The Confining String in Hyperbolic Space

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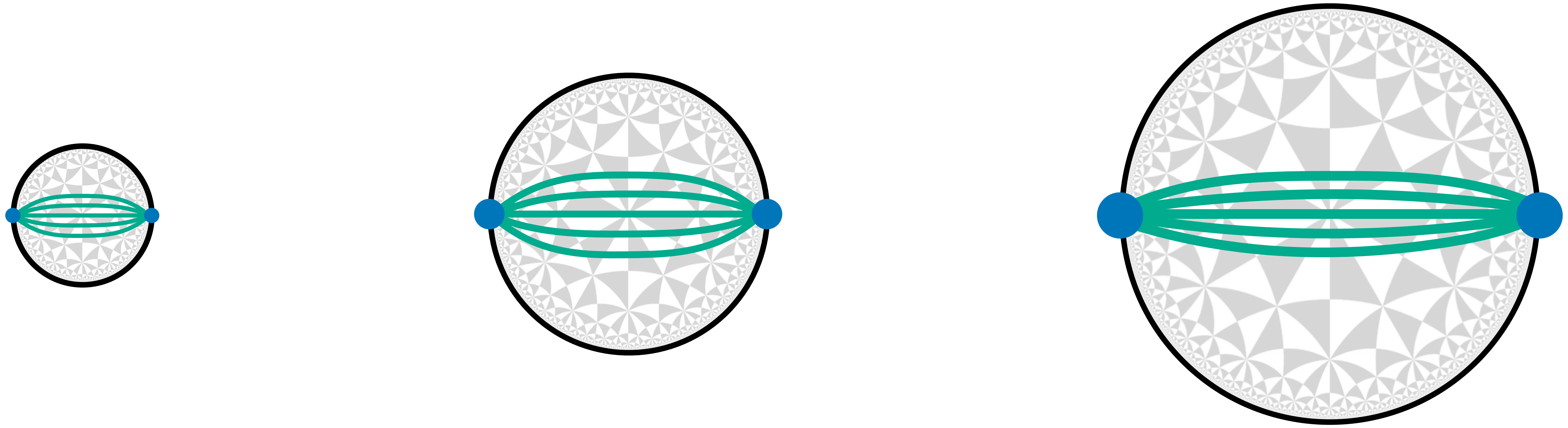

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Based on hep-th/2508.08250, hep-th/2602.16694, and work in progress

Yang-Mills flux tube

Effective string theory, lattice, and S-matrix bootstrap

Effective string theory

$$S = \int d^2\sigma \sqrt{h}\left[\ell_s^{-2} + \kappa(\ell_s^2 K_{\alpha\beta}K^{\alpha\beta})^2 + \dots\right]$$

[Luscher, Polchinski, Strominger, Weisz, Aharony, Komargodski, Dubovsky, Gorbenko, …]

$\longleftrightarrow$

Scattering excitations on a long open string

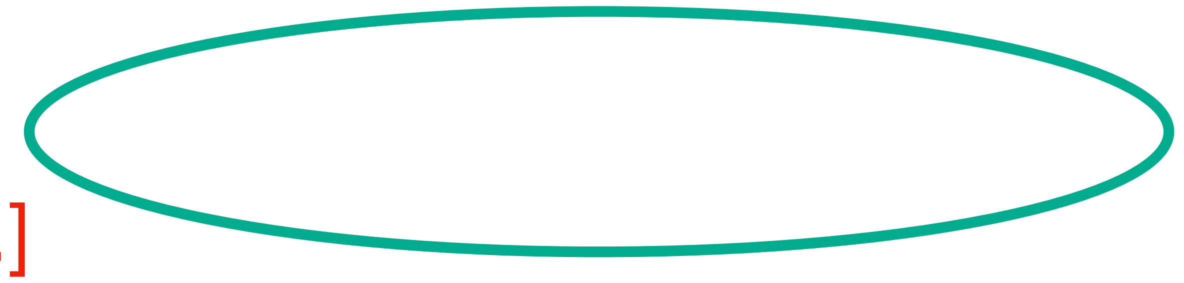
$$e^{2i\delta(s)}, \qquad 2\delta(s) = \frac{1}{4}s\ell_s^2 + \frac{\gamma_S}{768}\ell_s^6 s^3 + \dots$$

[Guerrieri, Miro, Hebbar, Penedones, Vieira, Homrich, Albert …]

$\updownarrow$

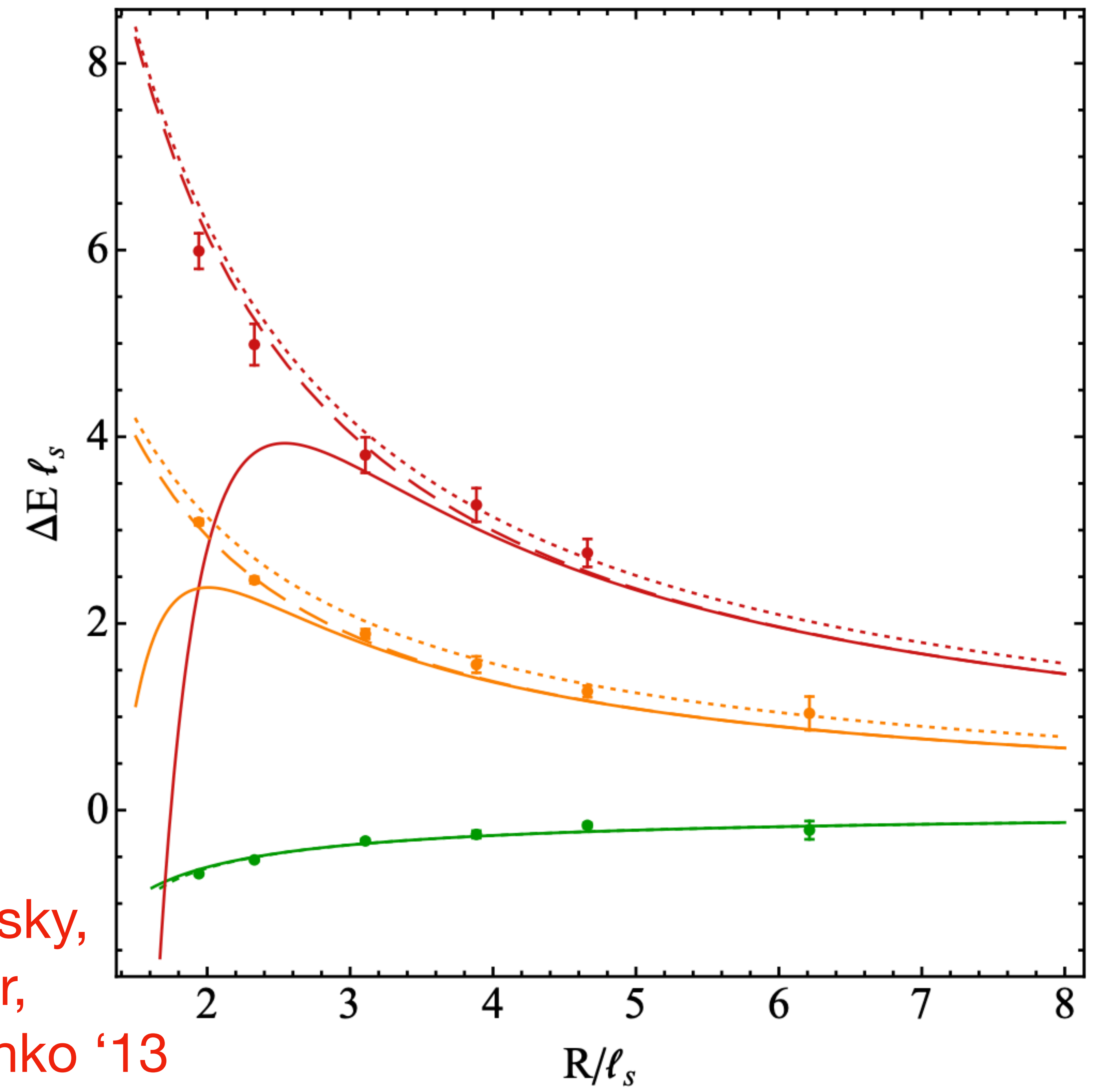
Spectrum on a winding closed string

[Caselle, Teper, Athenodorou, …]



$$E_n(R) = \frac{R}{\ell_s^2} + \frac{\#}{R} + \frac{\#\ell_s^2}{R^3} + \frac{\#\ell_s^4}{R^5} + \dots$$

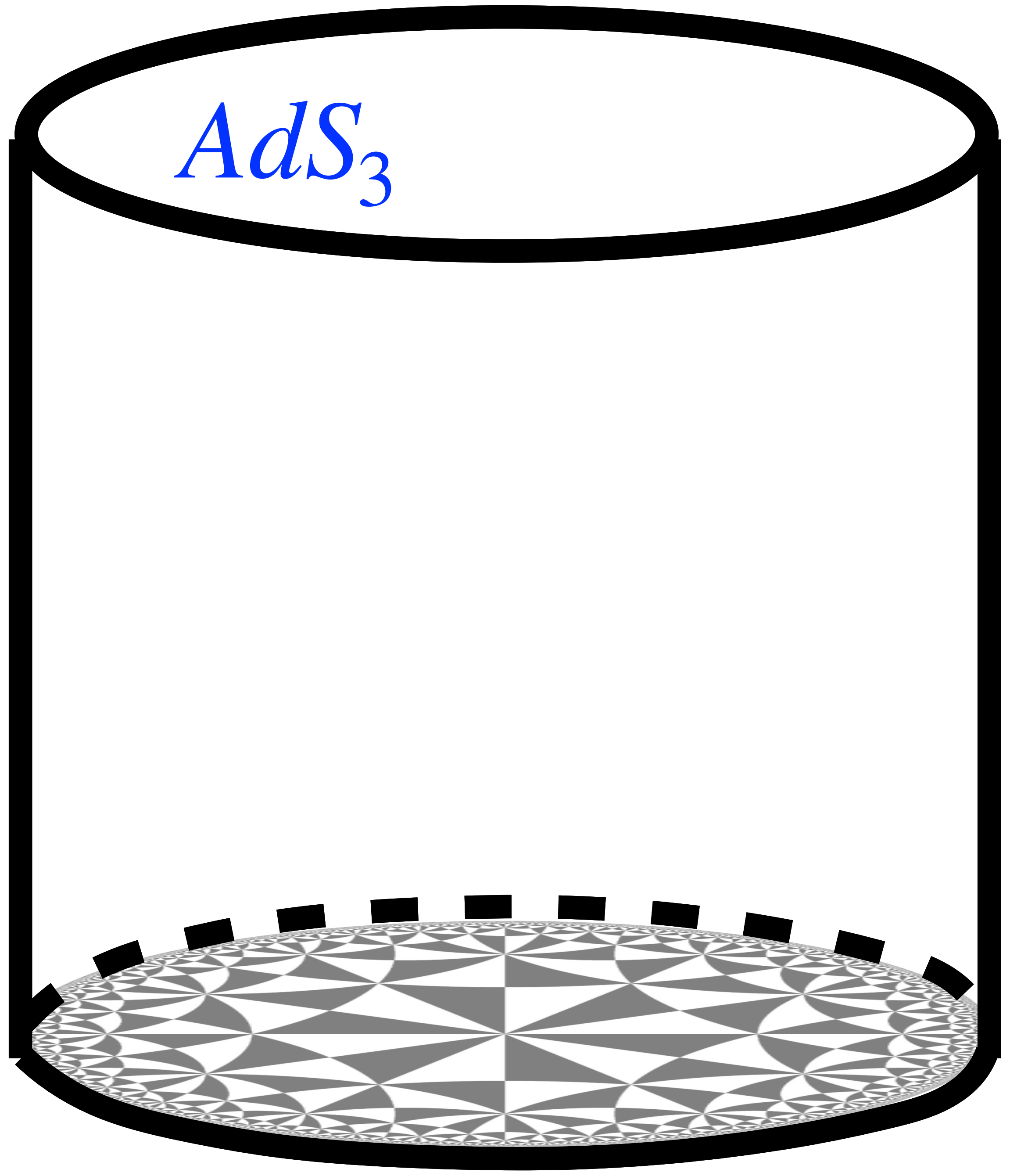
$\swarrow\!\!\nearrow$



From Dubovsky, Flauger, Gorbenko '13

Can one input perturbative YM into effective string theory and/or the S-matrix bootstrap?

Yang-Mills in AdS



- Basic idea: put large N Yang-Mills in AdS [Callan, Wilczek ’90; Aharony, Berkooz, Tong, Yankielowicz ’13; Ciccone, De Cesare, Di Pietro, Serone ’24, ’25; Di Pietro, Kousvos, Meineri, Piazza, Serone, Vichi ’25]

- This tames the IR without breaking any isometries

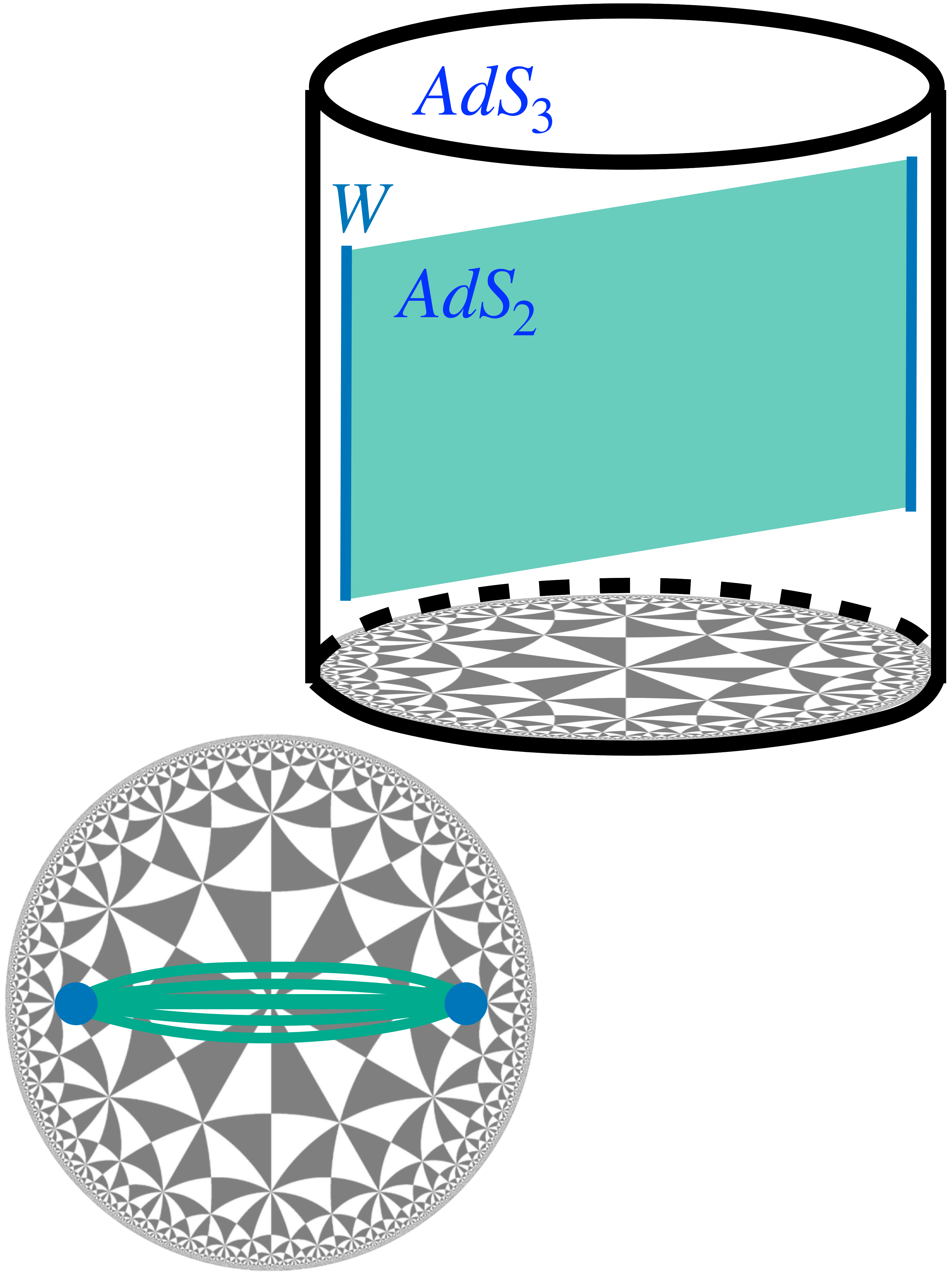
  - We can define $\lambda \equiv g_{\rm YM}^2 N \cdot R_{AdS}$ (or $\lambda = \Lambda_{\rm YM} R_{\rm AdS}$ in 4D), which interpolates between pYM regime ($\lambda = 0$) and flat space ($\lambda = \infty$).

- We get access to (most of) the tools of QFT in AdS and CFTs

  - Boundary correlators of gauge-invariant operators define a (non-local) CFT [Paulos, Penedones, Toledo, van Rees, Vieira ’16, …]

  - No holography, but we benefit from its example

Yang-Mills in AdS



- We impose Neumann BC on the gauge field
  - $A_i^{\text{bulk}}(x,z) \overset{z\to 0}{=} A_i(x) + \ldots, \quad F_{iz}\Big|_{z=0} = 0,\ F_{ij}\Big|_{z=0} \neq 0$
  - We conjecture that, with these BCs, the interpolation between small and large R_{AdS} is *smooth*. This is in contrast to Dirichlet BC
- The observable we focus on is the infinitely long flux tube, created by $W = \text{P}\exp\left(i\int A_\tau d\tau\right)$
  - The Wilson line preserves an SL(2,R) subset of isometries: conformal line defect on the boundary

Defect CFT in two descriptions

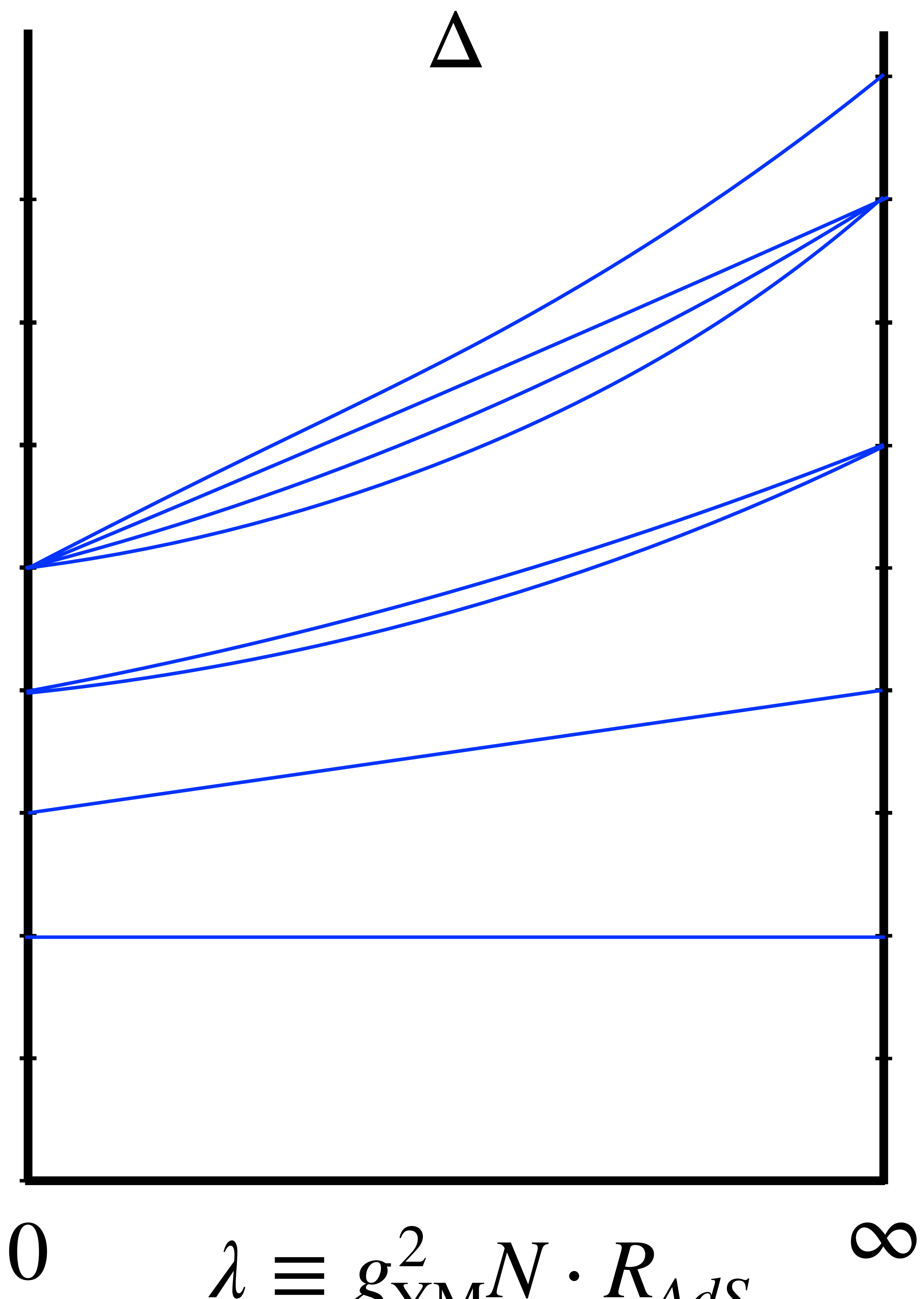
<u>Weakly coupled flux tube</u>

$$S_{\rm YM}[A_\mu] = \frac{1}{4g_{\rm YM}^2}\int d^3x\sqrt{g}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$$

$$\langle O(\tau_1)\ldots O(\tau_n)\rangle = \frac{\langle \mathrm{Tr\ P}e^{i\int A_\tau d\tau} O(\tau_1)\ldots O(\tau_n)\rangle}{\langle W\rangle}$$

Small AdS radius



$$\lambda \equiv g_{\rm YM}^2 N\cdot R_{AdS}$$

$$\lambda_s \equiv \ell_s^{-1}R_{AdS}$$

<u>Weakly coupled effective string</u>

$$S_{\rm EST}[X] = \int d^2\sigma\sqrt{h}\left[\ell_s^{-2} + \kappa\ell_s^2(K_{\alpha\beta}K^{\alpha\beta})^2 + \ldots\right]$$

$$Z[X_b] = \int_{X|_\partial = X_b} DXe^{-S[X]}$$

$$\langle X(\tau_1)\ldots X(\tau_n)\rangle = \frac{\delta^n Z[X_b]}{\delta X_b(\tau_1)\ldots\delta X_b(\tau_n)}$$

Large AdS radius

Outline

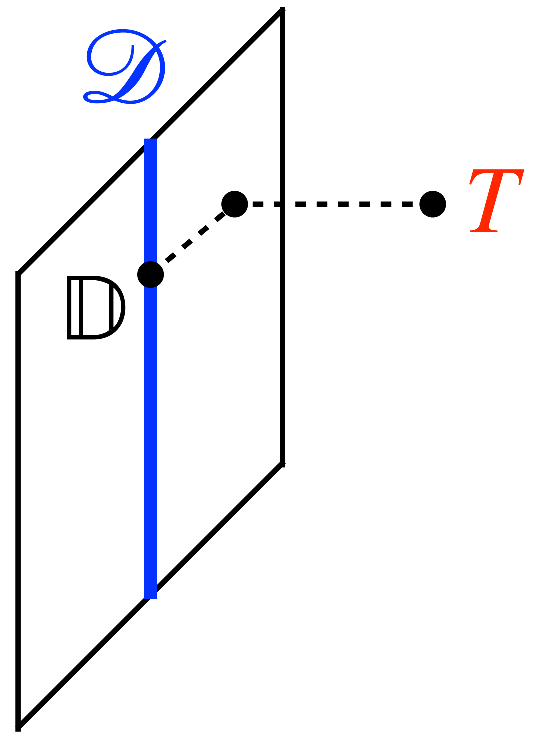
1. Introduction ✓
2. Gauge Theory Description ⟵

    [Based on 2508.08250 by B. Gabai, V. Gorbenko, J. Qiao]

3. Effective String Theory Description
4. Work in Progress and Future Directions

Displacement: existence and integral identities

1. A fundamental operator is the displacement: $\mathbb{D}_i = iF_{\tau i} = X_i$, with $\Delta_{\mathbb{D}} = 2$
2. The displacement <u>exists</u>. Not obvious for the WL on the bdy of AdS
   - In ordinary CFT:
     - $\partial_\mu T^{\mu i}(x, y) = \mathbb{D}^i(x)\delta(y)$
   - In QFT in AdS:
     - There is a $T_{\mu\nu}$ in the bulk, but not on the boundary
     - There are bulk-to-bdy and bdy-to-defect operator expansions, and one can show that $T_{zi}(z, x, y) = \sum c(z, y)\mathbb{D}_i(x) + \ldots$ [Qiao '26; Bianchi, de Sabbata, Meineri '26]



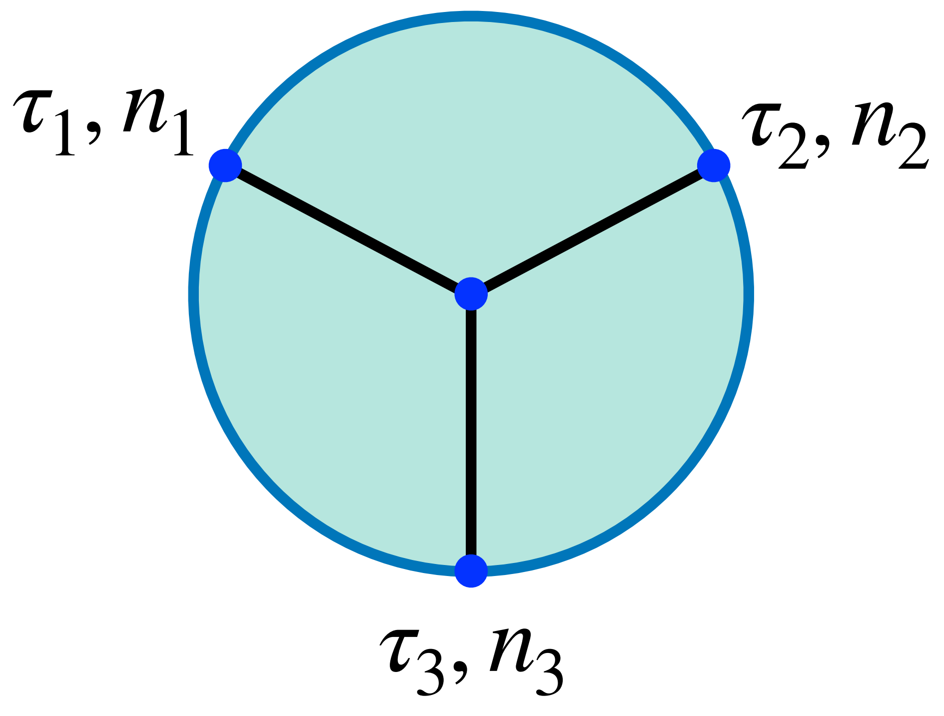
3. Correlators of the displacement satisfy <u>integral identities</u>, which are consequences of the symmetries broken by the line defect

$$0 = \int_0^1 d\chi K_\ell(\chi) G_{\mathbb{D}OOO}(\chi), \quad \int_0^1 d\chi L_\ell(\chi) G_{\mathbb{D}\mathbb{D}OO}(\chi) = \frac{\Delta_O}{c_D}$$

[Cavaglia, Gromov, Julius, Preti '22; Gabai, Sever, Zhong '25; Kong '25; Girault, Paulos, van Vliet '25; Drukker, Kong, Kravchuk '25]

CFT data at small AdS radius

[Gabai, Gorbenko, Qiao ’25]



- Method 1: perturbative Yang-Mills
    - One can compute OPE coeffs of single-Fs from a 3-pt diagram
- Method 2: integrated correlator bootstrap
    - The free four-point function naively does not satisfy the integral identities. Contributions of single-F operators are enhanced. This determines the anomalous dimensions and some OPE data of single-Fs
- These methods agree!

$$\gamma_{\partial_\perp^{n-1}F} = \frac{1}{2\pi}\left(\frac{2}{n+1} + (2\psi(n) + \gamma_E) - 1\right)\lambda + O(\lambda^2)$$

Outline

1. Introduction ✓
2. Gauge Theory Description ✓
3. Effective String Description ⟵

   [Based on 2602.16694 by B. Gabai, V. Gorbenko, BO]

4. Work in Progress and Future Directions

The displacement four point function

- We are interested in the branon four point function ($\mathbb{D} = X$)

$$\langle X(\tau_1)X(\tau_2)X(\tau_3)X(\tau_4)\rangle = \frac{1}{\tau_{12}^4\tau_{34}^4}G(\chi)$$

- We expand perturbatively in $\lambda_s \equiv R_{AdS}/\ell_s$:

$$G = \underbrace{G^{(0)}}_{\text{free}} + \underbrace{\lambda_s^{-2}G^{(1)}}_{\text{tree}} + \underbrace{\lambda_s^{-4}G^{(2)}}_{\text{1-loop}} + \underbrace{\lambda_s^{-6}G^{(3)}}_{\text{2-loop}} + \dots$$

- Via the conformal block expansion, we can read off OPE data of two-X operators:

$$G(\chi) = \sum_{O} \overset{c_{XXO}^2}{\overset{\downarrow}{a_O}} \mathfrak{f}_{\Delta_O}(\chi), \qquad \mathfrak{f}_\Delta(\chi) = \chi^\Delta {}_2F_1(\Delta, \Delta, 2\Delta, \chi)$$

$$\Delta_O = \Delta_O^{(0)} + \lambda_s^{-2}\gamma_O^{(1)} + \lambda_s^{-4}\gamma_O^{(2)} + \lambda_s^{-6}\gamma_O^{(3)} + \dots \qquad a_O = a_O^{(0)} + \lambda_s^{-2}a_O^{(1)} + \lambda_s^{-4}a_O^{(2)} + \lambda_s^{-6}a_O^{(3)} + \dots$$

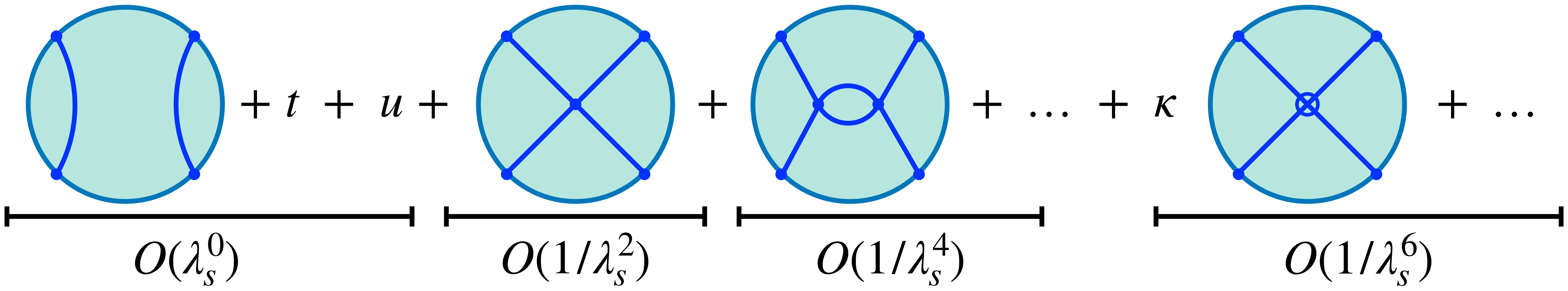
Tree level 4pt function from Witten diagrams

- Expand the effective action in powers of the branon field [Giombi, Roiban, Tseytlin '17]

$$S[X] = \int d^2\sigma\sqrt{h}\Big[\lambda^2 + \frac{1}{2}h^{\alpha\beta}\partial_\alpha X\partial_\beta X + X^2 - \frac{1}{8\lambda_s^2}(h^{\alpha\beta}\partial_\alpha X\partial_\beta X)^2 + \frac{1}{3\lambda_s^2}X^4$$
$$+\frac{\kappa}{\lambda_s^6}\left((\nabla_\alpha\nabla_\beta X\nabla^\alpha\nabla^\beta X)^2 - 4(h^{\alpha\beta}\partial_\alpha X\partial_\beta X)^2 + \frac{20}{3}X^4\right) + \ldots\Big]$$

- Expanding in $1/\lambda_s^2$:

$$\langle X(\tau_1)X(\tau_2)X(\tau_3)X(\tau_4)\rangle =$$



- Correlators and OPE data:

$$G^{(0)}(\chi) = 1 + \chi^4 + \frac{\chi^4}{(1-\chi)^4},$$

$$G^{(1)}(\chi) = r_1(\chi) + r_2(\chi)\log(1-\chi) + r_3(\chi)\log(\chi),$$

$$\gamma^{(1)}(\Delta^{(0)}, k) = -\frac{1}{8}c_{\Delta^{(0)}} + \frac{k}{4}$$

$\Delta^{(0)}$: $O(\lambda_s^0)$ dim.; k: # fields; $c_\Delta = \Delta(\Delta-1)$

One and two-loop: ansatz bootstrap

To get the four-point function at one and two loops, following [Ferrero, Ghosh, Sinha, Zahed '19; Ferrero, Meneghelli '21-'23, …; See also Mazac '18; Bonomi, Forini '24; Carmi, Ghosh, Sharma '24], we bootstrap:

1. Write an ansatz consisting of generalized logarithms: $\log(\chi), \log(1-\chi)$, $\mathrm{Li}_2(\chi)$, $\mathrm{Li}_3(\chi)$, …
2. Fix "highest logs" using lower order OPE data
3. Impose t-channel crossing and perturbative u-channel crossing: $(1-\chi)^4 G(\chi) = \chi^4 G(1-\chi)$ and $G\big(\chi/(\chi-1)\big) = G(\chi)|_{\log\chi \to \log|\chi|}$
4. Impose correct behavior in OPE limit
5. Fix remaining any undetermined coefficients:
    1. Set $\gamma_{X^2} = -1$ (freedom to define coupling)
    2. Integral constraints
    3. Impose correct behavior in Regge / flat space limit

$$2\delta(s) = -\pi \lim_{n,R\to\infty} \gamma_n, \quad \text{with } s \equiv \frac{4n^2}{R^2} \text{ fixed}.$$

[Paulos, Penedones, Toledo, van Rees, Vieira '16]

$$\chi^{-4} G(\chi)\Big|_{\chi=\frac{1}{2}+it} = \int_0^\infty dp_+ dp_- \psi(p_+)\psi(p_-) e^{2i\delta(s)}\Big|_{s=\frac{ip_+p_-t}{R^2}}$$

[Shenker, Stanford '14, …]

Two loop bootstrap

- We can repeat this procedure at two loops, but not beyond (without considerably more effort) due to operator mixing. Two particle: $c_{XX[X^2]_n} \sim 1/\lambda_s^0$, Multi particle: $c_{XX[X^4]_n} \sim 1/\lambda_s^2$.

- Now, there is one undetermined coefficient: $2\delta(s) = \frac{1}{4} s \ell_s^2 + \frac{\gamma_S}{768} \ell_s^6 s^3 + \ldots$

$$G^{(3)}(\chi) = r_1(\chi) + r_2(\chi)\log(1-\chi) + r_3(\chi)\log^2(\chi) + r_5(\chi)\log^3(1-\chi) + r_7(\chi)\mathrm{Li}_3(\chi) + r_8(\chi)S_{1,2}(\chi)$$

$$+\left[r_9(\chi)+r_{10}(\chi)\log(1-\chi) + r_{11}(\chi)\log^2(1-\chi) + r_{12}(\chi)\mathrm{Li}_2(\chi)\right]\log(\chi)$$

$$+\left[r_{13}(\chi) + r_{14}(\chi)\log(1-\chi)\right]\log^2(\chi) + r_{15}(\chi)\log^3(\chi)$$

Select OPE data

$$\Delta_X = 2, \qquad \text{(protected)},$$

$$\Delta_{X^2} = 4 - \varsigma, \qquad \text{(definition of coupling)},$$

$$\Delta_{[X^2]_2} = 6 - \frac{13}{4}\varsigma + \frac{9787}{8960}\varsigma^2 + \left[\frac{1994231}{7526400} - \frac{1485\pi^2\gamma_S}{512}\right]\varsigma^3 + O(\varsigma^4),$$

$$\frac{a_{[X^2]_4}\Delta_{[X^2]_4} + a_{[X^4]}\Delta_{[X^4]}}{a_{[X^2]_4} + a_{[X^4]}} = \Delta_{[X^2]_4} + O(\varsigma^3)$$

$$= 8 - \frac{13}{2}\varsigma + \frac{32713}{6720}\varsigma^2 - \left[\frac{641173}{1451520} + \frac{6105\pi^2\gamma_S}{256}\right]\varsigma^3 + O(\varsigma^4)$$

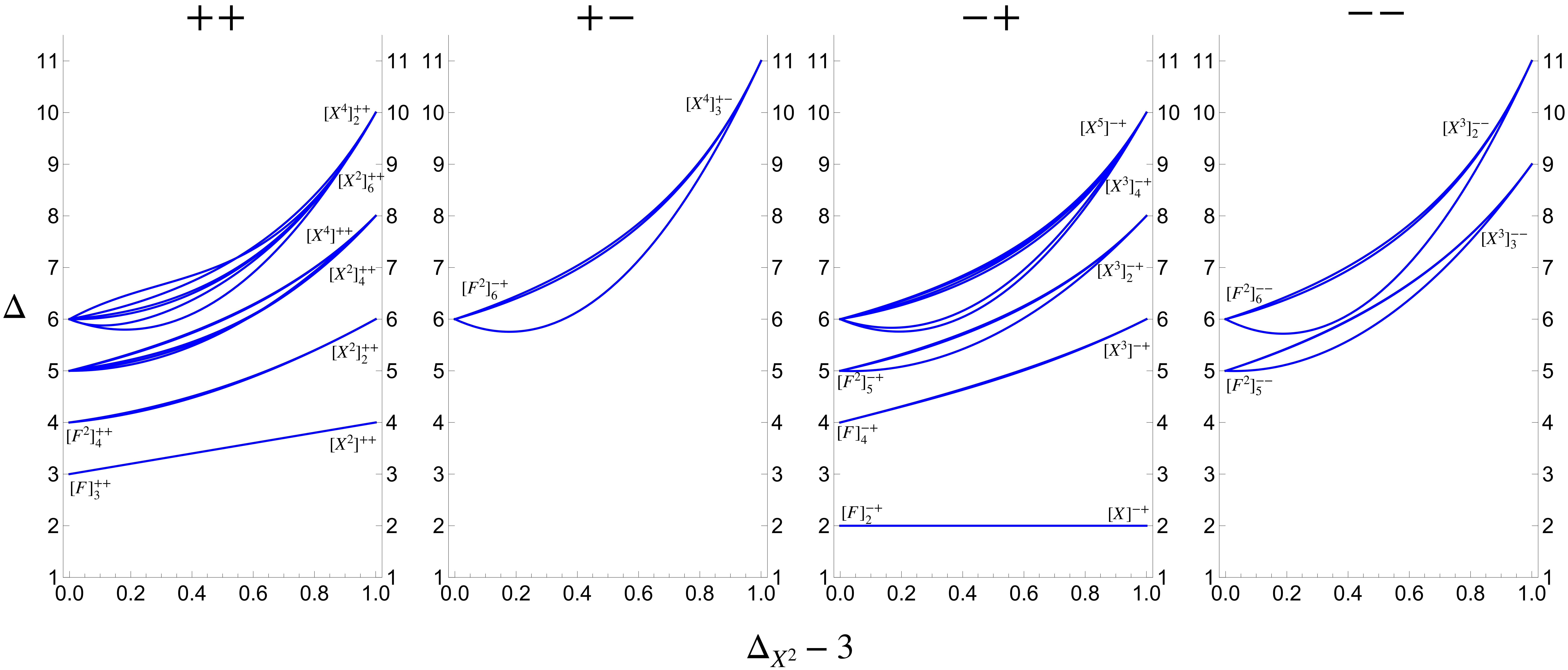
Select OPE data

$$a_{[X^2]} = 2 - \frac{251}{60}\varsigma + \frac{11011}{4800}\varsigma^2 + \left[\frac{1912081}{6912000} + \frac{\zeta(3)}{16} - \frac{165\pi^2\gamma_S}{448}\right]\varsigma^3 + O(\varsigma^4),$$

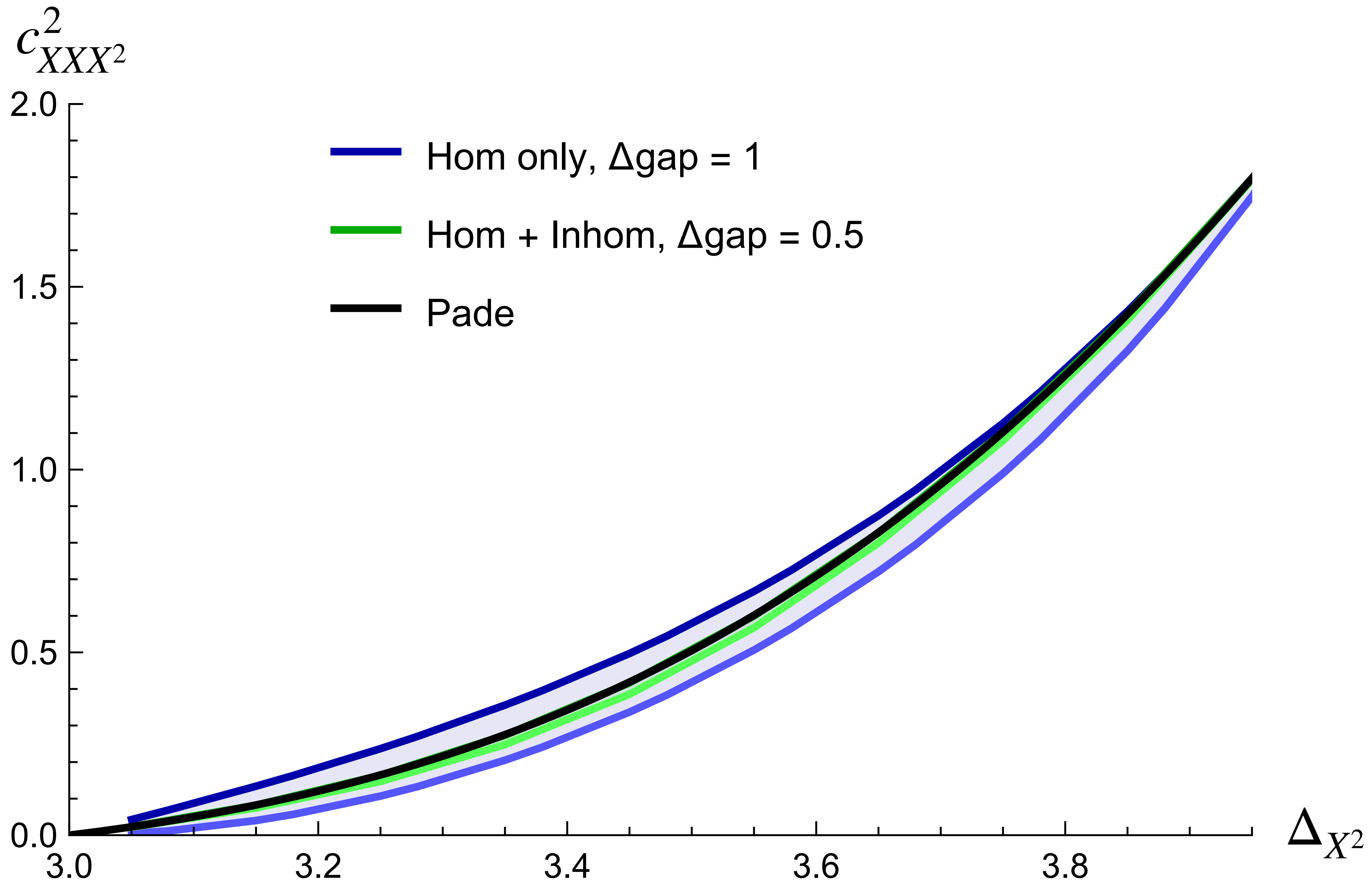
$$a_{[X^2]_2} = \frac{40}{9} - \frac{11455}{2268}\varsigma - \frac{1219193}{326592}\varsigma^2 + \left[\frac{438075464651}{92177326080} + \frac{695\zeta(3)}{144} - \frac{57695\pi^2\gamma_S}{3584}\right]\varsigma^3 + O(\varsigma^4),$$

$$a_{[X^2]_4} + a_{[X^4]} = a_{[X^2]_4} + O(\varsigma^2) = \frac{350}{143} + \frac{330295}{113256}\varsigma - \frac{1624478875}{194347296}\varsigma^2 + \left[-\frac{188283300608327}{12929536908288} + \frac{27125\zeta(3)}{2288} - \frac{130551925\pi^2\gamma_S}{5710848}\right]\varsigma^3 + O(\varsigma^4)$$

Padé interpolations



Comparison with bootstrap



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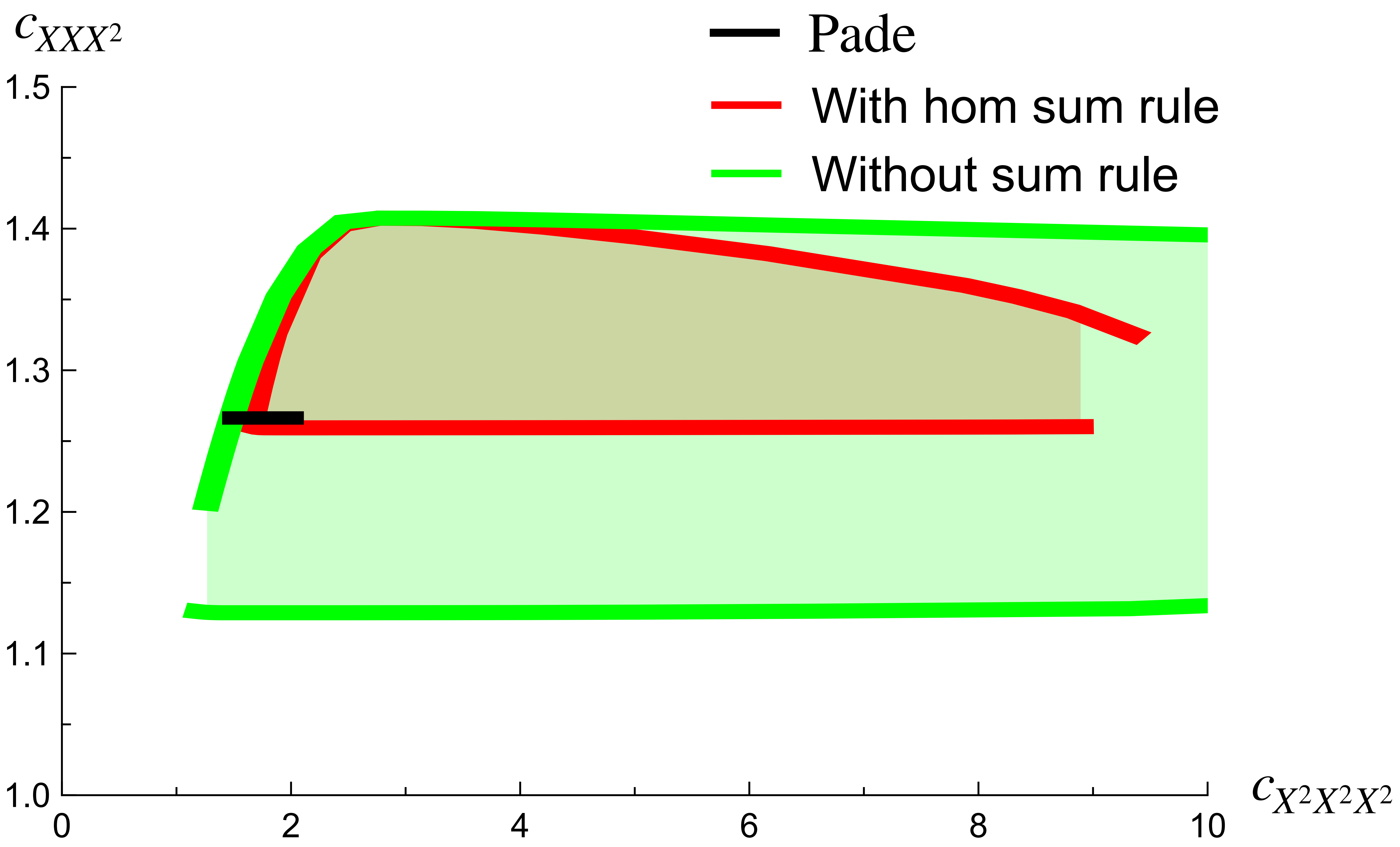
Comparison with bootstrap



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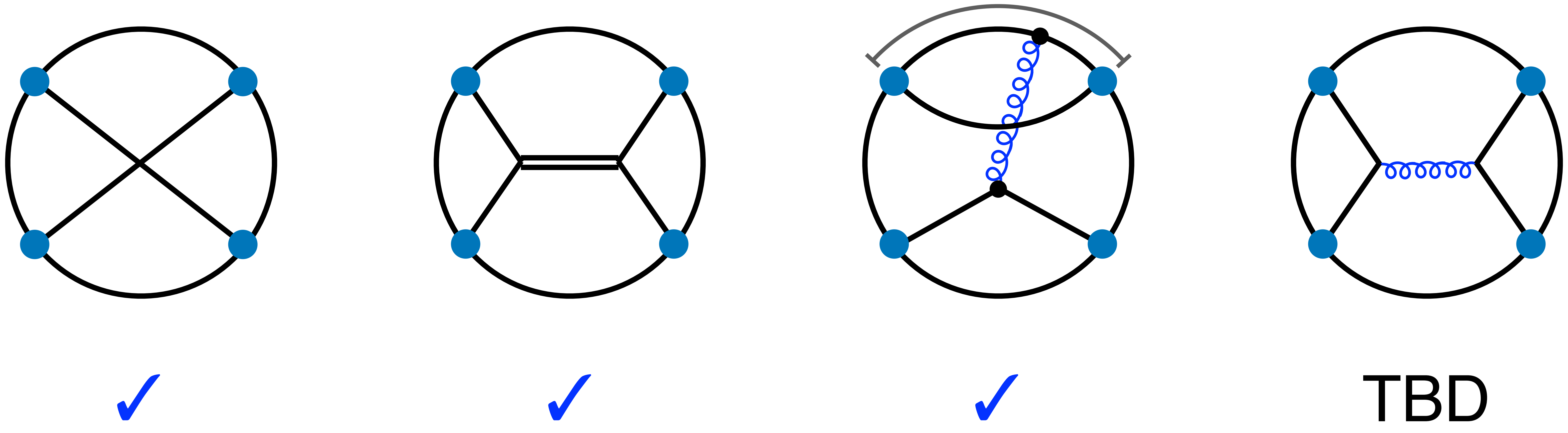
Summary

- Yang-Mills in AdS_3 with Neumann BCs is a nice set-up to study the confining flux tube analytically
- We can vary the AdS radius from small to large, interpolating between a Wilson line in weakly coupled Yang-Mills and a weakly coupled effective string. We conjecture the interpolation is smooth
- We have computed anomalous dimensions and OPE coefficients in both descriptions:
    - Perturbative Yang-Mills and the integrated-correlator bootstrap
    - Perturbative EST and the ansatz bootstrap
- Perturbative results look promising, especially when compared with numerical bootstrap!

WIP: extending pYM results

[also with Jonas Dujava (EPFL)]

- Knowing the slope at small R_{AdS} should significantly improve interpolations
- Four classes of diagrams:



- Analytic bootstrap for pYM?

WIP: flux tube in AdS4

[also with Jonas Dujava (EPFL)]

- EST
  - Tree-level bootstrap ✓ Two-loop bootstrap TBD
  - Three O(2) channels, each with a single tower of two-particle operators (have braiding symmetry and no-mixing at lowest orders)
- pYM
  - Potential simplifications: tree-level conformal symmetry + Yannie gauge
- Can we reproduce the Polchinski-Strominger term?
- Can we probe the world sheet axion?

WIP and future directions

[also with Jonas Dujava (EPFL)]

- Push numerical bootstrap further
  - Multi-correlator bootstrap: $\langle XXXX\rangle, \langle XXX^2X^2\rangle, \langle X^2X^2X^2X^2\rangle$ (in progress)
  - Use Padé results as extra input to get tighter bounds
- Can we say something about flat space physics?
  - Semi-analytic estimates of Wilson coefficients?
- Related set-ups:
  - Finite N, Wilson lines in higher representations, YM with adjoint matter
  - Null Wilson line and the spinning (GKP) string in Lorentzian AdS