A worldsheet for 2d YM

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Based on work in progress with

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Pure 2d YM

Consider pure 2d YM theory with action

$$S_{\rm YM} = \frac{1}{4\,g_{\rm YM}^2}\int_{\Sigma_T} d^2x\,\sqrt{h}\,{\rm tr}\big(F_{\mu\nu}F^{\mu\nu}\big)$$

- no propagating gluons (almost topological)
- nevertheless non-trivial on compact Σ_T
- exactly solvable [Migdal '75], [Gross '93]

Pure 2d YM

In particular, for gauge group U(N), partition function equals

$$Z_{\rm YM}(G,\lambda,N) = \sum_R (\dim R)^{2-2G} \exp\Big[-\frac{\lambda}{2N} C_2(R) \Big] ,$$

where

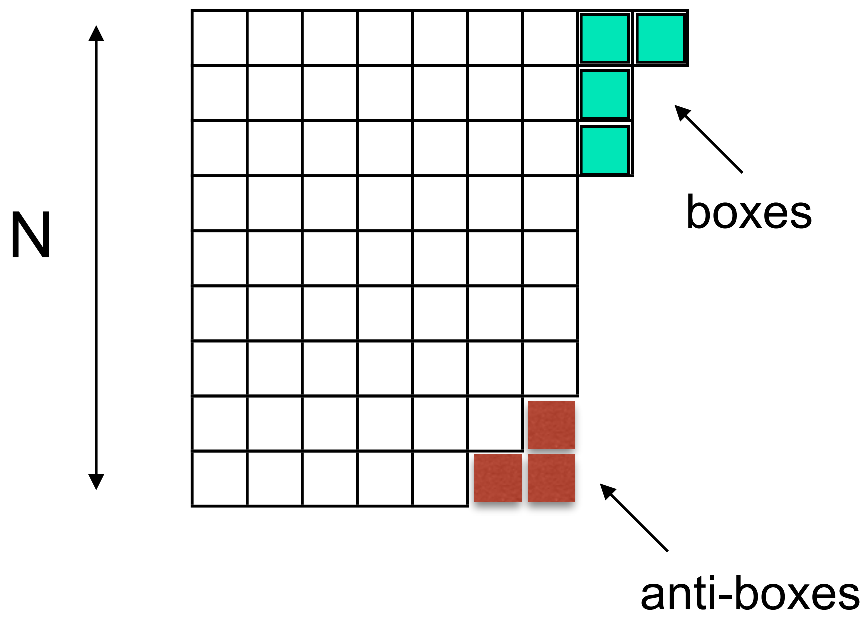
$\lambda = N g_{\rm YM}^2 A$: 2d 't Hooft coupling (A area)

G : genus of Σ_T

Consider 't Hooft limit $\lambda = N g^2 A = \text{fixed} \ , \quad N \to \infty$

Pure 2d YM

Representations of U(N) are described by pairs of Young diagrams



At large N, boxes and anti-boxes decouple: restrict to representations only consisting of boxes (no anti-boxes):

Chiral 2d YM

Pure 2d YM

In the free limit $(\lambda = 0)$ chiral YM partition function has very suggestive large N expansion:

[Gross '93] [Gross & Taylor, '93] [Cordes, Moore, Ramgoolam, '94]

$$Z^{+}_{\rm YM}(G,0,N) = \exp\left[\sum_{B=0}^{\infty}\sum_{L=0}^{B}\binom{2-2G}{L}\sum_{\Gamma\in H(g,G,B,L)_{\rm conn.}}\frac{1}{|{\rm Aut}(\Gamma)|}N^{2-2g}\right]$$

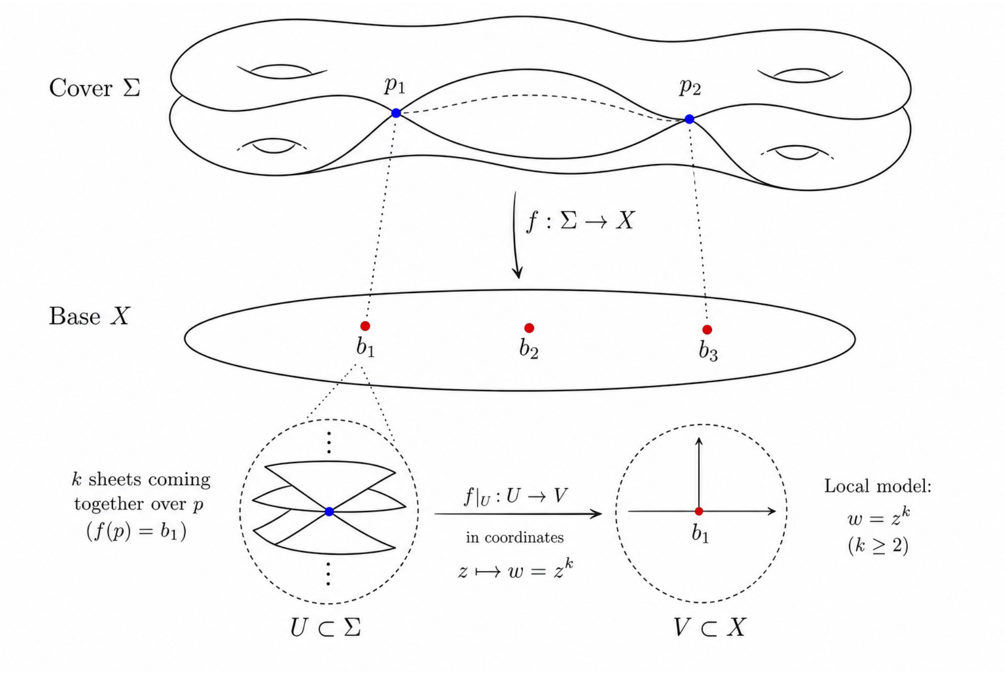
holomorphic covering maps

$$\Gamma : \Sigma_W(g) \mapsto \Sigma_T(G)$$

L: number of branch points
B: total branching

Covering maps

Illustrative example (L=2 branch points).



n: degree of covering map (# of generic preimages).

Each branch point defines permutation in S_n .

$$B = \sum_{i=1}^{L} (n - \#(\text{cycles})_i)$$

(total branching)

$$\boxed{2 - 2g = n(2 - 2G) - B}$$

Riemann-Hurwitz

Gross-Taylor formula

$$Z^{+}_{\mathrm{YM}}(G,0,N) = \exp\left[\sum_{B=0}^{\infty}\sum_{L=0}^{B}\binom{2-2G}{L}\sum_{\Gamma\in H(g,G,B,L)_{\mathrm{conn.}}}\frac{1}{|\mathrm{Aut}(\Gamma)|}N^{2-2g}\right]$$

Natural string interpretation: the large N behaviour is controlled by the genus of the covering map.

covering map == worldsheet

$$\frac{1}{N} \cong g_s$$

Contribution of each worldsheet: combinatorical factor.

→ **topological string**

Which string though?

Main question: which topological string theory describes 2d YM?

- Cordes-Moore-Ramgoolam ‘94: topological A-model on Σ_T
- Horava ‘95: `rigid’ topological string
- Aharony-Kundu-Shaeffer ’23-‘25: Polyakov-like generalisation of Horava string
- Komatsu-Maity ‘25: sigma model with composite dilaton field
- Benizri-Troost ’25: Gromov-Witten theory dual to Hurwitz TFT

Which string though?

Our approach: inspired by exact AdS/CFT duality

$$\mathrm{Sym}_N(\mathbb{T}^4) \quad = \quad \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

1 unit of NS-NS flux

[Eberhardt, MRG, Gopakumar, ’18-’19]

partition function/correlation functions can be calculated in terms of holomorphic covering maps

sum over covering maps is directly reproduced from above worldsheet theory!

AdS3 worldsheet theory

Key ingredient: worldsheet theory for tensionless case contains pair of symplectic bosons with OPE

$$\beta^{\pm}(z)\gamma^{\pm}(w) \sim -\frac{1}{z-w}$$

Spectrum includes spectrally flowed representations: characterised by $w \in \mathbb{N}$ with

$$\gamma^{-}(\zeta)\,V^{w}(\psi;z) \sim (\zeta-z)^{-\frac{w+1}{2}}V^{w}(\gamma_0^{-}\psi;z) + \mathcal{O}\big((\zeta-z)^{-\frac{w-1}{2}}\big)$$

$$\gamma^{+}(\zeta)\,V^{w}(\psi;z) \sim (\zeta-z)^{\frac{w-1}{2}}V^{w}(\gamma_0^{+}\psi;z) + \mathcal{O}\big((\zeta-z)^{\frac{w+1}{2}}\big)$$

dual to twisted sector state in symmetric orbifold: introduces branch point associated to permutation $\sigma_w = (1\cdots w)$

Localisation

Ward identities imply that worldsheet correlators satisfy

$$\left\langle \left(\gamma^+(z) + \Gamma(z)\gamma^-(z)\right) \prod_{i=1}^{n} V^{w_i}(\psi_i; x_i, z_i) \right\rangle_{\text{phys}} = 0$$

covering map $\Gamma = -\frac{\gamma^+}{\gamma^-}$

(homogenous coord. on $\mathbb{CP}^1$)

[Dei, MRG, Gopakumar, Knighton '20]

Implies that the correlators localise to the configurations that admit covering map

$$\Gamma(z) = x_i + a_i(z - z_i)^{w_i} \qquad z \sim z_i$$

Symmetric orbifold correlators

After integral over worldsheet moduli recover then

$$\int_{\mathcal{M}} d\mu(z_i)\, \langle \prod_{i=1}^{n} V^{w_i}(\psi; x_i, z_i) \rangle = \sum_{\Gamma} (\text{conf. factor})_{\Gamma} = \langle \prod_{i=1}^{n} V^{w_i}(x_i) \rangle_{\text{sym.orb.}}$$

This is very similar to what should happen for 2d YM!

Strategy:

- rewrite 2d YM as deformed symmetric orbifold
- imitate above construction to identify corresponding topological string theory

2d YM vs symmetric orbifold

The structure of the chiral 2d YM partition function

$$Z^{+}_{\mathrm{YM}}(G,0,N) = \exp\left[\sum_{B=0}^{\infty}\sum_{L=0}^{B}\binom{2-2G}{L}\sum_{\Gamma\in H(g,G,B,L)_{\mathrm{conn.}}}\frac{1}{|\mathrm{Aut}(\Gamma)|}N^{2-2g}\right]$$

is combinatorial, and this suggests that the corresponding symmetric orbifold has $c=0$.

So the natural candidate is the `trivial symmetric orbifold'

$\mathrm{Sym}_M(\cdot)$: seed theory has only vacuum state

c.f. [Dijkgraaf, Witten, '90] theory of S_M

defined on Σ_T (of genus G).

2d YM vs symmetric orbifold

As in the AdS/CFT case we take the symmetric orbifold in the grand canonical ensemble, i.e.

[Eberhardt, '20]

$$\mathrm{grand}_p(X) = \bigoplus_{M=0}^{\infty} p^M \, \mathrm{Sym}_M(X)$$

where p is related to string coupling g_s of dual string via

$$g_s^{2G-2} = p$$

Then partition function of trivial symmetric orbifold is

$$Z_{\mathrm{grand}(\cdot)} = \exp\left[\sum_{g=0}^{\infty} g_s^{2g-2} \sum_{\Gamma:\Sigma_W \to \Sigma_T} \frac{1}{|\mathrm{Aut}(\Gamma)|}\right]$$

genus of covering surface Σ_W

2d YM vs symmetric orbifold

However, this only reproduces part of $[g_s = \frac{1}{N}]$

$$Z^{+}_{\rm YM}(G,0,N) = \exp\left[\sum_{B=0}^{\infty}\sum_{L=0}^{B}\binom{2-2G}{L}\sum_{\Gamma\in H(g,G,B,L)_{\rm conn.}}\frac{1}{|{\rm Aut}(\Gamma)|}N^{2-2g}\right]$$

namely the terms with $B = L = 0$. To capture also branch points, we propose to perturb the symmetric orbifold, i.e. to consider

$$Z^{+}_{\rm YM}(G,0,N) = \left\langle \exp\left(\frac{g_s^{-1}}{4\pi}\int_{\Sigma_T} {\rm d}^2x\,\sqrt{h}\,R_h\,\sigma_2(x)\right)\right\rangle$$

↑ 2-cycle twisted sector ground state

[h: metric; R_h Ricci scalar.]

2d YM vs symmetric orbifold

We have confirmed this in perturbation theory

$$Z^{+}_{\mathrm{YM}}(G,0,N) = \left\langle \exp\left(\frac{g_s^{-1}}{4\pi} \int_{\Sigma_T} \mathrm{d}^2x \, \sqrt{h} \, R_h \, \sigma_2(x) \right) \right\rangle$$

where the string coupling is $g_s = \frac{1}{N}$.

The zero'th and first order calculations are straightforward; at second order

$$\frac{g_s^{-2}}{2!} \frac{1}{16\pi^2} \int_{\Sigma_T} \mathrm{d}^2x_2 \, \sqrt{h(x_2)} R_h(x_2) \int_{\Sigma_T} \mathrm{d}^2x_1 \, \sqrt{h(x_1)} R_h(x_1) \langle \sigma_2(x_1) \sigma_2(x_2) \rangle$$

we need to treat the regime where $x_1 \sim x_2$ carefully.

2d YM vs symmetric orbifold

To do so, we write

$$\frac{1}{4\pi}\int_{\Sigma_T} \mathrm{d}^2x_2\,\sqrt{h}\,R_h = \frac{1}{4\pi}\int_{\Sigma_T\setminus D_\varepsilon(x_1)} \mathrm{d}^2x_2\,\sqrt{h}R_h - \frac{1}{2\pi}\int_{\partial D_\varepsilon(x_1)} K$$
$$+\frac{1}{4\pi}\int_{D_\varepsilon(x_1)} \mathrm{d}^2x_2\,\sqrt{h}R_h + \frac{1}{2\pi}\int_{\partial D_\varepsilon(x_1)} K$$

[K extrinsic curvature of boundary $\partial D_\varepsilon(x_1)$.] The contribution for $x_1 \neq x_2$ gives then

$$\frac{1}{2!}\left(\frac{1}{4\pi}\int_{\Sigma_T\setminus D_\varepsilon(x_1)} \mathrm{d}^2x_2\,\sqrt{h}\,R_h - \frac{1}{2\pi}\int_{\partial D_\varepsilon(x_1)} K\right)\left(\frac{1}{4\pi}\int_{\Sigma_T} \mathrm{d}^2x_1\,\sqrt{h(x_1)}\,R_h(x_1)\right)$$
$$= \frac{(1-2G)(2-2G)}{2} = \binom{2-2G}{2}$$

and thus accounts for

$$\sum_{g=0}^{\infty} g_s^{2g-2}\binom{2-2G}{2}\sum_{\Gamma\in H(g,G,2,2)}\frac{1}{|\mathrm{Aut}(\Gamma)|}$$

2d YM vs symmetric orbifold

The higher order terms work similarly, and thus we recover chiral 2d YM!

Incidentally, we can also obtain, conversely, the symmetric orbifold by deforming 2d YM by

$$\delta S = -\frac{g_s^{-1}}{4\pi}\int_{\Sigma_T} \mathrm{d}^2x\,\sqrt{h}\,R_h\,V_{\mathcal{O}}(x)$$

Infinitesimal Wilson loop
in fundamental representation

The worldsheet theory

Given similarity with tensionless AdS/CFT duality, expect the dual string to be associated with AdS_3.

However, since the central charge vanishes, this should mean (via Brown-Henneaux) that

$$c = 6k = \frac{3\ell}{2G_3} = 0$$

↑

level of $\mathrm{sl}(2,\mathbb{R})$ algebra

The worldsheet theory

Can try to make sense of the level $k = 0$ $\mathrm{sl}(2,\mathbb{R})$ theory by using Wakimoto representation

$$\frac{1}{2\pi}\int_\Sigma \left(\beta\bar\partial\gamma + \bar\beta\partial\bar\gamma + \frac{1}{2}\partial\Phi\,\bar\partial\Phi - \frac{Q}{4}R\Phi - \mu\beta\bar\beta\, e^{-2\Phi/Q}\right)$$

where (β, γ) are symplectic bosons, and the level now enters via the background charge for the boson Φ

$$Q = \sqrt{\frac{2}{k-2}} \qquad c_{\mathrm{ws}} = 3 + 3Q^2 = \frac{3k}{k-2}$$

The worldsheet theory

At level $k=0$ the background charge becomes $Q=i$, and we can redefine $\phi = i\Phi$, to obtain

$$S = \frac{1}{2\pi}\int_\Sigma \left(\beta\bar\partial\gamma + \bar\beta\partial\bar\gamma - \frac{1}{2}\partial\phi\,\bar\partial\phi - \frac{1}{4}R\phi - \mu\beta\bar\beta\, e^{2\phi}\right)$$

boson with $Q=1$ and wrong sign kinetic term

Thus can fermionise into a first-order system of anti-commuting chiral fields: write $\phi = \varphi + \bar\varphi$ and define

$[h(p)=1,\ h(\theta)=0]$

$$p = e^{-\varphi}\,, \quad \theta = e^{\varphi}\,, \quad \bar p = e^{-\bar\varphi}\,, \quad \bar\theta = e^{\bar\varphi}$$

The worldsheet theory

Hence end up with two identical first order systems

$$(\beta, \gamma) \quad \text{and} \quad (p, \theta)$$

with opposite statistics and hence $c_{\rm ws} = 0$. Actually, can write $\mathrm{sl}(2,\mathbb{R})$ currents in terms of these free fields

$$J^+ = \beta\,, \quad J^3 = \beta\gamma + p\theta\,, \quad J^- = \beta\gamma^2 + 2\gamma p\theta$$

cf. [Dei, Knighton, Naderi, '23]

What is special about this set-up is that it actually contains a twisted $\mathcal{N} = 2$ algebra

$$G^+ = \beta\theta\,, \quad G^- = p\partial\gamma\ .$$

The worldsheet theory

Thus can quantise it as a topological string. Then the interaction term

$$\int_\Sigma \beta\bar{\beta}e^{2\phi} = \int_\Sigma \beta\bar{\beta}\theta\bar{\theta} = \frac{1}{2}\left\{Q_{\mathrm{BRST}} + \overline{Q}_{\mathrm{BRST}}, \int_\Sigma (p\bar{\beta}\theta\bar{\theta} + \beta\bar{p}\theta\bar{\theta})\right\}$$

is BRST exact, and can be dropped. Thus we end up with the `near boundary’ theory of [Knighton, Sriprachyakul, ’24] for which localisation can be directly understood (using path-integral methods).

[In fact, the symplectic bosons of this theory play essentially the same role as those we saw earlier.]

The worldsheet theory

By analogy to the tensionless AdS_3 worldsheet theory, take the spectrum to consist of usual free field representations and their spectrally flowed images.

↑ correspond to twisted sectors of dual symmetric orbifold

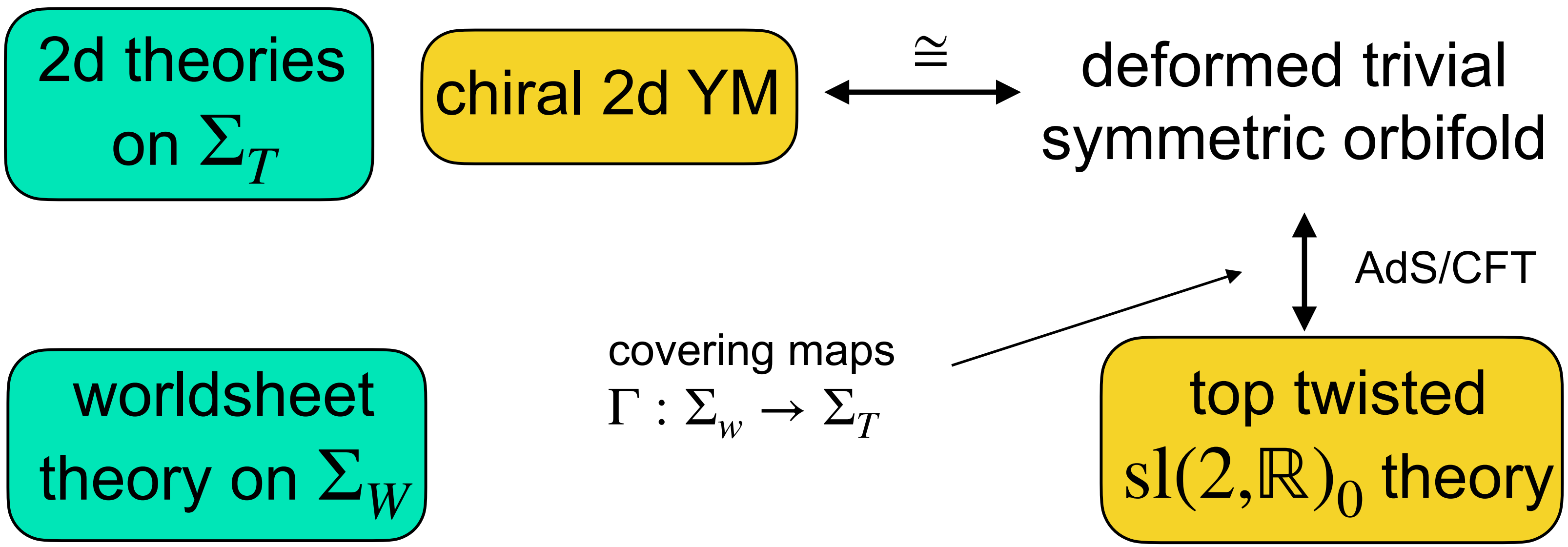
Topological twist: only the ground states in each spectrally flowed (=twisted) sector survives.

The worldsheet theory

Localisation of these spectrally flowed operators works as for the usual tensionless AdS/CFT case, and we thus reproduce the expected symmetric orbifold correlators, resp. partition function.

Deformation by 2-cycle twisted sector also natural from this perspective: in tensionless case the analogous operator switches on R-R flux in the dual AdS_3 background.

Conclusions and Outlook



This talk: concentrated on partition function.
Analysis works similarly for correlators.

Open questions

- what is special about 2d YM / symm. orbifold?
- extend to full 2d YM (chiral + anti-chiral)
- duality for non-zero ’t Hooft coupling
- relation to other proposals
- other versions: add matter, higher dim
- D-branes, defects, etc.
- …

Thank you!