi}^{T, \text{SMOM}}(\mu)$  as fiducial.
- $Z_{\text{SMOM}}^{\gamma_\mu \gamma_\nu \otimes \gamma_\mu \gamma_\nu}(\mu)$  (local tensor) determined in 2008.02024.
- $Z'_T = Z_T \sqrt{\frac{a_0 E'_0}{a'_0 E_0}}$ , where  $a_0, E_0$  are ground state amplitude/energy of the  $J/\psi$  from local and one-link tensor operators.

Analysis by Abhishek Samlodia (Syracuse)

$P \rightarrow V$  updates

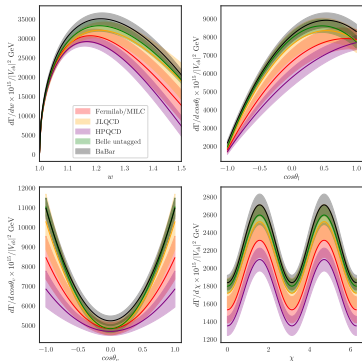
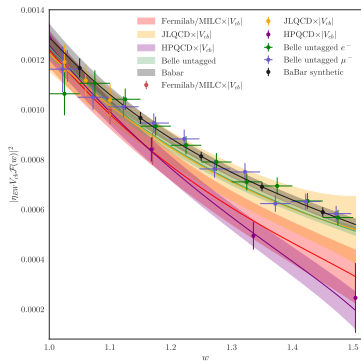
- Pioneering FNAL-MILC calculation beyond zero-recoil using FNAL  $b$  and  $c$  quarks.



Figs. courtesy A. Vaquero

- FNAL-HISQ analysis in progress (Vaquero).

# $B \rightarrow D^*$ comparisons



Lattice refs:

- JLQCD 2306.05657
- HPQCD 2304.03137
- FNAL/MILC 2105.14019

Figs. courtesy A. Vaquero

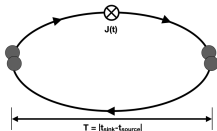
$$\frac{|V_{cb}| \times 10^3}{39.19(90)}$$

$$39.31(54)(51)$$

$$38.40(78)$$

## Extending allhisq to vector final states

Structurally, calculation is similar to  $P \rightarrow P$  – need to modify spin-taste at source/sink/current.



| Decay             | $\mathcal{O}_{H(s)}$                          | $\mathcal{O}_{H'}$          | $\mathcal{O}_J$                               | Matrix element                    |
|-------------------|---|-----------------------------|---|-----------------------------------|
| $P \rightarrow P$ | $\gamma_5 \otimes \gamma_5$                   | $\gamma_5 \otimes \gamma_5$ | $\gamma_i \otimes 1$                          | $\langle H'   V_i   H(s) \rangle$ |
| $P \rightarrow P$ | $\gamma_0 \gamma_5 \otimes \gamma_0 \gamma_5$ | $\gamma_5 \otimes \gamma_5$ | $\gamma_0 \otimes \gamma_0$                   | $\langle H'   V_4   H(s) \rangle$ |
| $P \rightarrow V$ | $\gamma_0 \gamma_5 \otimes \gamma_1 \gamma_3$ | $\gamma_1 \otimes \gamma_1$ | $\gamma_3 \otimes \gamma_3$                   | $\langle H'   V_3   H(s) \rangle$ |
| $P \rightarrow V$ | $\gamma_5 \otimes \gamma_5$                   | $\gamma_1 \otimes 1$        | $\gamma_5 \otimes \gamma_5$                   | $\langle H'   A_0   H(s) \rangle$ |
| $P \rightarrow V$ | $\gamma_5 \otimes \gamma_5$                   | $\gamma_3 \otimes \gamma_3$ | $\gamma_3 \gamma_5 \otimes \gamma_3 \gamma_5$ | $\langle H'   A_1   H(s) \rangle$ |
| $P \rightarrow V$ | $\gamma_5 \otimes \gamma_5$                   | $\gamma_1 \otimes \gamma_1$ | $\gamma_1 \gamma_5 \otimes \gamma_1 \gamma_5$ | $\langle H'   A_2   H(s) \rangle$ |

Normalize vector (axial vector) current using PCVC (PCAC).

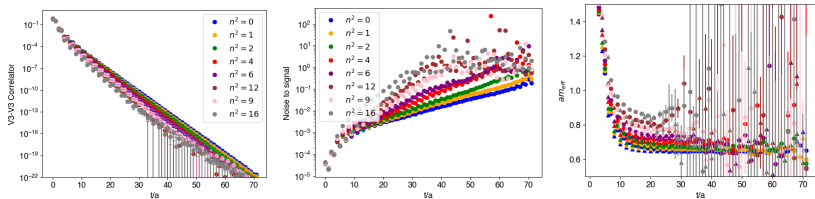
## Current year running update

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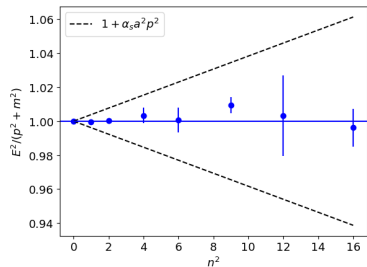
- 2023 allocation cycle: Awarded 3.8M Sky-core-hours on 1q.  $\sim 300$  confs on  $0.06\text{fm}-0.2m_s$  ensemble.
- April 18: 103% used (+1M Sky-core-hour jeopardy boost). Over 400 confs/data generated.
- Preliminary analysis on 326 confs. Expect to achieve 500 confs/data by end of allocation cycle.
- Tested run scripts on 1q2\_gpu. Ready for production here.

For next year's proposed running, we would like to extend data to  $0.09\text{fm}-0.1m_s$  (1q1\_cpu), and  $0.09\text{fm}-0.2m_s$  (1q2\_gpu)  $\rightarrow$  First look at discretization and sea-quark mass effects.

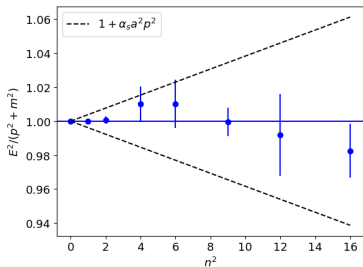
# Look at the $D^*(s)$



$D^*_s$



$D^*$



Figures courtesy of Akhil Chauhan (UIUC)

## Summary & Conclusion

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- Semileptonic decays are crucial sources of information for fundamental physics, e.g.  $|V_{ub}|$  and  $|V_{cb}|$ . Lattice results needed to support experimental physics programs at LHCb and Belle II.
  - ▶ Understand inclusive/exclusive discrepancies.
  - ▶ Pure SM predictions for  $R$ -ratios.
- The FNAL-MILC allHISQ- $b$  program aims to produce high quality form factor data for a range of phenomenologically important channels.
- Extending these calculations to vector final states, to obtain  $B_{(s)} \rightarrow D_{(s)}^*$  form factors over the full kinematic range.



Thank you!



## Normalization of axial vector currents

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$$\partial_\mu A_\mu^{\text{cons}} = (m_h + m_l)P$$

Applied to zero-momentum two-point correlators,

$$Z_{A^0} M'_H \langle 0 | A^0 | H' \rangle = (m_h + m_l) \langle 0 | P | H \rangle,$$

Here  $A^0$  has spin-taste  $\gamma^5 \gamma^0 \otimes \gamma^5 \gamma^0$ ,  $P$  has spin-taste  $\gamma_5 \otimes \gamma_5$ .  
We use that  $Z_{A^0} = Z_{A^i}$  up to polynomial discretization effects.

## $P \rightarrow V$ tensor insertions

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- Three correlators needed to extract three tensor form factors  $T_1, T_2, T_3$ .
- One-link current insertions can be used with same initial/final state interpolating operators to obtain vector and axial-vector form factors.

| Decay             | $\mathcal{O}_{H(s)}$                          | $\mathcal{O}_{H'}$          | $\mathcal{O}_J$                               | Matrix element                              |
|-------------------|---|-----------------------------|---|---|
| $P \rightarrow V$ | $\gamma_0 \gamma_5 \otimes \gamma_1 \gamma_3$ | $\gamma_3 \otimes \gamma_3$ | $\gamma_1 \gamma_2 \otimes \gamma_1$          | $\langle H'   T_1, T_2   H(s) \rangle$      |
| $P \rightarrow V$ | $\gamma_0 \gamma_5 \otimes \gamma_1 \gamma_3$ | $\gamma_1 \otimes \gamma_1$ | $\gamma_2 \gamma_3 \otimes \gamma_3$          | $\langle H'   T_1, T_2, T_3   H(s) \rangle$ |
| $P \rightarrow V$ | $\gamma_5 \otimes \gamma_5$                   | $\gamma_3 \otimes \gamma_3$ | $\gamma_0 \gamma_1 \otimes \gamma_3 \gamma_5$ | $\langle H'   T_1, T_2   H(s) \rangle$      |