



Science and  
Technology  
Facilities Council

# Development of vertical excursion FFA

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# Vertical excursion FFA (VFFA)

- Invented in 1955, rediscovered in 2013.
- Orbit moves vertically when the beam is accelerated.
- Constant path length over whole momentum range (zero momentum compaction factor for all orders).
- Isochronism for ultra-relativistic energies (slippage factor only dependent of Lorentz gamma, like a Linac).

# Motivation

- FFAs:

- **Flexibility:** Beam pulse only controlled by RF, allowing fast and sophisticated patterns
- **Sustainability:** energy efficient operation, enhanced with SC or permanent magnets, reduced operating cost
- **Reliability:** DC power supply simple and cheap, low failure rate and higher redundancy

- VFFA:

- Rectangular magnet considered, potentially **easier to manufacture** than spiral HFFA
- Tall magnet, but **smaller footprint** than HFFA

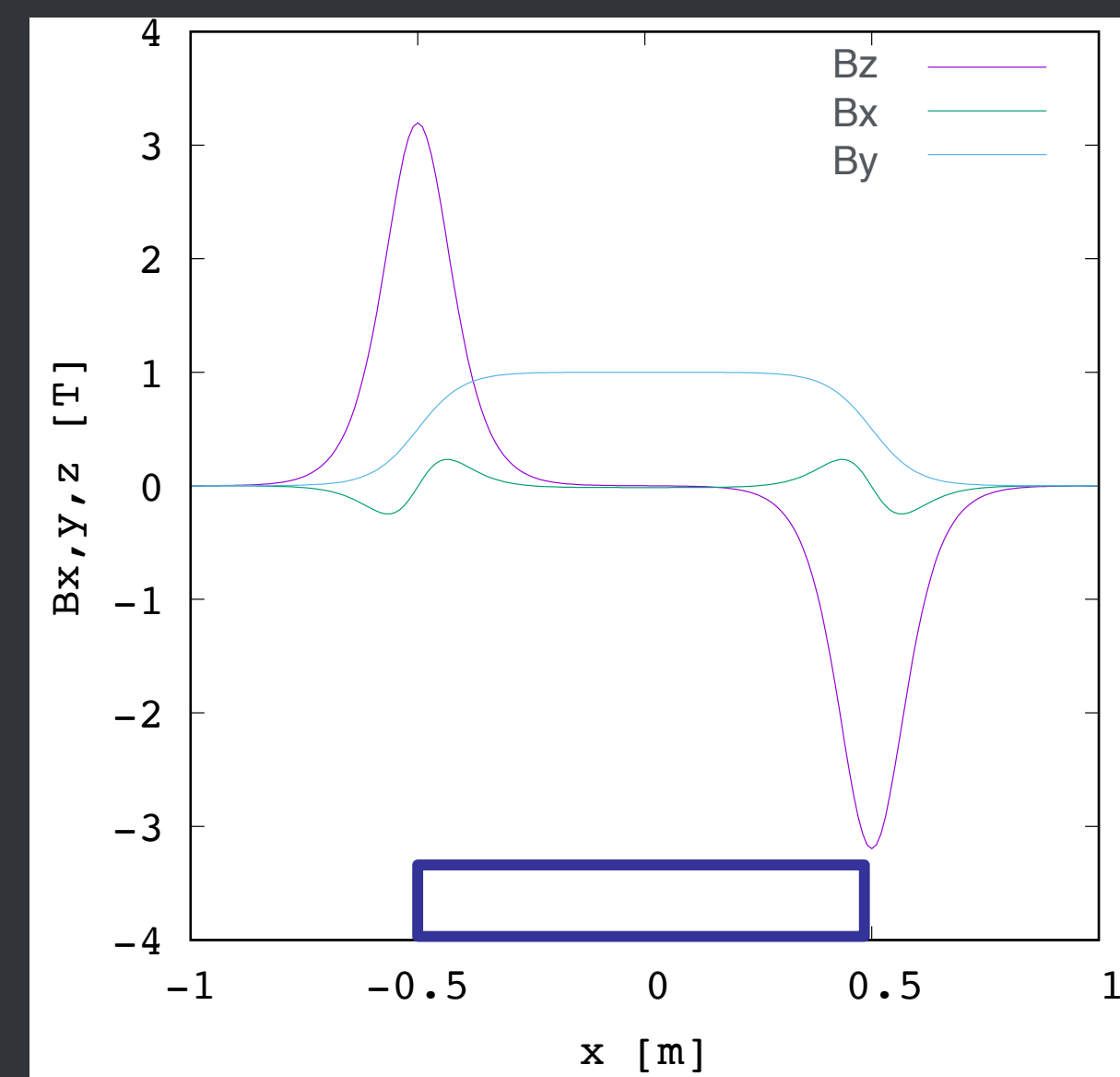
# Magnetic field in VFFA

- Exponentially increasing magnetic field to satisfy zero-chromatic conditions.

Cartesian coordinates  $x$  (hor.),  $y$  (vert.),  $z$  (long.)

$$\begin{cases} B_x(x, y, z) = B_0 e^{m(y-y_0)} \sum_i b_{xi}(z) (x-x_0)^i \\ B_y(x, y, z) = B_0 e^{m(y-y_0)} \sum_i b_{yi}(z) (x-x_0)^i \\ B_z(x, y, z) = B_0 e^{m(y-y_0)} \sum_i b_{zi}(z) (x-x_0)^i \end{cases}$$

- Non-zero longitudinal field on median plane.



- Importance of fringe field modelling, (more in small machines).

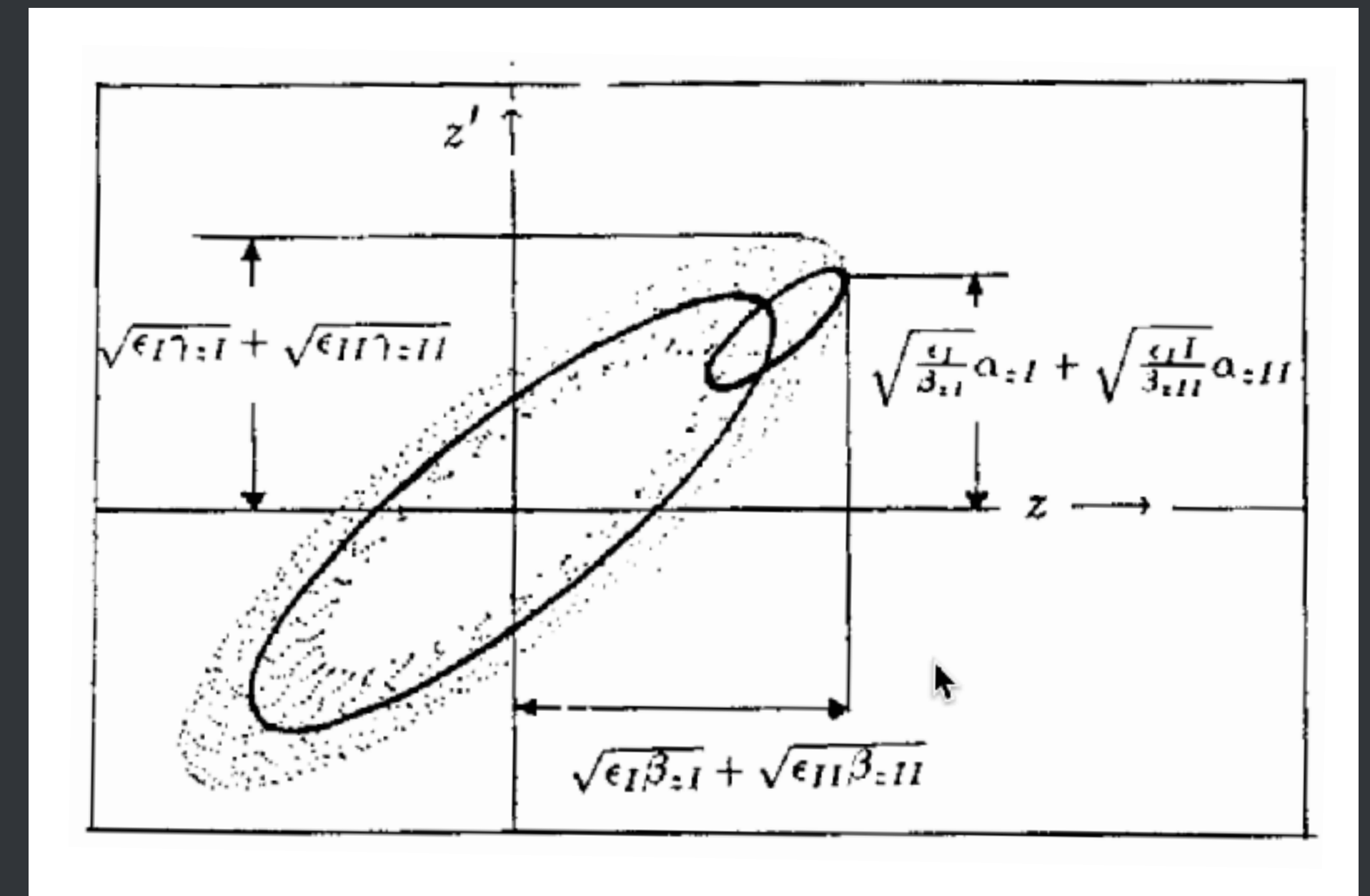
- Expansion of the field in the magnet shows alternance of normal and skew components.

➔ Strongly coupled optics

# VFFA Optics

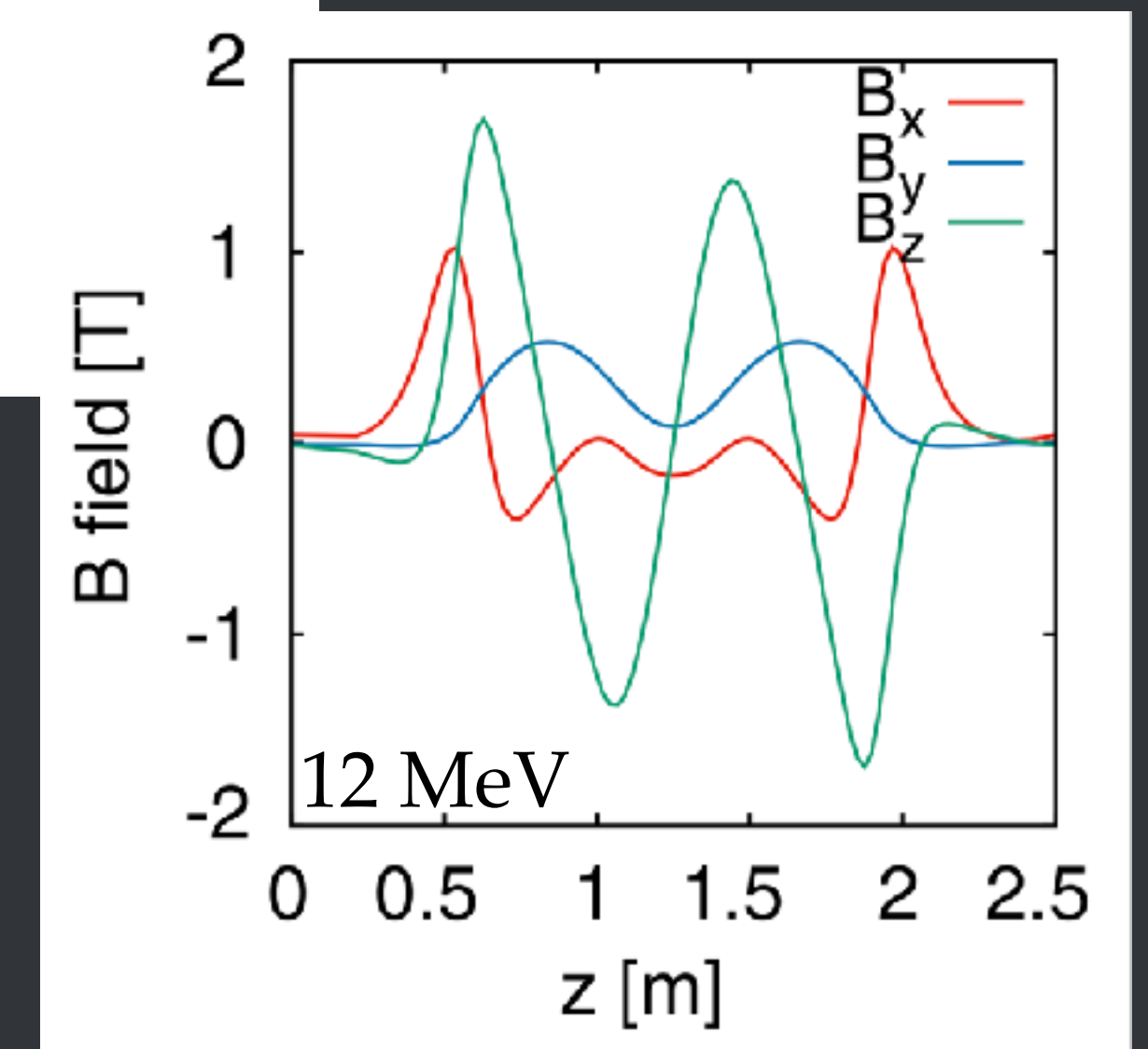
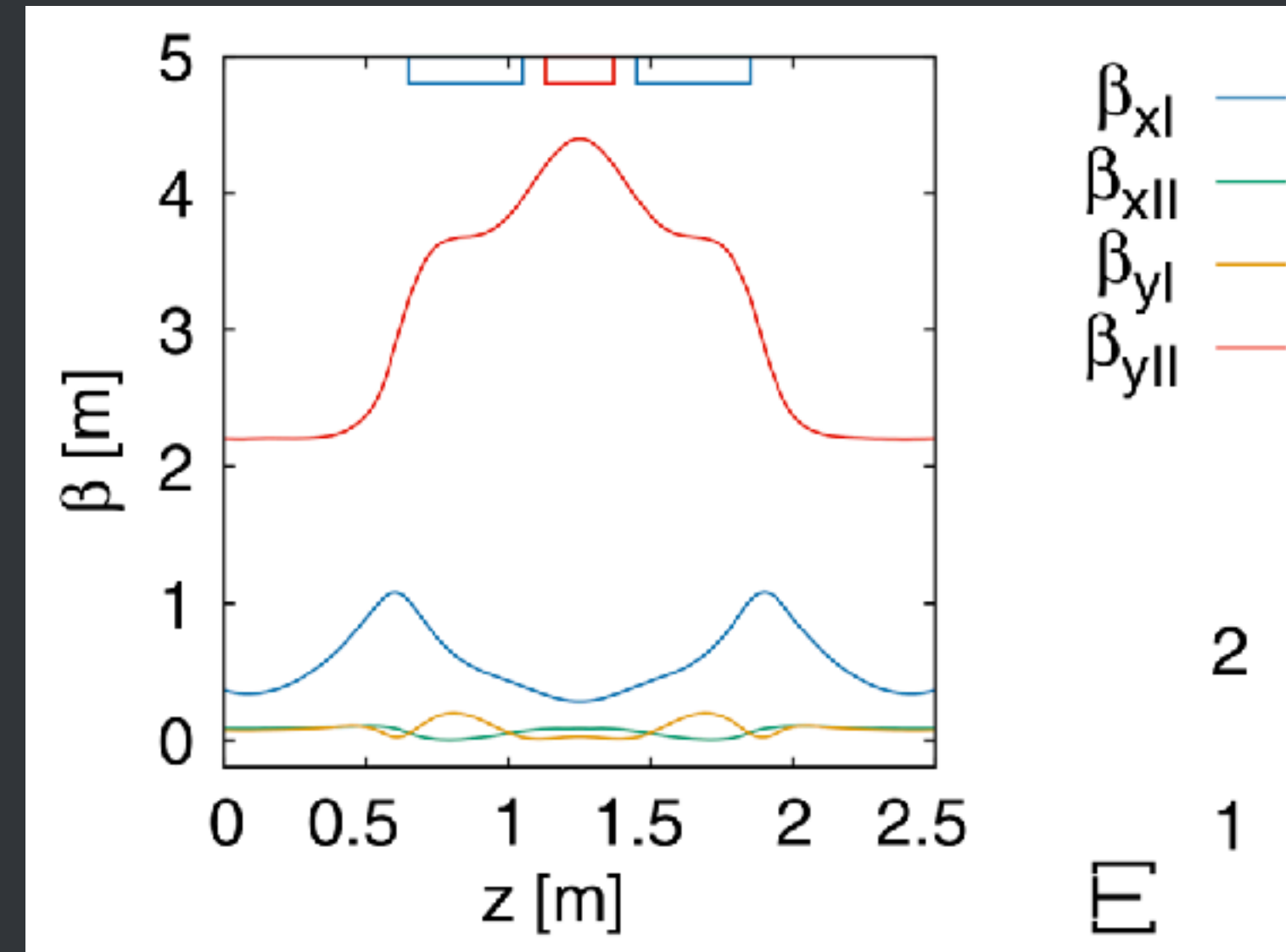
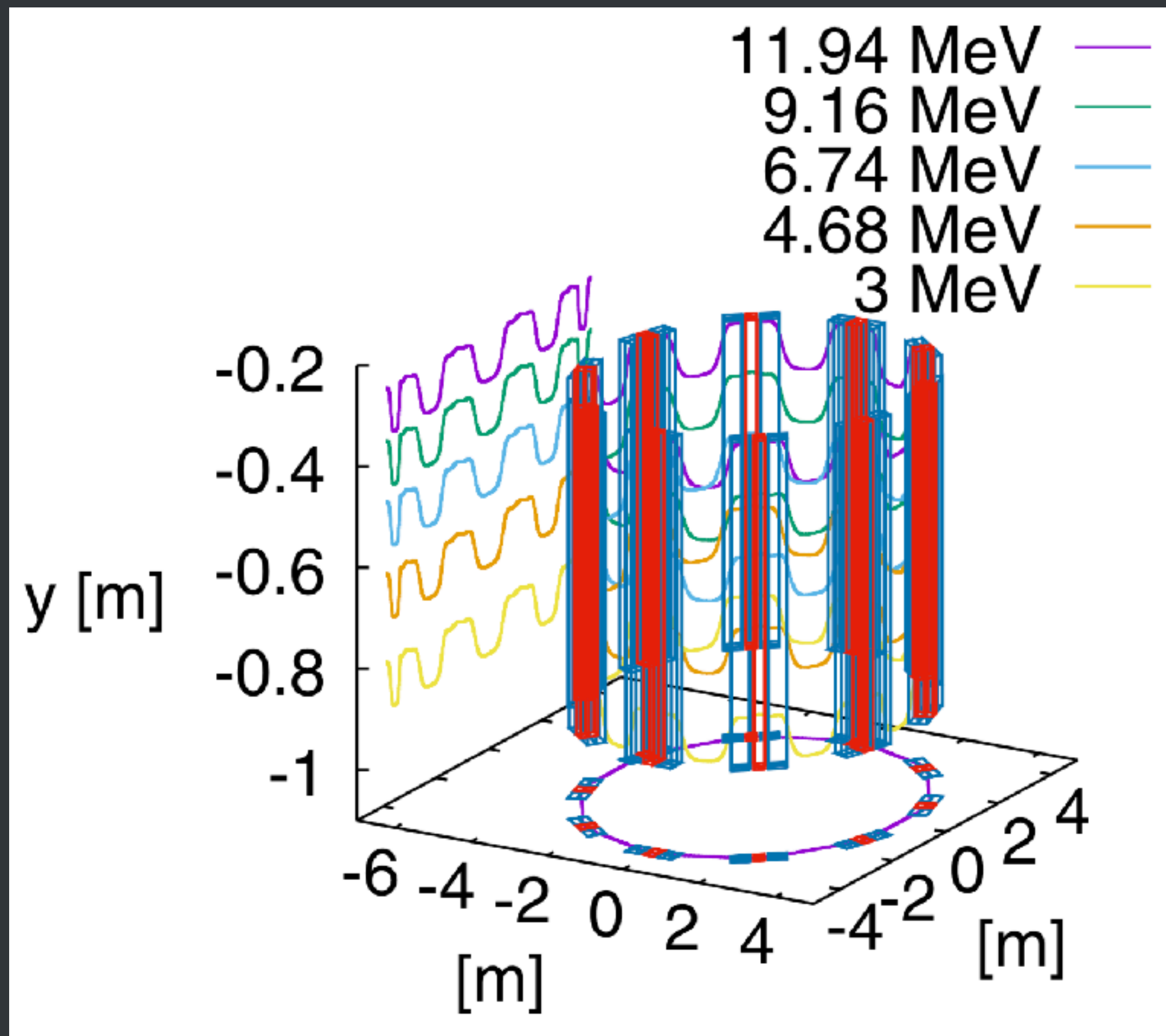
$$Z_{env} = \begin{pmatrix} x_{env} \\ x'_{env} \\ y_{env} \\ y'_{env} \end{pmatrix} = \begin{pmatrix} \sqrt{\epsilon_I} \sqrt{\beta_{xI}} + \sqrt{\epsilon_{II}} \sqrt{\beta_{xII}} \\ \sqrt{\epsilon_I} \sqrt{\gamma_{xI}} + \sqrt{\epsilon_{II}} \sqrt{\gamma_{xII}} \\ \sqrt{\epsilon_I} \sqrt{\beta_{yI}} + \sqrt{\epsilon_{II}} \sqrt{\beta_{yII}} \\ \sqrt{\epsilon_I} \sqrt{\gamma_{yI}} + \sqrt{\epsilon_{II}} \sqrt{\gamma_{yII}} \end{pmatrix}$$

- Tunes obtained from Eigenvalues of computed transfer matrices
- Beam envelope: beta-functions from Willeke-Ripken procedure



# VFFA test ring

Proof-of-principle ring (3-12 MeV proton) to be built by 2027.



# Coil configuration design

Coil designed based on Reverse Biot-Savart law

Biot-Savart law: 
$$B = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times \vec{r}}{|\vec{r}|^3}$$

Starting from a field model in the form of  $B_y = B_0 e^{m(y-y_0)} g(z)$

Passing in the frequency domain 
$$B_y = \left( \sum_i a_i e^{j\omega_{y_i} y} \right) \left( \sum_k b_k e^{j\omega_{z_i} z} \right)$$

The current density  $J_y$  and  $J_z$  of 2 infinite parallel current sheets separated by a gap  $\pm g$  are

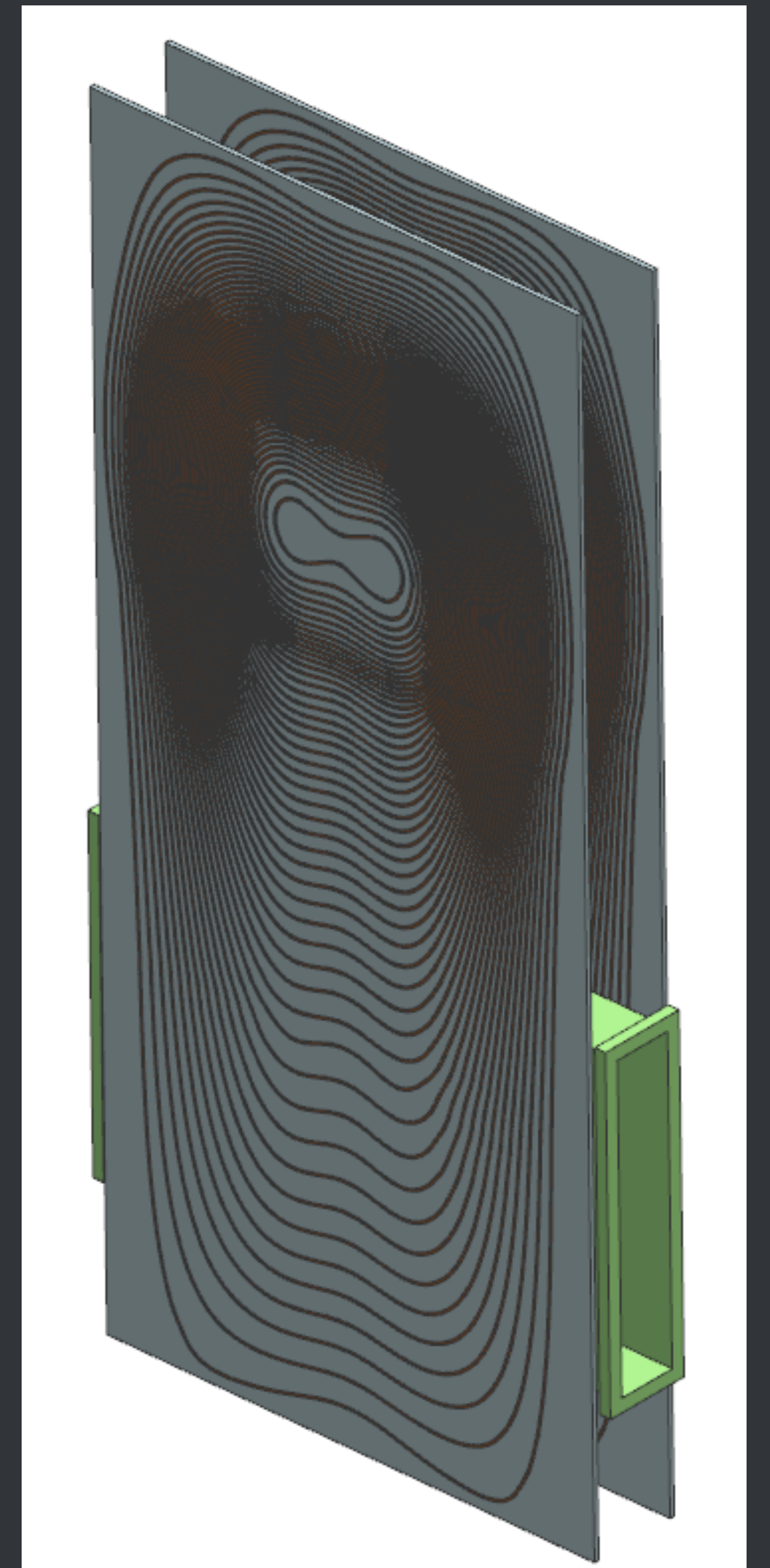
$$J_z = \sum_i \left( a_i e^{j\omega_{y_i} y} \sum_k b_k e^{j\omega_{z_i} z} e^{g\sqrt{\omega_{y_i}^2 + \omega_{z_i}^2}} \right) \text{ and } J_y = \int \frac{\partial J_z}{\partial z} dy$$

A 2D-FFT of  $B_y$  can then compute the current density.

# First prototype coil configuration

Prototype parameters:

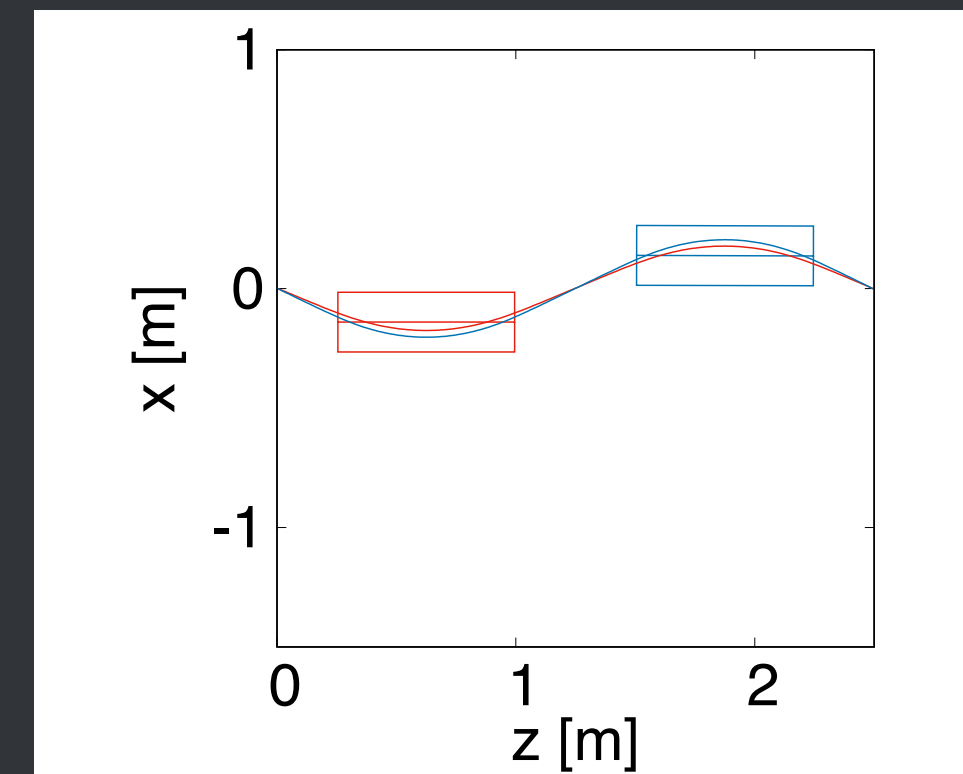
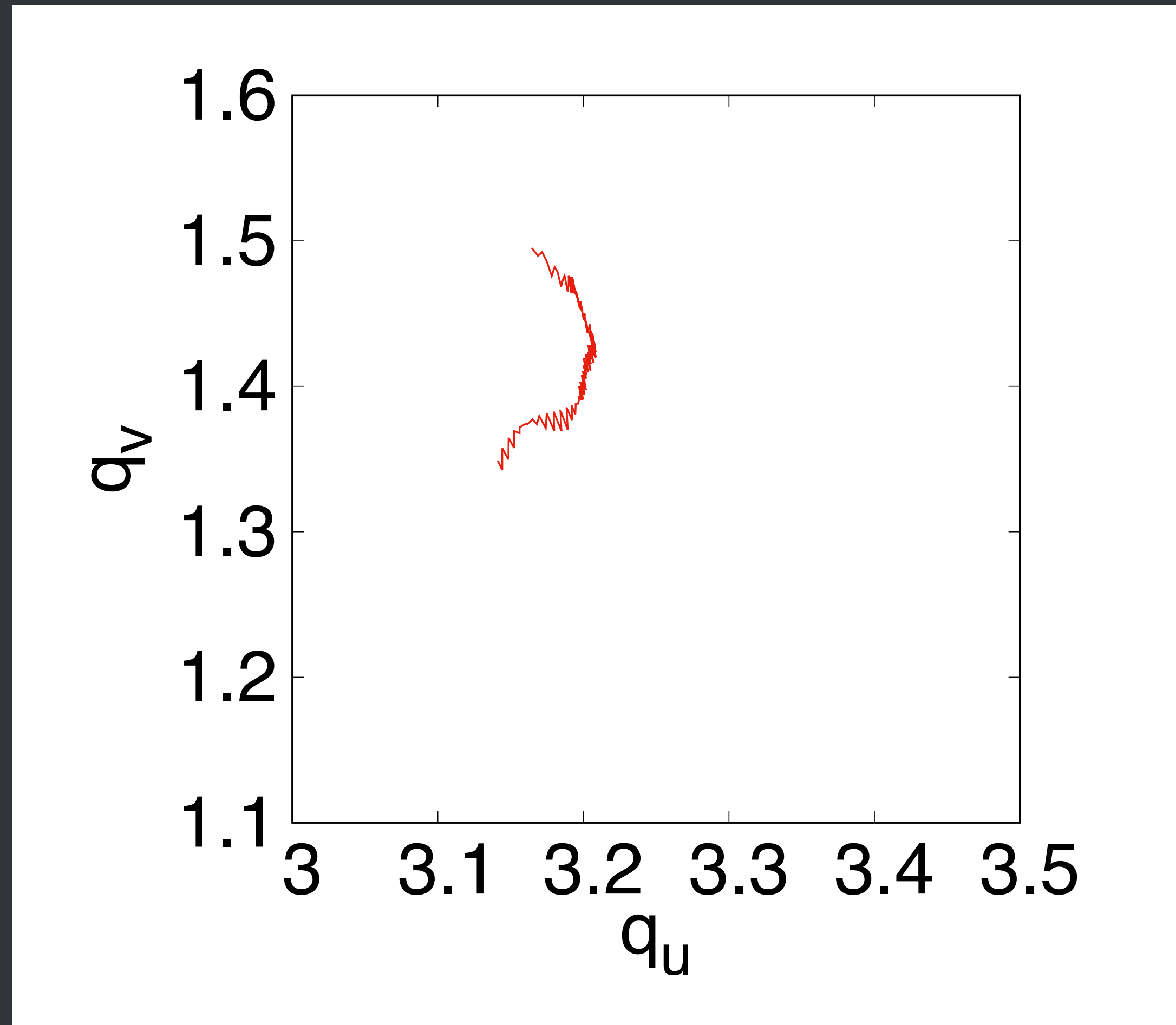
- Normal conducting with SC winding method
- 1 m-long magnet
- Normalised gradient  $m=1.3 \text{ m}^{-1}$ .
- 0.6 m vertical good field region
- 22 cm full gap size
- Coil made of 50 contours, each contour made of 16 turns
- 4.7 mm minimum spacing (centre coil to centre coil)



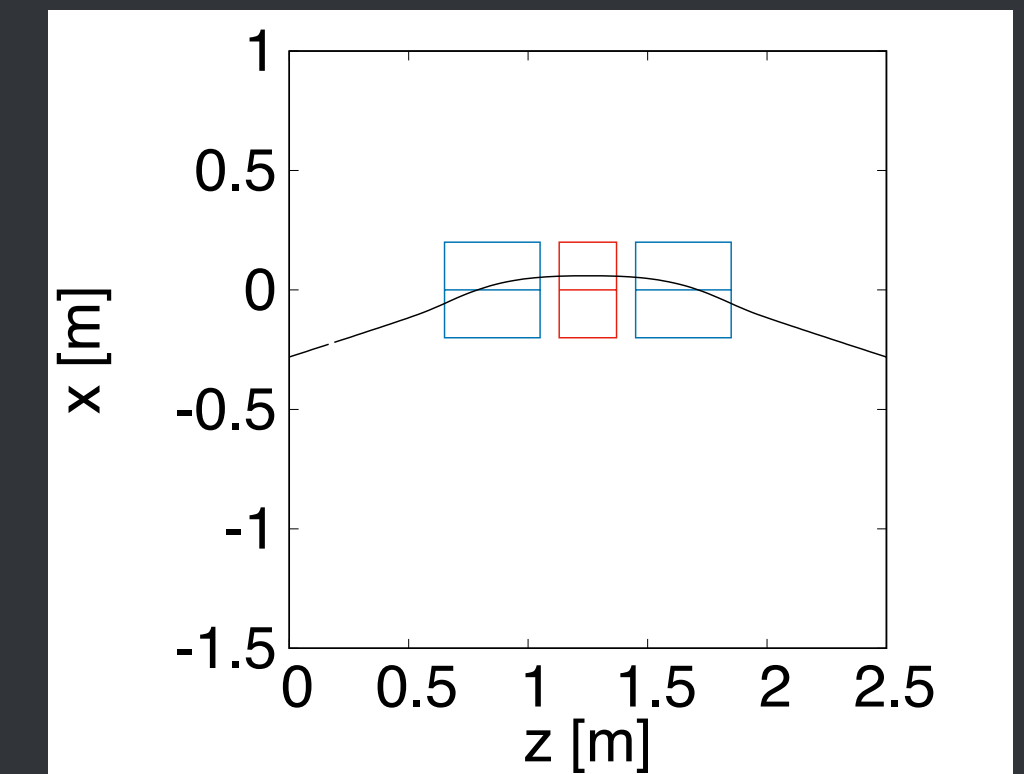


# Optics in more realistic magnet model

- Magnetic field model become available and optics is calculated based on 3-D field map.
- FODO cell lattice is taken to see how accurately magnetic field is created with realistic coil configuration.
- Tune should be constant during acceleration (scaling optics). Not fixed at the current magnet design.



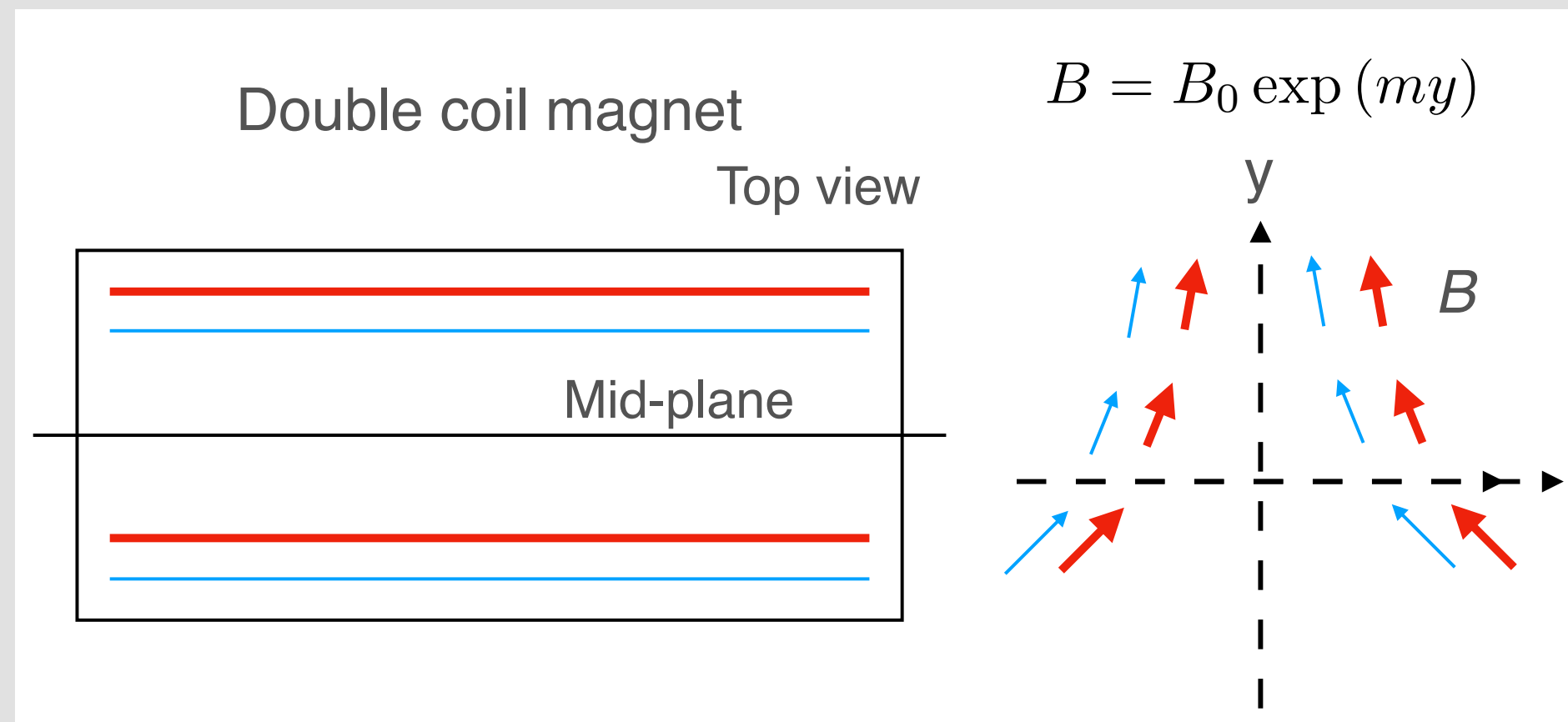
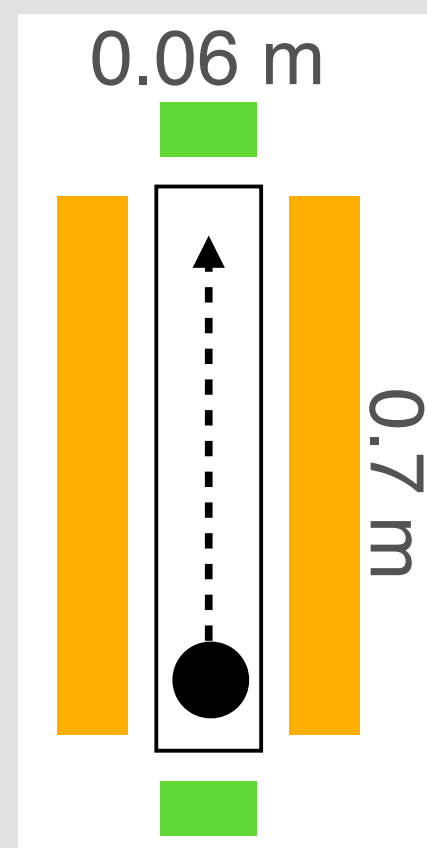
Lattice to check  
magnet accuracy



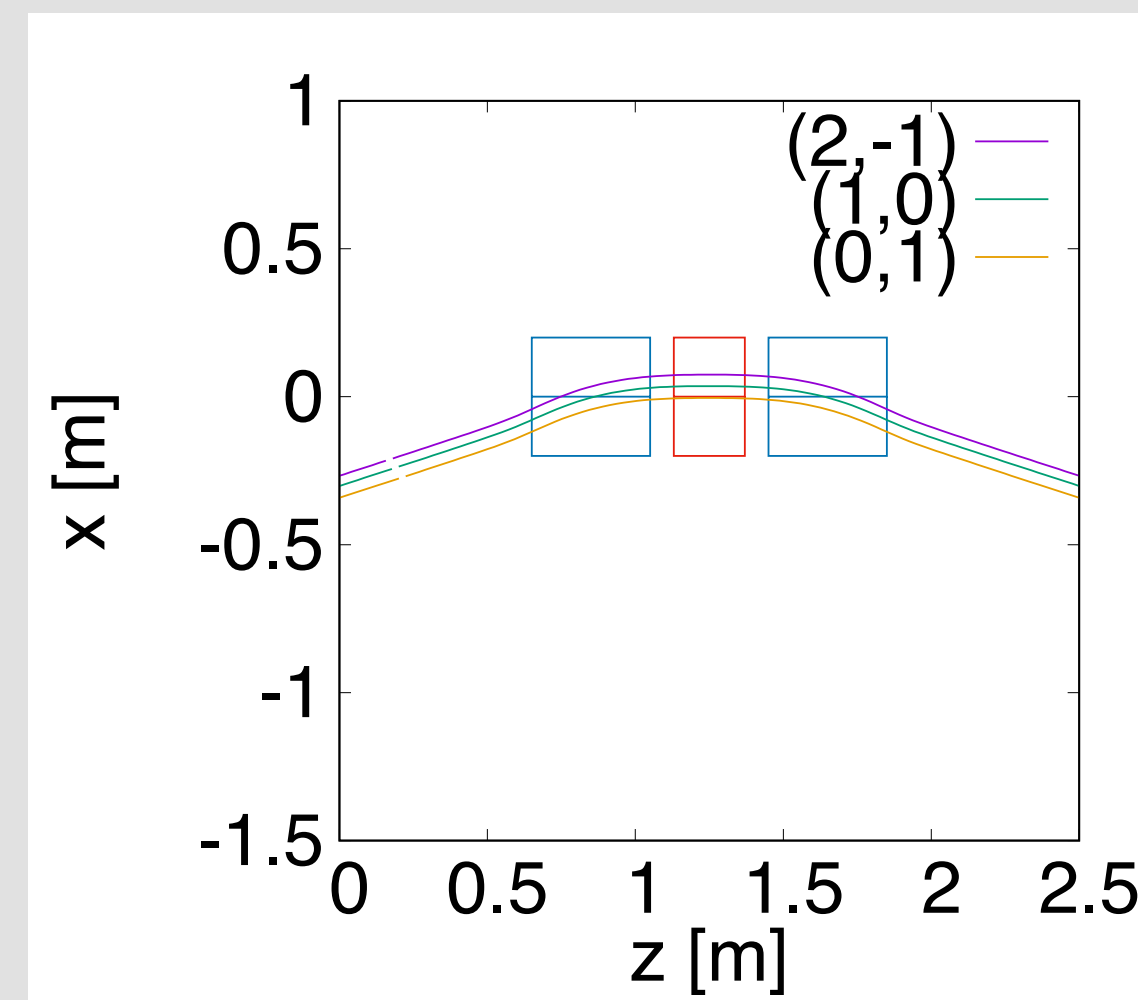
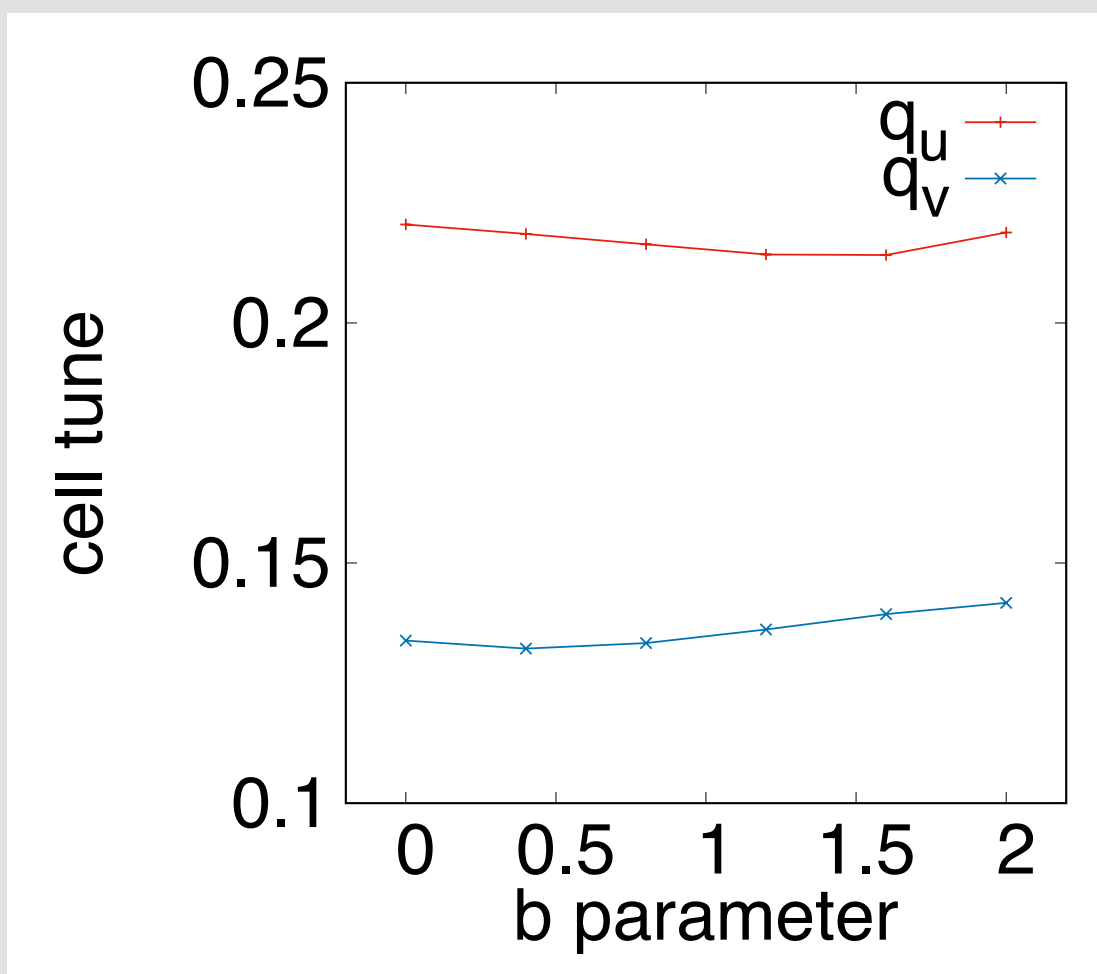
Lattice baseline

# Double coil design

## COD corrector



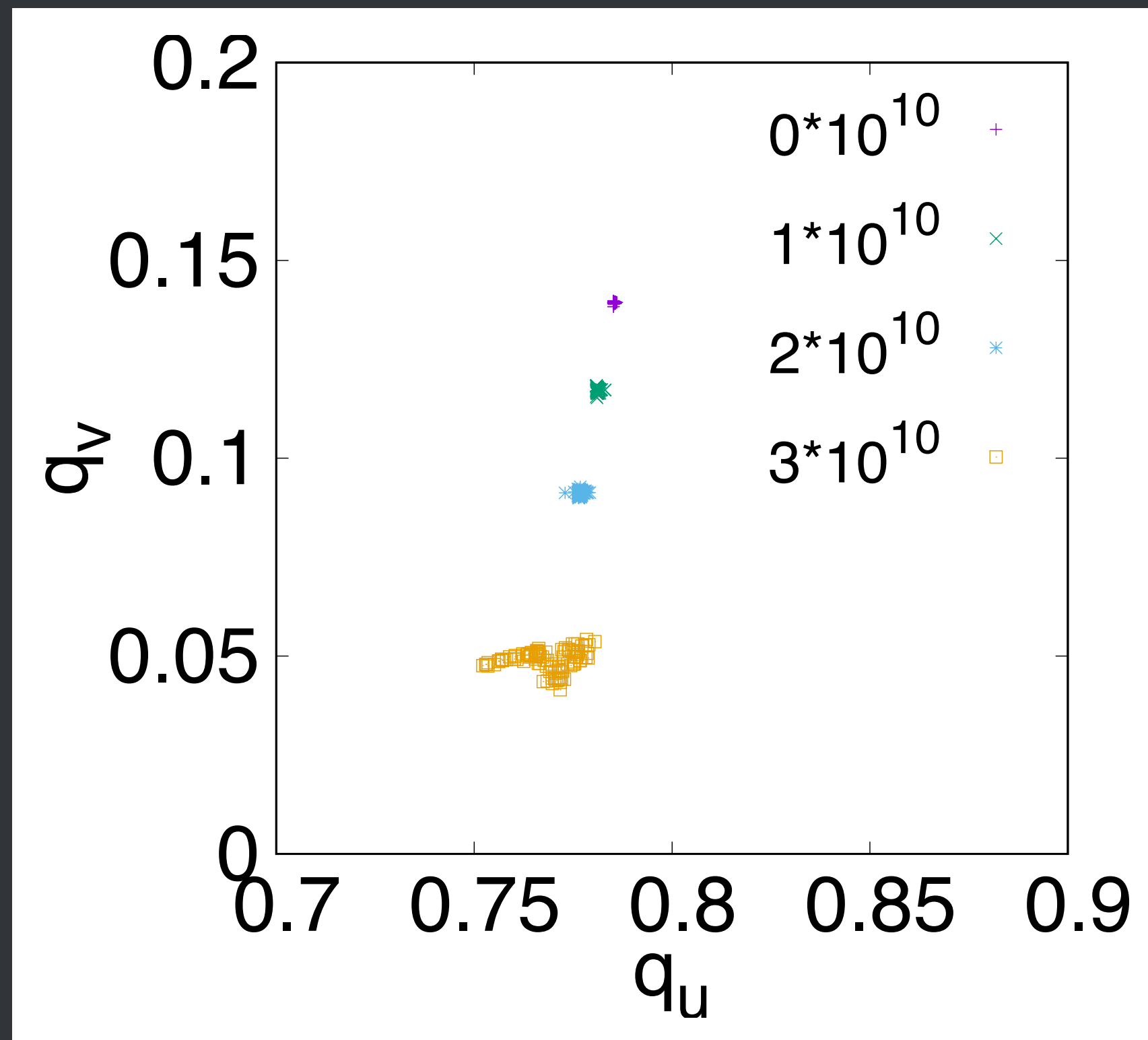
Horizontal orbit shift with double coil design



Several cm orbit movement with marginal tune change

# High intensity effects

## space charge tune shift



- Space charge tune shift from a simple model (uniform charge distribution, no longitudinal bunch structure):  $\Delta Q_u \sim -0.07$ ,  $\Delta Q_v \sim -0.30$

- Apply formula of tune shift per ring for decoupled optics:

- $a$  = horizontal beam size
- $b$  = vertical beam size
- Emittance = 0.25 pi mm mrad

$$\Delta Q_u = -\frac{n_t r_p R / Q_u}{\pi a (a + b) \beta^2 \gamma^3} = -0.23$$
$$\Delta Q_v = -\frac{n_t r_p R / Q_v}{\pi b (a + b) \beta^2 \gamma^3} = -0.43$$

➔ Reasonable tune shift, but needs more theoretical understanding.

# Summary

- Development of VFFA as a proton driver of a future spallation neutron source
- Proof of principle ring (3-12 MeV proton) planned by 2027
- Coil-based prototype magnet designed

Thank you for  
your attention