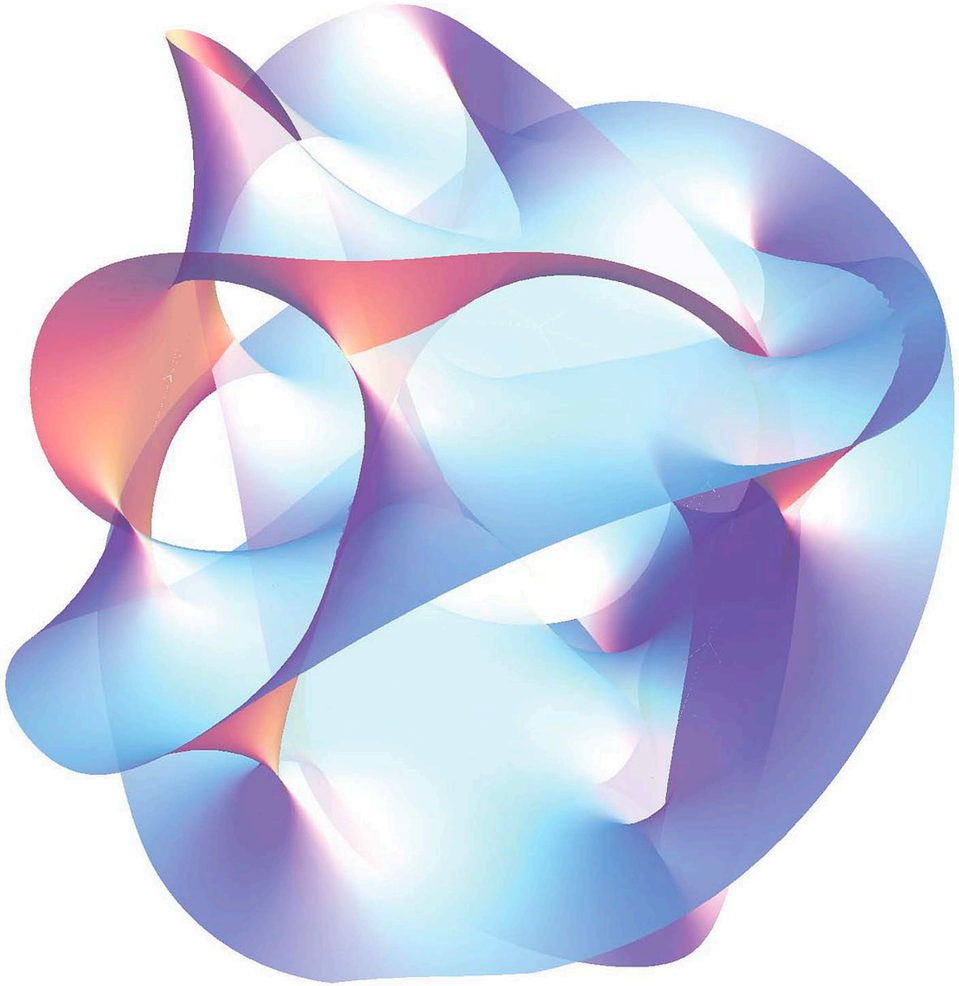
Computing with Calabi-Yaus

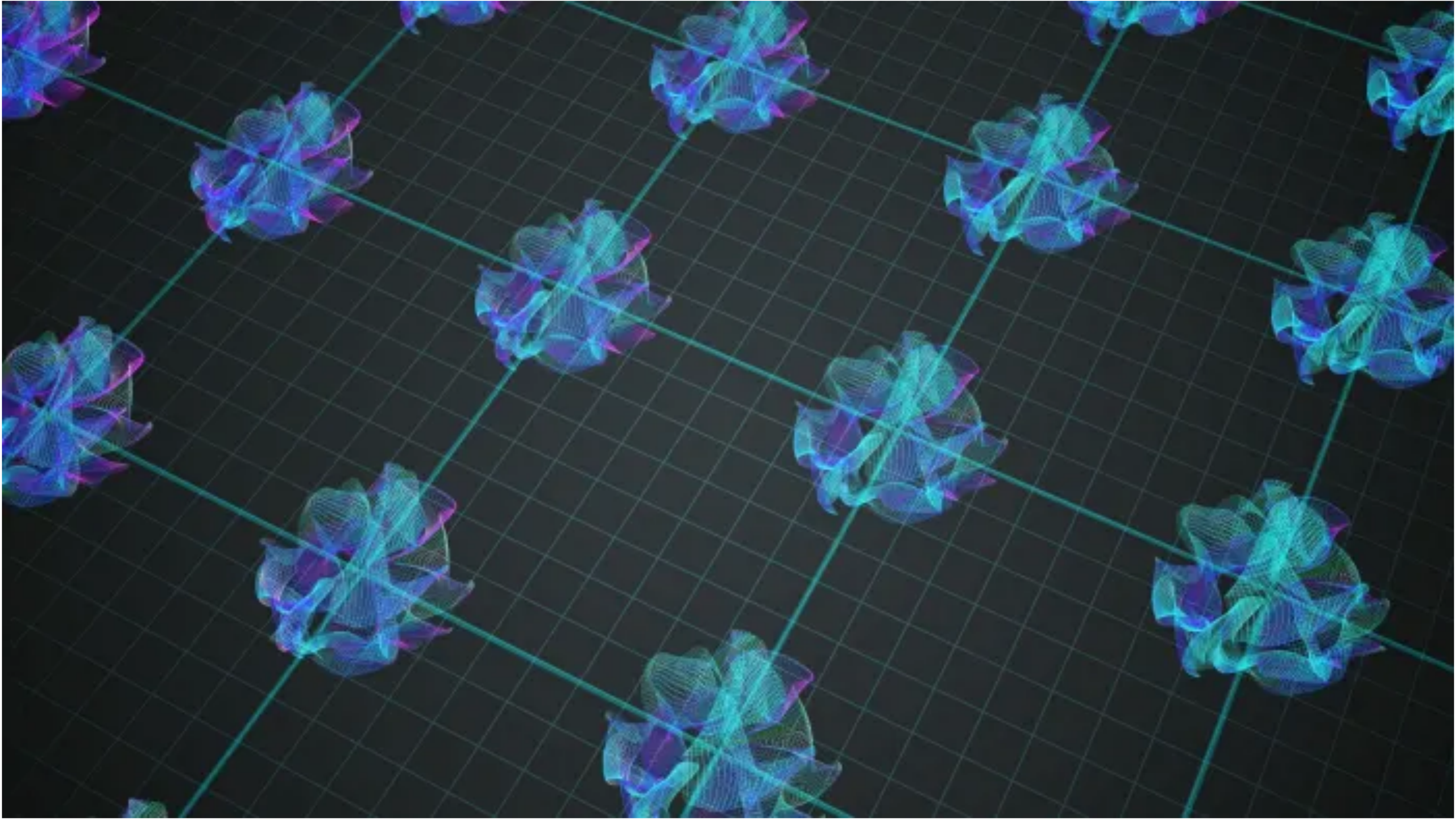
Richard Nally
subMIT User Meeting
Mar 11, 2025

Based on 2406.13751 w/ L. McAllister, J. Moritz, A Schachner
+ WIP with F. Abbasi and W. Taylor

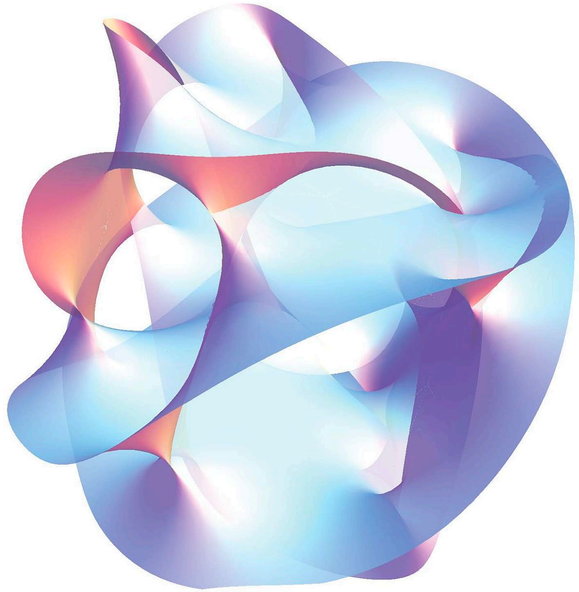
String Compactifications

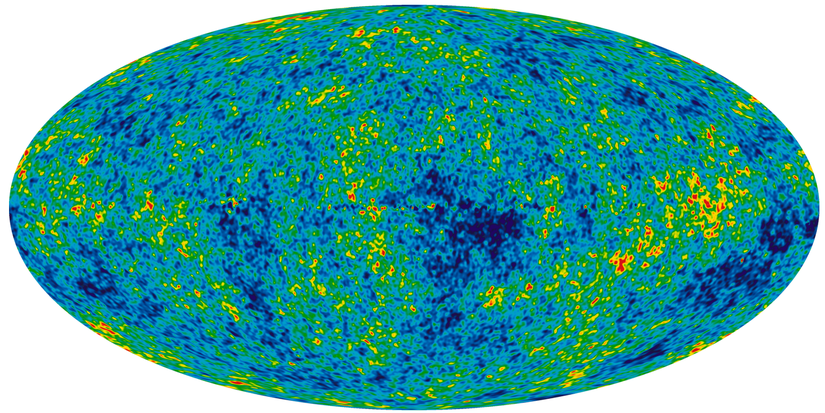
- Basic context: IIB string theory, a theory of quantum gravity in ten dimensions
- Compactification: solutions where six of the ten dimensions are compact and small
- Take the compact dimensions to be Calabi-Yau, special shapes that solve 10D Einstein equations
- Doing this gives an effective theory of supersymmetric gravity in four dimensions





String theory is a machine that turns six-dimensional shapes into universes!



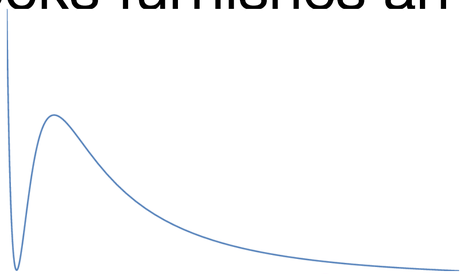


Obvious question: Do we come from one of these shapes?

The String Landscape

- String theory gives you a few basic types of building blocks:
  - Compact manifolds
  - Fluxes for 10D gauge fields
  - Dynamical branes

- Combining these ingredients gives the famous “landscape” of string theory
- Each choice of these building blocks furnishes an effective theory of matter coupled to gravity



- How much of this landscape can we explore? What properties do these effective field theories have?

String Pheno

- In particular, are there vacua of string theory that reproduce the features of our universe?
  - Particle physics?
  - Cosmology?

String Pheno

- In particular, are there vacua of string theory that reproduce the features of our universe?
  - Particle physics?
  - Cosmology?

String Pheno

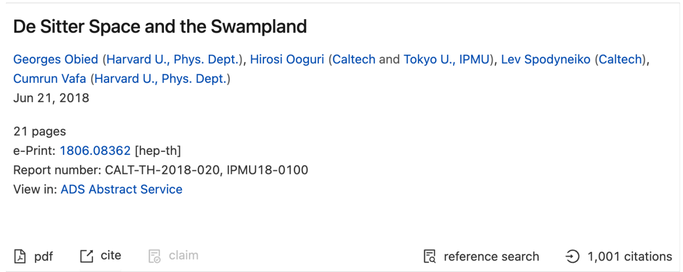
- In particular, are there vacua of string theory that reproduce the features of our universe?
  - Particle physics?
  - Cosmology?
- Since 1998, we have known the acceleration of the universe is accelerating
  - [Supernova Cosmology Team ’98, High-z Supernova Search Team ’98]
- Simplest explanation: empty space has an energy density, called the cosmological constant

$$G_{\mu\nu}+\Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$$
$$\Lambda \sim + 10^{-120} M_p^4$$

- Are there solutions of string theory with a positive cosmological constant?

de Sitter Vacua?

- Are there solutions of string theory with a positive cosmological constant?
  - These are called de Sitter (dS) vacua
- Nobody knows! Constructing them has been a major goal of string pheno for 20 years, but so far nobody has succeeded
- In fact, it is so hard that it has been conjectured that they don't exist!



De Sitter Space and the Swampland

Georges Obied (Harvard U., Phys. Dept.), Hirosi Ooguri (Caltech and Tokyo U., IPMU), Lev Spodyneiko (Caltech), Cumrun Vafa (Harvard U., Phys. Dept.)
Jun 21, 2018

21 pages
e-Print: 1806.08362 [hep-th]
Report number: CALT-TH-2018-020, IPMU18-0100
View in: ADS Abstract Service

pdf cite claim reference search 1,001 citations

- Why is it so hard? String theory is really, really good at making universes with supersymmetry, but dS can't be supersymmetric
- Need to break supersymmetry in a controlled way!

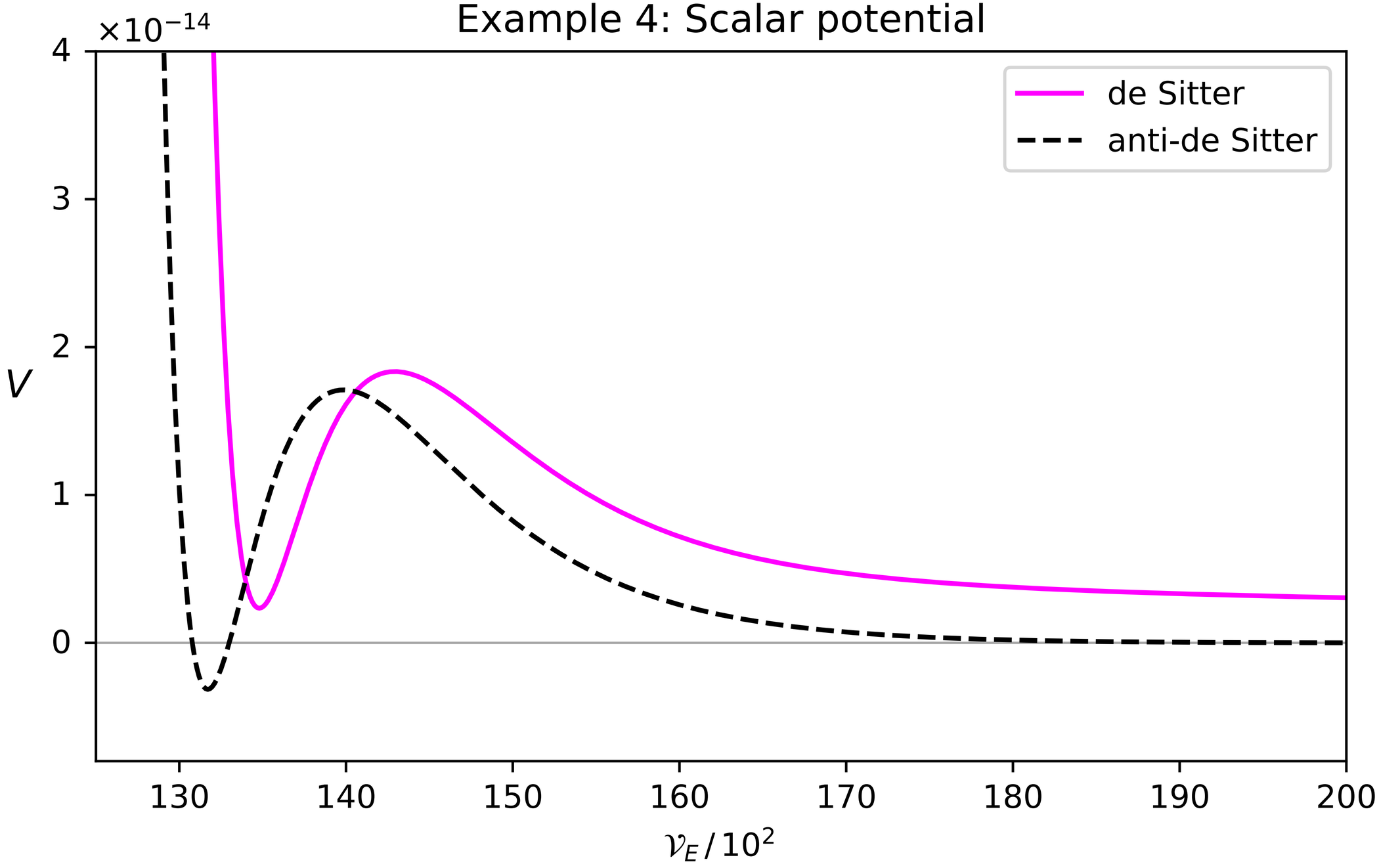
dS Candidates

- We found examples!

$h^{2,1}$	$h^{1,1}$	M	K'	g_s	W_0	g_sM	$\lvert z_{\rm cf}\rvert$	V_0
8	150	16	$\frac{26}{5}$	0.0657	0.0115	1.051	2.822×10^{-8}	$+1.937\times10^{-19}$
8	150	16	$\frac{93}{19}$	0.0571	0.00490	0.913	7.934×10^{-9}	$+1.692\times10^{-20}$
8	150	18	$\frac{40}{11}$	0.0442	0.0222	0.796	8.730×10^{-8}	$+4.983\times10^{-19}$
5	93	20	$\frac{17}{5}$	0.0404	0.0539	0.808	1.965×10^{-6}	$+2.341\times10^{-15}$
5	93	16	$\frac{29}{10}$	0.0466	0.0304	0.746	8.703×10^{-7}	$+2.113\times10^{-15}$

- First ever examples of candidate KKLT de Sitter vacua! [RAN et al ’24]
- Our approach: large scale computation
- Final dataset: ~50 core years on RedCloud

red cloud

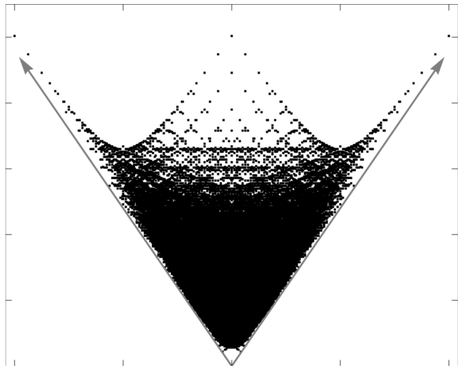

Example 4: Scalar potential
×10⁻¹⁴
4
3
2
1
0
V
de Sitter
anti-de Sitter
130
140
150
160
170
180
190
200
$\nu_E / 10^2$

Why now?

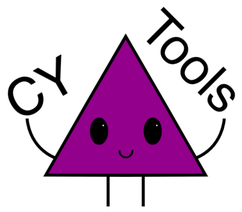
- KKLT proposal was 20 years ago. Why did it take so long?
- Tremendous progress has been made in recent years in actually building string vacua, at scale:
  - Efficient computation of topological data of CY3s [Demirtas et al ’22]
  - … and of GV invariants, i.e. period vectors [Demirtas et al ’23]
  - Large-scale processing of choices of flux vectors [RAN et al ’24, Dubey et al ’23]
  - Actually doing Kahler moduli stabilization [Demirtas et al ’21]
- Results come from harmoniously combining all of the above, plus others
- Could never have been done five years ago, much less 20
- String theory is becoming computational!

Toric Hypersurface CY3s

[Candelas et al '07]



- Key tool: practical computations with particular Calabi-Yaus
- Fix a 4D reflexive polytope Δ
  - Integer polytope = convex hull of a finite set of integer points in $\mathbb{R}^d$
  - Reflexive = dual polytope Δ° is also an integer polytope
  - [Kreuzer-Skarke '00]: exactly 473,800,776 4d reflexive polytopes
- [Batryev '94]: Sufficiently nice triangulations of Δ define a Calabi-Yau!
- Main appeal: geometry of CY3 is encoded by combinatorics of polytope, so you can easily compute topological data
  - Recently: CYTools, purpose built software for toric hypersurface CY3s [Demirtas et al '22, '23]

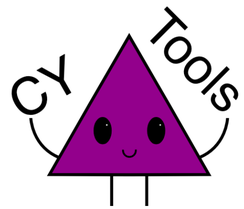


CYTools

- CYTools, purpose built software for toric hypersurface CY3s [Demirtas et al ’22, ’23]
  - Python library written by M. Demirtas, A. Rios-Tascon, N. MacFadden &c
  - Faster than general purpose math software, e.g. Sage, Mathematica
  - Dependencies: ORTools, flint, ~~TOPCOM~~, MOSEK, etc…

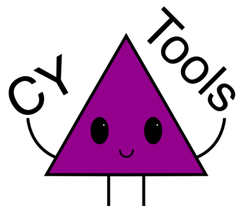
CYTools

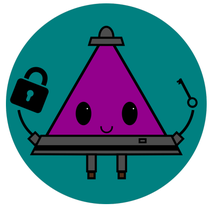
- CYTools, purpose built software for toric hypersurface CY3s [Demirtas et al ’22, ’23]
  - Python library written by M. Demirtas, A. Rios-Tascon, N. MacFadden &c
  - Faster than general purpose math software, e.g. Sage, Mathematica
  - Dependencies: ORTools, flint, ~~TOPCOM~~, MOSEK, etc…
- Comes in two flavors:
  - Public version
    - cy.tools or https://github.com/LiamMcAllisterGroup/cytools
    - Computes topological invariants of toric hyper surface CY3s
    - Singularity image easily installed on subMIT



CYTools

- CYTools, purpose built software for toric hypersurface CY3s [Demirtas et al ’22, ’23]
  - Python library written by M. Demirtas, A. Rios-Tascon, N. MacFadden &c
  - Faster than general purpose math software, e.g. Sage, Mathematica
  - Dependencies: ORTools, flint, ~~TOPCOM~~, MOSEK, etc…
- Comes in two flavors:
  - Public version
    - cy.tools or https://github.com/LiamMcAllisterGroup/cytools
    - Computes topological invariants of toric hyper surface CY3s
    - Singularity image easily installed on subMIT
  - Private version
    - Software tools specifically for ongoing projects in the Cornell group
    - Built specifically for RedCloud, not yet working on subMIT





Scanning Over Calabi-Yaus

- As we aim to build the string landscape, our task is to scan over CY3s
- Recall that we build CY3s from triangulations of polytopes
- Logic flow:

Scanning Over Calabi-Yaus

- As we aim to build the string landscape, our task is to scan over CY3s
- Recall that we build CY3s from triangulations of polytopes
- Logic flow:
  - Pick some polytopes

Scanning Over Calabi-Yaus

- As we aim to build the string landscape, our task is to scan over CY3s
- Recall that we build CY3s from triangulations of polytopes
- Logic flow:
  - Pick some polytopes
  - For each polytope on the list, make all of the Calabi-Yaus

Scanning Over Calabi-Yaus

- As we aim to build the string landscape, our task is to scan over CY3s
- Recall that we build CY3s from triangulations of polytopes
- Logic flow:
  - Pick some polytopes
  - For each polytope on the list, make all of the Calabi-Yaus
  - For each Calabi-Yau, try to make vacua

Scanning Over Calabi-Yaus

- As we aim to build the string landscape, our task is to scan over CY3s
- Recall that we build CY3s from triangulations of polytopes
- Logic flow:
  - Pick some polytopes
  - For each polytope on the list, make all of the Calabi-Yaus
  - For each Calabi-Yau, try to make vacua
  - For context, in [RAN et al ’24], we looked at:
    - 322 polytopes
    - 416 Calabi-Yaus
    - 240,480,254 choices of vacua

Scanning Over Calabi-Yaus

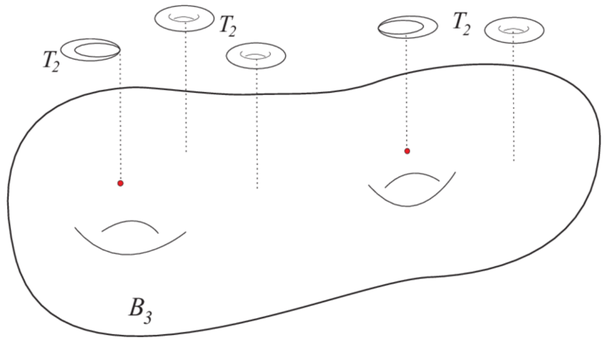
- As we aim to build the string landscape, our task is to scan over CY3s
- Recall that we build CY3s from triangulations of polytopes
- Logic flow:
  - Pick some polytopes
  - For each polytope on the list, make all of the Calabi-Yaus
  - For each Calabi-Yau, try to make vacua
  - For context, in [RAN et al '24], we looked at:
    - 322 polytopes
    - 416 Calabi-Yaus
    - 240,480,254 choices of vacua
- Parallelize over polytopes/CYs —> easy slurming!

Where Do We Go From Here?

- Some physics questions I hope to study in this way:
  - Making more, better de Sitter vacua [RAN et al WIP]
  - Low energy physics in these vacua
    - Inflation [Kleban-RAN-Norton WIP]
    - Holography [Engelhardt-RAN WIP]
  - 6D gauge theories [Abbasi-RAN-Taylor WIP]
  - …
- All of these will eventually be subMITted!

Building 6D Gauge Theories

- CY3s can define an interesting class of 6d theories, but only if the Calabi-Yau admits certain properties



[Leontaris '15]

- Our goal: which CY3s admit this "fibration" structure? [Abbasi-RAN-Taylor WIP]
- Which polytopes look like their CY3s might be fibered?
  - Scan over all 473,800,776 polytopes
- Of those polytopes, how many of the CY3s actually are fibered?
- Jobs ready to subMIT!

Conclusions

- [RAN et al ’24]: Fully explicit dS vacua with $\Lambda \sim +10^{-20} M_p^4$
  - Not phenomenologically viable- CC is 100 orders of magnitude too big
  - Still a big step towards realistic string compactifications
  - The plus sign was the hard part! It took 20+ years to get.
- Many open avenues for future research, and I expect subMIT to be a key tool in near-future projects
- Thanks for listening!