

Audible Axions

Wolfram Ratzinger

Based on:

[1811.01950](#) [2012.11584](#)

with Camila Machado, Pedro Schwaller and Ben Stefanek



July 28, 2021 Cambridge

Gravitational Waves and ALPs

- Superradiance
 - Probes presence of very light scalars due to BH spin-down
- GWs from phase transitions
 - Probes possible UV completions of ALP models
 - See Rachel Houtz talk
- This talk: GWs from axion dynamics after inflation
 - Axion coupling to dark photon causes instability

- Axion coupled to Dark Photon
 - Introduction to dynamics
 - Source of GWs
 - Numerics: Lattice Analysis
- Realisations in Model Building
 - Relaxion
 - Kinetic Misalignment

Axion Cosmology: Misalignment Mechanism

Axion Evolution in Expanding Universe

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 \quad (\phi \text{ homogeneous})$$

Pinned by Hubble Friction

$$H > m$$



ϕ displaced from minimum

Axion Cosmology: Misalignment Mechanism

Axion Evolution in Expanding Universe

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 \quad (\phi \text{ homogeneous})$$

Pinned by Hubble Friction

$$H > m$$



ϕ displaced from minimum

Oscillating

$$H < m$$



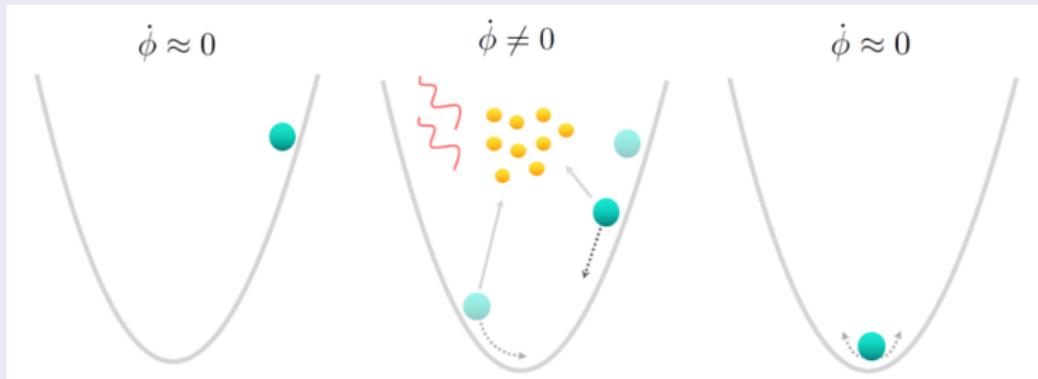
Hubble Friction \rightarrow Redshift
 \Rightarrow Cold Dark Matter

Additional Ingredient: Dark Photon

Dark Photon X + Coupling

$$\mathcal{L} \supset -\frac{\alpha}{4f}\phi X_{\mu\nu}\tilde{X}^{\mu\nu}$$

Dark Photon Production

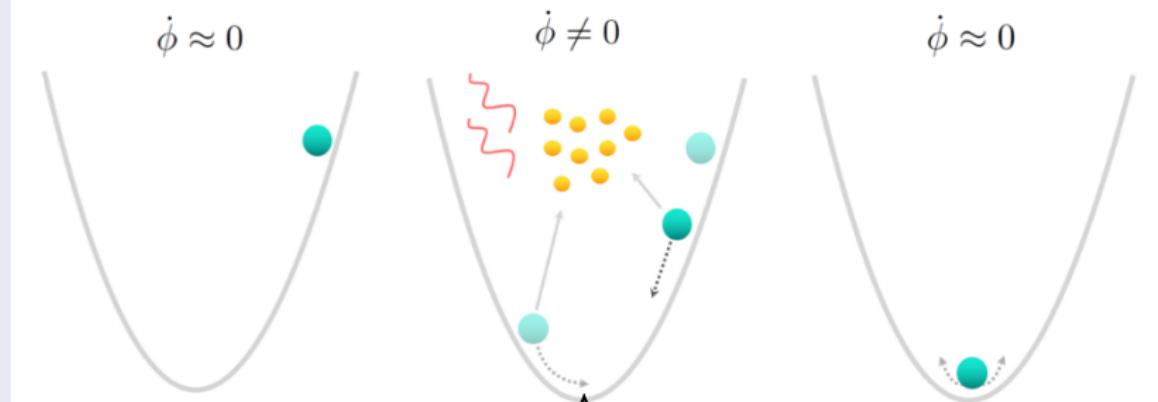


Motivation

- Deplete Axion Abundance
- Produce Vector DM

Agrawal et al '17, Kitajima et al '17
Agrawal et al '18

Production of Dark Photon



Dispersion of X :

$$\omega_{\pm}^2(t, k) = k^2 \mp \frac{\alpha}{f} \dot{\phi}(t) k < 0 \quad \text{for} \quad \begin{cases} \text{one helicity} \\ 0 < k < \alpha |\phi'| / f \end{cases}$$

Propagation \Rightarrow Exponential Growth $\propto \exp(|\omega_{\pm}| t)$

Gravitational Waves

Before Particle Production

Quantum Fluctuations
in Dark Photon Field:

$$v(\tau, k) = 1/\sqrt{2\omega} \exp(i\omega\tau)$$

Energy in Axion
→ homogeneous, isotropic



During Particle Production

Fluctuations grow
exponentially:

$$v \propto \exp(|\omega|\tau)$$

Energy in Dark Photon
→ inhomogeneous, anisotropic

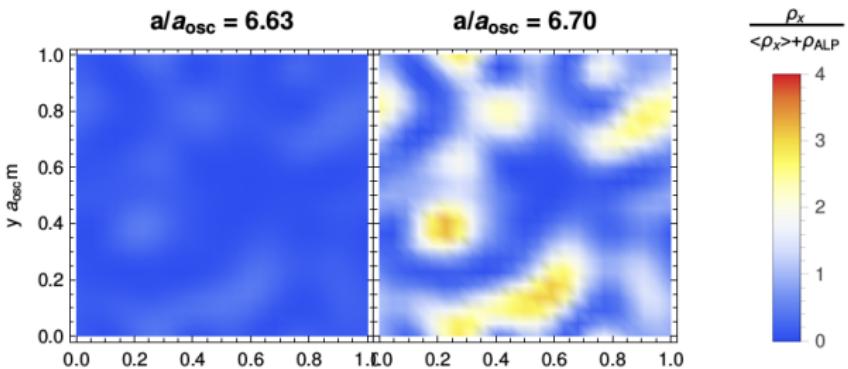
⇒ Particle Production leads to time-varying, anisotropic energy density that acts as source of Gravitational Waves:

Gravitational Wave → $h_{ij}''(\tau, k) + k^2 h_{ij}(\tau, k) = \frac{2}{m_{\text{pl}}^2} \Pi_{ij}(\tau, k)$ ↛ Anisotropic Stress

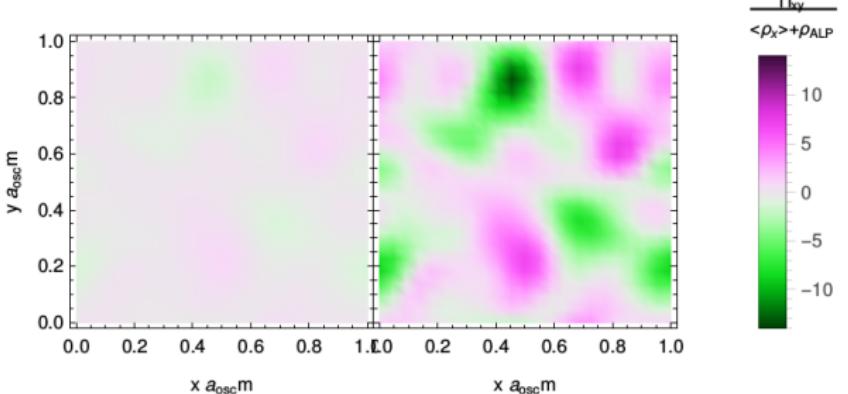
$$\Pi_{ij}(\tau, k) = -\frac{\Lambda_{ij,kl}}{a^2} \int \frac{d^3 q}{(2\pi)^3} [E_k(\tau, q) E_l(\tau, k-q) + B_k(\tau, q) B_l(\tau, k-q)]$$

Growth of Fluctuations

Energy Density
of Dark Photon

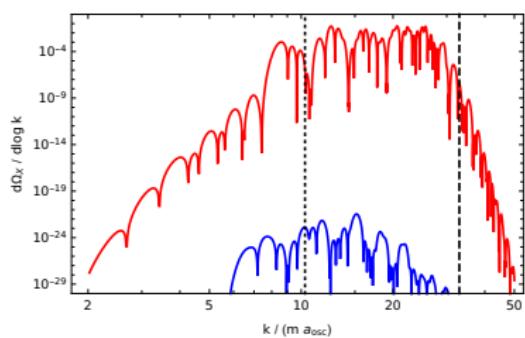


Anisotropic
Stress

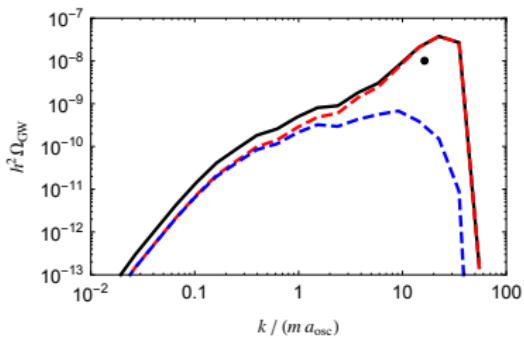


Gravitational Waves: Results

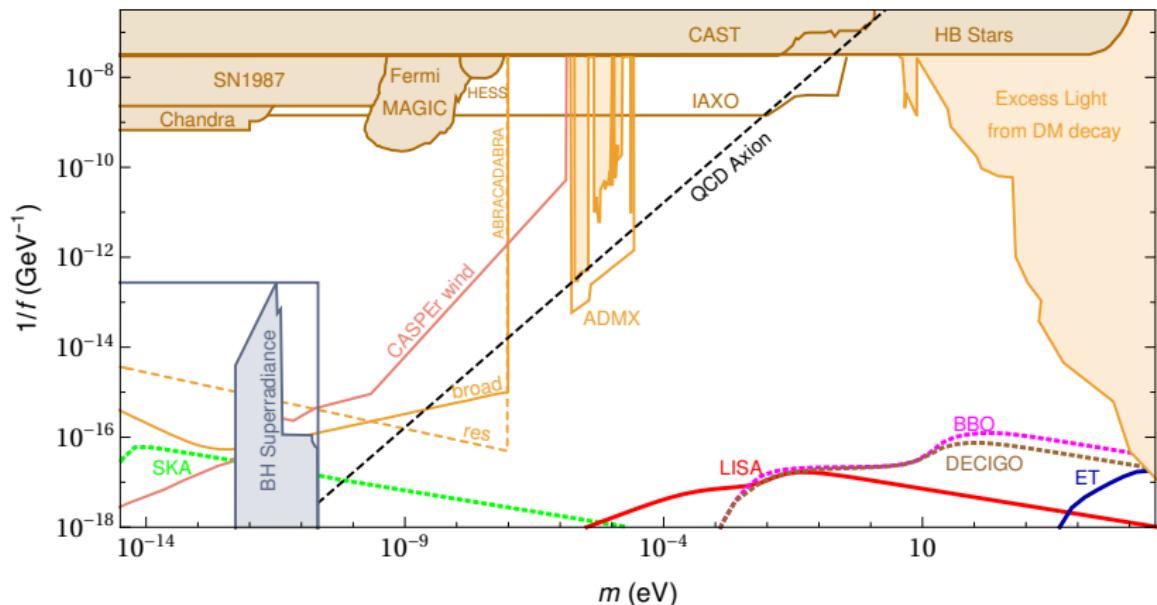
Rolling Axion
↓
Polarized dark photon
spectrum



Polarized GWs



Axion Discovery Potential



↑ Decay Constant f

$$\text{Source Strength } \Omega_\phi \approx \left(\frac{f}{m_{pl}} \right)^2$$

↑ Axion Mass m

Mass \Leftrightarrow Frequency \Leftrightarrow Detector

Lattice Results I: Less Axion Suppression

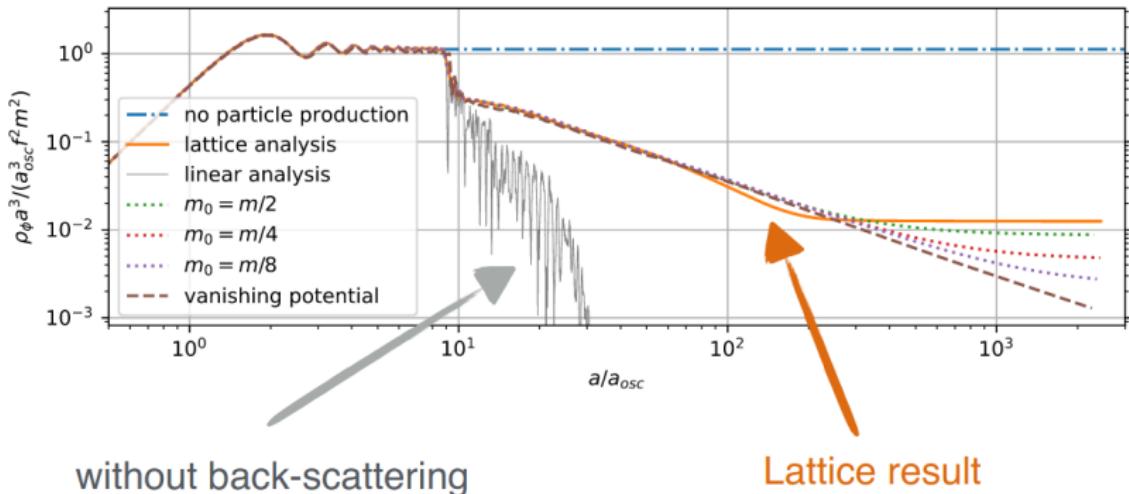
Old: Solve for DP mode functions, treat Axion as homogeneous

New: Solve E.O.M for discretized space-time

-include all the symmetries from the continuum

-includes full back-reaction onto axion

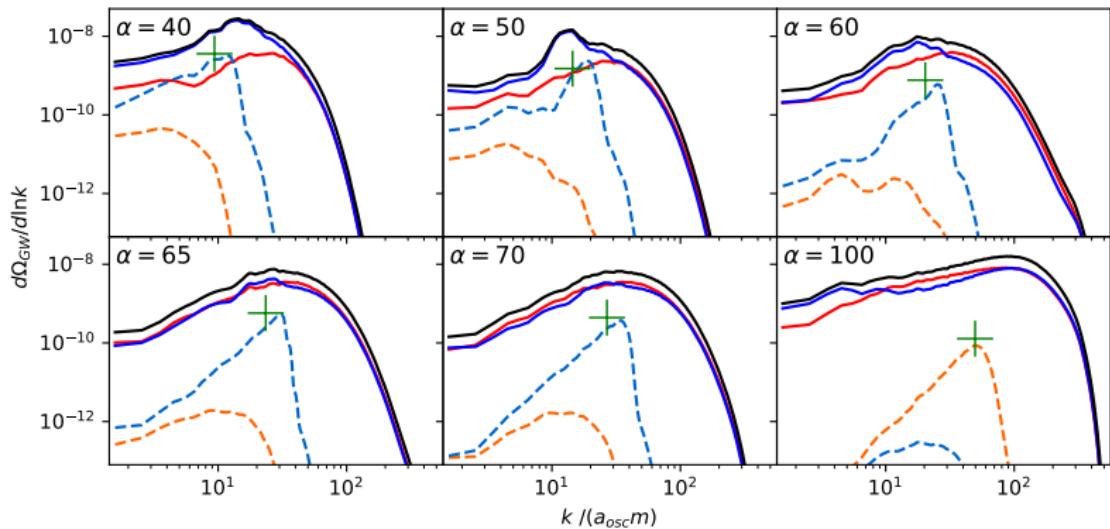
Figueroa et al '17



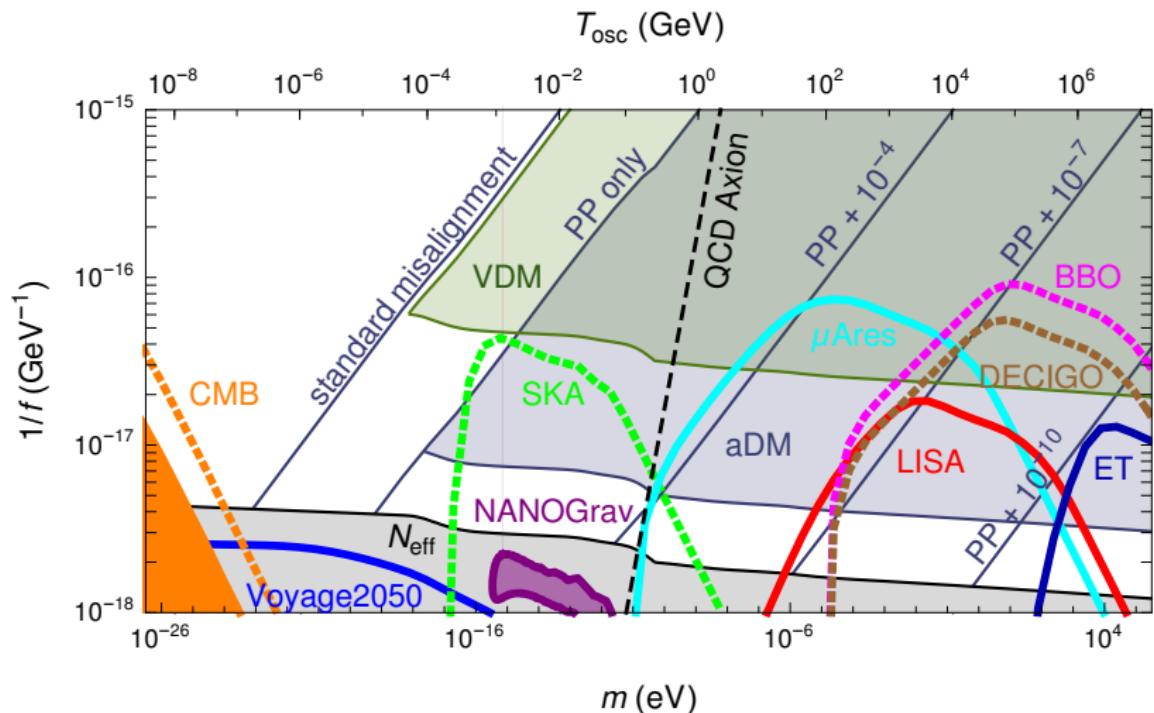
Axion inhomogeneities prohibit late time suppression! Agrawal et al '17

Lattice Results II: Less/ $\alpha + \theta$ -dependent Polarization

Subdominant Helicity from Re-Scatterings \Rightarrow Less Polarization

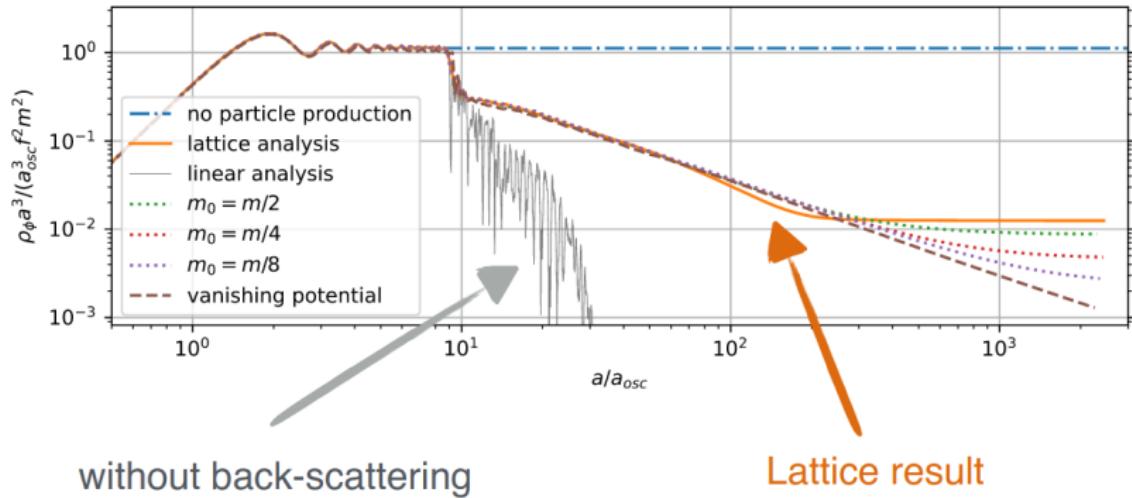


Detectable Region - Update



Most detectable parameters need extra axion suppression!

Achieving more Axion Suppression



- Time varying potential/mass
- Other source of $\dot{\phi}$ than potential

Audible Relaxion

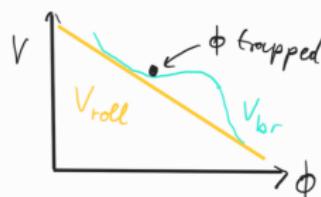
Setup:

$$-\mathcal{L} \supset V(H, \phi) + \frac{\alpha}{4} \frac{\phi}{f} X_{\mu\nu} \tilde{X}^{\mu\nu}$$

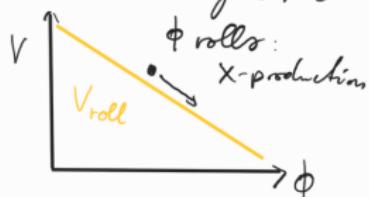
$$V(H, \phi) = V_{\text{roll}}(\phi) + \mu_H^2(\phi)|H|^2 + \lambda|H|^4 + V_{\text{br}}(H, \phi)$$

Evolution:

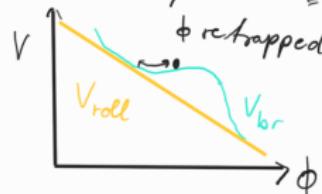
End of inflation: $\langle H \rangle = V_{EW}$



After reheating: $\langle H \rangle = 0$



At low temp.: $\langle H \rangle = V_{EW}$



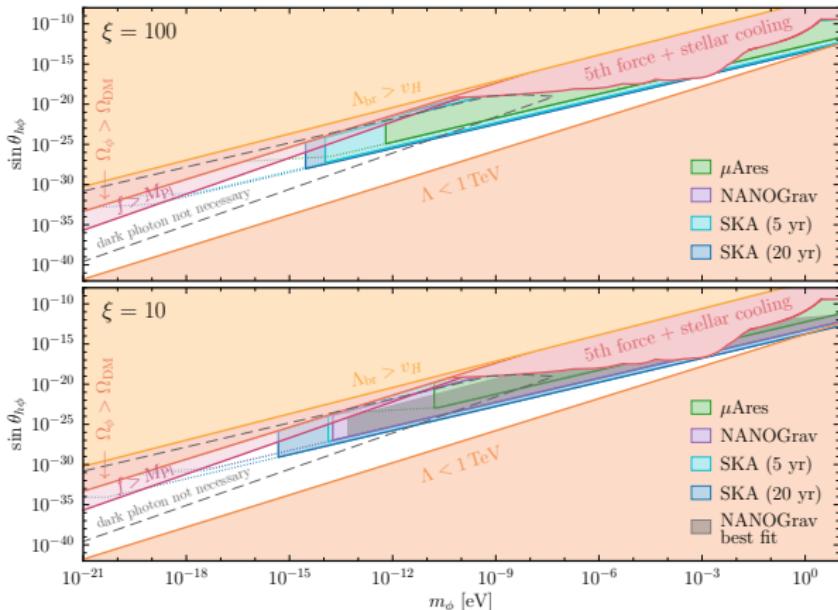
Results:

- Dark photon friction essential for retrapping
- Potentially observable GW signal

Audible Relaxion

Results:

- Dark photon friction essential for retrapping
- Potentially observable GW signal

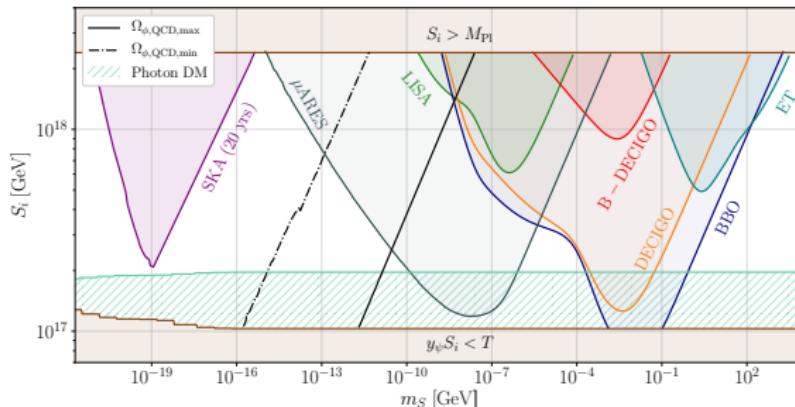


Kinetic Misalignment

Consider the case of large initial ϕ (Affleck–Dine):

- Detectable signal for smaller decay constants f / smaller couplings α
- Fix ALP mass to fit DM abundance
- Consistent with Axionogenesis

With $f = 5 \times 10^{13} \text{ GeV}$ and $\alpha = 10^{-2}$:



From Madge, WR, Schmitt, Schwaller (in preparation)

See also Co, Hall, Harigaya '20 and Co, Harigaya, Pierce '21

Conclusion

- Model: ALP + Dark Photon + Coupling $\frac{\alpha}{4f}\phi X_{\mu\nu}\tilde{X}^{\mu\nu}$
 - Tachyonic production of Dark Photon
- Tachyonic particle production frequently used in model building:
Inflation, Reheating, Relaxion, Axion Suppression, Vector DM
 - We now have precise numerical simulations
- GWs offer new way to probe axions/ALPs

Conclusion

- Model: ALP + Dark Photon + Coupling $\frac{\alpha}{4f}\phi X_{\mu\nu}\tilde{X}^{\mu\nu}$
 - Tachyonic production of Dark Photon
- Tachyonic particle production frequently used in model building:
Inflation, Reheating, Relaxion, Axion Suppression, Vector DM
 - We now have precise numerical simulations
- GWs offer new way to probe axions/ALPs

Thanks!

Backup

Axion Cosmology: Misalignment Mechanism

End of Inflation

- ϕ homogeneous



$T \sim f$

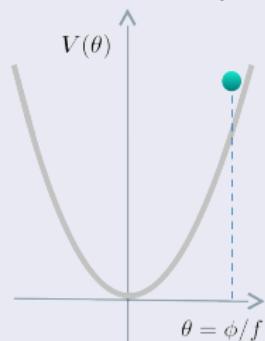
Spon. Symmetry Breaking
(Peccei-Quin Symmetry)



- Light Degree of Freedom:
Axion ϕ

$T \sim \Lambda$

Small Exp. Breaking (QCD Confines)



- Potential generated:
 $V(\phi) \sim \Lambda^4 (1 - \cos(\Theta + \phi/f))$
- Displacement from Minimum:
 $\phi = \Theta f$, $\Theta \sim 1$

Tachyonic Band

$$\omega_{\pm}^2(\tau, k) = k^2 \mp \frac{\alpha}{f} \phi'(\tau) k$$

$$\phi' \sim \phi_{osc} m \cdot a \left(\frac{a_{osc}}{a} \right)^{3/2} \cdot \cos(am \tau)$$

→ Produced Helicity changes

Efficient Tachyonic Growth:

Axion Oscillation Period $am <$ Growth Rate $|\omega_{\pm}|$

$$\omega_{\pm}^2 < 0 \rightarrow \omega_{\pm}^2 < -(am)^2$$

Tachyonic Band closes: $a/a_{osc} = (\theta\alpha/2)^{2/3}$

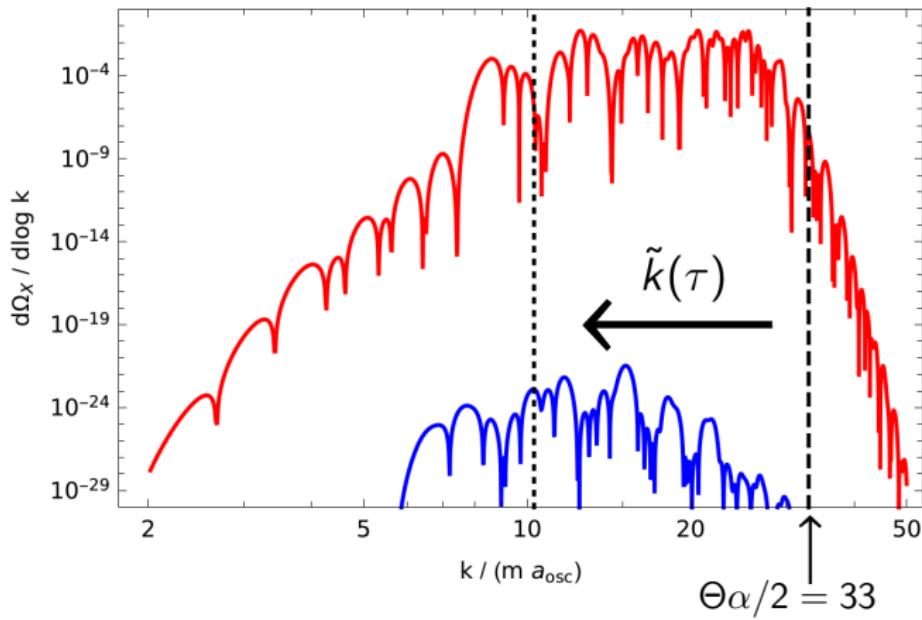
Fastest growing Mode - Peak in Photon Spectrum

$$\tilde{k}(\tau) = \frac{\alpha}{2f} \phi'(\tau) \approx \frac{\theta\alpha}{2} m \left(\frac{a_{osc}}{a} \right)^{3/2} a$$

Dark Photon Spectrum

Fastest growing Mode:

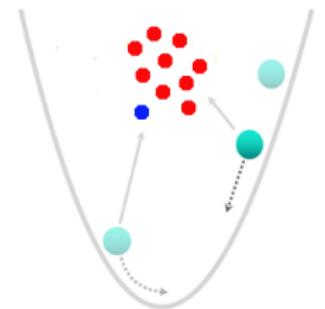
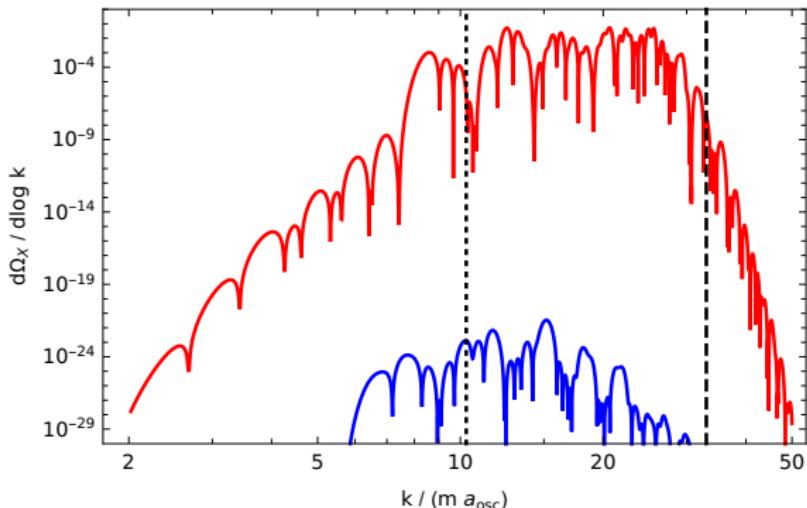
$$\tilde{k}(\tau) = \frac{\alpha}{2f} \phi'(\tau)$$



$$\Theta = 1.2, \quad \alpha = 55$$

Polarization of the Spectrum

$$\omega_{\pm}^2(\tau, k) = k^2 \mp \frac{\alpha}{f} \phi'(\tau) k \quad v_{\pm} \propto \exp(|\omega_{\pm}| \tau)$$



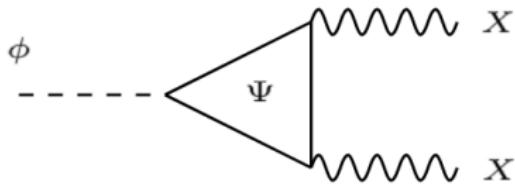
Parity Violation

$\langle \phi \rangle \neq 0 \longrightarrow \text{Polarized Spectrum}$

Axion - Dark Photon coupling

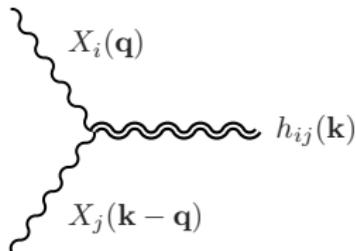
Starting from shift-symmetric coupling to fermions that carry dark charge e_d , using P.I. and fermion EoM:

$$\frac{1}{2} \frac{\partial^\mu \phi}{f} \bar{\Psi} \gamma_\mu \gamma_5 \Psi = -\frac{m_\Psi}{f} \phi \bar{\Psi} i \gamma_5 \Psi + \frac{N_\Psi e_d^2}{16\pi^2} \frac{\phi}{f} X_{\mu\nu} \tilde{X}^{\mu\nu}$$



⇒ Easiest way to get $\alpha > 1$, is large number of fermions N_Ψ

Features of the GW Spectrum



Peak Momentum/Frequency

$$k_{\text{peak}} \sim \sqrt{2}\tilde{k} \xleftarrow{\text{Dark Photon Peak}}$$

$$\sim m (\theta\alpha)^{2/3}$$

\hookrightarrow Axion Mass m determines Peak Frequency

Peak Amplitude

$$\frac{d\Omega_{\text{GW}}}{d \log k}(k_{\text{peak}}) \approx \Omega_X^2 \left(\frac{H}{k_{\text{peak}}} \right)^2 \approx \left(\frac{f}{m_{\text{pl}}} \right)^4 \left(\frac{\theta^2}{\alpha} \right)^{4/3}$$

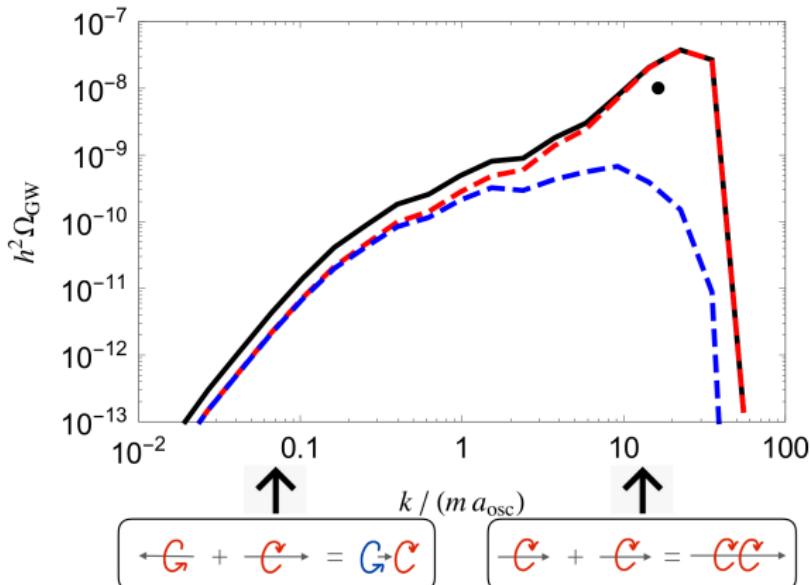
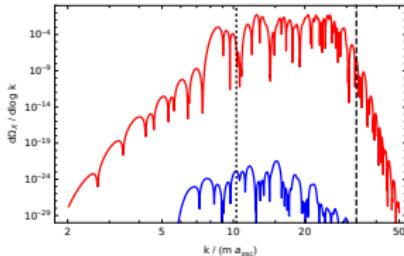
$$\Omega_X \approx \Omega_\phi \approx \left(\frac{\theta f}{m_{\text{pl}}} \right)^2$$

$\hookrightarrow f$ determines Peak Amplitude

$\hookrightarrow f \gtrsim 10^{17} \text{ GeV}$ for Detectable Signal

Features of the GW Spectrum: Chirality

Polarization of dark Photon Spectrum
causes the Peak of the GW Spectrum to be
polarized as well



Frequency

$$f_0 = \frac{k}{a_0} = \left(\frac{g_{s,\text{eq}}}{g_{s,\text{osc}}} \right)^{\frac{1}{3}} \left(\frac{T_0}{T_{\text{osc}}} \right) \frac{k}{a_{\text{osc}}}$$

For the peak:

$$\begin{aligned} f_0^{\text{peak}} &\approx (\theta\alpha)^{\frac{2}{3}} T_0 \left(\frac{g_{s,\text{eq}}}{g_{s,*}} \right)^{\frac{1}{3}} \left(\frac{m}{m_{\text{pl}}} \right)^{\frac{1}{2}} \\ &\approx 6 \times 10^{-4} \text{ Hz} \left(\frac{\alpha\theta}{66} \right)^{\frac{2}{3}} \left(\frac{m}{10 \text{ meV}} \right)^{\frac{1}{2}}. \end{aligned}$$

Amplitude

$$\begin{aligned} \Omega_{\text{GW}}^0 &= \Omega_{\text{GW}}^* \left(\frac{g_{s,\text{eq}}}{g_{s,*}} \right)^{\frac{4}{3}} \left(\frac{g_{\rho,*}}{g_{\rho,0}^\gamma} \right) \Omega_\gamma^0 \\ &\approx 1.67 \times 10^{-4} g_{\rho,*}^{-1/3} \Omega_{\text{GW}}^*. \end{aligned}$$