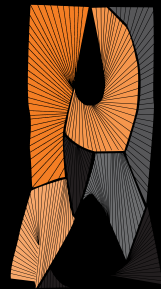


# Imprints of Asymptotic Freedom on Confining Strings



**Jan Albert**

Princeton Center for  
Theoretical Science

---

**Based on 2602.15097 with  
Alexandre Homrich**

# Setup

We consider pure  $SU(N)$  Yang-Mills theory in  $D = 3, 4$  dimensions in 't Hooft's large  $N$  limit,

$$\mathcal{L} = -\frac{1}{2 g_{\text{YM}}^2} \text{Tr}(F^2). \quad \left\{ \begin{array}{l} N \rightarrow \infty \\ \lambda \equiv g_{\text{YM}}^2 N \sim N^0 \end{array} \right.$$

# Setup

We consider pure  $SU(N)$  Yang-Mills theory in  $D = 3,4$  dimensions in 't Hooft's large  $N$  limit,

$$\mathcal{L} = -\frac{1}{2 g_{\text{YM}}^2} \text{Tr}(F^2). \quad \left\{ \begin{array}{l} N \rightarrow \infty \\ \lambda \equiv g_{\text{YM}}^2 N \sim N^0 \end{array} \right.$$

We expect it to be described by a theory of (confining) strings.

**Low energies:**

Effective string theory

**High energies:**

Asymptotic freedom?

# Setup

We consider pure  $SU(N)$  Yang-Mills theory in  $D = 3,4$  dimensions in 't Hooft's large  $N$  limit,

$$\mathcal{L} = -\frac{1}{2 g_{\text{YM}}^2} \text{Tr}(F^2). \quad \left\{ \begin{array}{l} N \rightarrow \infty \\ \lambda \equiv g_{\text{YM}}^2 N \sim N^0 \end{array} \right.$$

We expect it to be described by a theory of (confining) strings.

**Low energies:**

Effective string theory

**High energies:**

Asymptotic freedom?

**Outline:**

- **Review of effective string theory**
- **Observable for asymptotic freedom**
- **Imprints on string spectrum**
- **Imprints on worldsheet dynamics**

# Effective String Theory

Long **flux tubes** can be described with a worldsheet EFT:

[Nambu, Goto, Polyakov, Polchinski, Strominger, Dubovsky, Flauger, Gorbenko, Aharony, Komargodski, ...]

$$S_{EST} = - \int d^2\sigma \sqrt{-\det \partial X \partial \bar{X}} \left( \frac{1}{\ell_s^2} + \gamma_3 \ell_s^2 K(X)^4 + \dots \right)$$

**Wilson coefficients** capture the information of the underlying theory.

# Effective String Theory

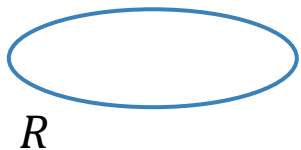
Long **flux tubes** can be described with a worldsheet EFT:

[Nambu, Goto, Polyakov, Polchinski, Strominger, Dubovsky, Flauger, Gorbenko, Aharony, Komargodski, ...]

$$S_{EST} = - \int d^2\sigma \sqrt{-\det \partial X \partial \bar{X}} \left( \frac{1}{\ell_s^2} + \underline{\gamma_3 \ell_s^2 K(X)^4} + \dots \right)$$

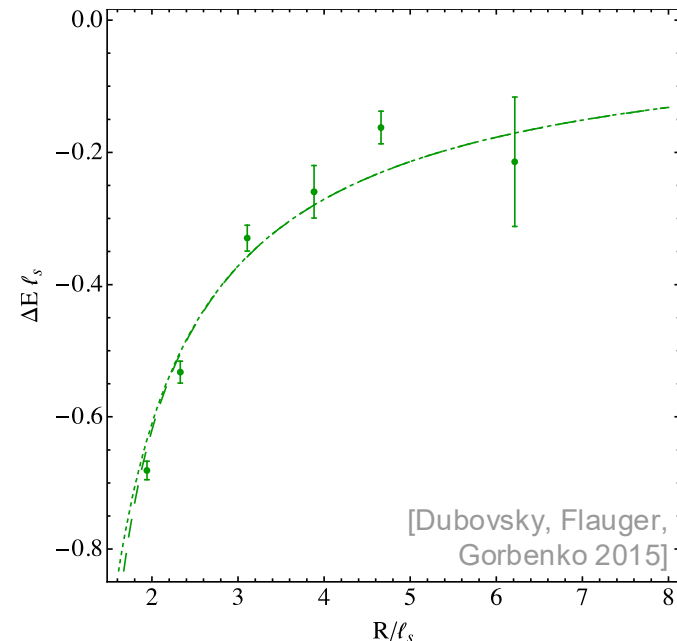
**Wilson coefficients** capture the information of the underlying theory.

They can be measured from the **ground state energy** of a closed string in  $\mathbb{R}^{D-1} \times S^1_R$ .



$$E_0(R) = \sqrt{\frac{R^2}{\ell_s^4} - \frac{4\pi(D-2)}{\ell_s^2} \frac{1}{12}} + \underbrace{\gamma_3 \frac{\ell_s^6}{R^7} + O(1/R^9)}_{\text{Corrections to Nambu-Goto result}}$$

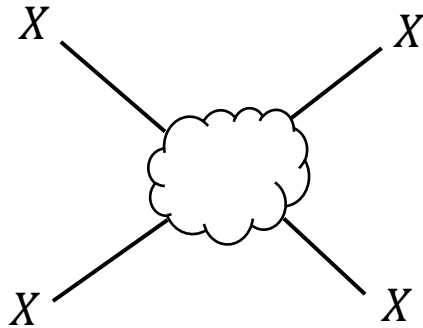
Corrections to  
Nambu-Goto result



# S-matrix (bootstrap)

The Wilson coefficients are also encoded in the low-energy limit of the Goldstone **S-matrix**. ( $D = 3, \ell_s = 1$ )

[Dubovsky, Flauger, Gorbenko 2012]



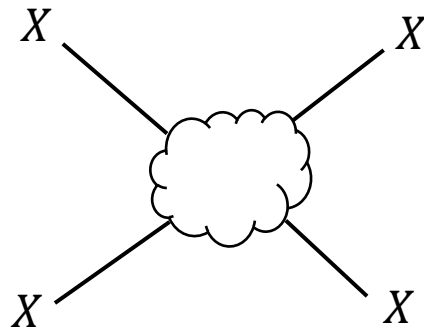
$$S(s) = \exp i \left( \underbrace{\frac{s}{4} + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7}_{\text{Elastic}} + \underbrace{i \gamma_8 s^8 + \dots}_{\text{Particle production}} \right)$$

Elastic

Particle production

# S-matrix (bootstrap)

The Wilson coefficients are also encoded in the low-energy limit of the Goldstone **S-matrix**. ( $D = 3, \ell_s = 1$ )



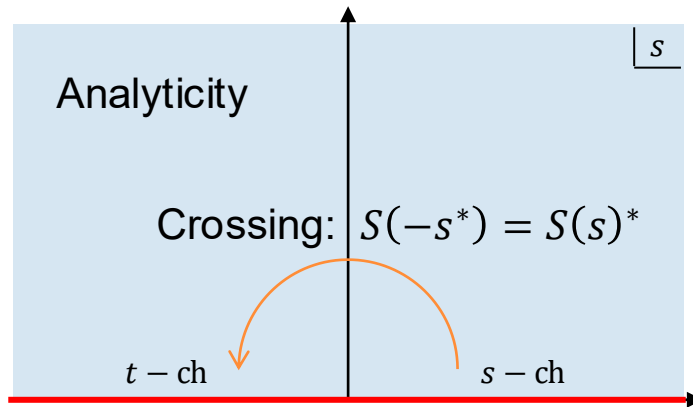
[Dubovsky, Flauger, Gorbenko 2012]

$$S(s) = \exp i \left( \underbrace{\frac{s}{4} + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7}_{\text{Elastic}} + i \underbrace{\gamma_8 s^8 + \dots}_{\text{Particle production}} \right)$$

Elastic

Particle production

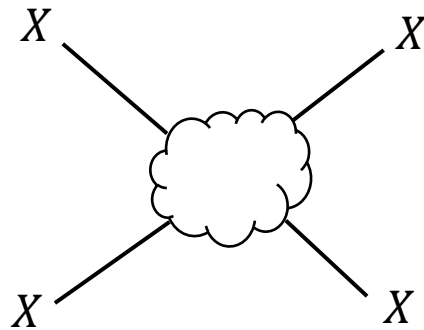
Consistency conditions for  $S(s)$ :



$$\text{Unitarity: } |S(\mu^2)| \leq 1$$

# S-matrix (bootstrap)

The Wilson coefficients are also encoded in the low-energy limit of the Goldstone **S-matrix**. ( $D = 3, \ell_s = 1$ )



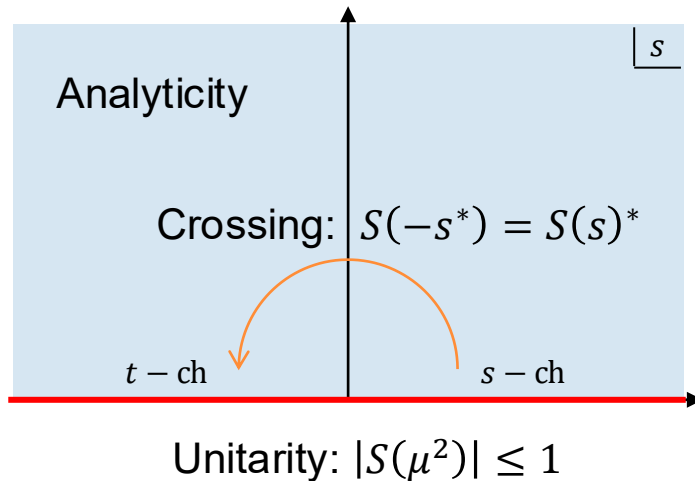
[Dubovsky, Flauger, Gorbenko 2012]

$$S(s) = \exp i \left( \underbrace{\frac{s}{4} + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7}_{\text{Elastic}} + i \underbrace{\gamma_8 s^8 + \dots}_{\text{Particle production}} \right)$$

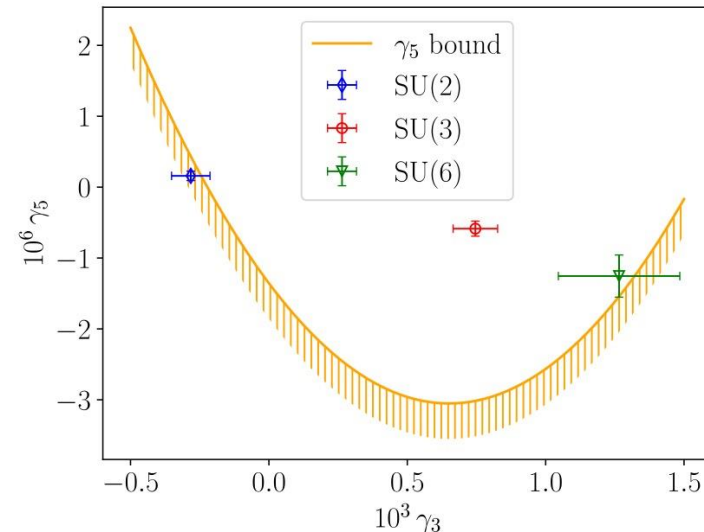
Elastic

Particle production

Consistency conditions for  $S(s)$ :



Bounds on Wilson coefficients:



[Caselle, Magnoli, Nada, Panero, Panfalone, Verzichelli 2024]

# Asymptotic Freedom

Finding an **observable** of the flux tube theory controlled by asymptotic freedom is hard:

1) Requires clear **map** world-sheet  $\leftrightarrow$  space-time.

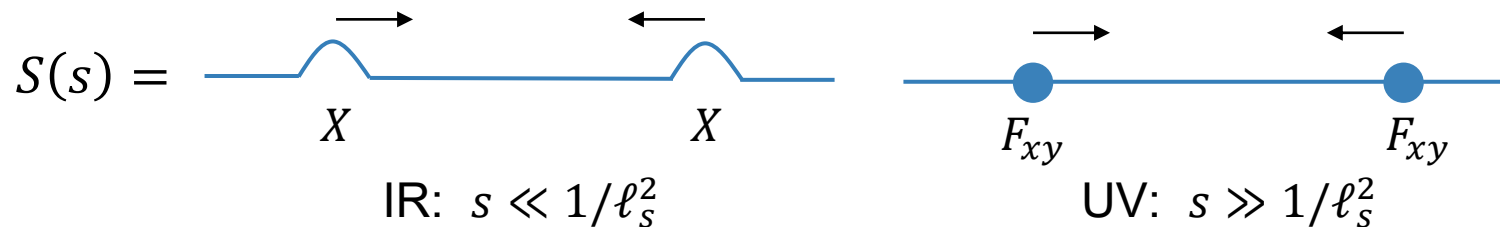
2) Requires **controllable limit** to the UV.

# Asymptotic Freedom

Finding an **observable** of the flux tube theory controlled by asymptotic freedom is hard:

1) Requires clear **map** world-sheet  $\leftrightarrow$  space-time.

e.g. the  $S$ -matrix? [Dubovsky 2019]



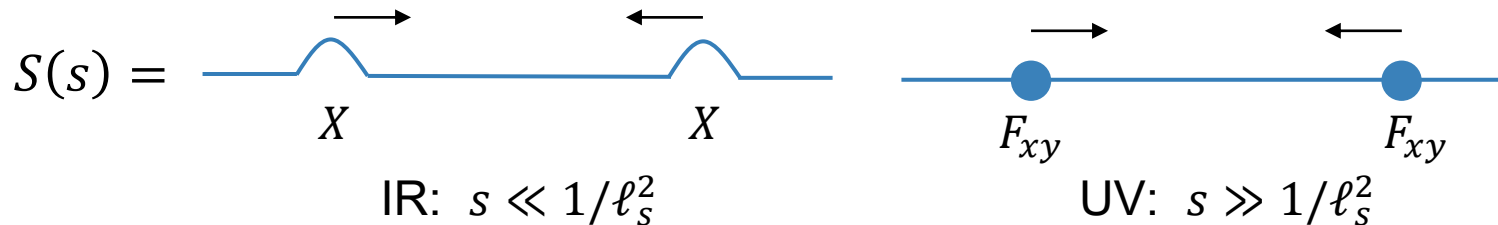
2) Requires **controllable limit** to the UV.

# Asymptotic Freedom

Finding an **observable** of the flux tube theory controlled by asymptotic freedom is hard:

1) Requires clear **map** world-sheet  $\leftrightarrow$  space-time.

e.g. the  $S$ -matrix? [Dubovsky 2019]

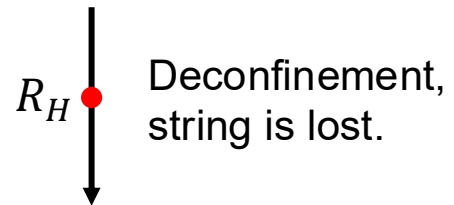


2) Requires **controllable limit** to the UV.

e.g. the torus partition function?



IR:  $R \gg \ell_s$

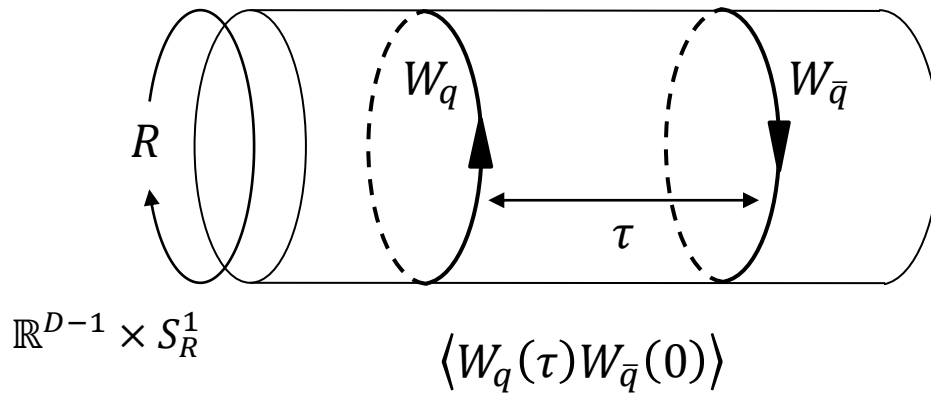


UV:  $R \ll \ell_s$

# Asymptotic Freedom

There is one such observable! The **Polyakov loop correlator**. ( $N \rightarrow \infty$ )

See also: [Gabai, Gorbenko,  
Offertaler, Qiao 2025-26]

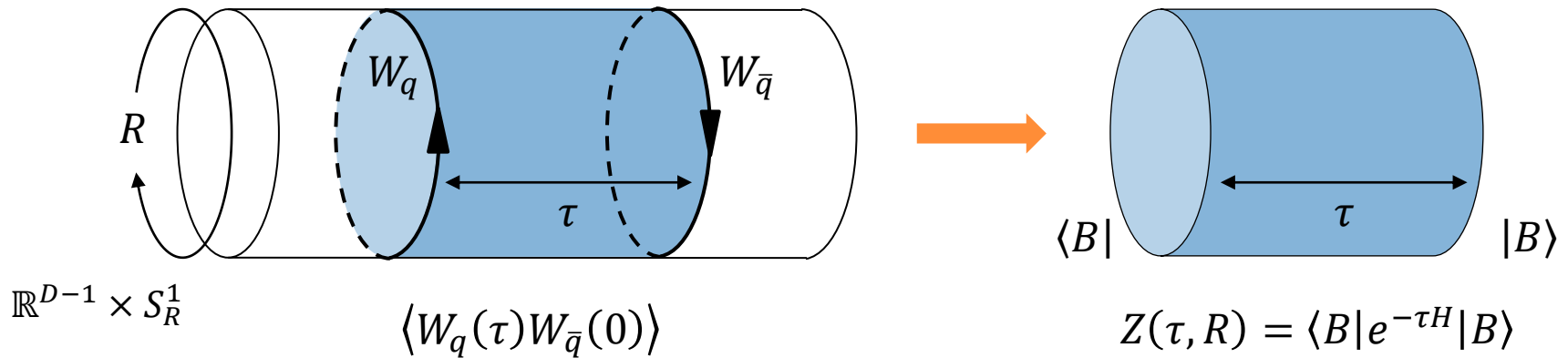


# Asymptotic Freedom

There is one such observable! The **Polyakov loop correlator**. ( $N \rightarrow \infty$ )

1) Map space-time  $\rightarrow$  world-sheet:

See also: [Gabai, Gorbenko, Offertaler, Qiao 2025-26]

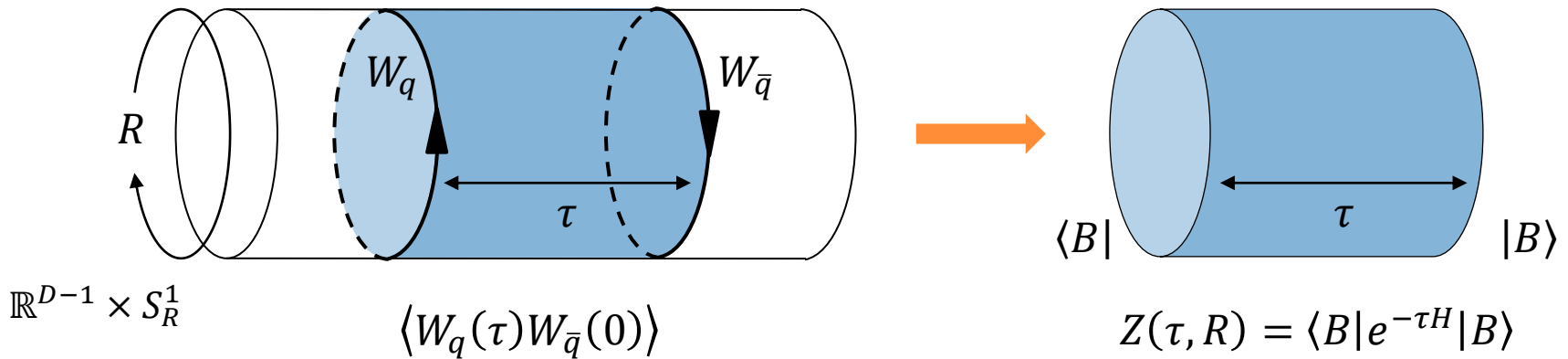


# Asymptotic Freedom

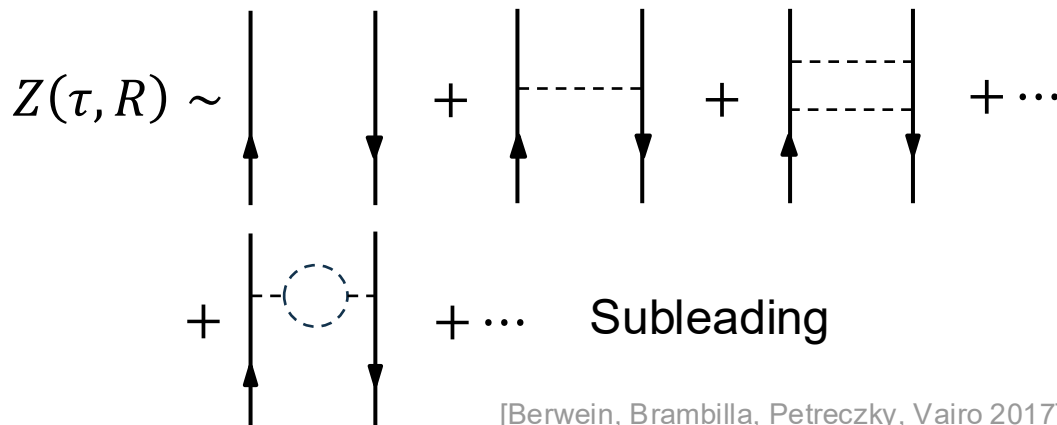
There is one such observable! The **Polyakov loop correlator**. ( $N \rightarrow \infty$ )

1) Map space-time  $\rightarrow$  world-sheet:

See also: [Gabai, Gorbenko, Offertaler, Qiao 2025-26]



2) Limit to the UV:  $\tau \ll \ell_s$

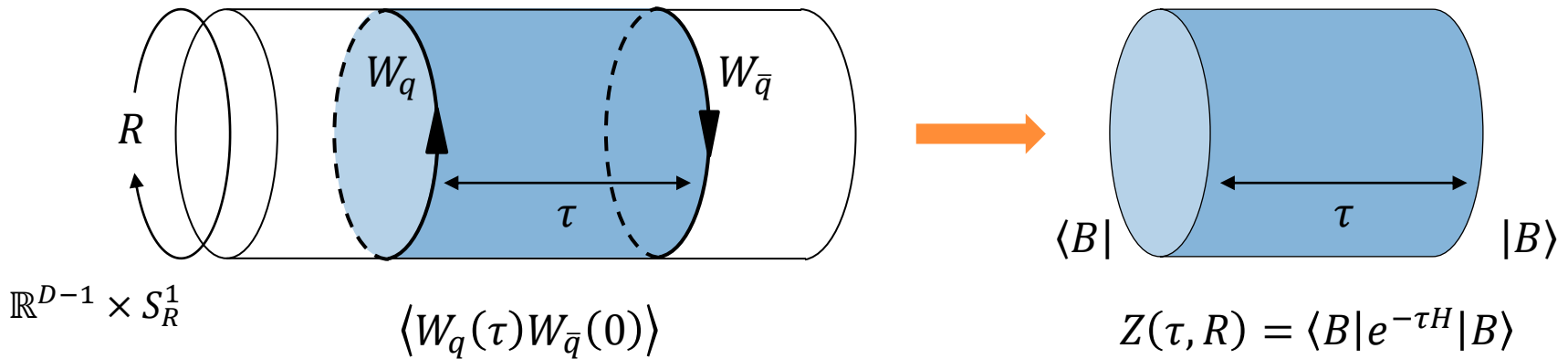


# Asymptotic Freedom

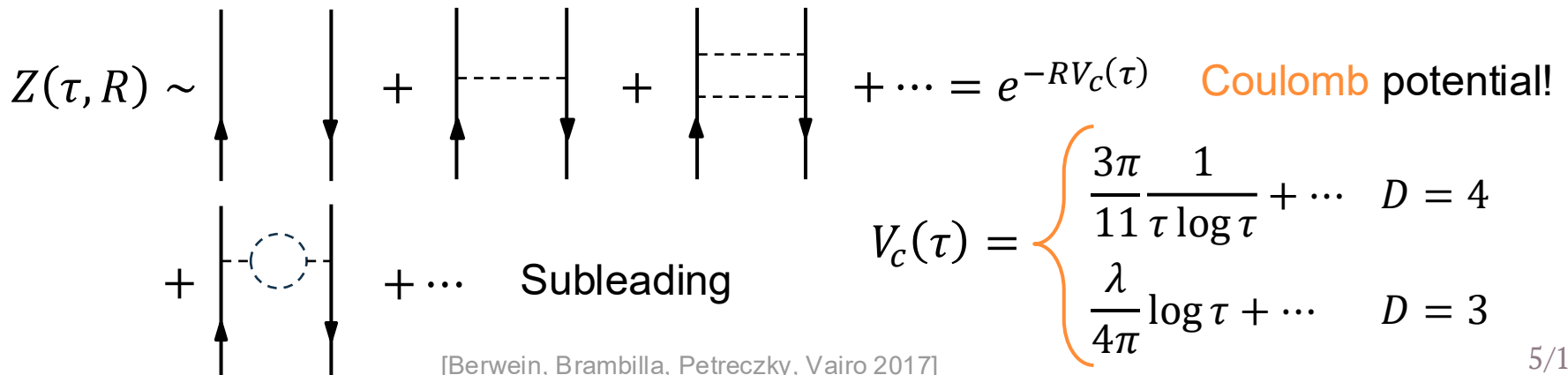
There is one such observable! The **Polyakov loop correlator**. ( $N \rightarrow \infty$ )

1) Map space-time  $\rightarrow$  world-sheet:

See also: [Gabai, Gorbenko, Offertaler, Qiao 2025-26]

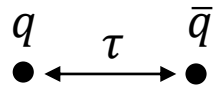


2) Limit to the UV:  $\tau \ll \ell_s$



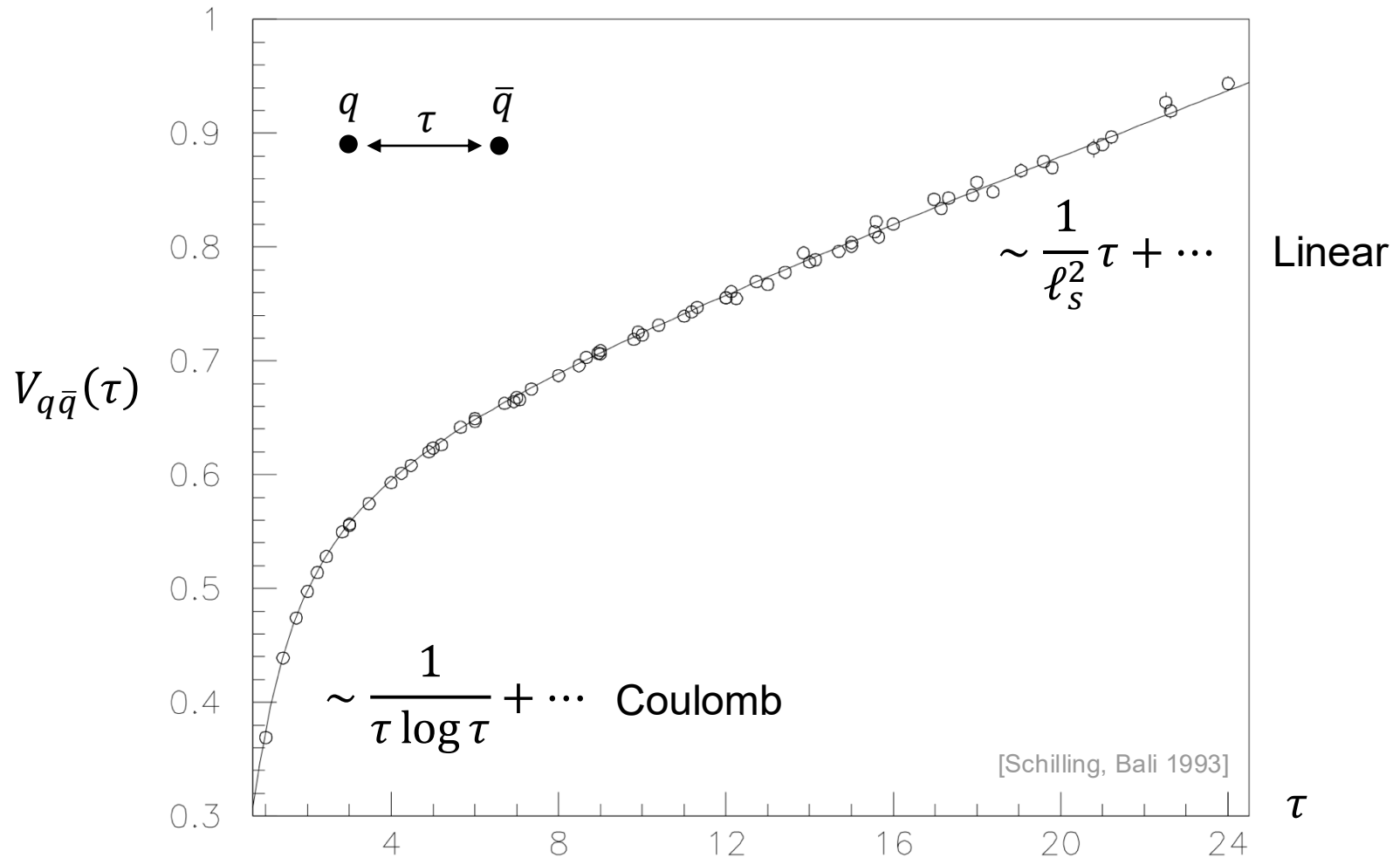
# Quark-antiquark V

As  $R \rightarrow \infty$ , we single out the **open string** ground state,  $Z \rightarrow e^{-RV_{q\bar{q}}(\tau)}$ .



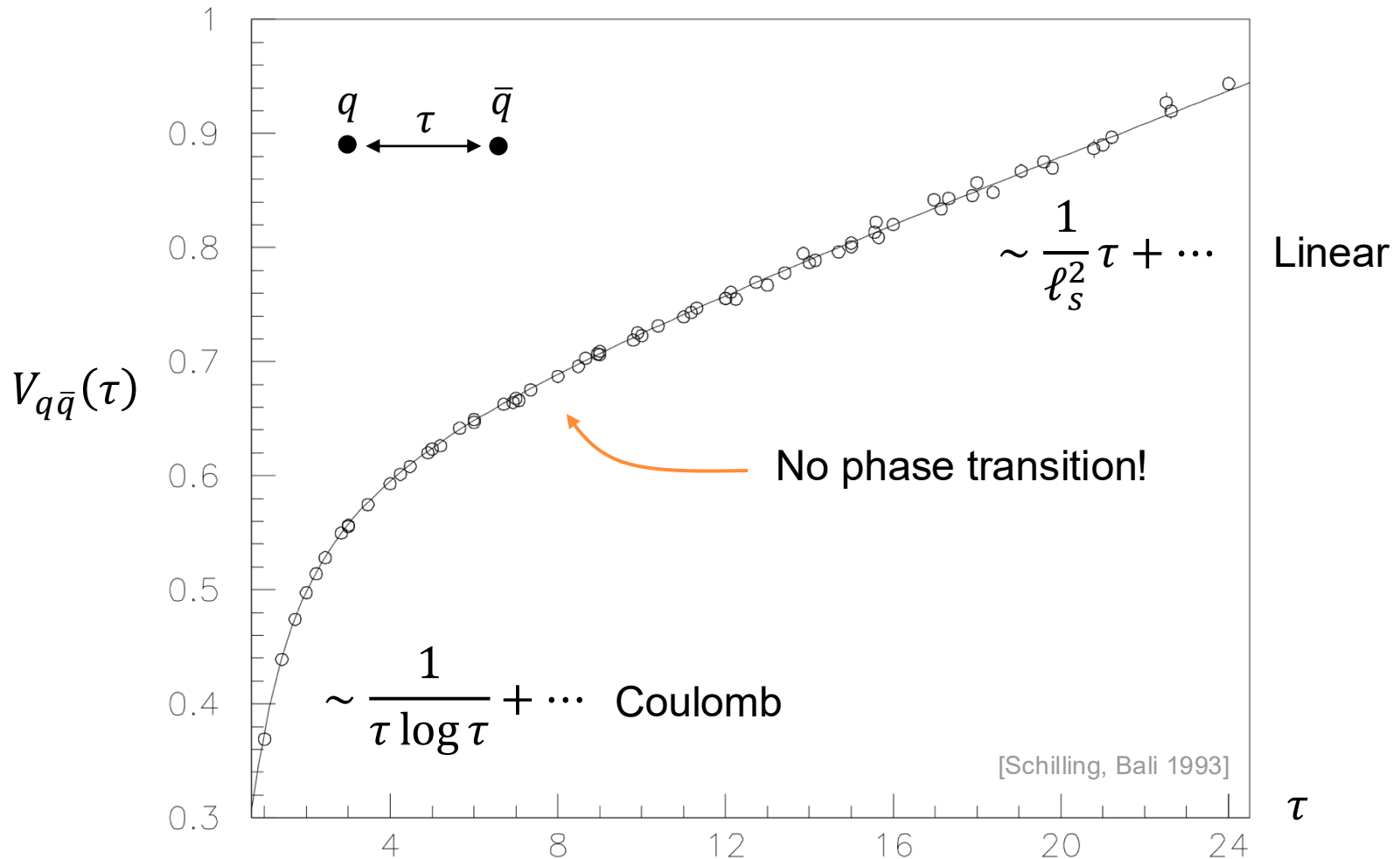
# Quark-antiquark V

As  $R \rightarrow \infty$ , we single out the **open string** ground state,  $Z \rightarrow e^{-RV_{q\bar{q}}(\tau)}$ .



# Quark-antiquark V

As  $R \rightarrow \infty$ , we single out the **open string** ground state,  $Z \rightarrow e^{-RV_{q\bar{q}}(\tau)}$ .



# • Imprints on the spectrum •

In the closed string channel,

$$Z(\tau, R) = \int_0^\infty dE \underbrace{\rho_W(E, R)} e^{-\tau E}.$$

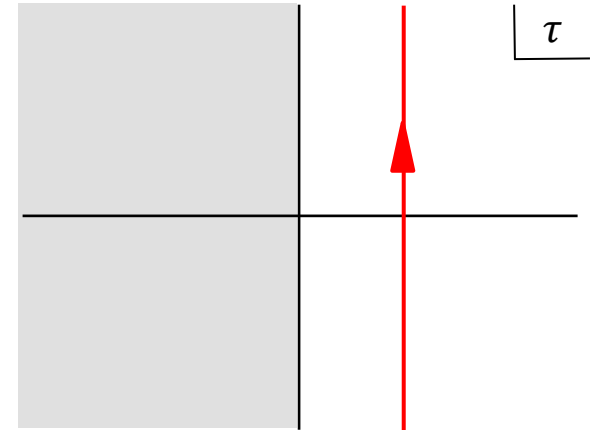
Spectral density:  $\sum_n |\langle W|n\rangle|^2 \delta(E - E_n)$   
 $E_n = \sqrt{m_n^2 + p_\perp^2}$

# • Imprints on the spectrum •

In the closed string channel,

$$Z(\tau, R) = \int_0^\infty dE \underbrace{\rho_W(E, R)} e^{-\tau E}.$$

Spectral density:  $\sum_n |\langle W|n\rangle|^2 \delta(E - E_n)$   
 $E_n = \sqrt{m_n^2 + p_\perp^2}$



An inverse Laplace transform on  $\tau \rightarrow 0$  gives its **asymptotics**,

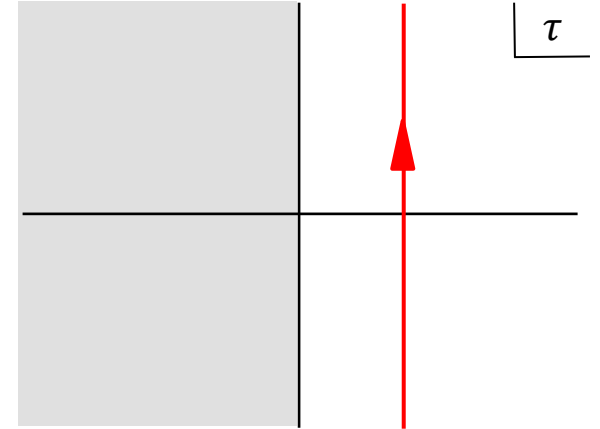
$$\rho_W(m, R) \rightarrow \begin{cases} \exp\left(\sqrt{\frac{24\pi}{11}} \frac{Rm}{\log m} + \dots\right) & D = 4 \\ m^{\frac{\lambda R}{4\pi} - 2} + \dots & D = 3 \end{cases}$$

# • Imprints on the spectrum •

In the closed string channel,

$$Z(\tau, R) = \int_0^\infty dE \underbrace{\rho_W(E, R)} e^{-\tau E}.$$

Spectral density:  $\sum_n |\langle W|n\rangle|^2 \delta(E - E_n)$   
 $E_n = \sqrt{m_n^2 + p_\perp^2}$



An inverse Laplace transform on  $\tau \rightarrow 0$  gives its **asymptotics**,

$$\rho_W(m, R) \rightarrow \begin{cases} \exp\left(\sqrt{\frac{24\pi}{11}} \frac{Rm}{\log m} + \dots\right) & D = 4 \\ m^{\frac{\lambda R}{4\pi} - 2} + \dots & D = 3 \end{cases}$$

Hagedorn growth,

$$\sum_n \delta(E - E_n) \sim e^{m/T_H},$$

so coupling must **decay**,

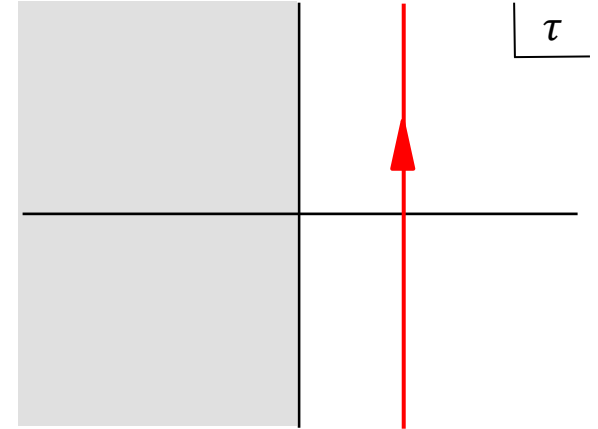
$$|\langle W|n\rangle|^2 \sim e^{-m/T_H} + \dots$$

# • Imprints on the spectrum •

In the closed string channel,

$$Z(\tau, R) = \int_0^\infty dE \underbrace{\rho_W(E, R)} e^{-\tau E}.$$

Spectral density:  $\sum_n |\langle W|n\rangle|^2 \delta(E - E_n)$   
 $E_n = \sqrt{m_n^2 + p_\perp^2}$



An inverse Laplace transform on  $\tau \rightarrow 0$  gives its **asymptotics**,

$$\rho_W(m, R) \rightarrow \begin{cases} \exp\left(\sqrt{\frac{24\pi}{11}} \frac{Rm}{\log m} + \dots\right) & D = 4 \\ m^{\frac{\lambda R}{4\pi} - 2} + \dots & D = 3 \end{cases}$$

Hagedorn growth,

$$\sum_n \delta(E - E_n) \sim e^{m/T_H},$$

so coupling must **decay**,

$$|\langle W|n\rangle|^2 \sim e^{-m/T_H} + \dots$$

In critical string theory, instead,  $|\langle D_p|N, \tilde{N}\rangle|^2 = 2^6 (4\pi^2 \alpha')^{p/2-6}$ .

# • Imprints on WS dynamics •

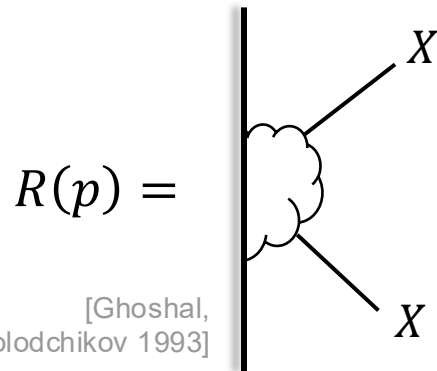
Microscopic details of the boundary are captured by the **R-matrix**:

$$R(p) = \text{[Diagram of a boundary interaction with two } X \text{ particles]}$$

[Ghoshal, Zamolodchikov 1993]

# • Imprints on WS dynamics •

Microscopic details of the boundary are captured by the **R-matrix**:



General relation  $R(p), S(s) \leftrightarrow Z(\tau, R)$  is unknown.

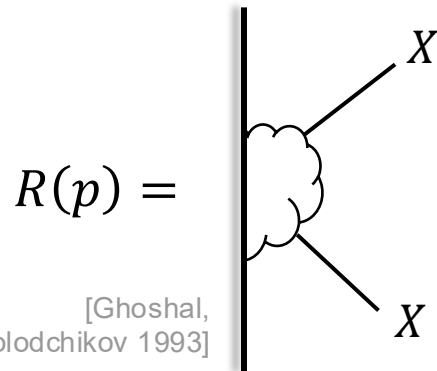
In **integrability**, it is given by TBA ( $D = 3, R \rightarrow \infty$ ):

$$V_{q\bar{q}}(\tau) = \tau \ell_s^{-2} + \frac{1}{2\pi} \int_0^\infty dp \log(1 - |R(ip)|^2 e^{-\epsilon(p)}) ,$$

$$\epsilon(p) = 2p\tau + \int \frac{dp'}{2\pi} \frac{1}{i} S^* \partial_{p'} S(p, p') \log(1 - |R(ip')|^2 e^{-\epsilon(p')}) .$$

# • Imprints on WS dynamics •

Microscopic details of the boundary are captured by the **R-matrix**:



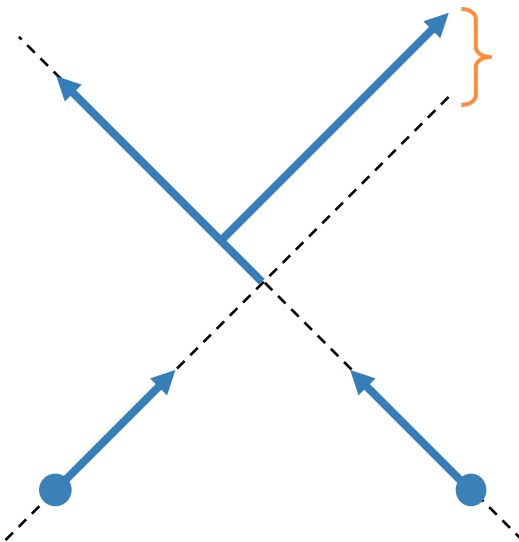
General relation  $R(p), S(s) \leftrightarrow Z(\tau, R)$  is unknown.

In **integrability**, it is given by TBA ( $D = 3, R \rightarrow \infty$ ):

$$V_{q\bar{q}}(\tau) = \tau \ell_s^{-2} + \frac{1}{2\pi} \int_0^\infty dp \log(1 - |R(ip)|^2 e^{-\epsilon(p)}) ,$$

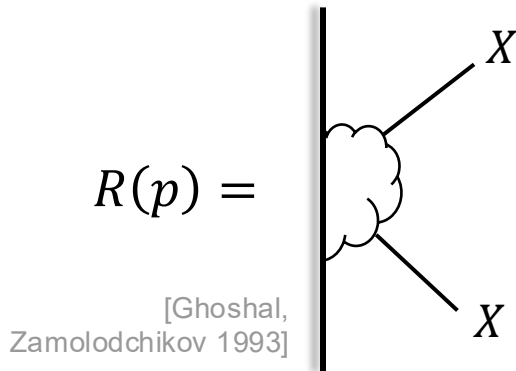
$$\epsilon(p) = 2p\tau + \underbrace{\int \frac{dp'}{2\pi} \frac{1}{i} S^* \partial_{p'} S(p, p')}_{\text{Wigner time delay}} \log(1 - |R(ip')|^2 e^{-\epsilon(p')}) .$$

Wigner **time delay**  $\Delta t \geq 0$ . Positivity can be proven from analyticity and unitarity of  $S(s)$ .



# • Imprints on WS dynamics •

Microscopic details of the boundary are captured by the **R-matrix**:



General relation  $R(p), S(s) \leftrightarrow Z(\tau, R)$  is unknown.

In **integrability**, it is given by TBA ( $D = 3, R \rightarrow \infty$ ):

$$V_{q\bar{q}}(\tau) = \tau \ell_s^{-2} + \frac{1}{2\pi} \int_0^\infty dp \log(1 - |R(ip)|^2 e^{-\epsilon(p)}) ,$$

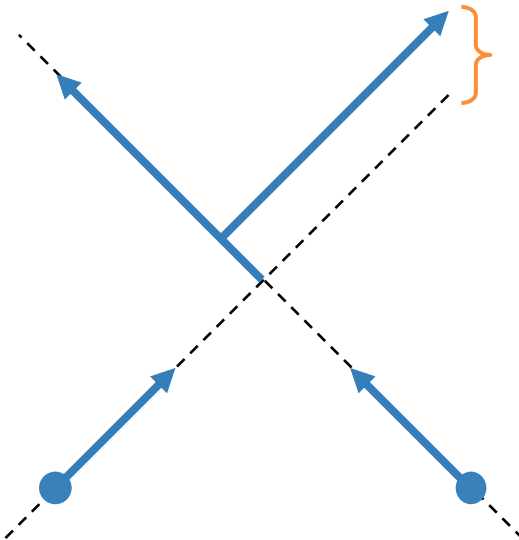
$$\epsilon(p) = 2p\tau + \underbrace{\int \frac{dp'}{2\pi} \frac{1}{i} S^* \partial_{p'} S(p, p')}_{\text{Wigner time delay}} \log(1 - |R(ip')|^2 e^{-\epsilon(p')}) .$$

Wigner **time delay**  $\Delta t \geq 0$ . Positivity can be proven from analyticity and unitarity of  $S(s)$ .

In absence of phase transitions from  $\tau = 0$ , we get **causality bounds** on thermodynamics.

$$\epsilon(p) \leq 2p\tau , \quad (\text{interactions are attractive})$$

$$\tau \ell_s^{-2} - V_{q\bar{q}}(\tau) \geq \frac{1}{2\pi} \int dp |R(ip)|^2 e^{-2p\tau} .$$



## • Imprints on WS dynamics •

Plugging the Coulomb potential ( $D = 3$ ) at  $\tau \rightarrow 0$ , shows that the R-matrix must **decay asymptotically**.

$$-\frac{\lambda}{4\pi} \log \tau \geq \frac{1}{2\pi} \int dp |R(ip)|^2 e^{-2p\tau} \quad \longrightarrow \quad |R(ip)|^2 \leq \frac{\lambda}{2p} \quad (p \rightarrow \infty)$$

## • Imprints on WS dynamics •

Plugging the Coulomb potential ( $D = 3$ ) at  $\tau \rightarrow 0$ , shows that the R-matrix must **decay asymptotically**.

$$-\frac{\lambda}{4\pi} \log \tau \geq \frac{1}{2\pi} \int dp |R(ip)|^2 e^{-2p\tau} \quad \longrightarrow \quad \boxed{|R(ip)|^2 \leq \frac{\lambda}{2p} \quad (p \rightarrow \infty)}$$

In critical string theory, instead,  $R(ip) = \pm e^{i\ell_s^2 p^2/2}$ ,  $|R(ip)| = 1$ .

## • Imprints on WS dynamics •

Plugging the Coulomb potential ( $D = 3$ ) at  $\tau \rightarrow 0$ , shows that the R-matrix must **decay asymptotically**.

$$-\frac{\lambda}{4\pi} \log \tau \geq \frac{1}{2\pi} \int dp |R(ip)|^2 e^{-2p\tau} \quad \longrightarrow \quad \boxed{|R(ip)|^2 \leq \frac{\lambda}{2p} \quad (p \rightarrow \infty)}$$

In critical string theory, instead,  $R(ip) = \pm e^{i\ell_s^2 p^2/2}$ ,  $|R(ip)| = 1$ .

**Bonus:** It has been proposed that at high energies the S-matrix **grows** as  $S(s) \sim e^{ic s}$ . [Dubovsky 2019]

## • Imprints on WS dynamics •

Plugging the Coulomb potential ( $D = 3$ ) at  $\tau \rightarrow 0$ , shows that the R-matrix must **decay asymptotically**.

$$-\frac{\lambda}{4\pi} \log \tau \geq \frac{1}{2\pi} \int dp |R(ip)|^2 e^{-2p\tau} \quad \longrightarrow \quad \boxed{|R(ip)|^2 \leq \frac{\lambda}{2p} \quad (p \rightarrow \infty)}$$

In critical string theory, instead,  $R(ip) = \pm e^{i\ell_s^2 p^2/2}$ ,  $|R(ip)| = 1$ .

**Bonus:** It has been proposed that at high energies the S-matrix **grows** as  $S(s) \sim e^{ic s}$ . [Dubovsky 2019]

Within integrability, this means

$$\begin{aligned} \epsilon(p) &\sim 2p\tau + \int \frac{dp'}{2\pi} 4cp \log(1 - |R(ip')|^2 e^{-\epsilon(p')}) \quad (p \rightarrow \infty) \\ &= p \left( 2(1 - 2c\ell_s^{-2})\tau + 4c V_{q\bar{q}}(\tau) \right) \sim -p \frac{2c\lambda}{\pi} \log \tau \quad (\tau \rightarrow 0) \end{aligned}$$

## • Imprints on WS dynamics •

Plugging the Coulomb potential ( $D = 3$ ) at  $\tau \rightarrow 0$ , shows that the R-matrix must **decay asymptotically**.

$$-\frac{\lambda}{4\pi} \log \tau \geq \frac{1}{2\pi} \int dp |R(ip)|^2 e^{-2p\tau} \quad \longrightarrow \quad \boxed{|R(ip)|^2 \leq \frac{\lambda}{2p} \quad (p \rightarrow \infty)}$$

In critical string theory, instead,  $R(ip) = \pm e^{i\ell_s^2 p^2/2}$ ,  $|R(ip)| = 1$ .

**Bonus:** It has been proposed that at high energies the S-matrix **grows** as  $S(s) \sim e^{ic s}$ . [Dubovsky 2019]

Within integrability, this means

$$\begin{aligned} \epsilon(p) &\sim 2p\tau + \int \frac{dp'}{2\pi} 4cp \log(1 - |R(ip')|^2 e^{-\epsilon(p')}) && (p \rightarrow \infty) \\ &= p \left( 2(1 - 2c\ell_s^{-2})\tau + 4c V_{q\bar{q}}(\tau) \right) \sim -p \frac{2c\lambda}{\pi} \log \tau && (\tau \rightarrow 0) \end{aligned}$$

A diverging pseudoenergy leads to a **phase transition** in  $\tau$ .

➡ Integrable system with  $S(s) \sim e^{ic s}$  is ruled out.

# Outlook

## Summary:

- Polyakov loop correlator opens a door to **asymptotic freedom**.
- Growth of asymptotic **spectral densities** is relatively mild.
- String **couplings** to Wilson loops must decay fast.
- Microscopically, this corresponds to a decaying **R-matrix**.
- Linear phase shift is ruled out in integrable setup.

# Outlook

## Summary:

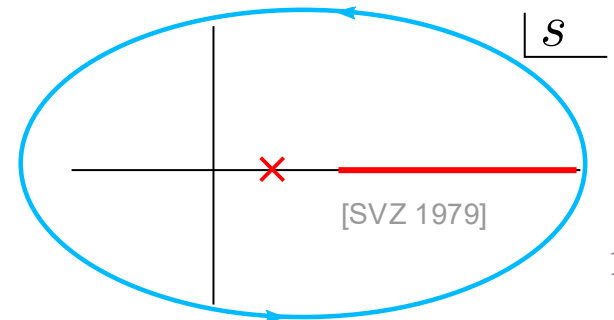
- Polyakov loop correlator opens a door to **asymptotic freedom**.
- Growth of asymptotic **spectral densities** is relatively mild.
- String **couplings** to Wilson loops must decay fast.
- Microscopically, this corresponds to a decaying **R-matrix**.
- Linear phase shift is ruled out in integrable setup.

## Future directions:

- Extend proofs beyond integrability, with a **DMB-like formula**.
- R-matrix **bootstrap** using asymptotic decay.
- **OPE**, non-perturbative corrections and QCD sum rules: [JA, Forsslund, Homrich, to appear]

$$\left| \begin{array}{c} D_1 \\ D_2 \\ \vdots \\ D_k \end{array} \right| \sim \bigoplus_k \left| \begin{array}{c} \times \exp \int dy \left( \frac{a_0}{\tau} + \sum_j a_j \tau^{\Delta_j - 1} \mathcal{O}_j \right) \end{array} \right|$$

[Cuomo, Komargodski, Kravchuk, Wang, ... 2024]



**Thank you!**



# Positive time delay

**Definition:** An *inner function* is a holomorphic function  $S: \mathbb{H} \rightarrow \mathbb{D}$  satisfying  $|S(z)| = 1$  (almost everywhere) for  $s \in \mathbb{R}$ .

Elastic massless S-matrices are examples of inner functions.

**Theorem (Blaschke-Nevanlinna):** The zeroes of an inner function satisfy the Blaschke condition  $\prod_k \frac{\text{Im } s_k}{|s_k|^2 + 1} < \infty$ . Moreover, it factorizes as

$$S(s) = B(s) \cdot f(s).$$

**Blaschke product:**

$$B(s) = \prod_k \frac{s - s_k}{s - s_k^*}$$

**Singular inner function** (no zeroes): It admits a dispersive representation,

$$f(s) = \exp\left(i \int_{\mathbb{R} \cup \infty} \frac{1 + t s}{t - s} \underbrace{\rho(t)}_{\geq 0} dt\right)$$

Computing the time delay,

$$\Delta t = -i \partial_s \log S(s) = \sum_k \frac{\text{Im } s_k}{|s_k|^2 + 1} + \int_{\mathbb{R} \cup \infty} \frac{1 + t^2}{(t - s)^2} \rho(t) dt \geq 0. \quad \square$$