Axions and Higher Group Global Symmetry

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Introduction

Axions are important!

- Strong CP problem
- Dark Matter
- Mechanism for Baryogenesis
- String Theory

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Introduction
Generalized Global Symmetries
Axion Electro-dynamics
Axion Yang-Mills
Generalizations and Outlook

- Important feature are axion strings:

$$\oint_\gamma da = 2\pi n f \qquad n \in \mathbb{Z}$$

tell us a lot about theory:

- Axion string confinement: color confinement scale
- String tension: upper bound on scale of UV physics

$$E_{string} \lesssim T_{string}^{1/2}$$

scale of string nucleation from vacuum.

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Introduction

Generalized Global Symmetries

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Generalizations and Outlook

Goal: Derive model independent constraints on UV completion of axion models

- Realize axion string stability in terms of generalized global symmetries
- Study implication of fact that realistic axion strings have finite tension

1. Introduction
2. Generalized Global Symmetries
3. Axion Electrodynamics
4. Axion Yang-Mills
5. Generalizations and Outlook

Global Symmetry

Consider d-dimensional theory

- $U(1)$ global symmetry $\Leftrightarrow$ conserved current

$$\partial_\mu j^\mu = 0 \quad \Longleftrightarrow \quad d * j_1 = 0$$

- Charged operators are local: $\mathcal{O}_q(x)$ charged with respect to $Q = \int_{M_{d-1}} *j_1$
- $U(1)$ symmetry group acts via exponentiated charge operators $U_\phi(M_{d-1}) = e^{iQ\phi}$

$$U_{\phi_1} U_{\phi_2} = U_{\phi_1+\phi_2} \quad U_\phi(M_{d-1})\, \mathcal{O}_q(x) = e^{iq\phi}\, \mathcal{O}_q(x)$$

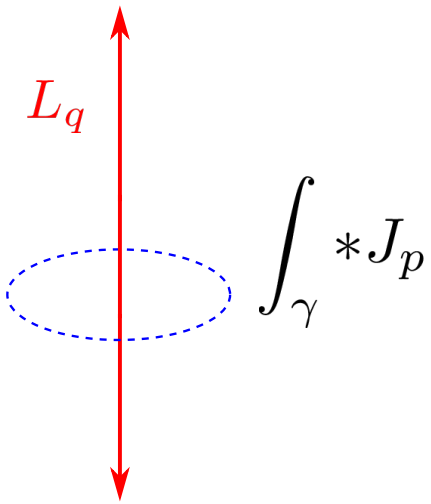
when x inside M_{d-1}.

Generalized Global Symmetry

- Continuous *higher form symmetries* correspond to conserved anti-symmetric tensor field "current"

$$\partial_{\mu_1} J^{\mu_1 \dots \mu_k} = 0 \quad \Longleftrightarrow \quad d * J_k = 0$$

- Integrating $*J_k$ over a closed $(d-k)$-manifold measures the charge of $(k-1)$-dimensional charged operator



Higher Form Global Symmetry

A $U(1)^{(p)}$ *p-form global symmetry* has:

- $(p+1)$-form conserved current J_{p+1}: $d * J_{p+1} = 0$
- Acts on charged p-dimensional operators by exponentiated charge operators or *symmetry defect operators* (SDO):

$$U_\phi(\Sigma) = e^{iQ[\Sigma]\phi} \quad Q[\Sigma] = \int_\Sigma *J_p$$

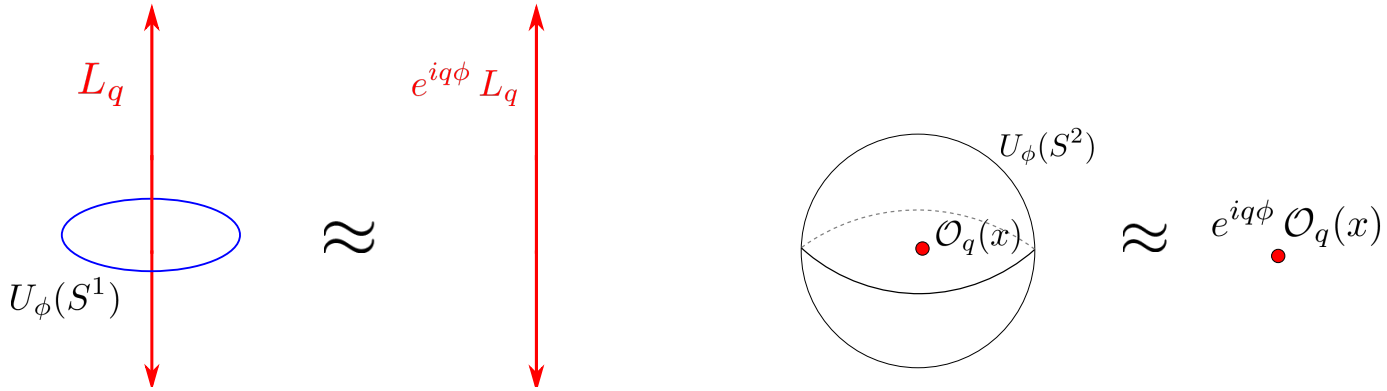
where $\phi \in \mathbb{R}$ and Σ is $(d-p-1)$-dimensional.

- $U_\phi(\Sigma)$ follow $U(1)$ group multiplication

$$U_{\phi_1}(\Sigma) \cdot U_{\phi_2}(\Sigma) = U_{\phi_1+\phi_2}(\Sigma)$$

Higher Form Global Symmetry (cont.)

- Symmetry defect operators act on charged p-dimensional operators by linking



Discrete Higher Form Symmetries

Discrete higher form symmetries will be important for axion physics

- No currents, but can still define notion of SDO:
  - Induce symmetry transformations on charged operators
- In axion models, many discrete symmetries descend from explicitly broken continuous symmetries
  - Can still use formalism of continuous symmetries restricted to discrete subgroup

Background Gauge Fields

Turning on background gauge fields is crucial for analyzing global symmetries

- p-form symmetry has $(p+1)$-degree differential form gauge field

$$S = \ldots + i \int A_{p+1} \wedge * J_{p+1} \quad \delta A_{p+1} = d\Lambda_p$$

- Discrete symmetries: inserting SDO's in path integral
  - Explicitly broken $U(1) \to \mathbb{Z}_N$, background gauge field can be realized as flat $U(1)$ connection with holonomies in $\mathbb{Z}_N$

$$\int_{\Gamma_{p+1}} A_{p+1} = \frac{2\pi k}{N}$$

- Anomalies

$$Z[A_{p+1} + d\Lambda_p] = Z[A_p]\, e^{i \int \mathcal{A}[A,\Lambda]}$$

Higher Group Global Symmetries

- Higher groups describe when higher form symmetries of different degree mix together to form a more general symmetry structure
- General Idea:
  - p-form and q-form symmetry have mixed anomaly ($p > q$)
  - Mixed anomaly canceled by Green-Schwarz mechanism
    - Modifying transformation properties of p-form background gauge field so that they depend on q-form transformation parameters

Higher Group Global Symmetries (cont.)

Example: 2-Groups

Simplest example is 2-Group: mixes 1-form and 0-form global symmetries

- QED with 4 Weyl fermions $\{\psi_i\}$

	$U(1)_{em}$	$U(1)_g^{(0)}$
ψ_1	1	1
ψ_2	1	-1
ψ_3	-1	q
ψ_4	-1	-q

- Mixed $U(1)_g^2 \times U(1)_{em}$-anomaly:

$$\mathcal{I}_6 = \frac{2-2q^2}{8\pi^2} \int F_{em} \wedge F_g \wedge F_g \ .$$

F_{em} is gauge field strength, $F_g = dA_1$ background field strength.

Higher Group Global Symmetries (cont.)

Example: 2-Groups

- $U(1)^{(1)}$ magnetic symmetry: magnetic line operators cannot be broken

$$*J_2 = \frac{F_{em}}{2\pi} \qquad S_l = \frac{i}{2\pi}\int B_2 \wedge F_{em}$$

- Realize anomaly as

$$\mathcal{I}_6 = \frac{(2-2q^2)i}{8\pi^2}\int F_g \wedge F_g \wedge *J_2 \ .$$

- Cancel anomaly by declaring B_2 transforms under background $U(1)_g^{(0)}$ transformations:

$$\delta A_1 = d\lambda_0 \qquad \delta B_2 = d\Lambda_1 - \frac{2-2q^2}{4\pi}\lambda_0 F_a$$

Higher Group Global Symmetries (cont.)

- More generally n-group can have 0-form, 1-form, ... $(n-1)$-form global symmetries that all mix together
  - Because of dimensionality, usually p-form only mixes with symmetries of higher degree (q-form for $q > p$)
- Nested Structure:

$$G_{n-1} \subseteq G_{n-2} \subseteq \ldots \subseteq G_0$$

G_p is n-group of p- and higher form symmetries form closed symmetry structure.

Higher Group Global Symmetries (cont.)

Consider emergent higher group symmetry

- $E_{p\text{-form}}$: energy scale where p-form symmetry emerges
- Nested structure of n-groups gives parametric hierarchy of scales:

$$E_{0\text{-form}} \lesssim E_{1\text{-form}} \lesssim \cdots \lesssim E_{n-1\text{-form}}$$

sames as case of ordinary nested group structure.

- Means parametric violation of hierarchy is not allowed: $E_{q\text{-form}} \not\gg E_{p\text{-form}}$ for $q < p$
  - Violation means (original) higher group symmetry does not emerge along RG flow

Axion Electrodynamics

Let us consider axion electrodynamics:

$$S = \int \frac{1}{2} da \wedge *da + \frac{1}{2g^2} F \wedge *F - \frac{iK}{8\pi^2 f} aF \wedge F$$

where $K \in \mathbb{Z}$ and $a \sim a + 2\pi f$.

- Symmetries associated with axion field:
  - $\mathbb{Z}_K^{(0)}$ 0-form shift symmetry
    - $U(1)^{(0)}$ broken by axion coupling

$$*j_1 = if * da \qquad \delta A_1 = d\lambda_0$$

- $U(1)^{(2)}$ 2-form winding symmetry
  - Stable axion strings

$$*J_3 = \frac{1}{2\pi f} da \qquad \delta A_3 = d\Lambda_2$$

Axion Electrodynamics (cont.)

$$S = \int \frac{1}{2} da \wedge *da + \frac{1}{2g^2} F \wedge *F - \frac{iK}{8\pi^2 f} aF \wedge F$$

- Symmetries associated with gauge field:
  - $\mathbb{Z}_K^{(1)}$ 1-form electric symmetry
    - Stable Wilson lines
    - $U(1)^{(1)} \to \mathbb{Z}_K^{(1)}$ broken by axion coupling

$$*J_e = i * \frac{F}{g^2} \qquad \delta B_e = d\Lambda_e$$

  - $U(1)_m^{(1)}$ 1-form magnetic symmetry
    - Stable monopole/'t Hooft defect

$$*J_m = \frac{F}{2\pi} \qquad \delta B_m = d\Lambda_m$$

Axion Electrodynamics (cont.)

Probe anomalies by turning on background gauge fields (and adding local counter terms)

$$S = \int \frac{1}{2}(da - fA_1) \wedge *(da - fA_1) + \frac{1}{2g^2}(F - B_e) \wedge *(F - B_e) - \frac{iK}{8\pi^2 f} a(F - B_e) \wedge (F - B_e) + \frac{i}{2\pi} F \wedge B_m + \frac{i}{2\pi f} A_3 \wedge da$$

Coupling to background fields can alternatively be written

$$\ldots + \int \frac{i}{2\pi} A \wedge dB_m + \frac{i}{2\pi f} a dA_3$$

Theory is not invariant under background gauge transformations

Axion Electrodynamics (cont.)

$$S = \int \frac{1}{2}(da - fA_1) \wedge *(da - fA_1) + \frac{1}{2g^2}(F - B_e) \wedge *(F - B_e)$$
$$- \frac{iK}{8\pi^2 f} a (F - B_e) \wedge (F - B_e) + \frac{i}{2\pi} A_{gauge} \wedge H_m + \frac{i}{2\pi f} a\, G_4$$

- Cancel variation terms with dynamical fields by modifying transformation

$$\delta A_3 = d\Lambda_2 - \frac{K}{2\pi} B_e \wedge d\Lambda_e - \frac{K}{4\pi} \Lambda_e \wedge d\Lambda_e$$
$$\delta B_m = d\Lambda_m + \frac{K}{2\pi} A_1 \wedge \Lambda_e - \frac{K}{2\pi} \lambda_0 B_e + \frac{K}{2\pi} d\lambda_0 \wedge \Lambda_e$$

and background field strengths

$$H_3 = dB_m + \frac{K}{2\pi} A_1 \wedge B_e \qquad G_4 = dA_3 + \frac{K}{4\pi} B_e \wedge B_e$$

Axion Electrodynamics (cont.)

$$S = \int \frac{1}{2}(da - fA_1) \wedge *(da - fA_1) + \frac{1}{2g^2}(F - B_e) \wedge *(F - B_e)$$
$$- \frac{iK}{8\pi^2 f} a(F - B_e) \wedge (F - B_e) + \frac{i}{2\pi} A_{gauge} \wedge H_m - \frac{i}{2\pi f} a\, G_4$$

- Remaining transformation encodes 't Hooft anomaly

$$\mathcal{I}_5 = \frac{i}{2\pi} \int A_1 \wedge G_4 + B_e \wedge H_m$$

- $\mathbb{Z}_K^{(0)} \times U(1)^{(2)}$-mixed anomaly
- $U(1)_m^{(1)} \times \mathbb{Z}_K^{(1)}$-mixed anomaly

Unsurprising, always mixed anomaly between dual global symmetries: cannot gauge electric and dual magnetic gauge field simultaneously.

Axion Electrodynamics (cont.)

3-Group structure:

- $\mathbb{Z}_K^{(1)} \hookrightarrow U(1)^{(2)}$: $E_{\text{1-form}} \lesssim E_{\text{2-form}}$

$$\delta A_3 = d\Lambda_2 - \frac{K}{2\pi} B_e \wedge d\Lambda_e - \frac{K}{4\pi} \Lambda_e \wedge d\Lambda_e$$

  - $U(1)^{(2)}$ is broken at $E_{\text{2-form}} \lesssim T_{string}^{1/2}$
  - $\Rightarrow \mathbb{Z}_K^{(1)}$ broken $E_{elec} \lesssim T_{string}^{1/2}$
  - Model independent for all axions!
- $\mathbb{Z}_K^{(0)} \times \mathbb{Z}_K^{(1)} \hookrightarrow U(1)_m^{(1)}$: $E_{elec}, E_{axion} \lesssim E_{mag}$

$$\delta B_m = d\Lambda_m + \frac{K}{2\pi} A_1 \wedge \Lambda_e - \frac{K}{2\pi} \lambda_0 B_e + \frac{K}{2\pi} d\lambda_0 \wedge \Lambda_e$$

  - If UV completion is non-abelian gauge theory with $\pi_1(G)$ finite then $U(1)_m^{(1)}$ broken at E_{GUT}
  - $\Rightarrow E_{elec}, E_{axion} \lesssim E_{GUT}$

Axion Yang-Mills

$SU(N)$ Axion Yang-Mills

$$S = \int \frac{1}{2} da \wedge *da + \frac{1}{2g^2} \mathrm{Tr}[F \wedge *F] - \frac{iK}{8\pi^2 f} a \, \mathrm{Tr}[F \wedge F]$$

- Axion Symmetries
  - $\mathbb{Z}_K^{(0)}$ axion shift symmetry
  - $U(1)^{(2)}$ axion winding symmetry
- Yang-Mills Symmetries
  - No magnetic 1-form symmetry
  - $\mathbb{Z}_N^{(1)}$ center symmetry

Axion Yang-Mills and Center Symmetry

- Wilson lines are labeled by $SU(N)$ representation R

$$W = \mathrm{Tr}_R \, P \, e^{i \int A_{gauge}}$$

R labeled by Young-Tableaux Y_R

- Since gluons charged, can partially screen Wilson lines
  - Only invariant is number of boxes in Y_R mod_N
  - Charge of representation R under $\mathbb{Z}_N = Z\Big(SU(N)\Big)$

Axion Yang-Mills and Center Symmetry (cont.)

- Can turn on background gauge field for $\mathbb{Z}_N^{(1)}$
  - Easy way to do this is to lift $SU(N)$ connection to $U(N)$ connection and then subtract off $U(1)$ component

$$F_{SU(N)} \to \hat{F}_{U(N)} - B_e \mathbb{1} \ .$$

- Subtraction requires 2-form Lagrange multiplier φ_2 in action

$$S = \int \frac{1}{2}(da - fA_1) \wedge *(da - fA_1) + \frac{i}{2\pi f} A_3 \wedge da$$
$$+ \frac{1}{2g^2}\mathrm{Tr}[(\hat{F} - B_e) \wedge *(\hat{F} - B_e)] + \frac{1}{2\pi}\int \varphi_2 \wedge \mathrm{Tr}[\hat{F} - B_e \mathbb{1}]$$
$$- \frac{iK}{8\pi^2 f} a \mathrm{Tr}[(\hat{F} - B_e) \wedge (\hat{F} - B_e)]$$

Axion Yang-Mills and Higher Group Symmetry

$$S = \int \frac{1}{2}(da - fA_1) \wedge *(da - fA_1) + \frac{i}{2\pi f} a\, G_4$$
$$+ \frac{1}{2g^2}\mathrm{Tr}[(\hat{F} - B_e) \wedge *(\hat{F} - B_e)] + \frac{1}{2\pi}\int \varphi_2 \wedge \mathrm{Tr}[\hat{F} - B_e \mathbb{1}]$$
$$- \frac{iK}{8\pi^2 f} a \mathrm{Tr}[(\hat{F} - B_e) \wedge (\hat{F} - B_e)]$$

- Again, we can cancel the variation of the action up to 't Hooft anomalies by adding local counter terms and modifying background transformations

$$\delta A_3 = d\Lambda_2 - \frac{KN(N-1)}{2\pi}\Lambda_1 \wedge B_e - \frac{KN(N-1)}{4\pi}\Lambda_1 \wedge d\Lambda_1$$
$$\delta B_e = d\Lambda_1$$
$$G_4 = dA_3 + \frac{KN(N-1)}{4\pi} B_e \wedge B_e$$

Axion Yang-Mills and Higher Group Symmetry (cont.)

- Again, 't Hooft anomaly between dual symmetries

$$\mathcal{I}_5 = \int A_1 \wedge G_4$$

3-group global symmetry:

$$\delta A_3 = d\Lambda_2 - \frac{KN(N-1)}{2\pi}\Lambda_1 \wedge B_e - \frac{KN(N-1)}{4\pi}\Lambda_1 \wedge d\Lambda_1$$

- $\mathbb{Z}_N^{(1)} \hookrightarrow U(1)^{(2)}$: $E_{\text{1-form}} \lesssim E_{\text{2-form}}$
  - $U(1)^{(2)}$ broken at $E_{\text{2-form}} \lesssim T_{string}^{1/2}$
  - $\Rightarrow E_{\text{1-form}} \lesssim T_{string}^{1/2}$

Axion Yang-Mills and Higher Group Symmetry (cont.)

Demonstrate with example: KSVZ model

- $SU(N)$ gauge theory, uncharged complex scalar φ, and fermions in fundamental (ψ_+) and anti-fundamental ψ_-

$$S = \int \frac{1}{2} d\varphi \wedge *d\bar{\varphi} + \frac{1}{2g^2}\mathrm{Tr}[F \wedge *F] + i\bar{\psi}_\pm \bar{\sigma}^\mu D_\mu \psi_\pm$$
$$-m^2(|\varphi|^2 - f^2)^2 + \lambda\bar{\varphi}\psi_+\psi_- + c.c.$$

- Theory has anomalous $U(1)_{PQ}$ symmetry $\psi_\pm \to e^{i\alpha}\psi_\pm$ and $\varphi \to e^{2i\alpha}\varphi$
- Flows in IR to axion-Yang-Mills
  - φ condenses $\Rightarrow$ $\psi_\pm$ acquire a mass and freeze out
  - $U(1)_{PQ}$ pseudo-goldstone is axion

Axion Yang-Mills and Higher Group Symmetry (cont.)

Theory has 3-Group global symmetry. Want to demonstrate

$$E_{\text{1-form}} \lesssim E_{\text{2-form}}$$

- UV theory has no $\mathbb{Z}_N^{(1)}$ center symmetry because $\psi_\pm$ charged:

$$E_{\text{1-form}} = m_{\psi_\pm} = \lambda f$$

- In this model, $E_{\text{2-form}}$ is scale above which axion strings can decay:
  - Energy for radial mode of φ to traverse over center of Mexican hat potential ($V(\varphi) = m^2(|\varphi|^2 - f^2)^2$)

$$E_{\text{1-form}} = \sqrt{m} f < T_{string}^{1/2} = f$$

Leads to inequality:

$$\lambda \lesssim \sqrt{m}$$

Axion Yang-Mills and Higher Group Symmetry (cont.)

Consider parametric violation: $\lambda >> \sqrt{m}$.

- Need to integrate out $\psi_\pm$ before radial mode of φ
  - Leads to large corrections to the effective potential

$$V_{eff} = \frac{\lambda^4 NK}{8\pi^2}\left(|\varphi|^4 \log\left(\frac{\lambda|\varphi|}{\Lambda_{UV}}\right) - \frac{|\varphi|^4}{4}\right) + m^2(|\varphi|^2 - f^2)^2$$

has minimum

$$\langle|\varphi|\rangle \approx \frac{\Lambda_{UV}}{\lambda} e^{-\frac{8\pi^2 m^2}{NK\lambda^4}} >> \Lambda_{UV}$$

- Leads to breakdown of effective field theory: theory flows to different IR fixed point.
- 3-Group Hierarchy satisfied!

Generalizations and Overview

- From pure symmetry considerations we were able to derive constraints on UV completions of axion models:

$$E_{\text{1-form}} \lesssim E_{\text{2-form}}$$

  - Able to demonstrate explicitly in example
- Analysis can also be applied to theories with low energy fermions: axion-QCD, axion-QED (see paper)
- Hopefully will be applicable in many phenomenologically interesting models

End.

The End.
Questions?