

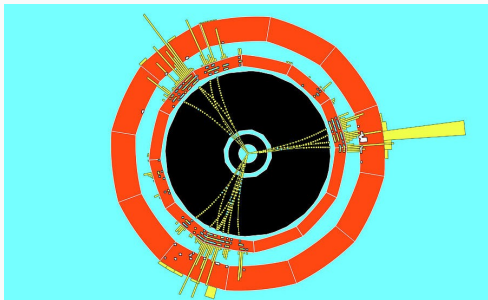
Positivity in energy correlators

Johan Henriksson (CERN)

19 May 2026

Simons Collaboration on Confinement and QCD Strings Workshop

Based on **work in progress** with A. Belin, N. Borak, R. Piron, and A. Zhiboedov

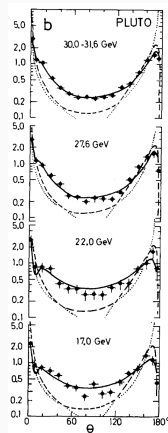


[ALEPH 1990's]

Observable: $\langle \Psi | \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_n) | \Psi \rangle$

$\mathcal{E}(\vec{n}) =$ energy flux in direction \vec{n}

Recently, lot of interest from theory and experiment!



[PLUTO 1981]

- Energy correlators are well-defined in almost any theory considered in high-energy physics
Collider physics, CFT, gravity

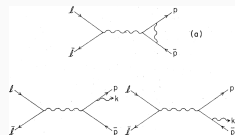
- Energy correlators are positive

pointwise $\mathcal{E}(\vec{n}) \geq 0$ & unitarity $C_J \geq 0$

ANEC $\langle \Psi | \mathcal{E}(\vec{n}_1) | \Psi \rangle \geq 0$ [Hofman, Maldacena 2008]

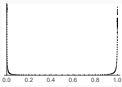
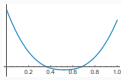
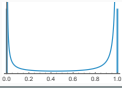
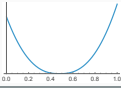
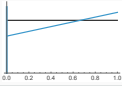
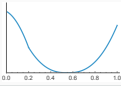
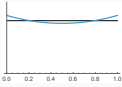
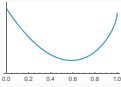
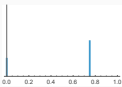
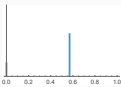
- Today: Towards a complete understanding of positivity

*Understand the **shape** of positive energy correlators*



[Basham, Brown, Lowell Ellis,
Love 1978]

Positive and not-so-positive two-point energy correlators

Allowed		Disallowed!	
	Experimentally measured (binned data)		Negative at some z_0
	1-loop QCD perturbative		Not attaining maximal value at $z = 0$
	Free particles at large multiplicity		C^k disc. at $z_0 \neq 0$ despite regular at $z = 0$
	Gravity in AdS		Counter example of theorem of [Devinatz 1959] [Gneiting 2011]
	Extremal solution maximizing C_3		Coefficient of $\delta(z - z_0)$ violating bound

1. Background
2. Positivity in energy correlators
3. Two-point energy correlators and atomic models
4. Constraints on real-world data
5. Summary

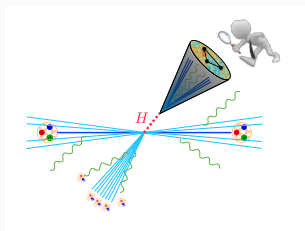
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Consider n -point energy correlators in state $|\Psi\rangle$

$$E^n C(\vec{n}_1, \dots, \vec{n}_n) = \langle \Psi | \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_n) | \Psi \rangle$$

Energy correlators are a rich set of observables

- Vary theory
- Vary state $|\Psi\rangle$
- Vary total energy Q
- Vary n
- Vary relative angles
- Replace \mathcal{E} (charge correlators, “on tracks,” etc.)



Nice review [Moult, Zhu 2025]

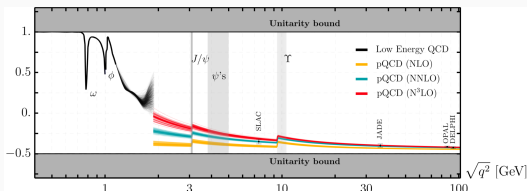
- One-point in rotationally-invariant state

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{Q}{4\pi}$$

- One-point in non-trivial state. Example: $|J_\mu\rangle$ [Hofman, Maldacena 2008]

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{Q}{4\pi} \left(1 - A(3 \cos^2 \theta - 1) \right) \geq 0 \quad -\frac{1}{2} \leq A \leq 1$$

Interpolates between free fermion and free boson. In QCD [Riembau, Son 2025]



- Two-point $\vec{n}_1 \cdot \vec{n}_2 = \cos \theta = 1 - 2z, \quad z \in [0, 1]$

$$EEC(z) = \int d^2 \vec{n}_1 d^2 \vec{n}_2 \delta(z - \frac{1}{2}(1 - \vec{n}_1 \cdot \vec{n}_2)) \langle \mathcal{E}(n_1) \mathcal{E}(n_2) \rangle = 1 + \sum_{J=1}^{\infty} C_J P_J(1-2z)$$

Consider momentum eigenstates $P^\mu |\Psi\rangle = q^\mu \delta^{\mu 0} |\Psi\rangle$, $q^\mu = (Q, 0)$

$$\langle \Psi' | \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_n) | \Psi \rangle = \delta(q - q') \langle \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_n) \rangle$$

and (unless specified) rotationally symmetric states with $Q = 1$.

- In a theory with particles $d\sigma_{\Psi \rightarrow N} \geq 0$

$$\langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \cdots \mathcal{E}(\vec{n}_k) \rangle = \frac{1}{\sigma} \sum_{N=2}^{\infty} \int d\sigma_{\Psi \rightarrow N} \prod_{i=1}^k \left(\sum_{j=1}^N E_j \delta\left(\frac{\vec{q}_j}{|\vec{q}_j|} - \vec{n}_i\right) \right)$$

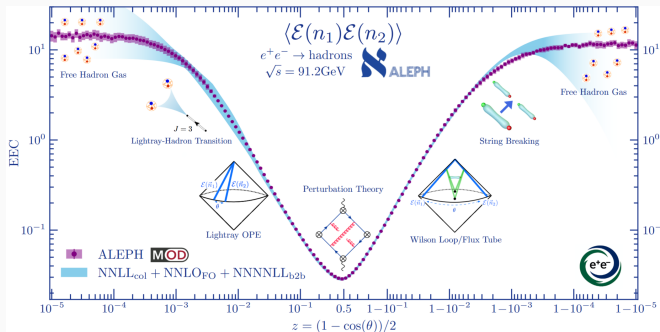
- In a general QFT (including CFT) [Korchemsky, Oderda, Sterman 1997]

$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} r^{d-2} \int_{-\infty}^{\infty} dt T_{0i} n^i(t, r\vec{n})$$

is the simplest **light-ray operator** [Balitsky, Braun 1989]

- Spectrum of light-ray operators in CFT \leftrightarrow analyticity in spin, [Kravchuk, Simmons-Duffin 2018], ...
- Light-ray OPE: $\mathcal{E}(\theta)\mathcal{E}(0) \sim \theta^{-2+\gamma(3)} \mathbb{O}_{J=3}$ [Hofman, Maldacena 2008], ...

Experimental two-point energy correlator



- Measurement in $e^+e^- \rightarrow \text{hadrons}$ at LEP, $Q = m_Z$
- Purple: Precise angular resolution using ALEPH archival data [Bossi et al 2025]
- Blue: Theoretical analysis [Jaarsma, Li, Moutl, Waalewijn, Zhu 2025]
- Probe of confinement at fixed energy: $\Lambda_{\text{eff}} \sim \theta Q$

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Main idea: $E^n C$ must fit into a “positive template”

Simplest conditions

- Energy and probability non-negative \Rightarrow pointwise positive (non-negative)
- Unitarity \Rightarrow lower bound $C_J \geq 0$ ($C_0 = 1$)

$$\begin{aligned}C_J &= \frac{2J+1}{2} \int_{-1}^1 d \cos \theta P_J(\cos \theta) \text{EEC} \left(\frac{1 - \cos \theta}{2} \right) \\&= \frac{1}{4\pi} \int d^2 \vec{n}_1 d^2 \vec{n}_2 \sum_{m=-J}^J Y_{Jm}^*(\vec{n}_1) \langle \Psi | \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) | \Psi \rangle Y_{Jm}(\vec{n}_2) \\&= \frac{1}{4\pi} \sum_{m=-J}^J \left| \int d^2 \vec{n} Y_{Jm}(\vec{n}) \mathcal{E}(\vec{n}) | \Psi \rangle \right|^2 \geq 0\end{aligned}$$

- Upper bound $C_J \leq 2J + 1$ [Fox, Wolfram 1978]. For even J saturated by

$$\text{EEC}(z) = \frac{1}{2} (\delta(z) + \delta(z - 1))$$

Positive hierarchy of energy correlators

Hierarchy of energy correlators satisfying

- $\langle \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_n) \rangle \geq 0$ (pointwise)
- $\langle \mathcal{E}(\vec{n}) \rangle = \frac{1}{4\pi}$
- Nested measure

$$\langle \mathcal{E}(\vec{n}_2) \cdots \mathcal{E}(\vec{n}_n) \rangle = \int d^2 \vec{n}_1 \langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \cdots \mathcal{E}(\vec{n}_n) \rangle$$

- Commutativity $[\mathcal{E}(\vec{n}_i), \mathcal{E}(\vec{n}_j)] = 0$

Hewitt–Savage theorem [1955] implies the existence of a fundamental probability measure μ (cross-section to give an event) on the space of probability measures (energy distribution per event)

$$d\mu \sim \frac{1}{\sigma} \sum_{N=2}^{\infty} \int d\sigma_{\Psi \rightarrow N}(\vec{n}_1, \dots, \vec{n}_N), \quad \vec{n}_i \in S^2$$

$E^n C$ are moments of this distribution (similar ideas in [Hofman, Maldacena 2008])

Exists even in a theory without particles

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Two-point energy correlators

- So far

$$\cos \theta = 1 - 2z$$

$$\text{EEC}(z) = 1 + \sum_{J=1}^{\infty} C_J P_J(1 - 2z), \quad 0 \leq C_J \leq 2J + 1$$

saturated for even J

- Non-trivial bounds for odd C_J [Dempsey, Karlsson, Pufu, Zahraee, Zhiboedov 2025] [Talk by Robin Karlsson]

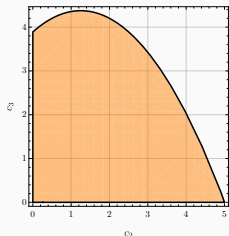
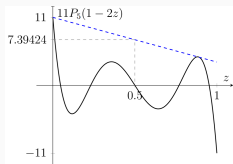
$$\max C_J \sim (2J + 1) \underbrace{\frac{1 - J_0(j_{1,1})}{2}}_{0.70138}, \quad \text{large } J \text{ odd}$$

- Primal insight: maximum $C_3 = 4.375$ attained by

$$\text{EEC}(z) = \frac{1}{3}(\delta(z) + 2\delta(z - 3/4))$$

corresponding to the fundamental measure

$$\mu = \mu_{\Delta} = \frac{1}{3}(\delta_{\phi_0} + \delta_{\phi_0 + 2\pi/3} + \delta_{\phi_0 + 4\pi/3})$$

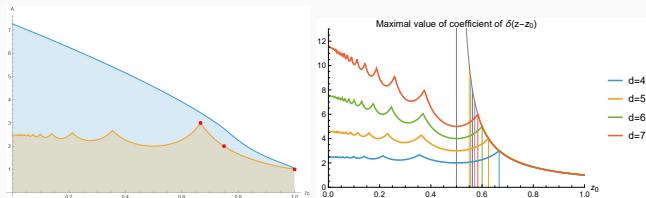


Two-atom model

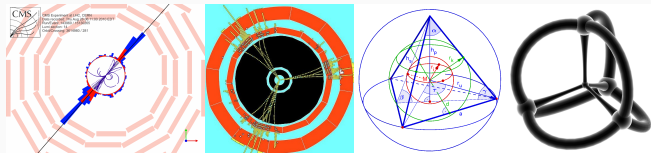
Relation to “atomic measures” [Winkler 1988]. Let us bound the simplest atomic model

$$\text{EEC}(z) \propto \delta(z) + A\delta(z - z_0)$$

Weak bound on A follows from $\langle \left(\int_{\delta\Omega_{\vec{n}_1}} d^2\vec{n} \mathcal{E}(\vec{n}) - \int_{\delta\Omega_{\vec{n}_2}} d^2\vec{n} \mathcal{E}(\vec{n}) \right)^2 \rangle \geq 0$



First peak in d dimensions: $(d - 1)$ -dimensional hypertetrahedron!



Positive-definite functions: $\sum_{i,j} c_i^* c_j f(x_i, x_j) \geq 0$ [Gneiting 2011] $\leftrightarrow C_J \geq 0$

The next result states conditions that prohibit membership in the class Ψ_1 , and therefore in any of the classes Ψ_d . Heuristically, the common motif can be paraphrased as follows:

If a positive definite function admits a certain degree of smoothness at the origin, it admits the same degree of smoothness everywhere.

Let $EEC(z) = f(z)$

1. $f(z) \leq f(0)$. Proof $P_J(1 - 2z) \leq P_J(1)$
2. If $f'(0) = 0$, then $f(z) = 1$ for all z
3. Bounds on contact terms
If $f(z) = A\delta(z) + \sum_i B_i\delta(z - z_i)$, then the B_i are bounded
4. Consequence of theorem 8a of [Gneiting 2011]
If $f(z)$ is regular at $z = 0$ but has C^k discontinuity elsewhere, then it is not positive-definite
5. Theorem 8d of [Gneiting 2011], from [Devinatz 1959]
If $f(\frac{1-\cos\theta}{2})$ extends to an even analytic function in θ of period $\neq 2\pi$, then it is not positive-definite

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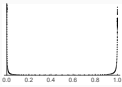
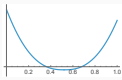
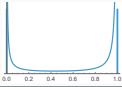
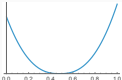
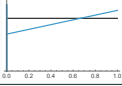
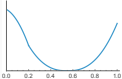
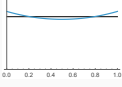
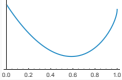


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	1-loop QCD perturbative		Not attaining maximal value at $z = 0$
	Free particles at large multiplicity		C^k disc. at $z_0 \neq 0$ despite regular at $z = 0$
	Planar $\mathcal{N} = 4$ SYM at strong coupling		Counter example of theorem of [Devinatz 1959] [Gneiting 2011]
	Extremal solution maximizing C_3		Coefficient of $\delta(z - z_0)$ violating bound

Restoring positivity

Assume a theoretical interpolation model: 1) $A + Bz^2$, 2) $C/z^{1-\gamma/2}$, 3) $D \text{ EEC}(z)_{1\text{-loop QCD}}$, 4) $E + F(1-z)^2$. (7 parameters)

Impose normalisation and $\mathcal{C}^0, \mathcal{C}^1$ continuity at $z = 0.01, 0.1, 0.88$

$$C_0 = 1, \quad C_1 = 0.00448, \quad C_2 = 0.3662, \quad C_3 = 0.094, \quad C_4 = 0.1440, \dots$$

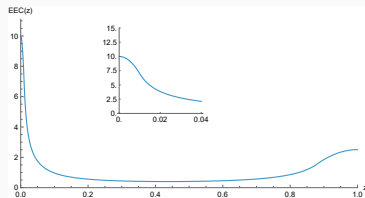
Violates some of our conditions!

Indeed: Find $C_{18} = -0.00163 < 0$ (!)

- Rescue 1: add contact term $A_\delta \delta(z)$

$$C_J \rightarrow \frac{1}{1 + A_\delta} (C_J + (2J + 1)A_\delta)$$

- Rescue 2: Force C_J positive for $J \geq 18$ (new convergent sum)
Result: Builds a peak at small z



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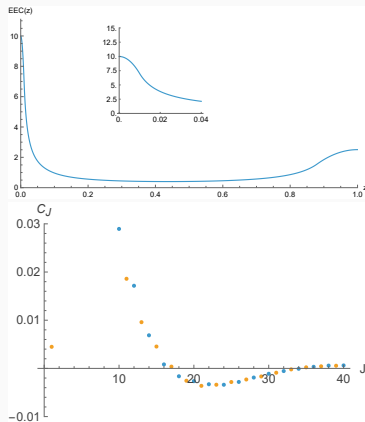
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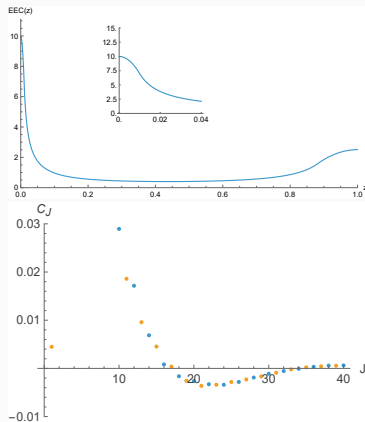
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Result: Builds a peak at small z



Restoring positivity

Assume a theoretical interpolation model: 1) $A + Bz^2$, 2) $C/z^{1-\gamma/2}$, 3) $D \text{EEC}(z)_{1\text{-loop QCD}}$, 4) $E + F(1-z)^2$. (7 parameters)

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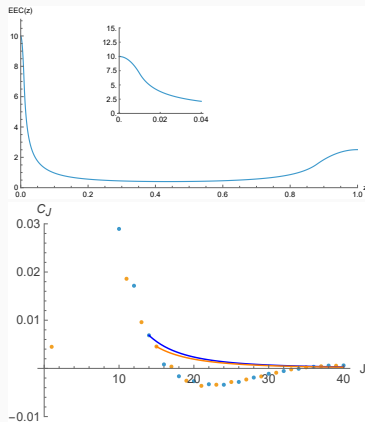
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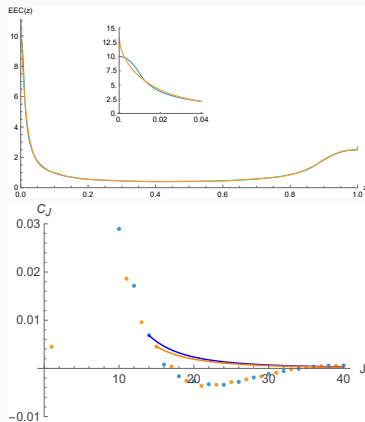
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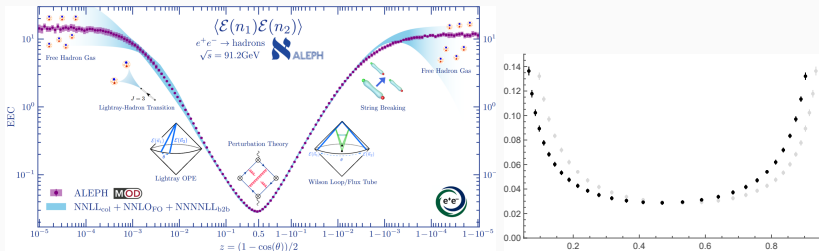
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1. Background
2. Positivity in energy correlators
3. Two-point energy correlators and atomic models
4. **Constraints on real-world data**
5. Summary

Constraining contact term



Measurements on tracks (only $q \neq 0$ particles) [Bossi et al 2025]

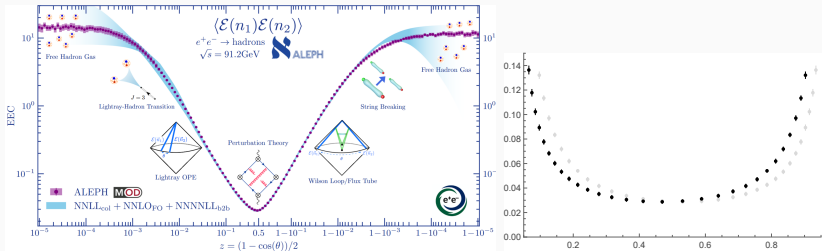
Measured distribution is skewed to the right

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and impose $C_J = (2J + 1)A_\delta + A_J \geq 0$

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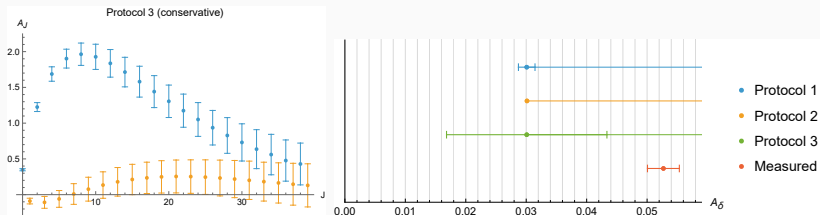
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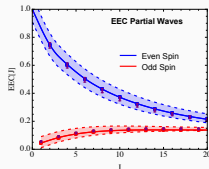
Strategy: Add A_δ and impose $C_J = (2J+1)A_\delta + A_J \geq 0$



$$A_\delta \gtrsim 0.030$$

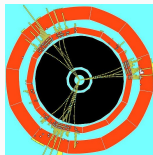
$$\nu_{q \neq 0, \text{eff}} = \sqrt{C_0} \gtrsim 0.63,$$

$$N_{q \neq 0, \text{eff}} = \frac{\sqrt{C_0}}{A_\delta} \lesssim 21.2$$



[Jaarsma, Li, Moutl, Waalewijn, Zhu 2025]

- Energy correlators are well-defined observables in almost any theory considered in high-energy physics
- They must fit in a consistent hierarchy deriving from a (hidden) probability distribution
- We derived general bounds on partial-wave coefficients C_J and shape of two-point energy correlators
- For experimental data, our bounds imply a lower bound on contact term
- For CFT, constraints on OPE coefficients and a species bound [\[In progress\]](#)



$$E^n C(\vec{n}_1, \dots, \vec{n}_n) = \langle \Psi | \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_n) | \Psi \rangle$$

Thank you for your attention!

