Feasibility Study for the Cylindrically Symmetric Magnetic Inflector

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On behalf of
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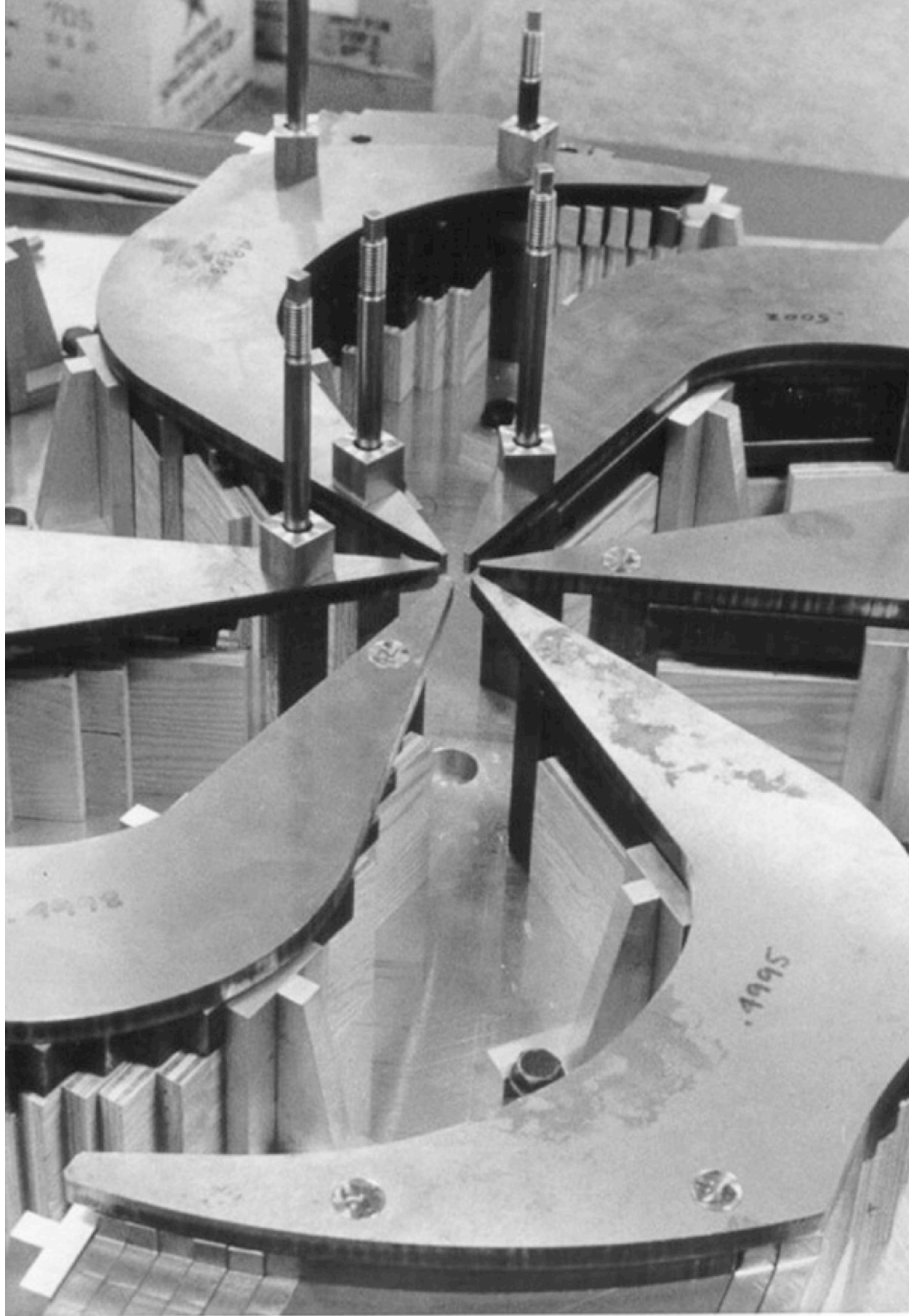
Sep, 2021



Discovery, accelerated

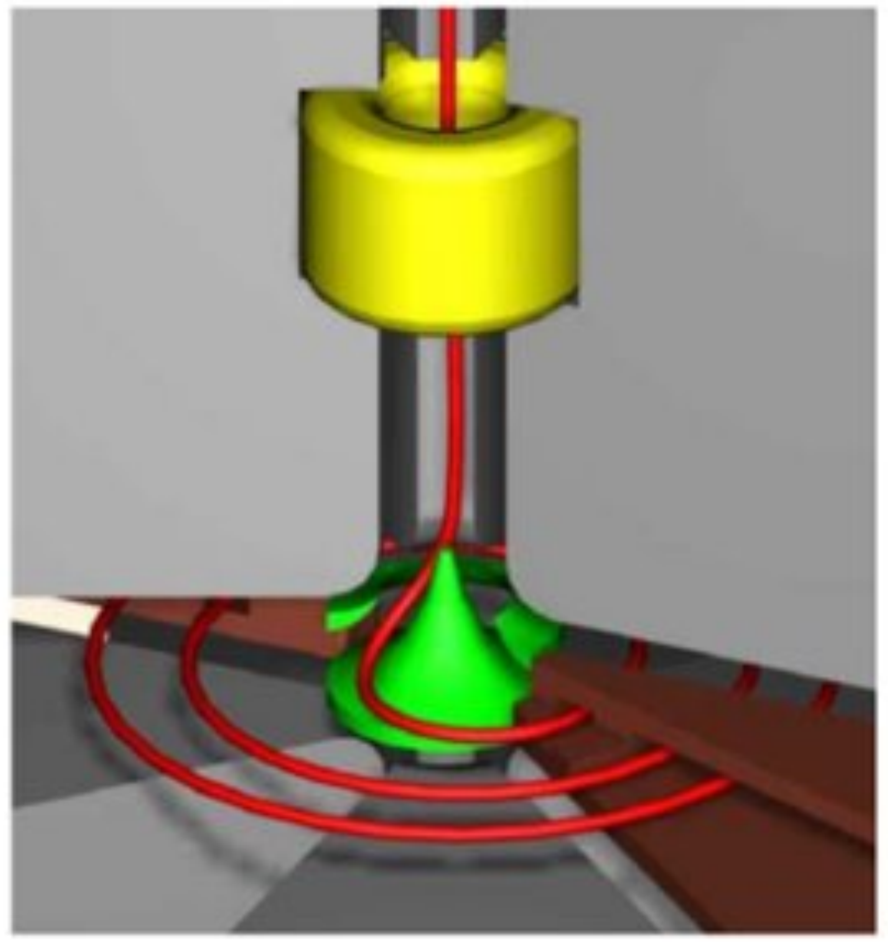
Outline

- Introduction
- TR100 magnet model
- Mirror like field and reference orbit
- Beam transport in the moving frame
- Different field parameters
- Conclusion

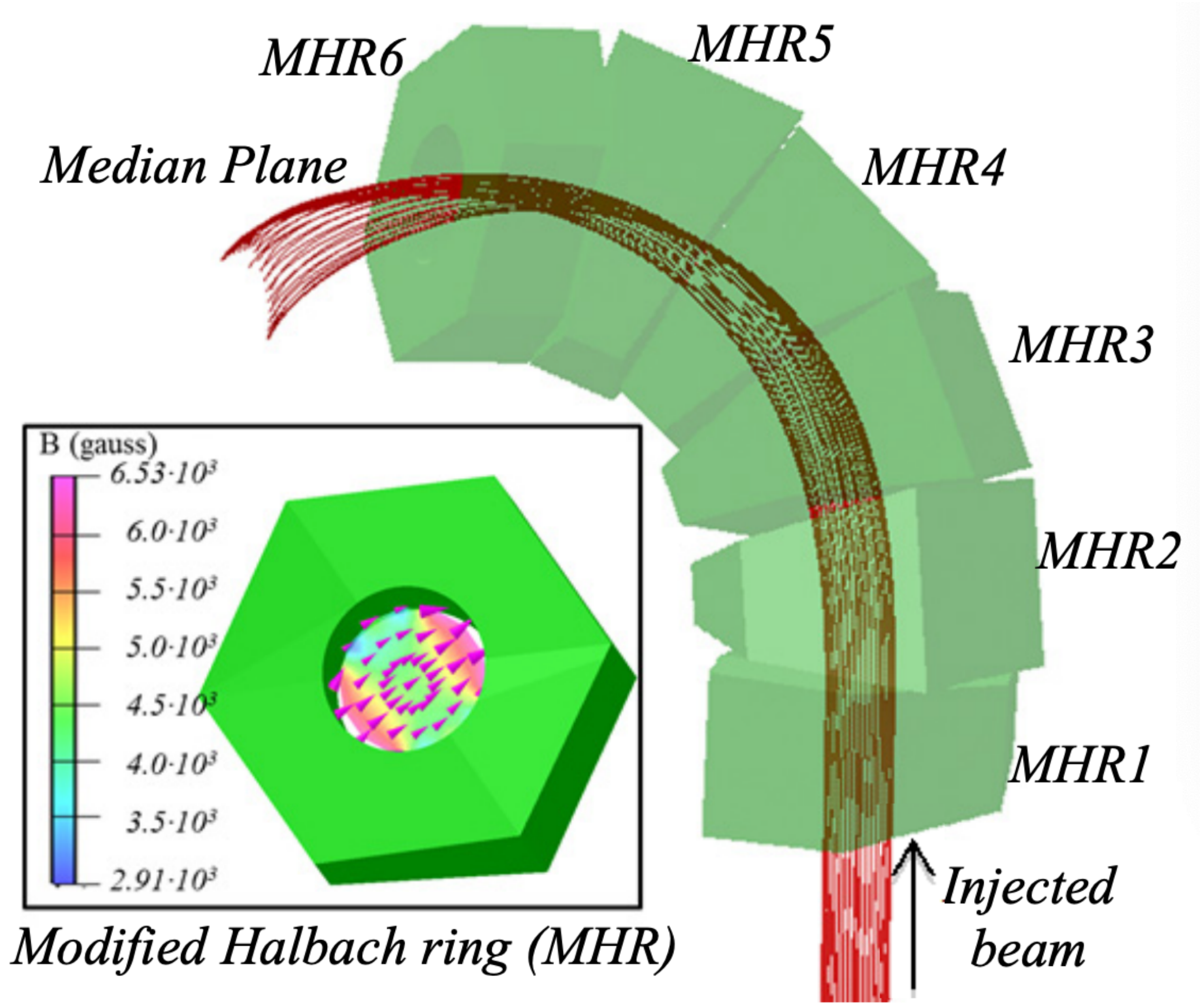
Introduction

- Inflector is used for axial injection into a cyclotron
- Conventional electrostatic inflector: injection energy is limited by the breakdown voltage between the 2 spiral electrodes and small aperture limits the injection intensity
- Magnetic inflector could overcome the disadvantages

Passive magnetic inflector using soft iron magnetized by the main magnetic field[1]



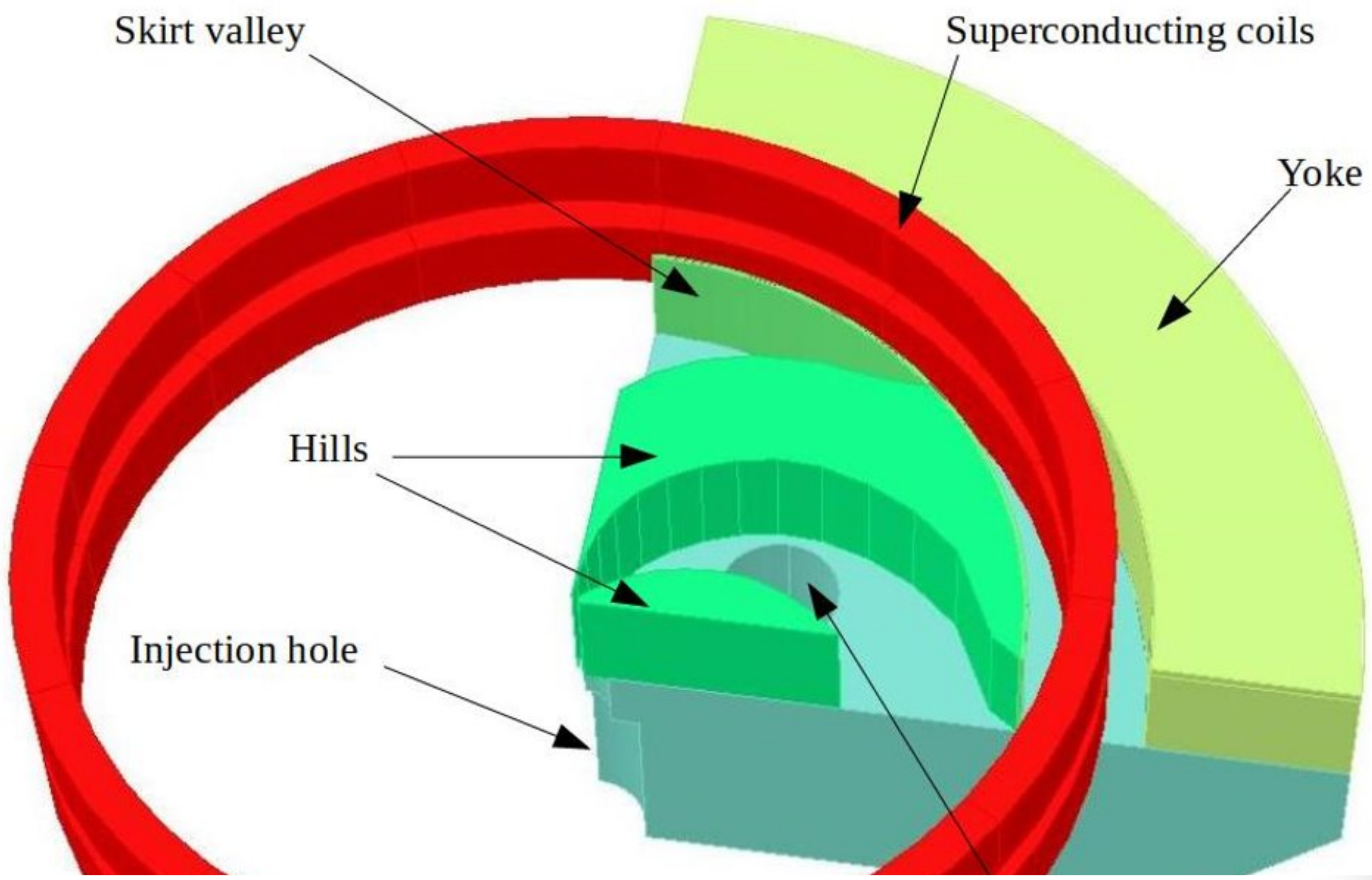
Active magnetic inflector using an array of permanent magnets[2]

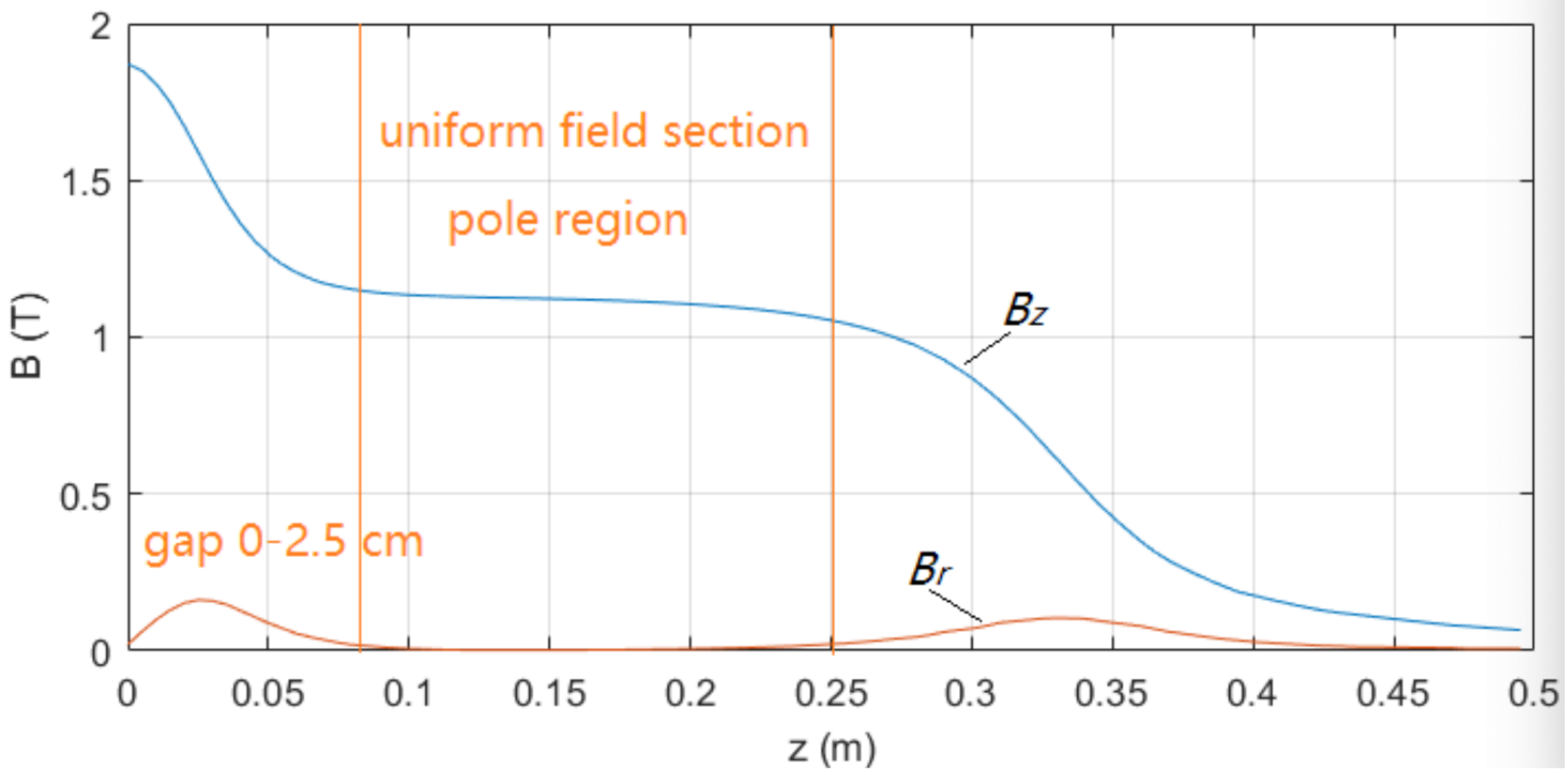


1. William Kleeven, CAS 2013
2. D. Campo, L. Calabretta & etc., IPAC'14

TR100 Main Parameters

Parameters	Value
Particle accelerated	H_2^+
Gap (cm)	5
Number of sectors	4
B0 at centre (T)	2.0
Pole radius (m)	1.65
Injection scheme	Axial + external ion source
Coils	2 superconducting coils





Magnetic mirror

The magnetic mirror is a component used to confine the charged particles. A vector potential used to define the axially symmetric magnetic field in a magnetic mirror is given as below

$$A_{theta} = \frac{A_1 \beta r}{2} - A_2 I_1(\beta r) \cos \beta z$$

The given magnetic field satisfies the Laplace's equation, which ensures that the field could be designed in a free space. The linear approximation of the vector potential is given as

$$A_{theta} = \frac{\beta r}{2}(A_1 - A_2 \cos \beta z)$$

Magnetic field is written as

$$B_r = -(\beta^2 r A_2/2) sin(\beta z)$$

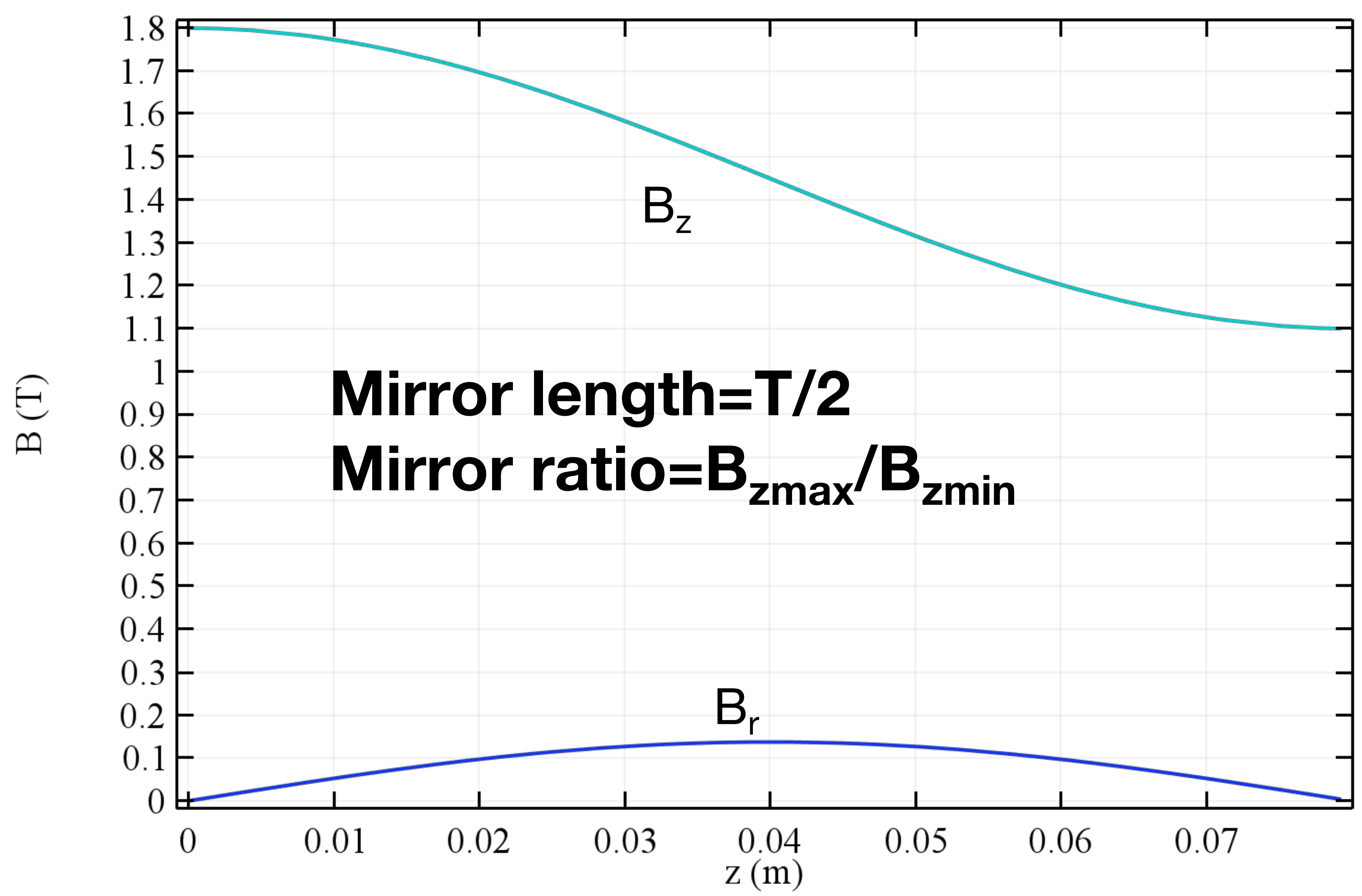
$$B_z = \beta(A_1 - A_2 cos(\beta z))$$

The motion equation could be written as a 2D form in r-z plane

$$P_r' = \frac{\partial U}{\partial r}$$

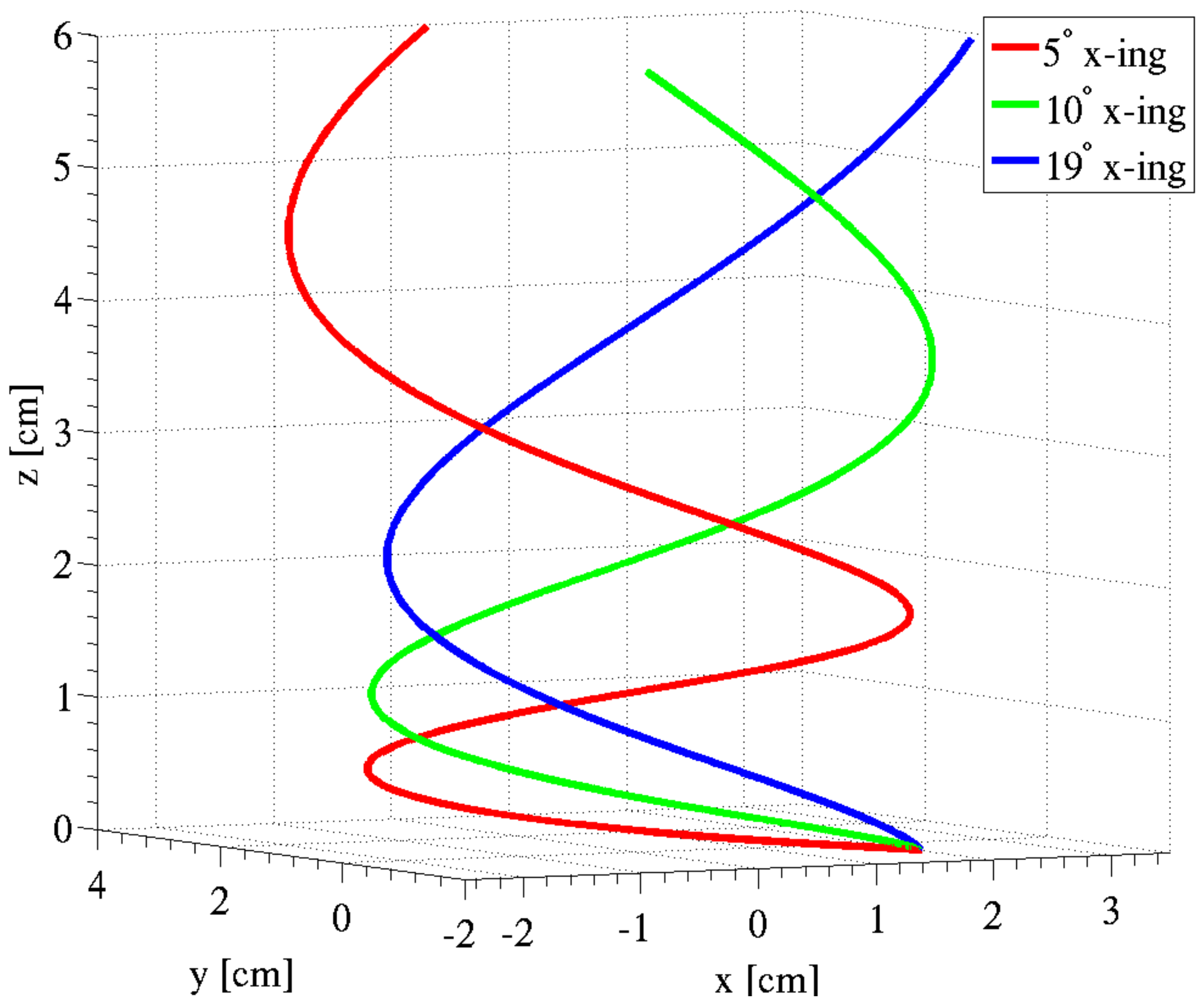
$$P_\theta' = 0$$

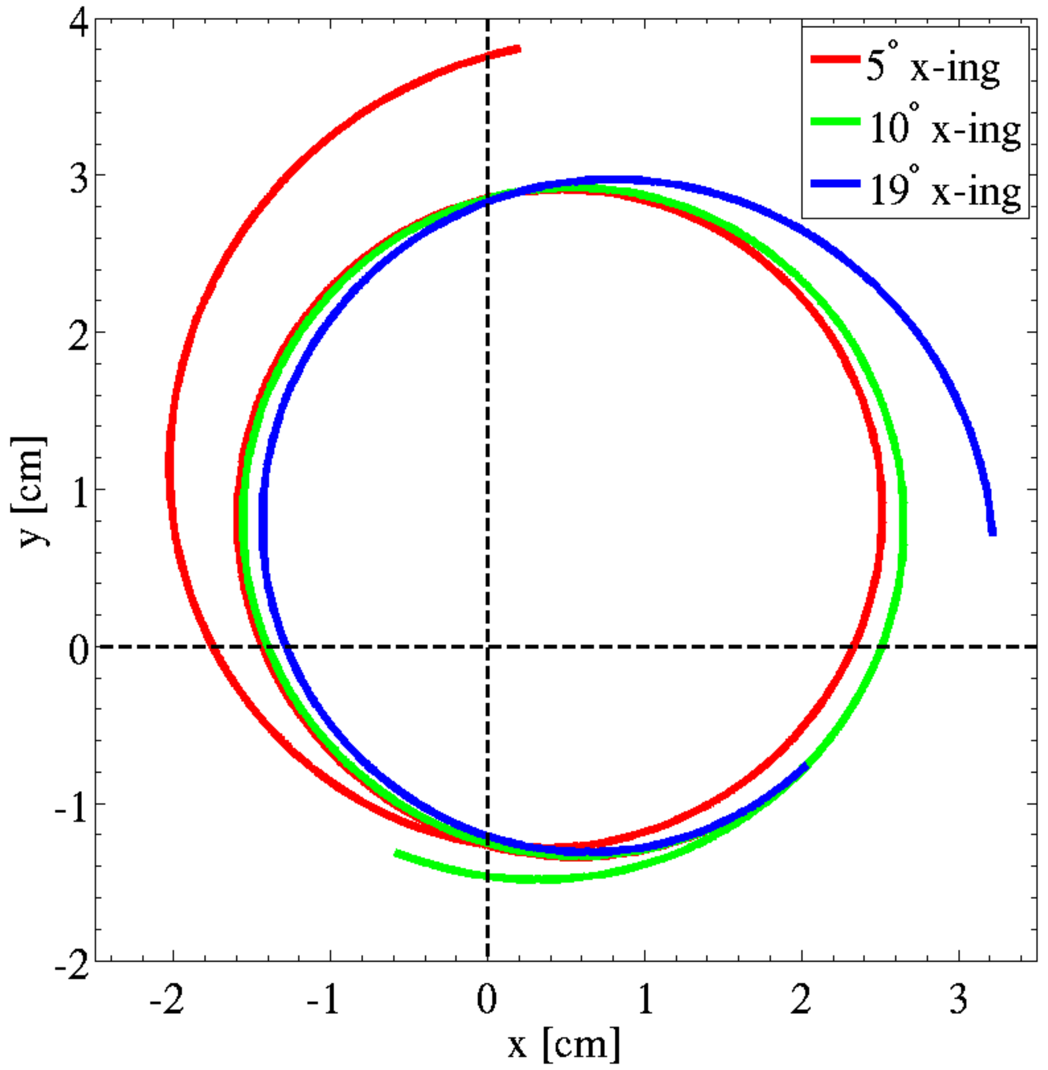
$$P_z' = \frac{\partial U}{\partial z}$$



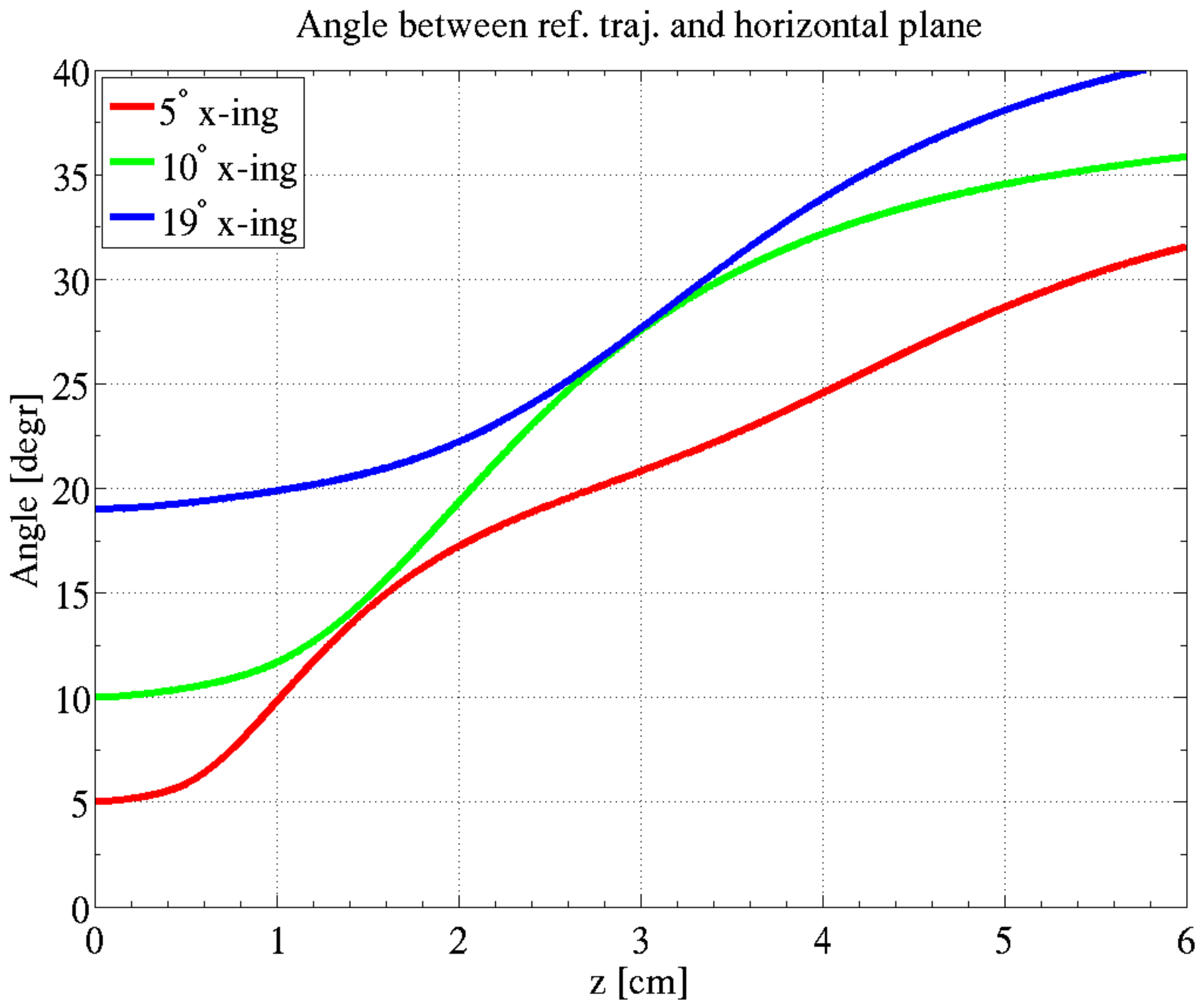
Reference orbit

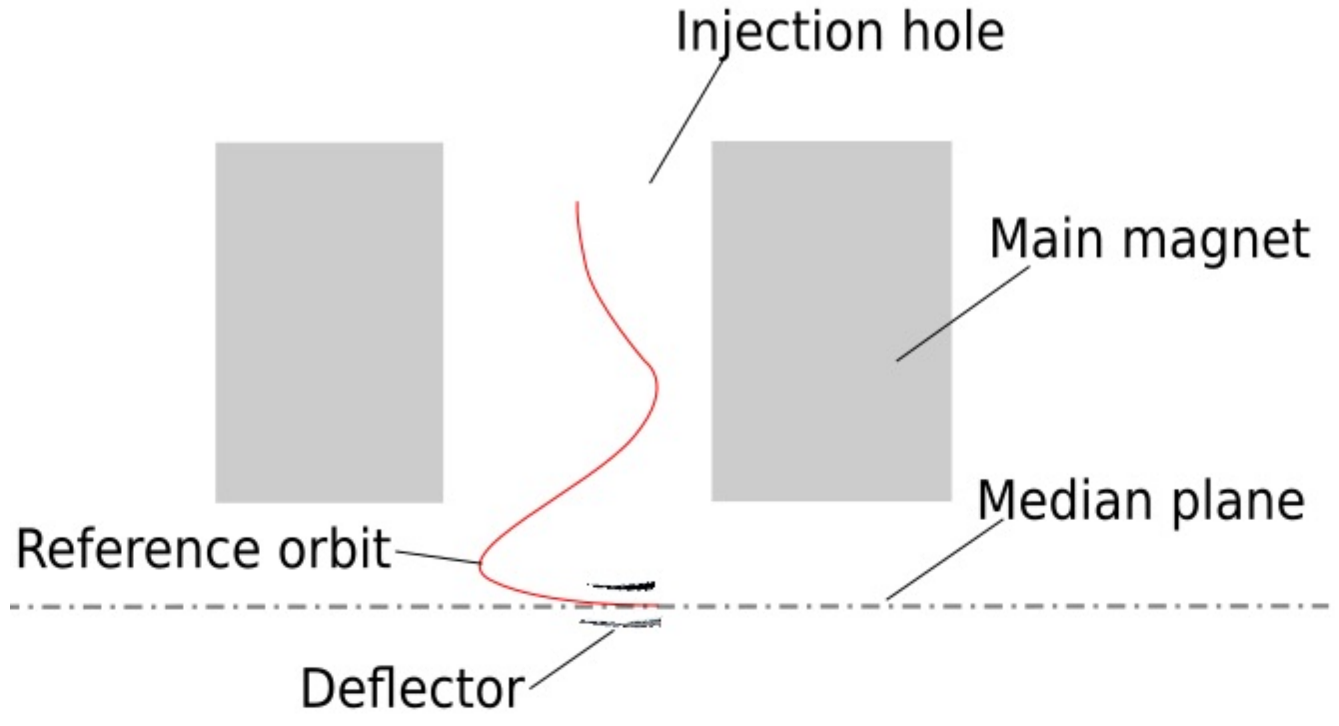
Track the particle reversely from the median plane to the injection point with different Pitch angles





The single B_r bump field near the median plane could reduce the pitch angle by about 20° from the injection point to the median plane



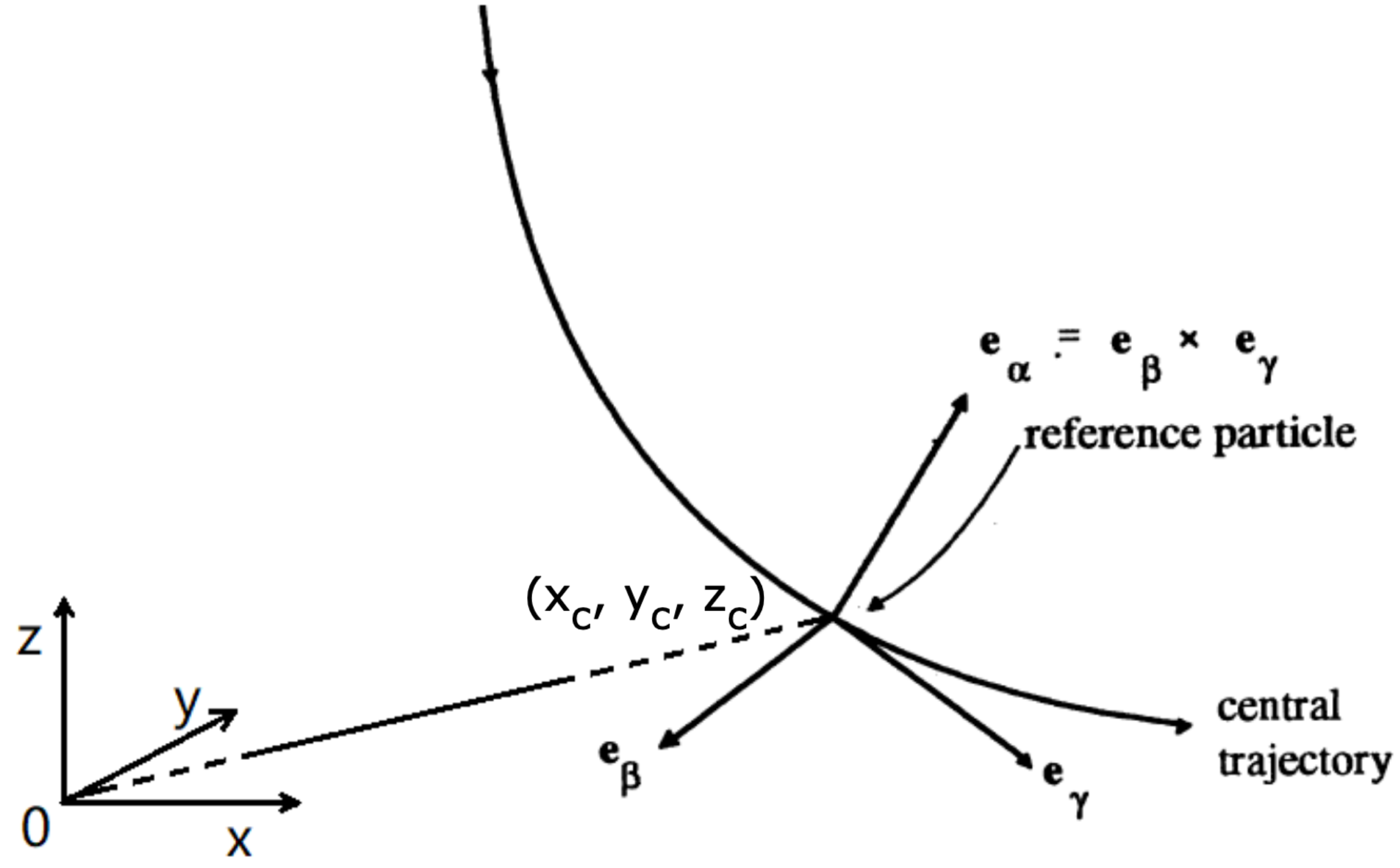


Moving frame

The moving frame $\alpha\beta\gamma$ is a right handed coordinate system. The γ direction is the same with the velocity of the reference particle. The β direction is perpendicular to the γ direction and parallel to the median plane.

The transformation from (x, P_x, y, P_y, z, P_z) to (α, P_α, β, P_β, γ, P_γ) is given by

$$\begin{bmatrix} \alpha \\ P_\alpha \\ \beta \\ P_\beta \\ \gamma \\ P_\gamma \end{bmatrix} = \mathrm{M}^T \begin{bmatrix} x - x_c \\ m_0 v_0 (x' - x'_c) \\ y - y_c \\ m_0 v_0 (y' - y'_c) \\ z - z_c \\ m_0 v_0 (z' - z'_c) \end{bmatrix} + q\mathrm{M}^T \begin{bmatrix} 0 \\ A_x - A_{x0} \\ 0 \\ A_y - A_{y0} \\ 0 \\ A_z - A_{z0} \end{bmatrix}$$



Transfer matrix

Calculate the transfer matrix by tracking 6 particles numerically with different initial coordinates and momenta

$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (1,0,0,0,0,0)$$
$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (0,1,0,0,0,0)$$
$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (0,0,1,0,0,0)$$
$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (0,0,0,1,0,0)$$
$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (0,0,0,0,1,0)$$
$$(\alpha, P_\alpha, \beta, P_\beta, P_\gamma) = (0,0,0,0,0,1)$$

Transfer matrix in the moving frame

$$\mathrm{R} = \begin{bmatrix} 1.9899 & 0.1493 & -1.6822 & -0.0167 & 0.3753 & 0.1340 \\ -5.0231 & 0.1862 & -0.2335 & -0.1780 & 3.8668 & 0.2139 \\ 0.5800 & 0.0223 & 0.8386 & 0.0260 & -0.5524 & -0.0134 \\ -13.7973 & -0.3713 & -8.0835 & 0.3925 & 1.9409 & -0.6353 \\ -0.0311 & 0.0394 & -0.3041 & 0.0195 & 0.6095 & 0.0989 \\ 5.3240 & 0.2201 & -12.4786 & 0.2672 & -5.4282 & 0.8583 \end{bmatrix}$$

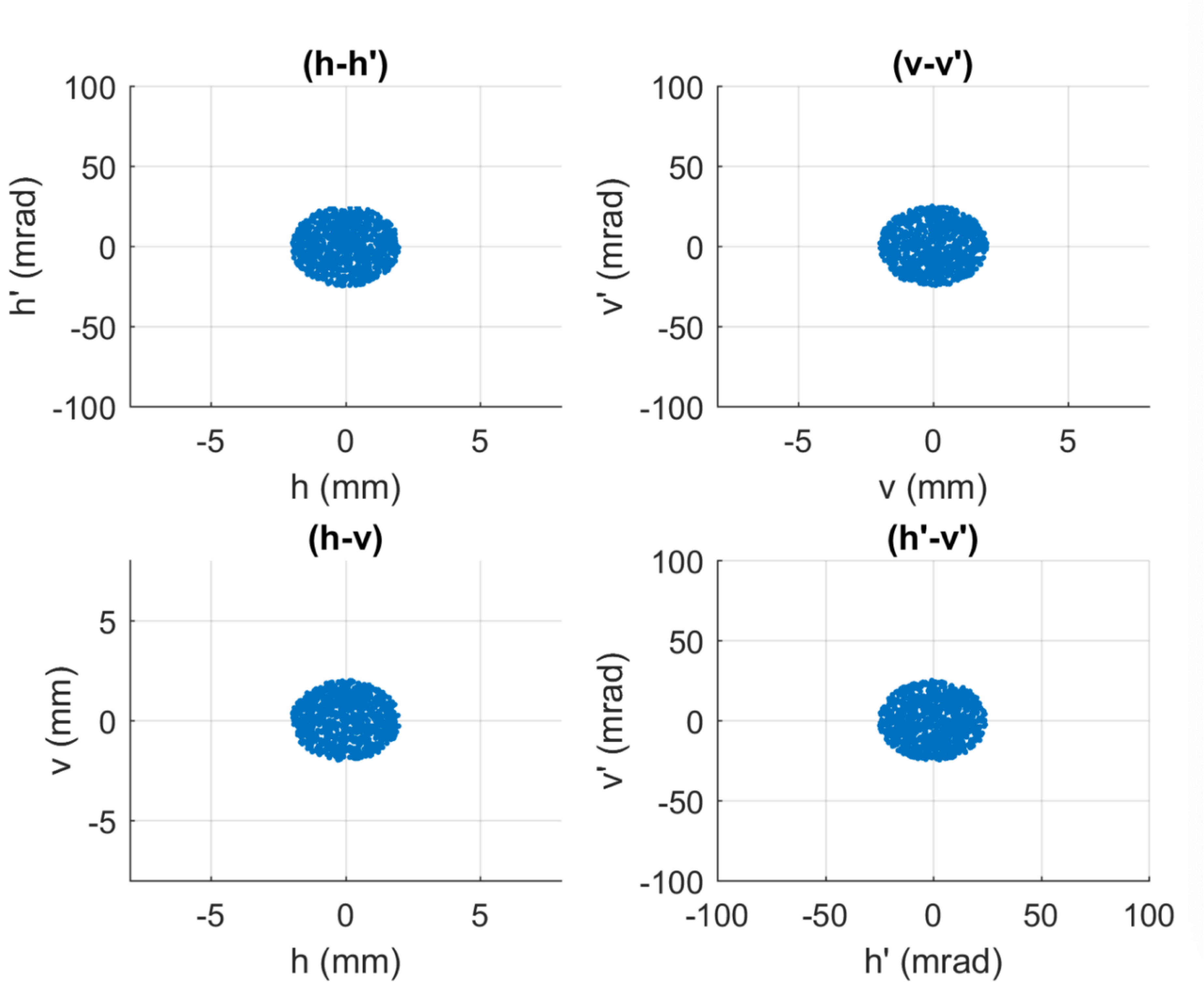
Transfer matrix symplectic

$$\mathrm{R^TJR - J} = \begin{bmatrix} 0 & -0.0043 & -0.0258 & 0.0364 & 0.0082 & -0.0084 \\ 0.0043 & 0 & -0.0151 & 0.0012 & -0.0026 & -0.0001 \\ 0.0258 & 0.0151 & 0 & -0.0024 & 0.0014 & 0.0036 \\ -0.0364 & -0.0012 & 0.0024 & 0 & 0.0007 & -0.0006 \\ -0.0082 & 0.0026 & -0.0014 & -0.0007 & -0.0000 & -0.0008 \\ 0.0084 & 0.0001 & -0.0036 & 0.0006 & 0.0008 & 0 \end{bmatrix}$$

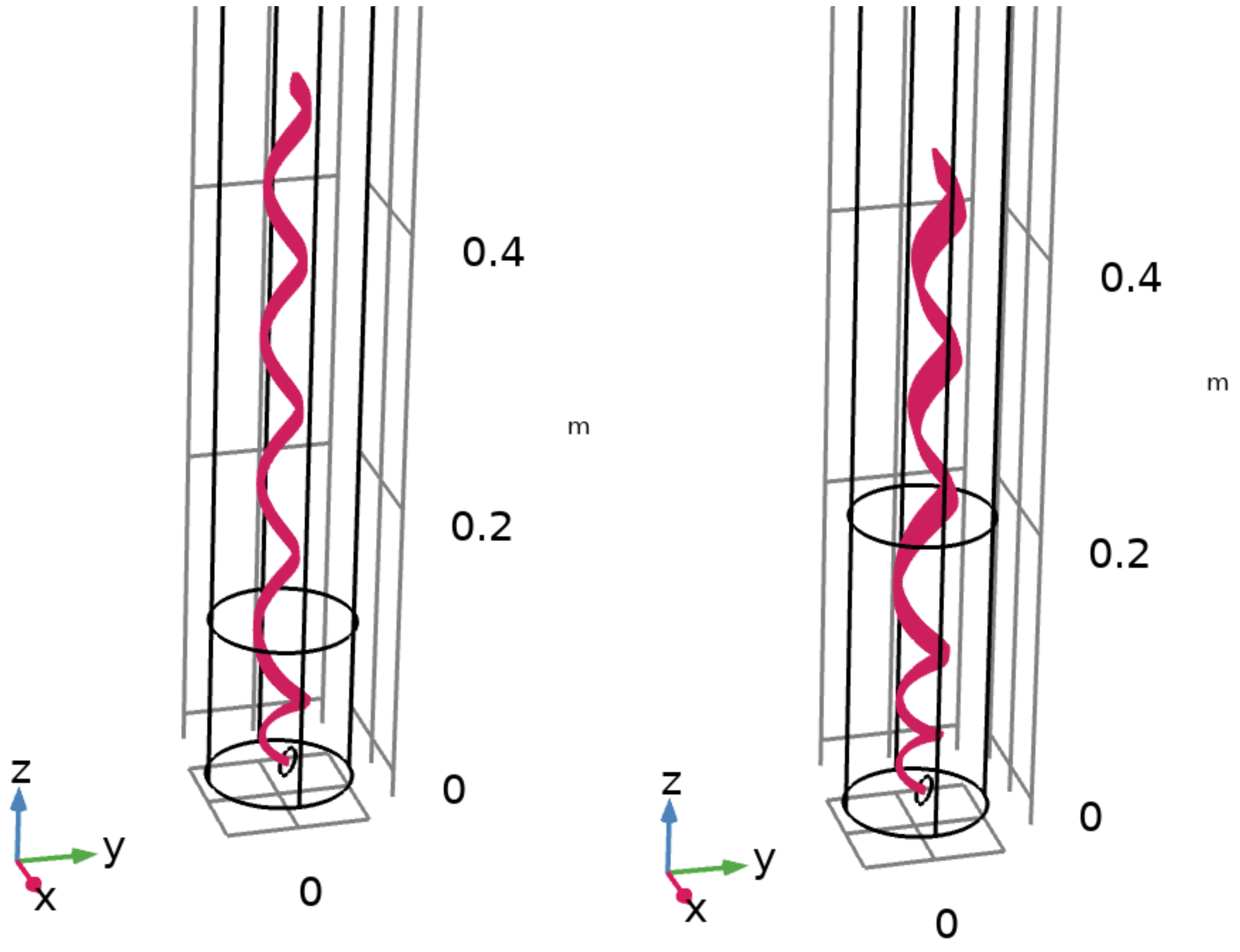
$$\mathrm{J} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

The error is between 10^{-3} and 10^{-2}, which may be resulted from the non-linear of the motion and the noise when tracking the particles numerically.

Beam focusing

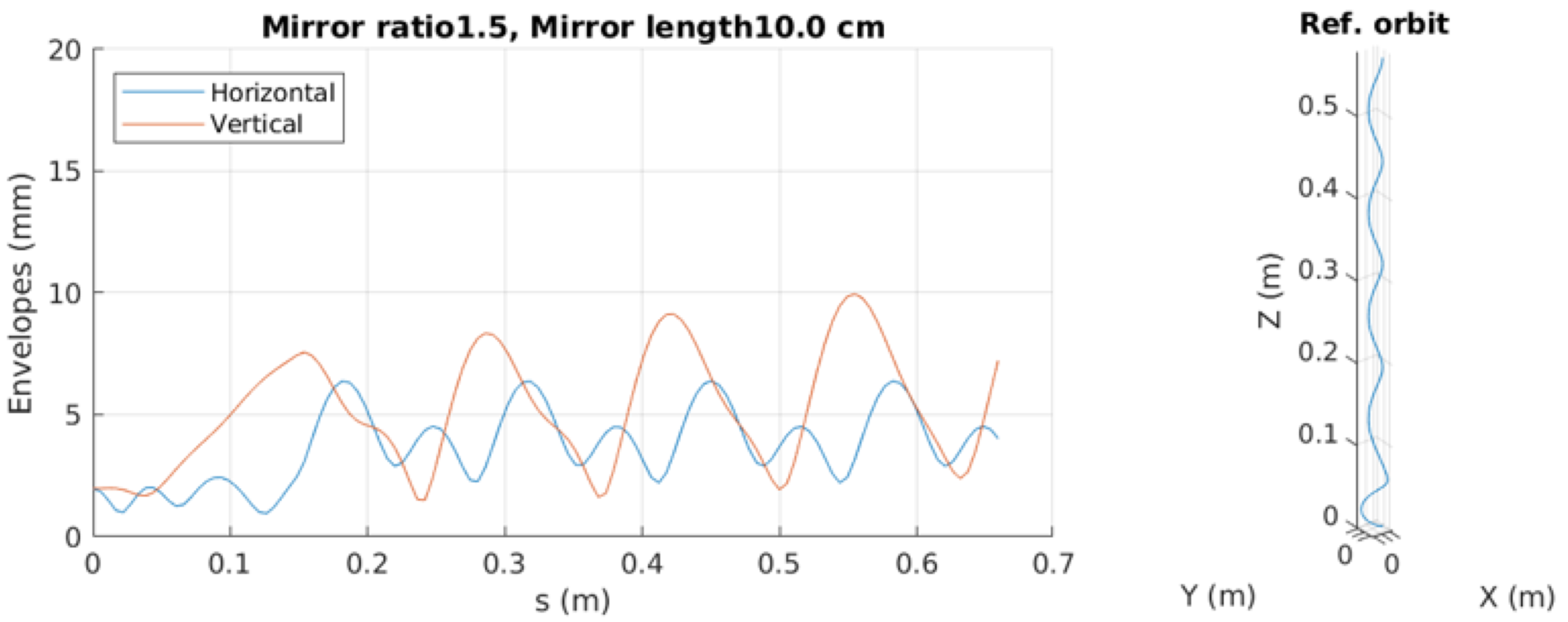


Initial K-V distribution beam at the matching point on the median plane



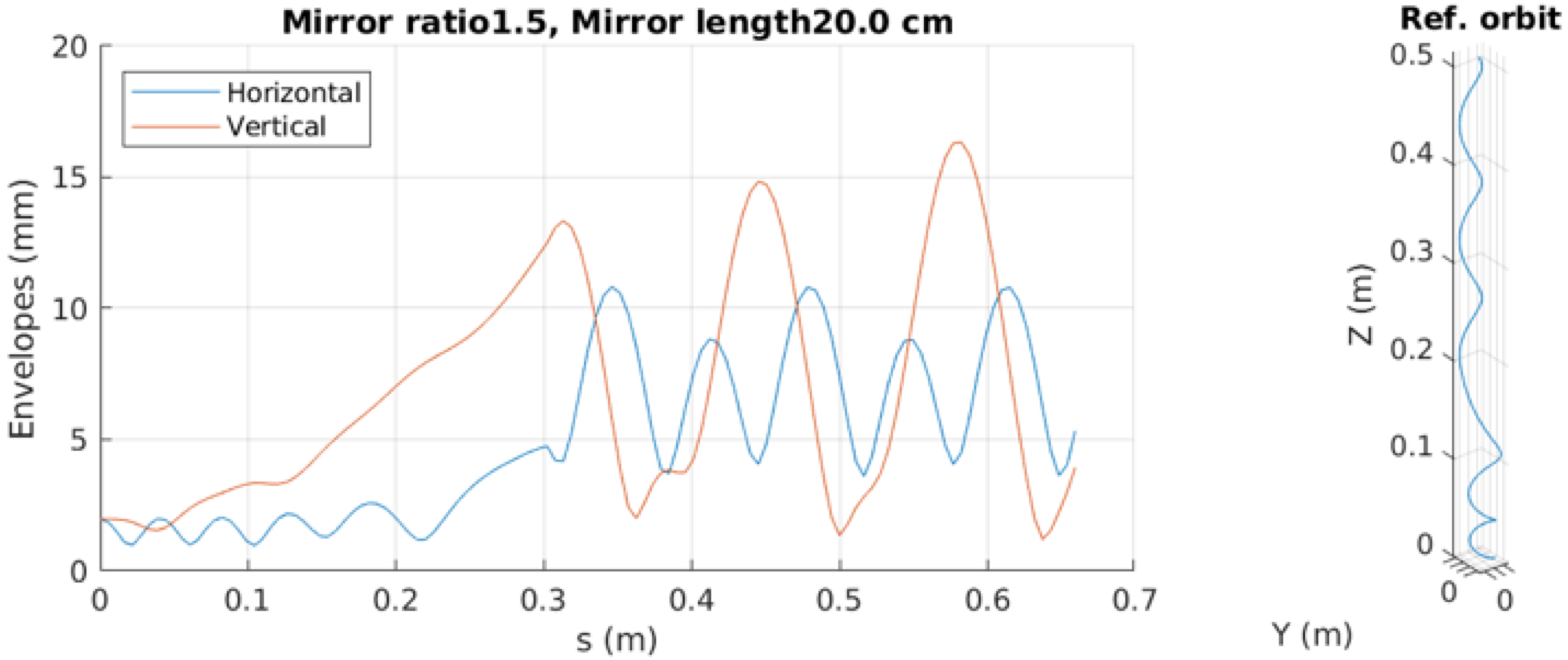
Particle orbits in cartesian frame

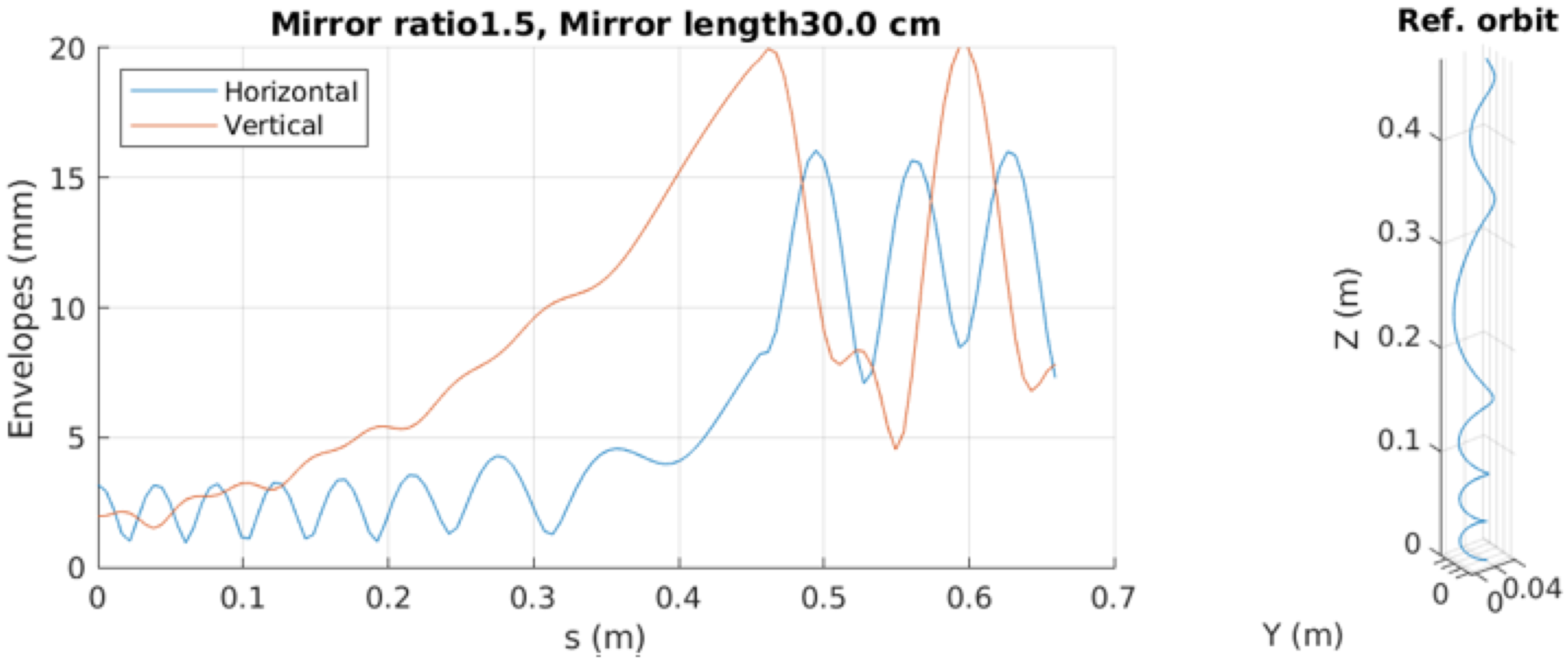
Mirror length



All the reference orbit are confined in the axial injection hole

With increasing the mirror length, the maximum beam size become larger

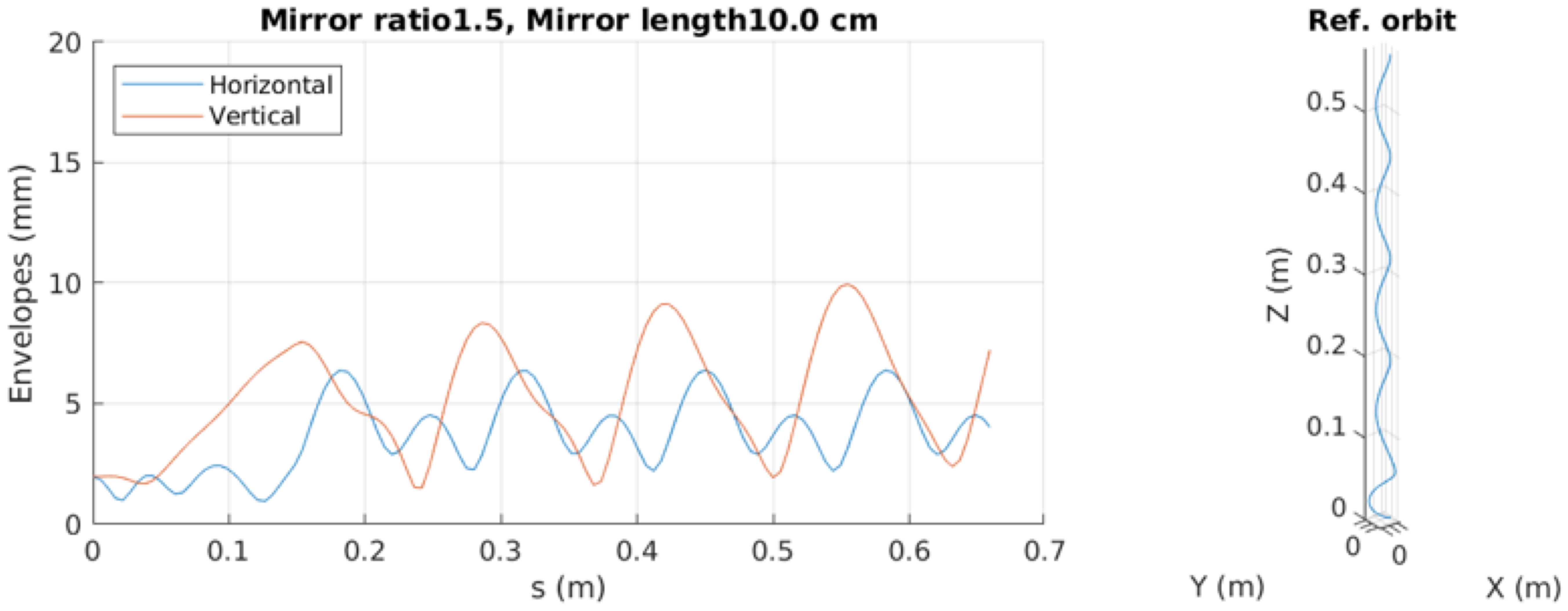


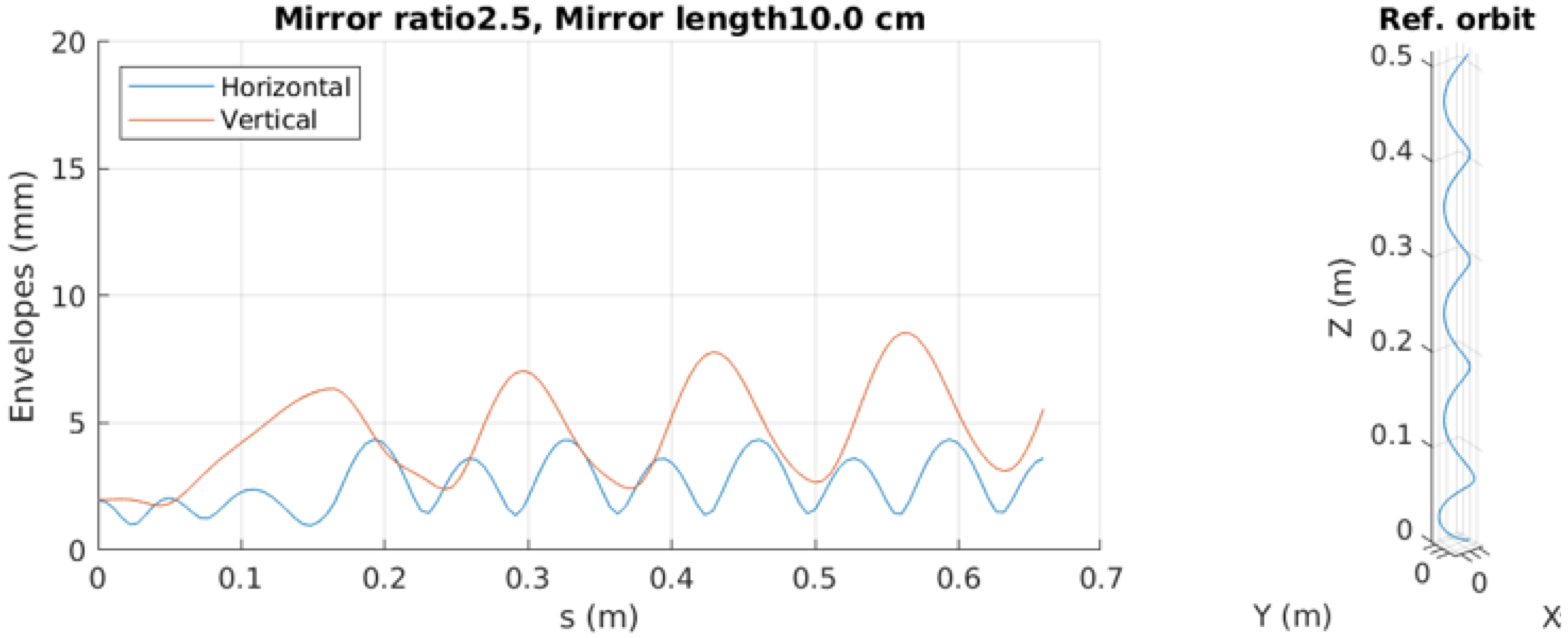


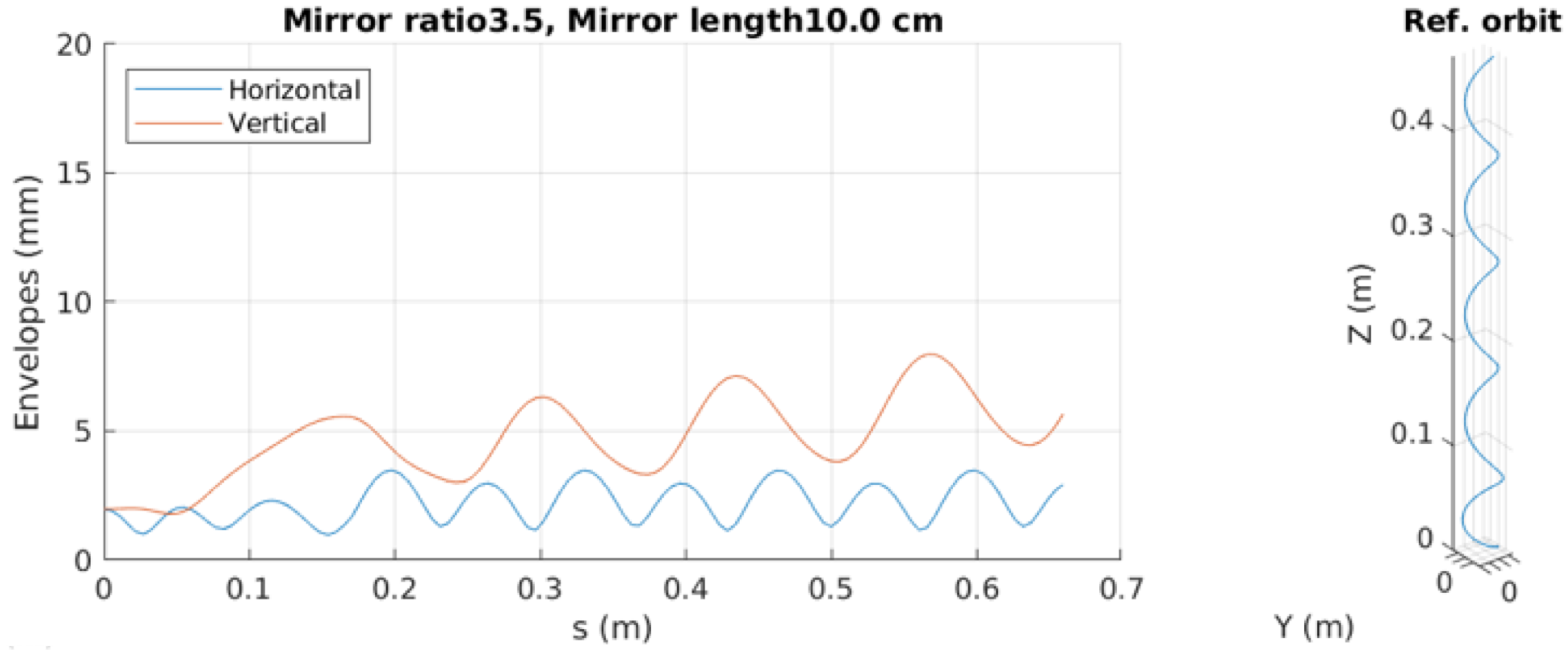
Mirror ratio

We use the optimal mirror length of 10 cm to study the envelopes vs. different mirror ratio

With increasing the mirror ratio, the maximum beam size become smaller

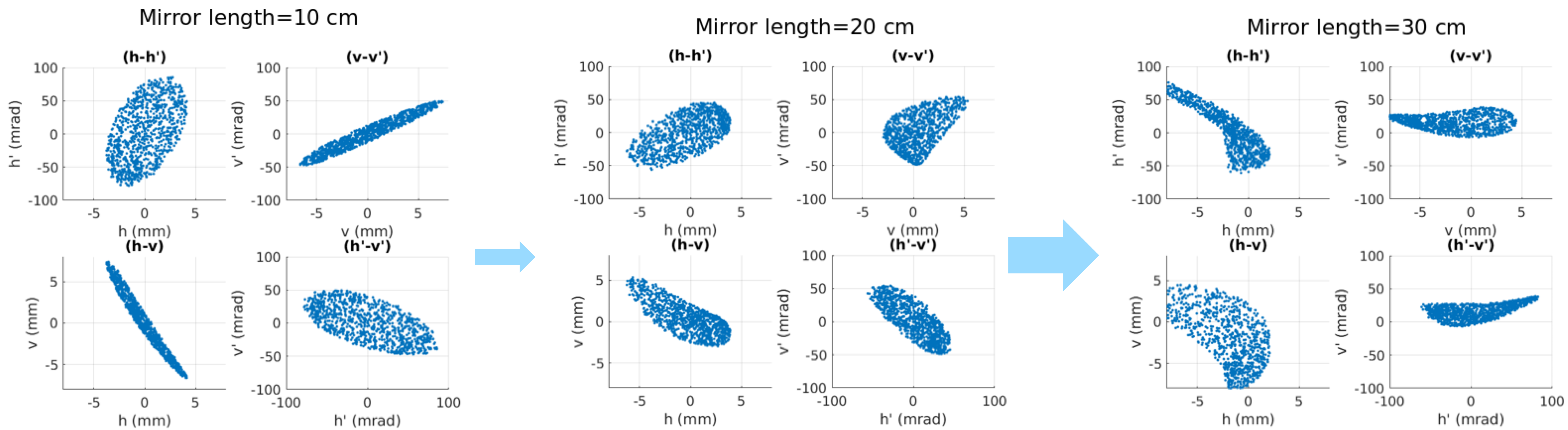




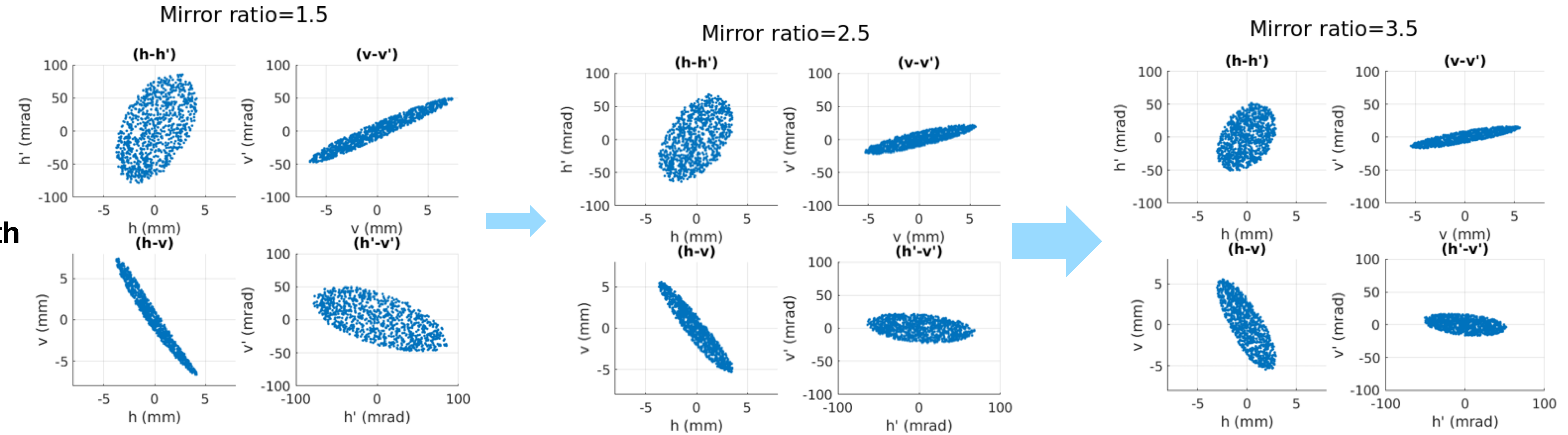


Linearity

Constant mirror ratio



Constant mirror length



Conclusion

- To keep the median plane symmetry, a electrostatic plate should be used at the end of magnetic inflector, which will finally deflect the beam into the median plane with 0 vertical momenta

- The beam could be focused both horizontally and vertically by optimizing the mirror ratio and mirror length

- Further study for how to design the iron shape in the injection hole that could produce the optimal mirror field

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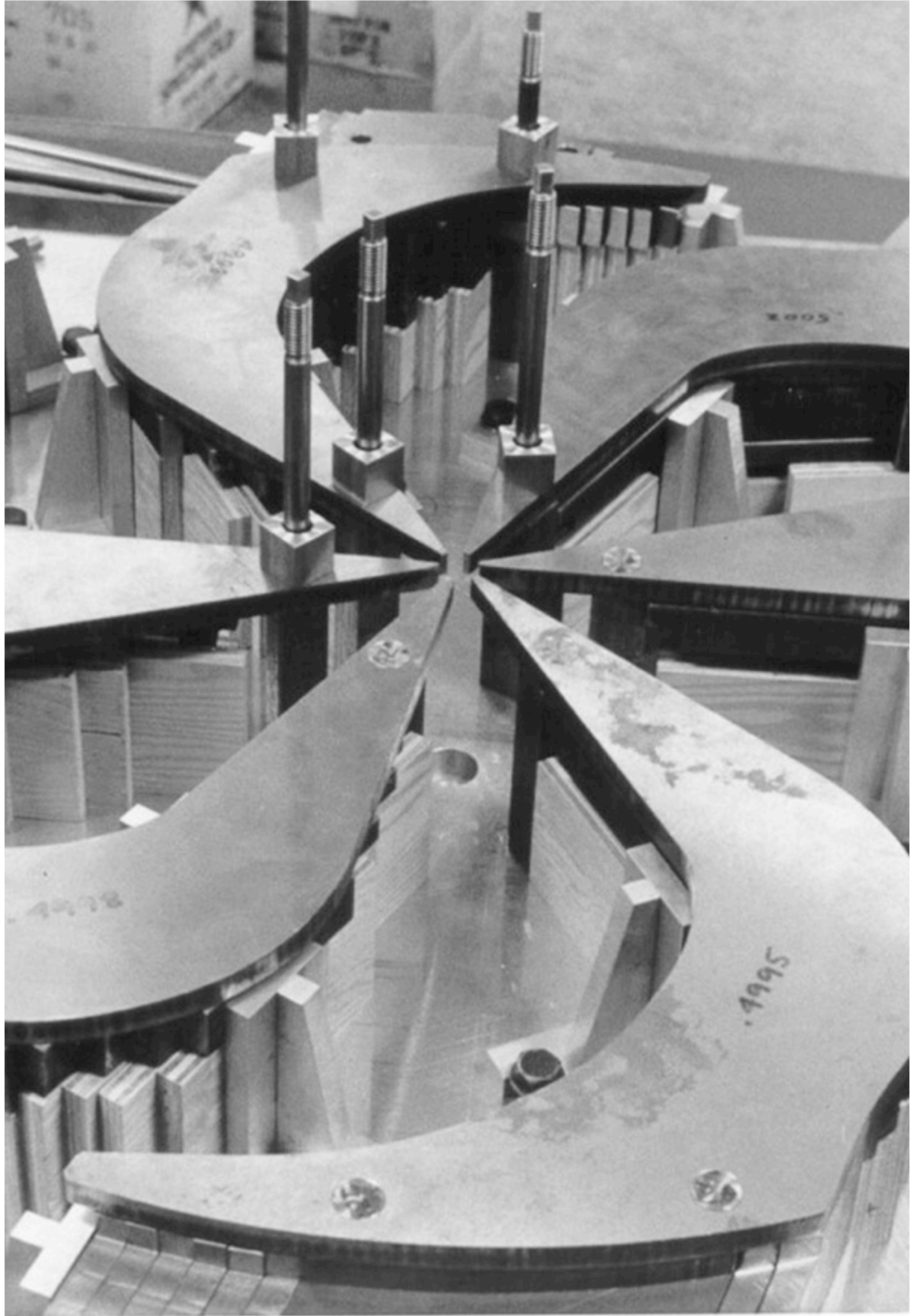

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