

New approach to determining radiative capture reaction rates at astrophysical energies

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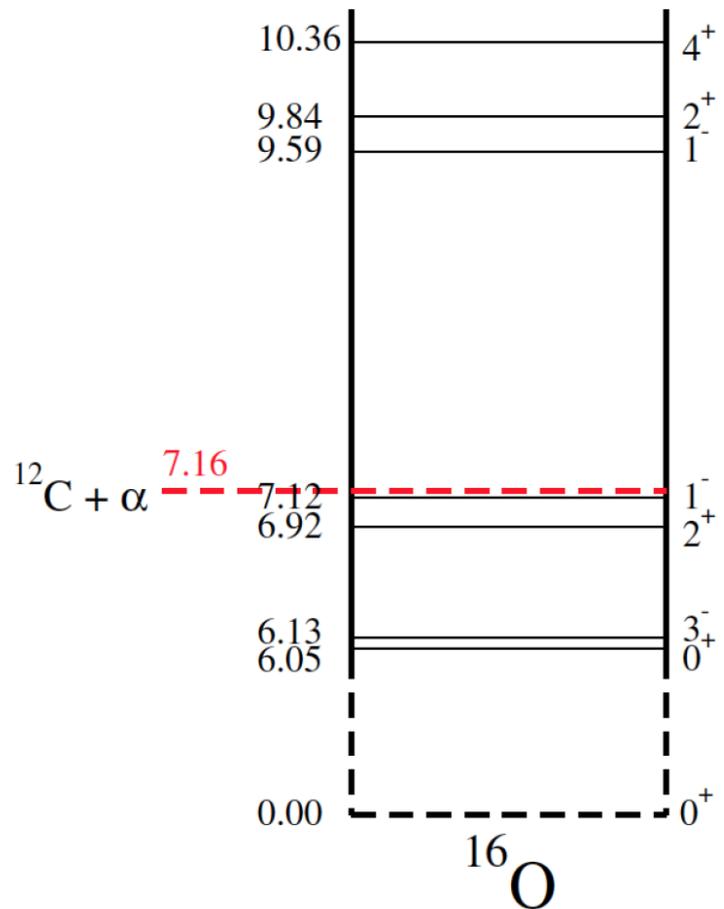


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Radiative capture reactions play a crucial role in stellar nucleosynthesis but have proved challenging to determine experimentally. In particular, the large uncertainty ($\approx 100\%$) in the measured rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction is the largest source of uncertainty in any stellar evolution model. With development of high-current energy-recovery linear accelerators (ERLs) and high-density gas targets, measurement of the $^{16}\text{O}(e, e'\alpha)^{12}\text{C}$ reaction close to threshold using detailed balance allows a new approach to determine the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate with significantly increased precision ($< 20\%$). We present the formalism to relate photo- and electrodisintegration reactions and consider the design of an optimal experiment to deliver increased precision. Once the new ERLs come online, an experiment to validate the approach we propose should be carried out. This approach has broad applicability to radiative capture reactions in astrophysics.

DOI: [10.1103/PhysRevC.100.025804](https://doi.org/10.1103/PhysRevC.100.025804)

$^{12}\text{C}/^{16}\text{O}$ abundance: $\alpha + ^{12}\text{C} \rightarrow \gamma + ^{16}\text{O}$



- Nucleosynthesis and evolution of massive stars
- White dwarfs: supernova type Ia
- End of stars: O-rich \rightarrow black hole, C-rich \rightarrow neutron star
T.A. Weaver and S.E. Woosley,
Phys. Rep. **227**, 335 (1993).
- At $T \approx 2 \times 10^8$ K, $E_g \approx 300$ keV, $\sigma \approx 10^{-5}$ pb
- The cross section at 300 keV is complicated
 - E1, 1^- : subthreshold state at 7.12 MeV and broad resonance at 9.59 MeV
 - E2, 2^+ : subthreshold state at 6.92 MeV and narrow resonance at 9.85 MeV

R.J. deBoer *et al.*, Rev. Mod. Phys. **89**, 035007 (2017).

S-factor: $S(E) = \sigma(E) E e^{2\pi\eta}$

- **Direct measurements:**

- a) α beam: γ -detection; S_{E1} and S_{E2}

- b) ^{12}C beam (inverse kin.) ^{16}O detection; S_{tot}

- **Indirect measurements:**

- a) β -decay of ^{16}N : $^{16}\text{O}^* \rightarrow \alpha + ^{12}\text{C}$; S_{E1}

- b) inverse reaction:

- (i) photodisintegration of ^{16}O

- bubble chamber

- R.J. Holt *et al.*, arXiv 1809.10176.

- time projection chamber

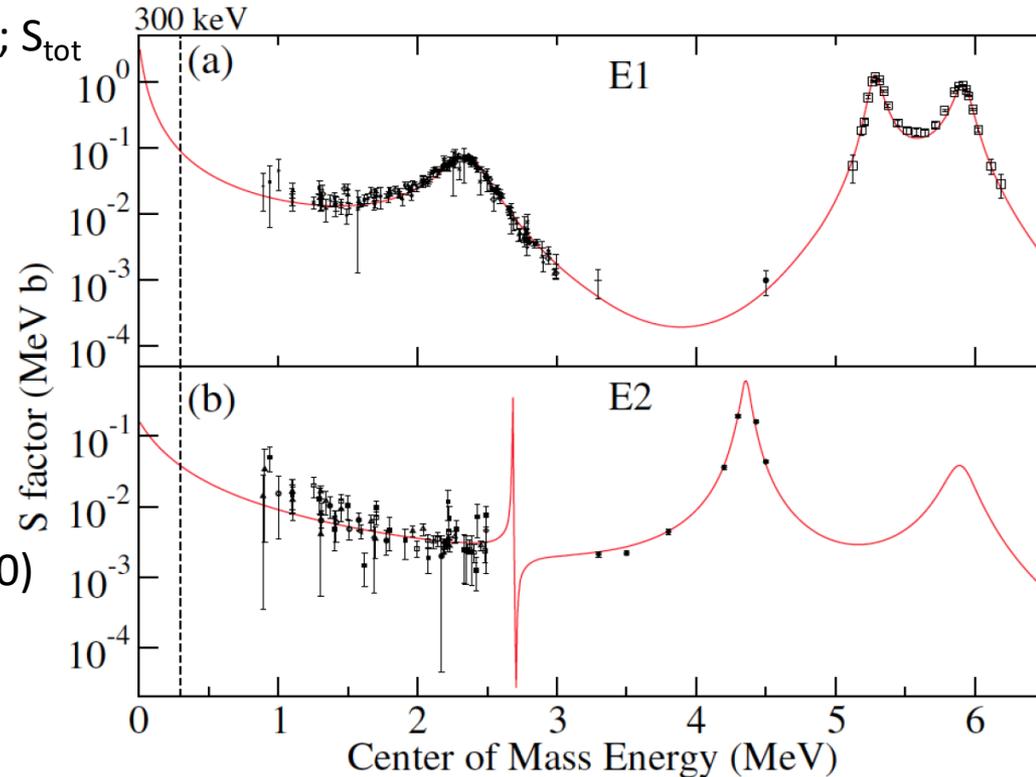
- M. Gai *et al.*, JINST 5, P12004 (2010)

- (ii) electrodisintegration of ^{16}O

- THIS TALK

- I. Frišćić, T.W. Donnelly and R.G.M.*

- arXiv:1904.05819



Previously proposed in electron storage ring
E. Tsentalovich, MIT-Bates PAC proposal 00-01.

Advantages of $^{16}\text{O}(e,e'\alpha)^{12}\text{C}$

- Inverse reaction => larger cross section
- Can suppress isotopic and chemical contamination using kinematics
- New generation of energy-recovery linear accelerators (ERLs) delivering ~ 100 MeV beams with intensity $I > 10\text{mA}$.

MESA, Mainz, Germany

F. Hug et al., Proc. Of LINAC'16 **28**, 313 (2017)

CBETA, Cornell, USA

D. Trbojevic et al., Proc. Of IPAC'17 **8**, 1285 (2017)

- Oxygen cluster gas-jet target with areal thickness $> 10^{18}$ atoms cm^{-2}

S. Grieser et al., NIM A **906**, 120 (2018)

Luminosity $> 10^{35}$ electron-atom $\text{cm}^{-2} \text{s}^{-1}$

Differential Cross Section: $^{16}\text{O}(e,e'\alpha)^{12}\text{C}$

A. S. Raskin and T. W. Donnelly, Ann. of Phys. 191 (1989)

$$\frac{d\sigma}{dE'_e d\Omega_e d\Omega_\alpha^{cm}} = \frac{M_\alpha M_{12C}}{8\pi^3 W} \frac{p_\alpha^{cm}}{(\hbar c)^3} \sigma_{Mott} (\tilde{v}_L R_L + \tilde{v}_T R_T + \tilde{v}_{LT} R_{LT} + \tilde{v}_{TT} R_{TT})$$

$$\rho \equiv |Q^2/q^2| = 1 - (\omega/q)^2$$

$$v_L = \rho^2$$

$$\tilde{v}_L = (W/M_{16O})^2 v_L$$

$$v_T = \frac{1}{2}\rho + \tan^2 \theta_e/2$$

$$\tilde{v}_T = v_T$$

Total invariant mass:

$$W = \sqrt{(M_{16O} + \omega)^2 - q^2}$$

$$v_{TL} = -\frac{1}{\sqrt{2}}\rho\sqrt{\rho + \tan^2 \theta_e/2}$$

$$\tilde{v}_{TL} = (W/M_{16O})v_T$$

$$E_\alpha^{cm} = W - W_{th}$$

$$v_{TT} = -\frac{1}{2}\rho$$

$$\tilde{v}_{TT} = v_{TT}$$

Response Functions for $J^\pi=0^+$ Nuclei

$$\begin{aligned}
 R_L = & P_0(\cos \theta_\alpha) \left(|t_{C0}|^2 + |t_{C1}|^2 + |t_{C2}|^2 \right) \\
 & + P_1(\cos \theta_\alpha) \left(2\sqrt{3}|t_{C0}||t_{C1}| \cos(\delta_{C1} - \delta_{C0}) + 4\sqrt{\frac{3}{5}}|t_{C1}||t_{C2}| \cos(\delta_{C2} - \delta_{C1}) \right) \\
 & + P_2(\cos \theta_\alpha) \left(2|t_{C1}|^2 + \frac{10}{7}|t_{C2}|^2 + 2\sqrt{5}|t_{C0}||t_{C2}| \cos(\delta_{C2} - \delta_{C0}) \right) \\
 & + P_3(\cos \theta_\alpha) \left(6\sqrt{\frac{3}{5}}|t_{C1}||t_{C2}| \cos(\delta_{C2} - \delta_{C1}) \right) \\
 & + P_4(\cos \theta_\alpha) \left(\frac{18}{7}|t_{C2}|^2 \right)
 \end{aligned}$$

$$\begin{aligned}
 R_T = & P_0(\cos \theta_\alpha) \left(|t_{E1}|^2 + |t_{E2}|^2 \right) \\
 & + P_1(\cos \theta_\alpha) \left(\frac{6}{\sqrt{5}}|t_{E1}||t_{E2}| \cos(\delta_{E2} - \delta_{E1}) \right) \\
 & + P_2(\cos \theta_\alpha) \left(-|t_{E1}|^2 + \frac{5}{7}|t_{E2}|^2 \right) \\
 & + P_3(\cos \theta_\alpha) \left(-\frac{6}{\sqrt{5}}|t_{E1}||t_{E2}| \cos(\delta_{E2} - \delta_{E1}) \right) \\
 & + P_4(\cos \theta_\alpha) \left(-\frac{12}{7}|t_{E2}|^2 \right)
 \end{aligned}$$

$$R_{TT} = -R_T \cos(2\phi_\alpha)$$

Matrix Elements and Coefficients

- Multipole matrix elements ($q_0 = 1.2 \text{ fm}^{-1}$):

$$t_{EJ} = \frac{\omega}{q} \left(\frac{q}{q_0} \right)^J a'_{EJ} \left[1 + \left(\frac{q}{q_0} \right)^2 b'_{EJ}(q) \right] e^{-\left(\frac{q}{q_0} \right)^2} \quad t_{CJ} = \left(\frac{q}{q_0} \right)^J a'_{CJ} \left[1 + \left(\frac{q}{q_0} \right)^2 b'_{CJ}(q) \right] e^{-\left(\frac{q}{q_0} \right)^2}$$

(t_{C0} leading dependence cannot occur due to orthogonality of initial and final state)

- Long wavelength limit ($q \rightarrow 0$) and continuity equation:

$$t_{EJ} \rightarrow -\sqrt{\frac{J+1}{J}} \left(\frac{\omega}{q} \right) t_{CJ} \quad a'_{EJ} = -\sqrt{\frac{J+1}{J}} a'_{CJ}$$

$$a'_{EJ} = \left(\frac{q_0}{\omega} \right)^J \sqrt{\frac{\hbar c \cdot p_\alpha^{cm} \cdot W}{2\alpha \cdot \omega \cdot M_\alpha M_{12C}} \frac{S_{EJ}(E_\alpha^{cm}) \cdot e^{-2\pi\eta(E_\alpha^{cm})}}{E_\alpha^{cm}}}; \quad J = 1, 2.$$

Theoretical Assumptions

- No knowledge about next to leading order coefficients $b'_{EJ,CJ}$ with $J = 1, 2$
→ Assuming $b'_{EJ,CJ} \approx 1$ and “+” sign
- No knowledge about C0 multipole and $b'_{C0} \cdot a'_{C0}$
→ Assuming $b'_{C0} \approx 1$ and “+” sign, **Case A** $a'_{C0} = a'_{E2}$ and **Case B** $a'_{C0} = 0.5a'_{E2}$
- For $E_{\alpha}^{cm} < 1.7$ MeV only Coulomb phase contributes:

$$\delta_{Cl} - \delta_{C0} = \delta_{El} - \delta_{E0} = \sum_{n=1}^l \arctan \frac{\eta}{l}$$

Experimental Simulation

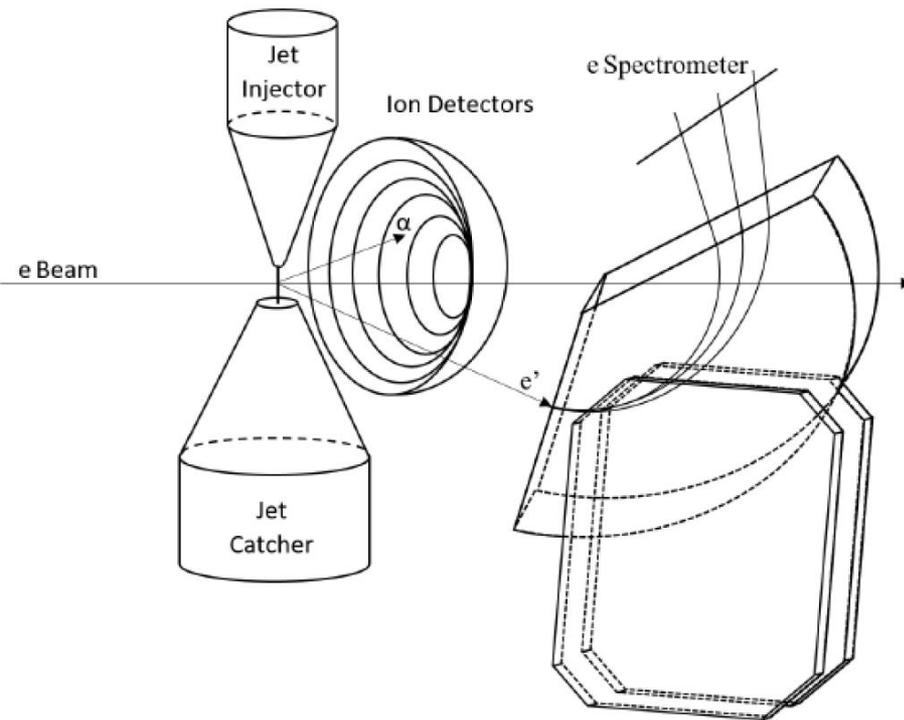


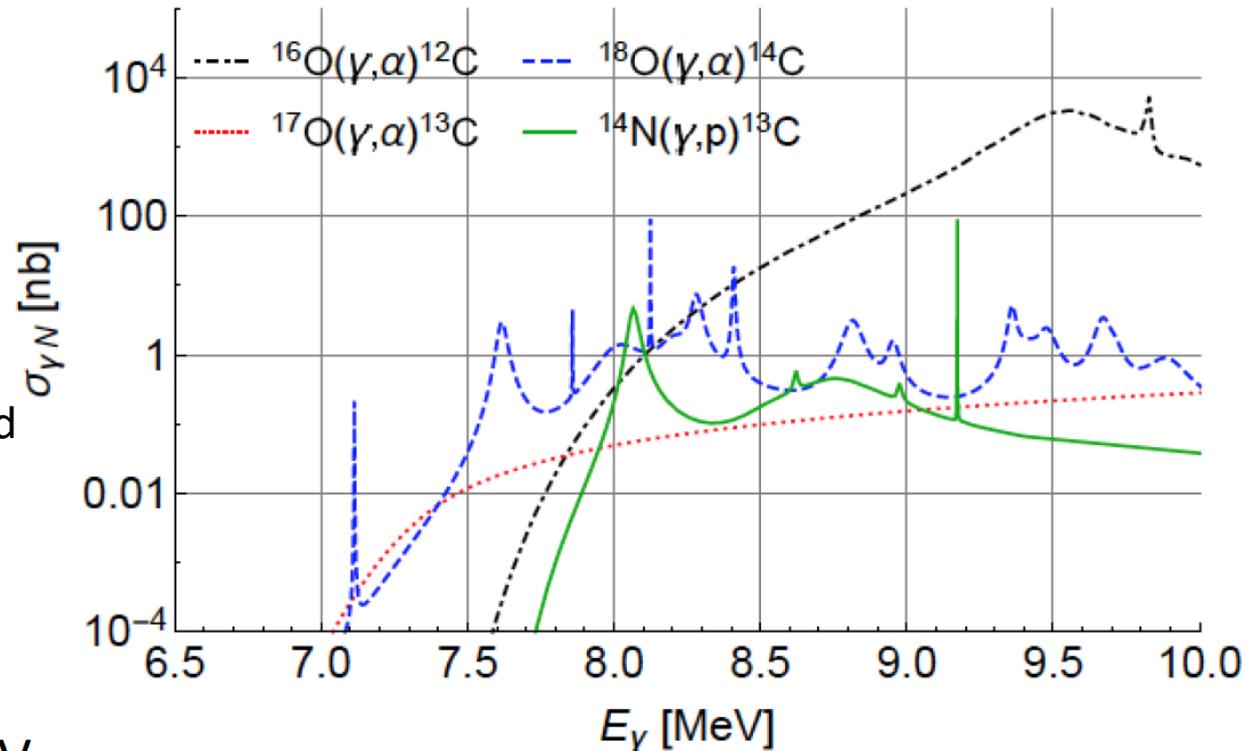
TABLE I. Summary of experimental parameters for the rate calculation.

Parameters		
Oxygen Target	Thickness	5×10^{18} atoms/cm ²
	Density	6.65×10^{-4} g/cm ³
Electron Beam	Current	40 mA
	Energies	78, 114, 150 MeV
Electron arm	In-plane acceptance	$\pm 2.08^\circ$
	Out-of-plane acceptance	$\pm 4.16^\circ$
	Solid angle acceptance	10.5 msr
α -particle arm	In-plane acceptance	60°
	Out-of-plane acceptance	360°
	Solid angle acceptance	3.14 sr
Luminosity		1.25×10^{36} cm ⁻² s ⁻¹
Integrated Luminosity (100 days)		1.08×10^7 pb ⁻¹
Central electron scattering angles		$15^\circ, 25^\circ, 35^\circ$
E_α^{cm} -range of interest		$0.7 \leq E_\alpha^{cm} \leq 1.7$ MeV

Isotopic and Chemical Contamination

- Oxygen isotopic contamination: ^{16}O 99.757%, ^{17}O 0.038% and ^{18}O 0.205%
K.J.R. Rosman, P.D.P. Taylor, Pure Appl. Chem. **71**, 1593 (1999)

- Photonuclear cross sections including natural abundances of oxygen isotopes and assuming depletion of ^{17}O and ^{18}O by a factor of 1000, and 5 ppmv for ^{14}N

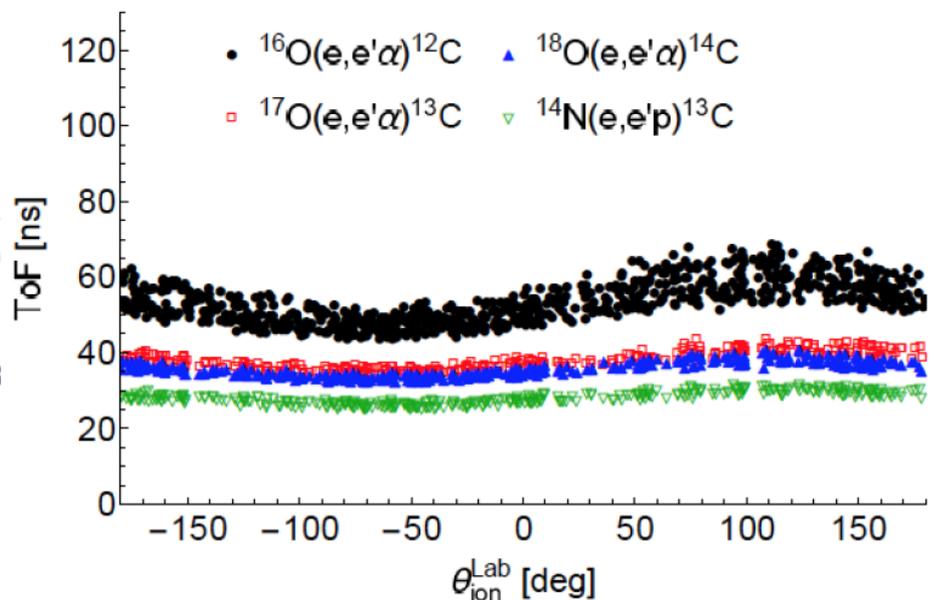
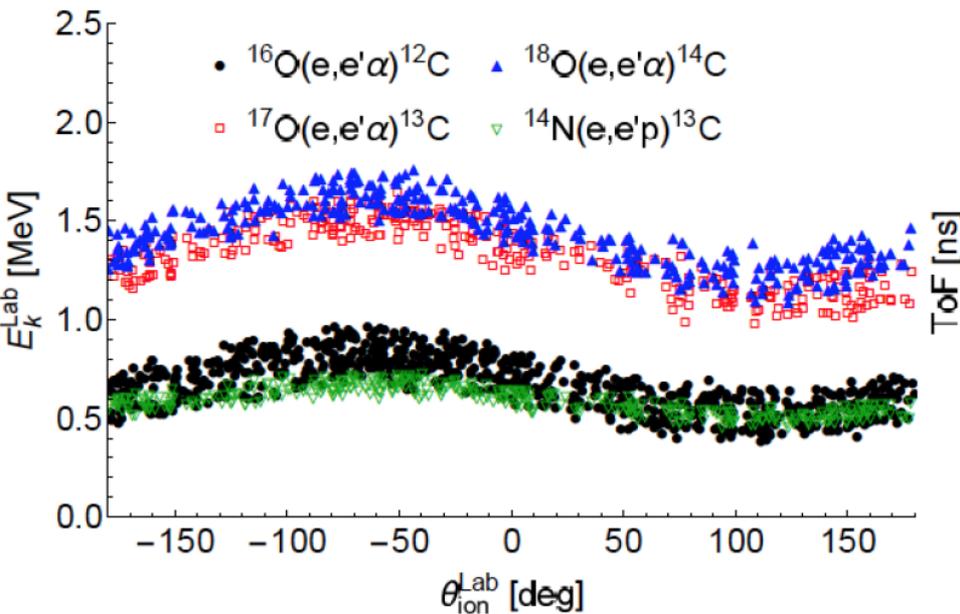


$$E_{\gamma} = E_{\alpha}^{cm} + 7.162 \text{ MeV}$$

https://wiki.jlab.org/ciswiki/index.php/Simulations_and_Backgrounds#Relevant_Theoretical_Cross_Sections

Reject Isotopic and Chemical Contamination Using Final-State Kinematics

- SRIM simulation: energy loss of α -particles in 2 mm wide oxygen jet, with a density of $6.65 \cdot 10^{-4} \text{ g/cm}^3$, $E_e = 114 \text{ MeV}$, $\theta_e = 15^\circ$, $1.0 \leq E_\alpha^{cm} \leq 1.1 \text{ MeV}$



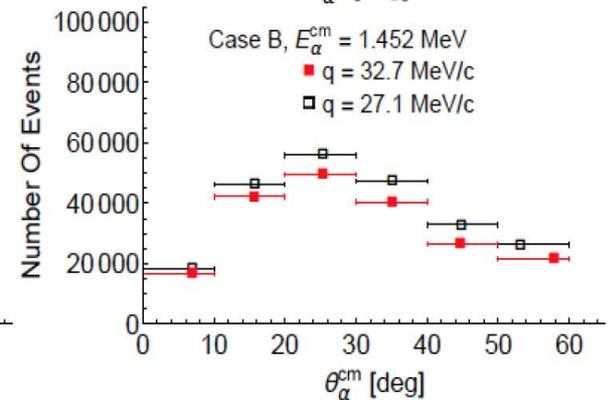
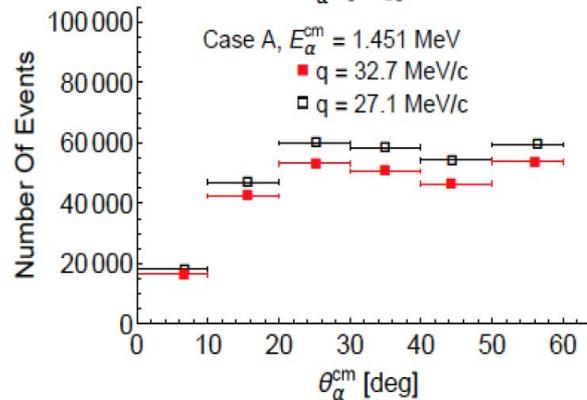
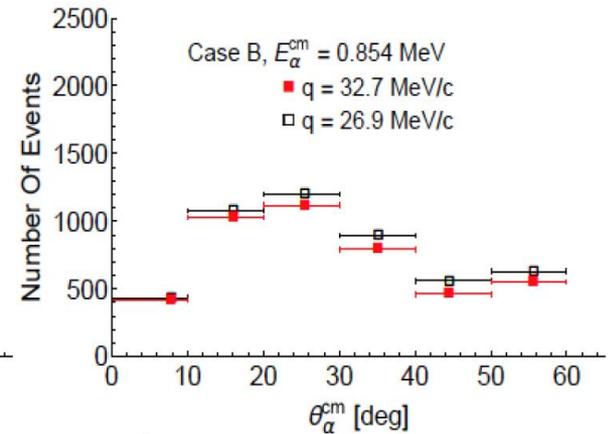
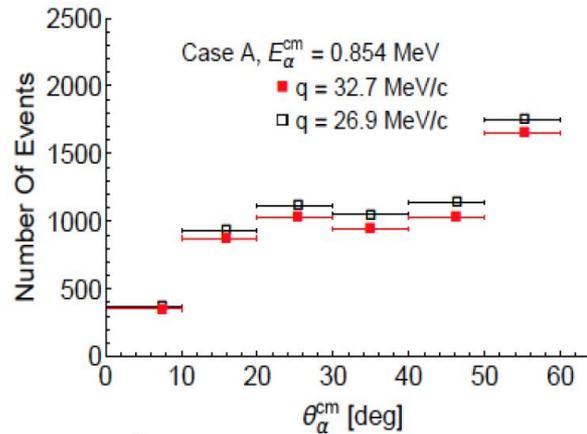
Number of Events after 100 Days

- Event were sorted in:
 - four 1.91 MeV wide q -bins
 - ten 100 keV wide E_{α}^{cm} -bins
 - six 10° wide θ_{α}^{cm} -bins

- $E_e = 114$ MeV, $\theta_e = 15^{\circ}$

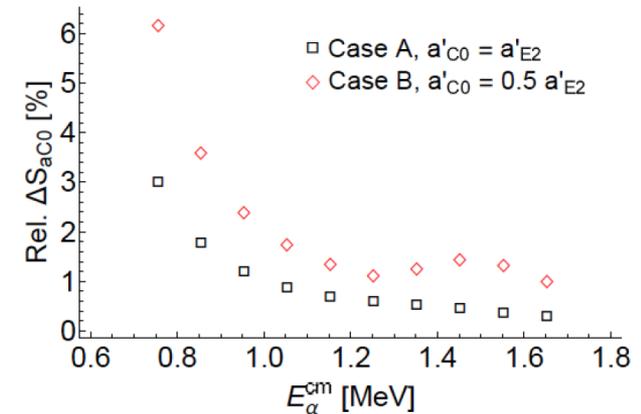
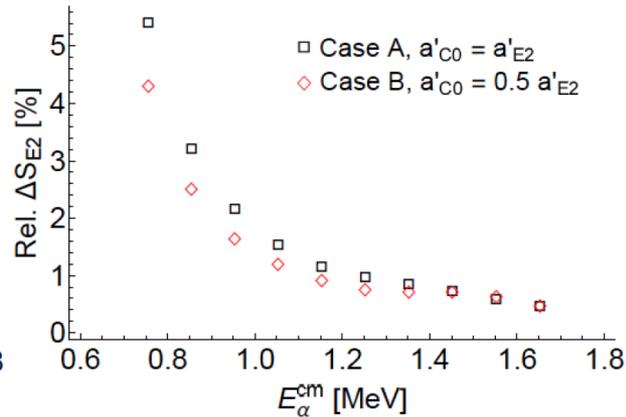
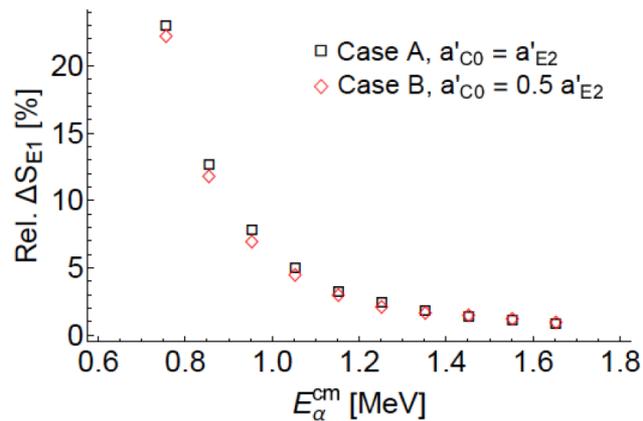
- Case A and Case B

- **Now we can compute statistical uncertainties**



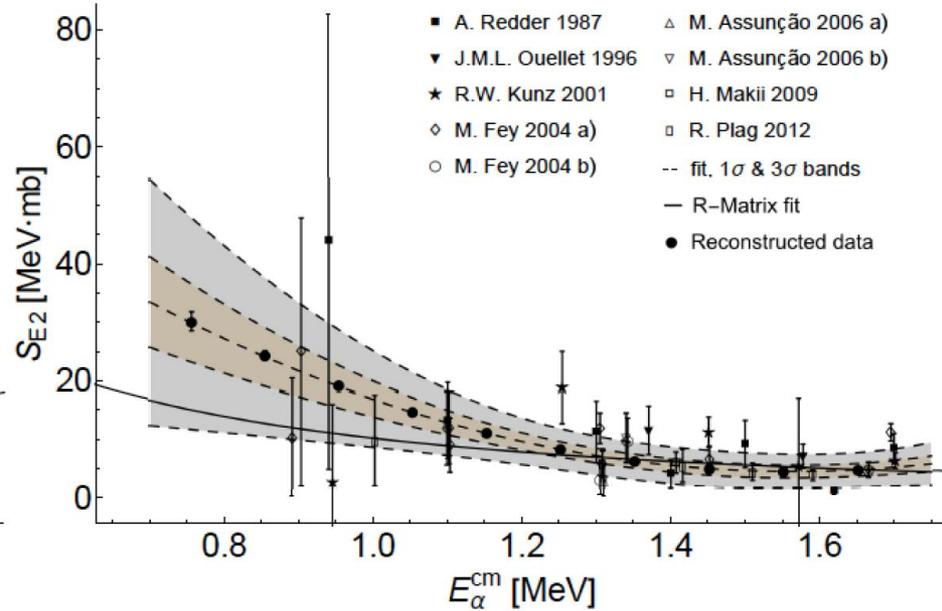
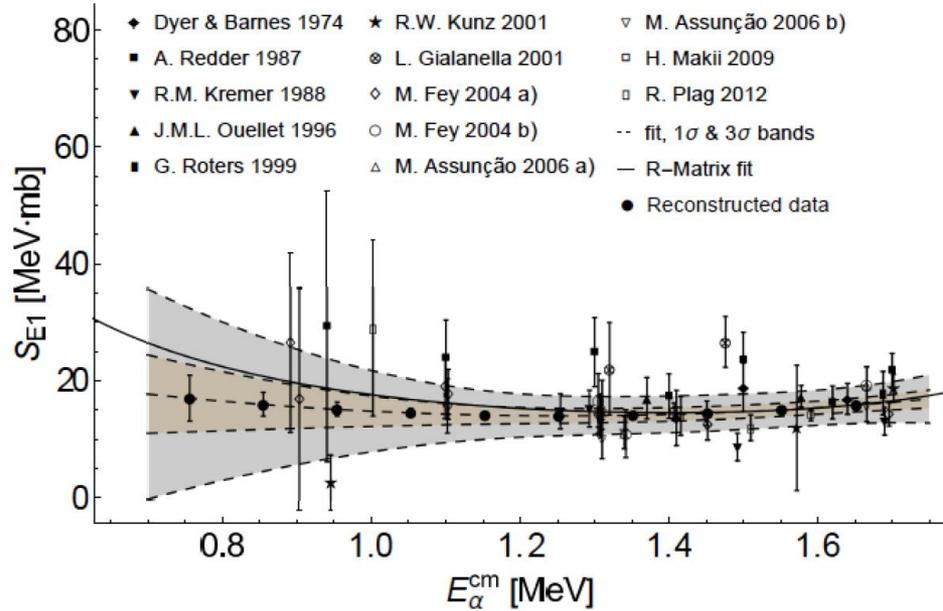
Projected Uncertainties in S-factors

- Three fitting parameters a'_{E1} , a'_{E2} and a'_{C0} \rightarrow S_{E1} , S_{E2} and S_{aC0} non-astrophysical factor
- a'_{C0} has a minor effect ($\sim 3\%$) on uncertainty of S_{E1} ;
- relative uncertainty of S_{E2} is $\sim 25\%$ larger for Case A
- Case B uncertainty of S_{aC0} is twice as large as in Case A



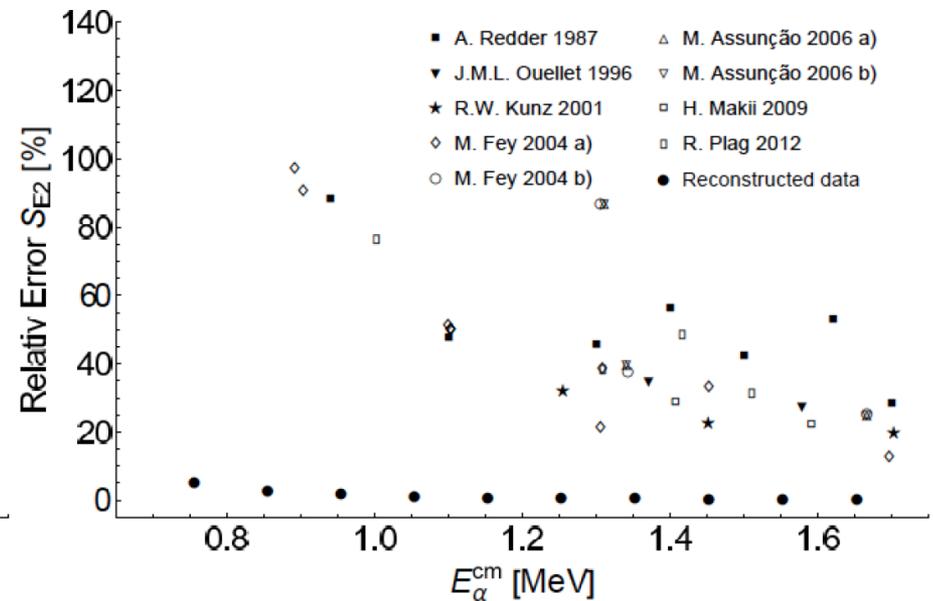
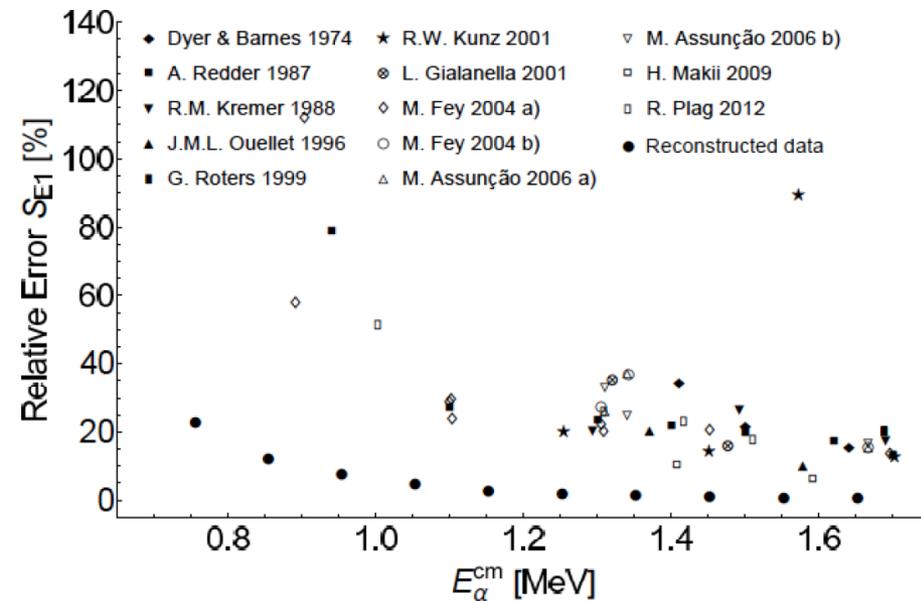
Comparison With World Data

- $E_e = 114 \text{ MeV}$, $\theta_e = 15^\circ$, Case A



Potential for Dramatic Reduction in Uncertainties

- $E_e = 114 \text{ MeV}$, $\theta_e = 15^\circ$, Case A
- Compared to most accurate measurements, statistical uncertainties of S_{E1} and S_{E2} are improved at least by factors 5.6 and 23.9, respectively



Impact of $^{16}\text{O}(e, e'\alpha)^{12}\text{C}$ measurements on the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ astrophysical reaction rate

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(Dated: September 2, 2019)

The $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, an important component of stellar helium burning, has a key role in nuclear astrophysics. It has direct impact on the evolution and final state of massive stars and also influences the elemental abundances resulting from nucleosynthesis in such stars. Providing a reliable estimate for the energy dependence of this reaction at stellar helium burning temperatures has been a longstanding and important goal. In this work, we study the role of potential new measurements of the reaction, $^{16}\text{O}(e, e'\alpha)^{12}\text{C}$ reaction, in reducing the overall uncertainty. A multilevel R -matrix analysis is used to make extrapolations of the astrophysical S factor for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction to the stellar energy of 300 keV. The statistical precision of the S -factor extrapolation is determined by performing multiple fits to existing $E1$ and $E2$ ground state capture data, including the impact of possible future measurements of the $^{16}\text{O}(e, e'\alpha)^{12}\text{C}$ reaction. In particular, we consider a proposed MIT experiment that would make use of a high-intensity low-energy electron beam that impinges on a windowless oxygen gas target as a means to determine the total $E1$ and $E2$ cross sections for this reaction.

SUMMARY

From this study it appears that OSEEA reaction data proposed by MIT could have a significant impact on the statistical precision of S(300 keV). The projected standard deviations for the 1000 fits to the $E1$ and $E2$ data with the proposed MIT data is significantly smaller than that without MIT data. The projected MIT results not only have superior statistical precision, but will also extend to lower energy than previous data.

Some rate estimates for a TRIUMF ARIEL experiment

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$^{16}\text{O}(e, e'\alpha)^{12}\text{C}$ measurements and the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ astrophysical reaction rate

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These results demonstrate that to achieve significant improvement in S factor uncertainties from measurements in the low energy range alone will require extraordinary control of both statistical and systematic uncertainties. This, of course, is already well known from the long history of experimental work seeking to extend measurements to ever lower energies. However, these results also show that significant progress can be made with measurements in the high energy range with much less stringent requirements on the experimental uncertainties. Moreover, experiments at higher energies are easier to perform because of the larger cross sections. This suggests that new experiments should not overlook making measurements at higher energies. For example, we find that the proposed OSEEA experiment over the full energy range could cut the uncertainty in half when combined with the existing data, assuming 10% systematic uncertainty and an additional 2% increase in statistical uncertainty. These results may give guidance in designing new experiments to study the CTAG reaction at stellar energies, and we look forward to new data that will improve our understanding of this important astrophysical reaction.

Electrodisintegration of ^{16}O at TRIUMF

Assumptions: eBeam \rightarrow 50 MeV, 10 mA

Electron-spectrometer at 15 deg

Electron arm \rightarrow In-plane acceptance ± 2.08 deg

\rightarrow Out-of-plane acceptance ± 4.16 deg

\rightarrow Solid angle acceptance 10.5 msr

α -particle arm \rightarrow In-plane acceptance 60 deg

\rightarrow Out-of-plane acceptance 180 deg

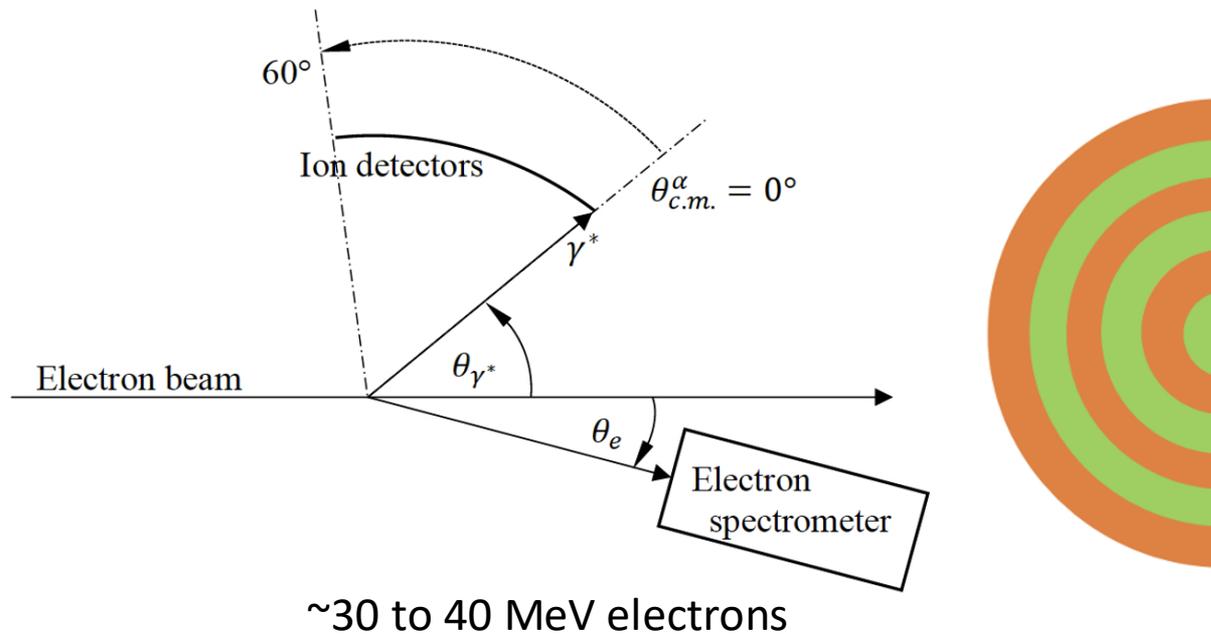
Oxygen target \rightarrow Thickness 5×10^{18} atoms/cm²

\rightarrow Density 6.65×10^{-4} g/cm³

Integrated luminosity (10 days) $\rightarrow 2.7 \times 10^5$ pb⁻¹

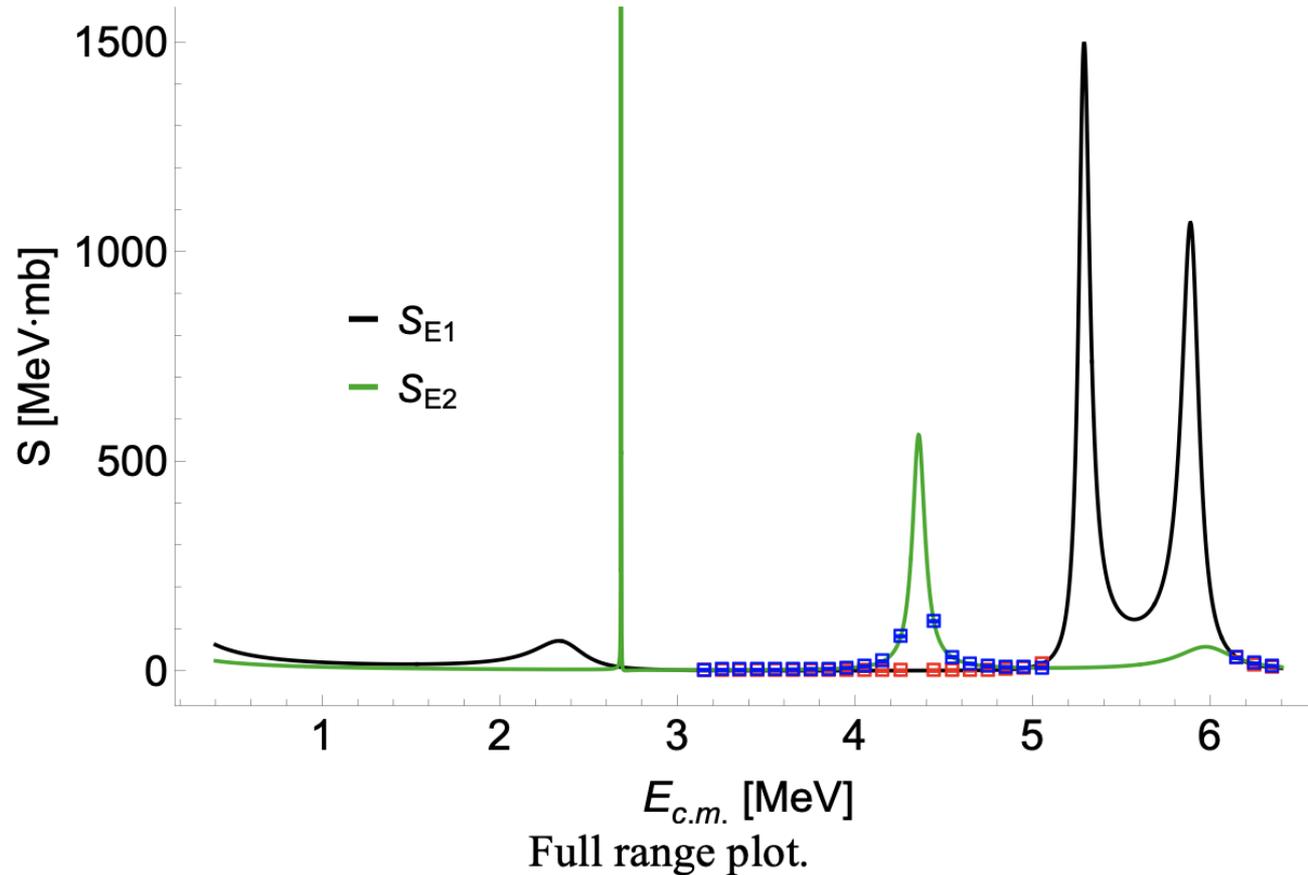
Integrated luminosity (100 days) $\rightarrow 2.7 \times 10^6$ pb⁻¹

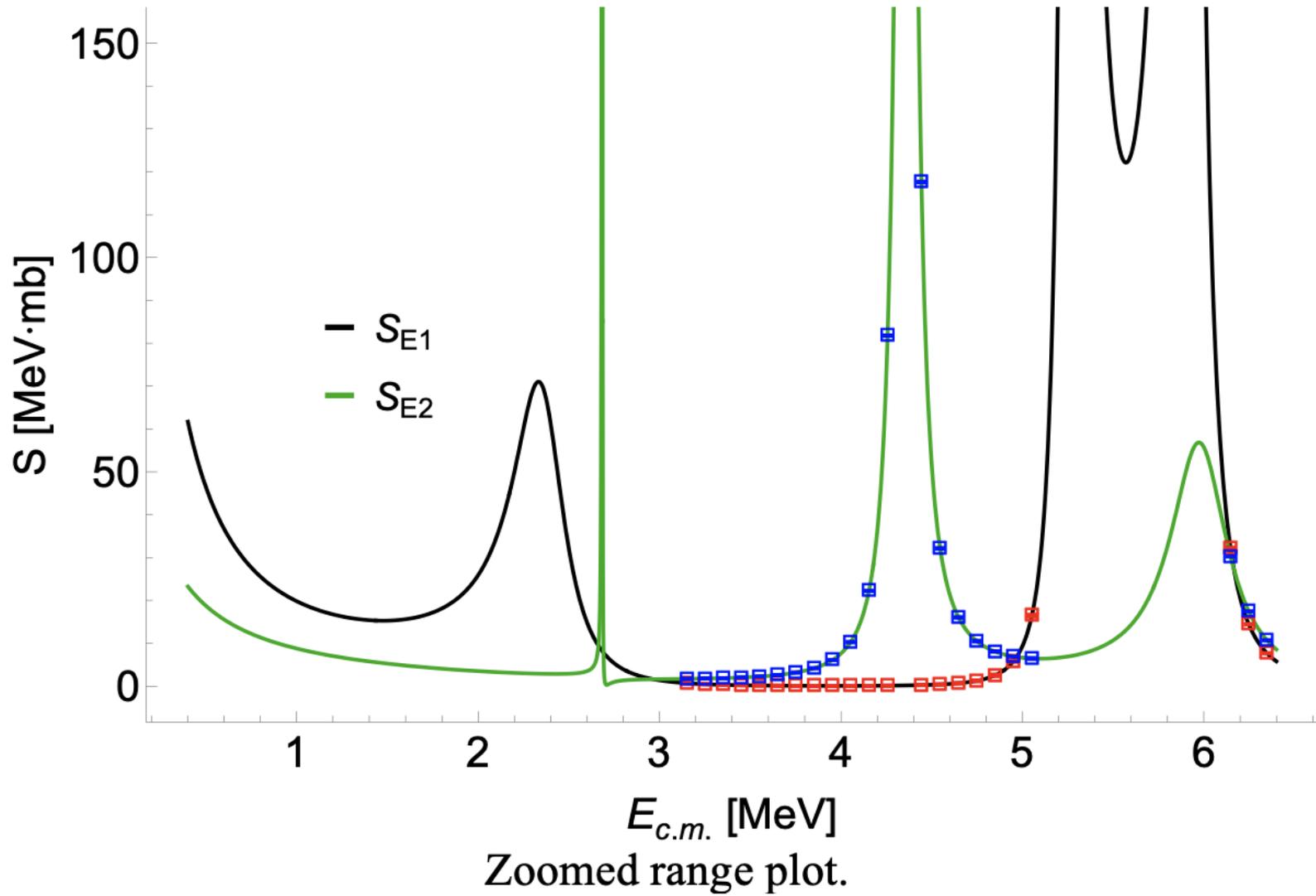
Possible Experiment



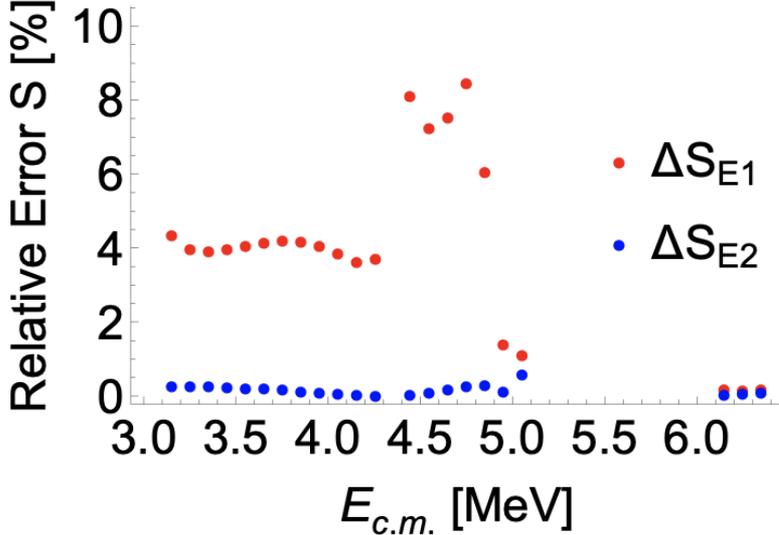
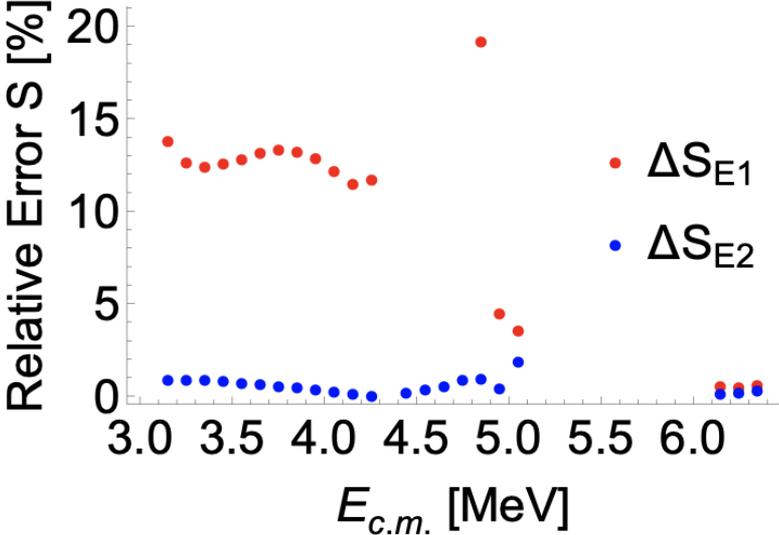
Schematics of the possible experiment. Alpha particles are detected only in left half-hemisphere since the θ_γ is too small and the full range of $0^\circ < \theta_{c.m.}^\alpha < 60^\circ$ of the right half-hemisphere would cut into the beamline.

S_{E1} and S_{E2} data provided by Roy Holt. α -particle range of interest $E_{c.m.} > 3$ MeV with step of 100 keV and the region of high energy resonances was omitted from the calculation. Full range and zoomed images are below:

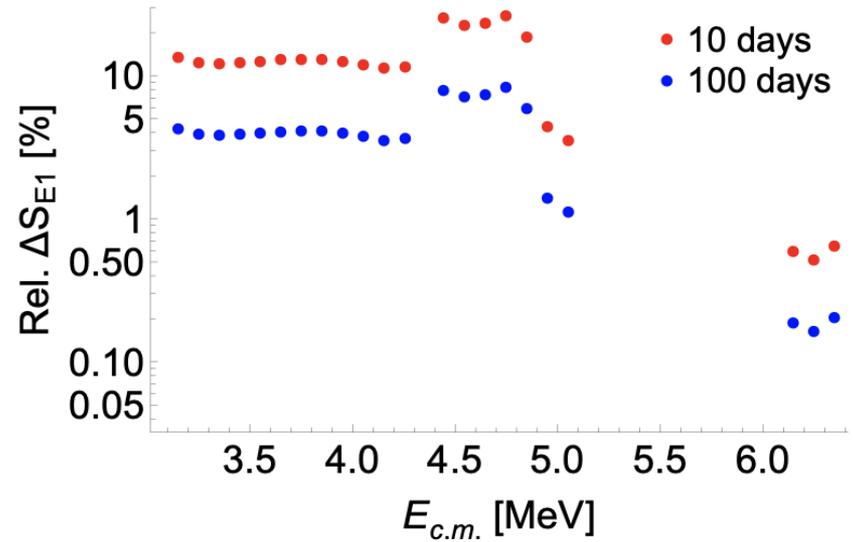
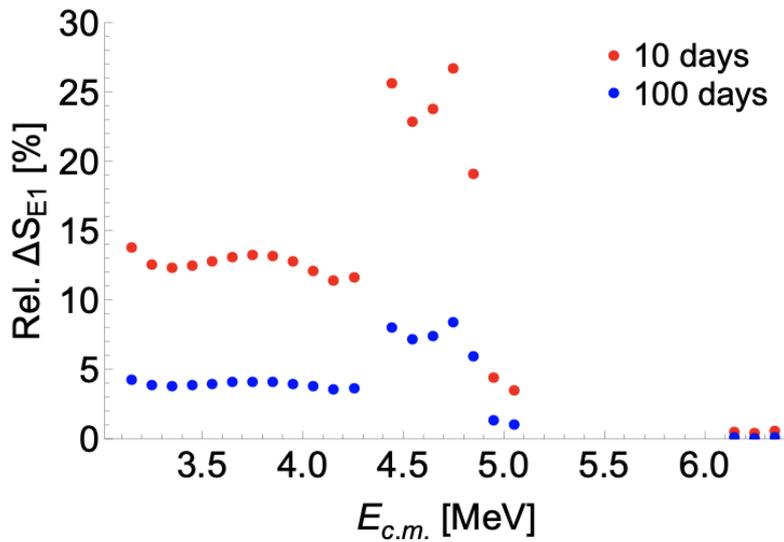




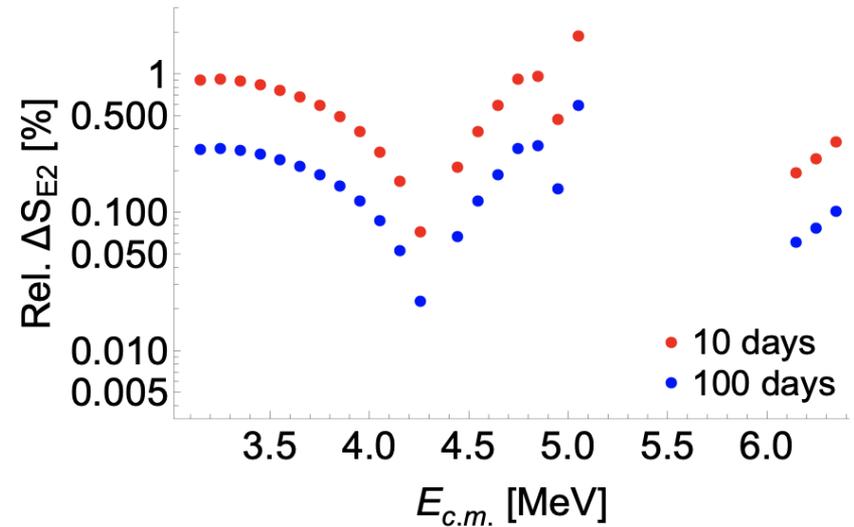
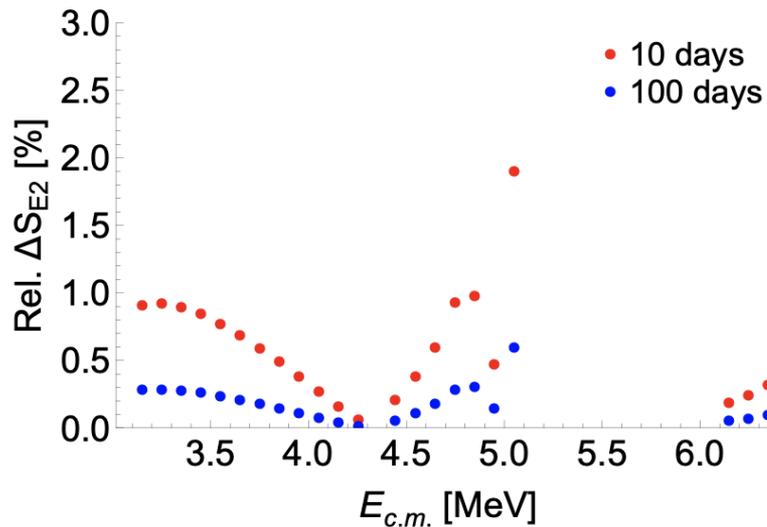
Estimated relative statistical uncertainties astrophysical S-factor after 10 days (left) and 100 days (right) of data taking:



Estimated relative statistical uncertainties of SE1 after 10 days and 100 days of data taking, linear plot (left) and logarithmic plot (right):



Estimated relative statistical uncertainties of SE2| after 10 days and 100 days of data taking, linear plot (left) and logarithmic plot (right):



Summary and Outlook

- Using a simple but fully realistic electroproduction model, and anticipated performance of new ERL electron accelerators and gas-jet targets, we have studied the rate of $^{16}\text{O}(e,e'\alpha)^{12}\text{C}$ reaction in the range $0.7 < E_{\alpha}^{\text{cm}} < 1.7$ MeV.
- We conclude that there is a clear potential to determine the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction rate with unprecedented precision.
- Detection of sub-MeV alpha-particle in electromagnetic background of high power beam-gas target interaction will be challenging.
- At $E_e = 114$ MeV and electron spectrometer at $\theta_e = 15^\circ$, with 10% E'_e acceptance, the full range $0 < E_e^{\text{cm}} < 10.2$ MeV is accessible.
- However, we propose an initial experiment focused on higher E_e^{cm} to validate our approach and to gain an experimental understanding of the technical challenges.
- TRIUMF ARIEL e-linac at higher energy and power looks very interesting.
- Full details can be found at **I. Frišćić, TWD, and RGM, Phys. Rev. C 100, 025804 (2019).**

Suggestion

- At an appropriate time, *i.e.* after our current experiment S2134 is installed, commissioned and producing data, we should organize a workshop to focus on scientific opportunities with a higher energy/power ARIEL e-linac.
- It could be beneficial to organize this jointly with Jefferson Lab.