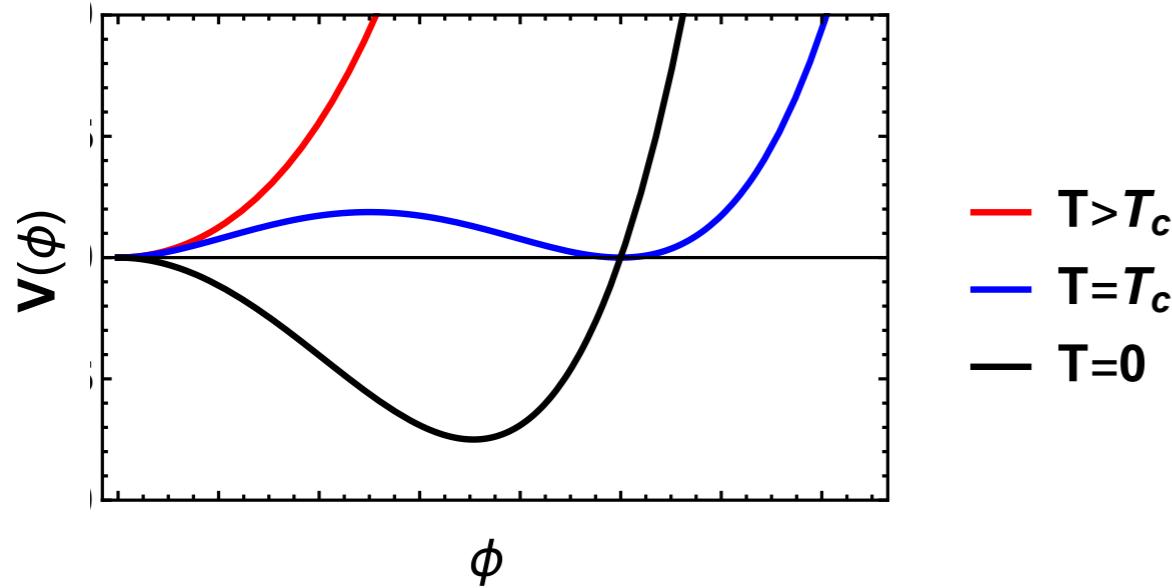

Dynamical Axions and Gravitational Waves



Rachel Houtz
Cambridge High
Energy Workshop
July, 2021

In Collaboration with D. Croon (TRIUMF & Durham) and
V. Sanz (U. Sussex & IFIC)

Gravitational Waves and Confining Sectors



- ❖ First order phase transitions can give a stochastic gravitational wave background

Witten (1984) Hogan (1986)

- ❖ Complementary probe of hidden sectors:

- Spontaneous symmetry breaking

Jaeckel, Khoze,

Spannowsky, 1602.03901

Schwaller, 1504.07263

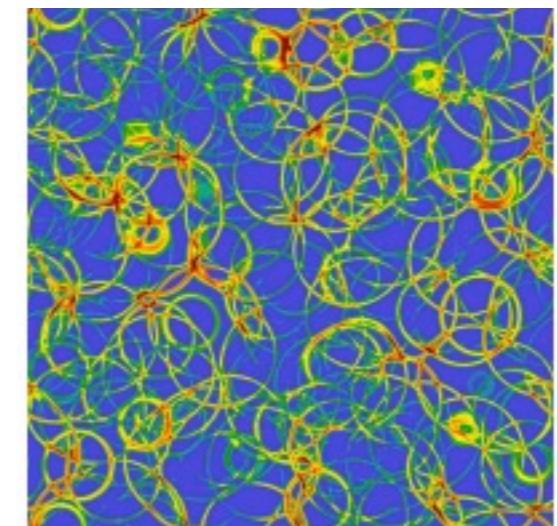
Croon, Sanz, White, 1806.02332

- Confining exotic color sectors

Helmboldt, Kubo, van der Woud, 1904.07891

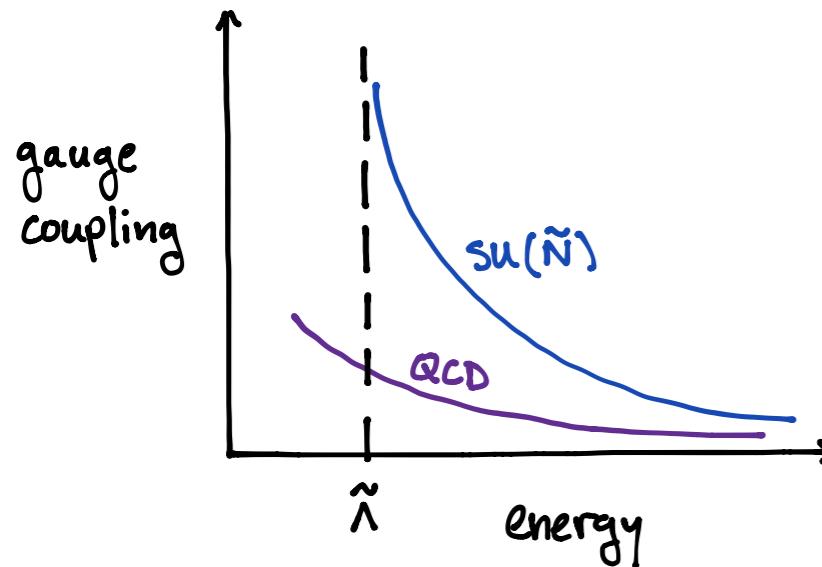
Bai, Long, Lu, 1810.04360

Sannino, et al, 2012.11614 Halverson et al, 2012.04071



Hindmarsh, Huber,
Rummukainen, Weir,
1504.0329

Gravitational Waves and Confining Sectors



- ❖ First order phase transition at confinement if $N_F \geq 3$
Pisarski, Wilczek (1984)
- ❖ Gravitational waves can probe confining exotic color sectors
- ❖ Use a low energy effective theory to try and parameterize the behavior of the potential at T_c
Helmboldt, Kubo, van der Woud, 1904.07891
Bigazzi, et al, 2011.08757
- ❖ Improve with lattice results for pure YM (or only heavy flavors)
Sannino, et al, 2012.11614 Halverson et al, 2012.04071 Brower, et al, 2006.16429
- ❖ Model building can motivate parameters in the low energy EFT

Dynamical Axions

- ❖ Solving the Strong CP problem by employing $U(1)_{PQ}$ results in the axion

$$\mathcal{L} \ni \frac{g^2}{32\pi^2} \frac{a}{f_a} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

Peccei, Quinn (1977)

Wilczek (1978)

Weinberg (1978)

Massless quark solution:

If $m_\psi = 0$, under chiral rotation of ψ

$$\theta \frac{g^2}{32\pi^2} G \tilde{G} \rightarrow (\theta - 2\alpha) \frac{g^2}{32\pi^2} G \tilde{G}$$

→ θ can be removed by field redefinition

- ❖ Massless quarks will form bound states, one of which is the dynamical axion



Motivation for Exotic Confining Groups

(1) Additional color interactions can alter the m_a, f_a relationship

Di Luzio, Giannotti, Nardi, Visinelli,
arXiv:2003.01100

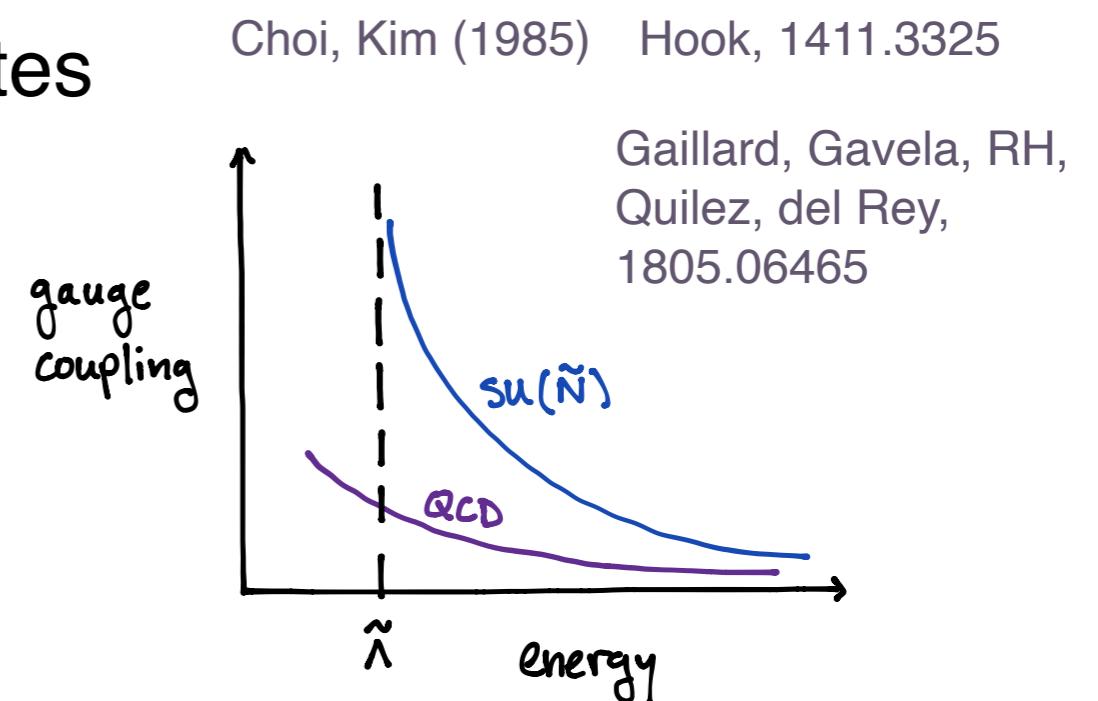
$$m_a^2 f_a^2 \approx m_\pi^2 f_\pi^2 \rightarrow + \sim \Lambda_{\text{new}}^4$$

(2) Hide massless quark in bound states



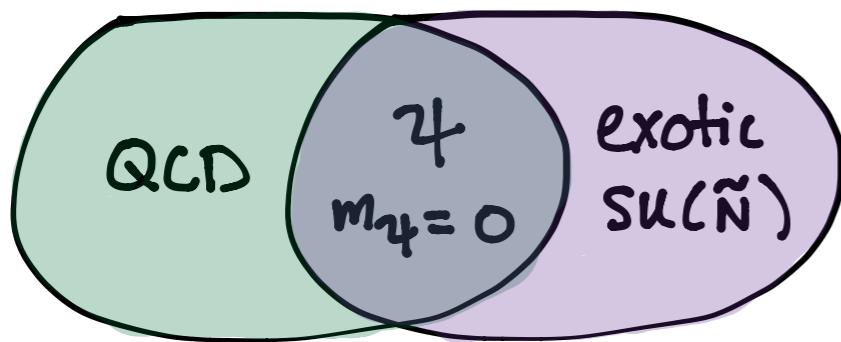
$$m_\psi = 0$$

$$\sim \Lambda_{\text{new}}$$



Generic Properties of Dynamical Axion Models

- ❖ Massless messenger fields



- ❖ $N_F \geq 3$ at $SU(\tilde{N})$ confinement

- ❖ First order PT at $\tilde{\Lambda} \sim 3$ TeV

Pisarski, Wilczek (1984)

- ❖ Quadratically divergent mass terms for pions

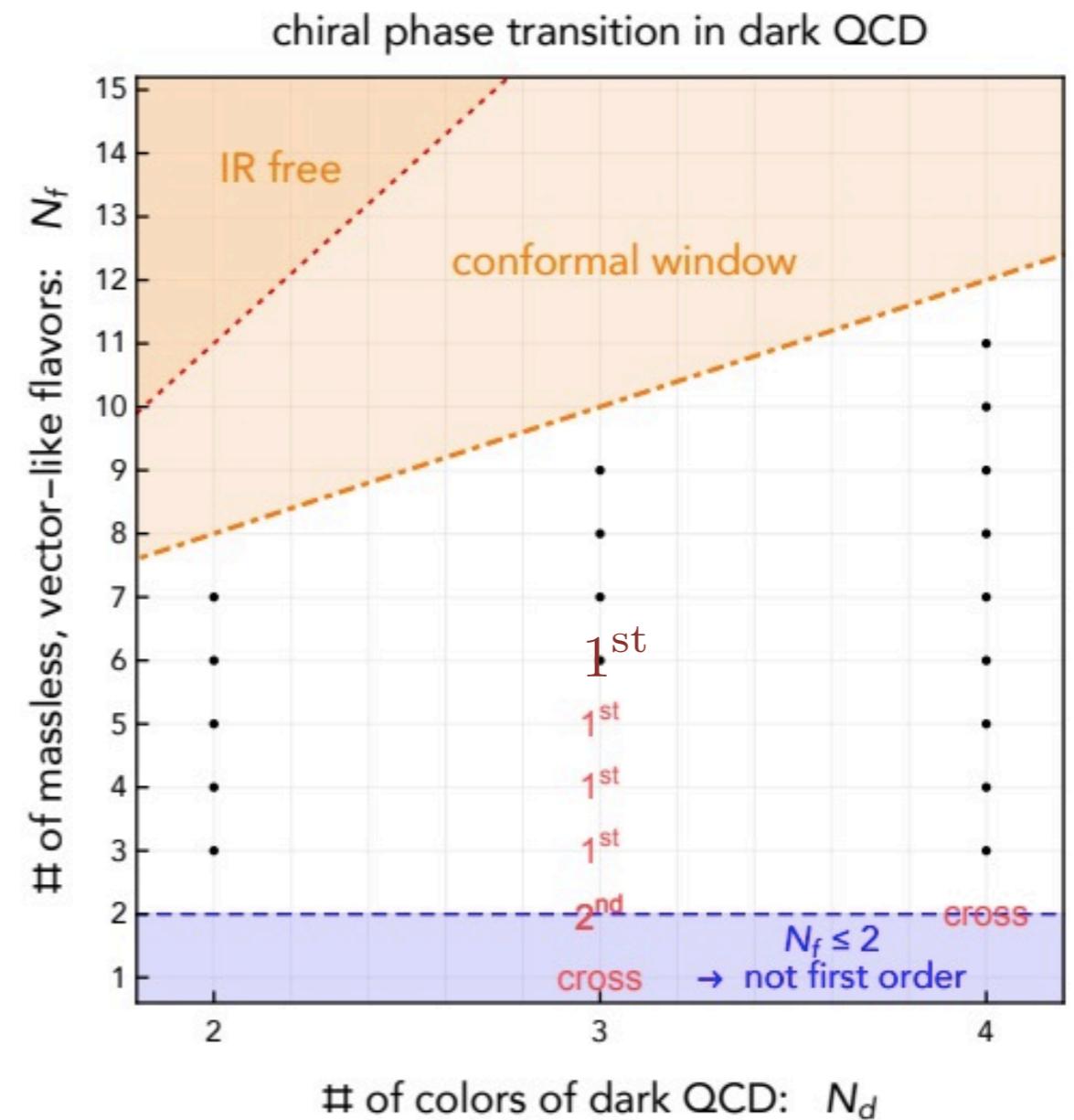


g_{QCD}

$$m^2(\pi) \sim g_{\text{QCD}}^2 \tilde{\Lambda}^2$$

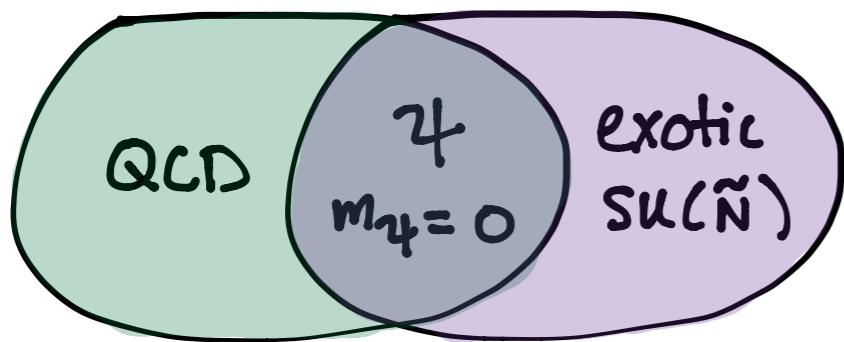
Plot lifted from: Bai, Long, Lu, arXiv:1810.04360

$N_F = 6$: Iwasaki, Kanaya, Sakai, Yoshié, hep-lat/9504019



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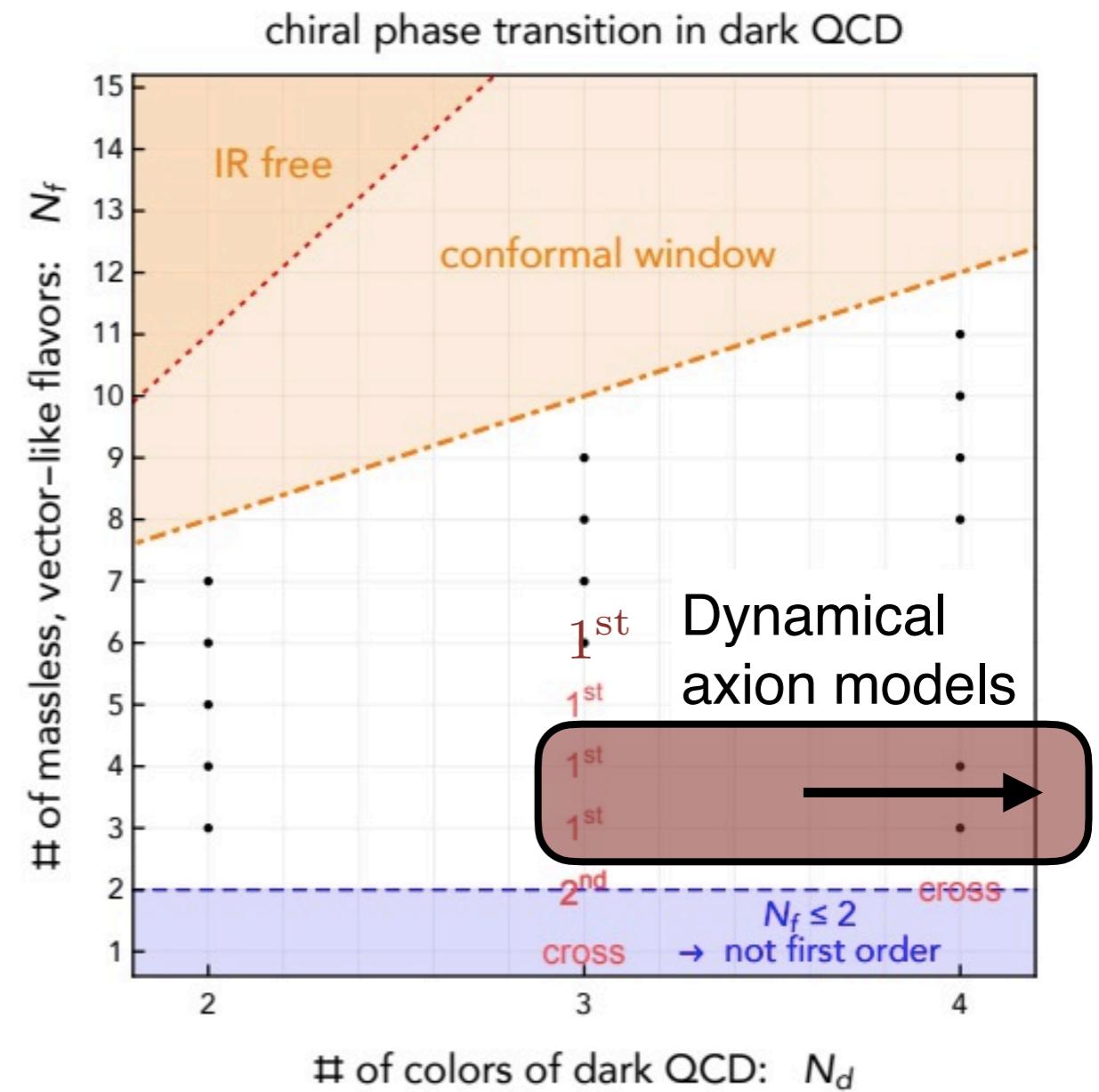


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Plot lifted from: Bai, Long, Lu, arXiv:1810.04360

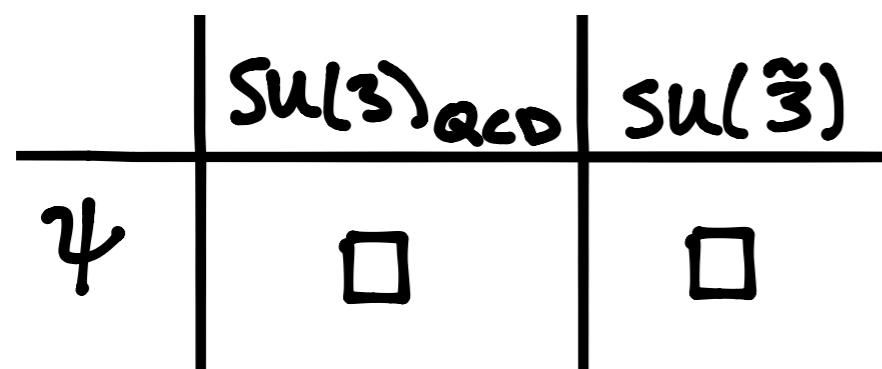
$N_F = 6$: Iwasaki, Kanaya, Sakai, Yoshié, hep-lat/9504019



Dynamical Axion Models

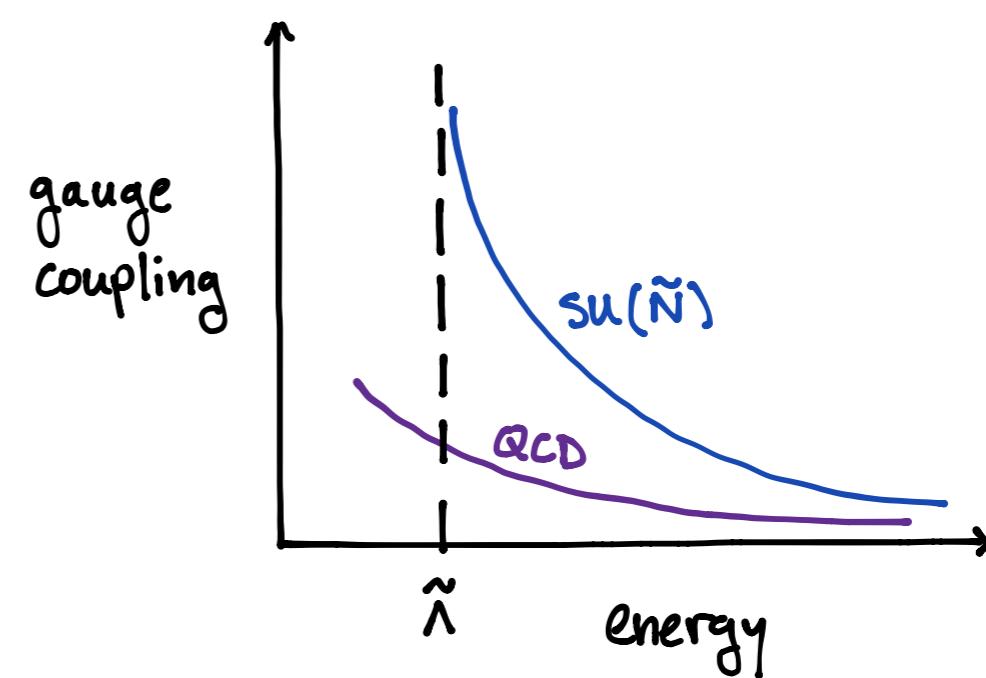
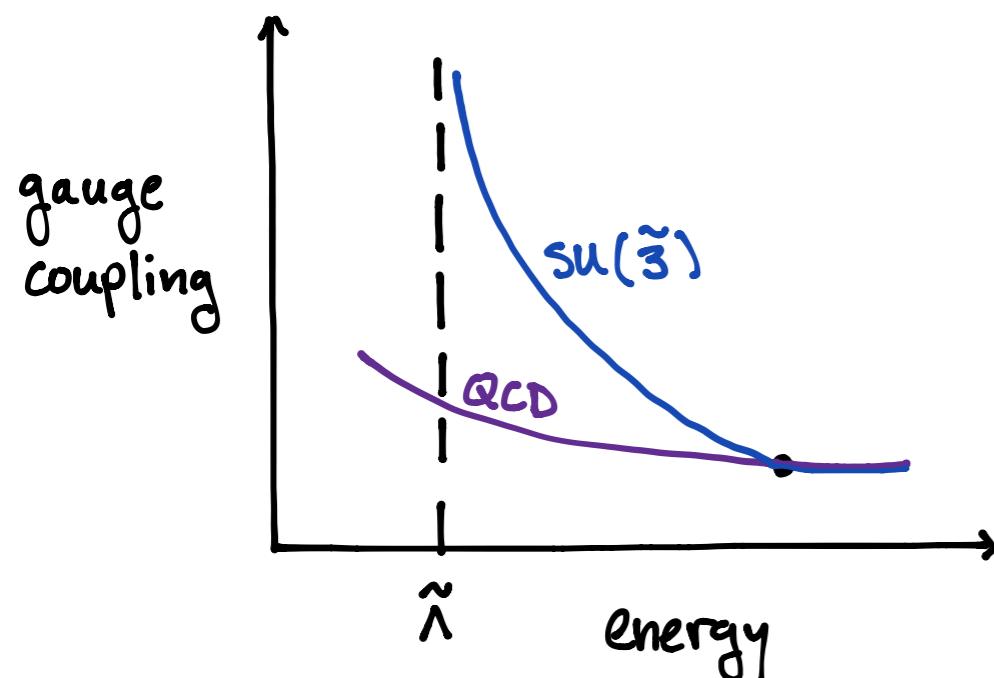
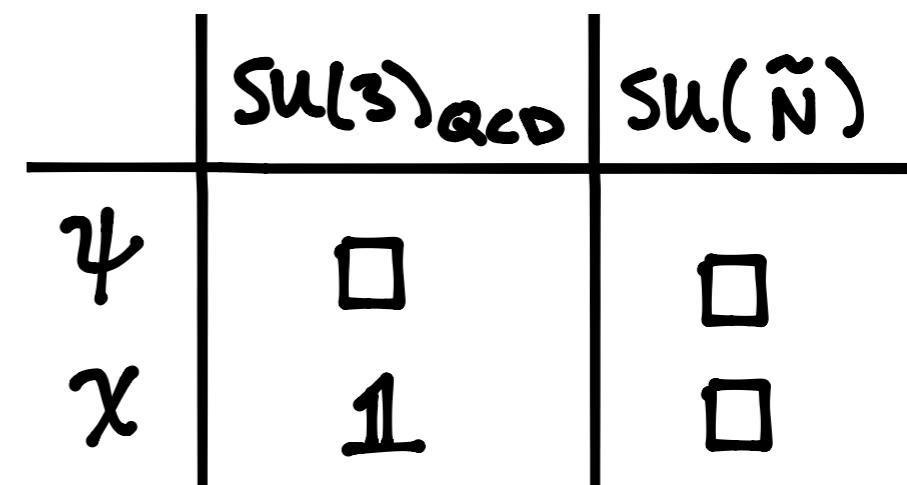
$$N_F = 3$$

Hook, 1411.3325



$$N_F = 4$$

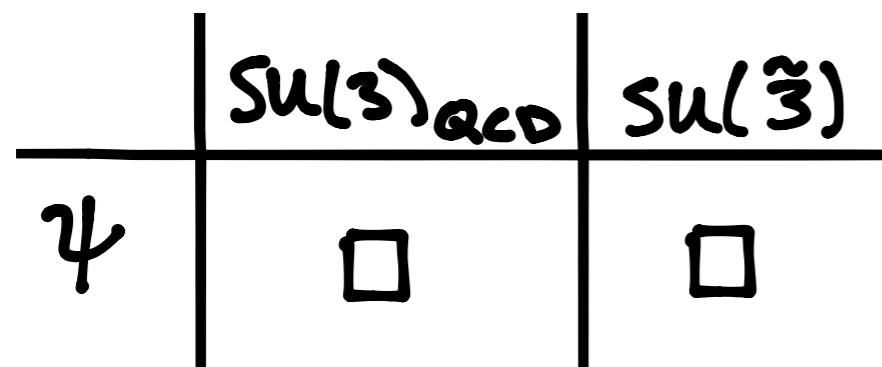
Choi, Kim (1985)



Dynamical Axion Models

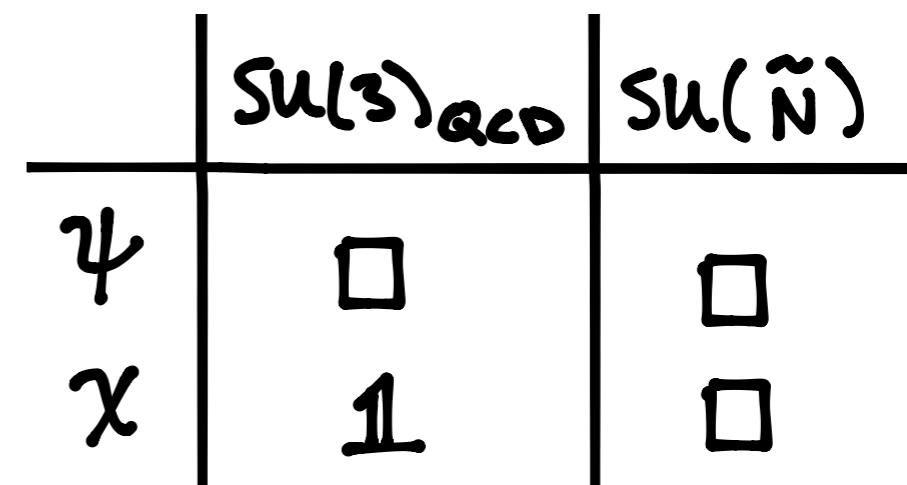
$$N_F = 3$$

Hook, 1411.3325



$$N_F = 4$$

Choi, Kim (1985)



- ❖ Chiral symmetry breaking pattern at confinement:

$$SU(N_F)_L \times SU(N_F)_R \rightarrow SU(N_F)_V$$

- Bound states composed of massless quarks live at $\tilde{\Lambda}$
- Possible light pNGB states

$SU(\tilde{3})$ Confinement, $N_F = 3$

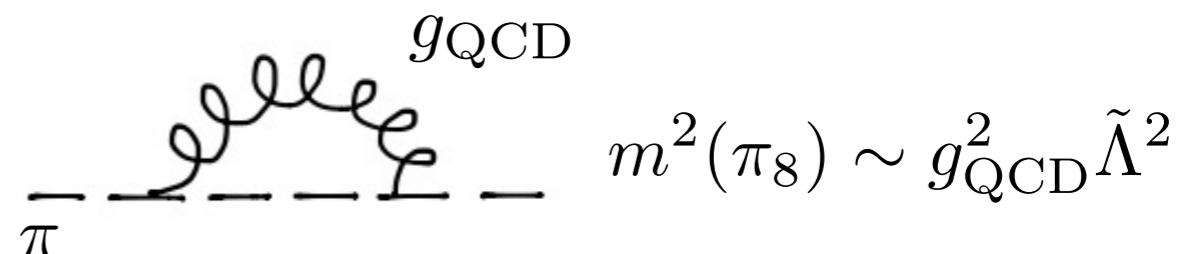
- ❖ Spontaneous chiral symmetry breaking:

$$SU(3)_L \times SU(3)_R \times U(1)_L \times U(1)_R \rightarrow SU(3)_V \times U(1)_V$$

- ❖ Resulting Goldstone Bosons: $9 \rightarrow 8_c + 1_c = \pi_8 + \eta'$

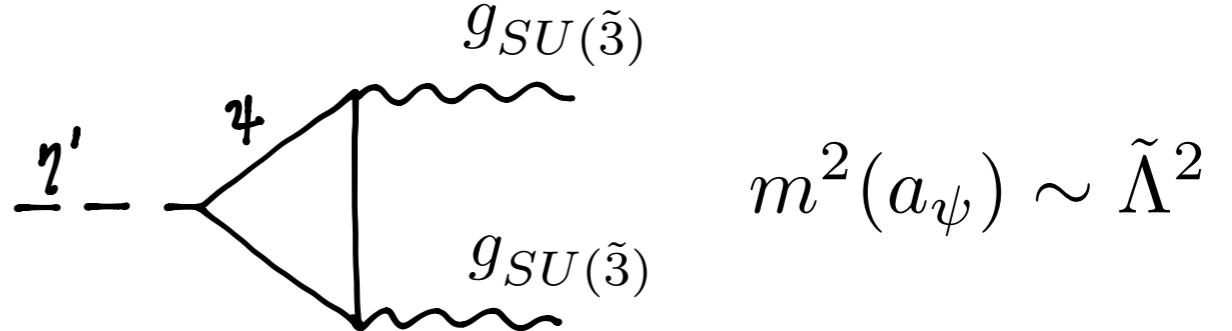
Explicit symmetry breaking effects:

- (1) QCD explicitly breaks $SU(3)_V$



- (2) $G\tilde{G}$ explicitly breaks $U(1)_A$

→ The η' is a **visible dynamical axion** a_ψ



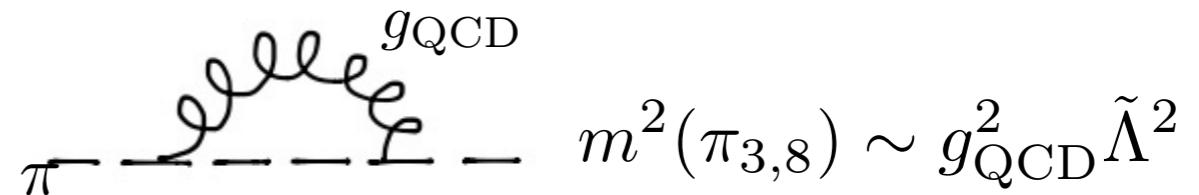
$SU(\tilde{N})$ Confinement, $N_F = 4$

Spontaneous symmetry breaking: $U(4)_L \times U(4)_R \rightarrow U(4)_V$

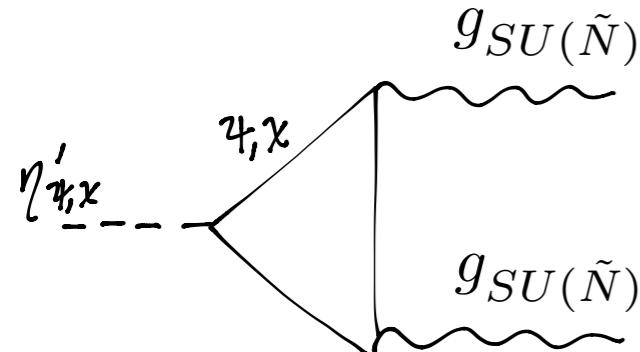
- ❖ Resulting Goldstone Bosons: $15 + 1 \rightarrow 8_c + 3_c + \bar{3}_c + 1_c + 1_c$
 $\vdots = \pi_8 + \pi_3 + \bar{\pi}_3 + \eta'_\psi + \eta'_\chi$

Explicit symmetry breaking effects:

(1) QCD explicitly breaks $SU(4)_V$



(2) $G\tilde{G}$ explicitly breaks $U(1)_A$



$$m^2(a_\psi) \sim \tilde{\Lambda}^2$$

$$m(a_\chi)\tilde{f} \sim m_\pi f_\pi$$

- ❖ The anomaly only gives mass to one axion a_ψ
- ❖ The light a_χ is an **invisible dynamical axion**

Visible Axion Models $N_F = 4$

- ❖ New physics at high energies can induce sizable instanton corrections to the axion mass

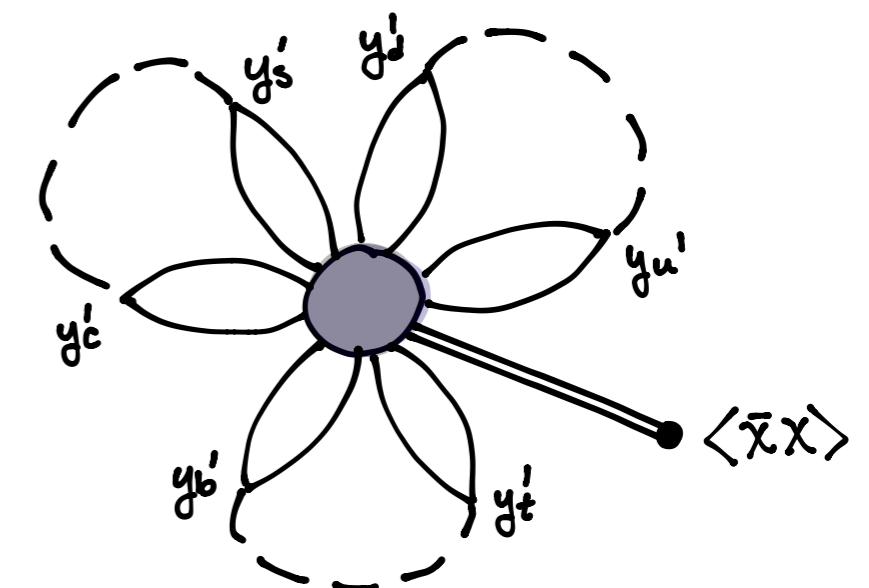
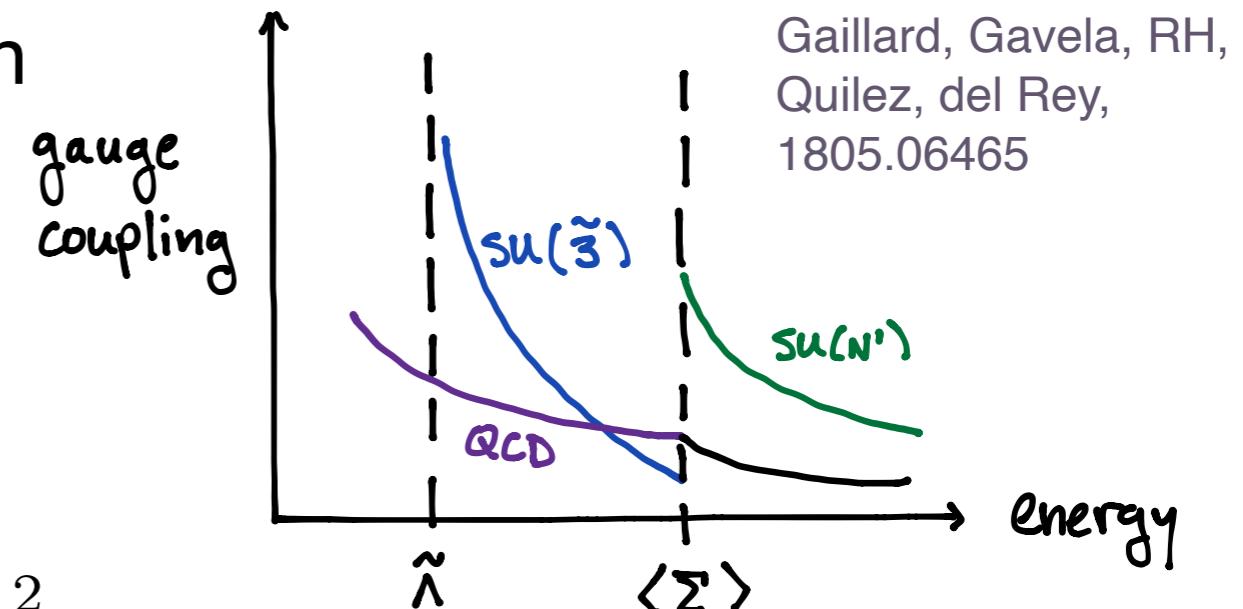
Agrawal, Howe, 1710.04213 & 1712.05803

Fuentes-Martin, Reig, Vicente, 1907.02550

$$m^2(a_\psi) \sim \tilde{\Lambda}^2 \quad \rightarrow \quad m^2(a_\psi) \sim \tilde{\Lambda}^2$$

$$m(a_\chi)\tilde{f} \sim m_\pi f_\pi \quad m^2(a_\chi) \sim \Lambda_{SSI}^2$$

- ❖ Possible to have a combination of anomalous effects raise the mass of the lightest $a_{\chi,\psi}$

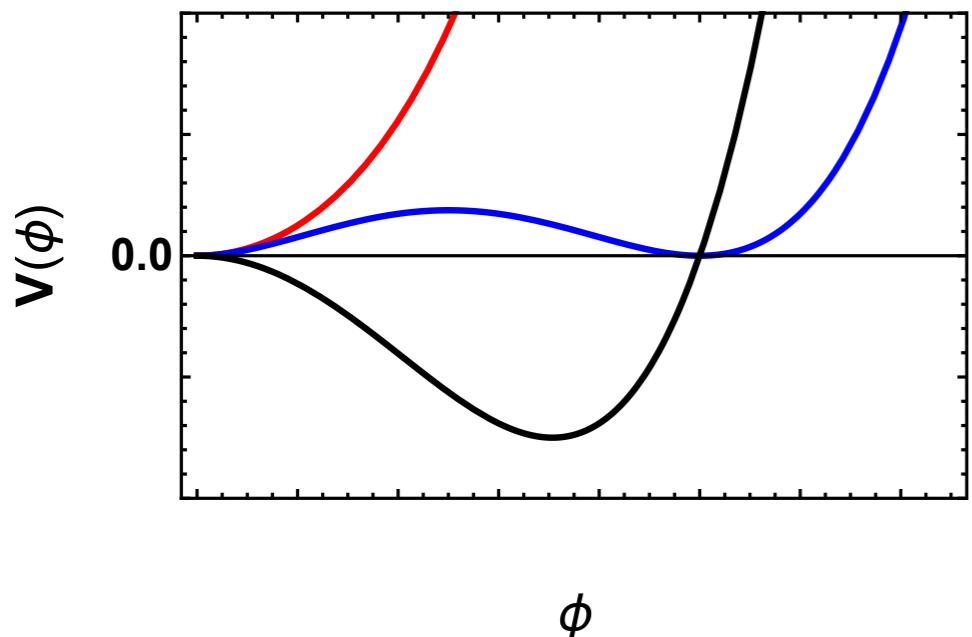


Phase Transition at Confinement

- ❖ Model the phase transition using the linear sigma model

$$V(\Sigma) = -m_\Sigma^2 \text{Tr} (\Sigma \Sigma^\dagger) + \frac{\lambda}{2} [\text{Tr} (\Sigma \Sigma^\dagger)]^2 + \frac{\kappa}{2} (\Sigma \Sigma^\dagger \Sigma \Sigma^\dagger),$$

- ❖ Spontaneous chiral symmetry breaking $\Sigma_{ij} \sim \langle \bar{\psi}_{Rj} \psi_{Li} \rangle$



$$\Sigma_{ij} = \frac{\varphi + i\eta'}{\sqrt{2N_F}} \delta_{ij} + X^a T_{ij}^a + i\pi^a T_{ij}^a$$

$\langle \varphi \rangle = 0, \quad T \gg 0$ Chiral symmetry restored

$\langle \varphi \rangle = f_\Sigma, \quad T \leq T_c$ Chiral symmetry

Phase Transition at Confinement

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Heavy fields corresponding to unbroken generators

$$\textcircled{\psi} \text{---} \textcircled{\psi} \sim \tilde{\Lambda}^2$$

Phase Transition at Confinement

- ❖ Model the phase transition using the linear sigma model

$$V(\Sigma) = -m_\Sigma^2 \text{Tr} (\Sigma \Sigma^\dagger) + \frac{\lambda}{2} [\text{Tr} (\Sigma \Sigma^\dagger)]^2 + \frac{\kappa}{2} (\Sigma \Sigma^\dagger \Sigma \Sigma^\dagger),$$

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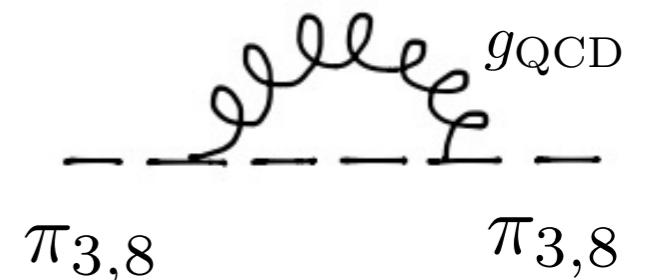
(pseudo) Goldstone Boson fields

- ➔ Masses due explicit symmetry breaking effects

Ingredients of the Potential of the gNGB States

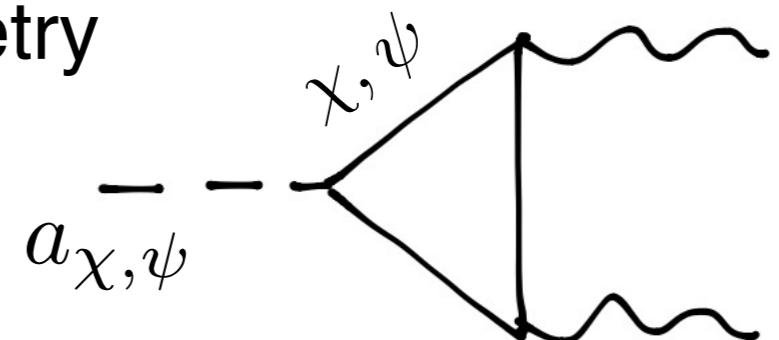
(1) QCD explicitly breaks the flavor symmetry

→ Add a mass term for the pions



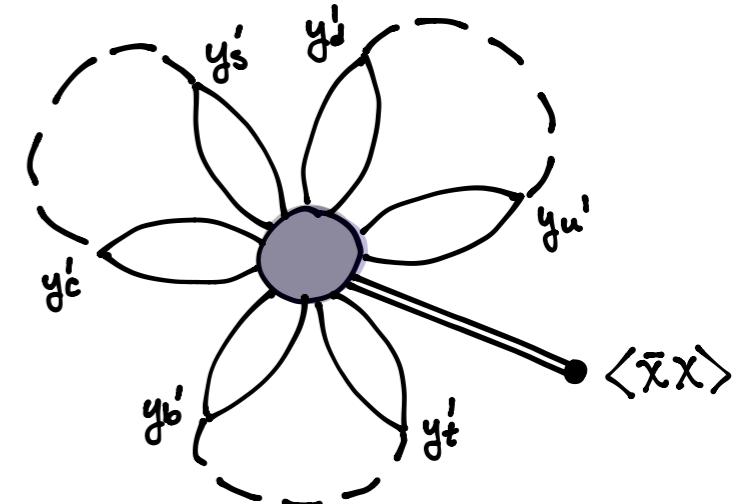
(2) $G\tilde{G}$ explicitly breaks the $U(1)_A$ symmetry

→ Add a mass term for the a_ψ



(3) Small sized instantons lift the mass of the a_χ

→ Add a tunable mass term for the a_χ

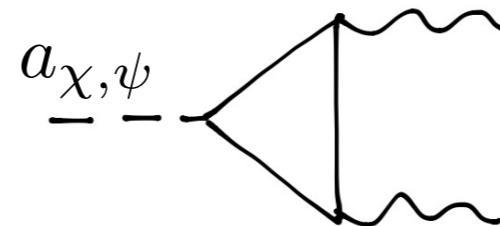


Phase Transition at Confinement

- ❖ Symmetry breaking parameters, μ_Σ , ξ , μ_{SSI} determine the masses of the pNGB's π and η'

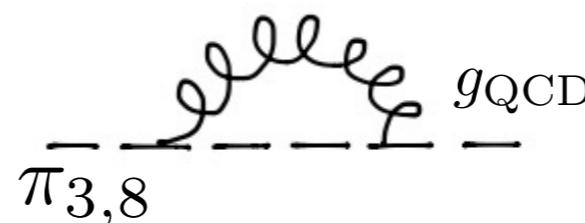
$$V(\Sigma) = -m_\Sigma^2 \text{Tr} (\Sigma \Sigma^\dagger) + \frac{\lambda}{2} [\text{Tr} (\Sigma \Sigma^\dagger)]^2 + \frac{\kappa}{2} (\Sigma \Sigma^\dagger \Sigma \Sigma^\dagger)$$

$$- (\mu_\Sigma \det \Sigma + h.c.)$$



~~$U(1)_A$~~

$$-\xi \text{Tr } Q^a \Sigma \Sigma^\dagger Q^{a\dagger}$$

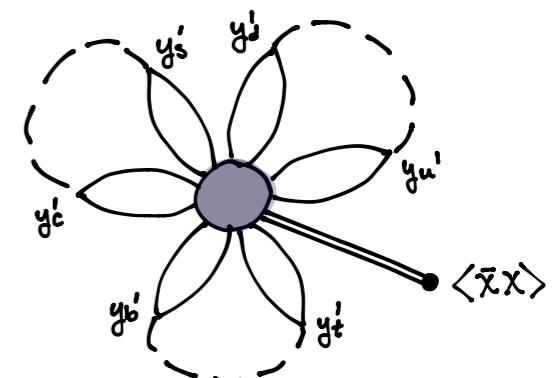


~~$SU(N_F)$~~

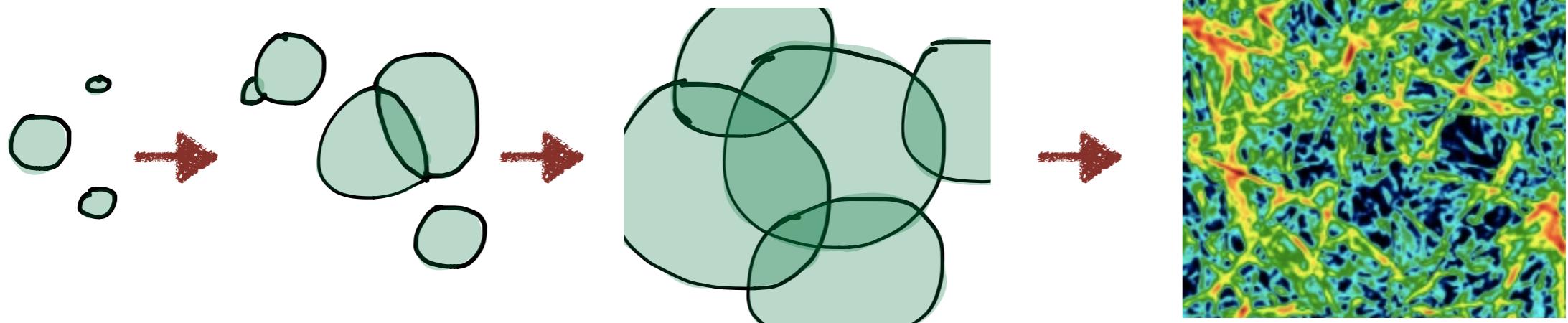
$$-\mu_{SSI} \text{Tr} (P_\chi \Sigma P_\chi \Sigma^\dagger)$$



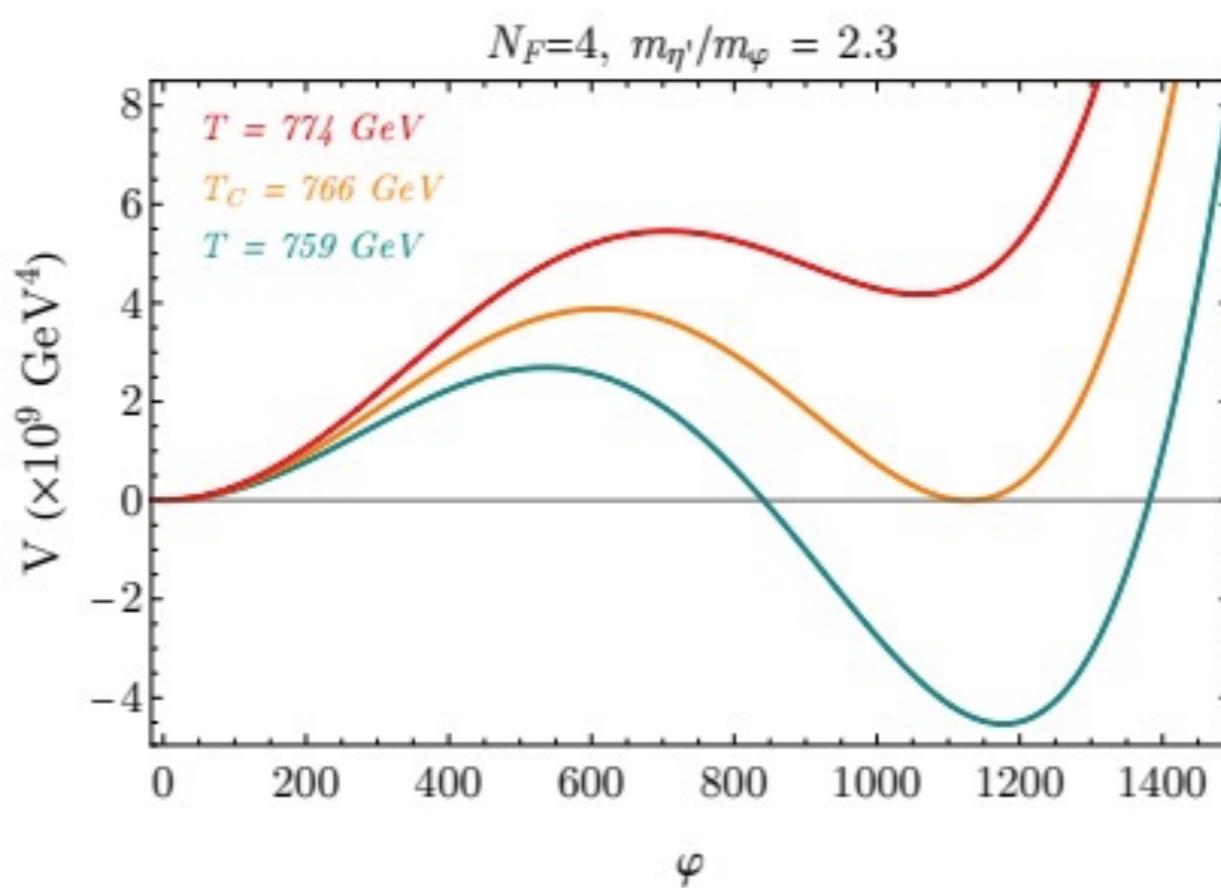
Include new
mass contributions from
small-sized instantons



Phase Transition in the Early Universe



Weir, "The sound of gravitational waves from a [confinement] phase transition," saoghal.net/slides/ectstar/

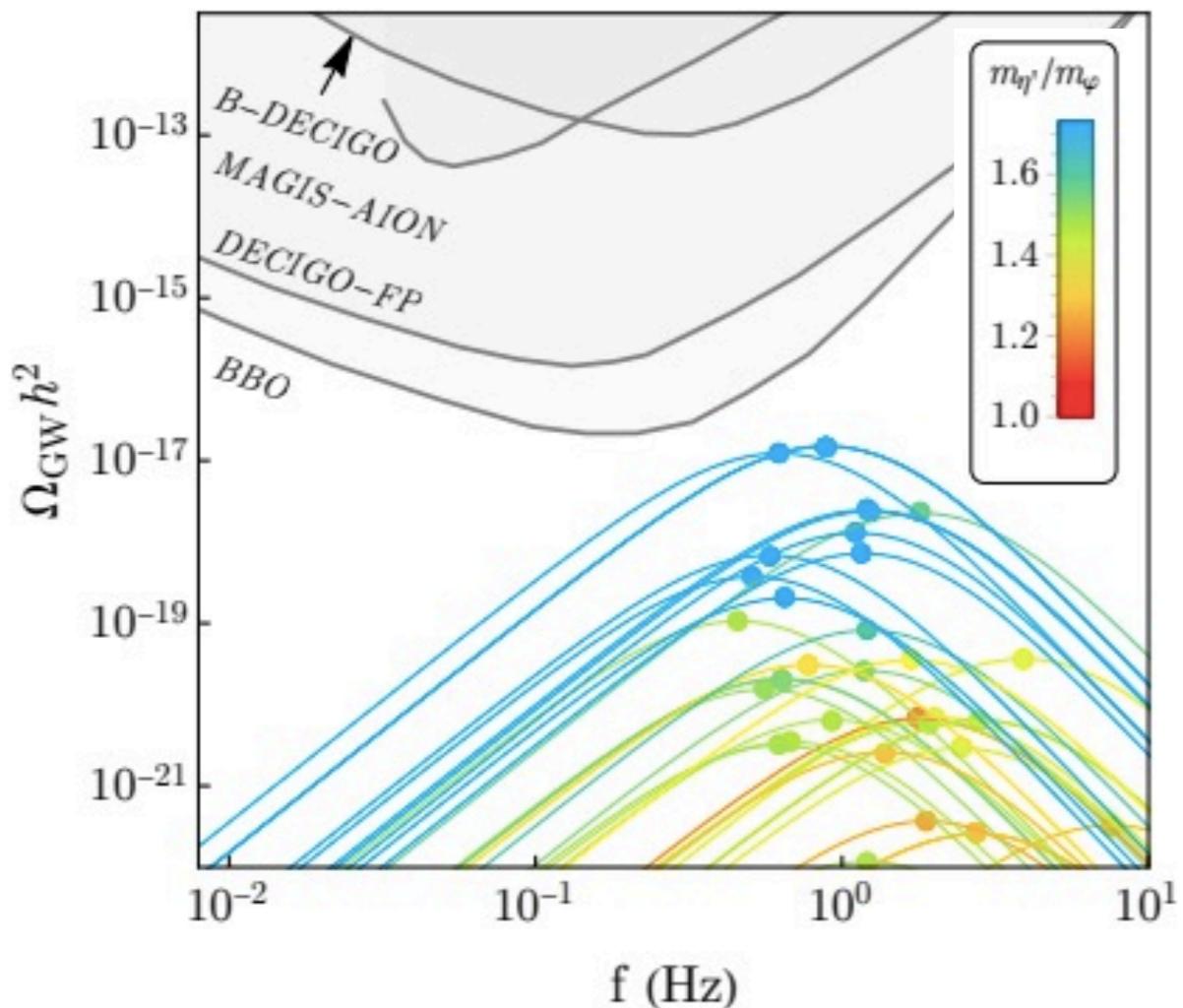


- ❖ Find the bounce solution to describe the tunneling from the false vacuum
- ❖ Calculate the frequency and power spectrum of the stochastic GW background

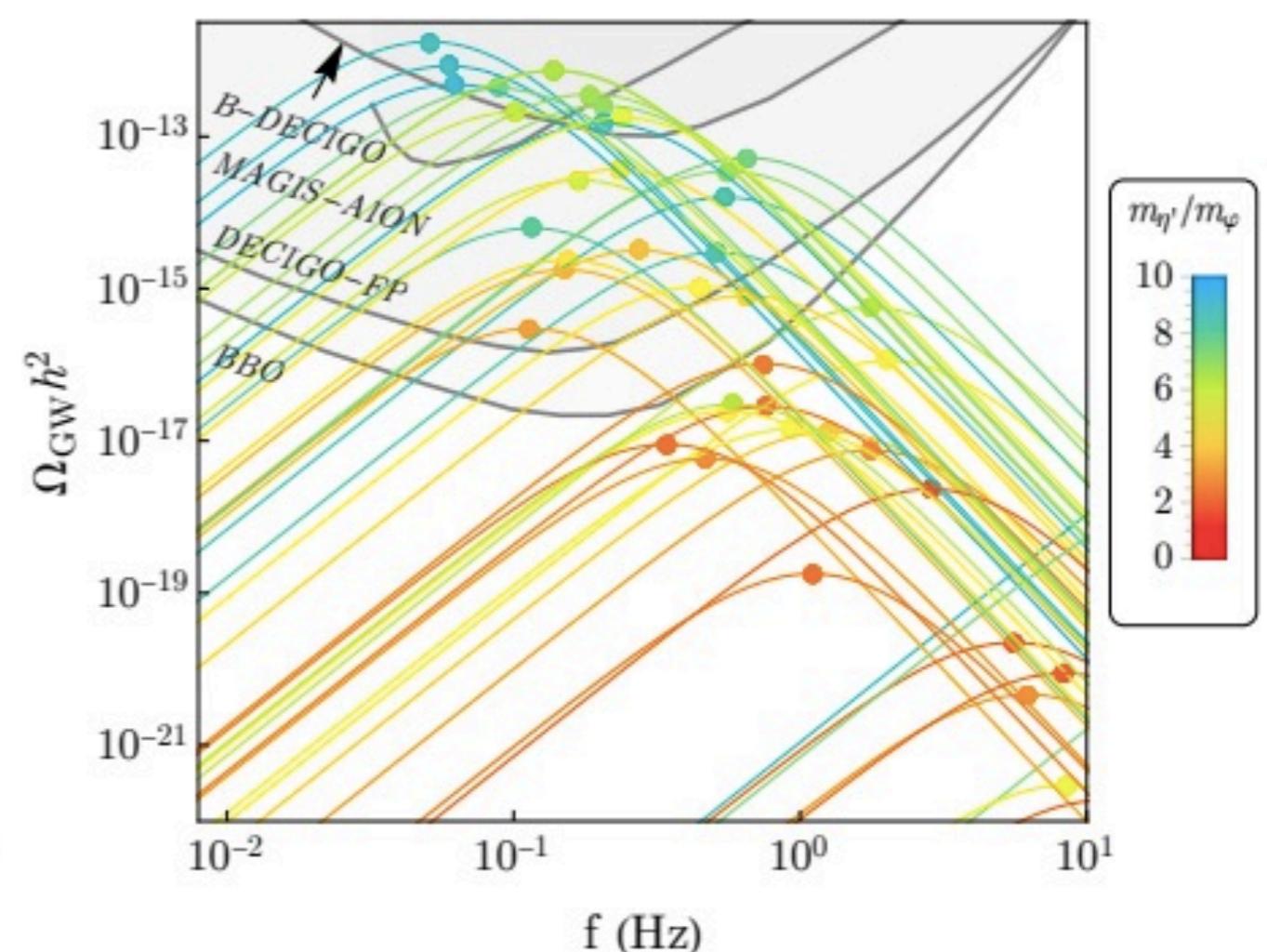
Gravitational Wave Signal

$\tilde{\Lambda} \sim 3 \text{ TeV}$ Croon, RH, Sanz, 1904.10967

$$N_F = 3$$

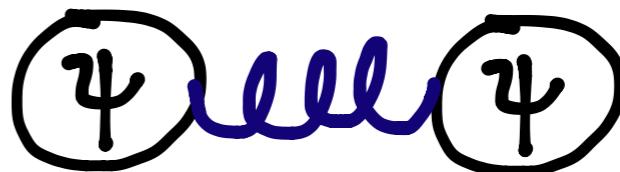


$$N_F = 4$$



Summary

- ❖ Prospects for gravitational wave signals for dynamical axion models with $\tilde{\Lambda} \sim 3$ TeV
- ❖ Features of dynamical axion models:
 - Massless quark messenger between QCD and $SU(\tilde{N})$
 - At least three light flavors and a first order phase transition
 - Generic parameters in the theory below $\tilde{\Lambda}$, for example the exotic pion masses due to gluon interactions
- ❖ The gravitational wave signature is sensitive to the anomalous effects that raise the axion mass in the linear sigma model



Temperature Dependence of μ_Σ

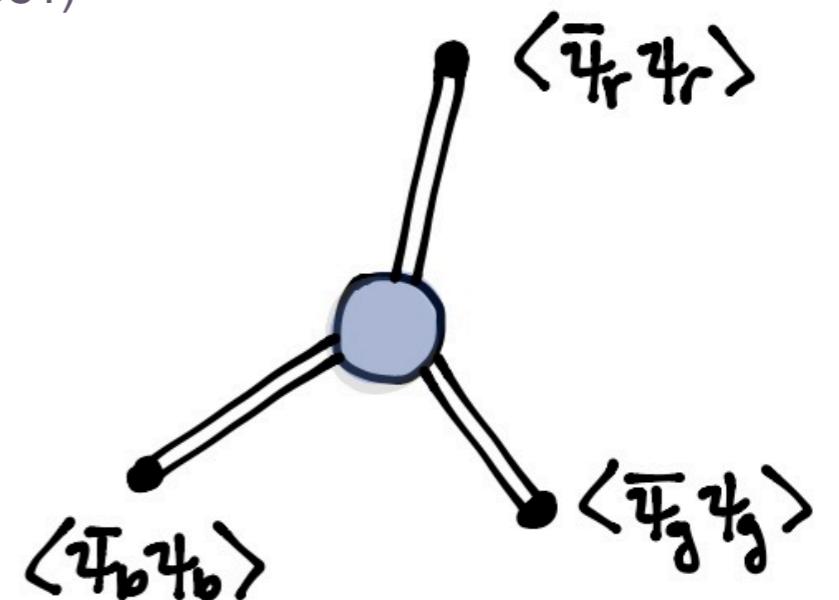
- ❖ The size of the $\mu_\Sigma \det\Sigma$ term is important for the GW signal

$$m_{\eta'} \sim \mu_\Sigma$$

- ❖ Explicit $U(1)_A$ breaking comes from instantons 't Hooft (1976)
- ❖ When $T \gg T_c$ and g_{QCD} is perturbative, the dilute instanton gas approximation holds Gross, Pisarski, Yaffe, (1981)

- ❖ In DGA, axion mass is lifted by small size instanton effects:

$$m_{\eta'}^2 \sim \frac{\Lambda_{SSI}^4}{\tilde{f}^2} \sim \frac{1}{\tilde{f}^2} \int \frac{d\rho}{\rho^5} d(\rho, T) \left(\frac{2}{3} \pi^2 \rho^3 \langle \bar{\psi} \psi \rangle \right)^3$$



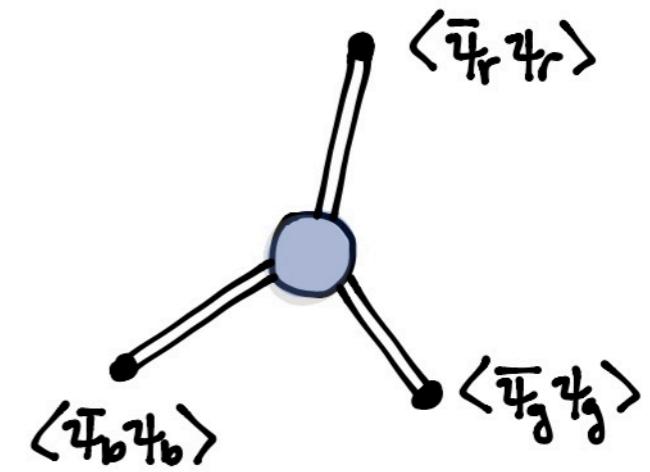
Shifman, Vainshtein, Zakharov (1980)

Callan, Dashen, Gross, (1978)

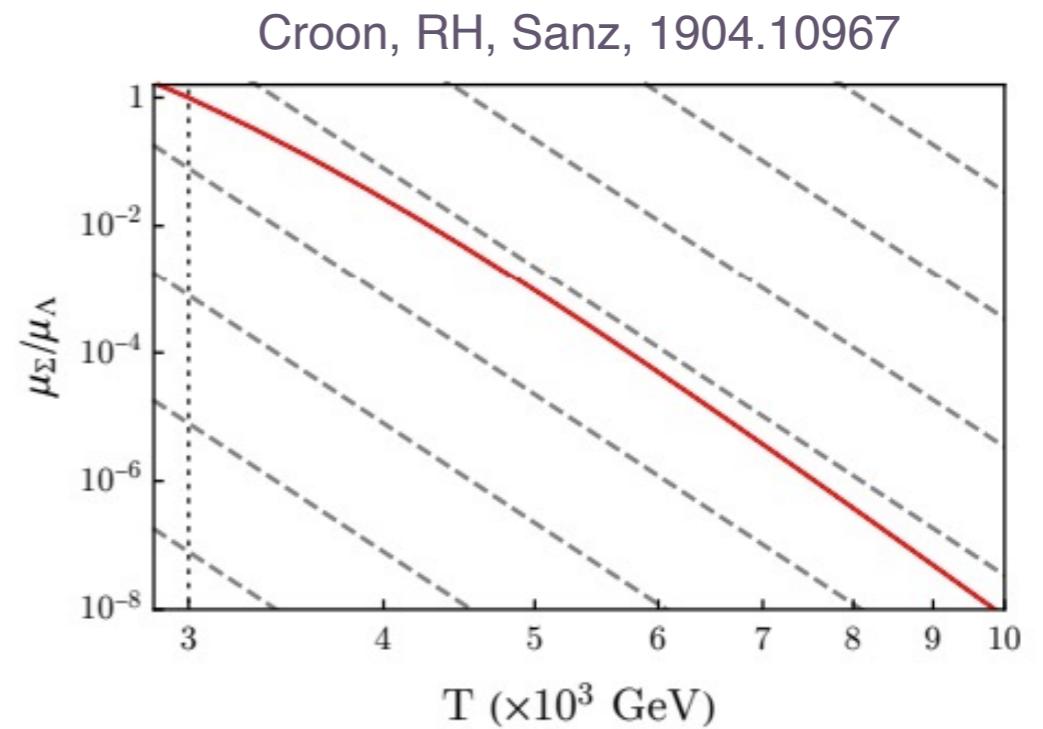
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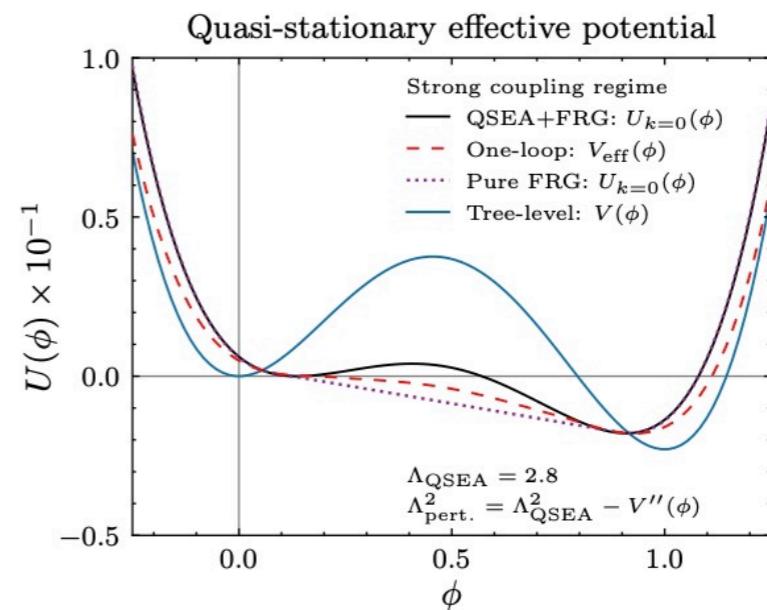
- ❖ DGA breaks down near confinement
- ❖ Does μ_Σ contribute significantly during the phase transition?



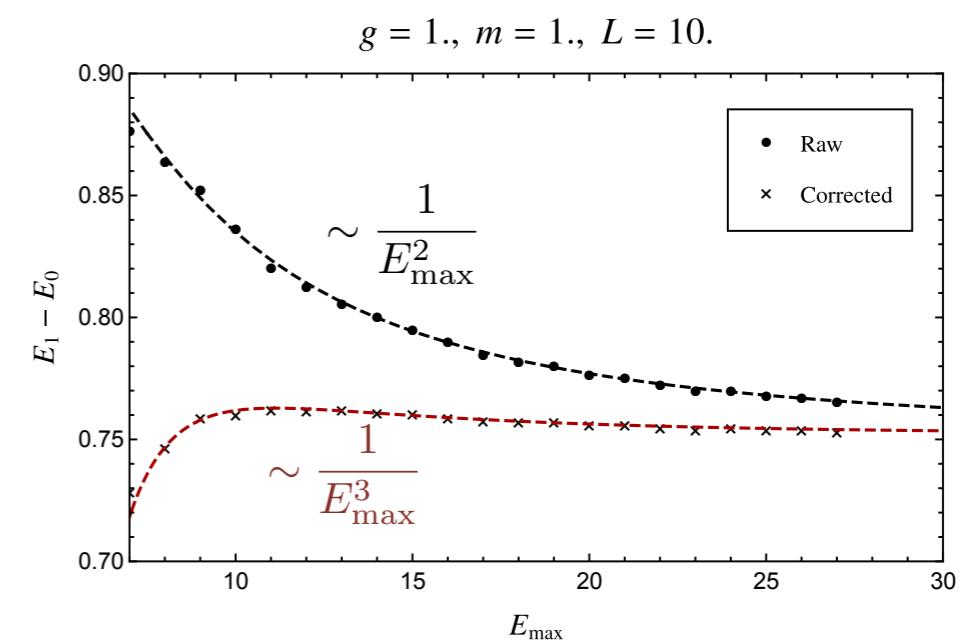
Strategies for Probing Confining Theories

- ❖ Nonperturbative methods like the quasi-stationary effective action
 - ❖ Using functional renormalization group methods to study axion couplings near QCD confinementCroon, Hall, RH, in progress

Plot lifted from Croon, Hall, Murayama, 2104.10687

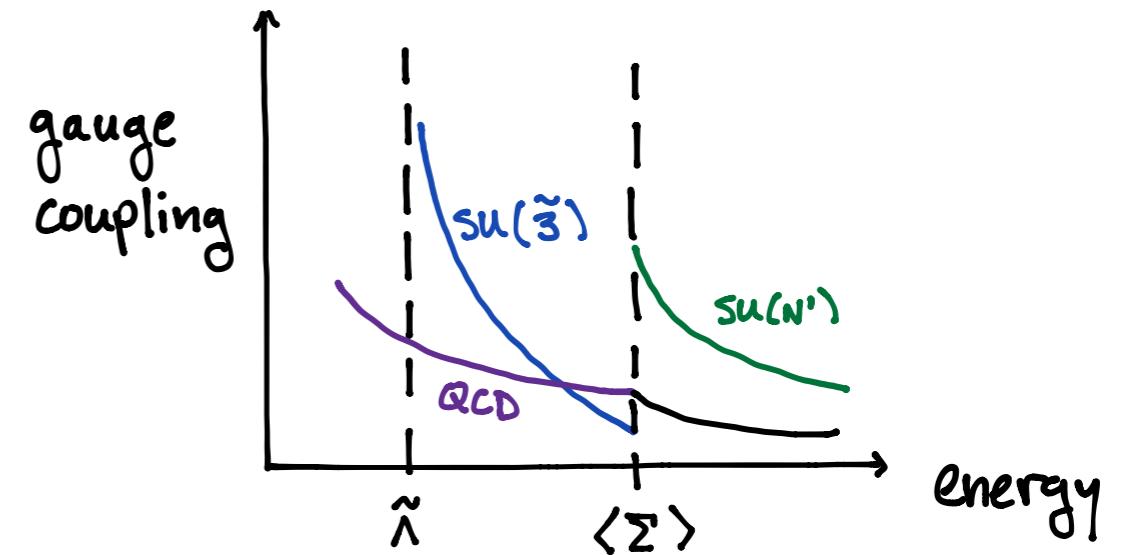


- ❖ Finite Energy Effective Field Theory
 - ❖ Consistent order-by-order scheme to improve Hamiltonian truncation to probe strongly coupled theoriesCohen, Farnsworth, RH, Luty, 21XX.XXXX



Conclusions

- ❖ Extra color groups allow for visible axions and dynamical axions
 - ❖ Visible axions $m_a^2 f_a^2 \approx m_\pi^2 f_\pi^2 \rightarrow + \sim \Lambda_{\text{new}}^4$
- ❖ Gravitational waves can probe exotic confining color groups
 - ❖ Symmetries of UV model inform the low energy EFT
 - ❖ EFT's break down near the confining PT
- ❖ GW signal favors models where high energy effects of extra color groups provide another source of axion mass

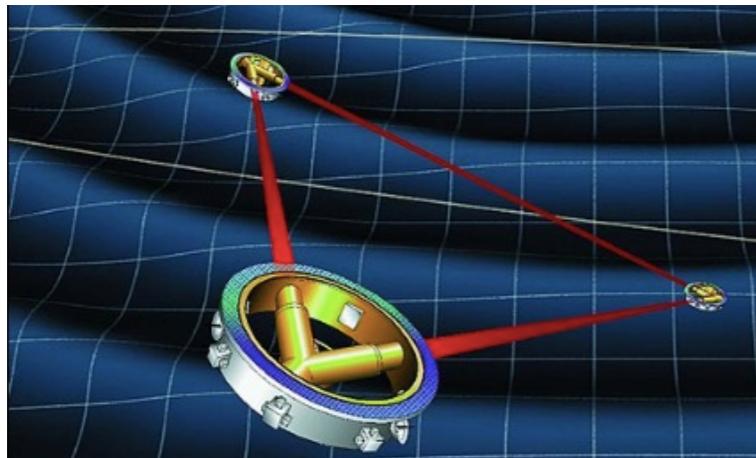


Thank you!

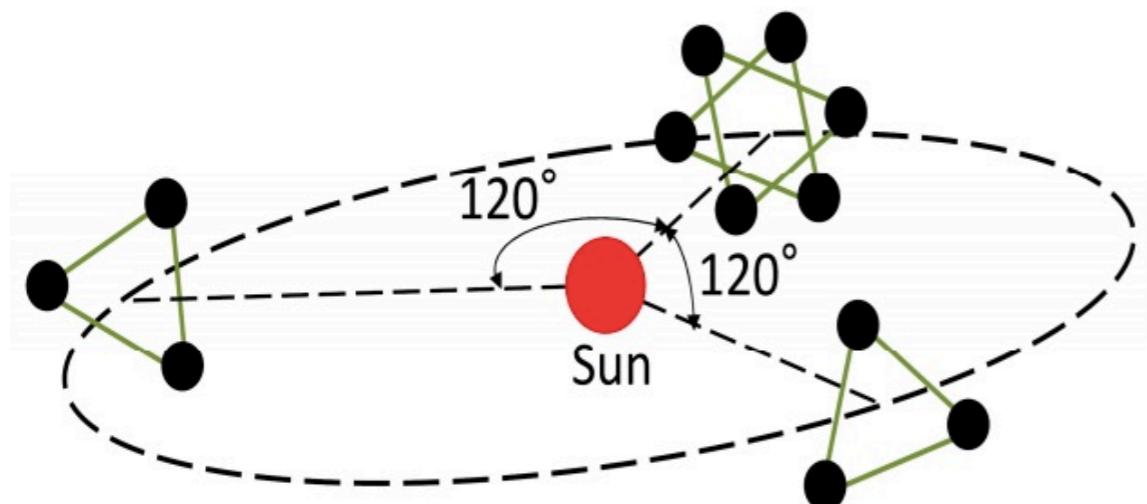
Back-up slides

Future Gravitational Wave Detectors

- ❖ Laser interferometers LISA, B-DECIGO, DECIGO and BBO

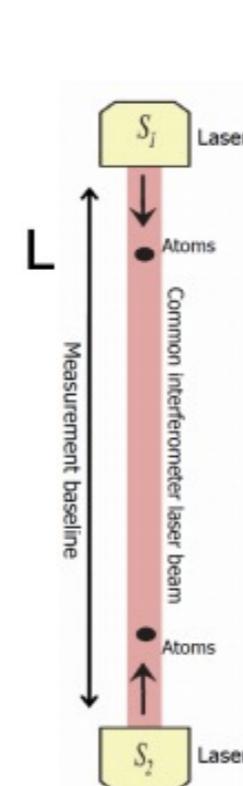


NASA illustration of
LISA

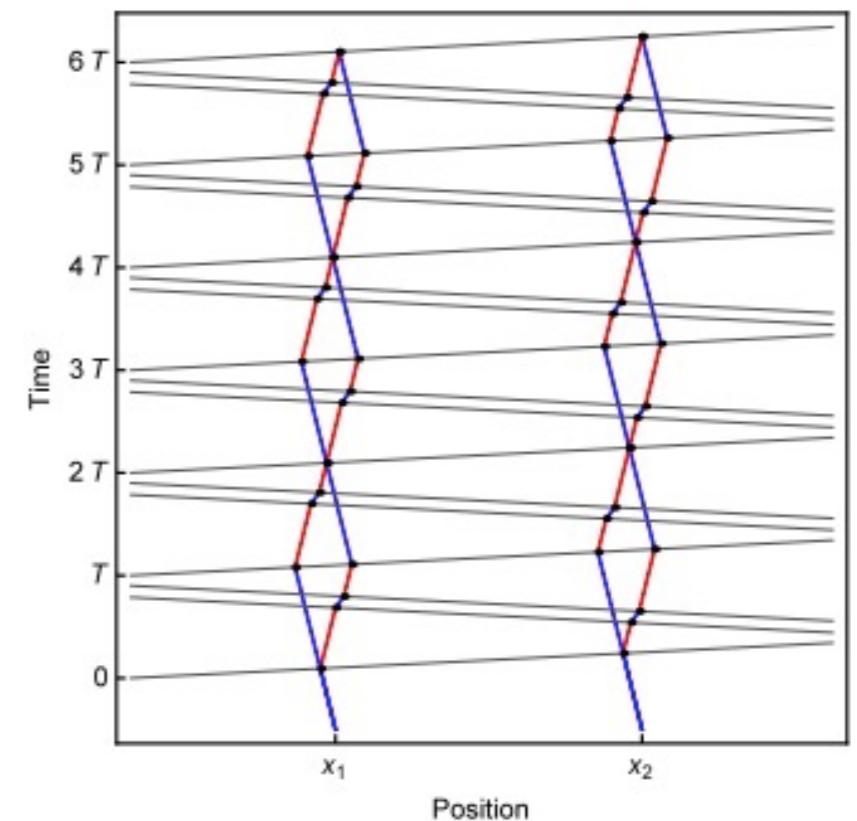


Yagi (2013)

- ❖ Atom interferometers AION and MAGIS



Graham, Hogan, Kasevich,
Rajendran (2016)

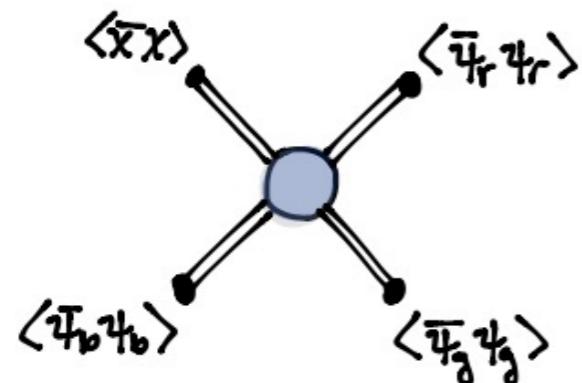
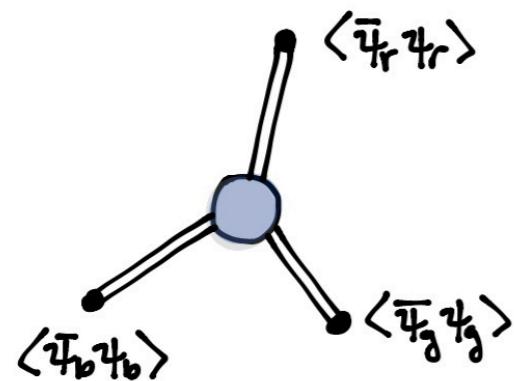


Buchmueller, “A UK AION for the exploration of ultra-light dark matter and mid-frequency gravitational waves” (2018)

Small Size Instantons with Fermions

- Adding fermion effects gives an instanton suppression

$$d(\rho, T) = C_{inst} \left(\frac{2\pi}{\alpha(1/\rho)} \right)^6 e^{-\frac{2\pi}{\alpha(1/\rho)}}$$

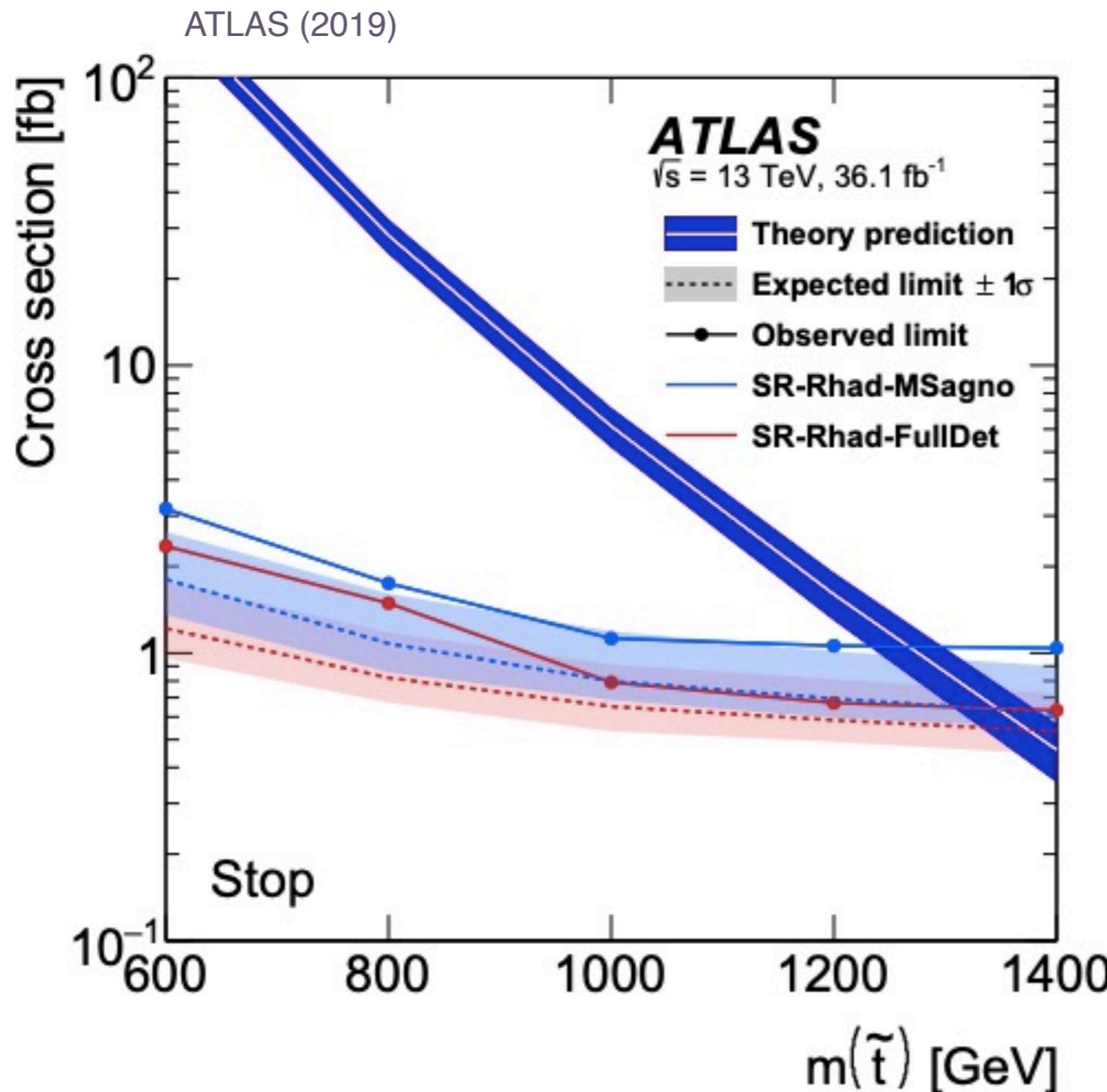


$$\Lambda_{SSI}^4 = \int \frac{d\rho}{\rho^5} d(\rho, T) \left(\frac{2}{3} \pi^2 \rho^3 \langle \bar{\psi} \psi \rangle \right)^3$$

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$$\mathcal{L}_{eff} = \Lambda_{SSI}^4 \cos \left(2 \frac{\eta'_\chi}{f_d} \right) + \Lambda_{diag}^4 \cos \left(2 \frac{\eta'_\chi}{f_d} + \sqrt{6} \frac{\eta'_\psi}{f_d} \right) + \Lambda_{QCD}^4 \cos \left(\sqrt{6} \frac{\eta'_\psi}{f_d} \right)$$

Collider Phenomenology: R-Hadron Searches



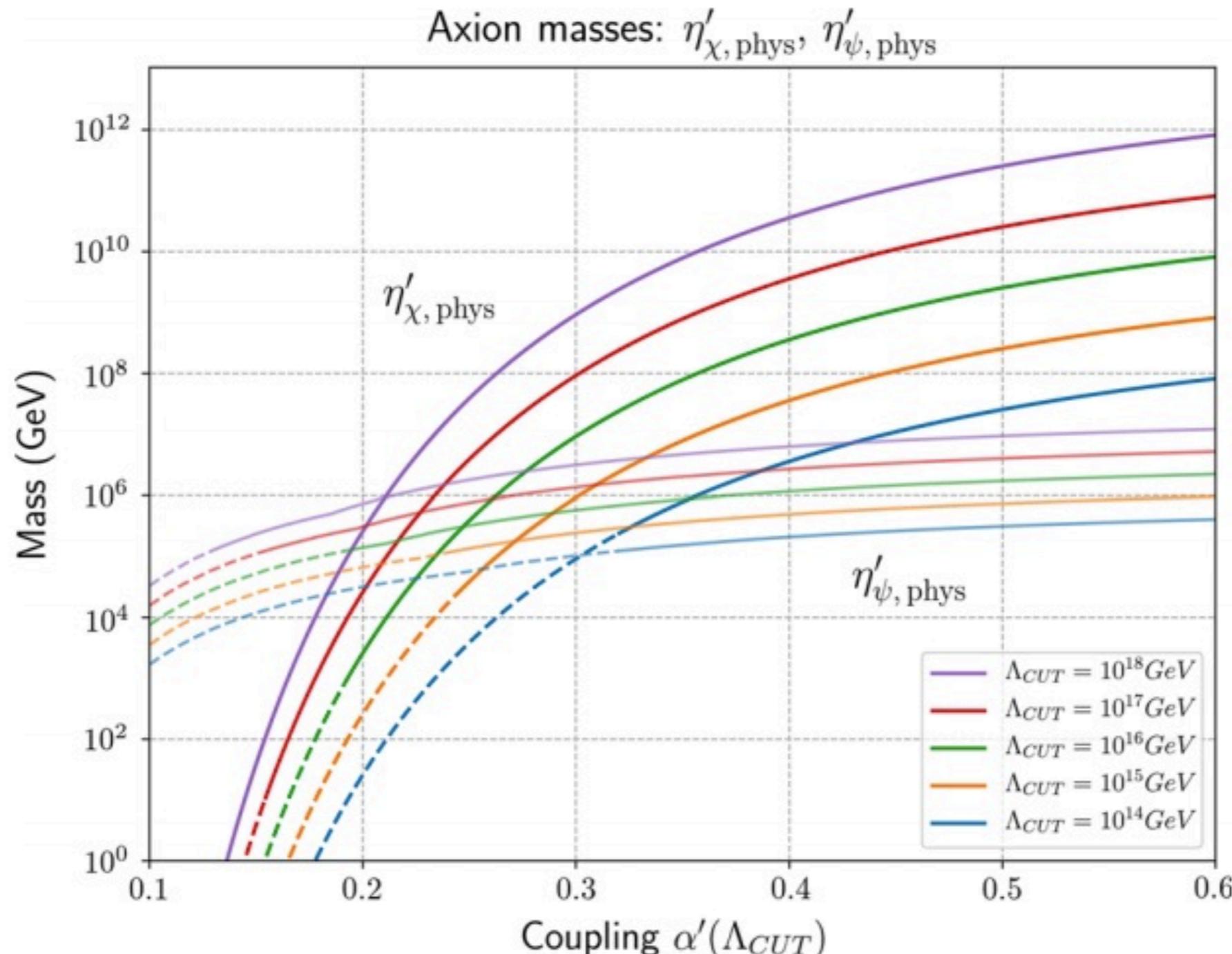
- ❖ We have an updated bound on color triplet scalars

$$m(\pi_d) \gtrsim 1345 \text{ GeV}$$

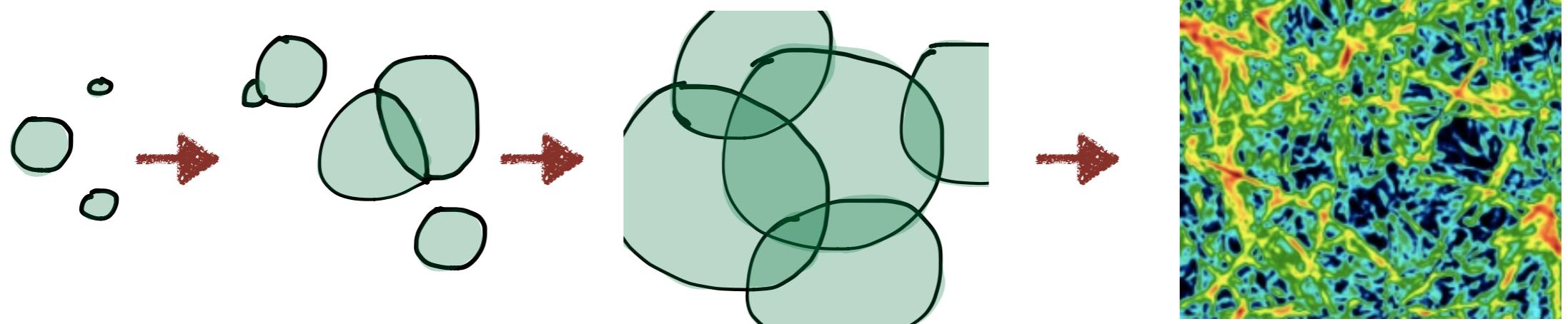
$$m^2(3_c) \approx \frac{\alpha_c}{\pi} \Lambda_{\text{diag}}^2$$

$$\Lambda_{\text{diag}} \gtrsim 7 \text{ TeV}$$

Plausible Range of Dynamical Axion Masses



Phase Transition in the Early Universe



Weir, "The sound of gravitational waves from a [confinement] phase transition," saoghal.net/slides/ectstar/

- ❖ Dynamics from tunneling to the $T = 0$ vacuum give:

$$\Omega_{GW}|_{\text{peak}} = \Omega_{GW} \left(\alpha, \frac{\beta}{H} \right)$$

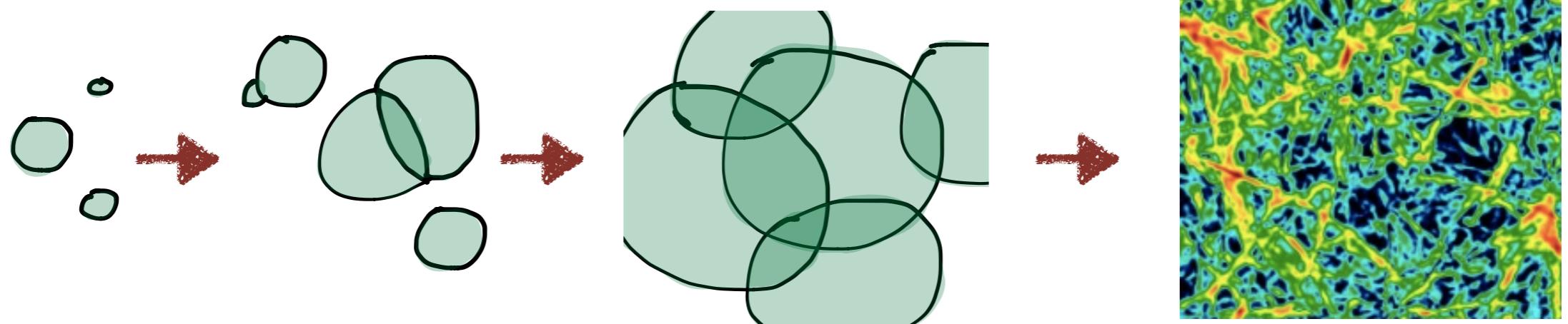
$$f_{GW}|_{\text{peak}} = f_{GW} \left(T_N, \frac{\beta}{H} \right)$$

α = Latent heat, $\frac{\Delta \mathcal{L}}{\rho_{\text{rad}}}$

$\frac{\beta}{H}$ = Parameterizes speed of the phase transition

T_N = Nucleation temperature

Phase transition in the Early Universe



Weir, "The sound of gravitational waves from a [confinement] phase transition," saoghal.net/slides/ectstar/

- ❖ Dynamics from tunneling to the $T = 0$ vacuum give:

$$\Omega_{GW}|_{\text{peak}} = \Omega_{GW} \left(\alpha, \frac{\beta}{H} \right) \quad \alpha \sim \frac{1}{\rho_N} \left(\Delta V - \frac{T}{4} \Delta \frac{dV}{dT} \right) \Big|_{T=T_N}$$

$$f_{GW}|_{\text{peak}} = f_{GW} \left(T_N, \frac{\beta}{H} \right) \quad \frac{\beta}{H} \sim T \frac{d(S_E/T)}{dT} \Big|_{T=T_N}$$