

Axion assisted Schwinger effect

Yohei Ema

DESY → U. of Minnesota (this fall)

CHEW on Axion Physics 29.07.2021

Based on [2101.05192](#) with V. Domcke (CERN) and K. Mukaida (KEK)

See also [1910.01205](#)



Motivation

This talk: study Schwinger pair production with axion.

— WHY? —

- **Axion inflation:**

$\phi F\tilde{F}$ induces tachyonic gauge field production.

[Anber, Peloso 09; ...]

→ Interesting phenos: non-gaussianity, gravitational wave, etc..

→ Fermion production stops gauge field production, can be a game changer.

- **Friction term to relaxion:**

Tachyonic gauge field production used for friction.

[Hook, Marques-Tavares 16]

→ Fermion production can again change the story.

Outline

1. Motivation

2. Review of Schwinger effect

3. Axion assisted Schwinger effect

4. Summary

Outline

1. Motivation

2. Review of Schwinger effect

3. Axion assisted Schwinger effect

4. Summary

Schwinger effect

Schwinger effect: pair production in a strong electric field.

- Assume constant electric field in z -direction $\rightarrow A_\mu = (0, 0, 0, Et)$.

$$\rightarrow \Omega^2(t) = (p_z + gEt)^2 + p_T^2 + m^2, \quad p_T^2 = p_x^2 + p_y^2.$$

- Frequency depends on time \rightarrow positive and negative frequency modes mixed.

$$ue^{-i\Theta} \rightarrow \alpha ue^{-i\Theta} + \beta ve^{i\Theta}, \quad \Theta = \int \Omega dt.$$

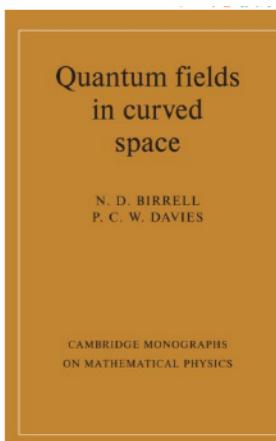
- Time evolution of Bogoliubov coefficients derived from Dirac equation:

$$\dot{\alpha} = -\frac{m_T \dot{\Pi}_z}{2\Omega^2} e^{2i\Theta} \beta, \quad \dot{\beta} = \frac{m_T \dot{\Pi}_z}{2\Omega^2} e^{-2i\Theta} \alpha, \quad \Pi_z = p_z + gEt.$$

- Interpreted as production from time-dependent background.

occupation number: $n_p = 2 |\beta|^2$.

chirality



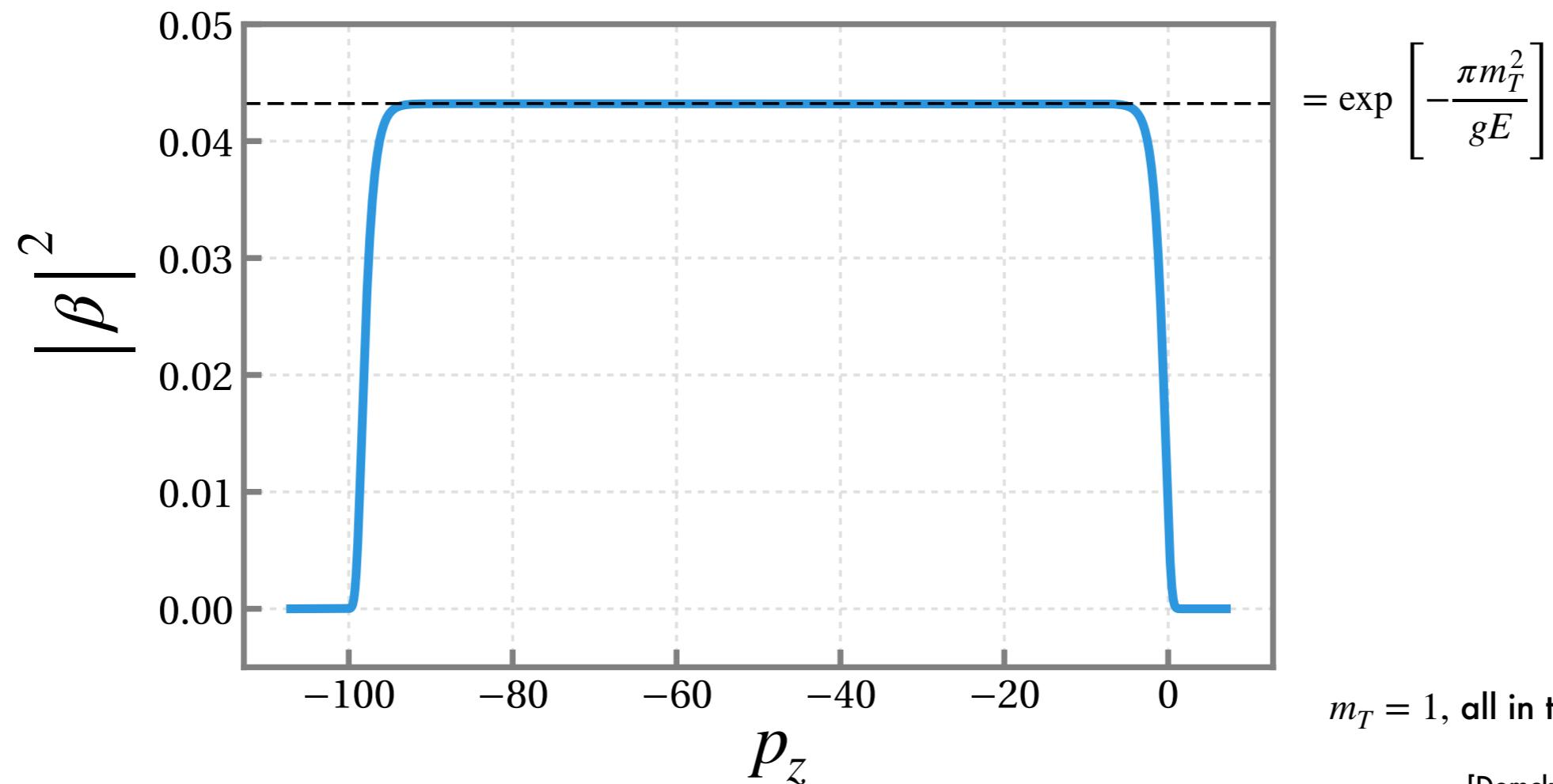
Numerical results

- Impose electric field for $0 < t < 100$ (turned on and off adiabatically).
- Develop a plateau for $-100 < p_z < 0$, modes that pass $\Pi_z = 0$.

$$* \quad \Pi_z = p_z + gEt.$$

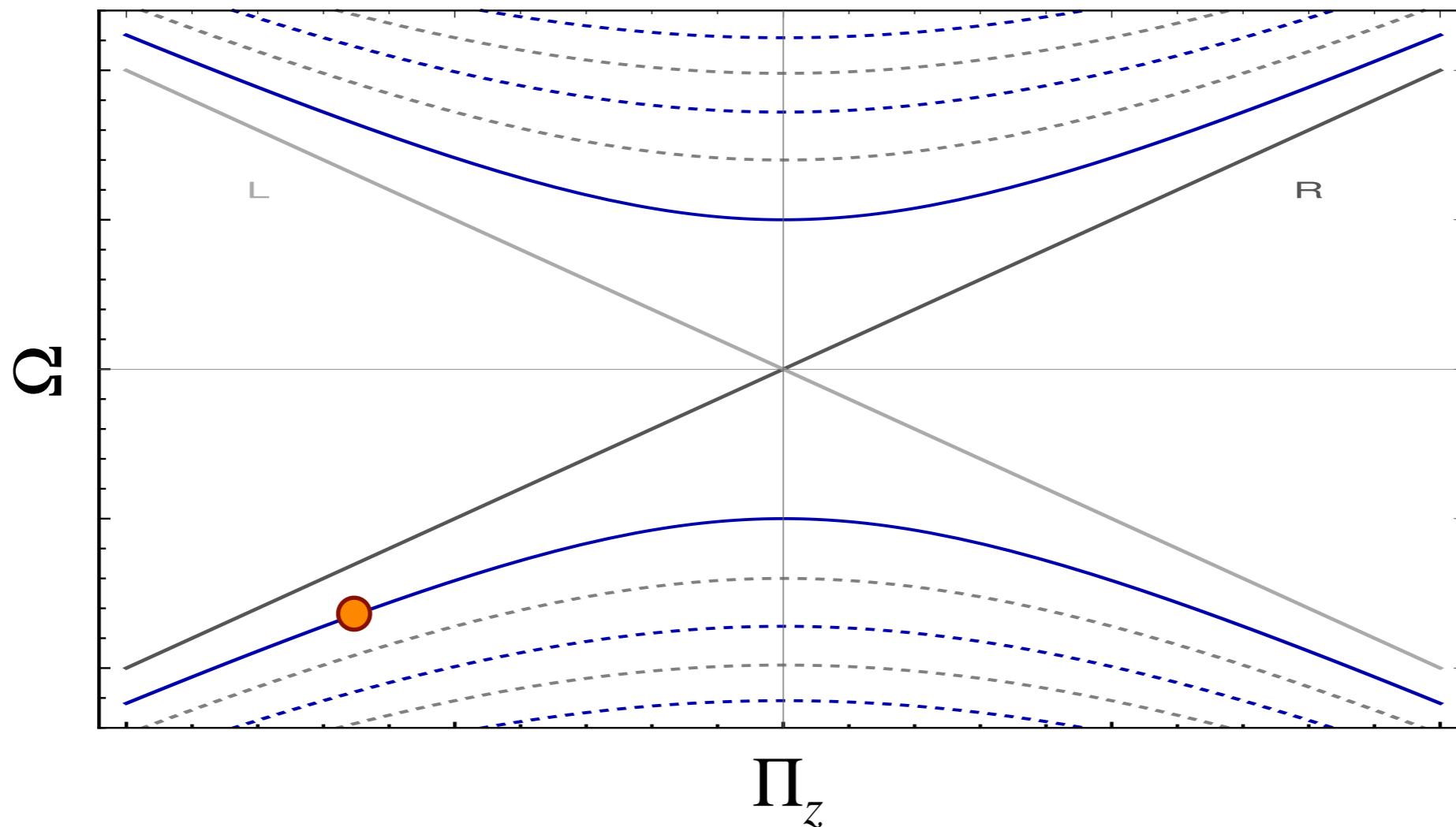
- Height of the plateau:

$$|\beta|^2 \simeq \exp \left[-\frac{\pi m_T^2}{gE} \right], \quad m_T^2 = p_T^2 + m^2.$$



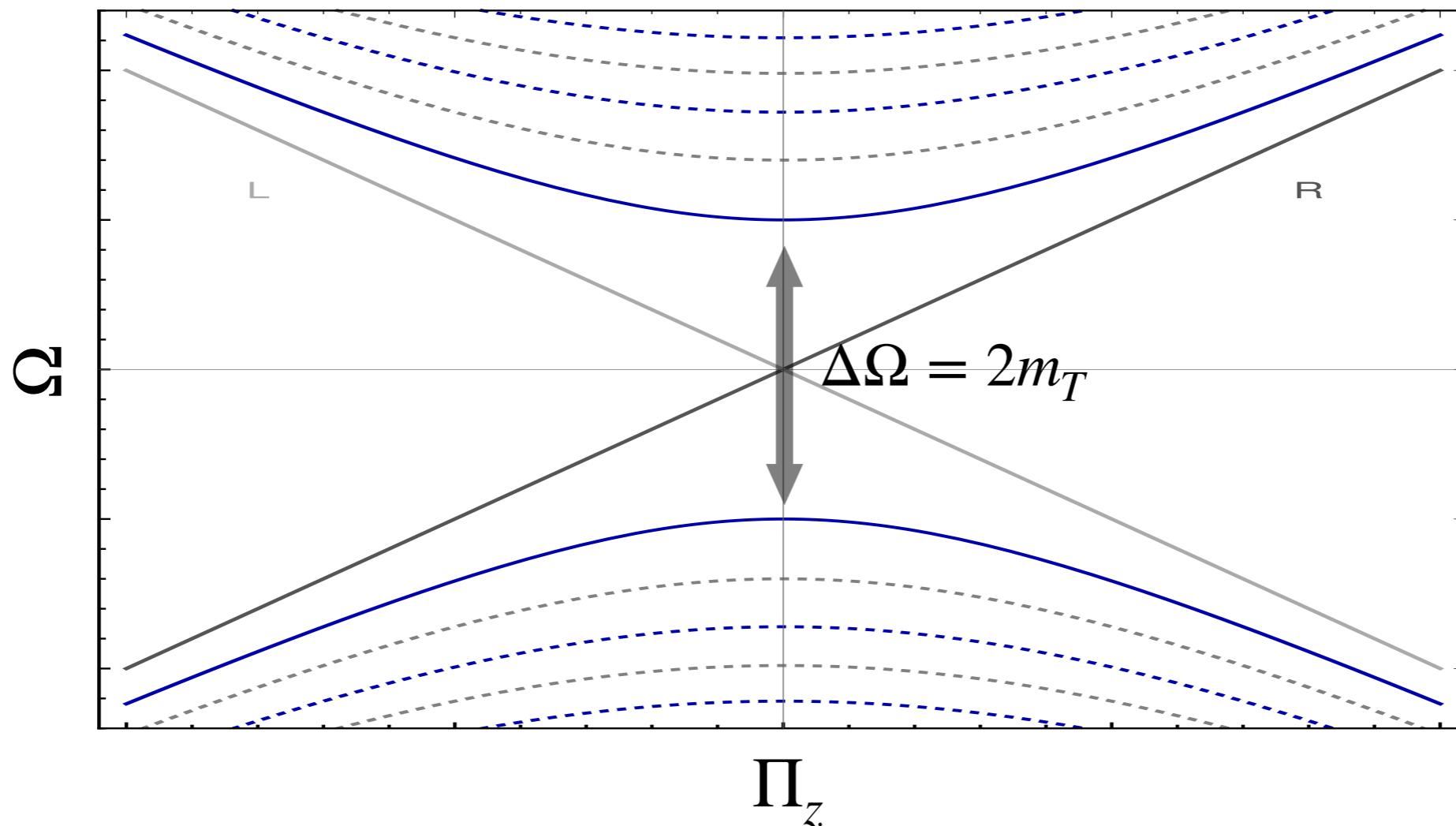
Intuitive picture

- Particles in Dirac sea accelerated by electric field.
- Tunneling most probable for $\Pi_z = 0$ due to the smallest gap.
 - * $\Pi_z = p_z + gEt$.
- Gap size $= 2m_T \rightarrow$ suppression controlled by this parameter.



Intuitive picture

- Particles in Dirac sea accelerated by electric field.
- Tunneling most probable for $\Pi_z = 0$ due to the smallest gap.
 - * $\Pi_z = p_z + gEt$.
- Gap size $= 2m_T \rightarrow$ suppression controlled by this parameter.



Phase integral method

- One can quantify this intuitive picture by a semiclassical method. [Dumlu, Dunne 11; ...]

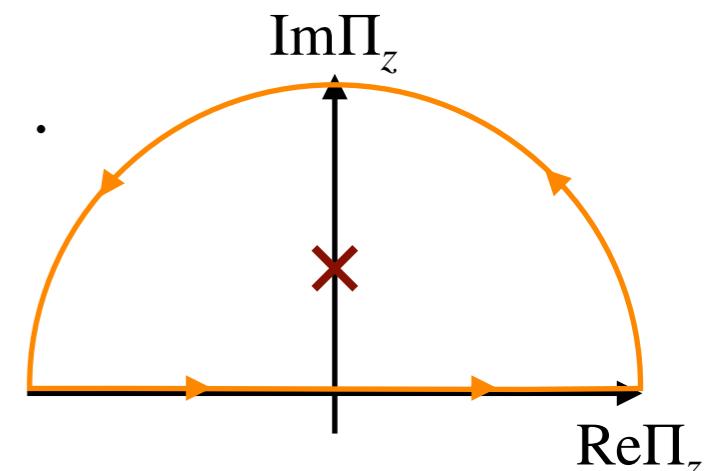
- Production exponentially suppressed \rightarrow Born approximation: $\alpha \simeq 1$.

$$\Rightarrow \dot{\beta} \simeq \frac{m_T \dot{\Pi}_z}{2\Omega^2} e^{-2i\Theta}, \text{ integrated as } \beta \simeq \int_{-\infty}^{\infty} dt \frac{m_T \dot{\Pi}_z}{2\Omega^2} e^{-2i\Theta}.$$

- Close the integral contour and apply residue theorem:

$$\beta \simeq \frac{2\pi i}{gE} \sum_{\Pi_z = \Pi_\otimes} \text{Res}_{\Pi_\otimes} \left[\frac{m_T \dot{\Pi}_z}{2\Omega^2} e^{-2i \int^{\Pi_z} d\Pi_z \frac{\Omega}{gE}} \right].$$

poles: $\Omega = 0$ (turning points) $\rightarrow \Pi_\otimes = \pm im_T$.



- Suppression = “distance” of the pole from the real axis (\sim gap size).

$$|\beta|^2 \sim \exp \left[-4 \int_0^{m_T} \sqrt{m_T^2 - \Pi_z^2} \frac{d\Pi_z}{gE} \right] = \exp \left[-\frac{\pi m_T^2}{gE} \right].$$

Outline

1. Motivation

2. Review of Schwinger effect

3. Axion assisted Schwinger effect

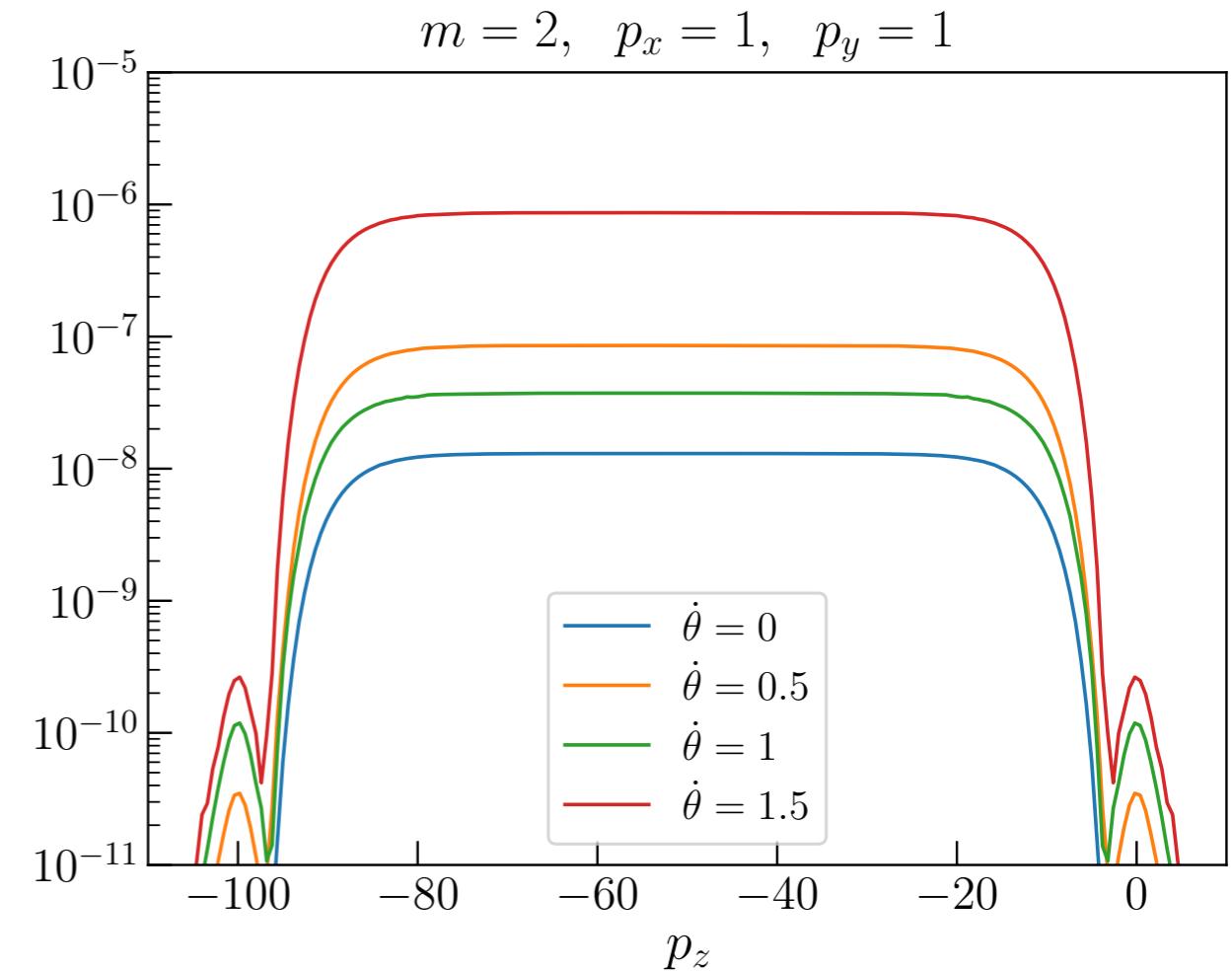
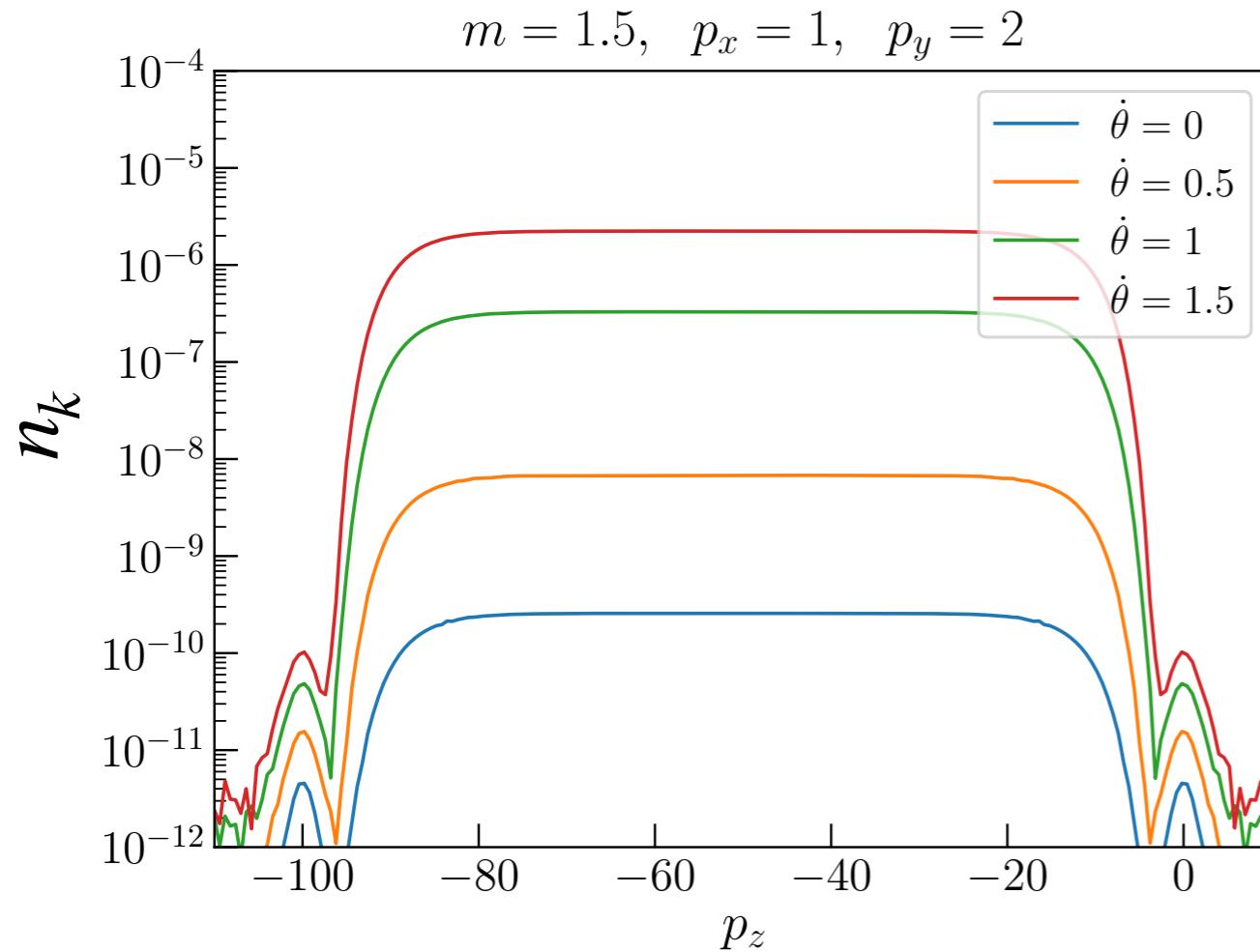
4. Summary

Axion coupling: spectrum

- Include all dim-5 operators of axion-fermion-gauge field system.

Dirac eqn:
$$\left[i\gamma^\mu D_\mu - me^{2ic_m\gamma_5\phi/f_a} + c_5 \frac{\partial_\mu \phi}{f_a} \gamma^\mu \gamma_5 \right] \psi = 0.$$

- Impose constant electric field and axion velocity.
- Height of the plateau: enhanced and strongly dependent on $\dot{\theta}_{5+m} \equiv (c_5 + c_m)\dot{\phi}/f_a$ (!)

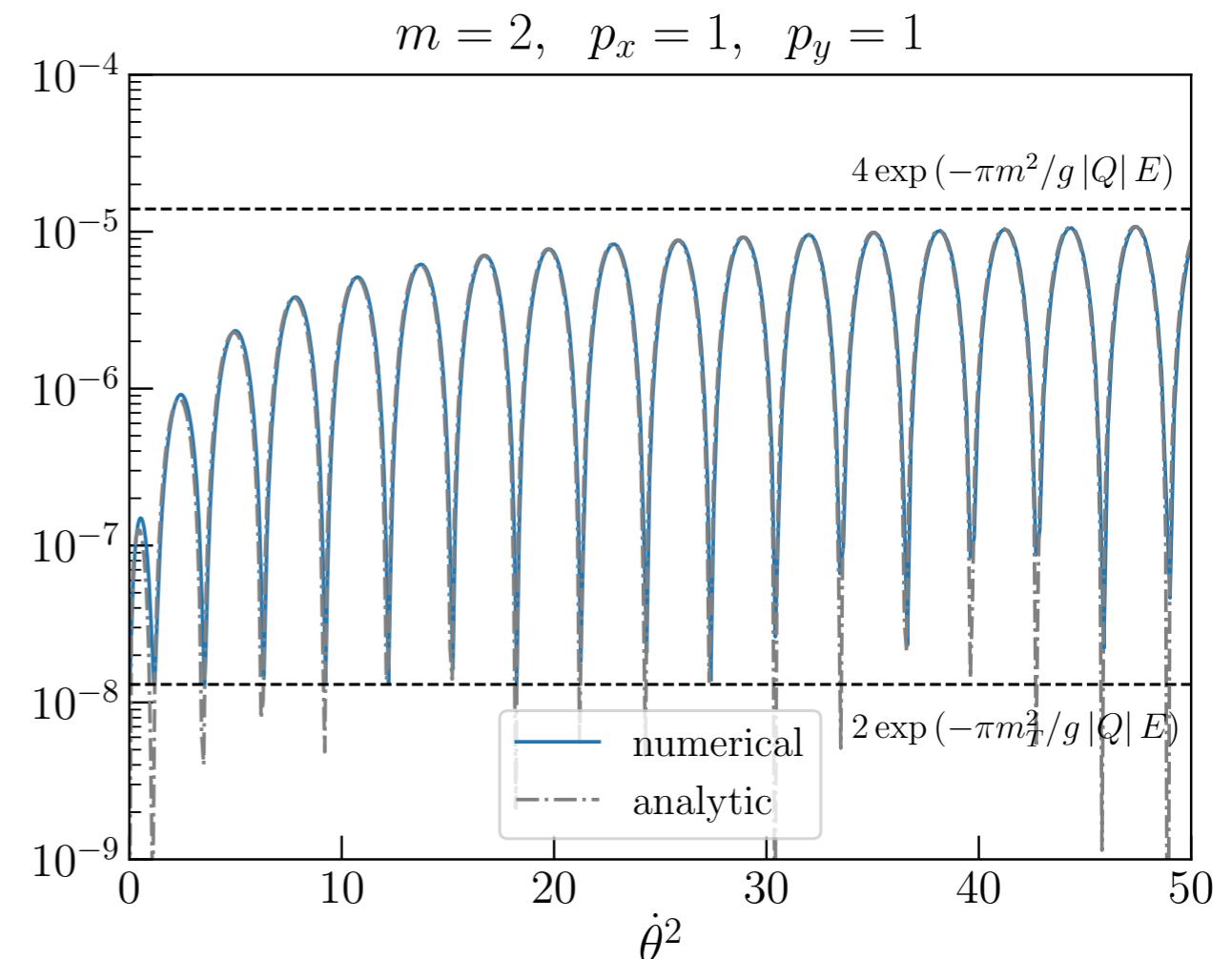
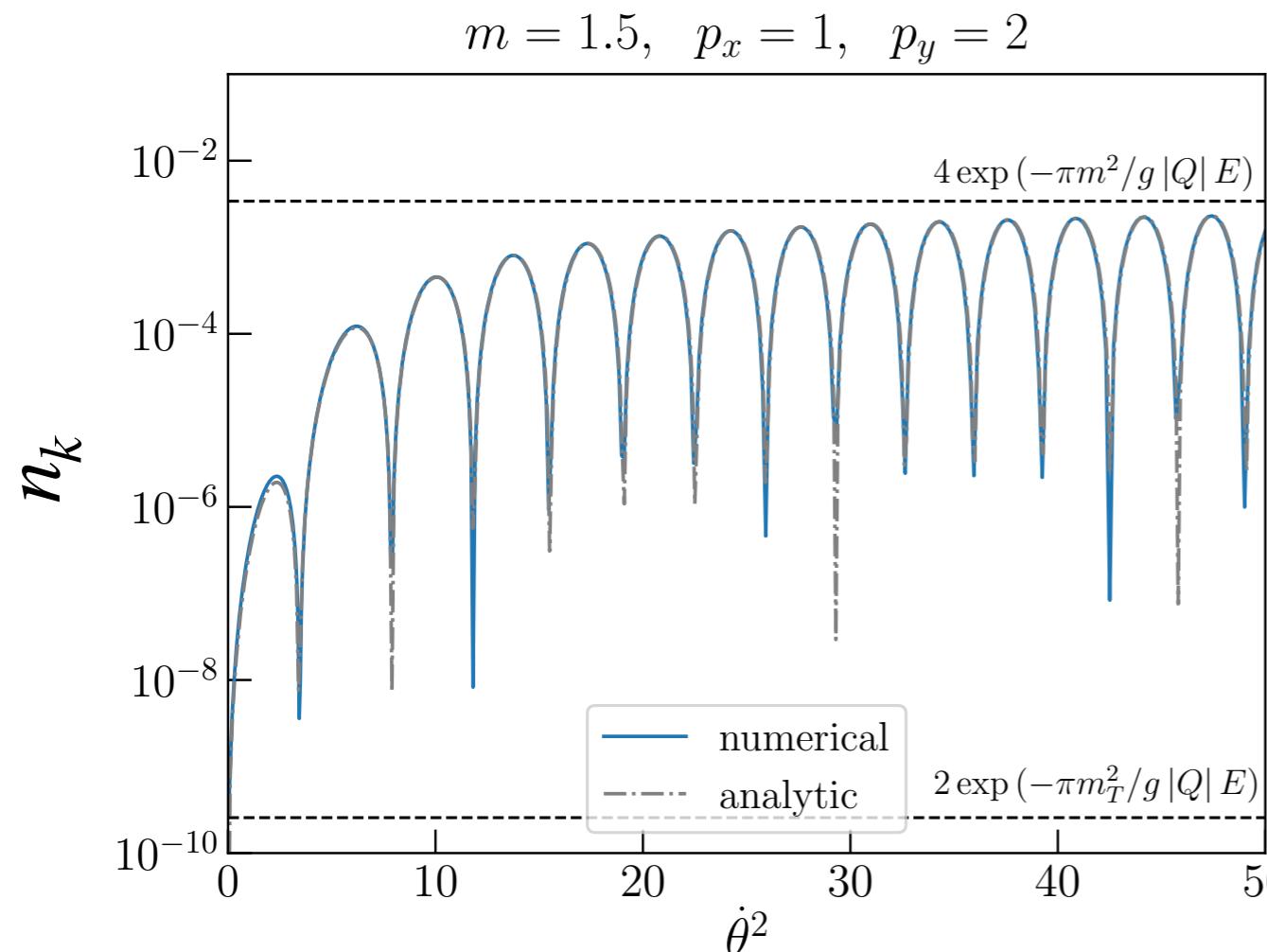


Axion coupling: height

- Plot the occupation number versus $\dot{\theta}_{5+m}$ for $p_z = -50$ (middle of the plateau).
- Occupation number for large enough $\dot{\theta}_{5+m}$:

$$n_k \sim \exp \left[-\frac{\pi m^2}{g E} \right], \text{ or suppression from } p_T \text{ is gone!}$$

- On top of the enhancement, the height oscillates with $\dot{\theta}_{5+m}$.



Interpret: spin-momentum coupling

- Take the non-relativistic limit $m^2 \gg gE, \dot{\theta}_{5+m}^2, p_T^2$:

$$\mathcal{L}_\eta = \eta^\dagger i\partial_0 \eta + \frac{1}{2m} \eta^\dagger \left(\vec{\Pi}^2 - 2\dot{\theta}_{5+m} \vec{\Pi} \cdot \vec{\sigma} + \dot{\theta}_{5+m}^2 \right) \eta + \mathcal{O}\left(\frac{1}{m^2}\right), \quad \vec{\Pi} = \vec{p} + g\vec{A}.$$

 axion induces spin-momentum interaction.

- Diagonalizing the positive frequency two component spinor η gives

$$\tilde{\Omega}_{\text{NR}}^\pm = \frac{1}{2m} \left(\sqrt{\Pi_z^2 + p_T^2} \pm \dot{\theta}_{5+m} \right)^2.$$

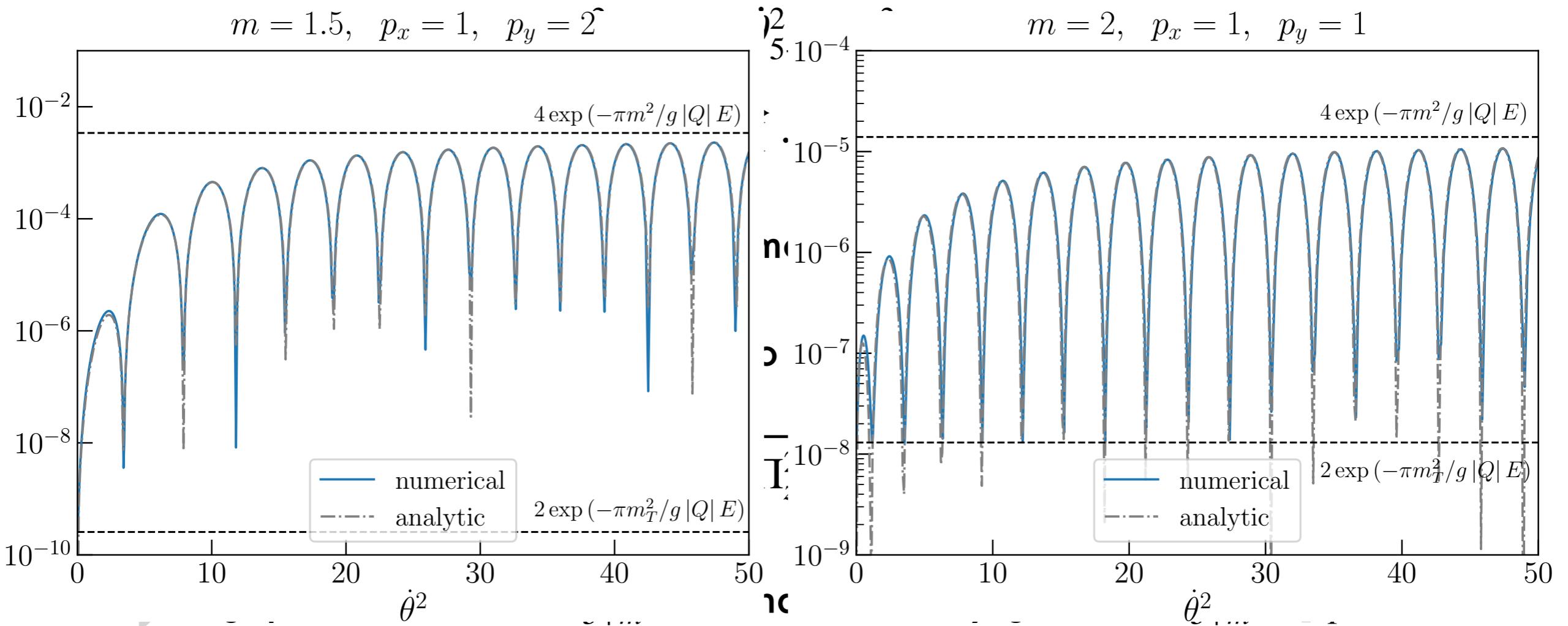
 “gap” is smaller for $\dot{\theta}_{5+m} \neq 0$ and eventually gone for $\dot{\theta}_{5+m} > p_T$.

- Indeed empirical formula reproduces numerical results for $m, p_T \gtrsim gE$:

$$n_k \simeq 2 \left| \exp \left(2i \int_0^{\Pi_+} \frac{d\Pi_z}{gE} \tilde{\Omega}^- \right) \right|^2 - 2\text{Re} \left[\exp \left(2i \int_{\Pi_-}^{\Pi_+} \frac{d\Pi_z}{gE} \tilde{\Omega}^- \right) \right],$$

where $\tilde{\Omega}^- = \sqrt{\left(\sqrt{\Pi_z^2 + p_T^2} - \dot{\theta}_{5+m} \right)^2 + m^2}$, Π_\pm defined as $\tilde{\Omega}^- \Big|_{\Pi_z=\Pi_\pm} = 0$: turning points.

Interpret: spin-momentum coupling

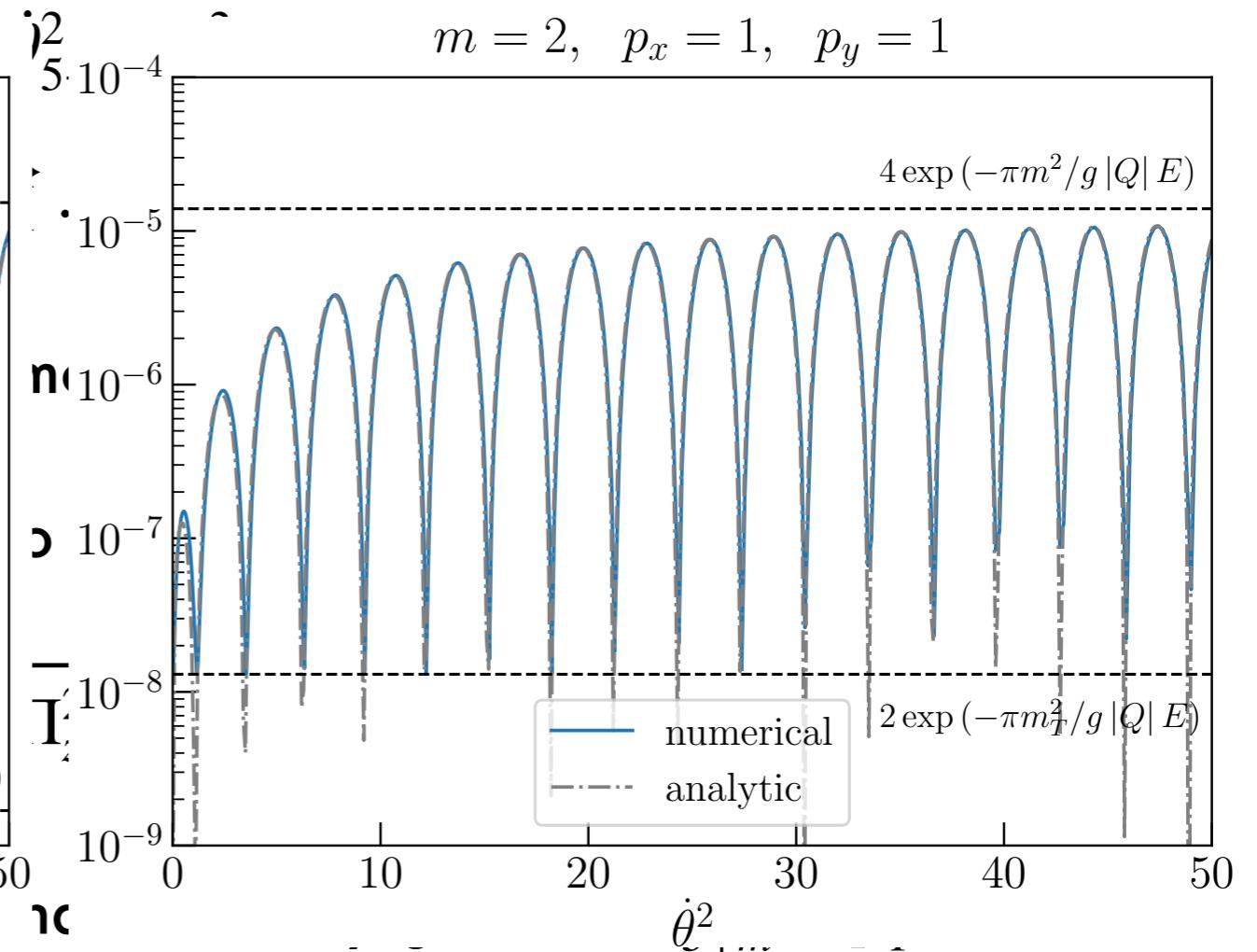
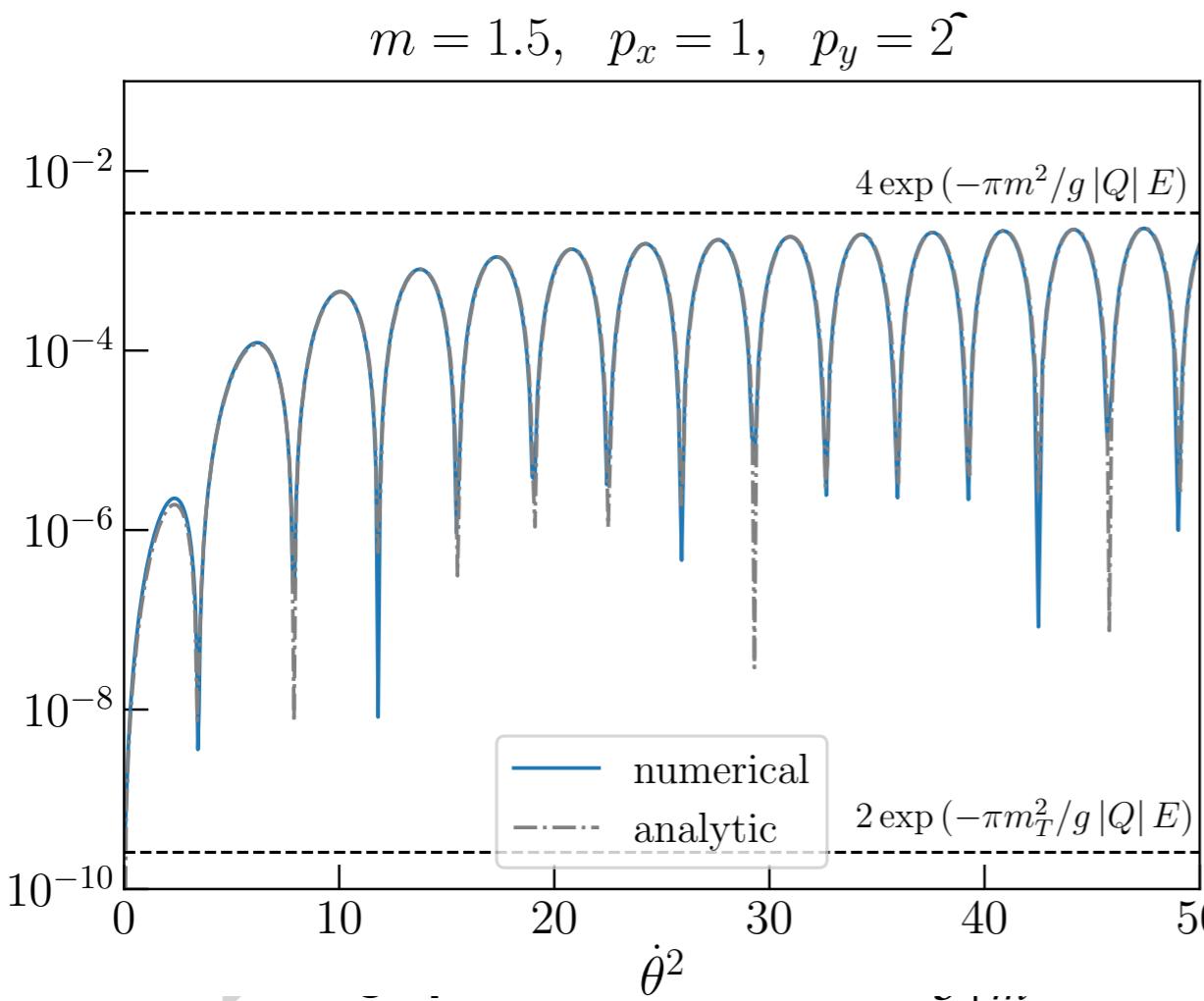


- Indeed empirical formula reproduces numerical results for $m, p_T \gtrsim gE$:

$$n_k \simeq 2 \left| \exp \left(2i \int_0^{\Pi_+} \frac{d\Pi_z}{gE} \tilde{\Omega}^- \right) \right|^2 - 2\text{Re} \left[\exp \left(2i \int_{\Pi_-}^{\Pi_+} \frac{d\Pi_z}{gE} \tilde{\Omega}^- \right) \right],$$

where $\tilde{\Omega}^- = \sqrt{\left(\sqrt{\Pi_z^2 + p_T^2} - \dot{\theta}_{5+m} \right)^2 + m^2}$, Π_\pm defined as $\tilde{\Omega}^- \Big|_{\Pi_z=\Pi_\pm} = 0$: turning points.

Interpret: spin-momentum coupling



- Indeed empirical formula reproduces numerical results for $m, p_T \gtrsim gE$:

$$n_k \simeq 2 \left| \exp \left(2i \int_0^{\Pi_+} \frac{d\Pi_z}{gE} \tilde{\Omega}^- \right) \right|^2 - 2 \text{Re} \left[\exp \left(2i \int_{\Pi_-}^{\Pi_+} \frac{d\Pi_z}{gE} \tilde{\Omega}^- \right) \right],$$

where $\tilde{\Omega}^- = \sqrt{\left(\sqrt{\Pi_z^2 + p_T^2} - \dot{\theta}_{5+m} \right)^2 + m^2}$, Π_{\pm} defined as $\tilde{\Omega}^- \Big|_{\Pi_z=\Pi_{\pm}} = 0$: turning points.

cause oscillation: interference

Outline

1. Motivation

2. Review of Schwinger effect

3. Axion assisted Schwinger effect

4. Summary

Summary

- Rate of standard Schwinger effect:

$$(\text{rate}) \propto \exp \left[-\frac{\pi (m^2 + p_T^2)}{gE} \right] \text{ with } p_T^2 = p_x^2 + p_y^2 \text{ for } \vec{E} \propto \hat{z}.$$

- Axion velocity assists the production, resulting in

$$(\text{rate}) \propto \exp \left[-\frac{\pi m^2}{gE} \right] \text{ for } \dot{\theta}_{5+m} \gg m, p_T.$$

- Can be interpreted as axion induced spin-momentum interaction.

Back up

Theoretical aspects

- Chiral rotation invariance:

✓ c_5 and c_m always appears in the combination $c_5 + c_m$ in our computation.

- Anomaly equation:

$$\partial_\mu J_5^\mu = 2im\bar{\psi}e^{2i\theta_m\gamma_5}\gamma_5\psi - \frac{g^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

→ non-trivial check of computation for $\vec{E} \parallel \vec{B}$ ($F\tilde{F} = -4\vec{E} \cdot \vec{B}$).

✓ Follows from EoM of the Bogoliubov coefficients.

- Euler-Heisenberg Lagrangian:

[Domcke, YE, Mukaida 19]

α and β contain terms suppressed by power, not exp, of m for $\dot{E} \neq 0$.

✓ Identified as higher dimensional operators, i.e., Euler-Heisenberg terms.

(Gone after turning off time-dependence, not to be confused with particle production.)

Preliminary

- Include all dim-5 operators of axion-fermion-gauge field system.

$$\text{Dirac eqn: } \left[i\gamma^\mu D_\mu - me^{2ic_m\gamma_5\phi/f_a} + c_5 \frac{\partial_\mu \phi}{f_a} \gamma^\mu \gamma_5 \right] \psi = 0.$$

- Assume constant electric field + constant axion velocity.



Again positive and negative frequency modes mixed.

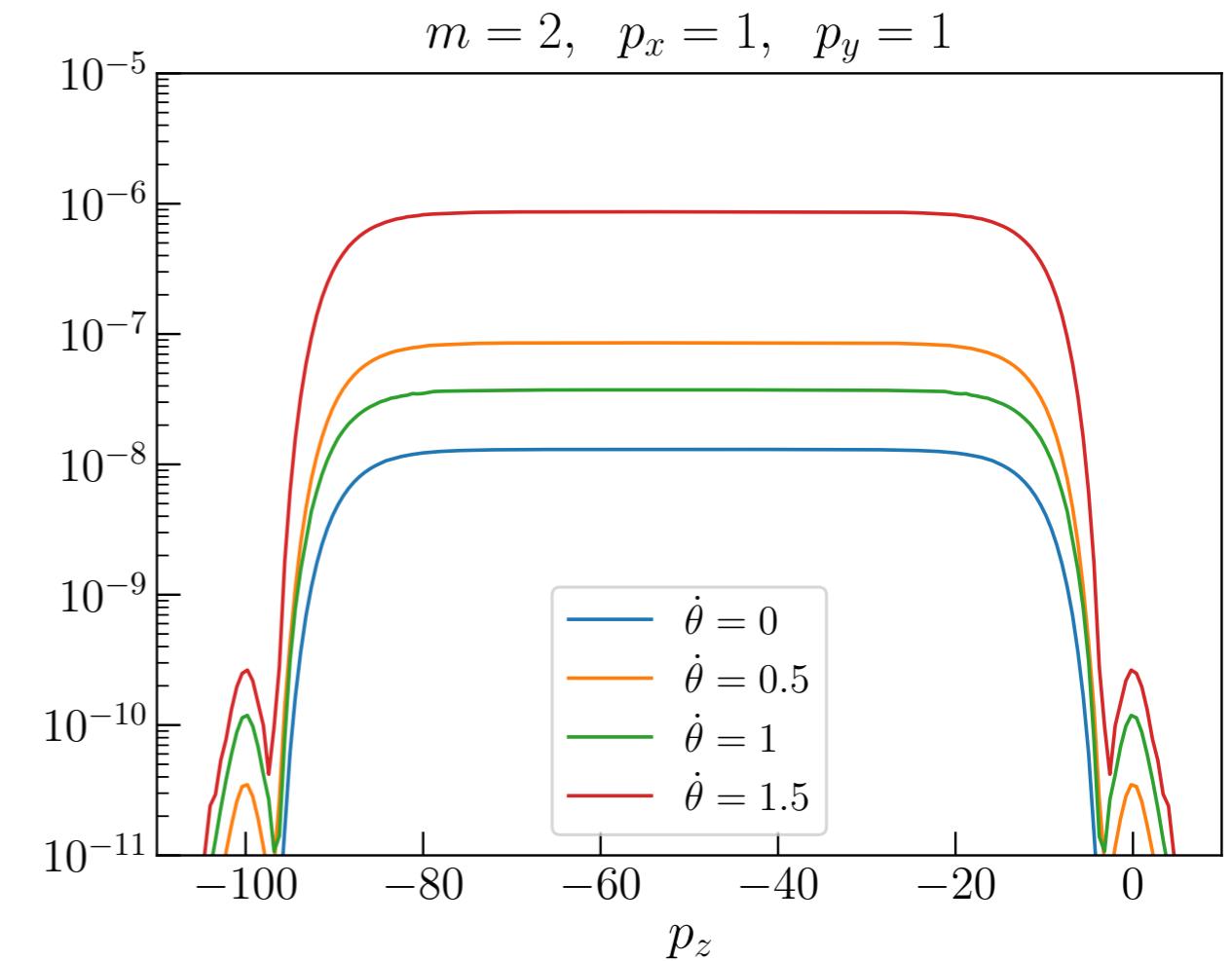
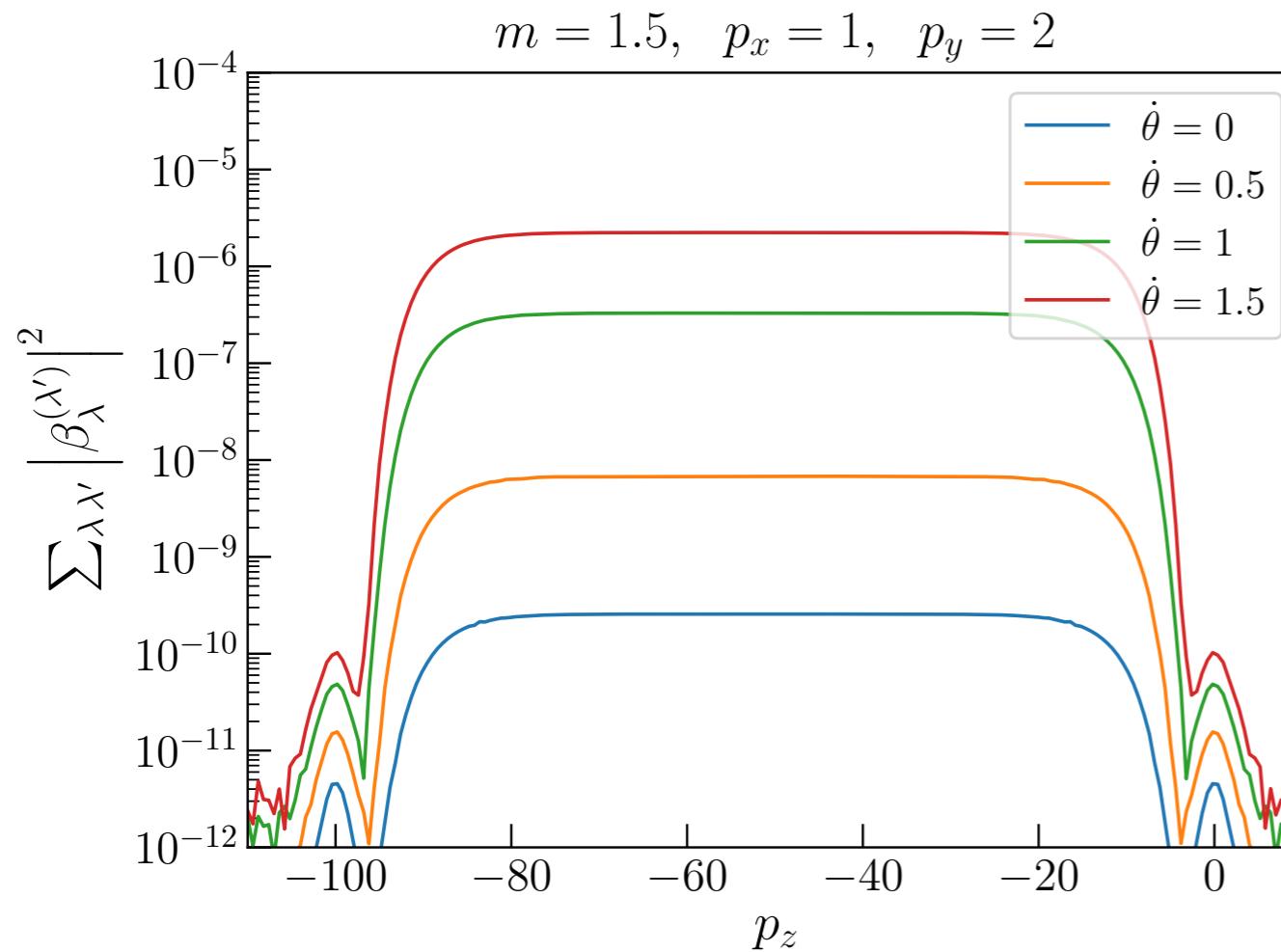
- Time evolution of Bogoliubov coefficients again from Dirac eqn:

$$\begin{pmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{\beta}_1 \\ \dot{\beta}_2 \end{pmatrix} = i\dot{\theta}_{5+m} \begin{pmatrix} -\frac{m}{m_T} \frac{\Pi_z}{\Omega} & \frac{p_T}{m_T} & \frac{m}{\Omega} e^{2i\Theta} & 0 \\ \frac{p_T}{m_T} & \frac{m}{m_T} \frac{\Pi_z}{\Omega} & 0 & -\frac{m}{\Omega} e^{2i\Theta} \\ \frac{m}{\Omega} e^{-2i\Theta} & 0 & \frac{m}{m_T} \frac{\Pi_z}{\Omega} & \frac{p_T}{m_T} \\ 0 & -\frac{m}{\Omega} e^{-2i\Theta} & \frac{p_T}{m_T} & -\frac{m}{m_T} \frac{\Pi_z}{\Omega} \end{pmatrix} + \frac{m_T \dot{\Pi}_z}{2\Omega^2} \begin{pmatrix} 0 & 0 & -e^{2i\Theta} & 0 \\ 0 & 0 & 0 & -e^{2i\Theta} \\ e^{-2i\Theta} & 0 & 0 & 0 \\ 0 & e^{-2i\Theta} & 0 & 0 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{pmatrix},$$

where $\theta_{5+m} = (c_5 + c_m) \frac{\phi}{f_a}$ and 1,2: remnants of helicities.

Numerical results: spectrum

- Impose electric field and axion velocity for $0 < t < 100$ (turned on and off adiabatically).
- Again develop a plateau for $-100 < p_z < 0$.
- Height of the plateau: enhanced and strongly dependent on $\dot{\theta}_{5+m}$ (!)

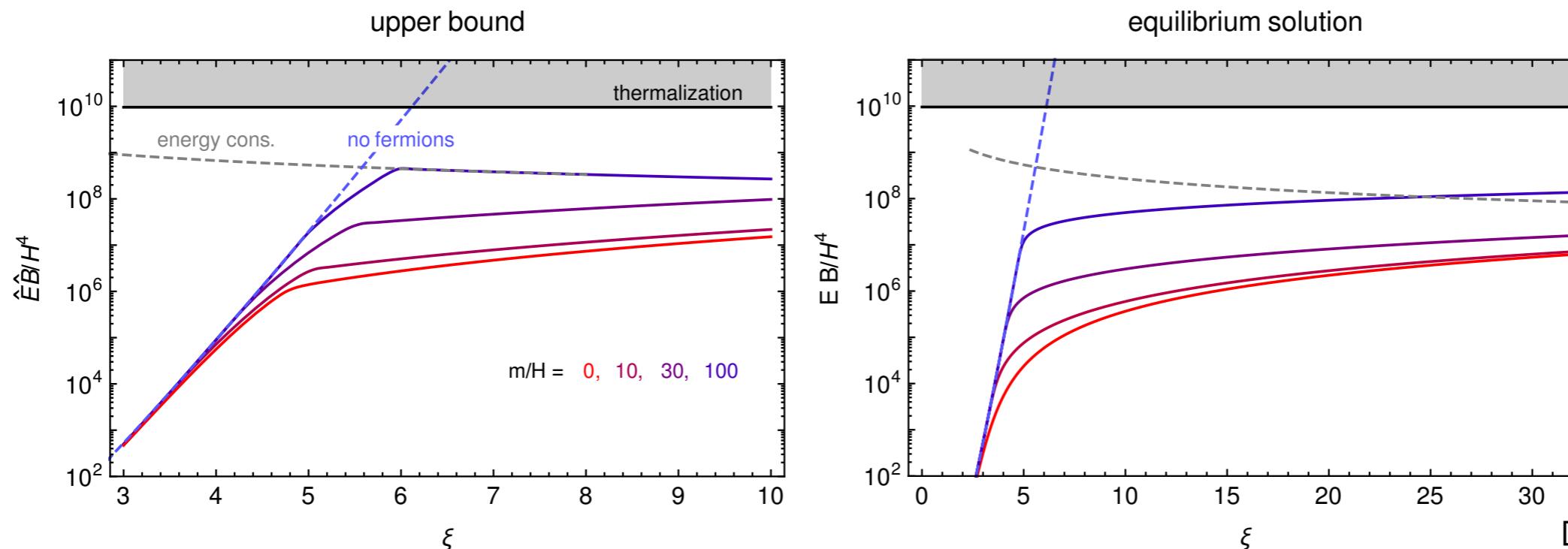


Phenomenology

- Axion-gauge field coupling induces tachyonic gauge field production.

$$\langle \vec{E}^2 \rangle \simeq 10^{-4} \frac{e^{2\pi\xi}}{\xi^3} H^4, \quad \langle \vec{B}^2 \rangle \simeq 10^{-4} \frac{e^{2\pi\xi}}{\xi^5} H^4, \quad \xi = c_A \frac{\dot{\phi}}{H f_a}.$$

- Fermion can suppress gauge boson production, even without axion coupling.



[Domcke, YE, Mukaida 19]

- Axion coupling enhances induced current, can be more effective:

$$g\langle J_z \rangle \sim \tau \times \frac{g^3 E^2}{2\pi^2} e^{-\frac{\pi m^2}{gE}} \times \max \left[\frac{B}{E} \coth \left(\frac{\pi B}{E} \right), \frac{\dot{\theta}_{5+m}^2}{\pi m^2} \right], \quad \tau : \text{duration of electric field.}$$

Gauge boson production

- Axion velocity induces tachyonic production of gauge boson: [Anber, Sorbo 09; ...]

$$0 = \left[\frac{d^2}{d\eta^2} + k(k \pm 2\lambda aH\xi) \right] A_{\pm}(\eta, \vec{k}), \quad \xi = \frac{\alpha |\dot{\phi}|}{2\pi f_a H}, \quad \lambda = \text{sgn}(\dot{\phi}).$$

→ one helicity exponentially enhanced: $A_{-\lambda} \simeq \frac{1}{\sqrt{2k}} \left(\frac{k}{2\xi aH} \right)^{1/4} e^{\pi\xi - 2\sqrt{2k\xi/aH}}$.

- Helical gauge boson production at the end of inflation:

$$\langle \vec{E}^2 \rangle \simeq 2.6 \times 10^{-4} \frac{e^{2\pi\xi}}{\xi^3} H^4, \quad \langle \vec{B}^2 \rangle \simeq 3.0 \times 10^{-4} \frac{e^{2\pi\xi}}{\xi^5} H^4,$$

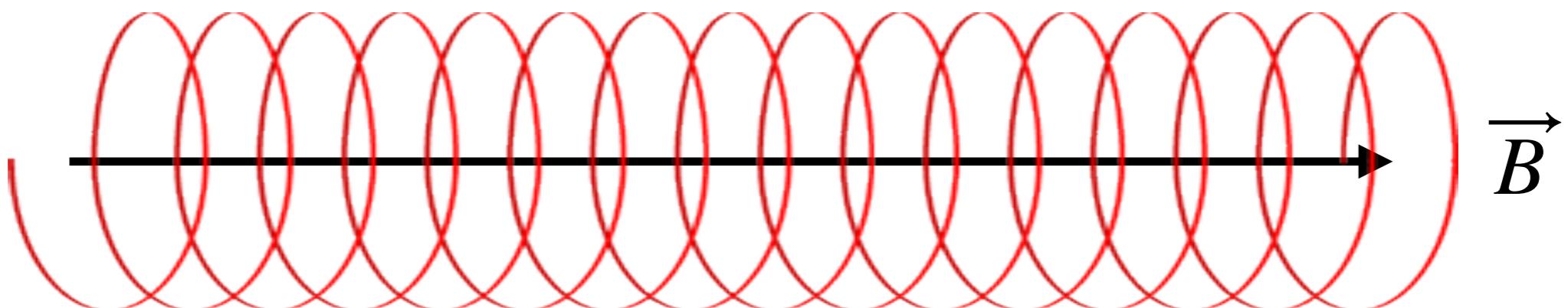
$$\langle \vec{E} \cdot \vec{B} \rangle \simeq 2.6 \times 10^{-4} \lambda \frac{e^{2\pi\xi}}{\xi^4} H^4.$$

Homogenous within each Hubble patch and $\vec{E} \parallel \vec{B}$.

→ $F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\vec{E} \cdot \vec{B} \neq 0$, nontrivial Chern-Simons term.

Magnetic field

- Charged particles classically spiral under magnetic field.



- Transverse motion quantum-mechanically discretized: Landau levels.

transverse momentum p_T^2 replaced by $m_B^2 = 2ng|QB|$ with $n = 0, 1, 2, \dots$

→ $\begin{cases} n = 0 : \text{Lowest Landau level (moving parallel to } B\text{).} \\ n \geq 1 : \text{higher Landau level (moving with transverse momentum).} \end{cases}$

Equation of motion

- Lowest Landau level: only one component appears.

$$\dot{\alpha}_0 = i\dot{\theta}_{5+m} \frac{\Pi_z}{\Omega_0} \alpha_0 - \left(\frac{m\dot{\Pi}_z}{2\Omega_0^2} + i\dot{\theta}_{5+m} \frac{m}{\Omega_0} \right) e^{2i\Theta_0} \beta_0,$$

$$\dot{\beta}_0 = -i\dot{\theta}_{5+m} \frac{\Pi_z}{\Omega_0} \alpha_0 + \left(\frac{m\dot{\Pi}_z}{2\Omega_0^2} - i\dot{\theta}_{5+m} \frac{m}{\Omega_0} \right) e^{2i\Theta_0} \beta_0,$$

where $\Omega_0 = \sqrt{\Pi_z^2 + m^2}$, $\Theta_0 = \int dt \Omega_0$.

- Higher Landau levels: the same as without B after $p_T \rightarrow m_B$.

$$\begin{pmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{\beta}_1 \\ \dot{\beta}_2 \end{pmatrix} = i\dot{\theta}_{5+m} \begin{pmatrix} -\frac{m}{m_T} \frac{\Pi_z}{\Omega} & \frac{m_B}{m_T} & \frac{m}{\Omega} e^{2i\Theta} & 0 \\ \frac{m_B}{m_T} & \frac{m}{m_T} \frac{\Pi_z}{\Omega} & 0 & -\frac{m}{\Omega} e^{2i\Theta} \\ \frac{m}{\Omega} e^{-2i\Theta} & 0 & \frac{m}{m_T} \frac{\Pi_z}{\Omega} & \frac{m_B}{m_T} \\ 0 & -\frac{m}{\Omega} e^{-2i\Theta} & \frac{m_B}{m_T} & -\frac{m}{m_T} \frac{\Pi_z}{\Omega} \end{pmatrix} + \frac{m_T \dot{\Pi}_z}{2\Omega^2} \begin{pmatrix} 0 & 0 & -e^{2i\Theta} & 0 \\ 0 & 0 & 0 & -e^{2i\Theta} \\ e^{-2i\Theta} & 0 & 0 & 0 \\ 0 & e^{-2i\Theta} & 0 & 0 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{pmatrix}.$$

Anomaly equation

- Anomaly equation should hold in any background spacetime.

$$\partial_\mu J_5^\mu = -\frac{g^2 Q^2}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + 2im \bar{\psi} e^{2i\theta_m \gamma_5} \gamma_5 \psi, \quad J_5^\mu = \bar{\psi} \gamma^\mu \gamma_5 \psi.$$

→ spatially averaged version: $\dot{q}_5 = \frac{g^2 Q^2 E B}{2\pi^2} + 2im \langle \bar{\psi} e^{2i\theta_m \gamma_5} \gamma_5 \psi \rangle$.

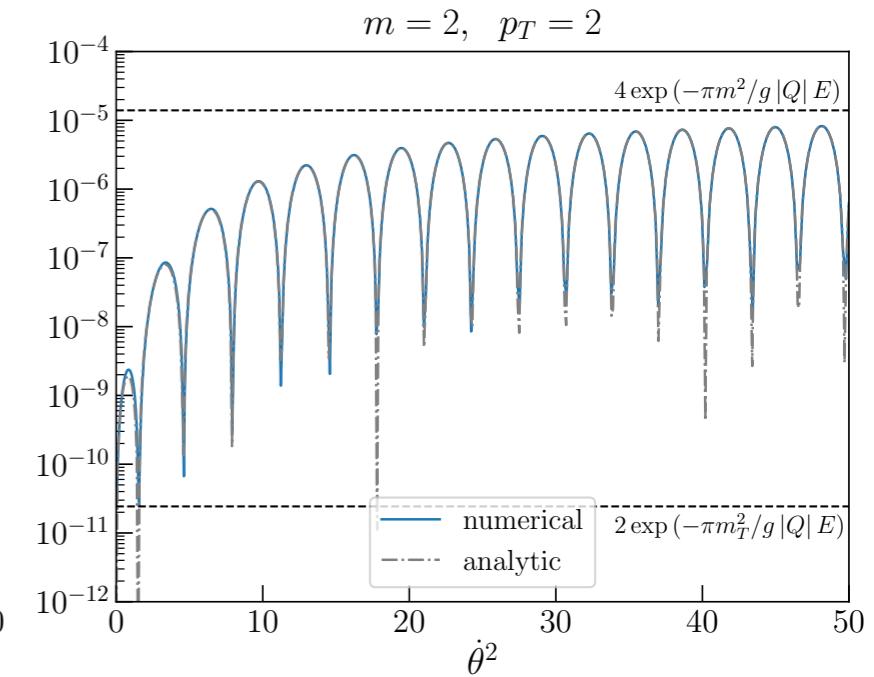
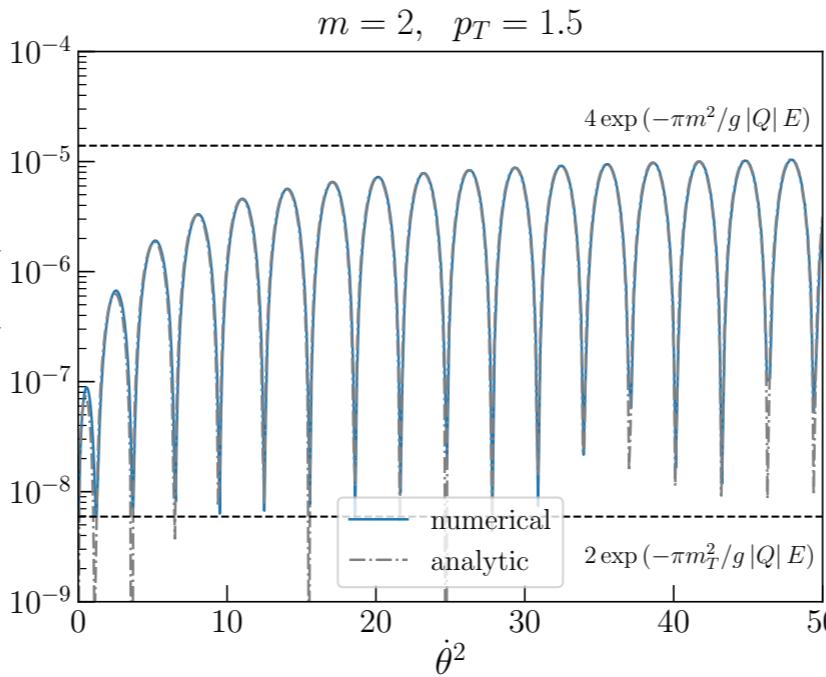
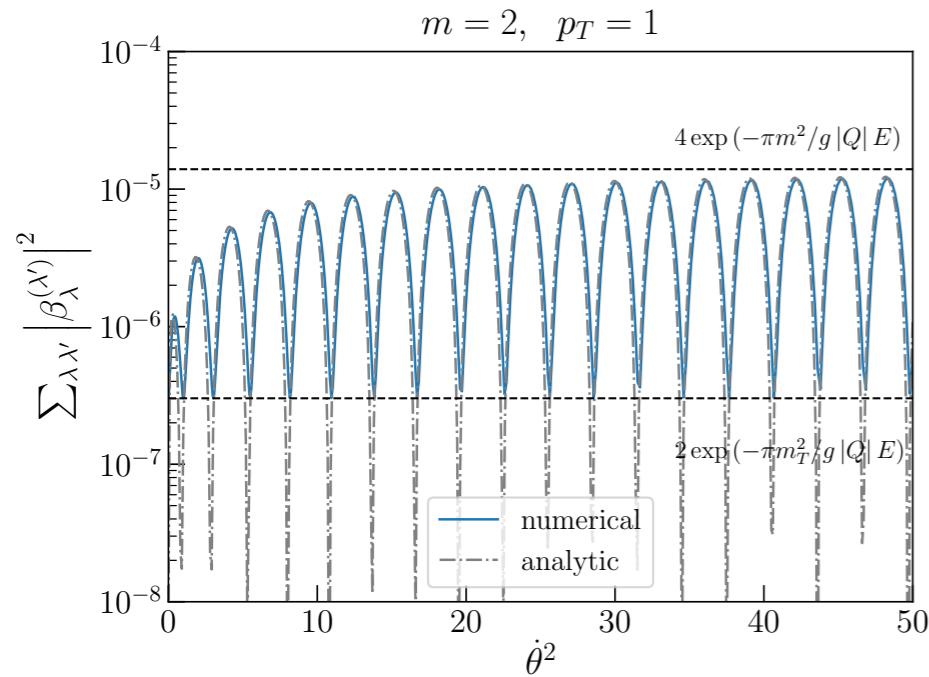
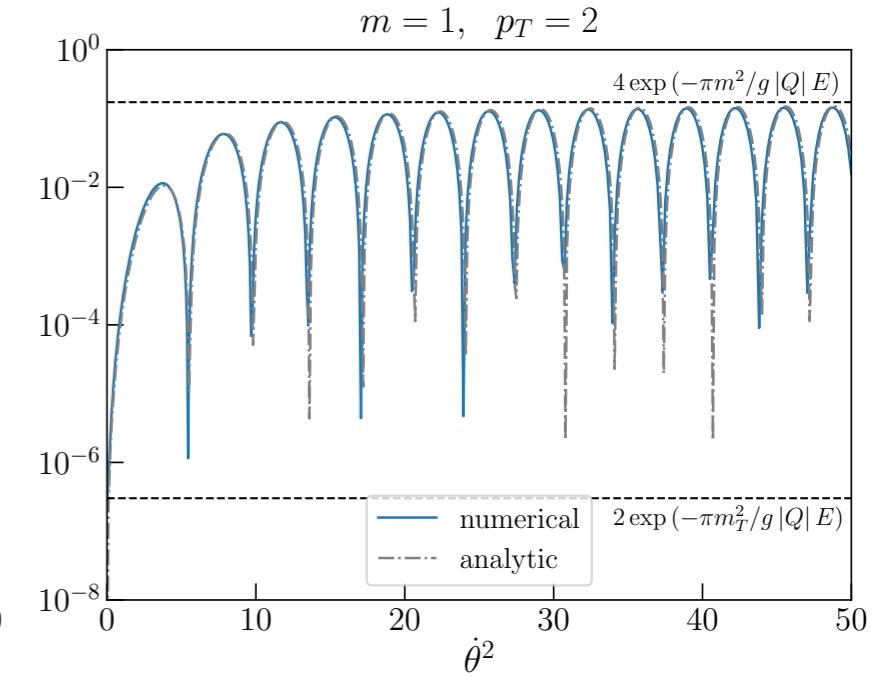
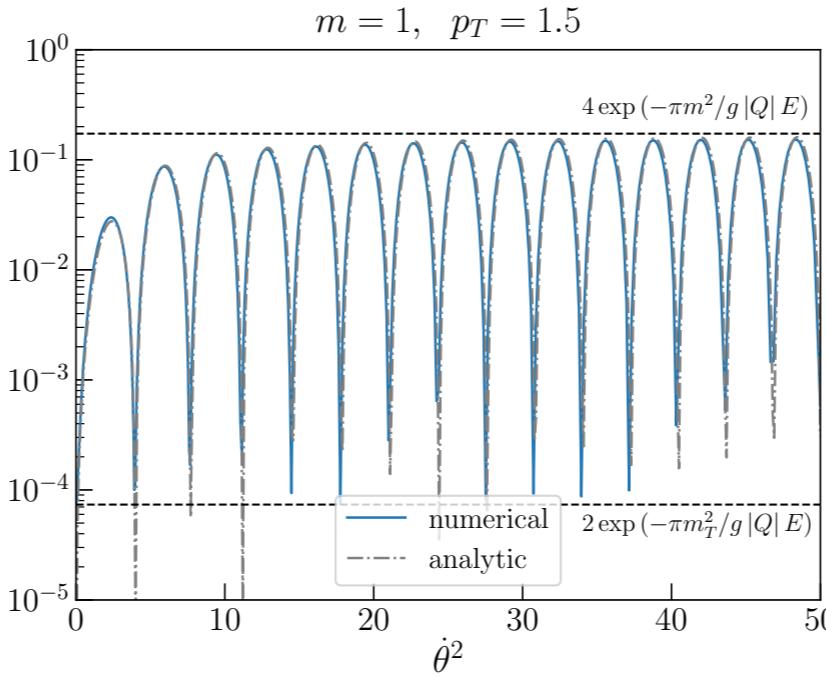
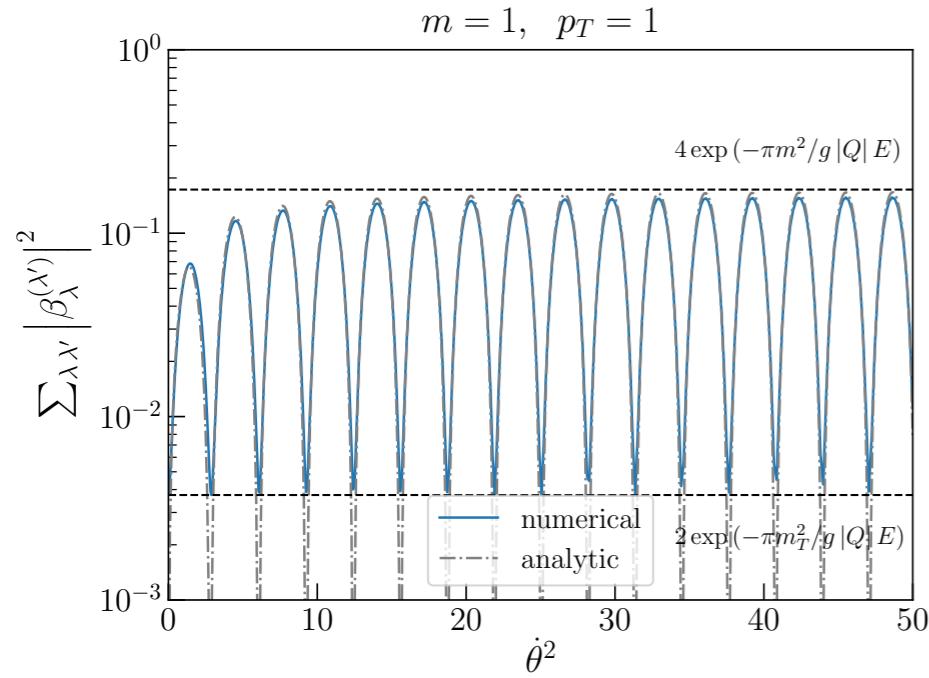
- Follows directly from equations of motion of α and β in the current case.

$$\left\{ \begin{array}{l} \text{lowest Landau level: } \dot{q}_{5,0} = \frac{g^2 Q^2 E B}{2\pi^2} + 2im \langle \bar{\psi} e^{2i\theta_m \gamma_5} \gamma_5 \psi \rangle \Big|_{\text{LLL}} \\ \text{higher Landau level: } \dot{q}_{5,n} = 2im \langle \bar{\psi} e^{2i\theta_m \gamma_5} \gamma_5 \psi \rangle \Big|_{\text{HLL},n} \end{array} \right.$$

→ Chern-Simons term served entirely by the lowest Landau level.

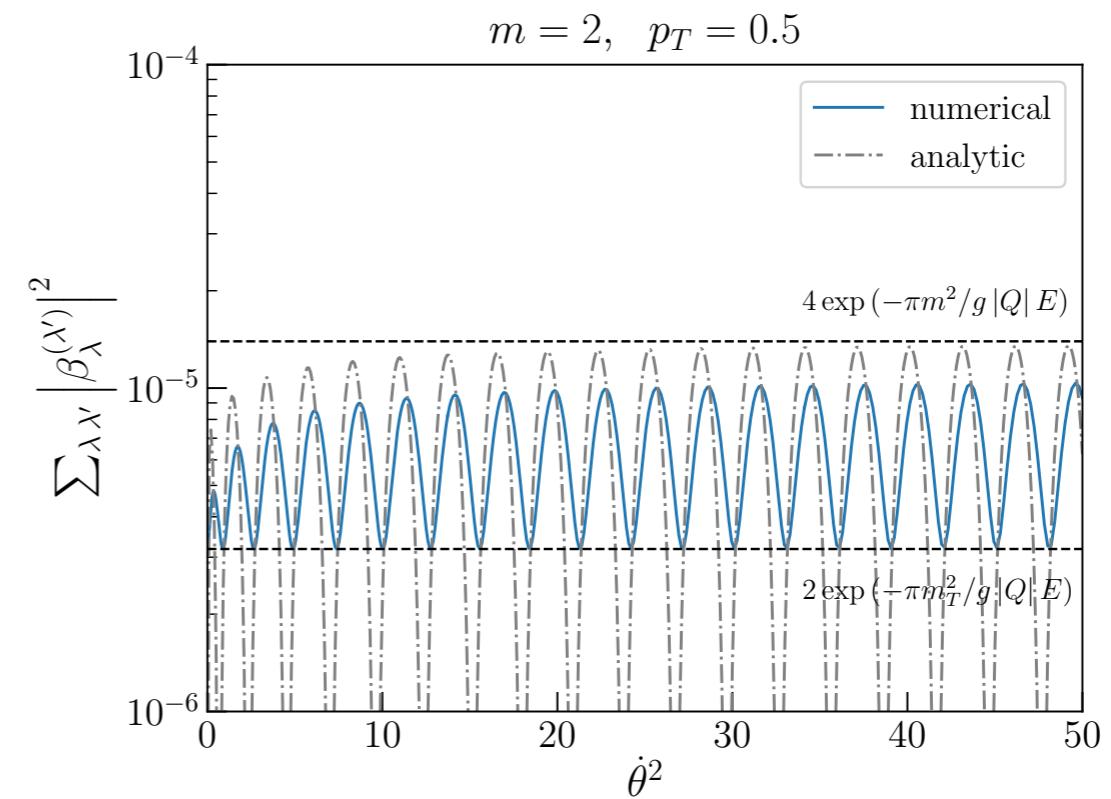
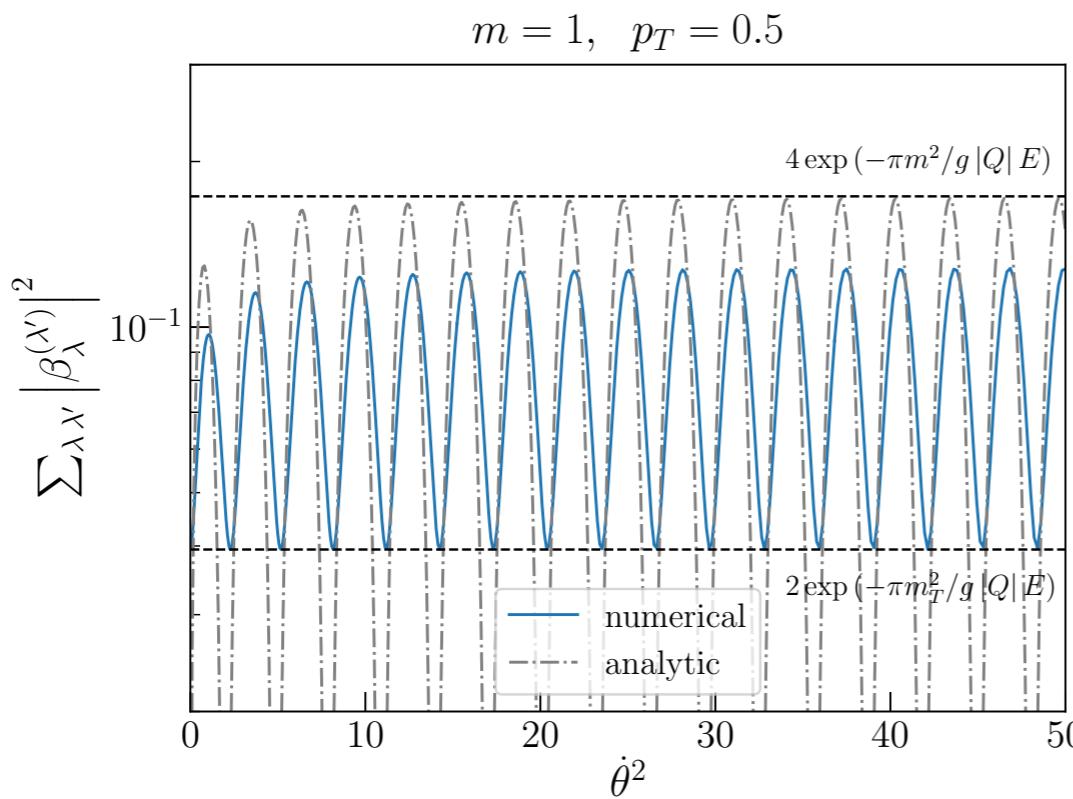
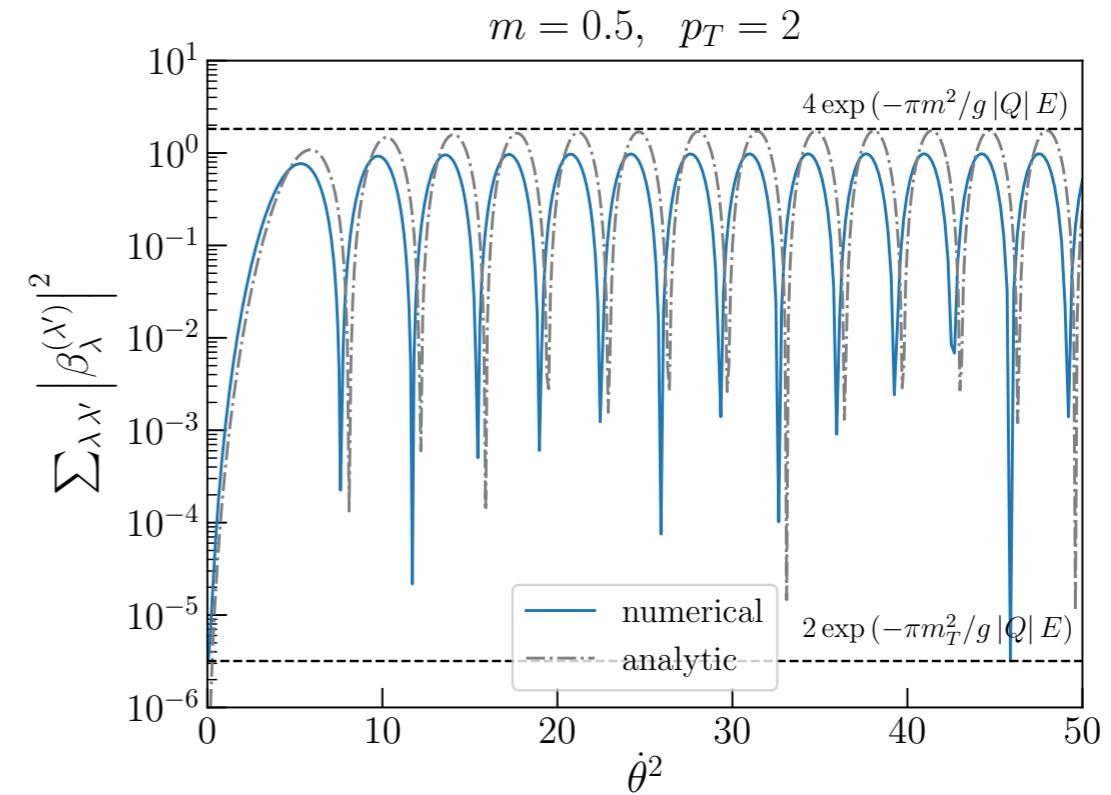
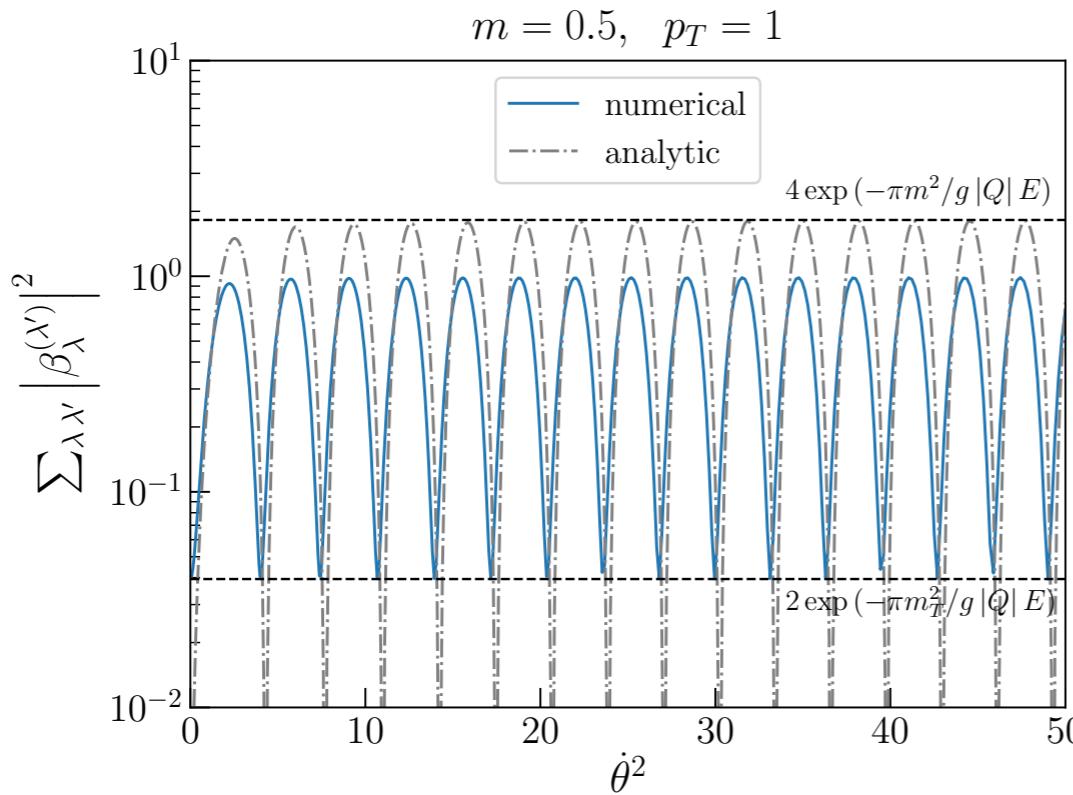
Empirical formula

Well reproduces numerical results for $m, p_T \gtrsim gE$.



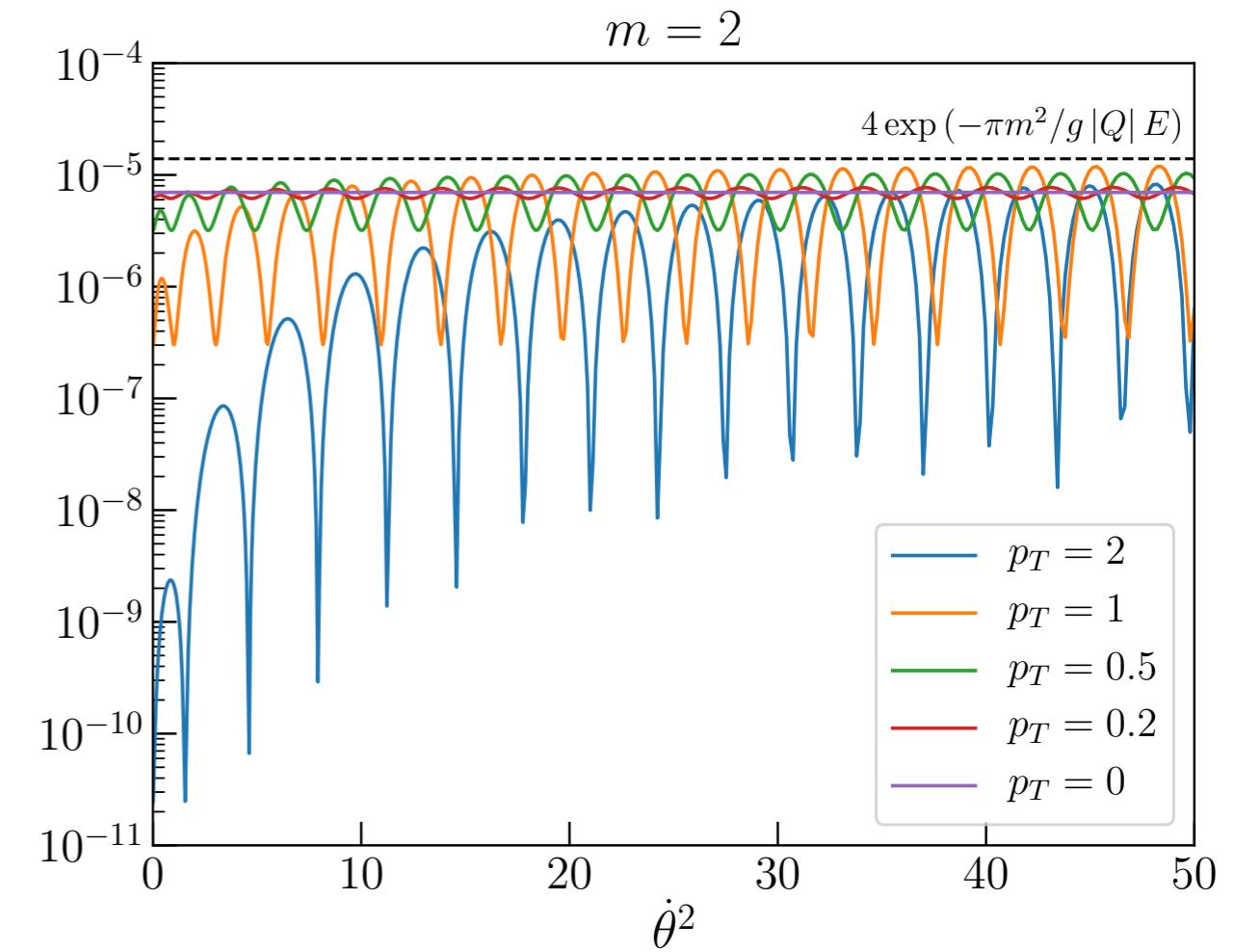
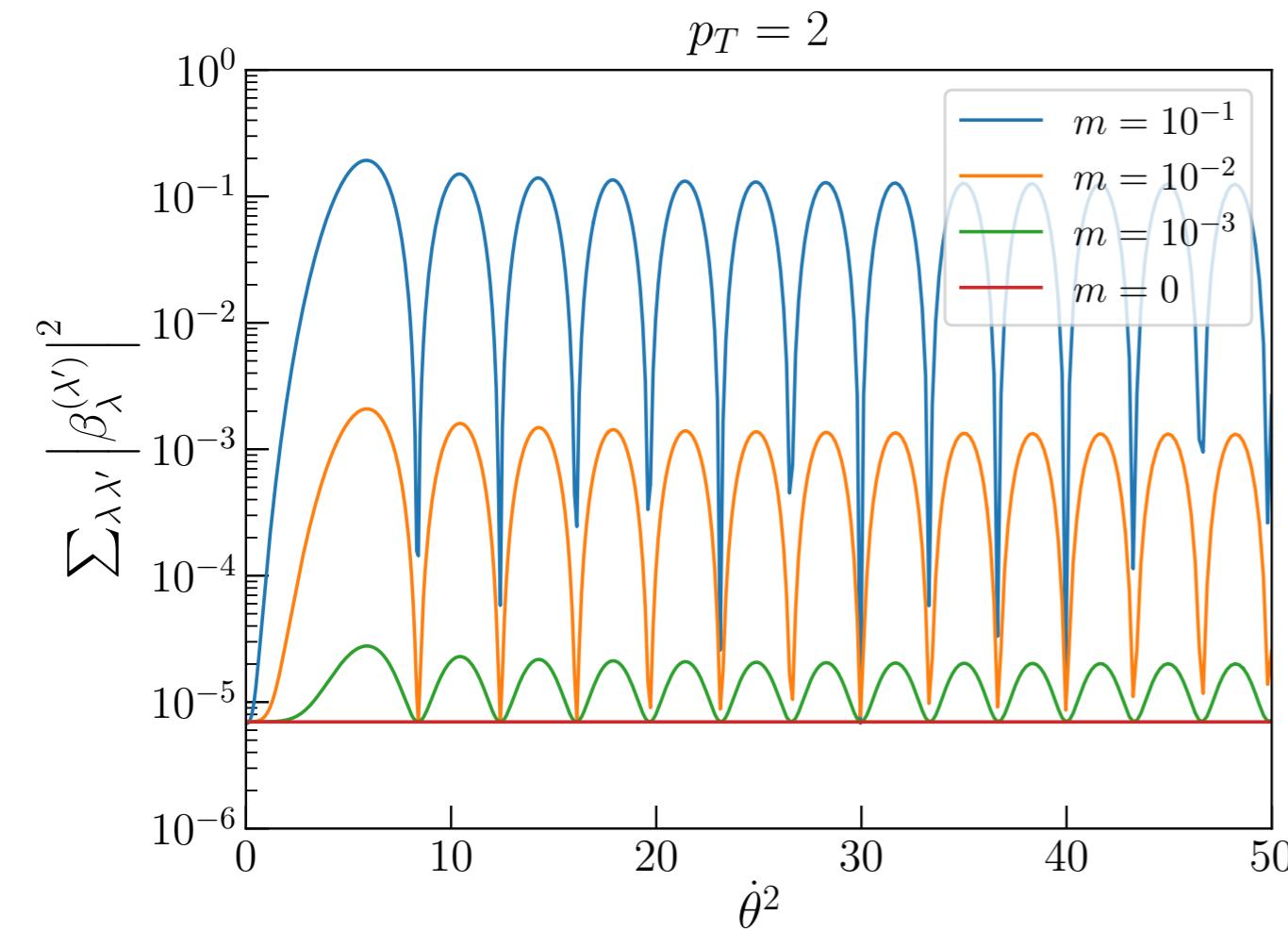
Limitation

Does not work well for $m, p_T \lesssim gE$.



Small m or p_T limit

- Enhancement is gone in the limit $m \rightarrow 0$.
Should be the case since $c_5 + c_m$ is unphysical in this limit.
- Enhancement is more important for larger p_T .



Larger axion velocity

- Gap of $\tilde{\Omega}^-$: takes minimum at $\Pi_z = \pm \sqrt{\dot{\theta}_{5+m}^2 - p_T^2}$, not at $\Pi_z = 0$.

Remember that $\tilde{\Omega}^- = \sqrt{\left(\sqrt{\Pi_z^2 + p_T^2} - \dot{\theta}_{5+m}\right)^2 + m^2}$.

- Can be seen also in the numerical results.

