Predictions of Matrix String Theory Duality for 2D super Yang-Mills

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Gauge-gravity Duality

- AdS/CFT (Maldacena '97, ...)

  e.g. IIB string in $AdS_5 \times S^5 \longleftrightarrow$ 4d $\mathcal{N}=4$ SYM

- Matrix dualities (Banks, Fischler, Shenker, Susskind '96 ...)

SYM $\longleftrightarrow$ string theory in Minkowski spacetime

Coulomb branch physics $\longleftrightarrow$ S-matrix

Matrix Dualities

0+1 D
- BFFS matrix quantum mechanics

$\longleftrightarrow$ M-theory in $\mathbb{R}^{1,10}$

1+1 D
- 2d $(8,8)$ $U(N)$ SYM

$\longleftrightarrow$ IIA string theory in $\mathbb{R}^{1,9}$

- 2d $(0,8)$ $O(N)$ SYM

$\longleftrightarrow$ $SO(32)$ heterotic string theory in $\mathbb{R}^{1,9}$

⋮

2D (8,8) U(N) SYM

$$S = \frac{1}{g_{YM}^2}\int d^2x\, \mathrm{tr}\Big(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D_\mu\phi^i D^\mu\phi^i + \frac{1}{4}[\phi^i,\phi^j]^2$$
$$- \lambda_{\alpha+}D_-\lambda_{\alpha+} - \lambda_{\dot\alpha-}D_+\lambda_{\dot\alpha-} - \lambda_{\alpha+}\gamma^i_{\alpha\dot\alpha}[\phi^i,\lambda_{\dot\alpha-}]\Big)$$

- $x^1 \sim x^1 + 2\pi R$
- $so(8)_R$: vec i, chiral α, anti-chiral $\dot\alpha$

$g_{YM}R \gg 1$

$$\Big\downarrow \longleftarrow \Delta S = \frac{a_0}{g_{YM}}\int d^2x\, \overbrace{\Sigma(x)}^{\text{DVV twist field}} + \cdots$$

$\mathrm{Sym}^N(\mathbb{R}^8)$ SCFT

2D (8,8) U(N) SYM

Center : $U(1) \supset \mathbb{Z}_N$ $\rightsquigarrow$ N-ality of $SU(N)$

$\Rightarrow$ Electric flux sector

Q. Spectrum in k-flux sector?

Resonances & decay rate?

$\rightsquigarrow$ predictions from matrix duality

Matrix String Theory (MST) Duality

(Motl '97, Banks, Seiberg '97, Dijkgraaf, Verlinde, Verlinde '97)

2D (8,8) U(N) SYM

$\uparrow$ decoupling

N D1's in IIB $\longleftrightarrow$ Black 1-brane sol' in IIB

$\swarrow$ decoupling

$$ds^2_{str} = \left(\tilde{f}_1(r)\right)^{-1/2}\left(-dt^2+dx^2\right) + \left(\tilde{f}_1(r)\right)^{1/2}\left(dr^2+r^2 d\Omega_7^2\right)$$

$$e^{\Phi} = \left(\tilde{f}_1(r)\right)^{1/2}, \qquad C_2 = \tilde{f}_1^{-1}\, dt\wedge dx,$$

$$g_{YM}^2 = \frac{g_B}{2\pi \ell_B^2}$$

$$\tilde{f}_1(r) = \frac{c_1 N}{r^6}, \qquad c_1 = 32\pi^2 g_B \ell_B^6$$

Matrix String Theory (MST) Duality

- S-dual $\rightsquigarrow \widetilde{\mathrm{IIB}}$: pure NSNS

$$ds^2_{str} = (\tilde{f}_1(r))^{-1}(-dt^2 + dx^2) + dr^2 + r^2 d\Omega_7^2, \qquad x \sim x + 2\pi R$$

$$e^{\Phi} = (\tilde{f}_1(r))^{-1/2}, \qquad B_2 = \tilde{f}_1^{-1} dt \wedge dx \qquad (\tilde{g}_B = g_B^{-1}, \ \tilde{l}_B = g_B^{1/2} l_B)$$

- T-dual $\rightsquigarrow$ IIA: pure geometry

$$ds^2_{str} = 2dt\,d\tilde{x} + \frac{c_1 N}{r^6} d\tilde{x}^2 + dr^2 + r^2 d\Omega_7^2, \qquad \tilde{x} \sim \tilde{x} + \frac{2\pi l_A^2}{R}$$

$$\left(l_A = \tilde{l}_B, \ g_A = \tilde{g}_B \frac{\tilde{l}_B}{R}, \ c_1 = 32\pi^2 g_A^2 l_A^4 R^2\right)$$

- $g_A = \dfrac{1}{\sqrt{2\pi}\, g_{YM} R} \Rightarrow$ IIA string pert' $\leftrightsquigarrow$ near IR of SYM

Matrix String Theory (MST) Duality

$$ds^2_{str} = 2\,dt\,d\tilde{x} + \frac{c_1 N}{r^6} d\tilde{x}^2 + dr^2 + r^2 d\Omega_q^2 \qquad \tilde{x} \sim \tilde{x} + \frac{2\pi l_A^2}{R}$$

- Proper length of $\tilde{x}$ circle $\sim \dfrac{g_A l_A^4 N^{1/2}}{r^3}$

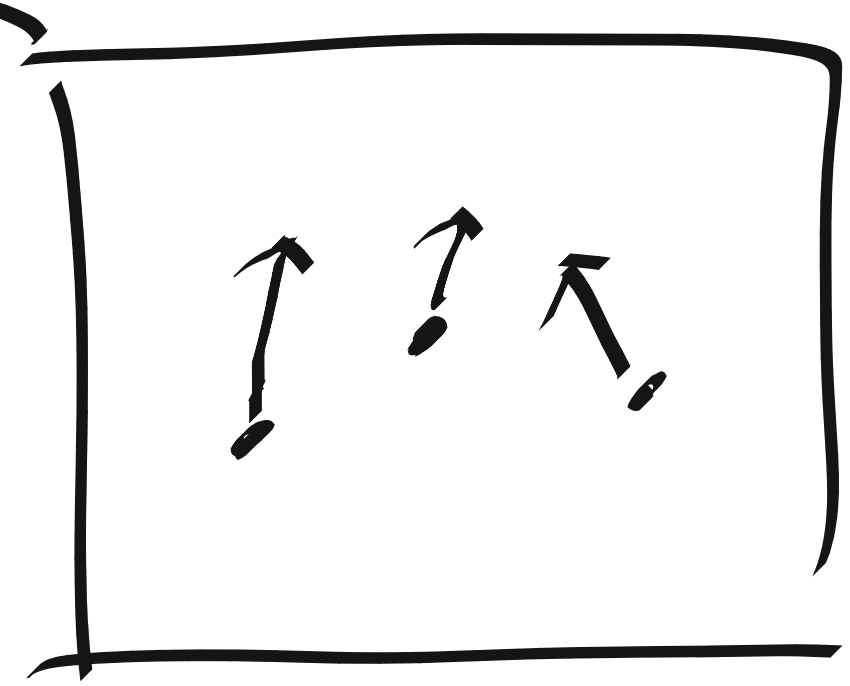
$\Rightarrow$ sugra valid for $1 \ll \dfrac{r}{l_A} \ll g_A^{1/3} N^{1/6}$

$\rightsquigarrow$ Take $N \to \infty$

Matrix String Theory (MST) Duality

· Multi-core : $\frac{N}{r^6} \longrightarrow \sum_a \frac{N_a}{|\vec{x}-\vec{x}_a|^6}$

$\mathbb{R}^8$



IIA in $\mathbb{R}^{1,9}$: $\underbrace{\text{graviton}}$ S-matrix

$$k_a^+ \sim N_a$$

$$k_a^- \sim E_a^{SYM}$$

SYM : $U(N) \rightsquigarrow \underbrace{U(N_1)} \times U(N_2) \times \cdots \to U(\tilde{N}_1) \times U(\tilde{N}_2) \times \cdots$

$$U(1) \rightsquigarrow k_a^i$$

$g_{YM}R \gg 1$: $\mathrm{Sym}^N(\mathbb{R}^8) + \mathrm{DS}$ $\left(g_A \sim \frac{1}{g_{YM}R} \right)$

$$(X^i_a,\ \theta^\alpha_a,\ \tilde{\theta}^{\dot\alpha}_a)/S_N$$

- Twisted sectors $\longleftrightarrow$ $N = \sum_{k\geq 1} n_k k$, $n_k \in \mathbb{Z}_{\geq 0}$

$k \sim (12\cdots k)$

$$\mathcal{H} = \bigoplus_{\sum n_k k = N} \left[\bigotimes_{k \geq 1} \mathrm{Sym}^{n_k} \underbrace{\mathcal{H}_k}_{} \right]$$

$\hookrightarrow$ max' twisted sector of Sym^k

$$\mathcal{H}_k \hookrightarrow \mathcal{H}_1$$

- SUSY : only (R,R) sector

$$\sim e^{ik^+ \cdot X} \otimes (\psi^i, S^{\dot\alpha}) \otimes (\tilde{\psi}^i, \tilde{S}^\alpha) \to \text{sugra multiplet}$$

$g_{YM}R \gg 1$: $\mathrm{Sym}^N(\mathbb{R}^8)$ + OS

· 1-graviton : $|N, k^\perp, ij\rangle \sim \frac{1}{\sqrt{N!}} \sum_{h \in S_N} \left[e^{ik^\perp \cdot X} \psi^i \tilde{\psi}^j \right]_{h(1\cdots N)h^{-1}}$

$$\Rightarrow \quad E = H_{cyl} = \frac{(k^\perp)^2}{2NR} \qquad \left(E \sim k^-, \quad NR \sim k^+ \right)$$

· 2-graviton : $|N_1, k_1^\perp, i_1 j_1 ; N_2, k_2^\perp, i_2 j_2\rangle \qquad (N_1 + N_2 = N)$

$$\sim \frac{1}{\sqrt{N!}} \sum_{h \in S_N} \left[e^{ik_1^\perp \cdot X} \psi^{i_1} \tilde{\psi}^{j_1} \right]_{h(1\cdots N_1)h^{-1}} \left[e^{ik_2^\perp \cdot X} \psi^{i_2} \tilde{\psi}^{j_2} \right]_{h(N_1+1\cdots N)h^{-1}}$$

$\vdots$

$g_{YM}R \gg 1$: $\mathrm{Sym}^N(\mathbb{R}^8) + \Delta S$

$$\Delta S = \frac{a_0}{g_{YM}} \int d^2x \, \underbrace{\Sigma(x)}_{\text{DVV twist field} \, = \, \sum_{I<J} \Sigma_{(IJ)}} + \cdots$$

← should preserve all symmetries

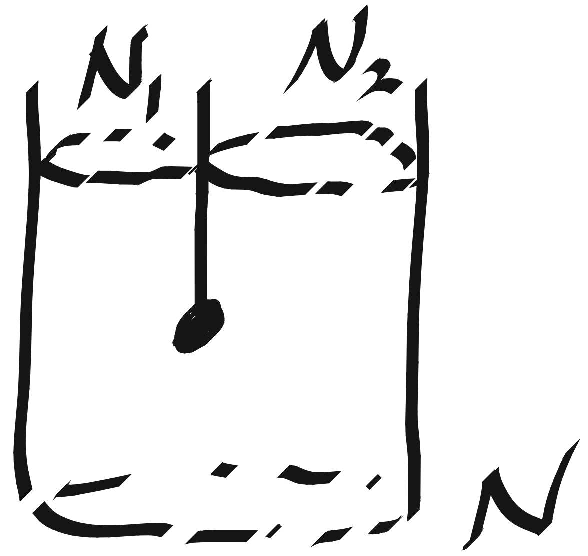
$$\Sigma_{(IJ)} = 4\, G^{\dot\alpha}_{-\frac{1}{2}} \tilde{G}^{\alpha}_{-\frac{1}{2}} \left[S^{\dot\alpha} \tilde{S}^{\alpha} \right]_{(IJ)}$$

(no sum over $\alpha, \dot\alpha$)

$$\sim \left[\psi^i \partial X^i \, \tilde{\psi}^j \bar\partial X^j \right]_{(IJ)}$$

$$A^{3pt,(1)}_{MST} = -\frac{a_0}{g_{YM}R} \int d^2x \, \langle 1\text{-grav} | \Sigma(x) | 2\text{-grav} \rangle$$

covering space



(Lunin, Mathur '00)

$g_{YM} R \gg 1$: $\mathrm{Sym}^N(\mathbb{R}^8) + \Delta S$

$$\Delta S = \frac{a_0}{g_{YM}} \int d^2x\, \Sigma(x) + \cdots$$

- $$A^{3pt,(1)}_{MST} = \frac{A^{3pt,g=0}_{IIA}\Big|_{LC}}{\delta(k_3^+ - k_1^+ - k_2^+)} \Rightarrow a_0 = -2(2\pi)^{3/2}$$

- $A^{4pt,(2)}_{MST}$: Arutyunov, Frolov '97

- $A^{2pt,(2)}_{MST}$: W.I.P.

Flux Sector

2d (8,8) U(N) SYM in k-flux sector

$\}$ (Witten '95) e.g. $k=1$: $\mathrm{tr}_f \mathcal{P}\, e^{i\oint A}$

(k, N) - string (IIB)

$\xrightarrow{S}$ k D1 charge ($\widetilde{\text{IIB}}$) $\xrightarrow{T}$ k D0 charge (IIA)

NCOS (Klebanov, Maldacena '00, (DVV '97)

Herzog, Klebanov '00)

- $E_k^{g.s.} = \dfrac{k^2 g_{YM}^2}{2N} 2\pi R = \dfrac{\alpha'}{2NR}(k M_{D0})^2$, $\quad M_{D0} = \dfrac{1}{g_A \sqrt{\alpha'}}$

$\left(p^+ = \dfrac{NR}{\alpha'},\ p^- = E_k\right)$

Flux Sector ($k=1$)

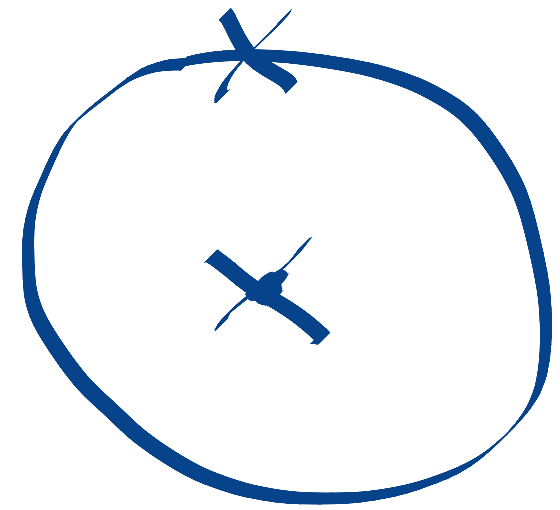
- Open strings on D0 $\rightsquigarrow$ excitations of D0 itself
  - massless $\longleftrightarrow$ $U(1)$
  - massive $\longleftrightarrow$ excitations in $k=1$ sector of $SU(N)$

$$\Rightarrow \Delta E = \frac{\alpha'}{2NR}\left[\left(M_{D0} + \frac{\sum_i n_i \sqrt{l_i}}{\sqrt{\alpha'}}\right)^2 - M_{D0}^2\right]$$

$$= \frac{\sqrt{2\pi}\, g_{YM}}{N}\left[\sum_i n_i \sqrt{l_i} + \mathcal{O}\left(\frac{1}{g_{YM} R}\right)\right]$$

Flux Sector $(k=1)$ $\left(\ell=1,\ M=M_{D0}+\frac{1}{\sqrt{\alpha'}}\right)$

- Decay into D0 + $\underbrace{\text{closed string}}_{p^{+\prime}=\frac{nR}{\alpha'}}$, $n \ll N$



(when kinematically allowed)

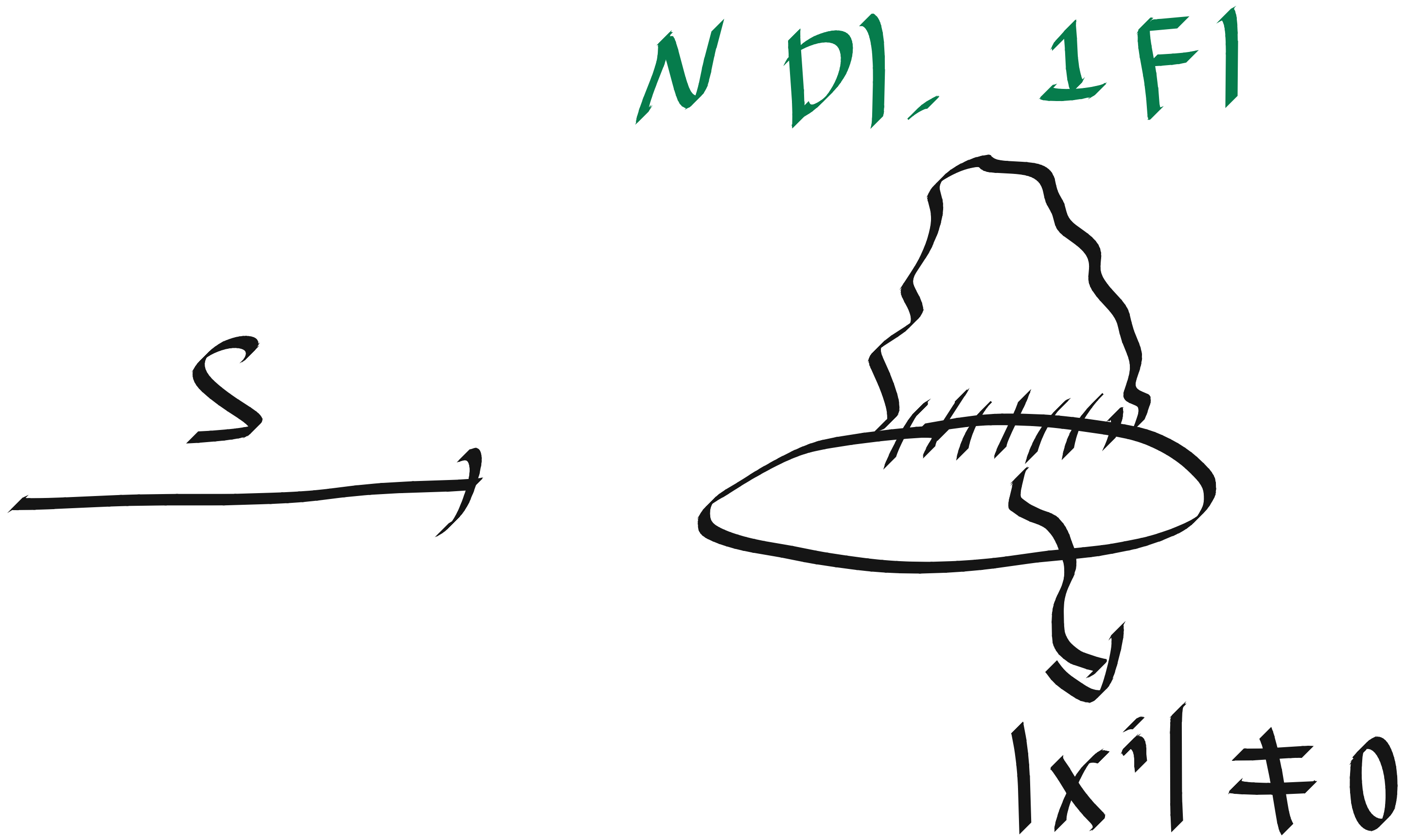
$$\updownarrow$$

Massive exc' $\sim \frac{g_{YM}}{N}$ $\longrightarrow$ $U(N-n)$ w/ $k=1$ $\times$ $U(n)$

$\hookrightarrow$ resonance w/ $\Gamma \sim \frac{1}{NR}$

Flux Sector $(k=1)$ $(n=1)$

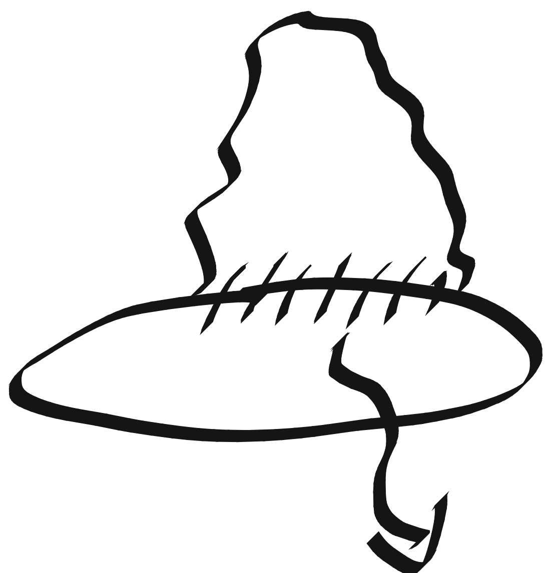
IIA $\qquad\qquad\qquad\qquad$ $\widetilde{\text{IIB}}$



IIB

$$\phi^i = \begin{pmatrix} -\frac{x^i}{N-1} I_{N-1} & 0 \\ 0 & x^i \end{pmatrix} \in su(N)$$

Flux Sector $(k=1)$ $(n=1)$

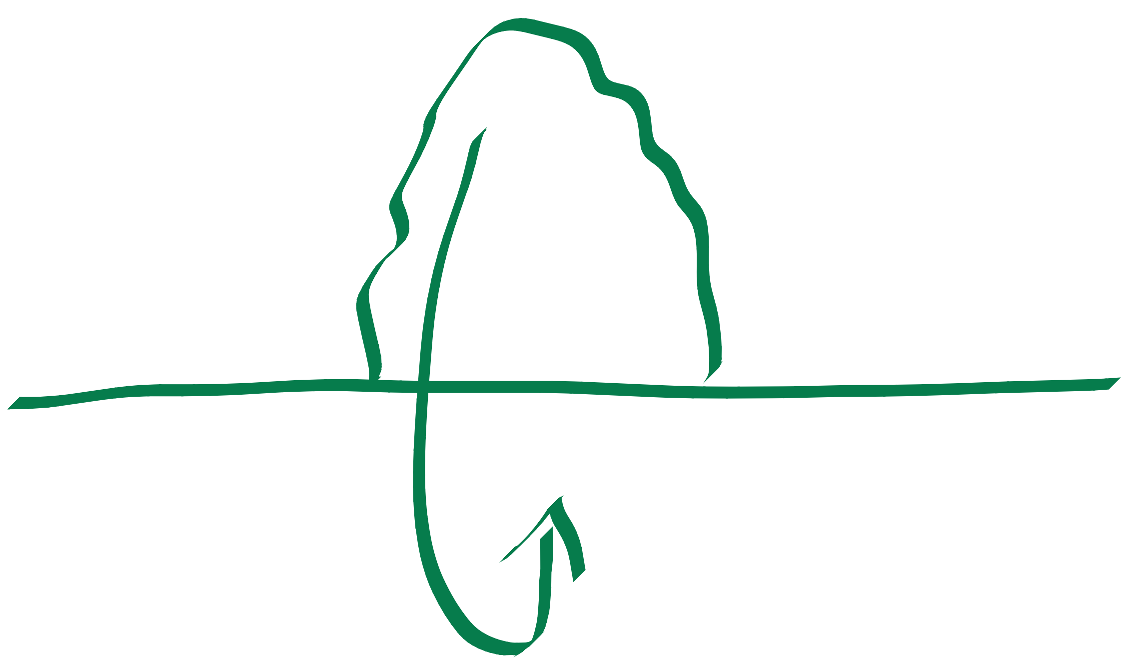


$|x^i| \neq 0$

large $|x^i|$ $\rightsquigarrow$ massless EFT

$\updownarrow$

IIA F1



$\longleftrightarrow$ leading Regge of IIA F1 on D0

$$E = \frac{g_{YM}}{N}\sqrt{2\pi J}$$

Heterotic SO(32) MST

$$\text{2d } (0,8) \ O(N) \text{ SYM} \longleftrightarrow \text{het } SO(32) \text{ in } \mathbb{R}^{1,9}$$

$$g_H \sim \frac{1}{g_{YM} R}$$

Lightest SO(32) spinor : massive, but stable!

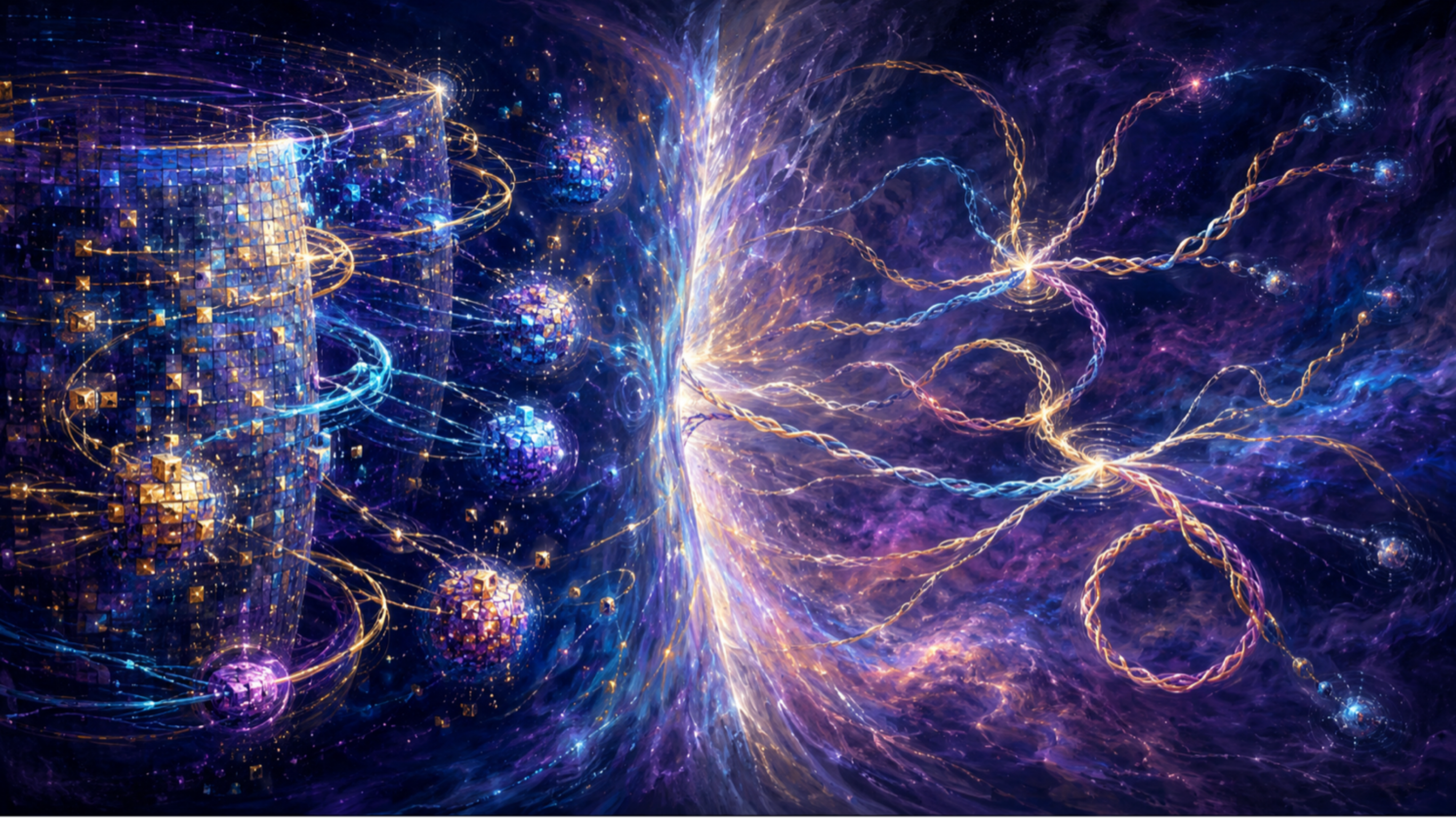
$$M_{\text{spinor}} \sim \frac{1}{l_H} f(g_H), \quad f(g_H) \sim \begin{cases} 1, & g_H \ll 1 \\ \# \, g_H^{1/2}, & g_H \gg 1 \end{cases}$$

(Sen '13)

$$\Rightarrow \boxed{E_{\text{spinor}} \sim \frac{1}{NR} f^2(g_H)}$$

Summary & Outlook

- Matrix dualities make predictions for SYM
- SYM $\rightsquigarrow$ nonperturbative string theory S-matrix
- Other angles : NCOS, NRCS (Gomis, Ooguri '00)
- Other examples : $k=1$ ABJM $\hookrightarrow$ IIB in $\mathbb{R}^{1,9}$



(credit to ChatGPT)

Thank You!!