The 3d Ising Field Theory from the Fuzzy Sphere

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Simons Collaboration on Confinement and QCD Strings Workshop, MIT

Based on 2409.02998 and 2602.04958 with Fitzpatrick and Katz
and WIP with Fitzpatrick, Katz and Xin

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Introduction and motivations

How we can try to formulate QFT **non perturbatively**?

$$\mathrm{CFT}_{\mathrm{UV}} \xrightarrow{+\ \text{relevant deformation}} \mathrm{QFT}$$

$$S_{\mathrm{QFT}} = S_{\mathrm{CFT}} + \sum_{\Delta_i<d} \int d^d x\, g_i \mathcal{O}_i(x)$$

This is a two step procedure:

1. Solve the CFT
2. Numerically solve the RG flow

Introduction and motivations

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Ising CFT

$+\, \boldsymbol{\sigma},\, \boldsymbol{\epsilon}$

$\Delta_\sigma^{2d} = \frac{1}{8},\ \Delta_\sigma^{3d} = 0.52$
$\Delta_\epsilon^{2d} = 1,\ \Delta_\epsilon^{3d} = 1.41$

Ising Field Theory

$$S_{\mathrm{IFT}} = S_{\mathrm{Ising}} + \int d^d x (m\epsilon + h_z \sigma)$$

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Ising Field Theory

$$S_{\mathrm{IFT}} = S_{\mathrm{Ising}} + \int d^d x (m\epsilon + h_z \sigma)$$

2d: very rich dynamics [Wu, McCoy, Fonseca, Yurov, Zamolodchikov]

$$\eta = \frac{m}{|h_z|^{\frac{8}{15}}}$$

$\eta \gg 1$ High T
Weakly coupled fermion

$\eta \gg -1$ Low T
Tower of mesons

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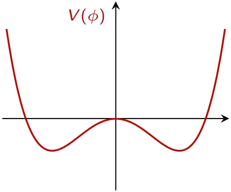

$\eta \to -\infty \ (h_z = 0)$

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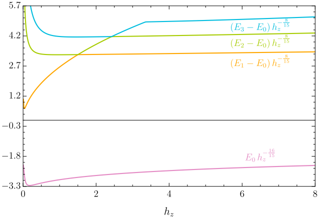
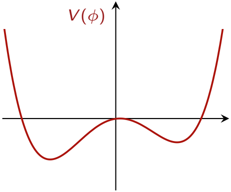


$$S_{\rm IFT} = S_{\rm Ising} + \int d^d x (m\epsilon + h_z \sigma)$$

2d: very rich dynamics [Wu, McCoy,Fonseca, Yurov,Zamolodchikov]



$\eta \ll -1\ (h_z \neq 0)$

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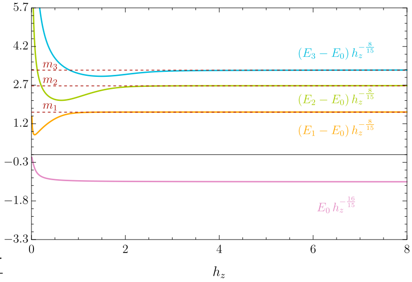
Ising Field Theory

$$S_{\mathrm{IFT}} = S_{\mathrm{Ising}} + \int d^d x (\cancel{m}\epsilon + h_z \sigma)$$

2d: very rich dynamics [Wu, McCoy,Fonseca, Yurov,Zamolodchikov]

$$m_1\,, \qquad m_2 = 2m_1 \cos\frac{\pi}{5}\,,$$
$$m_3 = 2m_1 \cos\frac{\pi}{30}\,, \qquad m_4 = 2m_2 \cos\frac{7\pi}{30}\,,$$
$$m_5 = 2m_2 \cos\frac{2\pi}{15}\,, \qquad m_6 = 2m_2 \cos\frac{\pi}{30}\,,$$
$$m_7 = 4m_2 \cos\frac{\pi}{5}\cos\frac{7\pi}{30}\,, \qquad m_8 = 4m_2 \cos\frac{\pi}{5}\cos\frac{2\pi}{15}$$



$\eta = 0\ h_z \neq 0$: integrable E_8 theory

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$3d$ Ising Field theory: Two paths

Truncated Conformal Space Approach

$$\langle \mathcal{O}_i | H | \mathcal{O}_j \rangle = \underbrace{\langle \mathcal{O}_i | H_{\mathrm{CFT}} | \mathcal{O}_j \rangle}_{\Delta_i / R \delta_{ij}}$$

$$+ h_z \int \underbrace{d^2x \langle \mathcal{O}_i | \sigma(x) | \mathcal{O}_j \rangle}_{\sim R^{2-\Delta_{\mathcal{O}}} f_{\mathcal{O}_i \sigma \mathcal{O}_j}}$$

Diagonalize H matrix w/ $\Delta_{\mathcal{O}_i} < \Delta_{\max}$

Full Fuzzy Sphere realization

Ising CFT can be obtained from a system of non-relativistic fermions on a Fuzzy Sphere, tuned to the fixed point [Zhu et al (22)]. Add a $\mathbb{Z}_2$-**odd** deformation

$$H_{IFT} = H_{\mathrm{Ising}}^{(\mathrm{FS})} + h_z \int d\Omega \, \psi^\dagger(\Omega) \sigma_z \psi(\Omega)$$

3*d* Ising Field theory: Two paths

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Advantages:

The starting point is exactly the Ising CFT

Challenges:

Need to get a lot of **OPE data** (Δ_i, 3-pt functions) to set $\Delta_{\max}$ high enough (e.g. from numerical bootstrap or fuzzy sphere [GF, Fitzpatrick, Katz (26)])

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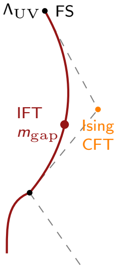
Advantages:

Sparse matrix techniques, extract lowest-E states only

Challenges:

Double-hierarchy of scales
Narrow window of valid h_z



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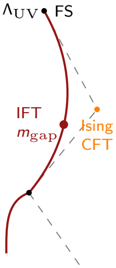

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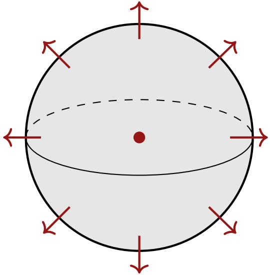
Fuzzy sphere setup

Non-relativistic fermions on S^2 in the presence of a magnetic monopole at the center (homogeneous $|B| = s/R^2$ with $2s \in \mathbb{Z}$).

$$H_{\text{free}} = \frac{(p-A)^2}{2MR^2}$$

Spherical Landau Levels

$$E_n = n(n+1) + (2n+1)s$$



$A = s\cos\theta d\phi \quad \int dA = 4\pi s$

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$$\psi_{LLL}(\Omega) = \frac{1}{R} \sum_{m=-s}^{s} \Phi_m(\Omega) \begin{pmatrix} c_{m,\downarrow} \\ c_{m,\uparrow} \end{pmatrix}$$

Monopole Harmonics

Flavor

Spherical Landau Levels

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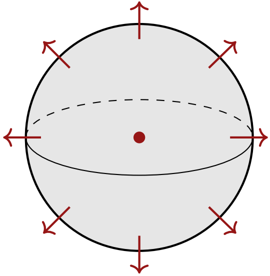
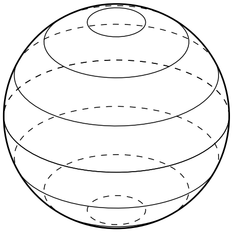


[Haldane (83), Greiter (11)]

Lowest Landau Level

$(2s+1)$-fold degenerate



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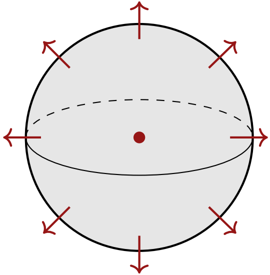
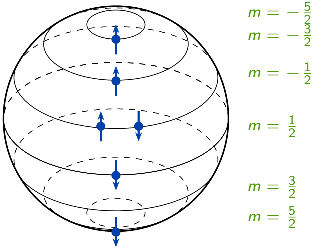


[Haldane (83), Greiter (11)]

$$\xRightarrow{\text{Lowest Landau Level}}$$

$N_e = 2s + 1$

Half filling



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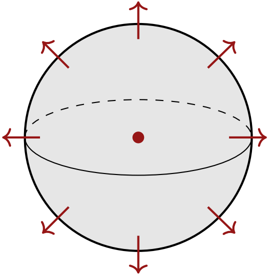

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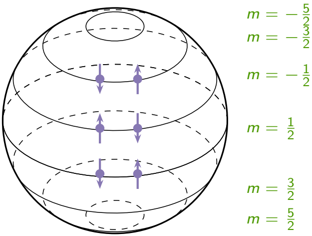

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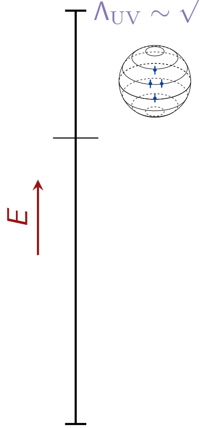
3d Ising CFT: Fuzzy sphere realization

"Lattice" scale
$\Lambda_{\mathrm{UV}} \sim \sqrt{N}$

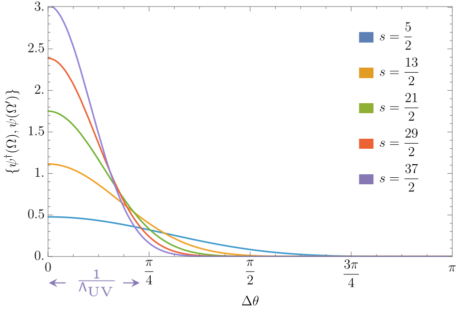


$$H = R^2 \int_{S^2} \mathcal{H}\,,\ \mathcal{H} = \sum_{k=0,1} \lambda_k \left(n_0 \nabla^{2k} n_0 - n_z \nabla^{2k} n_z \right) - h n_x$$

$$n_i = \psi^\dagger \sigma_i \psi$$

Degrees of freedom are non-relativistic fermion bilinears

$$\{\psi^\dagger(\Omega), \psi(\Omega')\} = \sum_{-s}^{s} \Phi_m^*(\Omega)\Phi_m(\Omega') = \delta_{\Lambda_{\mathrm{UV}}}(\Omega \cdot \Omega')$$



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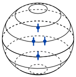


$$n_i = \psi^\dagger \sigma_i \psi$$

Degrees of freedom are non-relativistic fermion bilinears

E

Ising phase transition

$$h \gg 0\,, \quad |\psi_x\rangle = \prod_m |{+\hat{x}}\rangle_m$$

$$h = 0\,, \quad |\psi_\pm\rangle = \prod_m |{\pm\hat{z}}\rangle_m$$

Ising symmetries

- SO(3) invariance
- $\mathbb{Z}_2$ symmetry: $\uparrow \leftrightarrow \downarrow$
- Time Reversal: particle $\leftrightarrow$ hole

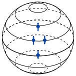
$\Lambda_{\mathrm{IR}}(\sim \text{size } \mathrm{E}_{\mathrm{gap}}) = 1\,(R = 1)$

In the IR $(2+1)d$ Ising CFT on S^2

3d Ising CFT: Fuzzy sphere realization

"Lattice" scale

$\Lambda_{\rm UV} \sim \sqrt{N}$



$$H = R^2 \int_{S^2} \mathcal{H}\,,\; \mathcal{H} = \sum_{k=0,1} \lambda_k \left(n_0 \nabla^{2k} n_0 - n_z \nabla^{2k} n_z \right) - h n_x$$

$$n_i = \psi^\dagger \sigma_i \psi$$

Degrees of freedom are non-relativistic fermion bilinears

E

[Zhu, Han, Huffman, Hofmann, He (22)] @ $N = 16$

	σ	σ'	$\sigma_{\mu_1\mu_2}$	$\sigma'_{\mu_1\mu_2}$	$\sigma_{\mu_1\mu_2\mu_3}$	$\sigma_{\mu_1\mu_2\mu_3\mu_4}$
Bootstrap	0.518	5.291	4.180	6.987	4.638	6.113
Fuzzy sphere	0.524	5.303	4.214	7.048	4.609	6.069
	ϵ	ϵ'	ϵ''	$T_{\mu\nu}$	$T'_{\mu\nu}$	$\epsilon_{\mu_1\mu_2\mu_3\mu_4}$
Bootstrap	1.413	3.830	6.896	3	5.509	5.023
Fuzzy sphere	1.414	3.838	6.908	3	5.583	5.103

Tuned at the **Critical Point**:

$\lambda_0 = \lambda_0^*,\ \lambda_1 = \lambda_1^*,\ h = h^*$

$\Lambda_{\rm IR} = 1\,(R = 1)$

In the IR $(2+1)d$ Ising CFT on S^2

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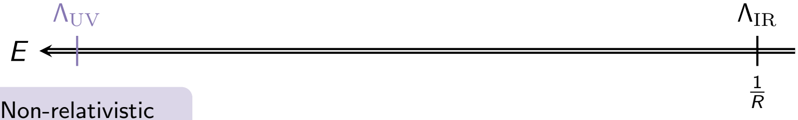
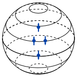
General overview



Non-relativistic fermions on a FS



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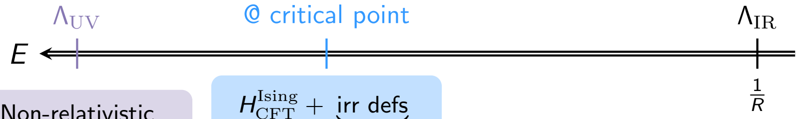
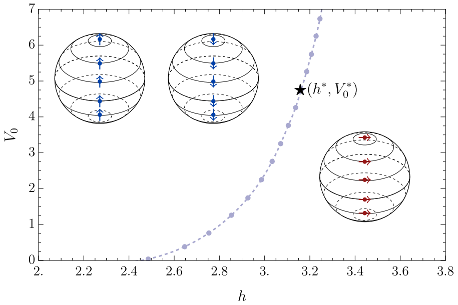


Non-relativistic fermions on a FS

$H_{\mathrm{CFT}}^{\mathrm{Ising}} + \underbrace{\text{irr defs}}_{\sim N^{\frac{3-\Delta_{\mathcal{O}}}{2}}}$



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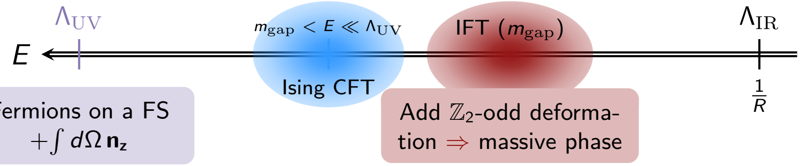
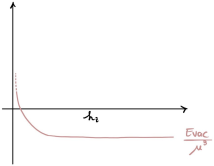
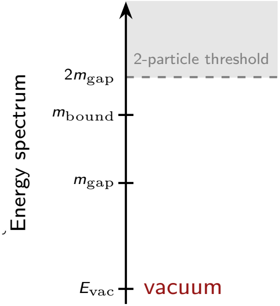


$$S = S_{\mathrm{CFT}} + h_z \int d^3x\, \sigma(x)$$

Only one **scale** $\mu = h_z^{\frac{1}{3-\Delta_\sigma}}$ at ∞ volume

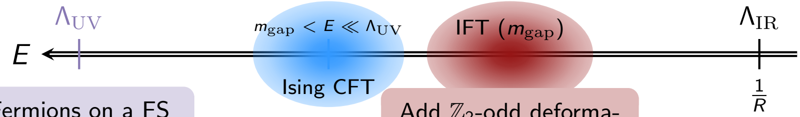




Vacuum energy density

$$E_{\mathrm{vac}} \sim \mu^d = h_z^{\frac{3}{3-\Delta_\sigma}}$$

General overview



Fermions on a FS $+\int d\Omega\, \mathbf{n_z}$

Add $\mathbb{Z}_2$-odd deformation $\Rightarrow$ massive phase

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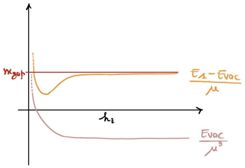
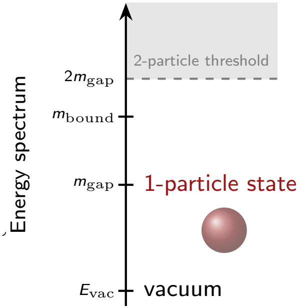
Only one **scale** $\mu = h_z^{\frac{1}{3-\Delta_\sigma}}$ at ∞ volume

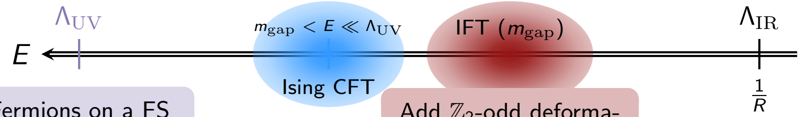




$$S = \frac{1}{2}\int \left(\dot{\phi}^2 - m^2\phi^2 - (\nabla_{S^2}\phi)^2 - c(\nabla^2_{S^2}\phi)^2 + \cdots\right)$$

$$(E - E_{\rm vac})^2 = m_{\rm gap}^2 + \ell(\ell+1) + c\ell^2(\ell+1)^2$$

General overview



Fermions on a FS
$+\int d\Omega\, \mathbf{n_z}$

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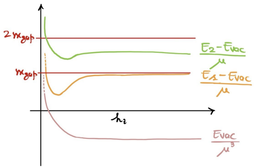

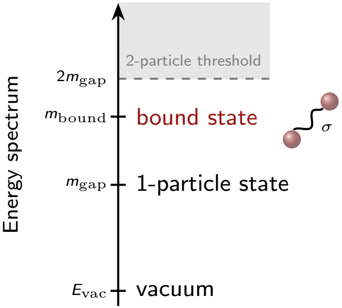

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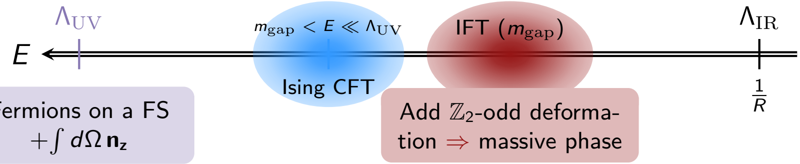
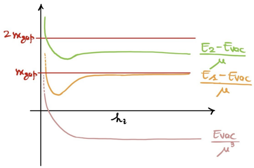
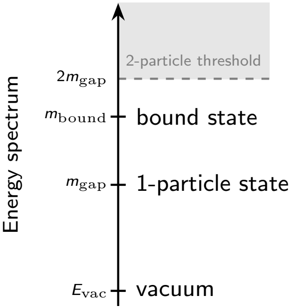


$$\Lambda_{\rm UV} \ll m_{\rm gap} \ll \frac{1}{R} \quad \longleftrightarrow \quad \text{small range of } h_z$$





At **finite** volume

Curvature corrections

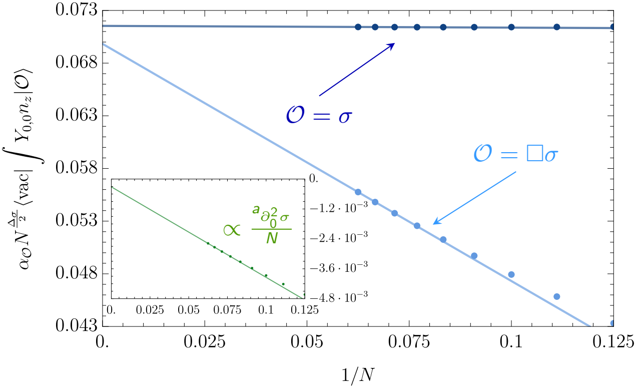
$$E_{\rm gap} - E_{\rm vac} =$$
$$m_{\rm gap}\left(1 + \frac{f}{m_{\rm gap}^2 R^2} + \cdots\right)$$
$$= A_1 h_z^{\frac{1}{3-\Delta_\sigma}}\left(1 + \frac{\tilde{f}}{h_z^{\frac{2}{3-\Delta_\sigma}}}\right)$$

$n_z = \sigma + \cdots$

$$\mathcal{H} \to \mathcal{H}(V_0, V_1, h) + \tilde{h}_z n_z(\Omega)\,, \qquad n_z(\Omega) \equiv \psi^\dagger(\Omega)\sigma^z\psi(\Omega)$$

$$n_z = \frac{a_\sigma}{N^{\frac{\Delta_\sigma}{2}}}\sigma + \frac{a_{\partial_0^2\sigma}\partial_0^2\sigma + a_{\nabla^2\sigma}\nabla^2\sigma}{N^{\frac{\Delta_\sigma+2}{2}}} + \frac{a_{\sigma_2}}{N^{\frac{\Delta_{\sigma_2}}{2}}}\sigma_{00} + \cdots$$

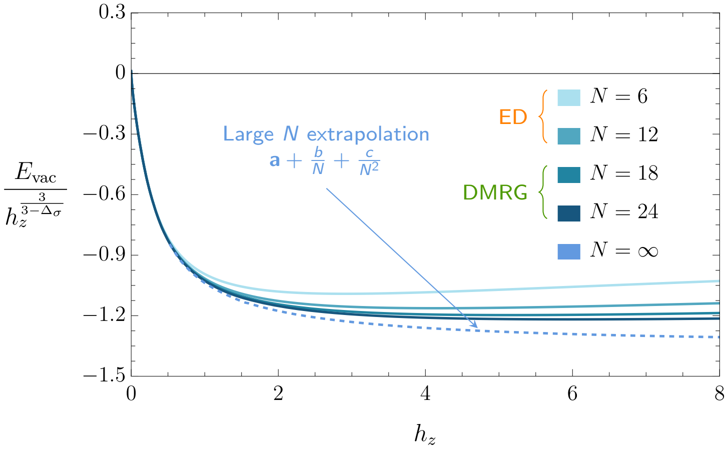
Fixed by matching condition



Vacuum energy density

$$\frac{E_{\mathrm{vac}}}{h_z^{\frac{3}{3-\Delta_\sigma}}} = \mathcal{E}_0 + \frac{\tilde{\mathcal{E}}_0}{h_z^{\frac{2}{3-\Delta_\sigma}}}$$

$\frac{1}{R^2}$ curvature corrections



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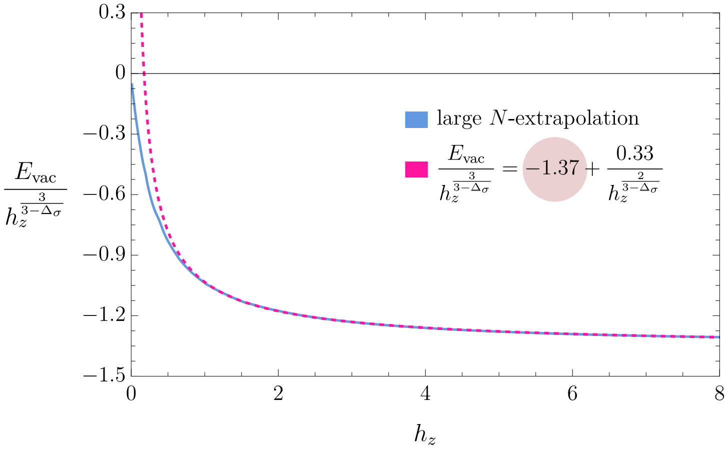
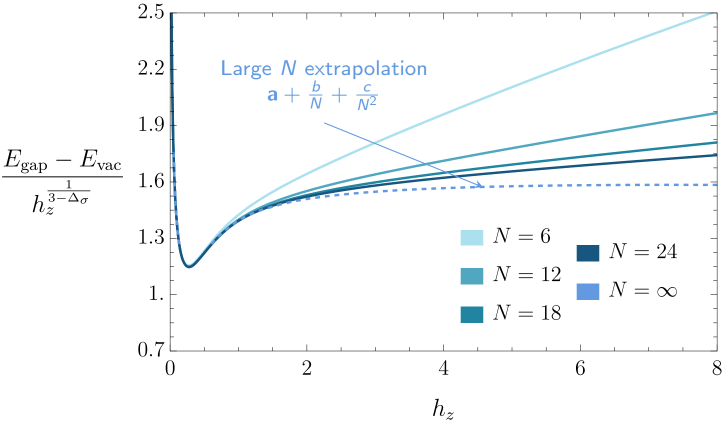


One particle state

$$\frac{E_{\rm gap} - E_{\rm vac}}{h_z^{\frac{1}{3-\Delta_\sigma}}} = m_{\rm gap} + \frac{f}{h_z^{\frac{2}{3-\Delta_\sigma}}}$$

$\frac{1}{R^2}$ curvature corrections



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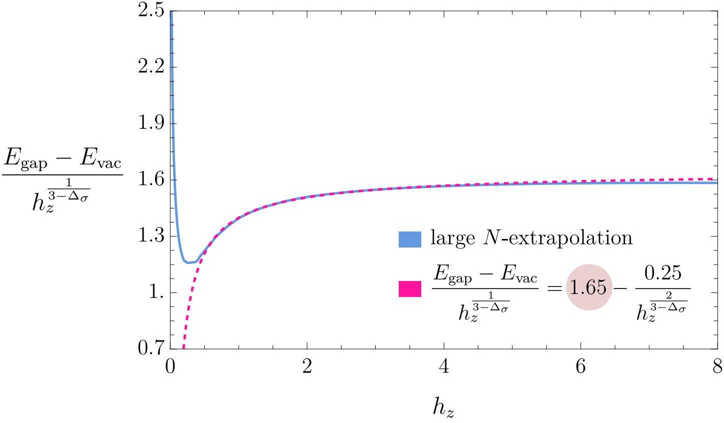
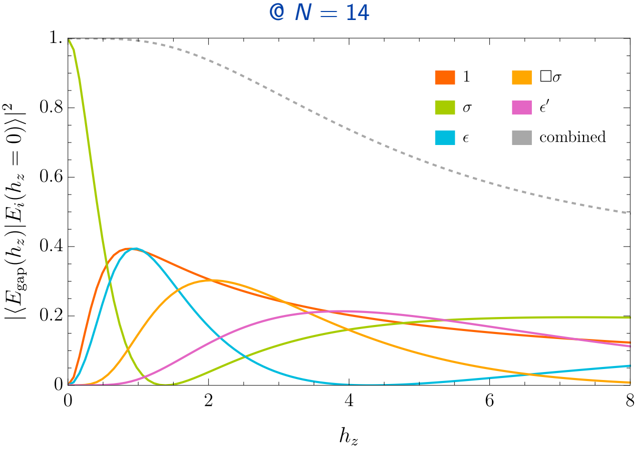


One particle state: composition

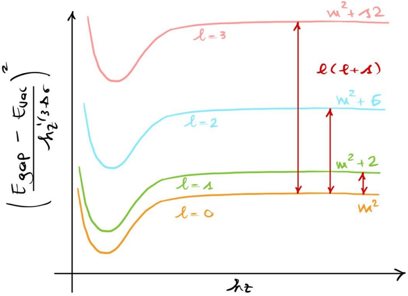
$|\langle E_{\text{gap}}(h_z)|E_i(h_z=0)\rangle|^2,$ where $|E_i(h_z=0)\rangle$ corresponds to a CFT operator



Interlude: Spinning sectors

$$S = \frac{1}{2}\int \left(\dot{\phi}^2 - m^2\phi^2 - (\nabla_{S^2}\phi)^2 - c(\nabla^2_{S^2}\phi)^2 + \cdots\right)$$

$$E_{\mathrm{gap}}(\ell) = E_{\mathrm{vac}} + \sqrt{m^2_{\mathrm{gap}} + \ell(\ell+1) + c\ell^2(\ell+1)^2}$$



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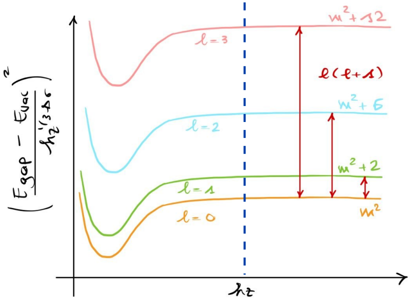
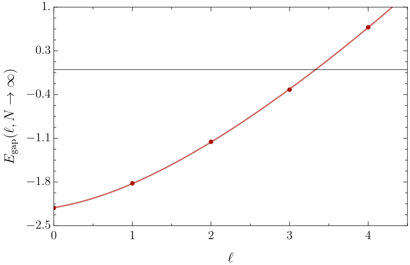


@ $h_z = 3$



$$-1.23\, h_z^{\frac{3}{3-\Delta_\sigma}} + \sqrt{\left(1.56\, h_z^{\frac{1}{3-\Delta_\sigma}}\right)^2 + \ell(\ell+1) + 0.006\,\ell^2(\ell+1)^2}$$

Interlude: Spinning sectors

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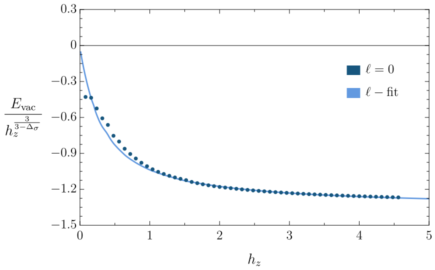
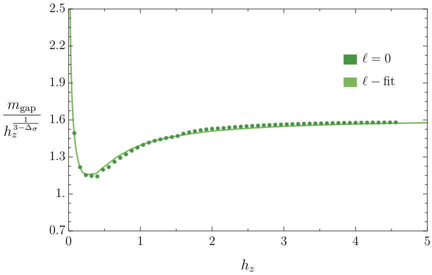


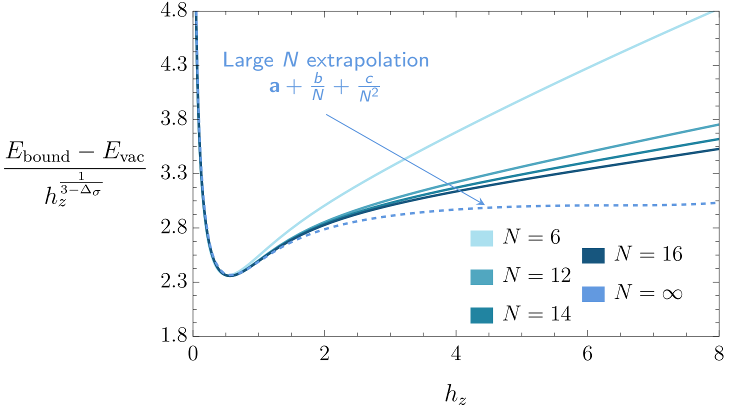


Large N extrapolation + fit with $\ell = 0, 1, \ldots, 4$

Bound state

$$\frac{E_{\text{bound}} - E_{\text{vac}}}{h_z^{\frac{1}{3-\Delta_\sigma}}} = m_{\text{b}} + \frac{f_b}{h_z^{\frac{2}{3-\Delta_\sigma}}}\,, \qquad m_b < 2m_{\text{gap}}$$

$\frac{1}{R^2}$ curvature corrections



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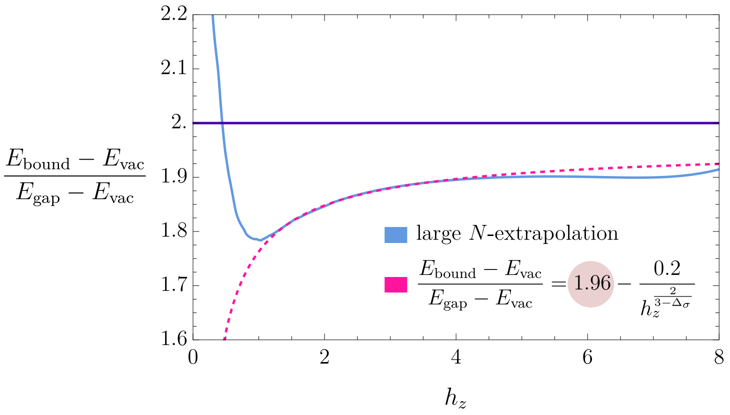


Fuzzy Sphere vs Truncated Conformal Space Approach

$$\begin{pmatrix} \Delta_1 & & \\ & \ddots & \\ & & \Delta_n \end{pmatrix} \qquad \left(\begin{array}{c:c} H_{\mathrm{CFT}}^{\mathrm{even}} & h_z V \\ \hdashline h_z V^T & H_{\mathrm{CFT}}^{\mathrm{odd}} \end{array}\right)$$

$$V_{ij} = f_{\mathcal{O}_i \sigma \mathcal{O}_j}$$

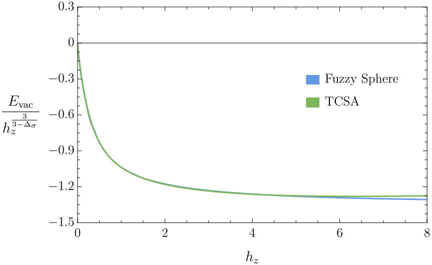
Connects $\mathbb{Z}_2$ odd and $\mathbb{Z}_2$ even

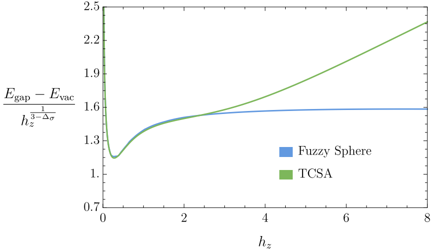
Diagonalize a 51×51 matrix for $\Delta_i < 11.7$ (new OPE data needed [GF, Fitzpatrick, Katz (26)])

Fuzzy Sphere vs Truncated Conformal Space Approach

TCSA: $\left(\begin{array}{c|c} H_{\rm CFT}^{\rm even} & h_z V \\ \hline h_z V^T & H_{\rm CFT}^{\rm odd} \end{array}\right)$ $\longrightarrow$ $V_{ij} = f_{\mathcal{O}_i \sigma \mathcal{O}_j}$

Connects $\mathbb{Z}_2$ odd and $\mathbb{Z}_2$ even





$$\frac{E_{\rm vac}}{h_z^{\frac{3}{3-\Delta_\sigma}}} = \begin{cases} -1.369 + 0.336 h_z^{-\frac{2}{3-\Delta_\sigma}} \\ -1.372 + 0.334 h_z^{-\frac{2}{3-\Delta_\sigma}} \end{cases}$$

$$\frac{E_{\rm gap} - E_{\rm vac}}{h_z^{\frac{1}{3-\Delta_\sigma}}} = \begin{cases} 1.65 - 0.25 h_z^{-\frac{2}{3-\Delta_\sigma}} \\ 1.66 - 0.28 h_z^{-\frac{2}{3-\Delta_\sigma}} \end{cases}$$

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- Further study of the bound state: mass ratio, separation from the two-particle state.
- Refine TCSA analysis (increase $\Delta_{\max}$, understand the dependence in $\Delta_{\max}$, spinning sectors)
- Add a ϵ-deformation, generic IFT phase diagram
- Other $3d$ QFTs from the fuzzy sphere (gauge theories?)

Thank you!