Generalized Anomalies and Axion Physics

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<u>Based on</u> arXiv: 2001.03631 with Erich Poppitz & 2008.05491 with Stephen Baker

Outline

- Higher-form symmetries
- ’t Hooft anomalies of higher-form symmetries (generalized anomalies)
- Axions and domain walls: deconfinement on the walls
- Models of axion inflation in the light of generalized anomalies

Higher-form symmetries

- Global 0-form symmetry: acts at a point
- E.g. global $U(1)$ symmetry: $\phi(x) \to e^{i\alpha}\phi(x), \quad 0 \leq \alpha < 2\pi, \quad \text{K.E.} = |\partial_\mu \phi|^2$
- Gauging a 0-form symmetry: $\phi(x) \to e^{i\alpha(x)}\phi(x)\,, \quad \text{K.E.} \neq |\partial_\mu \phi|$
- Introduce a 1-form gauge field $B_\mu \equiv B^{(1)}\,, \quad \partial_\mu \phi \to D_\mu \phi = (\partial_\mu - iB_\mu)\phi$

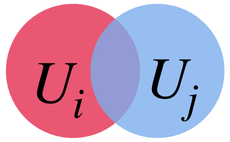
Higher-form symmetries

- Global 1-form symmetry: acts on a 1-d object.
- E.g. Wilson line $\mathscr{W}$.
- $\mathbb{Z}_N^{(1)}$ $1-$form center symmetry of $SU(N)$ acting on Wilson lines.
- Closed Wilson line is order parameter for confinement.
- **Gauging 1-form symmetry**: **2-form gauge field** $B_{\mu\nu} \equiv B^{(2)}$.

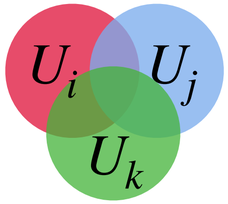
Gaiotto, Kapustin, Seiberg, Willett, 2014

Higher-form symmetries

- Turn on **background** gauge fields: **compatibility condition**.
- Consider a $\mathbb{M}^4$ with charts $\{U_i\}$ with fermions χ transforming under $R(G)$.
- On $U_i \cap U_j$: $\chi_i = R\left(\mathscr{K}\right)_{ij} \chi_j$, $\mathscr{K}$ the transition function.



- On $U_i \cap U_j \cap U_k$: $R(\mathscr{K})_{ij} R(\mathscr{K})_{jk} R(\mathscr{K})_{ki} = 1$ (**compatibility condition**)



Higher-form symmetries

- Given R, we may relax: $R(\mathscr{K})_{ij}R(\mathscr{K})_{jk}R(\mathscr{K})_{ki} = 1 \to \mathscr{K}_{ij}\mathscr{K}_{jk}\mathscr{K}_{ki} = Z(G)$. Then, we can **turn on** $B^{(2)}$ of $Z(G)$.
- E.g. take χ in the **adjoint** of $SU(N)$, then $\mathscr{K}_{ij}\mathscr{K}_{jk}\mathscr{K}_{ki} = e^{i\frac{2\pi}{N}n_{ijk}}$.

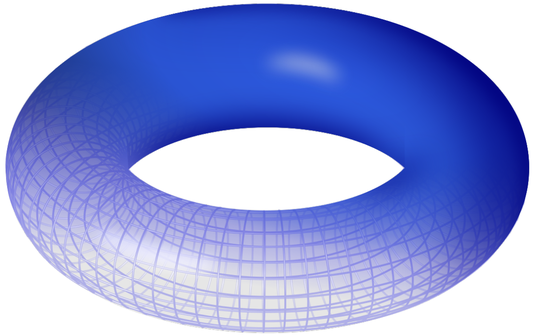
N boxes

$R(\mathscr{K}_{ij}) \sim \left(e^{i\frac{2\pi}{N}}\right)^N \sim 1.$

- Thus, we may turn on $B^{(2)}$ of $\mathbb{Z}_N^{(1)}$.
- $B^{(2)}$ sources a **fractional flux** on 2-cycles of $\mathbb{M}^4$:

$$\oint_{\mathbb{T}^2} B^{(2)} = \frac{2\pi}{N}, \quad Q = \frac{N}{8\pi^2}\int_{\mathbb{T}^4} B^{(2)} \wedge B^{(2)} = \frac{1}{N}$$

Kapustin, Seiberg, 2014



$\mathbb{T}^2 \subset \mathbb{T}^4$

Generalized anomalies

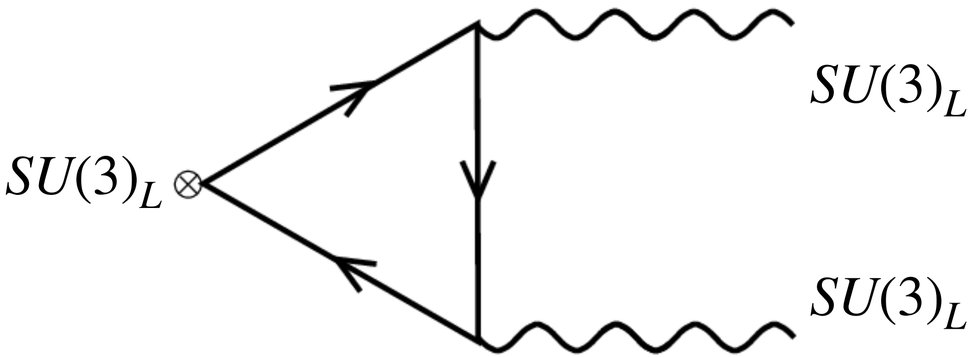
- 't Hooft anomaly is the obstruction to **gauging** a global symmetry: $\mathcal{Z} \to e^{i\gamma}\mathcal{Z}$
- UV/IR matching: constraints on the IR phases of strongly coupled gauge theories.
- Discovered in the 80s: anomalies of 0-form symmetries (**integer fluxes:**

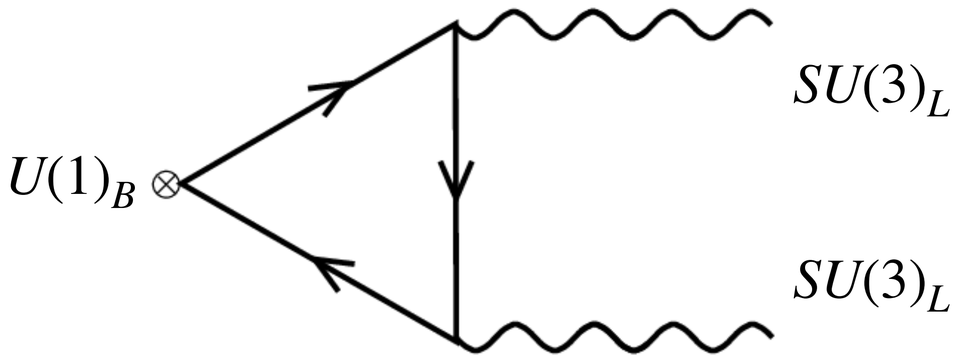
$$Q = \frac{1}{8\pi^2}\int_{\mathbb{M}^4} F^{SU(N_f)_L} \wedge F^{SU(N_f)_L} \in \mathbb{Z}\ )$$

't Hooft, 1980
Frishman, Schwimmer, Banks, Yankielowicz 1981

Generalized anomalies

- E.g. in QCD with fundamentals. Global symmetry $SU(3)_L \times SU(3)_R \times U(1)_B$





- The anomaly $U(1)_B\left[SU(3)_L\right]^2$ can be understood as: $\mathcal{Z}\left[F^{(2)}_{SU(3)_L}\right] \xrightarrow{U(1)_B} e^{i\gamma}\mathcal{Z}\left[F^{(2)}_{SU(3)_L}\right]$.
- Matching in the IR: Goldstones, composite fermions, CFT.
- E.g., in QCD with fundamental: $SU(3)_L \times SU(3)_R \times U(1)_B \to SU(3)_V \times U(1)_B$

Witten, 1983

Generalized anomalies

- How this generalizes to fractional fluxes $B^{(2)}$? These are the **generalized 't Hooft anomalies**.
- Consider $SU(N)$ with N_f adjoints. There is a discrete chiral $\mathbb{Z}_{2NN_f}$: $\mathcal{Z}[B^{(2)}] \xrightarrow{\mathbb{Z}_{2NN_f}} e^{i\frac{2\pi}{N}}\mathcal{Z}[B^{(2)}]$.
- Matching the anomaly in IR: breaking $\mathbb{Z}_{2NN_f} \to \mathbb{Z}_{2N_f}$. Thus we have N vacua.

Axions and strong dynamics

- We consider vector-like gauge theories.

Anber, Poppitz, 2019
Anber, Poppitz 2020

	$SU(N)$	$SU(N_f)_1$	$SU(N_f)_2$	$U(1)_B$	$\mathbb{Z}_{2N_f T_R}$
χ	R	$\square$	1	1	1
$\tilde{\chi}$	$\overline{R}$	1	$\overline{\square}$	-1	1

Axions and strong dynamics

- Cocycle conditions: we can turn on fractional $B^{(2)c}, B^{(2)f}, B^{(2)u}$ fluxes.
- This leads to a baryon-color-flavor (BCF) anomaly:
  $\mathcal{Z}\left[B^{(2)c}, B^{(2)f}, B^{(2)u}\right] \xrightarrow{\mathbb{Z}_{2N_f T_R}} e^{i\gamma}\mathcal{Z}\left[B^{(2)c}, B^{(2)f}, B^{(2)u}\right], \quad \gamma \in \mathbb{Z}_p \subseteq \mathbb{Z}_{N_f T_R}$
- What are the implications for axion dynamics?
- We couple the theory to a complex scalar $\Phi = \rho e^{i\frac{a}{f}}, \quad f \gg \Lambda$:
  $\mathcal{L} \supset y\,\Phi\tilde{\chi}\chi + \text{h.c.} + |\partial_\mu \Phi|^2 - V(|\Phi|)$
- Symmetries: $SU(N_f)_1 \times SU(N_f)_2 \times U(1)_B \times \mathbb{Z}_{2N_f T_R} \to SU(N_f)_V \times U(1)_B \times \mathbb{Z}_{2N_f T_R}$
- $\mathbb{Z}_{2N_F T_R}: \quad \Phi \to e^{i\frac{4\pi}{2N_f T_R}}\Phi, \quad \chi \to e^{-i\frac{2\pi}{2N_f T_R}}\chi, \quad \bar{\chi} \to e^{-i\frac{2\pi}{2N_f T_R}}\tilde{\chi}$

Axions and strong dynamics

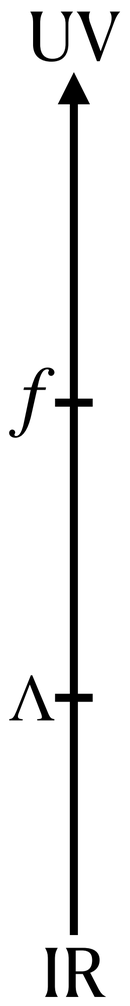
- The theory is still sensitive to the BCF anomaly.
- What goes wrong with the conventional wisdom of EFT?

$$\mathscr{L} = \text{YM} + y\,\Phi\tilde{\chi}\chi + \text{h.c.} + |\partial_\mu \Phi|^2 - V(|\Phi|)$$

$$\Phi = f e^{i\frac{a}{f}}, \quad \mathscr{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f}\frac{N_f T_R}{8\pi^2} F^{(2)c} \wedge F^{(2)c} + \text{YM} + \text{heavy quarks}$$

(approximate) $\mathbb{Z}_N^{(1)}$

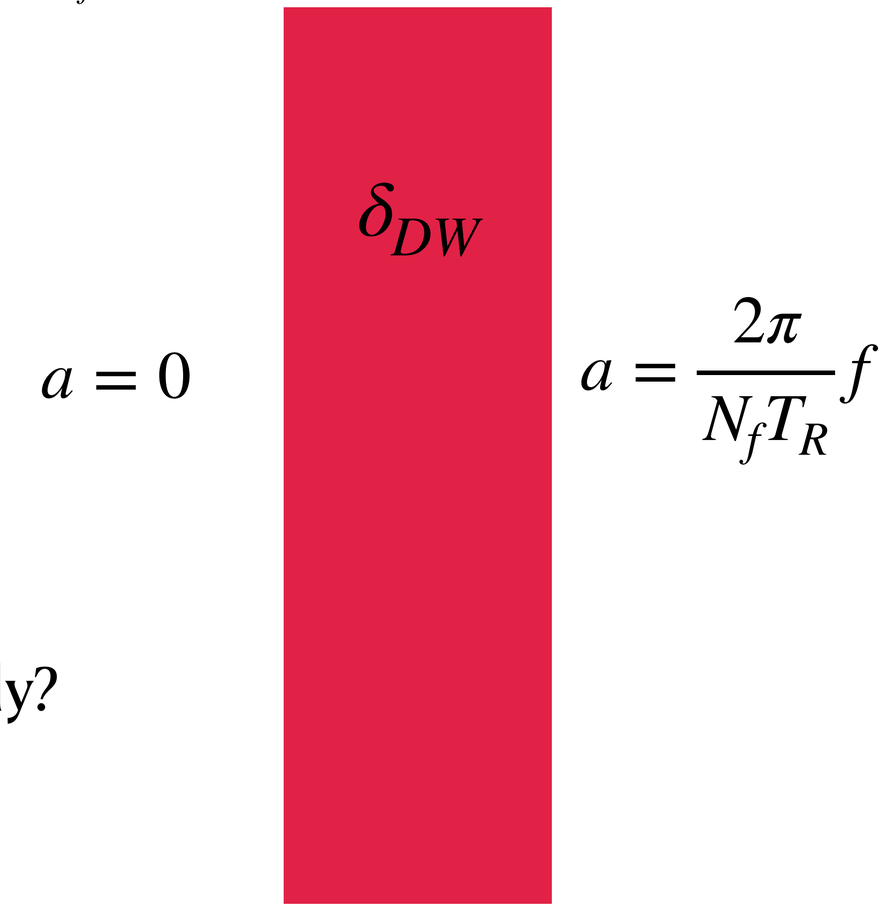
$$\mathscr{L} = \frac{1}{2}\left(\partial_\mu a\right)^2 + \Lambda^4 \left[1 - \cos\left(\frac{N_f T_R\, a}{f}\right)\right]$$



Axions and strong dynamics

- The potential will cause $\mathbb{Z}_{2N_f T_R} \to \mathbb{Z}_2$. Thus, $N_f T_R$ vacua separated by domain walls.

- $\delta_{DW} \sim \dfrac{f}{\Lambda^2} \gg \Lambda^{-1}$



- Where is the BCF anomaly?

Axions and strong dynamics

- Anomalies cannot disappear: anomaly inflow (which involves $B^{(2)c}$)
- Chern Simons (CS) theory lives on the wall.
- $\delta_{DW} \sim \dfrac{f}{\Lambda^2} \gg \Lambda^{-1} \qquad \delta_{CS} \sim \Lambda^{-1}\,.$
- CS theory is gapped (no long range force).
- Thus quarks must be deconfined on the wall.

Axions and strong dynamics

- In the bulk approximate $\mathbb{Z}_N^{(1)}$ symmetry

Axions and strong dynamics

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Axions and strong dynamics

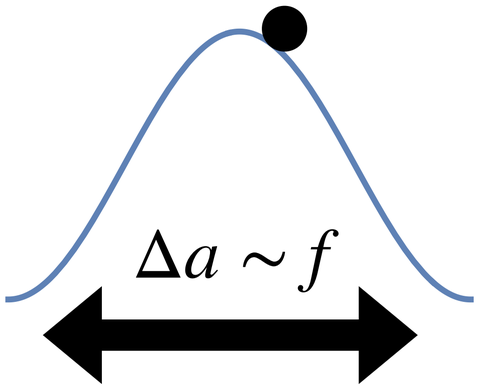
- The Lagrangian $\frac{1}{2}\left(\partial_\mu a\right)^2 + \Lambda^4\left[1 - \cos\left(\frac{N_f T_R\, a}{f}\right)\right]$ breaks down inside the wall!

$\Delta a \sim 2\pi f$

$\Delta V \sim \Lambda^4$

$$a = 0\,, \quad \Delta V \sim \Lambda^4\left(\frac{\Delta a}{f}\right)^4 \ll \Lambda^4 \qquad a = \frac{2\pi}{N_f T_R} f\,, \quad \Delta V \sim \Lambda^4\left(\frac{\Delta a}{f}\right)^4 \ll \Lambda^4$$

Axions and strong dynamics

- There is **intertwining** between two scales Λ and f as a makes large excursion.
- This also has implications for **axion inflation**:
- 

$$V = \Lambda^4 \left[1 - \cos\left(\frac{a}{f} \right) \right]$$
- One needs to modify the potential.

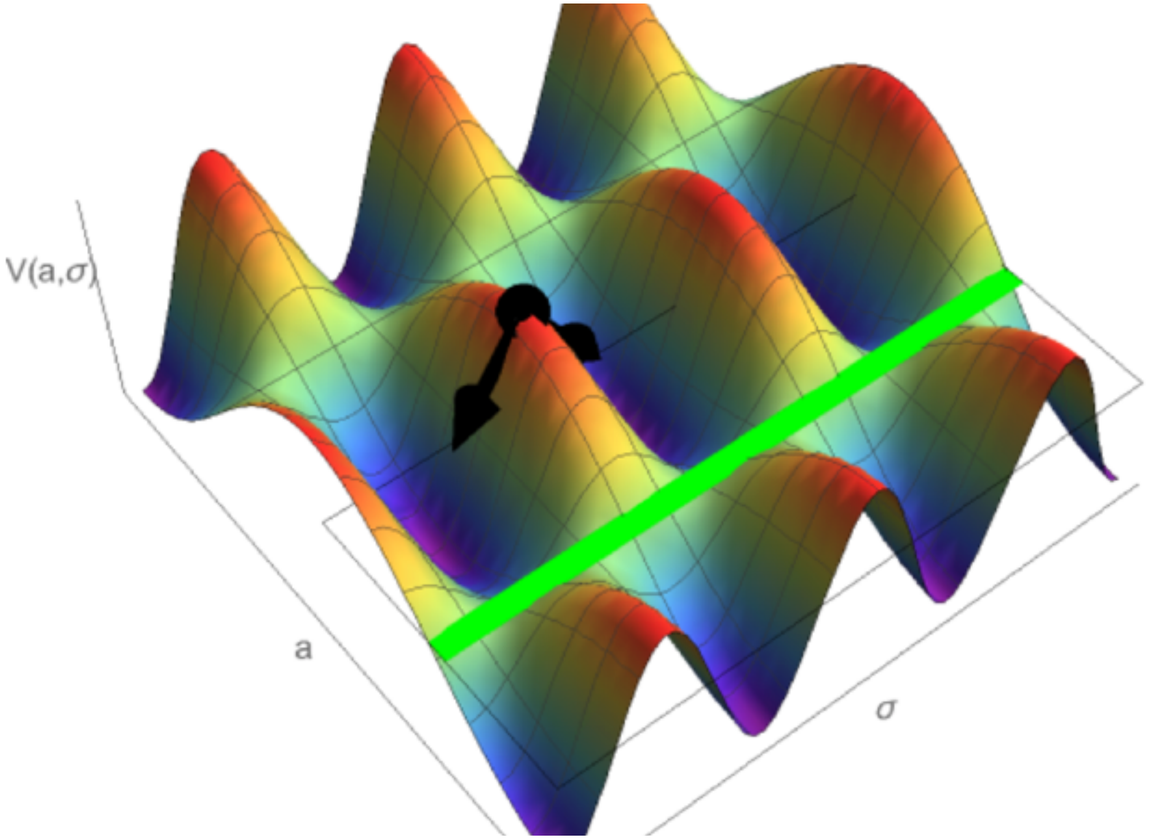
Axions and strong dynamics

Anber, Baker 2020

- A model that captures the intertwining

$$V(a,\sigma) = \Lambda^4 \left[2 - \cos\left(\frac{a}{f} + \frac{\sigma}{\Lambda}\right) - \cos\left(\frac{a}{f} - \frac{\sigma}{\Lambda}\right) \right]$$

- Heavy DOF spoil inflation.
- There is a safe zone:
- $f > 9M_P$ yielding $50 - 60$ e-foldings
- Power spectrum and r consistent with CMB



Conclusion

- ’t Hooft anomalies are far from a finished business.
- Generalized anomalies can provide new insight about old physics.
- They also can teach us about new phenomena.
- Many other applications: implications for composites (Anber, Poppitz 2020), condensates (Anber, 2021)
- Are important to study chiral gauge theories (work in progress).
- A comprehensive study to catalogue all anomalies is underway.