Numerical Relativity:

Solving Einstein's equations on a computer

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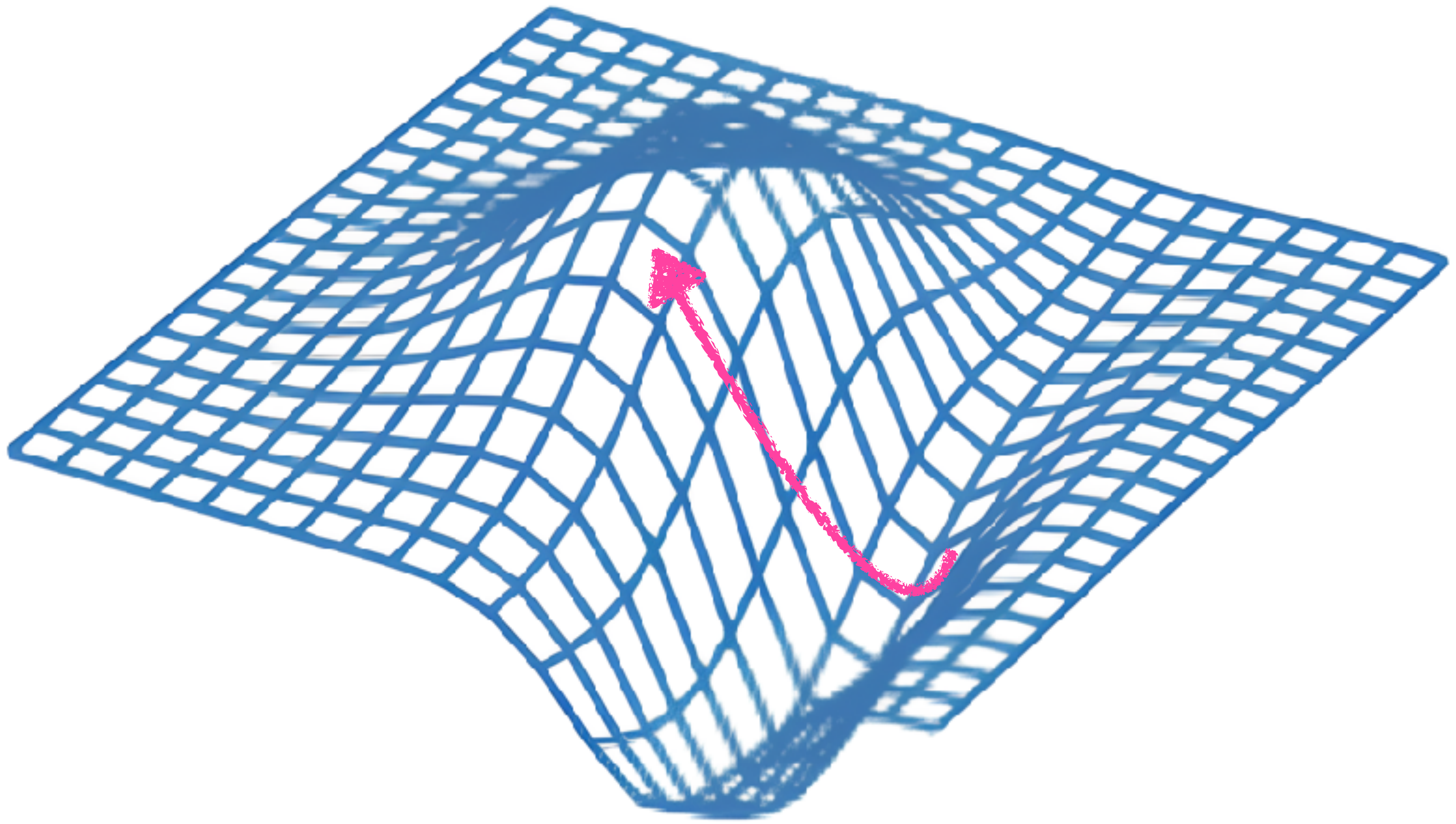
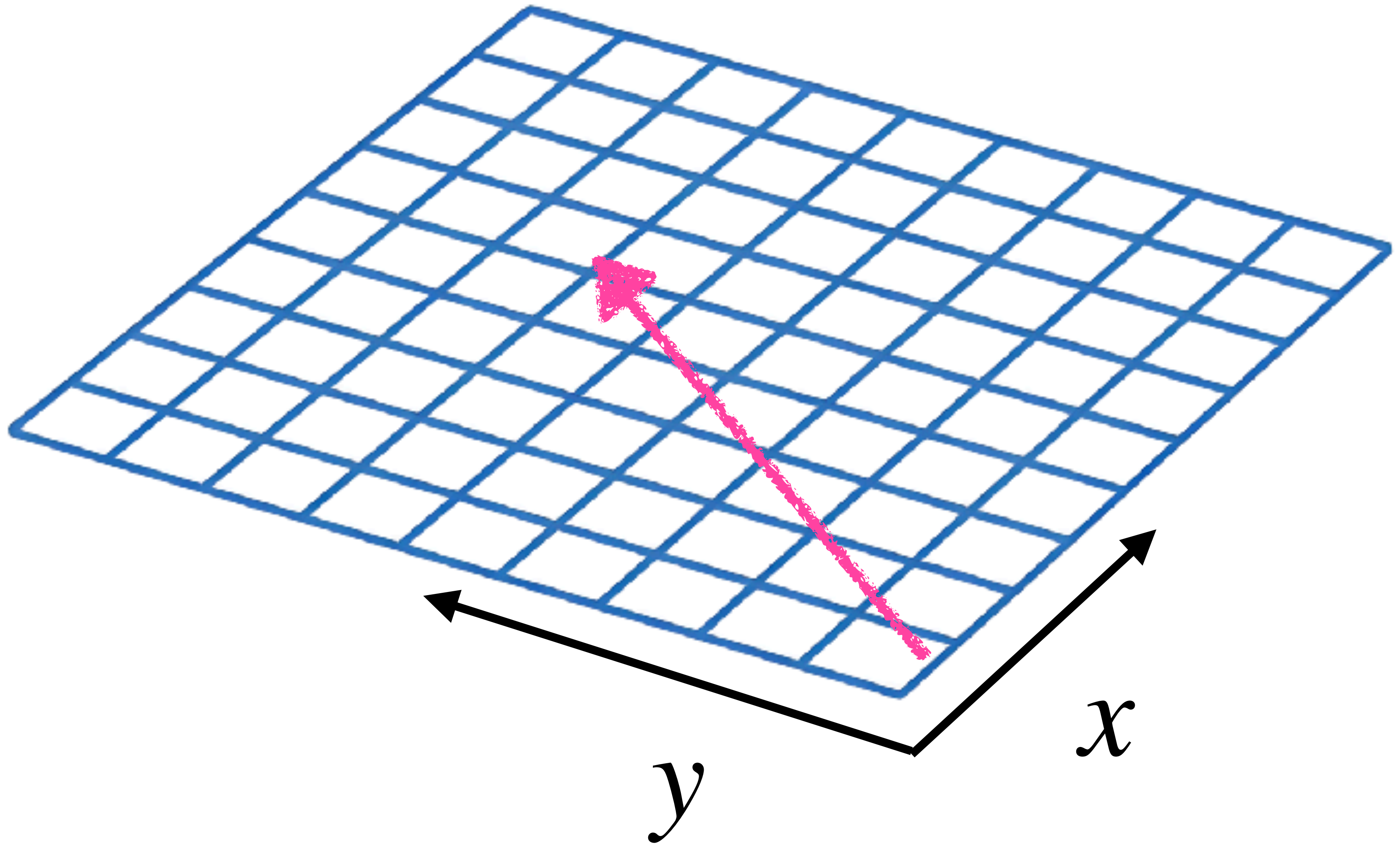
We want to know the spacetime metric $g_{\mu\nu}(t, \overrightarrow{x})$

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 8\pi G T_{\mu\nu}$$



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Flat space

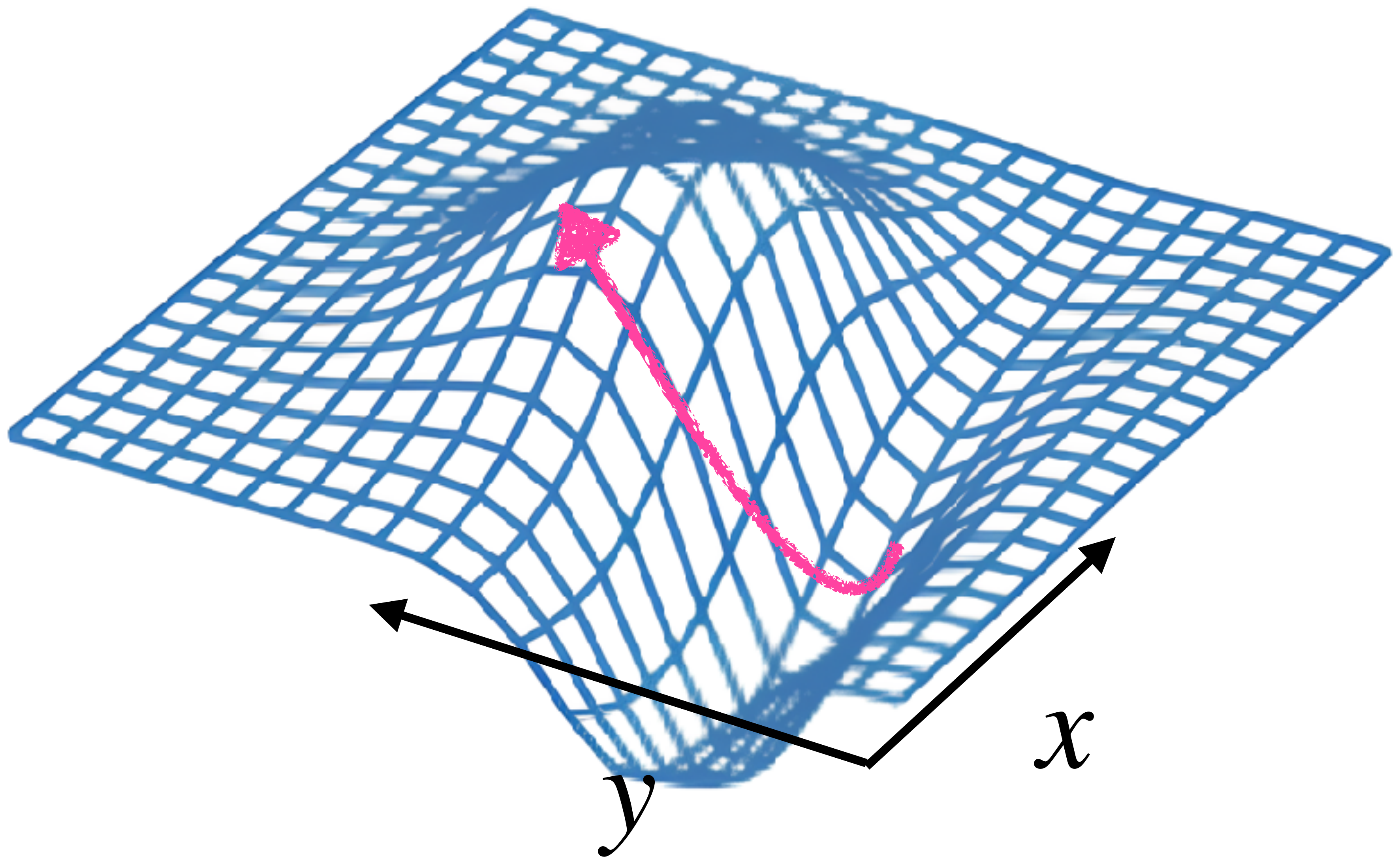


$$dl^2 = dx^2 + dy^2$$

$$dl^2 = (dx \quad dy) \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\delta_{ab}} \begin{pmatrix} dx \\ dy \end{pmatrix}$$

"The metric"

Curved space

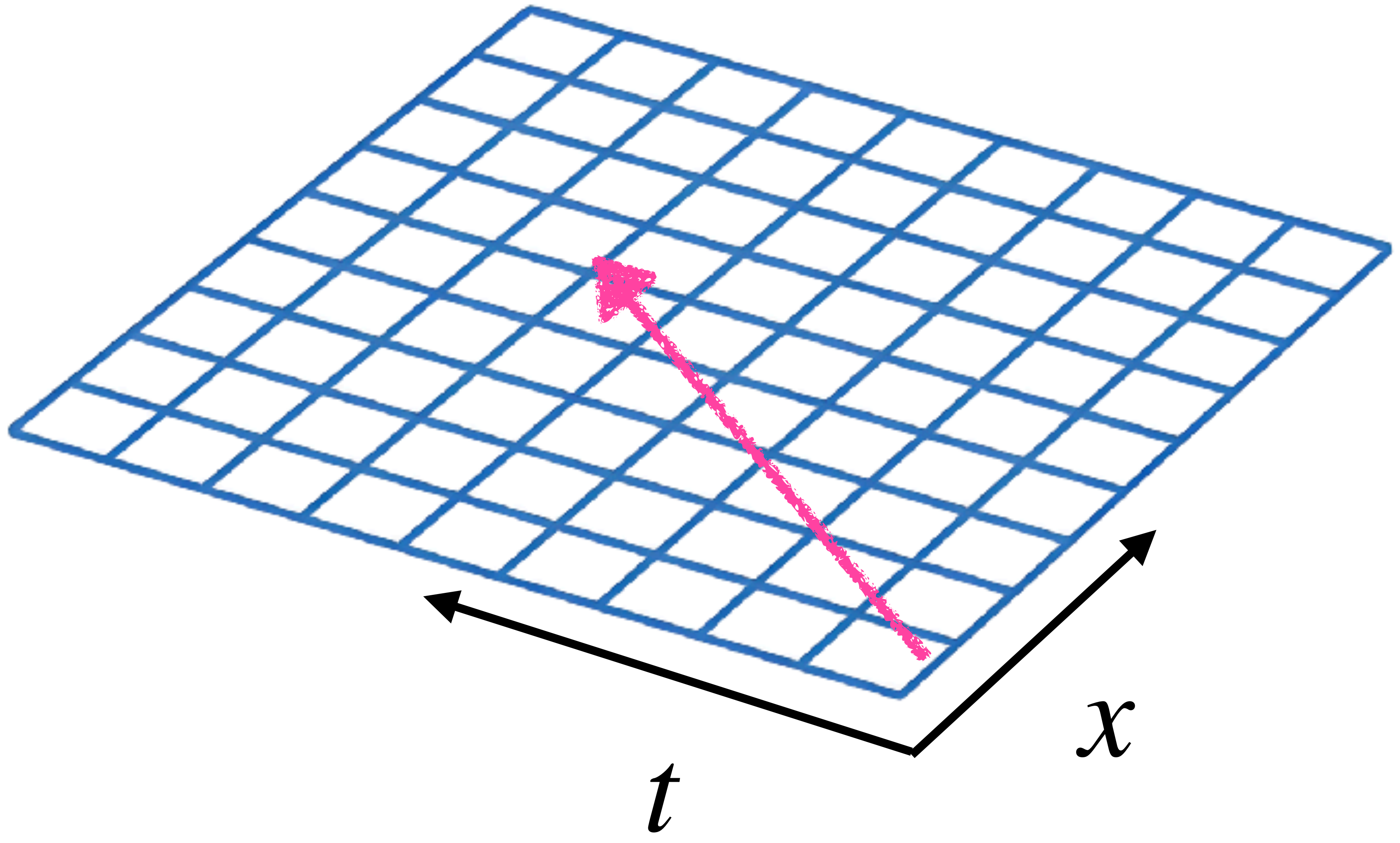


$$dl^2 = f(x,y)dx^2 + g(x,y)dy^2$$

$$dl^2 = (dx \quad dy) \begin{pmatrix} f(x,y) & 0 \\ 0 & g(x,y) \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix}$$

Figures from Katy Clough

Flat spacetime

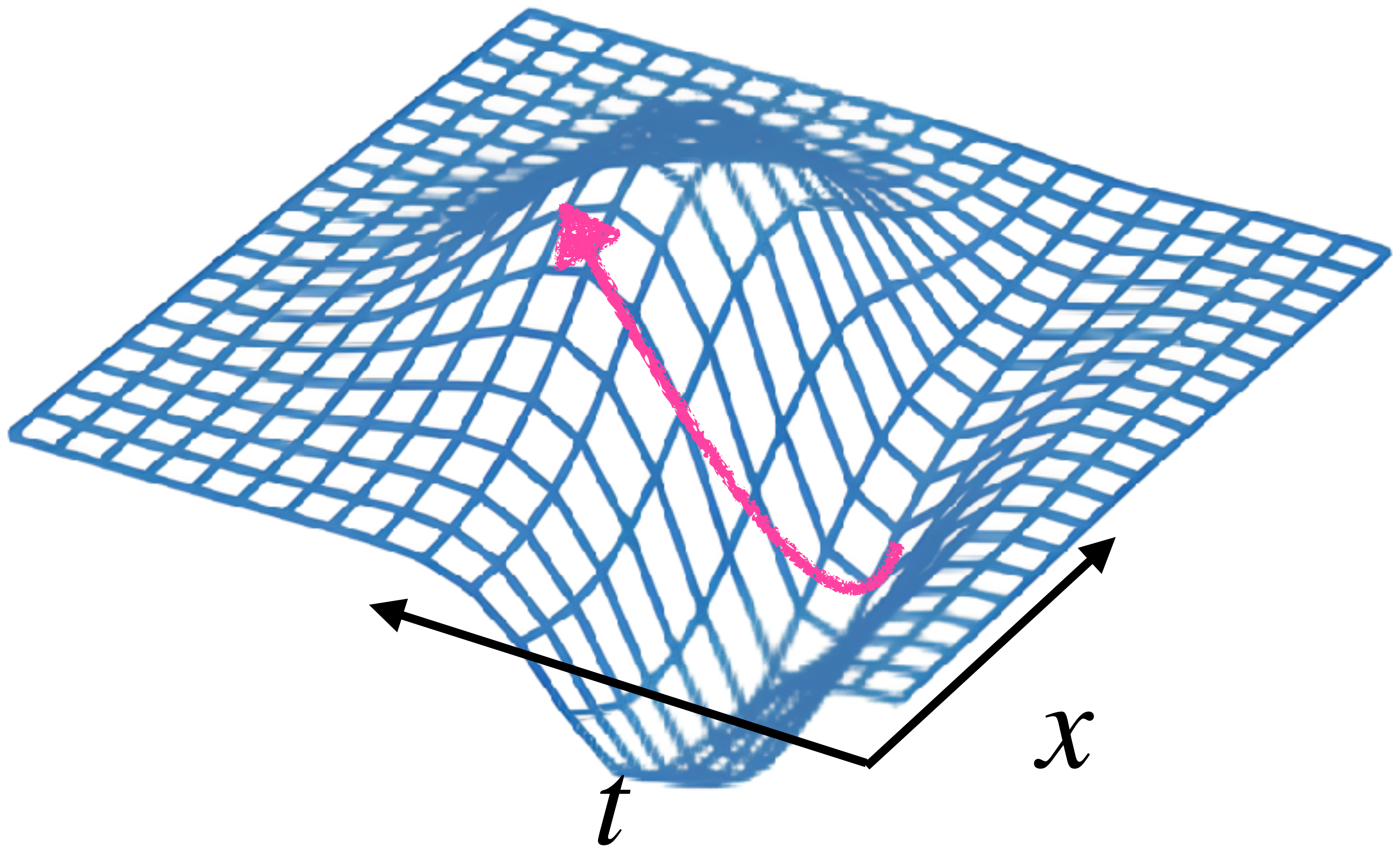


$$ds^2 = -c^2dt^2 + dx^2$$

$$ds^2 = (dt \quad dx)\underbrace{\begin{pmatrix} -c^2 & 0 \\ 0 & 1 \end{pmatrix}}_{\eta_{\mu\nu}}\begin{pmatrix} dt \\ dx \end{pmatrix}$$

"The spacetime metric"

Curved spacetime



$$ds^2 = f(t,x)dt^2 + g(t,x)dx^2 + 2h(t,x)dtdx$$

$$ds^2 = (dt \quad dx)\begin{pmatrix} f(t,x) & h(t,x) \\ h(t,x) & g(t,x) \end{pmatrix}\begin{pmatrix} dt \\ dx \end{pmatrix}$$

Figures from Katy Clough

$$ds^2 = \begin{pmatrix} dt & dx & dy & dz \end{pmatrix} \underbrace{\begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix}}_{g_{\mu\nu}(t, \vec{x})} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$

“The spacetime metric”

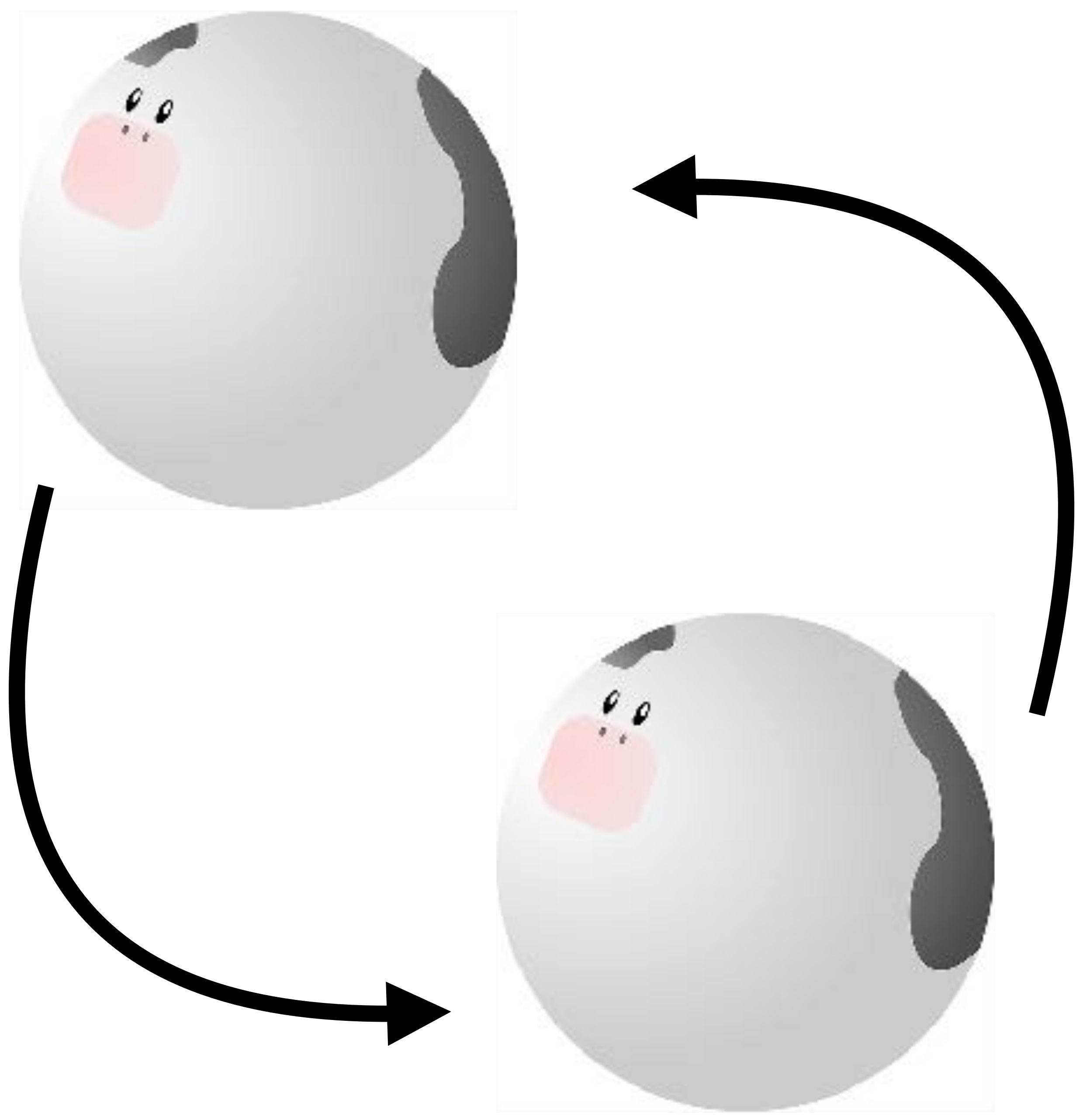
$$\underbrace{R_{\mu\nu} - \frac{R}{2} g_{\mu\nu}}_{f(\partial_t^2 g_{\mu\nu},\, \partial_t g_{\mu\nu},\, g_{\mu\nu})} = \underbrace{8\pi G T_{\mu\nu}}_{f(\rho, S_i)}$$

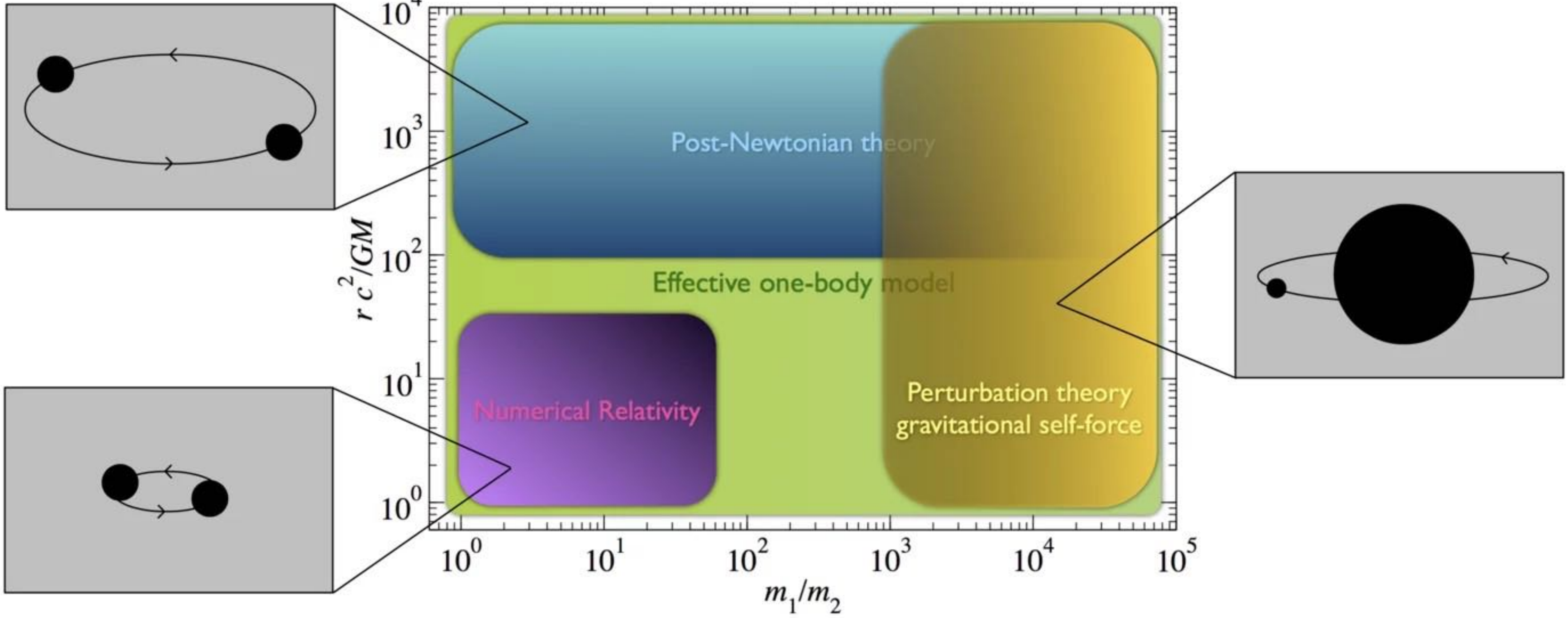
“Curvature” **“Energy-Momentum”**

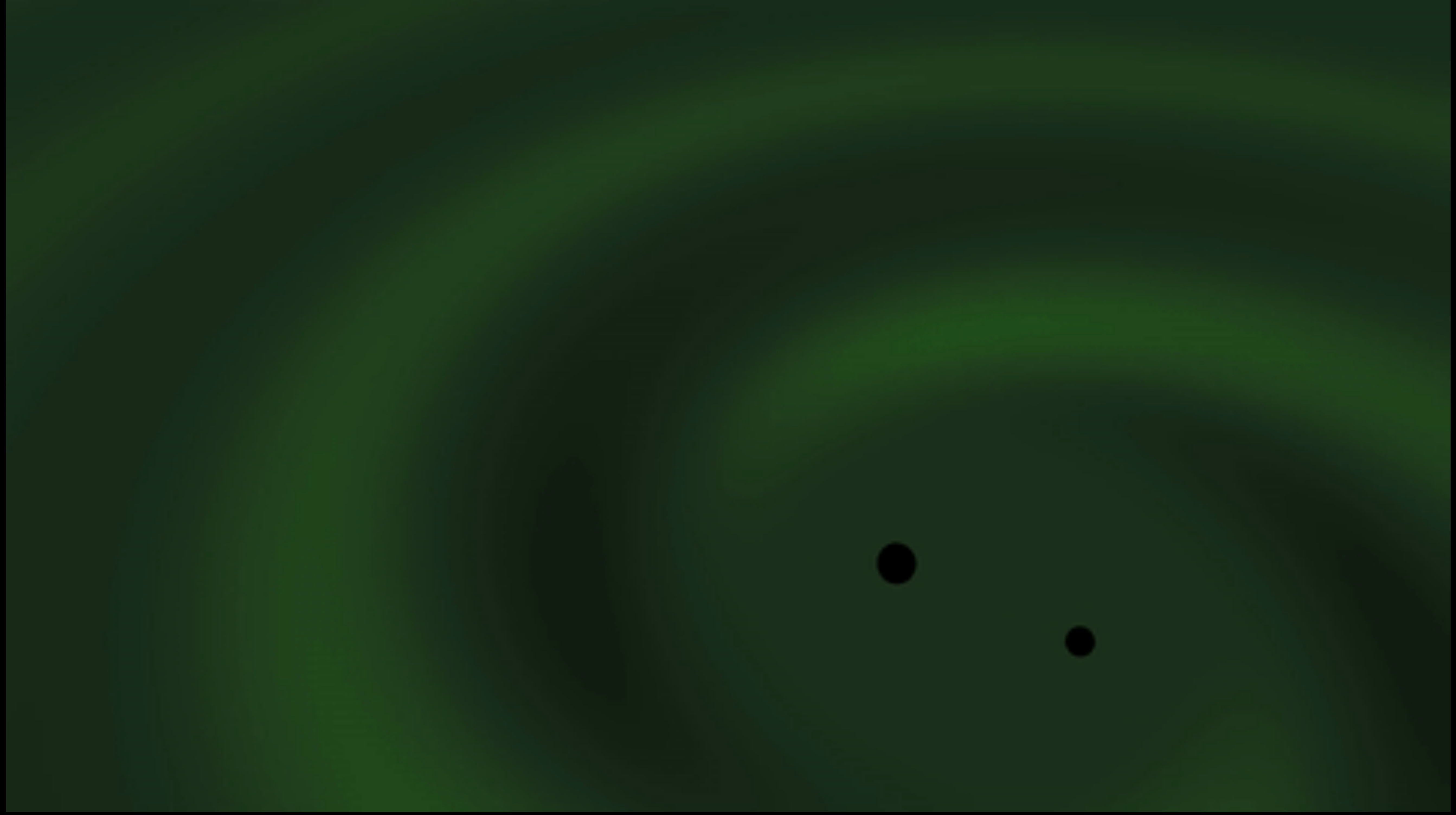
Schwarzschild solution

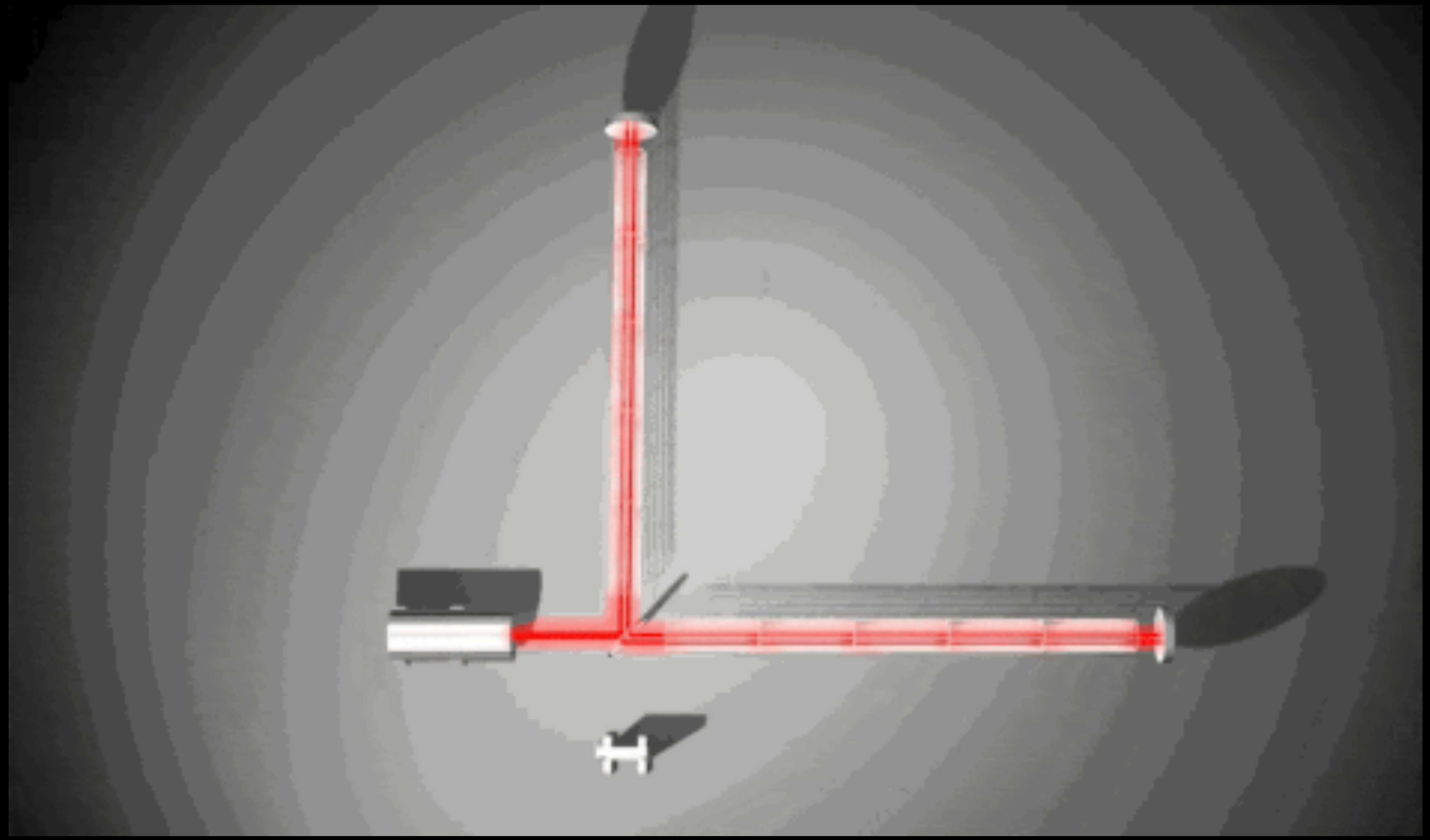
$[T_{\mu\nu} = 0, \;\; g_{\mu\nu} = g_{\mu\nu}(r), \;\; \partial_t g_{\mu\nu} = 0]$

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

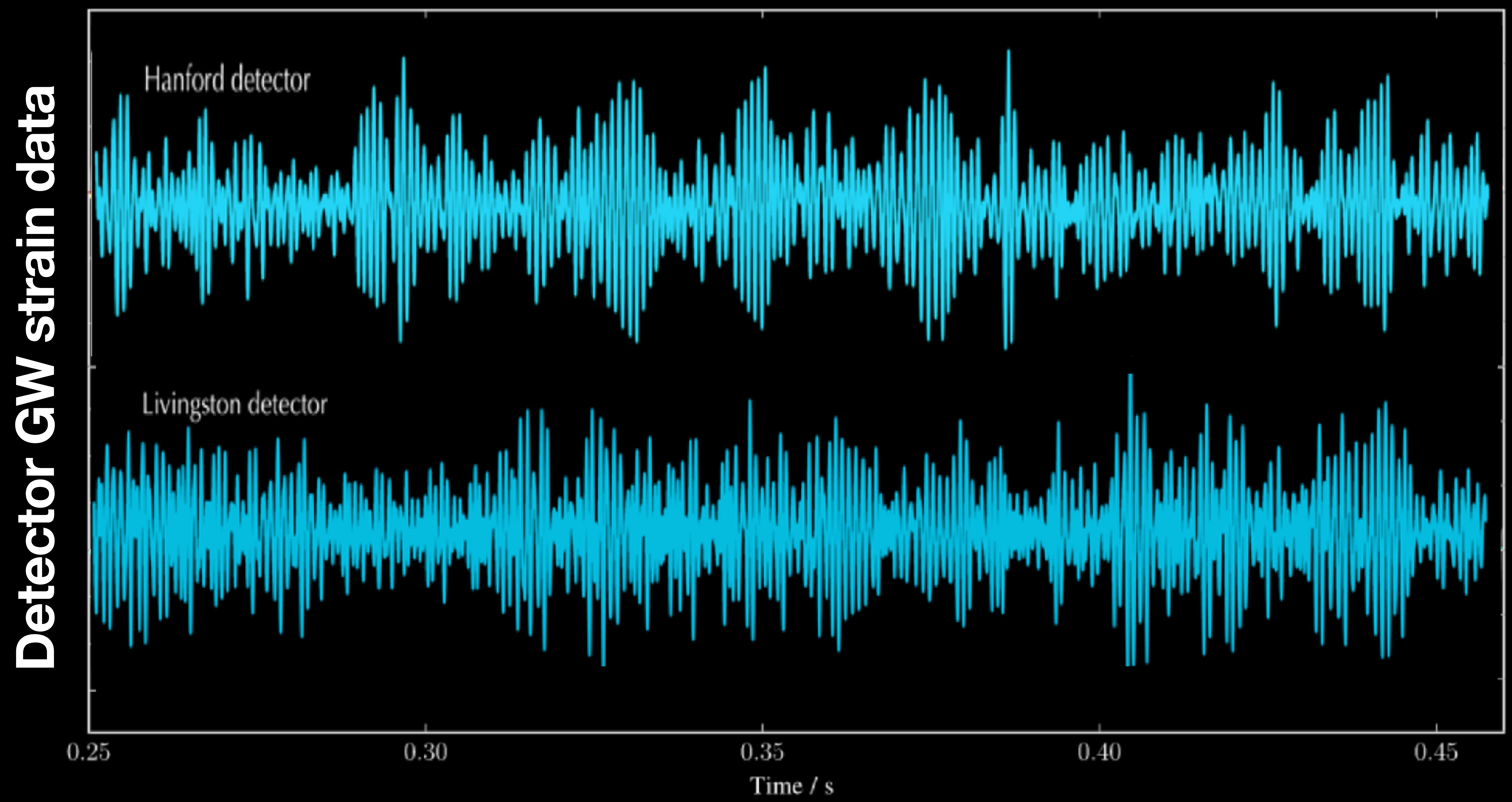



Post-Newtonian theory
Effective one-body model
Numerical Relativity
Perturbation theory
gravitational self-force
$r c^2/GM$
m_1/m_2
10^0
10^1
10^2
10^3
10^4
10^5

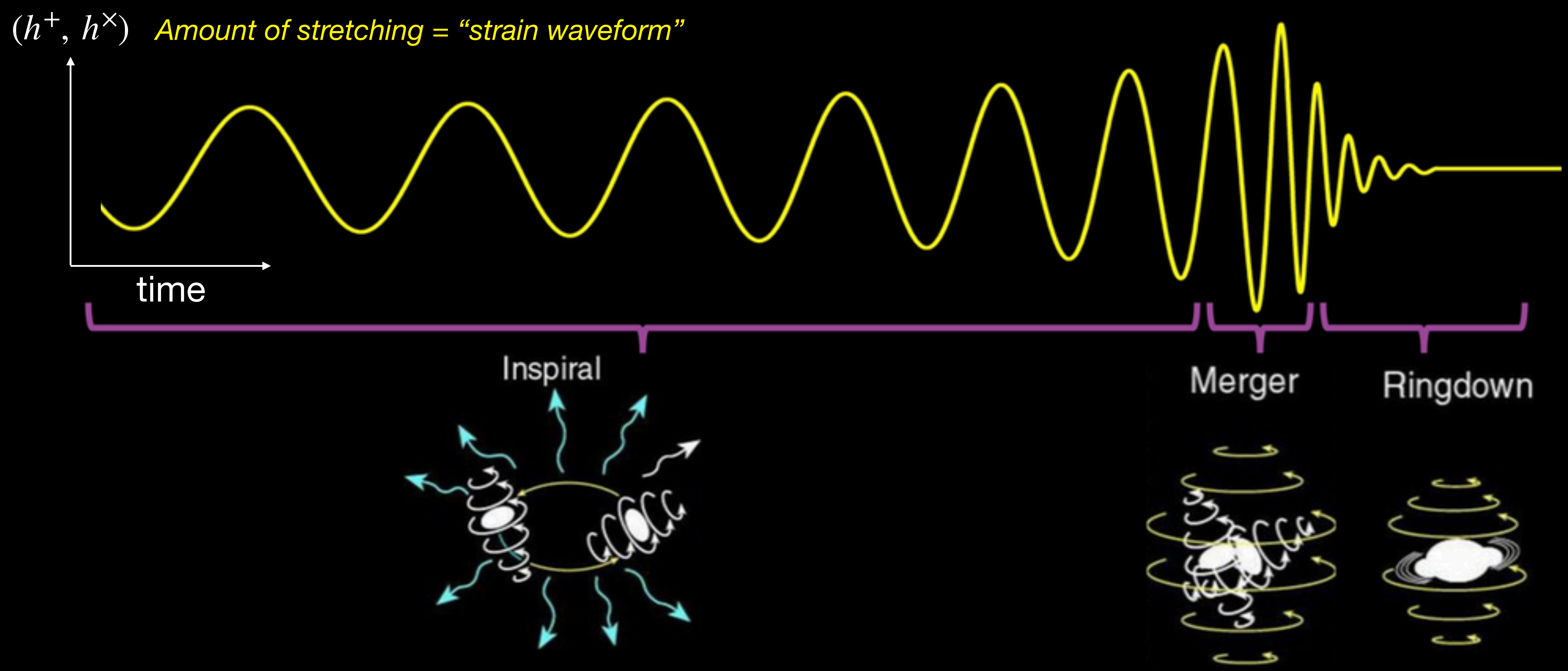


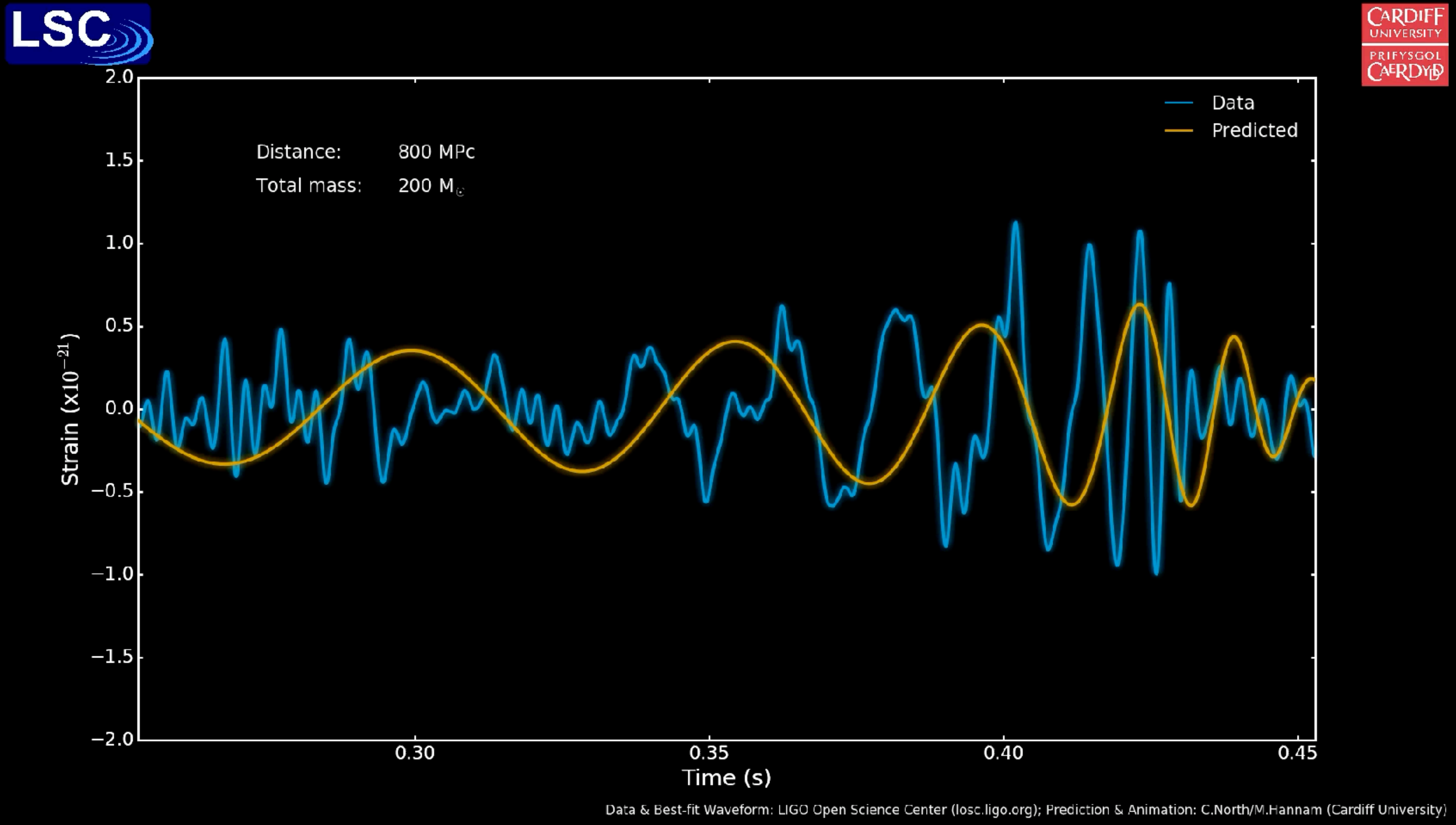


Ingredient 1: Data



Ingredient 2: Theory




LSC
CARDIFF UNIVERSITY
PRIFYSGOL CAERDYDD
Data
Predicted
Distance: 800 MPc
Total mass: 200 M☉
Strain (x10⁻²¹)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.30
0.35
0.40
0.45
Time (s)
Data & Best-fit Waveform: LIGO Open Science Center (losc.ligo.org); Prediction & Animation: C.North/M.Hannam (Cardiff University)

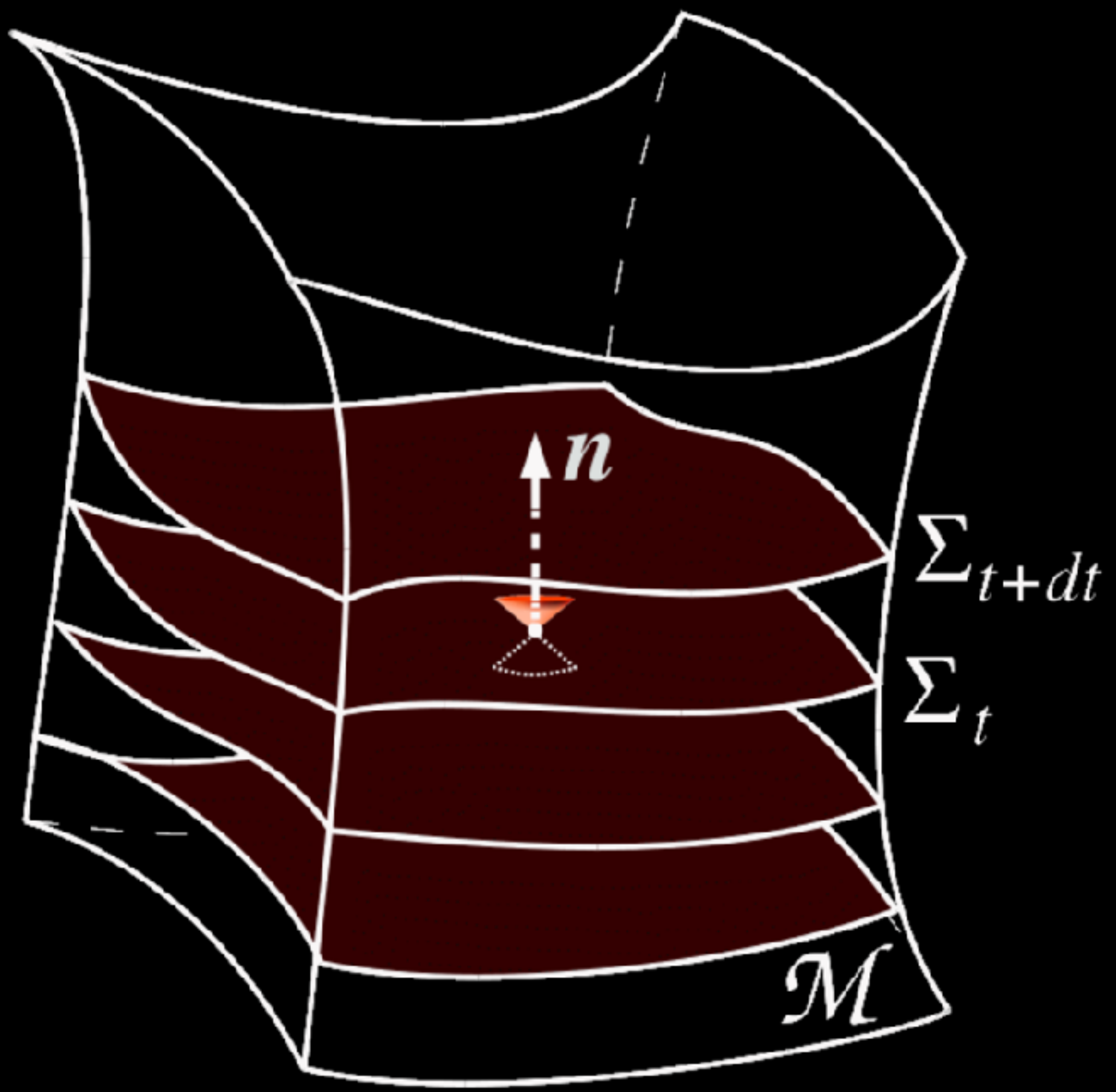
Solving GR on a computer:

Covariant form

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

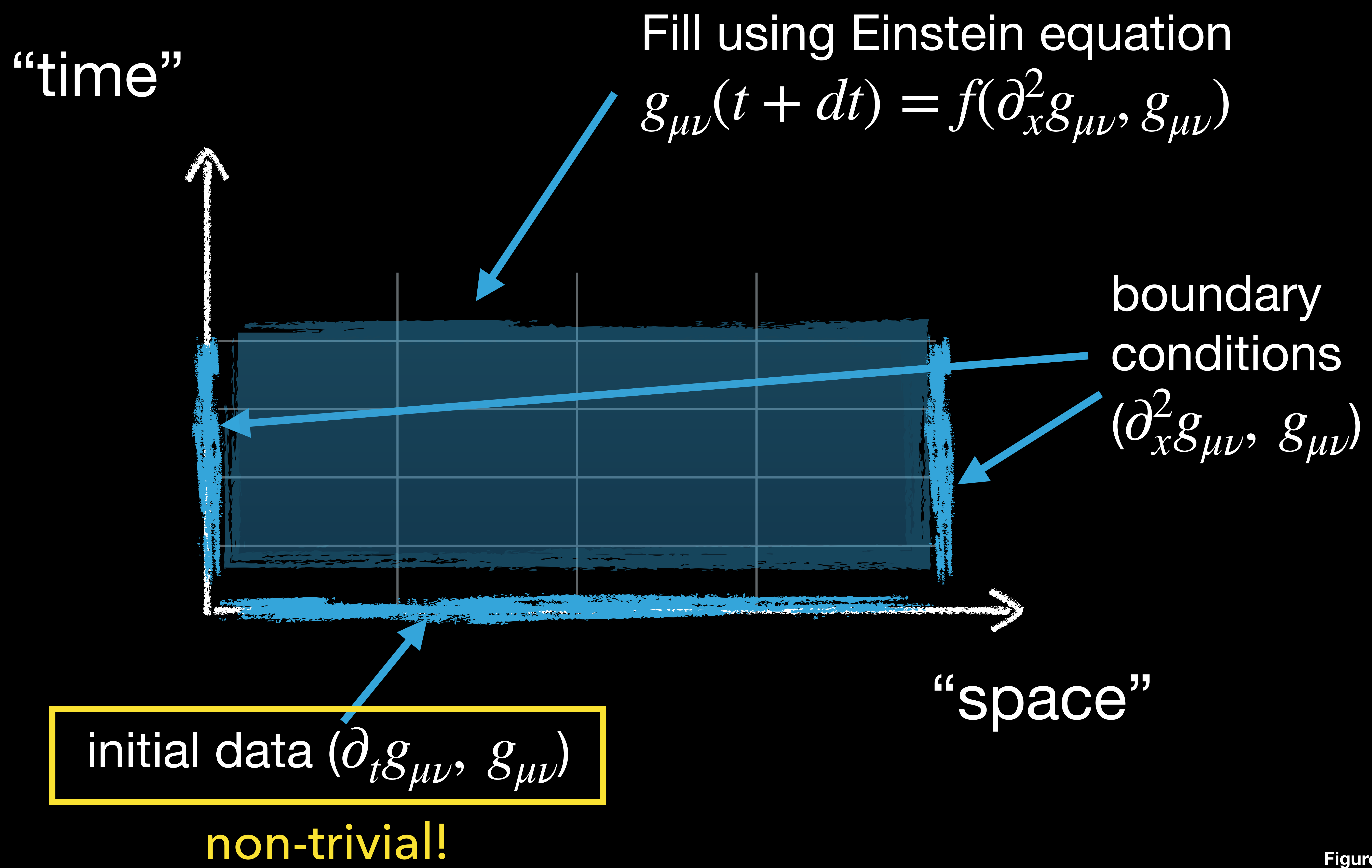
$$\partial_t g_{\mu\nu} = \ldots$$

Initial value form



Arnowitt, Deser, Misner
Baumgarte, Shapiro, Shibata, Nakamura

Solving GR on a computer:



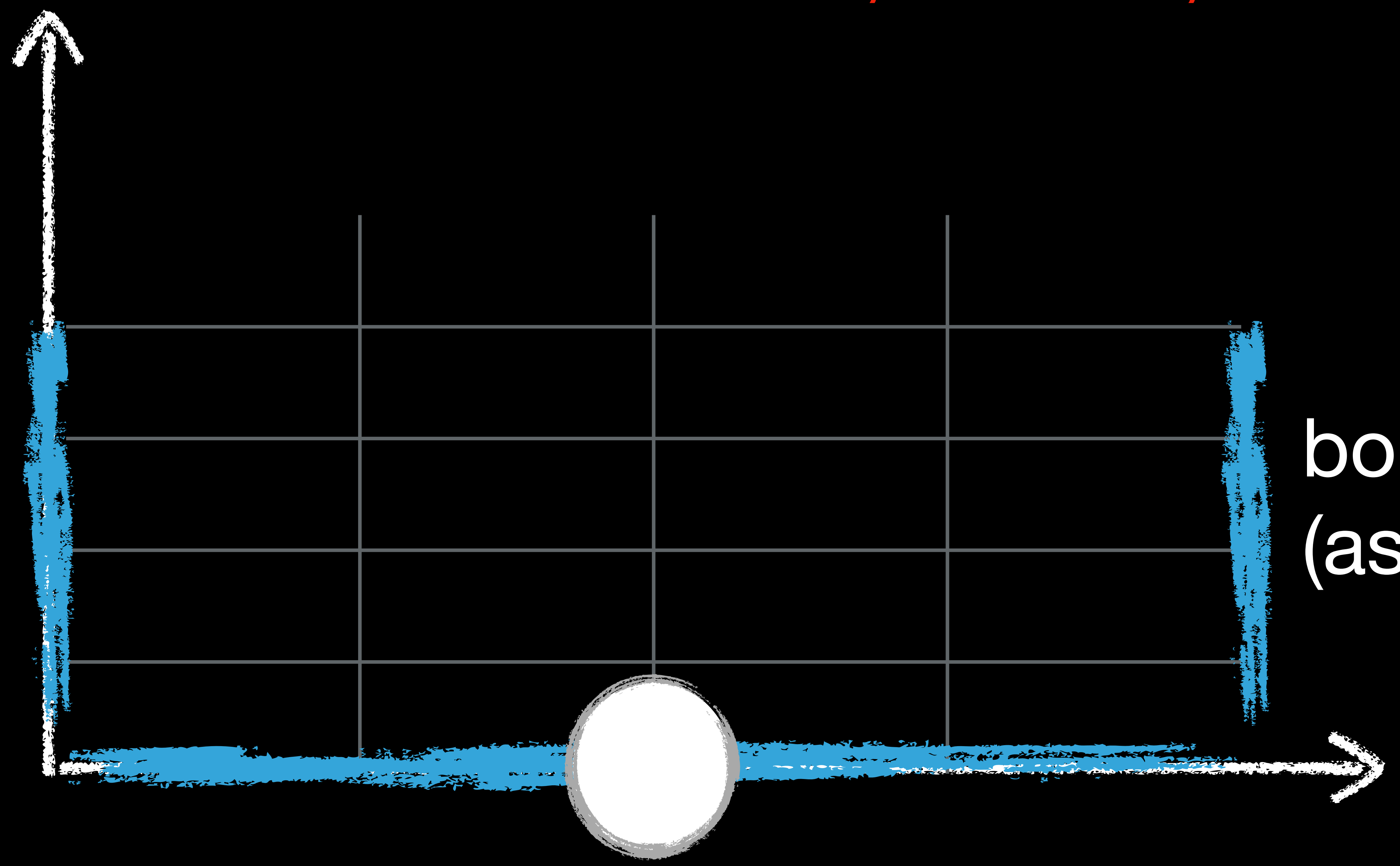
Figures from Katy Clough

Solving GR on a computer:

Fill using Einstein equation

$\partial_t g_{\mu\nu} = \partial_t^2 g_{\mu\nu} = 0$ (boring!)

“time”



boundary conditions (asymptotically flat)

“space”

initial data (Schwarzschild metric)

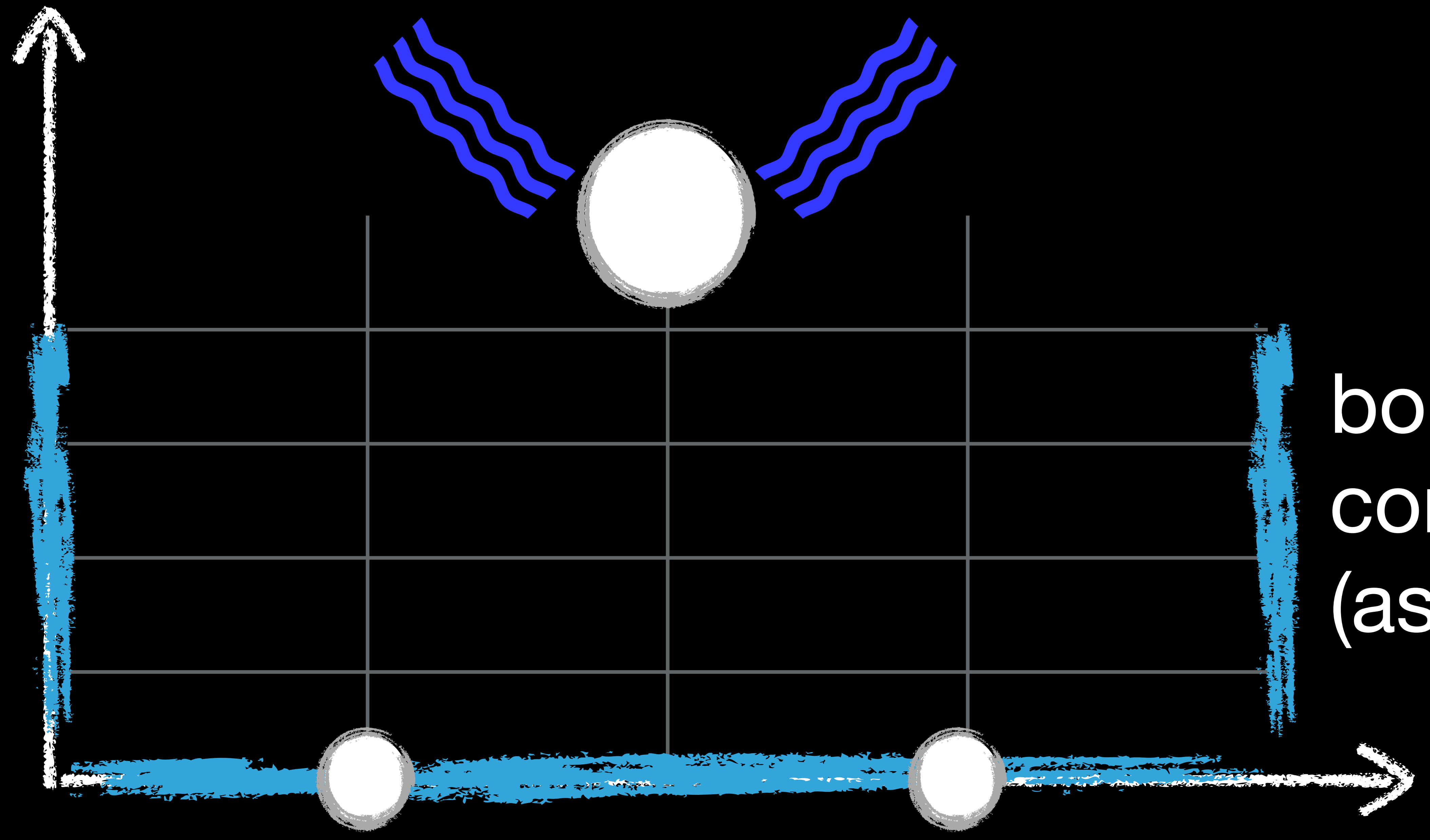
Figures from Katy Clough

Solving GR on a computer:

Fill using Einstein equation

$\partial_t g_{\mu\nu} \neq \partial_t^2 g_{\mu\nu} \neq 0$ (interesting!)

"time"



boundary conditions (asymptotically flat)

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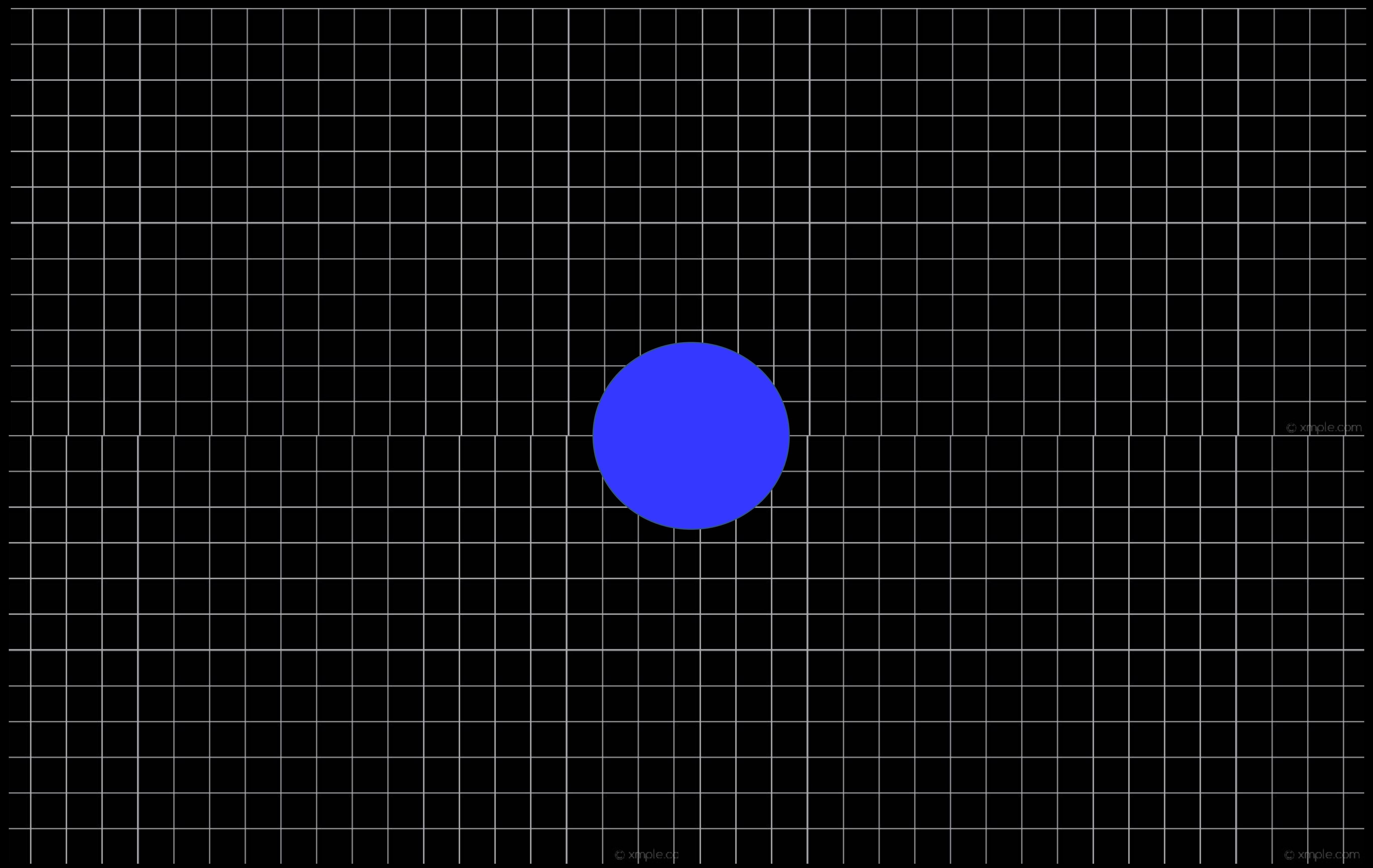
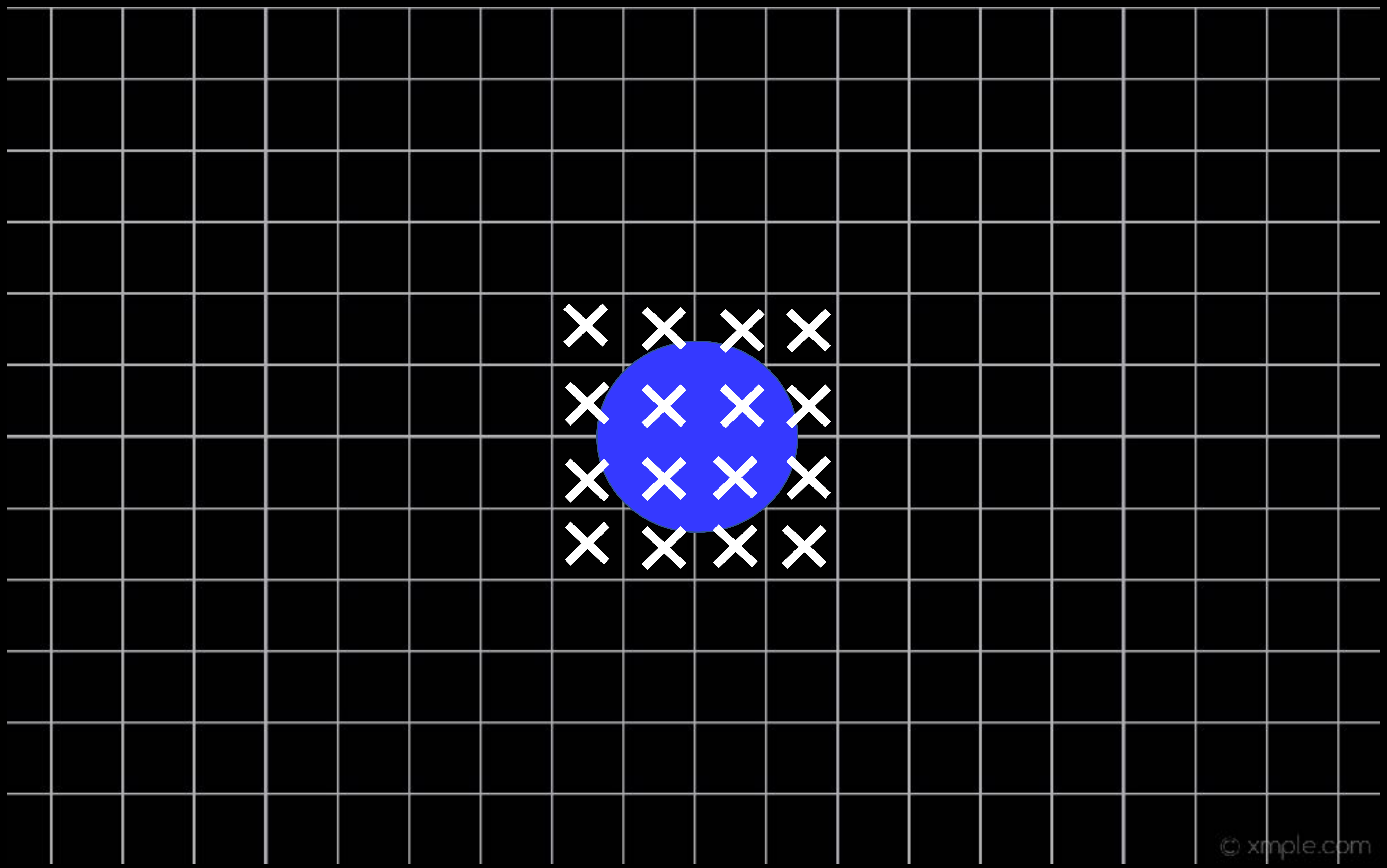
initial data (Schwarzschild metric)

Figures from Katy Clough

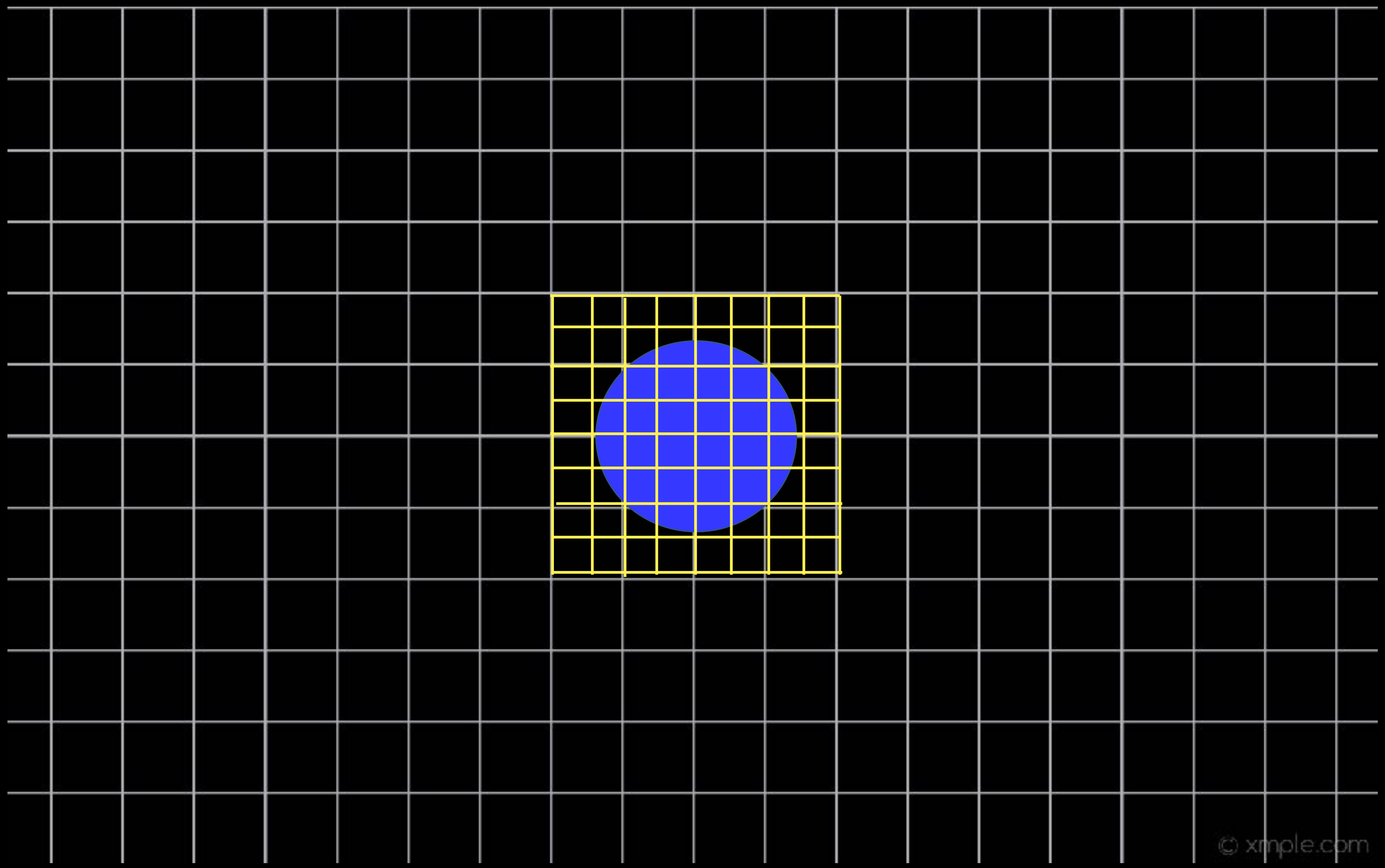
Adaptive Mesh Refinement



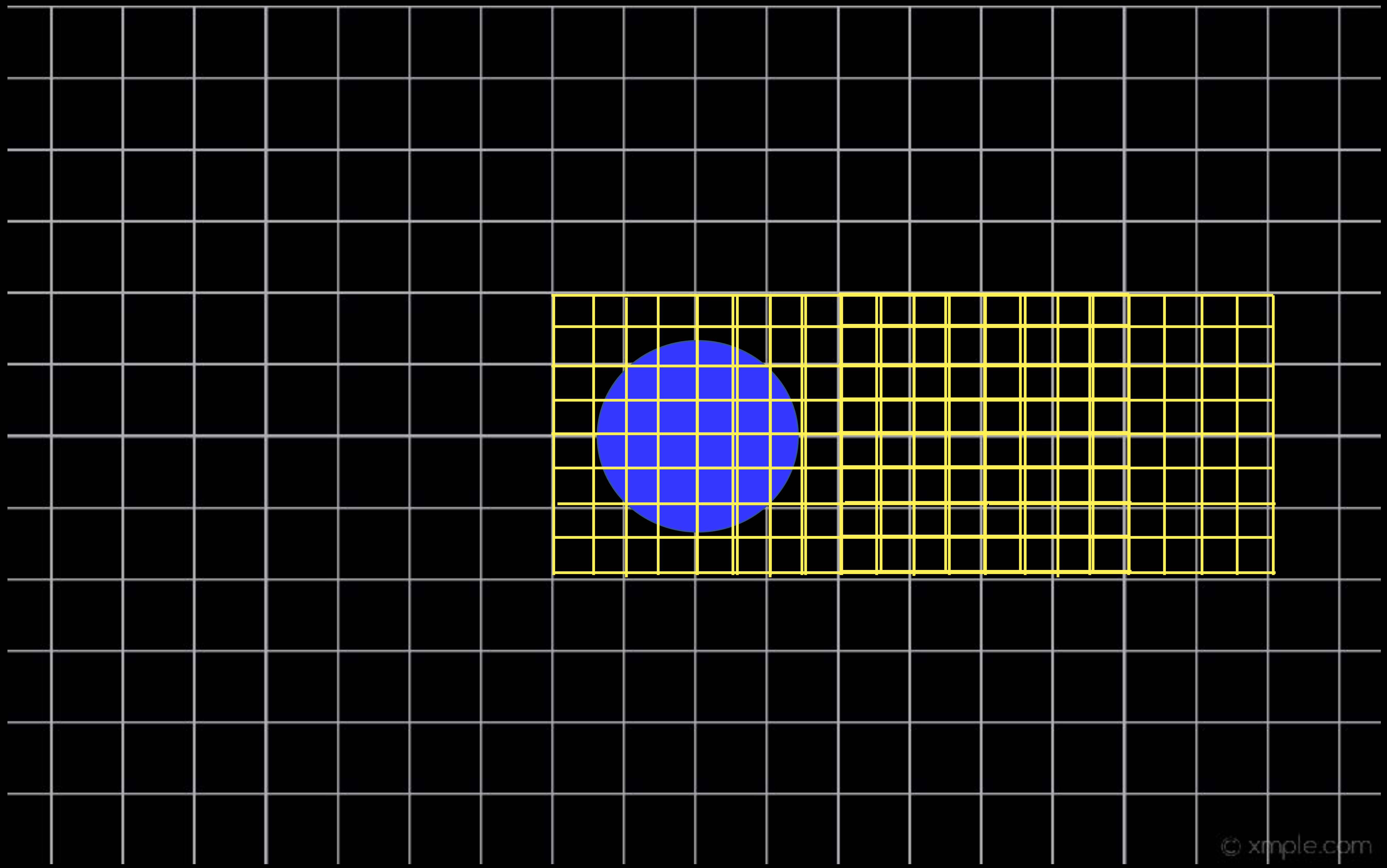
Adaptive Mesh Refinement



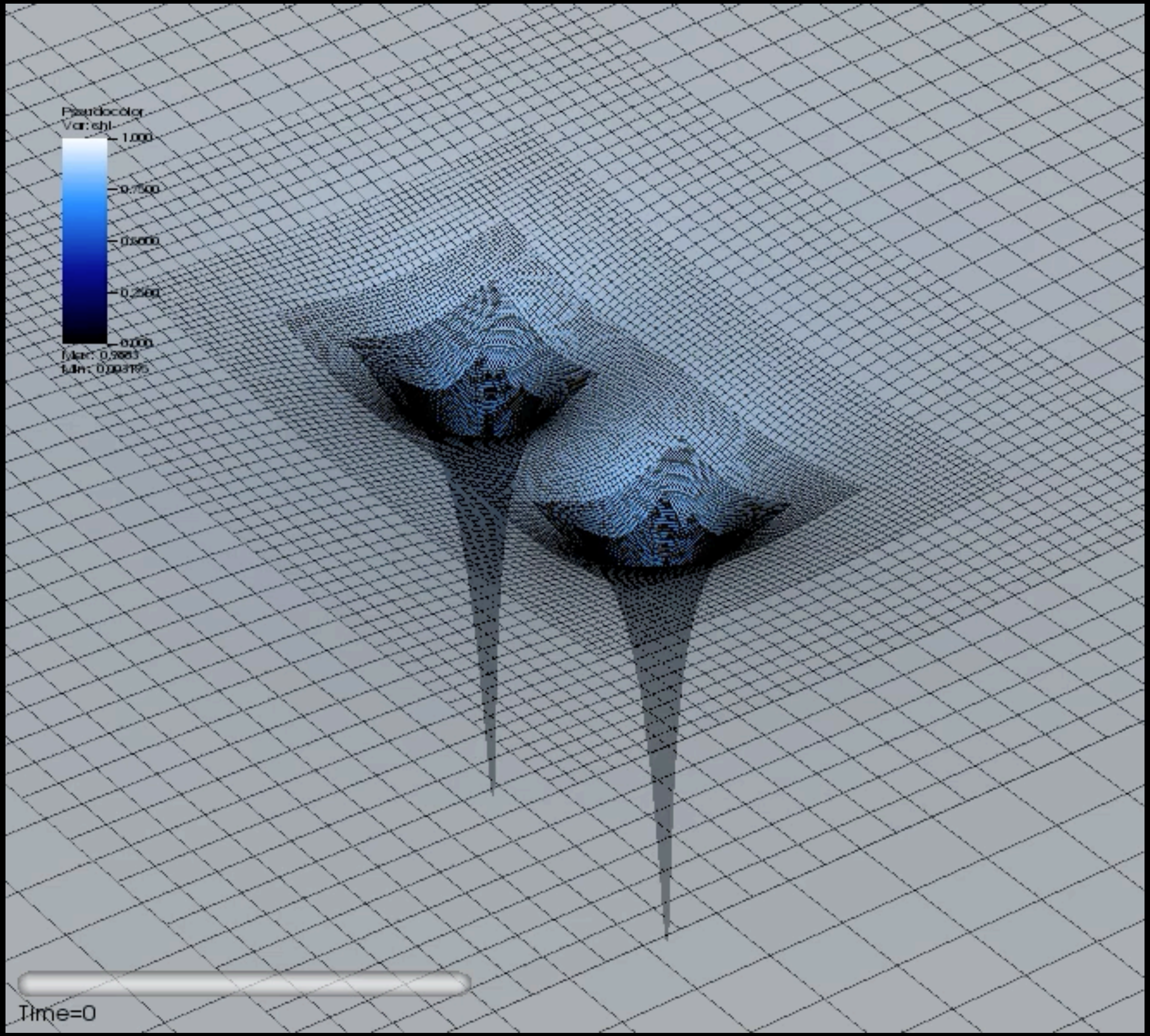
Adaptive Mesh Refinement



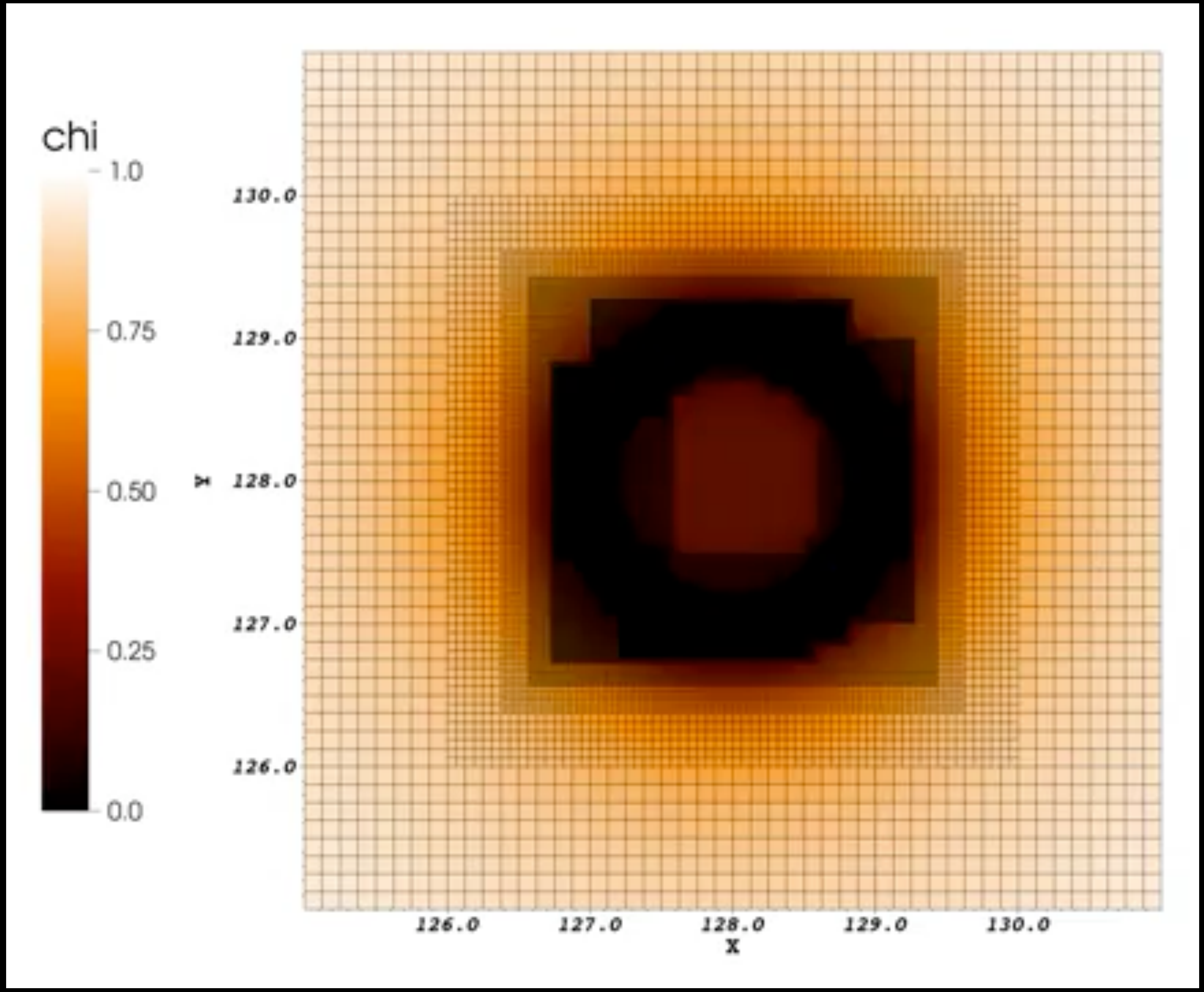
Adaptive Mesh Refinement



Adaptive Mesh Refinement



Binary black hole



Higher dimensional black ring

Numerical relativity with AMR

www.grtlcollaboration.org

- Open source
- Hybrid MPI/OpenMP
- Vectorised simd AVX512
- (Ported to GPUs)

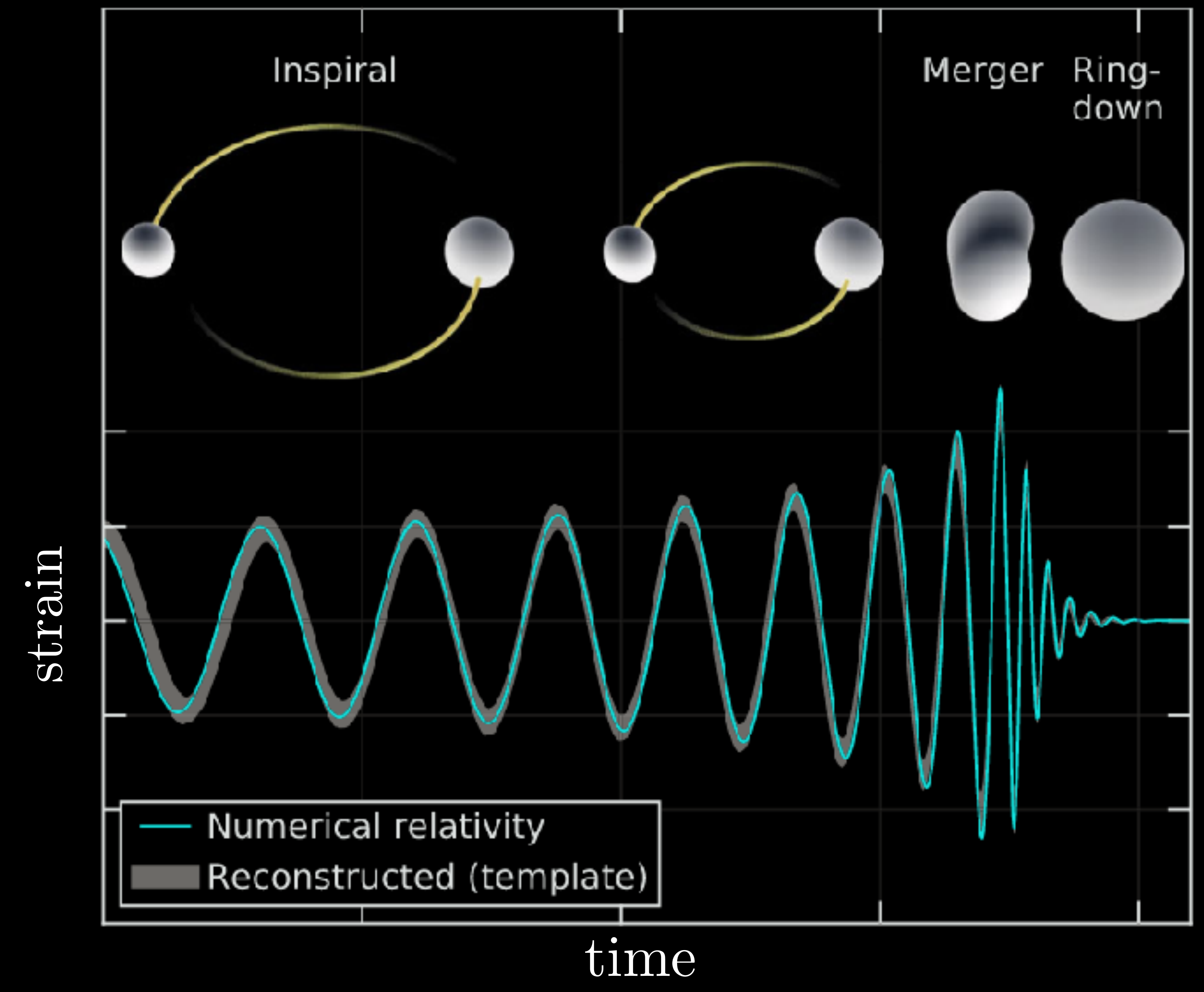
prerequisites:
gcc/intel, fortran, mpi, hdf5, blas, lapack

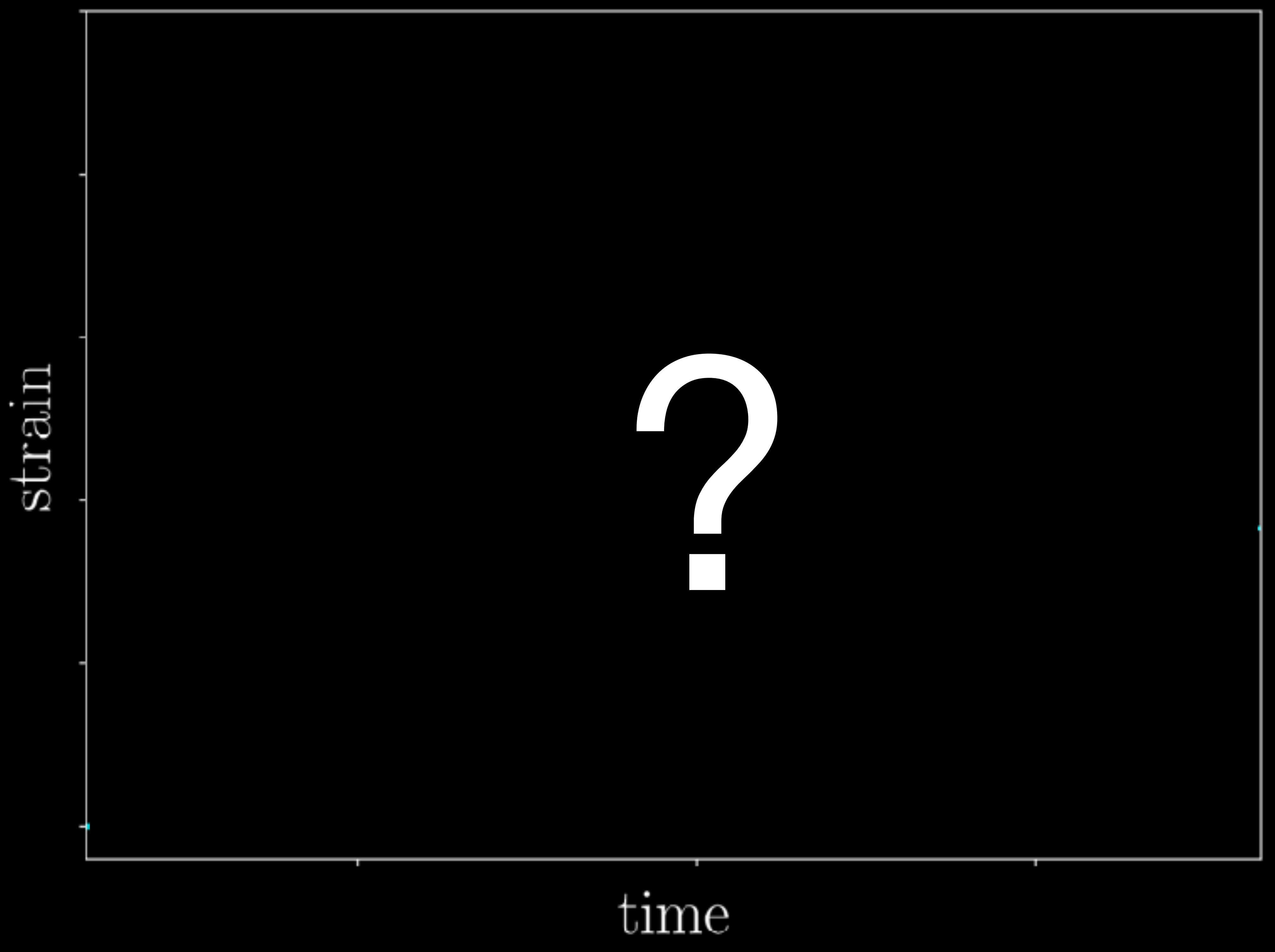
GRDzhadzha (Fixed background)

enGRenage (1D code in python)

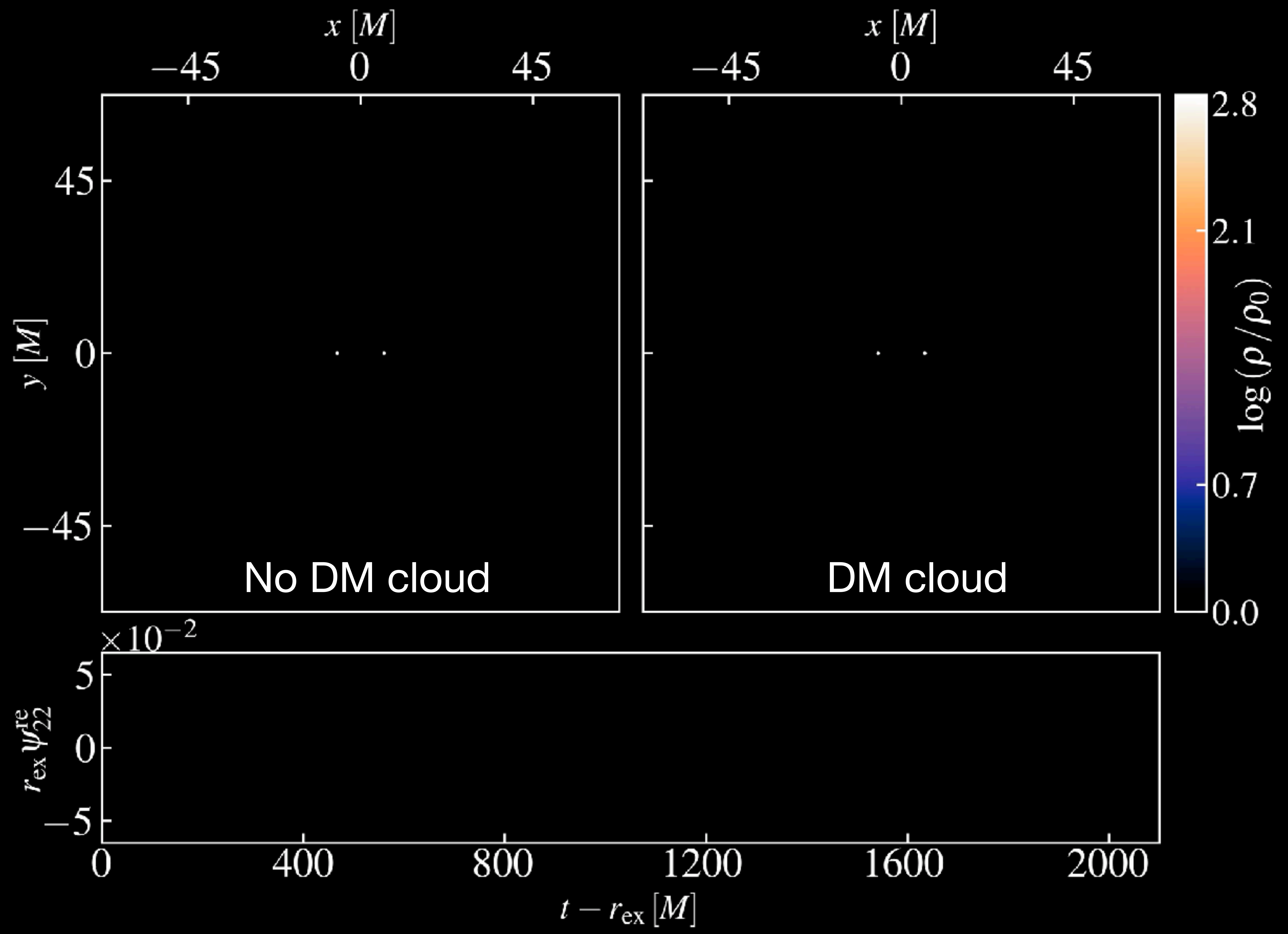
Some examples

Black Holes vs New Physics



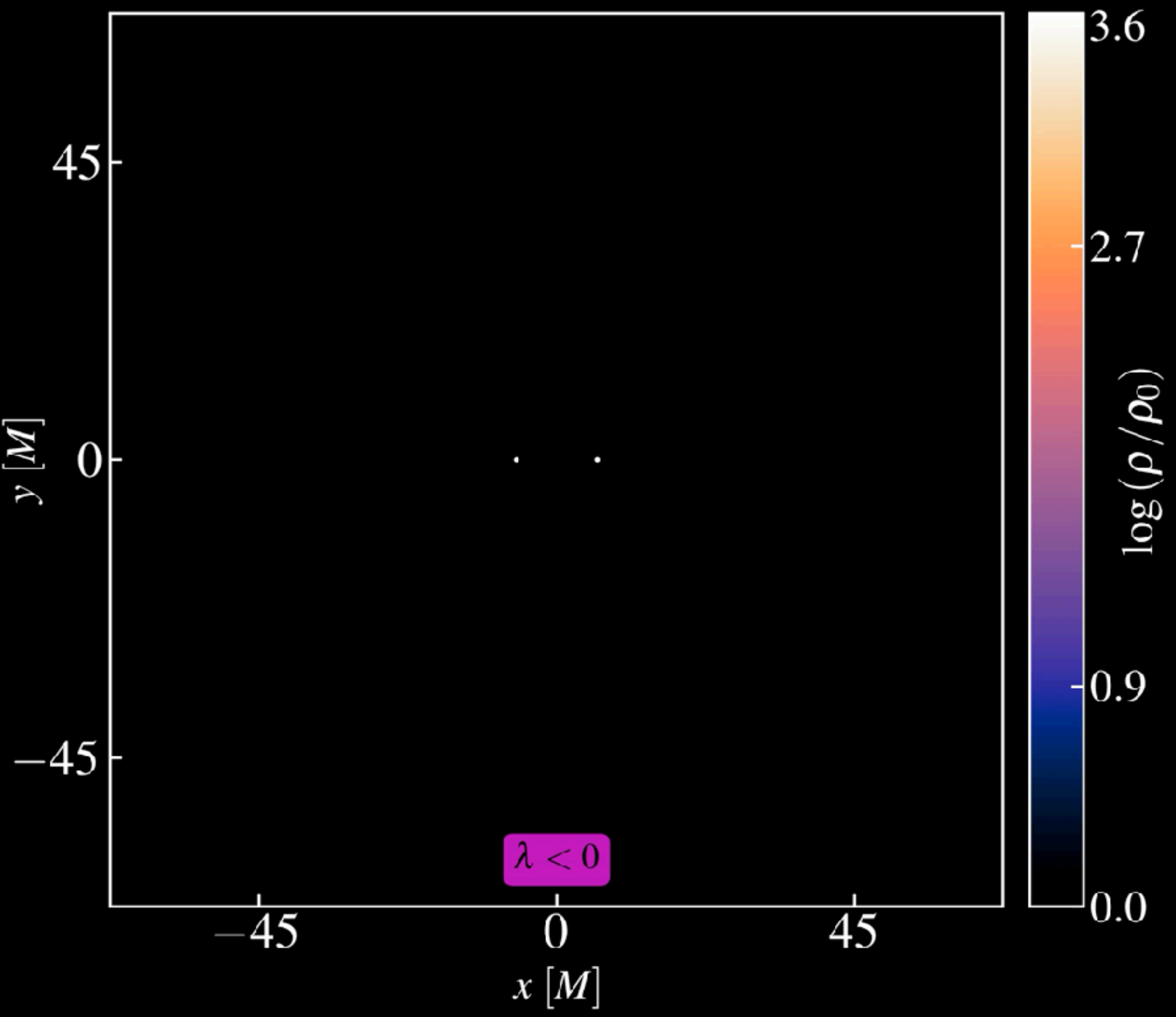


Dark Matter around black holes



Aurrekoetxea+ (PRL, 2024)

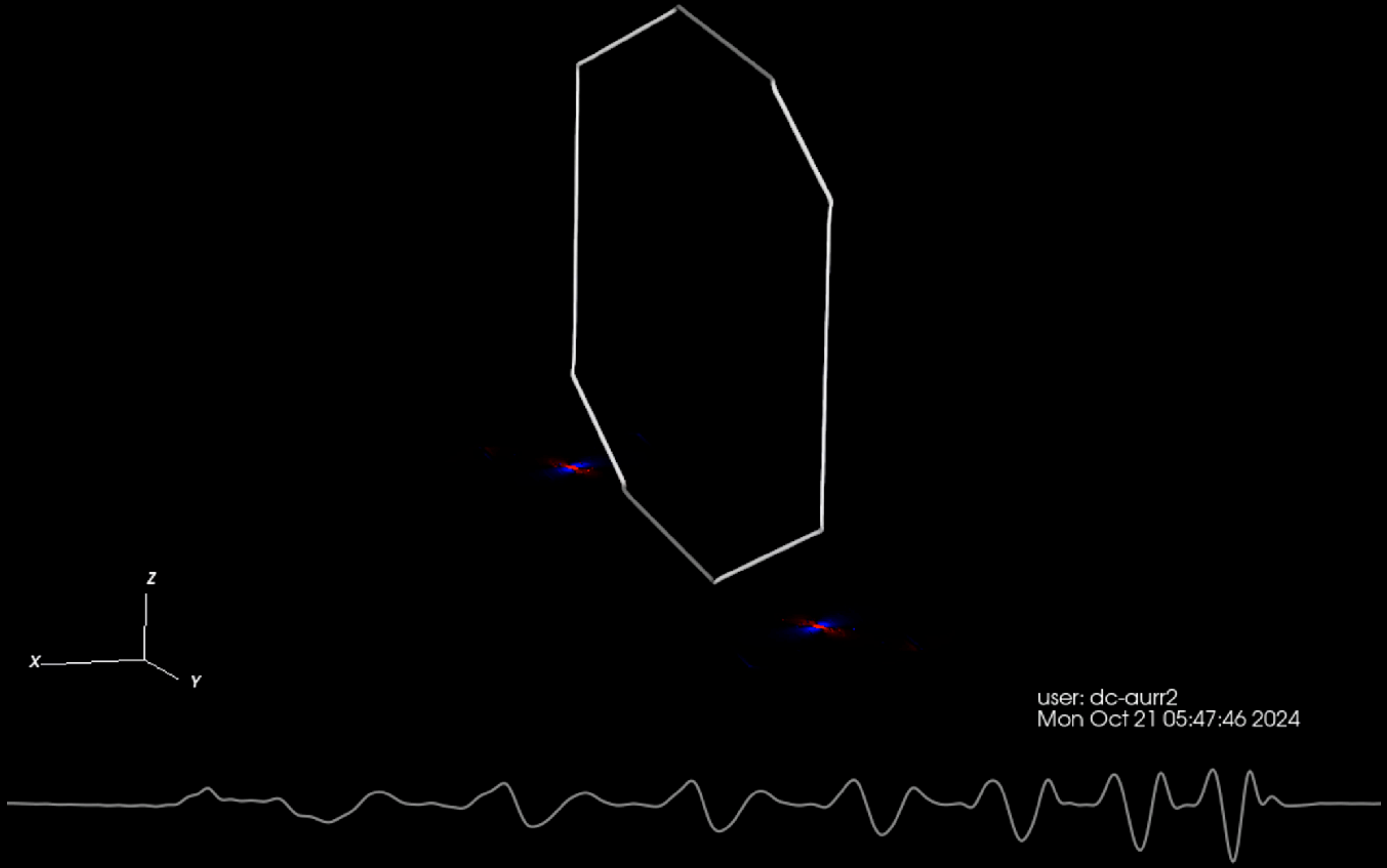
Dark Matter around black holes



Bosenova explosion

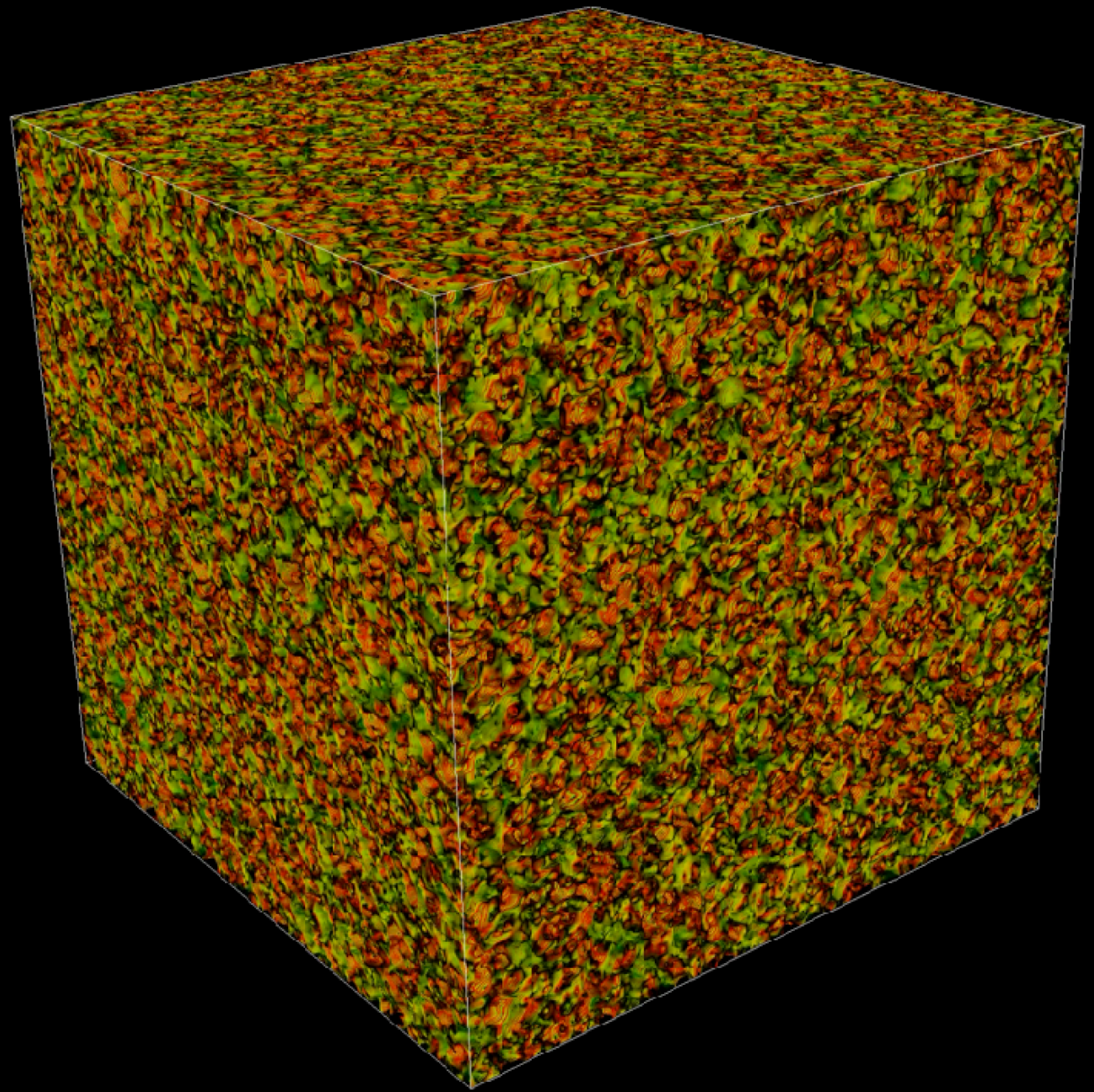
Aurrekoetxea+ (PRD, 2024)

GWs from Cosmic Strings



Aurrekoetxea+ (in preparation)

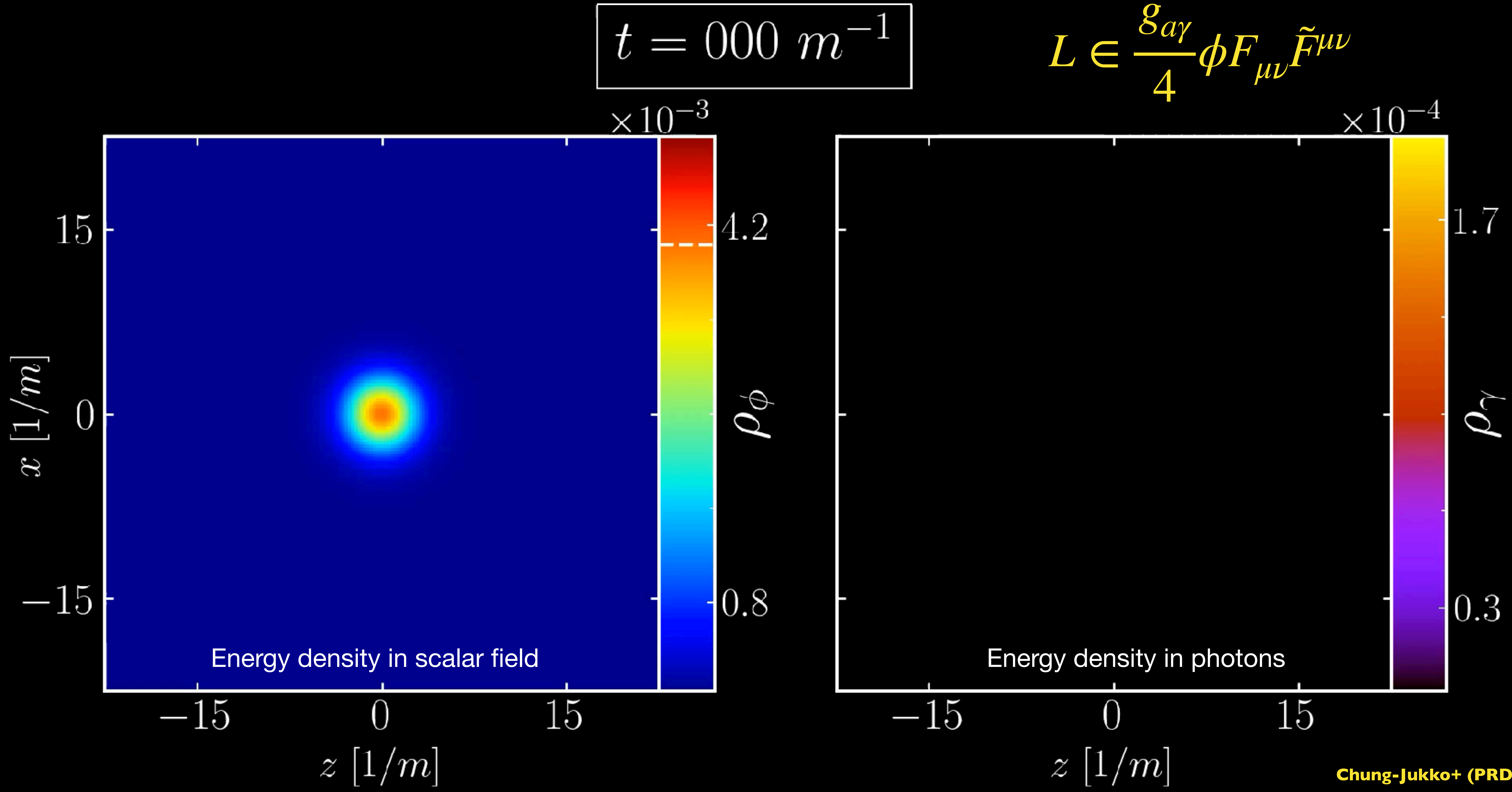
Formation of compact objects



Aurrekoetxea+ (PRD, 2023)

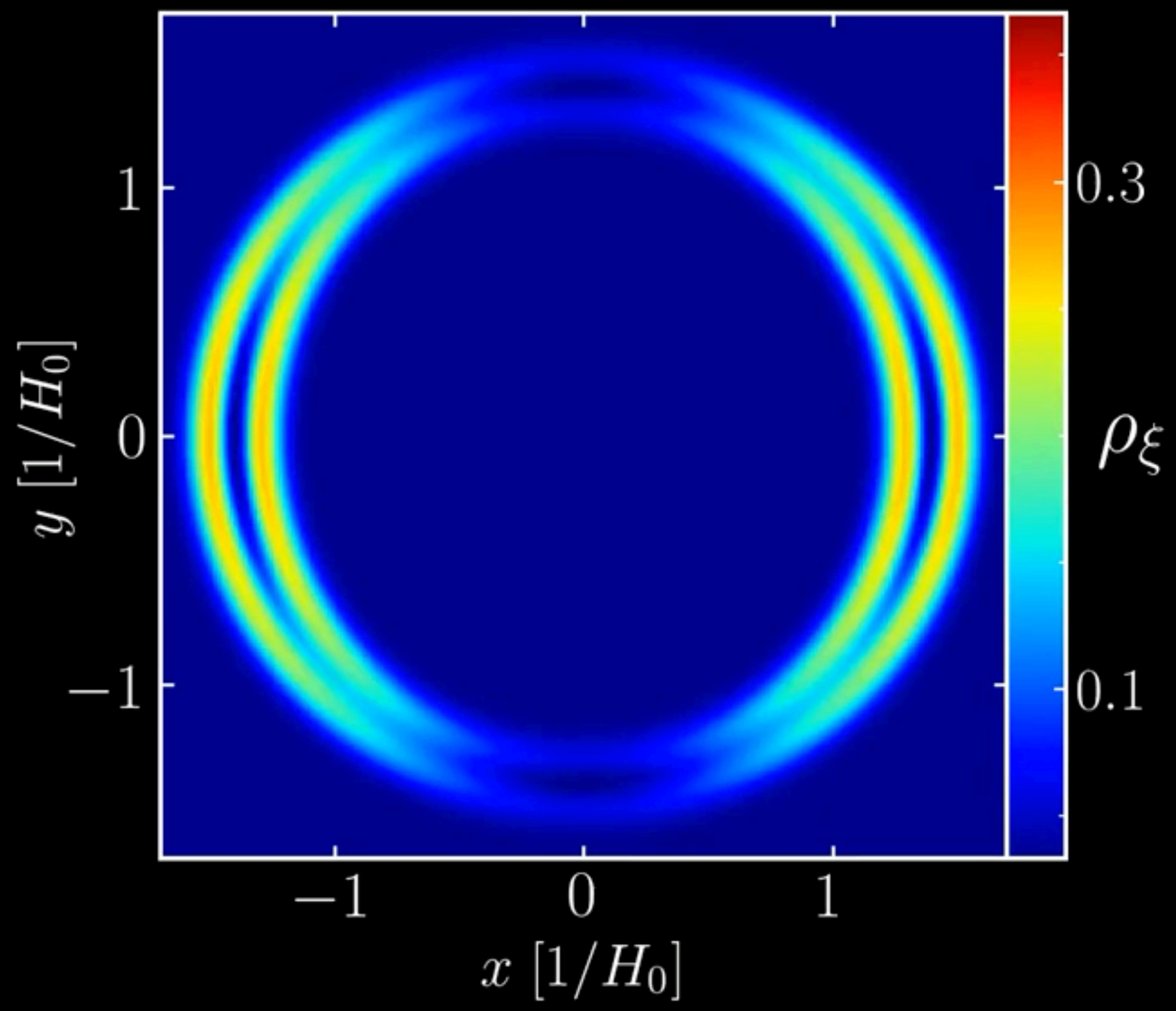
Axion Star Explosions

$$L \in \frac{g_{a\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$



Chung-Jukko+ (PRD, 2023)

Spinning Primordial Black Holes

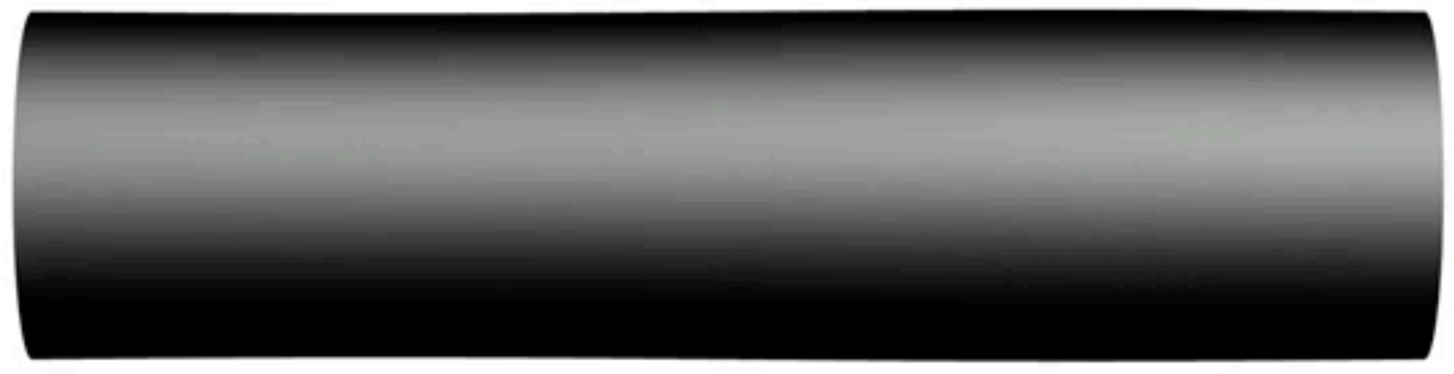


DeJong+ (JCAP, 2023)

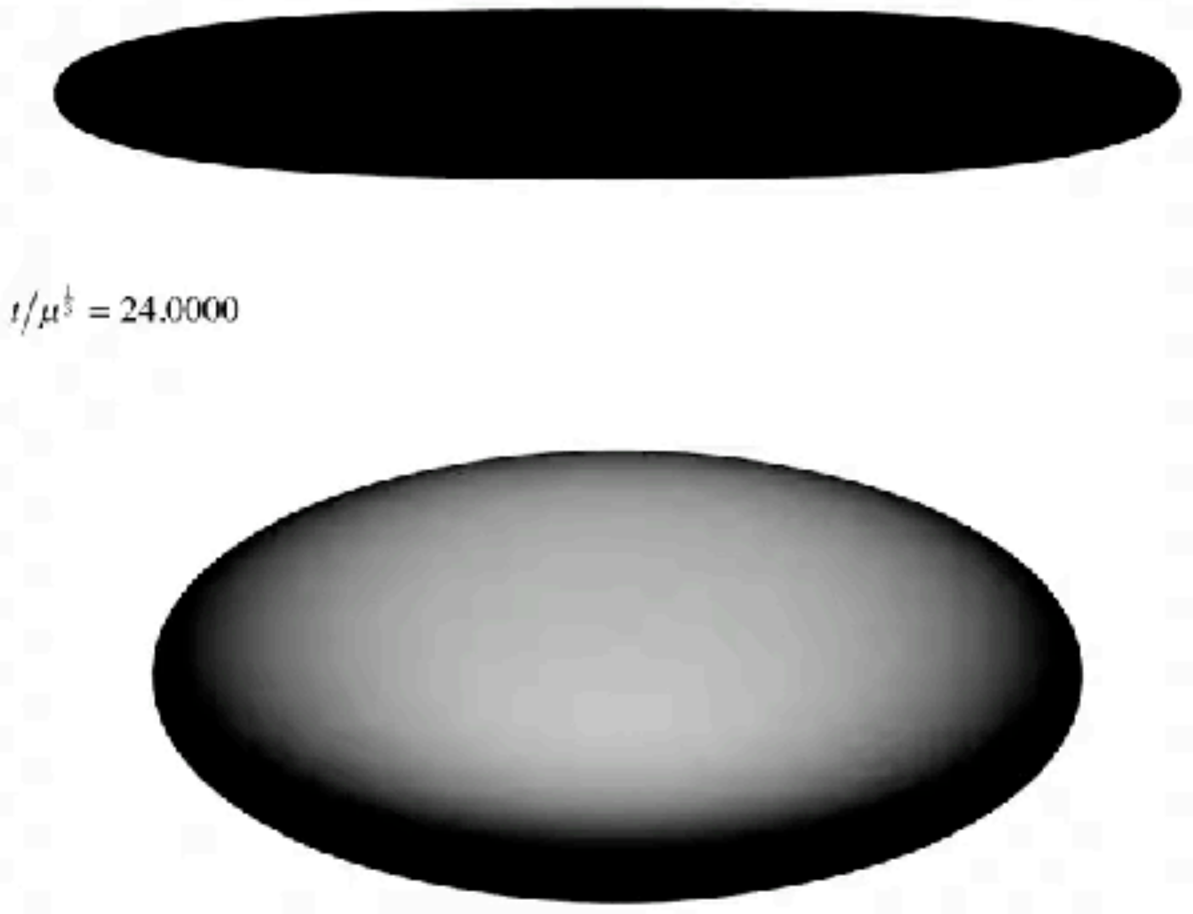
Weak Cosmic Censorship

t=0.00

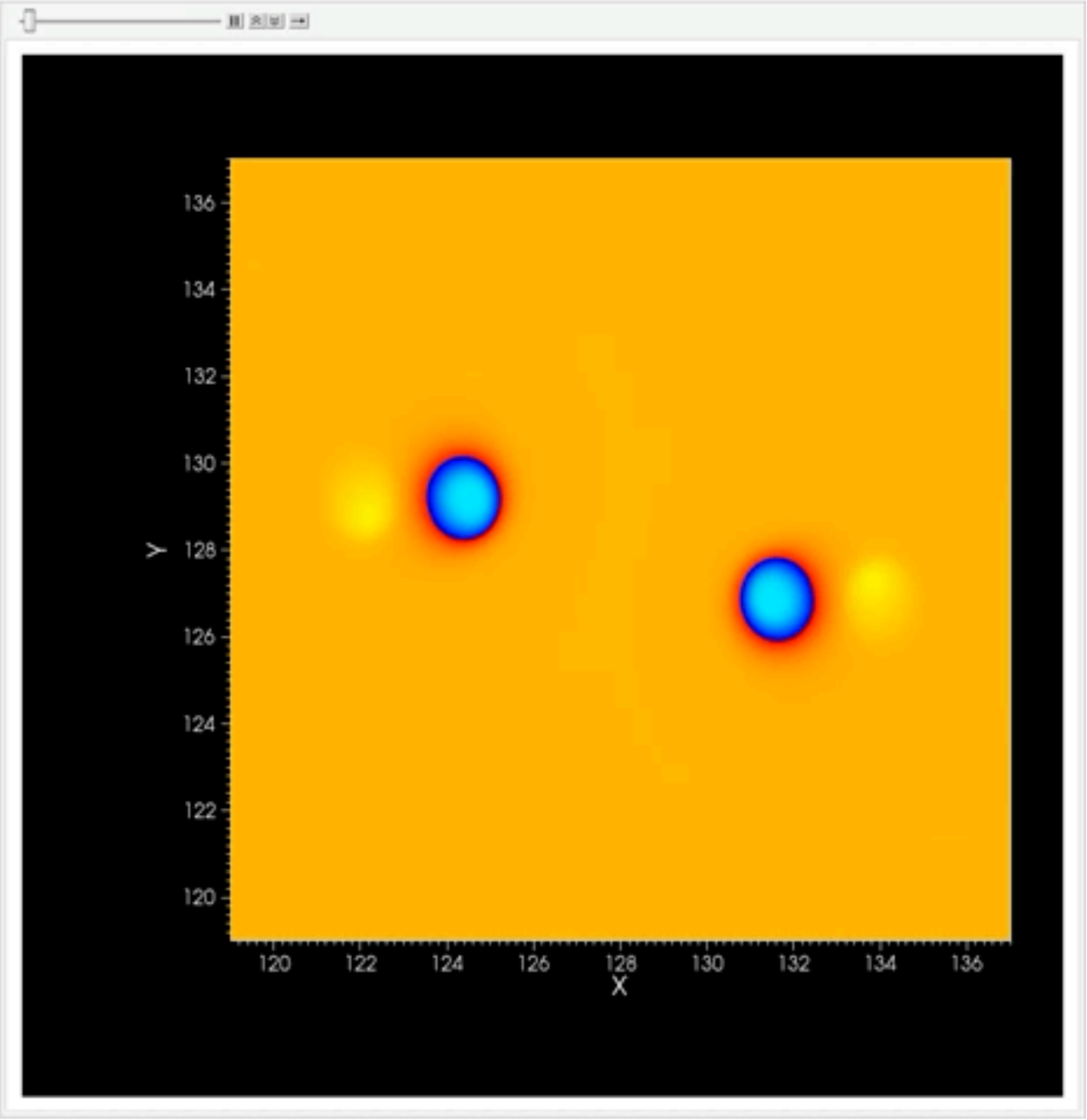
Black string



Myers-Perry BH

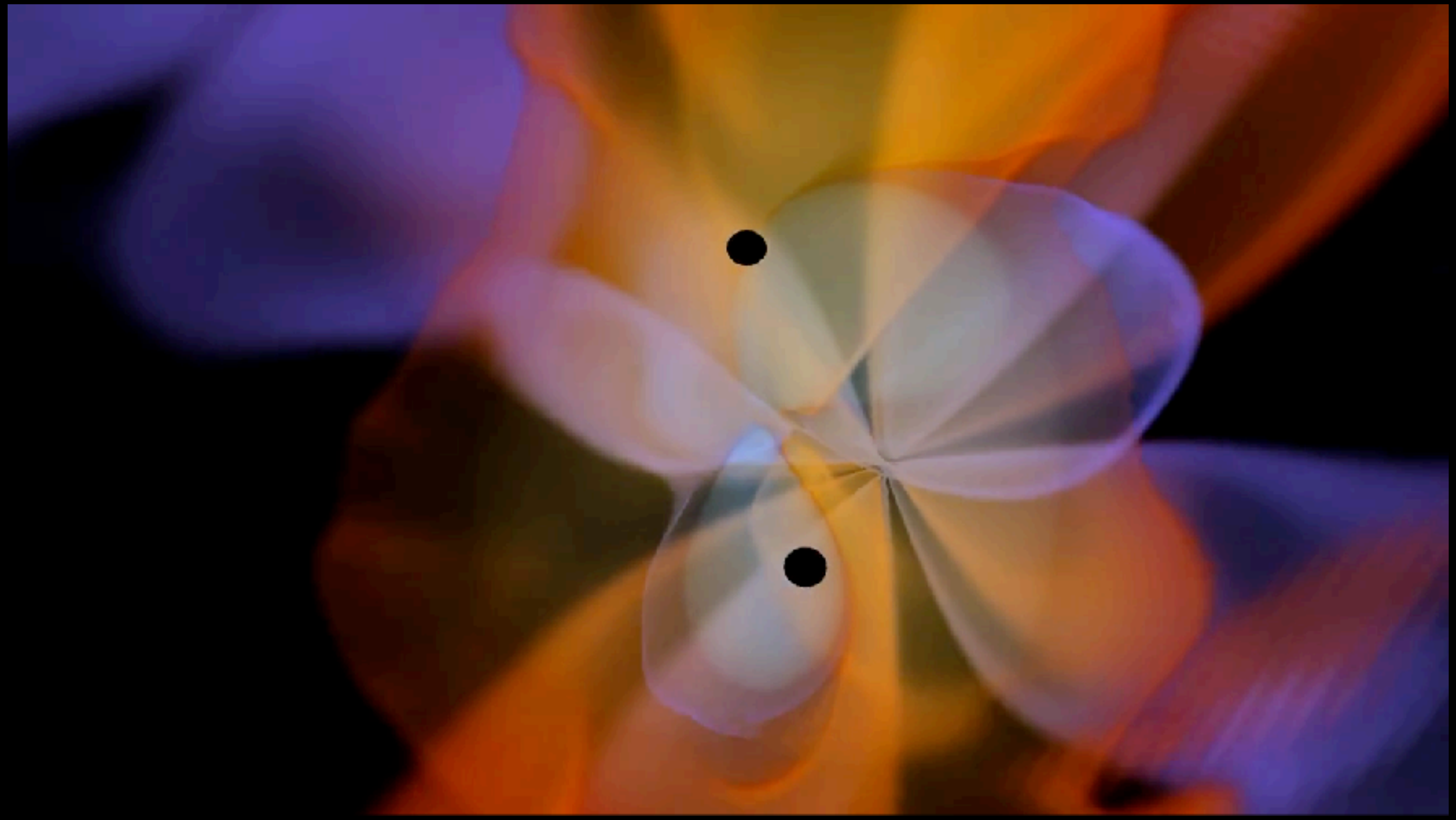


BH collisions in higher dimensions



Figueras+ (2017, 2020, 2022)

Numerical relativity with AMR



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