3d Ising CFT from AdS EFT

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Based on 2605.xxxx
2403.07079, 2508.20160, 2508.20158 with Giulia Fardelli and Liam Fitzpatrick

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BOSTON UNIVERSITY

Outline

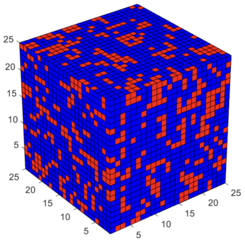
Motivation & Introduction to the problem

The method: Large Spin EFT in AdS

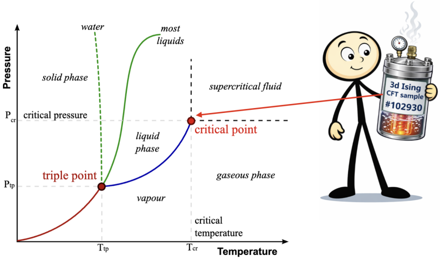
Result

Motivation

3d Ising CFT describes many physical systems at criticality



3d Ising model at criticality



Boiling Water

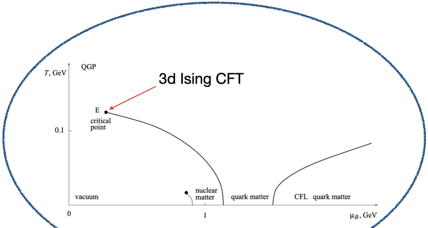
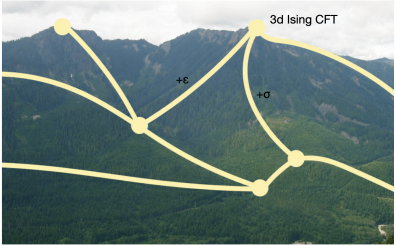


Fig. 1. QCD phase diagram.

[Jackson's talk]

By solving 3d Ising CFT, one can solve nearby QFTs by deforming relevant operators

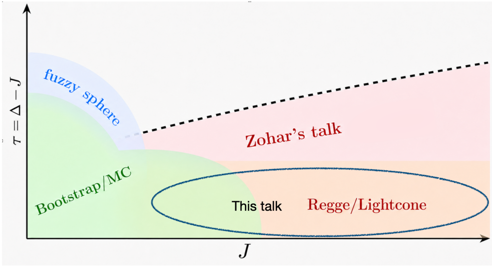


$$S_{\text{Ising Field Theory}} = S_{\text{3d Ising CFT}} + h \int d^3x \sigma(x) + g \int d^3x \epsilon(x)$$

[Giulia's talk]

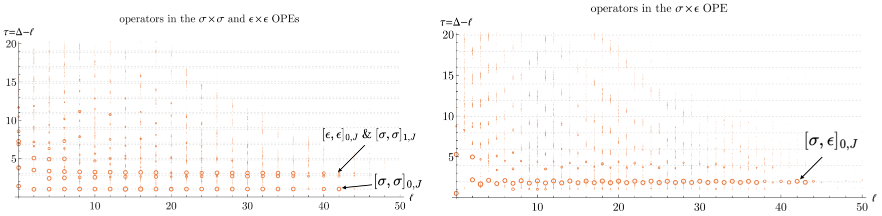
Spectrum of 3d Ising CFT

- CFTs are characterized by the two set of data $\{\Delta_i, \lambda_{ijk}\}$
- We have powerful analytic and numeric methods to compute the spectrum Δ_i in 3d Ising CFT



- Powerful numerical method at low twist $\tau = \Delta - J$ low spin region: Numerical conformal Bootstrap [24' Chang et al.], Fuzzy Sphere [Giulia's talk][23' Zhu et al.], Monte Carlo [02' Pelissetto, Vicari]...
- At Regge region (Large J fixed τ), universality of CFT spectrum are shown by analytic bootstrap methods: (Light-cone bootstrap [12' Fitzpatrick et al.] [12' Komargodski, Zhiboedov], and Lorentzian Inversion Formula [17' Caron-huot]). Recently, it was rigorously proven by [22' Pal et.al] [24' van Rees]

CFT Spectrum at Regge Region



3d Ising CFT spectrum from numerical bootstrap [16' Simmons-duffin]

Analytic bootstrap [12' Fitzpatrick et al.] [12' Komargodski, Zhiboedov] tells us that for any local operator $\mathcal{O}_A, \mathcal{O}_B$, there exist families of operators $[\mathcal{O}_A, \mathcal{O}_B]_{n,J}$ whose twist $\tau = \Delta - J$ will asympto $\tau_{\mathcal{O}_A} + \tau_{\mathcal{O}_B}$,

$$\lim_{J\to\infty} \tau\Big([\mathcal{O}_A, \mathcal{O}_B]_{n,J}\Big) = \tau_{\mathcal{O}_A} + \tau_{\mathcal{O}_B} + 2n + \frac{\#}{J^{\tau_{\text{min}}}} \text{(Known analytically)}$$

In free theory, those operators roughly are $\mathcal{O}_A \partial^{\mu_1} \ldots \partial^{\mu_J} \Box^n \mathcal{O}_B$. In interacting theory, it will have anomalous dimension perturbatively in $\frac{1}{J}$

Low-twist $\mathbb{Z}_2$-odd spectrum

We can get a rough understanding of the spectrum just based on twist accumulation points. Start with the lowest $\mathbb{Z}_2$-odd scalar operator $\tau_\sigma = 0.51815$

$$\tau([\sigma,\sigma]_{0,J=0,2,4,\dots}) = \{\underbrace{1.4126}_{\epsilon},\ \underbrace{1}_{T^{\mu\nu}},\ \underbrace{1.02267}_{[\sigma,\sigma]_{0,J=4}},\dots,\ \underbrace{1.0363}_{[\sigma,\sigma]_{0,J=\infty}}\ \}$$

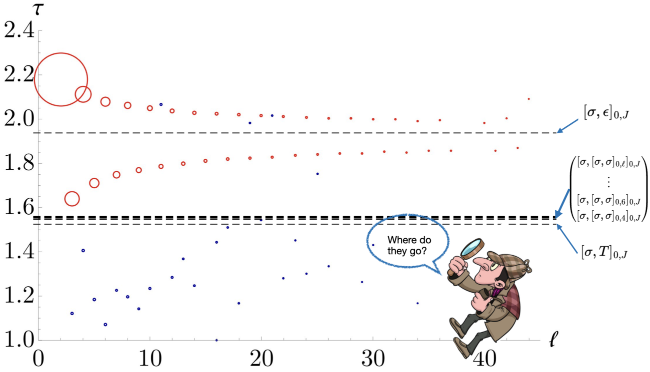
Let's look at the twist accumulation points of $[\sigma,[\sigma,\sigma]_{0,J=2,4,\dots,\infty}]_{0,J}$

$$\lim_{J\to\infty}\begin{pmatrix}\tau([\sigma,\epsilon]_{0,J})\\ \tau([\sigma,T^{\mu\nu}]_{0,J})\\ \tau([\sigma,[\sigma,\sigma]_{0,4}]_{0,J})\\ \vdots\\ \tau([\sigma,[\sigma,\sigma]_{0,\infty}]_{0,J})\end{pmatrix} = \begin{pmatrix}\tau_\sigma+\tau_\epsilon\\ \tau_\sigma+\tau_T\\ \tau_\sigma+\tau_{[\sigma,\sigma]_{0,4}}\\ \vdots\\ 3\tau_\sigma\end{pmatrix} = \begin{pmatrix}1.9308\\ 1.5182\\ 1.5408\\ \vdots\\ 1.5546\end{pmatrix}$$

We should see them in the $\mathbb{Z}_2$-odd sector. But...

Where do they go?

Let's zoom in to the low-twist ($\tau < 2$) region of the $\mathbb{Z}_2$-odd plot,



$\Lambda_{\text{max}} = 43$ spectrum from [22' Ning Su] from mixed σ, ϵ bootstrap

- Numerical bootstrap is the **best numerical method** in Regge region. Yet it seems to only access to one trajectory $[\sigma, \epsilon]_{0,J}$...
- **Goal of this talk: Present a new method to the spectrum via AdS EFT**

Large Spin Effective Field Theory (EFT) in AdS

How can AdS EFT describes 3d Ising CFT???

- For large-N CFT$_d$, there is **scale separation** between planck scale ℓ_{pl} and AdS unit scale ℓ_{AdS}

$$\lim_{N\to\infty} \frac{\ell_{\text{AdS}}}{\ell_{\text{pl}}} \sim c_T^{\frac{1}{d-1}} \to \infty$$

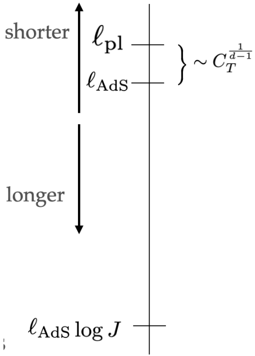
Gravity is weakly coupled at unit scale

- For 3d Ising CFT, there is **NO scale separation** between planck scale ℓ_{pl} and AdS unit scale ℓ_{AdS}

$$\frac{\ell_{\text{AdS}}}{\ell_{\text{pl}}} \sim \sqrt{c_T} \sim \mathcal{O}(1)$$

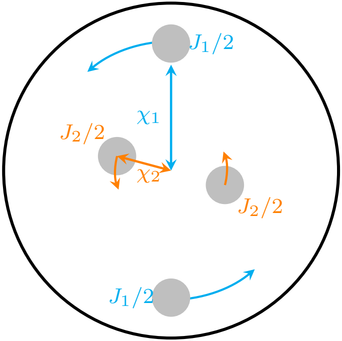
Gravity is strongly coupled at unit scale!

- **Resolution**: At long distance $\ell \geq \ell_{\text{pl}}, \ell_{\text{AdS}}$, gravity should be weakly coupled again. **What does long distance AdS EFT describes?**

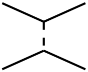


Large J= Large Separation

Consider 2-particle state in AdS,



$$\chi \sim \frac{1}{2}\ell_{\mathsf{AdS}} \log \frac{J}{2\Delta}$$

Adding interaction: $V_{ex}(\chi) =$ 

$$\langle \Psi_{\mathsf{free}} | V_{ex}(\chi) | \Psi_{\mathsf{free}} \rangle \sim \exp(-\tau_{ex}\chi)$$

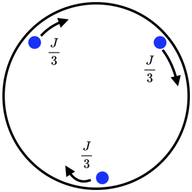
- **Long distance** AdS EFT describes **large spin** 2-pt state
- The conformal dimension is related to the energy in AdS

$$\Delta = \frac{E_{tot}}{\ell_{\mathsf{AdS}}} = 2\Delta_\phi + J + \# e^{-\tau_{\mathsf{ex}}\chi} = 2\Delta_\phi + J + \frac{\#}{J^{\tau_{\mathsf{ex}}}}$$

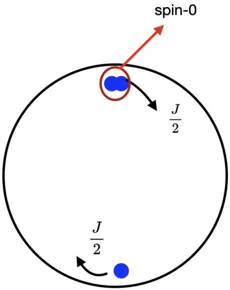
(matches the analytic bootstrap result! [14' Fitzpatrick, Kaplan, Walters])

Problem for multi-particle states EFT

Let's look at two examples of three-particle states,



$\sim [\sigma, [\sigma, \sigma]_{0,\infty}]_\infty$



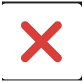
$\sim [\sigma, \sigma^2]_J$

$$\lim_{J\to\infty} E_{tot} = 3\Delta_\sigma + \frac{\#}{J^\#}$$

(long-range interaction)

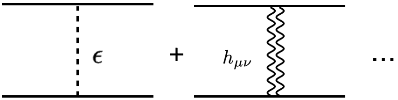
$$\lim_{J\to\infty} E_{tot} = \Delta_\sigma + \Delta_{\sigma^2} + \frac{\#}{J^\#} \quad \text{(long-ranged)}$$

Since $\Delta_{\sigma^2} = \Delta_\epsilon \neq 2\Delta_\sigma$, we need CFT input for the binding energy Δ_{σ^2}



Ingredients for simplest AdS EFT

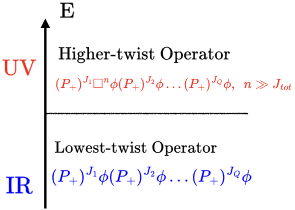
- Long-range interaction



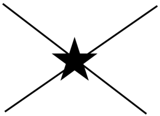
Mass of the exchange: $m^2 = \Delta(\Delta - d)$

Coupling $\alpha \sim$ OPE coefficient $\lambda_{\phi\phi\epsilon}$

- IR modes of AdS EFT



- Short-distance interaction

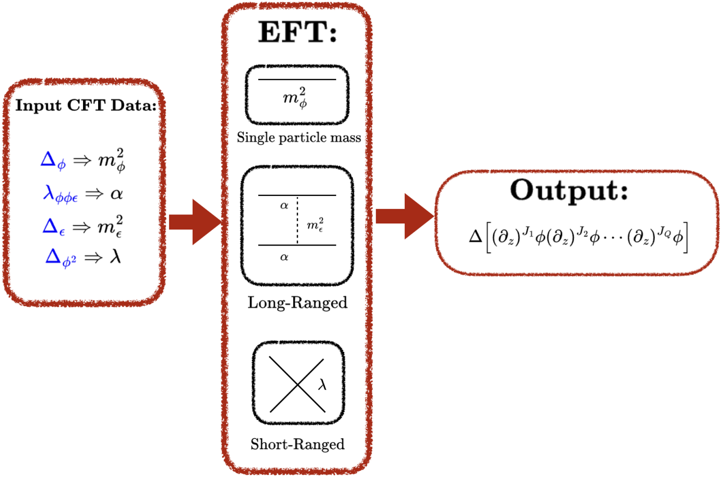


$\subset g_{4,0}\phi^4,\ g_{4,2}\nabla_\nu\nabla_\mu\phi\nabla^\mu\phi\nabla^\nu\phi\ldots$

g are fixed by CFT dimension of small spin bound state (e.g. $g_{4,0}$ fixed by Δ_{ϕ^2})

Building large spin EFT: Pipeline

In summary, our EFT use small set of CFT datas as input, and the output is scaling dimensions of lowest-twist operators



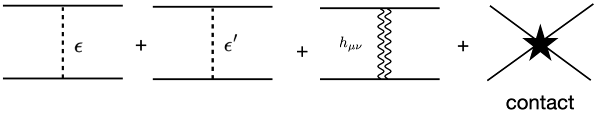
Application to 3d Ising CFT

AdS EFT for 3d Ising CFT

We will consider following EFT in AdS$_4$,

$$S=\frac{1}{\kappa^2}\int_{\text{AdS}} d^4x\sqrt{-g}\Big(\frac{R-\Lambda}{2}-\frac{1}{2}(\partial\sigma)^2-\frac{1}{2}m_\sigma^2\sigma^2-g_\epsilon\epsilon\sigma^2-g_{\epsilon'}\epsilon'\sigma^2$$
$$-\frac{\lambda_{4,0}}{4!}\sigma^4-\frac{\lambda_{4,2}}{4!}\sigma\nabla_\mu\sigma\nabla_\nu\sigma\nabla^\mu\nabla^\nu\sigma+\dots\Big)$$

Bulk Model:

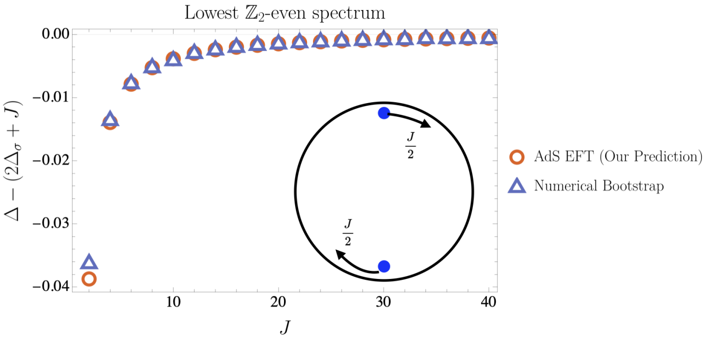


We take following CFT data from numerical bootstrap [24' Chang et al.] [16' Simmons-duffin][18' Poland, Rychkov, Vichi]:

- Long-range interaction: $\Delta_\sigma=0.51815$, $\Delta_\epsilon=1.41263$, $\Delta_{\epsilon'}=3.82968$, $\Delta_T=3$, $\lambda_{\sigma\sigma\epsilon}=1.0519$, $\lambda_{\sigma\sigma\epsilon'}=0.05301$, $\lambda_{\sigma\sigma T}=0.32614$
- Short-range interaction: $\Delta_\epsilon=1.41263$, $\Delta_T=3$, $\Delta_{[\sigma,\sigma]_{0,4}}=5.02267,\dots,\Delta_{[\sigma,\sigma]_{0,J}}$

2-particle states: EFT without contact term

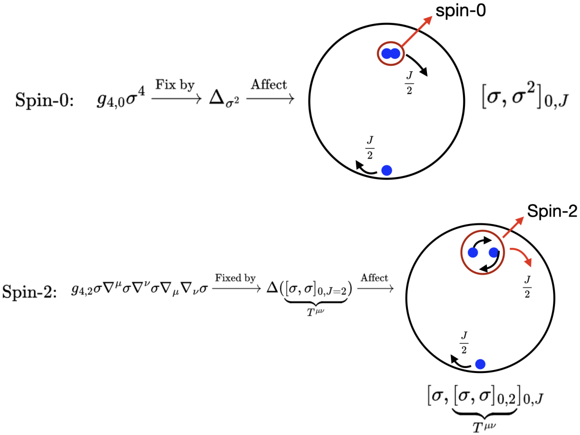
For two-particle states, at each spin J we have exactly one primary state.
It corresponds to lowest $\mathbb{Z}_2$-even state $[\sigma, \sigma]_{0,J}$ in 3d Ising CFT



Result with just long-ranged interaction for $J \geq 2$

Contact terms

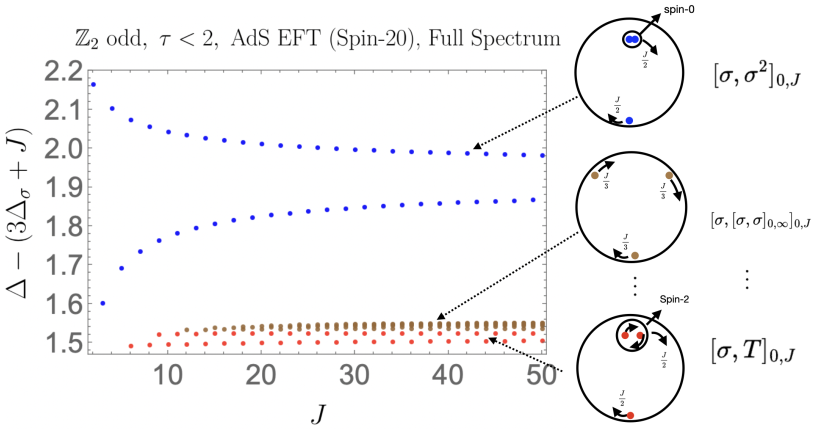
Contact terms will only have finite J effect. But they will have a large J effect on 3-pt states! **Willson coefficients are fixed by** $\Delta([\sigma,\sigma]_{0,J})$



Strategy: Fixed the up to spin-$J \leq J_{\text{max}}$ contact terms with CFT input $\Delta\left([\sigma,\sigma]_{0,J\leq J_{\text{max}}}\right)$. **The result should converge very fast in** J_{max}

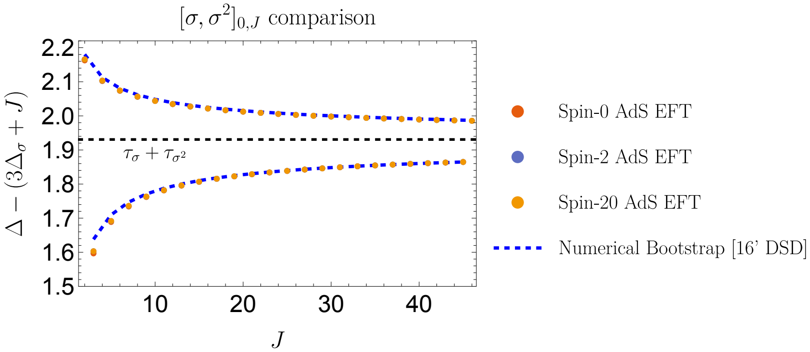
3-particle states: Full spectrum

For 3-pt states, AdS EFT spectrum should match with low-twist spectrum in the $\mathbb{Z}_2$-odd sector of 3d Ising CFT



We can compare highest eigenstate ($[\sigma, \sigma^2]_J$) with numerical bootstrap [16' Simmons-duffin]

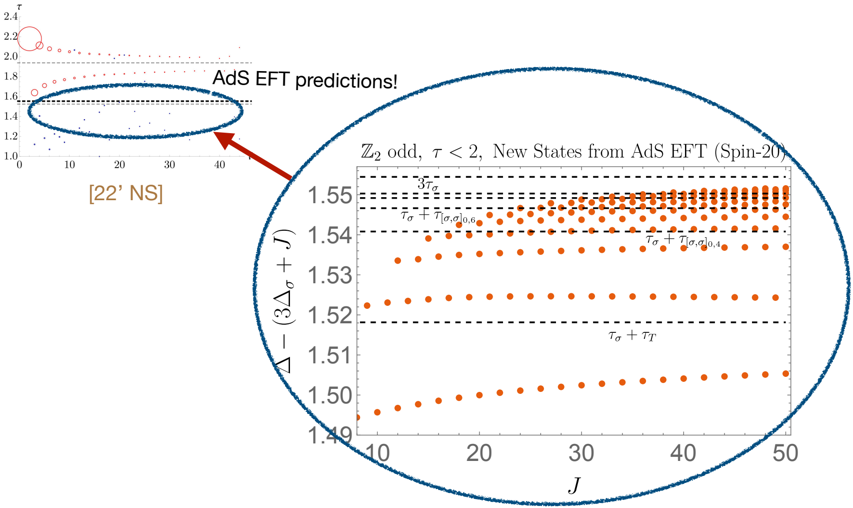
Comparison with bootstrap : $[\sigma, \sigma^2]_{0,J}$



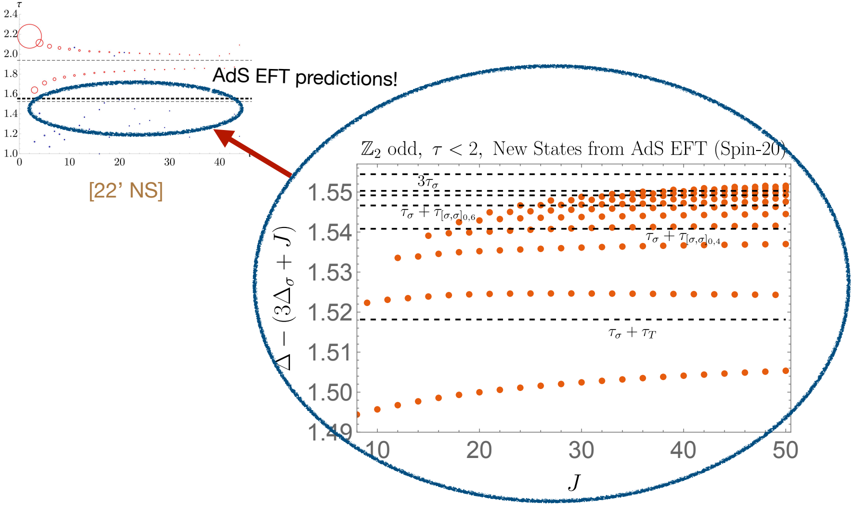
- Excellent agreement with numerical bootstrap even to small spin!
- Converge as we add higher spin contact terms

New prediction for 3d Ising CFT

New prediction for 3d Ising CFT

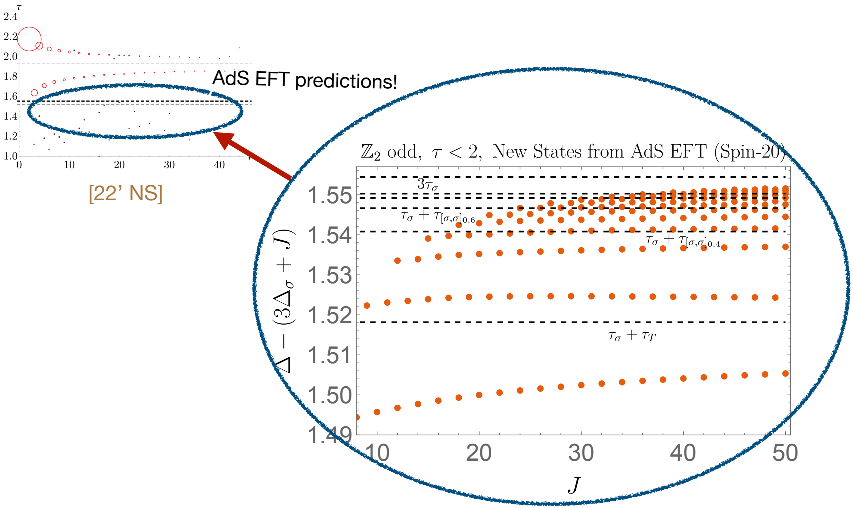


New prediction for 3d Ising CFT



So far, we have nothing to compare to...

New prediction for 3d Ising CFT



So far, we have nothing to compare to... **Untill last summer in TASI...**


FARRAND FIELD
Me
TASI 2025
Threads in a Theory Tapestry


FARRAND FIELD
Matt Mitchell
Me
TASI 2025
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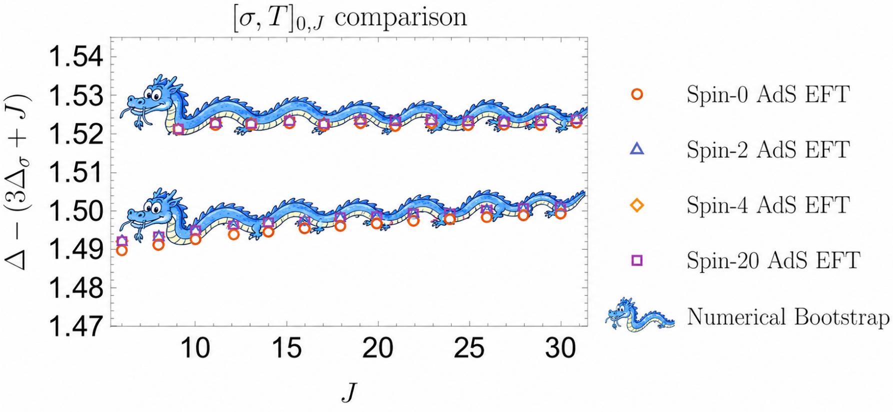
FARRAND FIELD
Matt Mitchell
Me
I got new prediction!
TASI 2025
Threads in a Theory Tapestry


FARRAND FIELD
Matt Mitchell
Me
I got new prediction!
I can check it!
TASI 2025
Threads in a Theory Tapestry

Compare $[\sigma, T]_{0,J}$ with bootstrap



AdS EFT result of $[\sigma, T]_{0,J}$ is consistent with **preliminary data** of the stress tensor bootstrap[26xx.xxxx, Erramilli, Dommes, Mitchell, Poland, Simmons-duffin, Kravchuk] at large J

Future directions

- Understand the rule for systematically improving the EFT
- Multi-particle spectrum in holographic CFT (e.g. $\mathcal{N} = 4$ SYM at large N and large t'hooft coupling λ) [2606.xxxx, Fardelli, Fitzpatrick, WL, Mei, Novikova]

Thank you!

Back up

Anomalous dimension from eigenvalues of H_{AdS}

In AdS, the dilatation operator D is the generator of time translation,

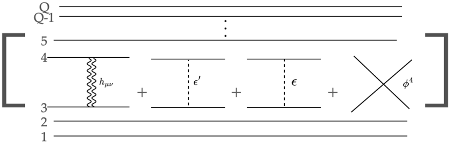
$$\boxed{\text{CFT spectrum } \Delta_i \;=\; \text{Eigenvalue of } H_{\text{AdS}}}$$

Concretely for this EFT, our AdS Hamiltonian $H_{\text{AdS}}^{\text{3d Ising}}$ will be,

$$H_{\text{AdS EFT}}^{\text{3d Ising}} = \left[\underbrace{(\Delta_\sigma + k)\delta_{k,\ell} a_k a_\ell^\dagger}_{\text{free}} + \underbrace{f_{2\to 2}(k_1, k_2; \ell_1, \ell_2) a_{k_1} a_{k_2} a_{\ell_1}^\dagger a_{\ell_2}^\dagger}_{\text{interaction}}\right]$$

Therefore we can get the operator anomalous dimension by diagonalizing H_{AdS}

$$\gamma_{m\times m} = (\boldsymbol{\Delta})_{m\times m} - (Q\Delta_\phi + J)\mathbf{1}_{m\times m} \quad = \text{Eigensystem}$$



2-to-2 Interaction matrix

The 2-to-2 Interaction matrix $f_{2\to 2}(k_1, k_2; \ell_1, \ell_2)$ is basically,

$$f_{2\to 2}(k_1, k_2; \ell_1, \ell_2)$$
$$= \langle \text{vac}| \Big((K_-)^{k_1}\sigma\Big)\Big((K_-)^{k_2}\sigma\Big) \ |V_{int}| \ \Big((P_+)^{\ell_1}\sigma\Big)\Big((P_+)^{\ell_2}\sigma\Big)|\text{vac}\rangle$$

when acting on lowest-twist sector, for **all the tree-level diagrams+contact terms** can be written as,

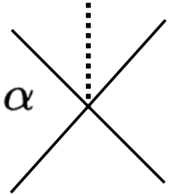
$$f_{2\to 2}(k_1, k_2; \ell_1, \ell_2) = \sum_{J=0}^{k_1+k_2} \mathcal{N}(\Delta_\sigma, k_1, k_2; \ell_1, \ell_2)\gamma([\sigma, \sigma]_{0,J})$$

$\mathcal{N}(\Delta_\sigma, k_1, k_2; \ell_1, \ell_2)$ is known function and don't independent of CFT data. The key point is the 2-to-2 matrix element just depends on anomalous dimension $\gamma([\sigma, \sigma]_{0,J})$. Therefore when computing the higher-spin contact term, we don't care about which contact terms explicitly.

Inputting OPE coefficients

We should be also able to have CFT input of **OPE coefficients**. For example, we can input $\lambda_{\epsilon\epsilon\epsilon}$ by this following diagram

$$\alpha \int_{\mathrm{AdS}} \mathrm{d}^4 x [\sigma(x)]^4 \epsilon(x) =$$



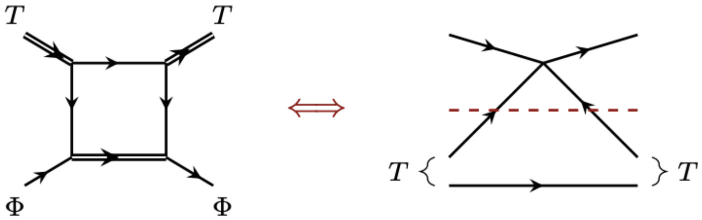
α is fixed by $\lambda_{\epsilon\epsilon\epsilon}$

Why not put $\epsilon = \sigma^2$ as fundamental field?

In the current approach we are treating ϵ as permutation 2-pt state and only fix the binding energy by contact term σ^4,

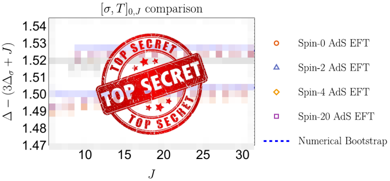
$$|\sigma^2\rangle \Leftrightarrow \epsilon$$

We think this approach is much more efficient than having an additional bulk field ϵ. The reason is that we think part of the loop diagrams, **in particular the box diagram is contained as tree-level diagram in our computation**



We indeed see this happening when we apply our AdS EFT to $d = 4 - \epsilon, O(2)$, WF fixed point. [25' Fardelli, Fitzpatrick, WL]

Data to Dragon



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