Vaccum structure in gapped QCD_2 theories from the infinite Hamiltonian lattice

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with Ross Dempsey, Silviu S. Pufu, and Benjamin T. Søgaard

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Upshot

- Generically, gapped (1+1)d QCD-like theories have multiple degenerate vacua related by (non-)invertible symmetries

$$|0\rangle_1 \overset{\text{non-inv.}}{\sim} |0\rangle_2$$

- Lots of progress constraining vacuum structure and spectrum of QCD_2 from symmetry arguments [Komargodski, Ohmori, Roumpedakis, and Seifnashri 2021; Delmastro, Gomis, Yu 2021; Córdova, García-Sepúlveda, and Holfester 2024; Damia, Galati, Tizzano 2025, ...]

- On the lattice: non-invertible symmetries broken, different vacua mix on **finite lattice** :(

$$|0\rangle^{\text{lat}} \propto \text{a}\, |0\rangle_1^{\text{lat}} + \text{b}\, |0\rangle_2^{\text{lat}}$$

- On the **infinite lattice:** lattice analogs of all degenerate continuum vacua :) despite broken non-invertible symmetries
[Dempsey, AMG, Pufu, Søgaard 2025]

$$|0\rangle_1^{\text{lat}} \qquad |0\rangle_2^{\text{lat}}$$

Upshot

This talk

- ✓ **insights about nature of continuum vacua from the infinite lattice**
- ✓ **numerically confirm particle-soliton degeneracies predicted in** [Córdova, García-Sepúlveda, and Holfester 2024]

Roadmap

1. QCD_2 action and properties
2. Lattice Hamiltonian
3. Vacua on the infinite lattice and the lattice decay rule
4. Particle and soliton degeneracies

QCD_2 action and properties

Gauge group SU(N_c) with massless **Majorana fermion in real representation λ in (1+1)d**

$$S = \int d^2x \left(-\frac{1}{4g^2} F^A_{\mu\nu} F^{A\mu\nu} + \frac{i}{2} \bar{\psi}^\alpha \gamma^\mu (D_\mu \psi)^\alpha \right)$$

- ψ^α Majorana fermions
- A_μ gauge field, $F_{\mu\nu}$ field strength
- D_μ covariant derivative

- $A = 1, \ldots N_c^2 - 1$
- $\alpha = 1, \ldots, \dim \lambda$

- Invertible 0-form symmetries can include: $(\mathbb{Z}_2)_F$, $(\mathbb{Z}_2)_\chi$, $(\mathbb{Z}_2)_C$
- If λ is invariant under center of SU(N_c): **1-form center symmetry** $\mathbb{Z}_{N_c}^{(1)}$

  ➡ theory splits into N_c distinct **flux-tube sectors** $p = 0, \ldots, N_c - 1$
- IR is determined by coset CFT, for gapped theories: TQFT

  ➡ determines # **of vacua**

QCD$_2$ action and properties

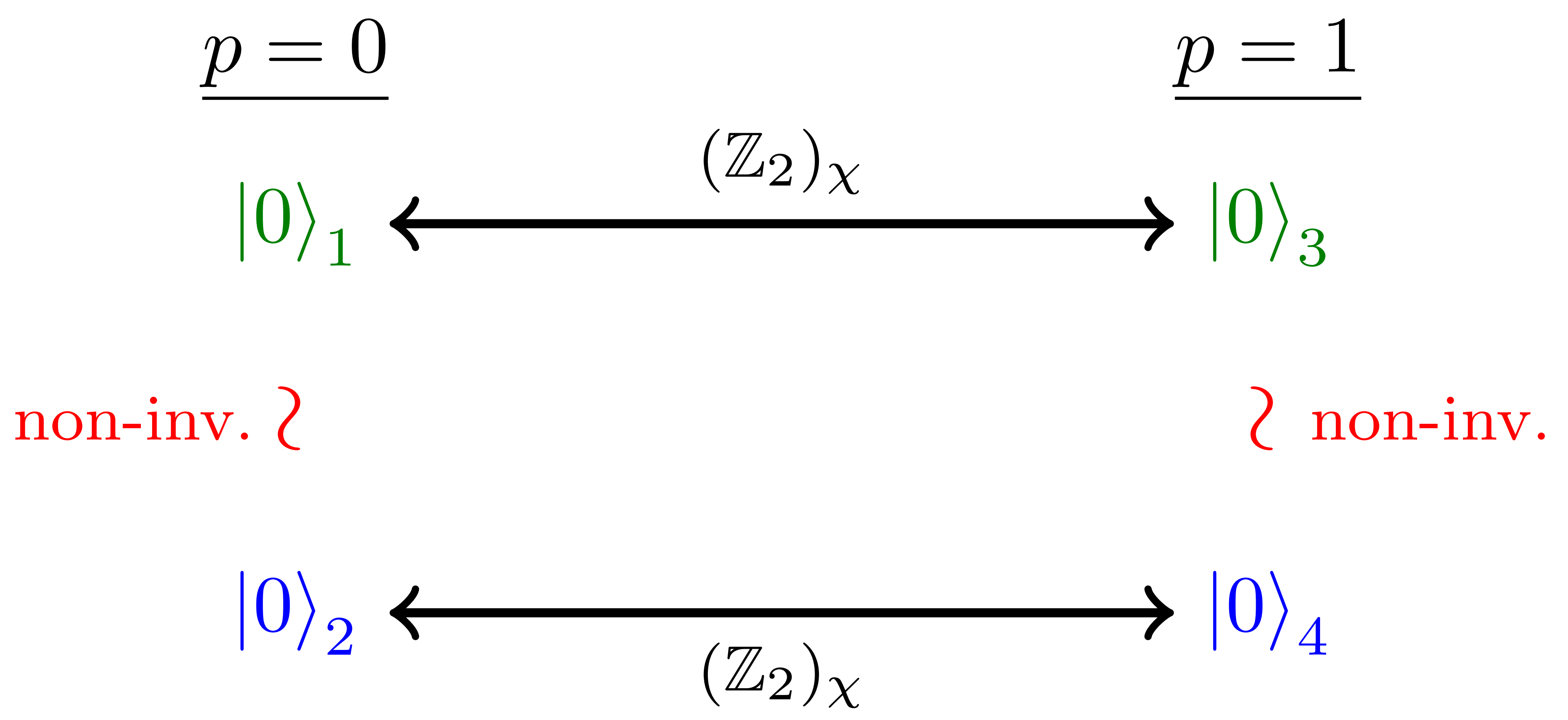
e.g. SU(2)+5 (SU(2) with Majorana fermions in $j = 2$ irrep of SU(2))

$$S = \int d^2x \left(-\frac{1}{4g^2} F^A_{\mu\nu} F^{A\mu\nu} + \frac{i}{2} \bar{\psi}^\alpha \gamma^\mu (D_\mu \psi)^\alpha \right)$$

- ψ^α Majorana fermions
- A_μ gauge field, $F_{\mu\nu}$ field strength
- D_μ covariant derivative

- $A = 1,\ldots,3$
- $\alpha = 1,\ldots,5$

- 0-form symmetries: $(\mathbb{Z}_2)_F$, $(\mathbb{Z}_2)_\chi$
- 1-form symmetry $\mathbb{Z}_2^{(1)}$ (mixed anomaly with $(\mathbb{Z}_2)_\chi$) $\Rightarrow$ flux-tube sectors $p = 0,1$
- Gapped in the IR, coset predicts 2 vacua in each flux tube sector



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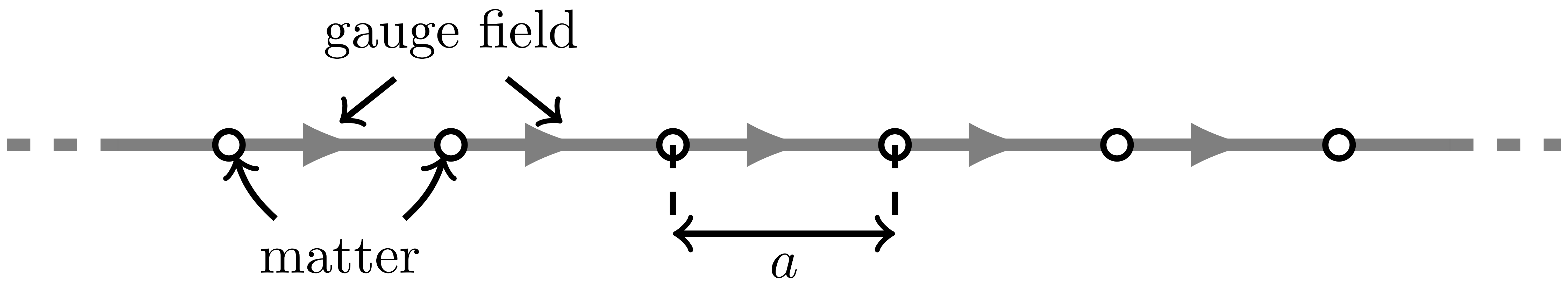
Infinite Hamiltonian Lattice for Gauge Theories

- Hamiltonian lattice:

  [Kogut, Susskind, 1975]



Discretized space,
Continuous time

$$H = \sum_n \left[\frac{ag^2}{2} L_n^A L_n^A - \frac{i}{2a} \chi_n^\alpha U_n^{\alpha\beta} \chi_{n+1}^\beta \right] \quad \text{with Gauss law} \quad L_n^A - R_{n-1}^A = Q_n^A = \frac{1}{2} \chi_n^\alpha (T^A)_{\alpha\beta} \chi_n^\beta$$

[Dempsey, Klebanov, Pufu, Søgaard, 2024]

- χ_n^α lattice Majorana fermion operator, $\{\chi_n^\alpha, \chi_m^\beta\} = \delta_{nm}\delta^{\alpha\beta}$
- L_n^A, R_n^A left- and right-acting field strength operators
- $U^{\alpha\beta}$ gauge connection

- exact invertible symmetries on the lattice, e.g. chiral symmetry = translation by one site
- no known non-invertible symmetries

  ➡ vacuum degeneracies due to these symmetries lifted

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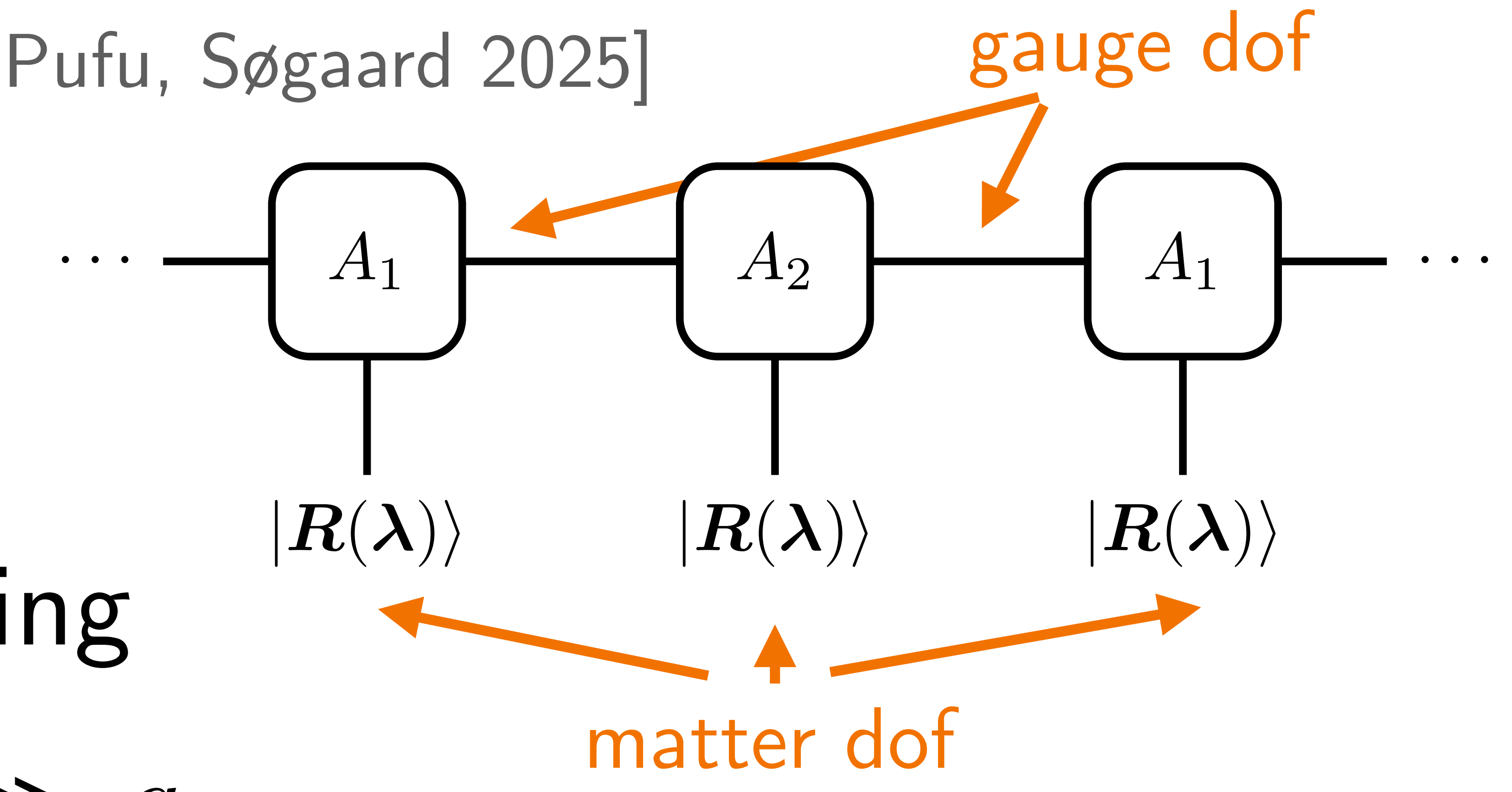
Infinite Hamiltonian Lattice for Gauge Theories

- Infinite Symmetric Matrix Product State Ansatz [Vidal 2007, Pérez-García et al, 2008, Sanz et al, 2009, ...] for translation-invariant non-abelian gauge theories [Dempsey, AMG, Pufu, Søgaard 2025]

Put non-abelian theory directly on ∞ lattice

$|\psi\rangle \approx$



- Infinite Matrix Product States are generically cluster decomposing

$$\langle\psi|\mathcal{O}_1(x)\mathcal{O}_2(y)|\psi\rangle \to \langle\psi|\mathcal{O}_1(x)|\psi\rangle\langle\psi|\mathcal{O}_2(y)|\psi\rangle \quad \text{for } |x-y| \gg a$$

Despite the lack of non-invertible symmetries on the lattice, cluster decomposition forbids mixing of lattice states in different symmetry sectors!

➡ Find states corresponding to **ALL** continuum vacua at finite lattice spacing a as **local minima** of the energy density with variational optimization algorithms

Infinite Hamiltonian Lattice for Gauge Theories

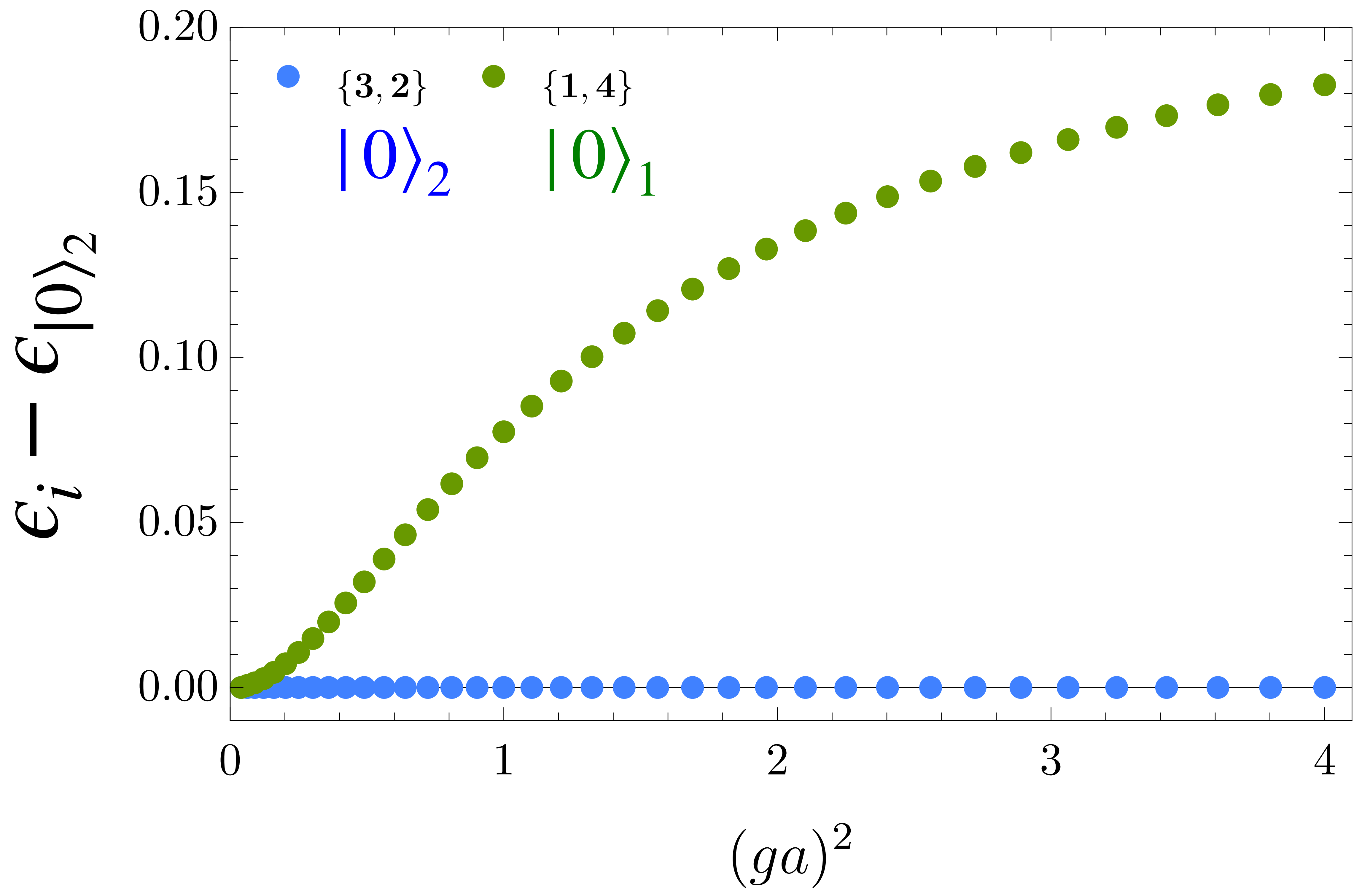
- SU(2)+5:

$\underline{p = 0}$

$|0\rangle_1$

non-inv. $\wr$

$|0\rangle_2$



Degeneracy lifted at finite lattice spacing but still clearly separated "lattice vacua"

VEV for SU(2)+5

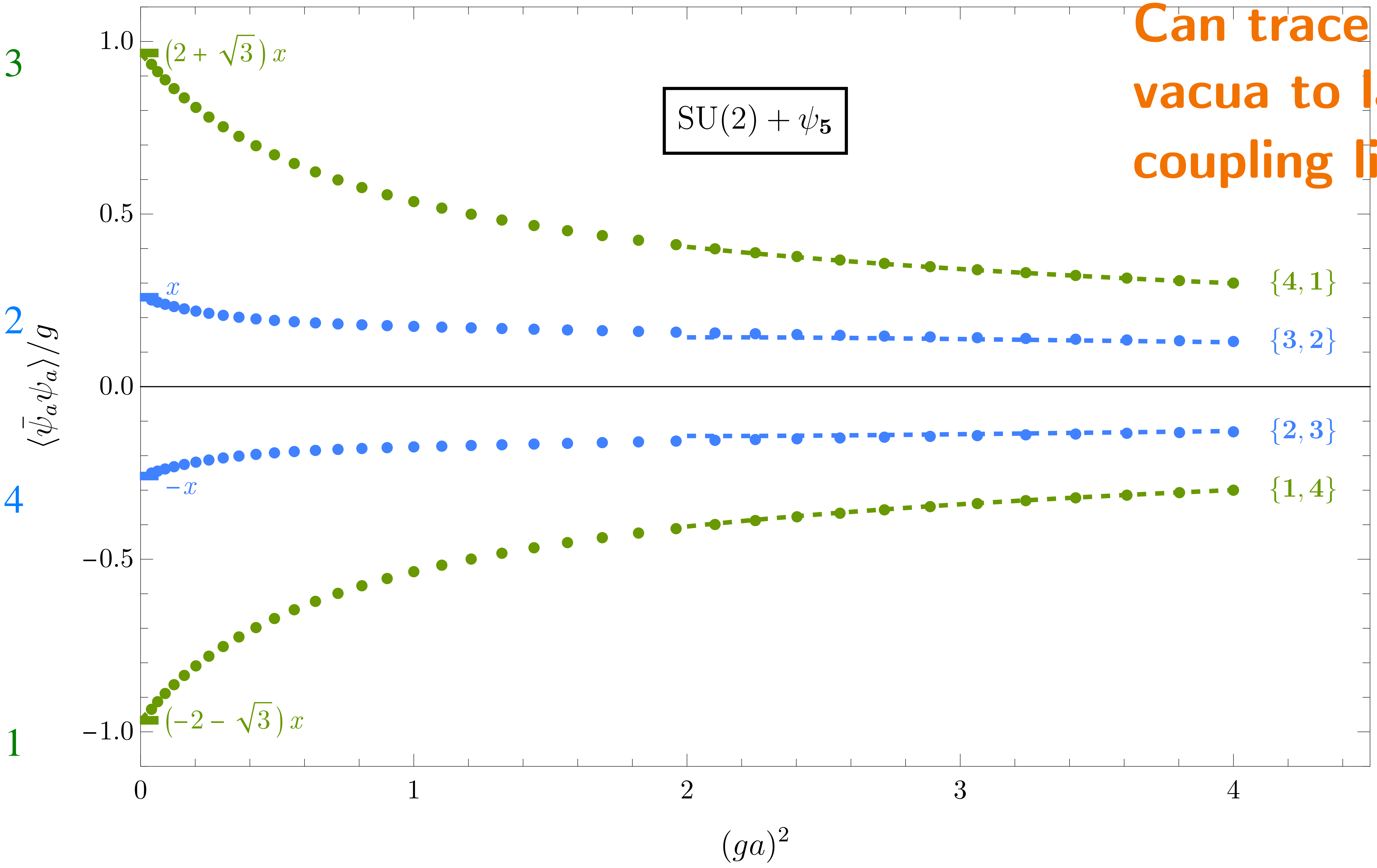
Agree with $\langle\bar{\psi}^\alpha\psi^\alpha\rangle/g$ **with continuum predictions** [Córdova, García-Sepúlveda, and Holfester 2024]

$p = 1,\ |0\rangle_3$

$p = 0,\ |0\rangle_2$

$p = 1,\ |0\rangle_4$

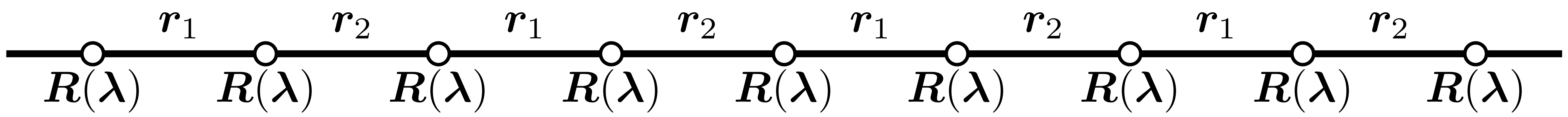
$p = 0,\ |0\rangle_1$



Can trace back continuum vacua to lattice strong-coupling limit $ag \to \infty$**!**

Strong Coupling Limit

- Lattice strong-coupling Hamiltonian: $H_{\text{sc}} \overset{ag\to\infty}{\to} \dfrac{ag^2}{2}\sum_n L_n^A L_n^A$
- Eigenstates characterized by pairs of representations on adjacent links:



$$\epsilon(\mathbf{r}_1, \mathbf{r}_2) = \frac{g^2}{4}\Big(C_2(\mathbf{r}_1) + C_2(\mathbf{r}_2)\Big)$$

- SU(2)+5:

$p=0$		$p=1$
$\vert 0\rangle_1$	$\longleftrightarrow$ $(\mathbb{Z}_2)_\chi$	$\vert 0\rangle_3$
non-inv. ≀		≀ non-inv.
$\vert 0\rangle_2$	$\longleftrightarrow$ $(\mathbb{Z}_2)_\chi$	$\vert 0\rangle_4$

Continuum, $ag = 0$,
energies degenerate

$ag \to \infty$ ⟶

?

$p=0$		$p=1$	
$\{\mathbf{1},\mathbf{4}\}$	$\longleftrightarrow$ $(\mathbb{Z}_2)_\chi$	$\{\mathbf{4},\mathbf{1}\}$	$\epsilon(\mathbf{1},\mathbf{4}) = \epsilon(\mathbf{4},\mathbf{1}) = \dfrac{15g^2}{16}$
$\{\mathbf{3},\mathbf{2}\}$	$\longleftrightarrow$ $(\mathbb{Z}_2)_\chi$	$\{\mathbf{2},\mathbf{3}\}$	$\epsilon(\mathbf{2},\mathbf{3}) = \epsilon(\mathbf{3},\mathbf{2}) = \dfrac{11g^2}{16}$

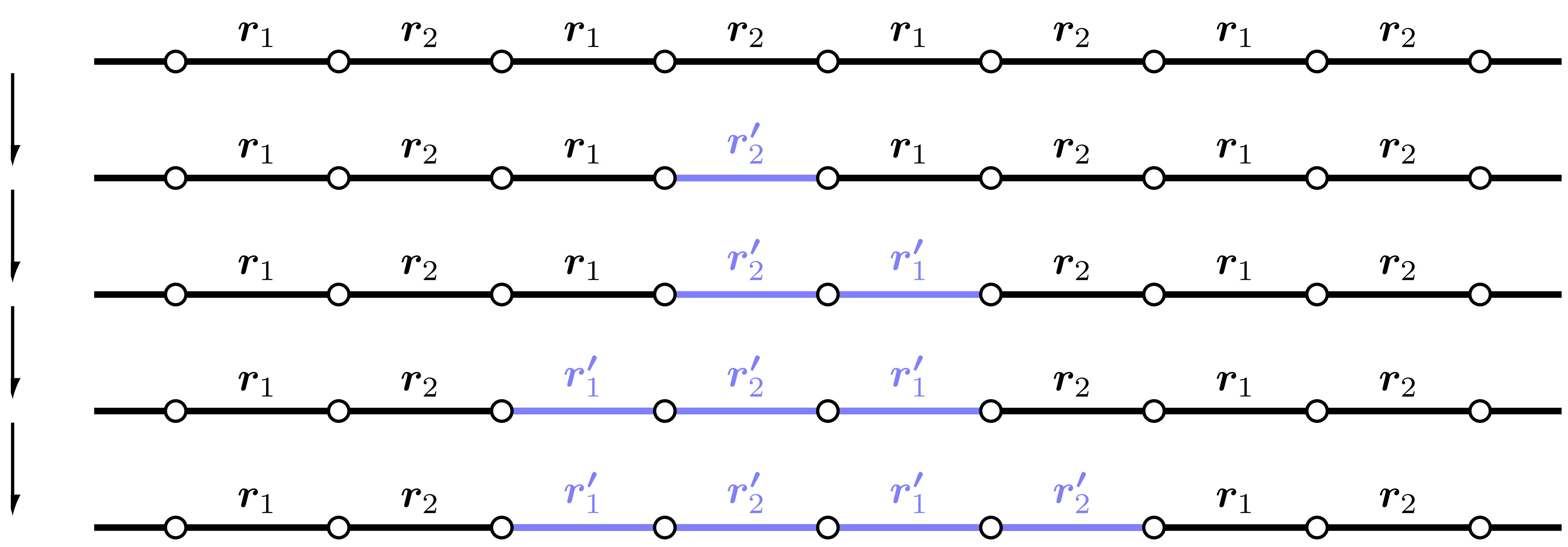
LSC, $ag \to \infty$

ground states
as $ag \to \infty$

Lattice Decay Rule

Can we determine from the strong-coupling limit which pairs of representations in the lattice strong-coupling limit are connected to degenerate continuum vacua?

Intuition: Decay through bubble nucleation



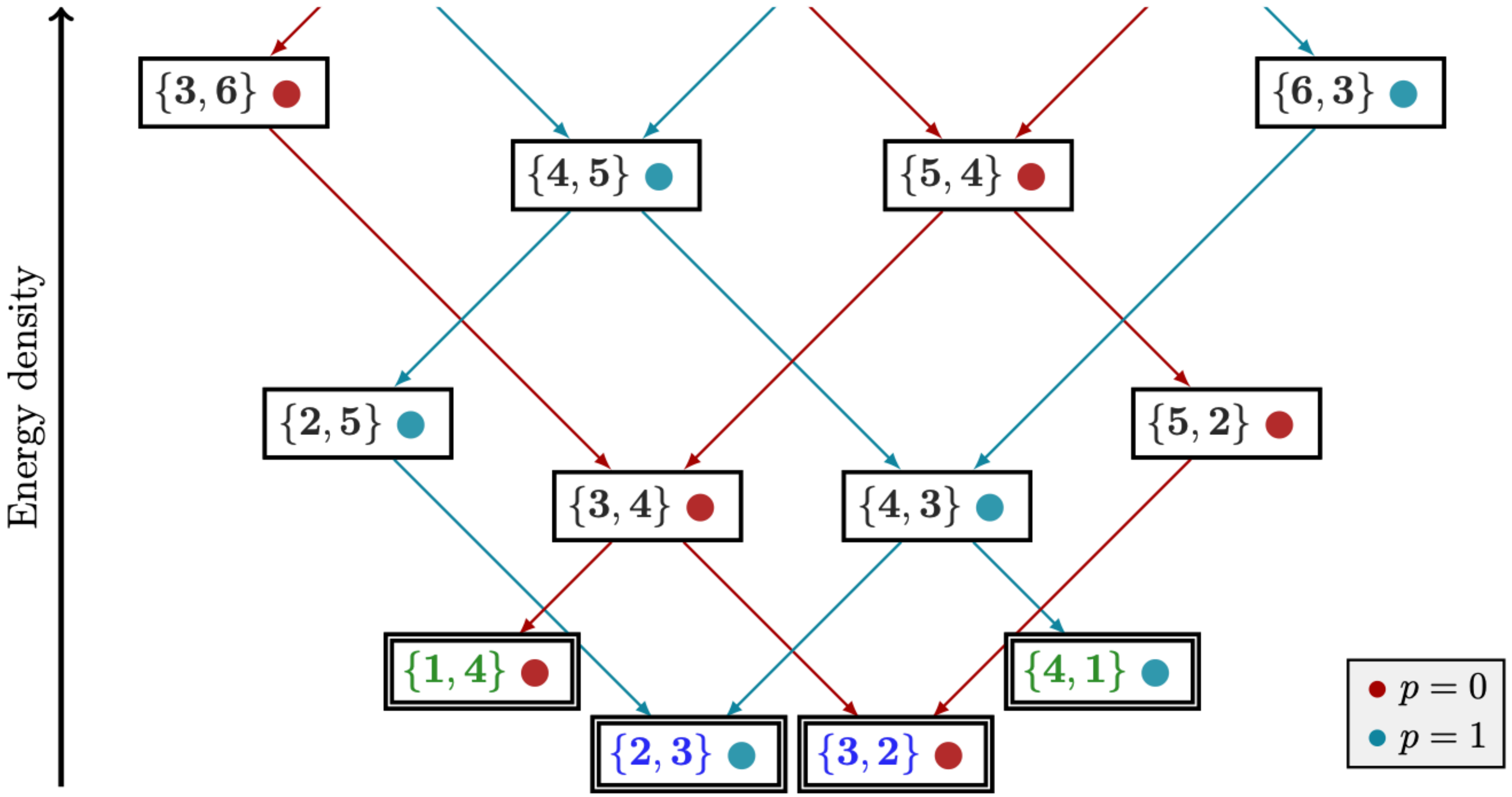
- Kinematically possible if $\mathbf{r}'_2 \in \mathbf{r}_1 \otimes \mathbf{R}(\lambda)$ and $\mathbf{r}'_1 \in \mathbf{r}_2 \otimes \mathbf{R}(\lambda)$
- Dynamical constraint: $C_2(\mathbf{r}'_1) \le C_2(\mathbf{r}_1)$ and $C_2(\mathbf{r}'_2) \le C_2(\mathbf{r}_2)$

Lattice Decay Rule:

Endpoints of decay chains are the strong-coupling limits of the degenerate vacua in the continuum.

Lattice Decay Rule

SU(2)+5:



Insight on degenerate continuum vacua from easy lattice picture!
Predict which lattice states will become degenerate vacua!

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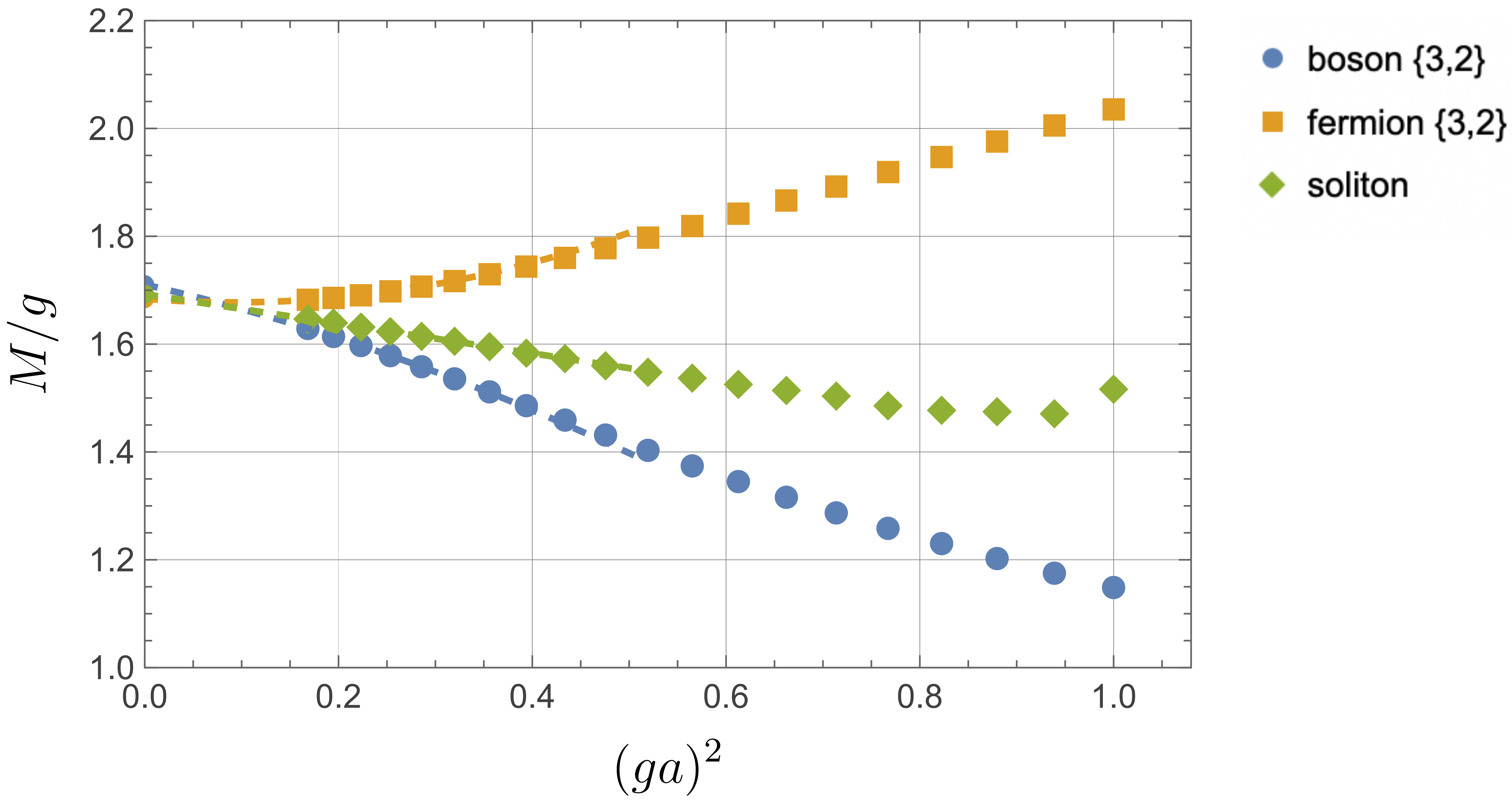
Particle-Soliton degeneracy

- **Degenerate multiplet of one soliton and two particles** in SU(2)+5
  [Córdova, García-Sepúlveda, and Holfester 2024]
- Confirmed with our simulations: (work in progress)



Take-home message and next steps

- **Cluster-decomposition on the infinite lattice** allows the numerical study of all vacua in gapped QCD-like theories, even if non-invertible symmetry is broken on the lattice
- **Lattice decay rule** gives intuition about degenerate vacua from the lattice strong-coupling limit
- Numerically confirm **particle-soliton degeneracies**

- Next Steps:
  - Understand the particle-soliton degeneracy also in other examples (work in progress with Clay Córdova and Nicholas Holfester)

Back-up slides

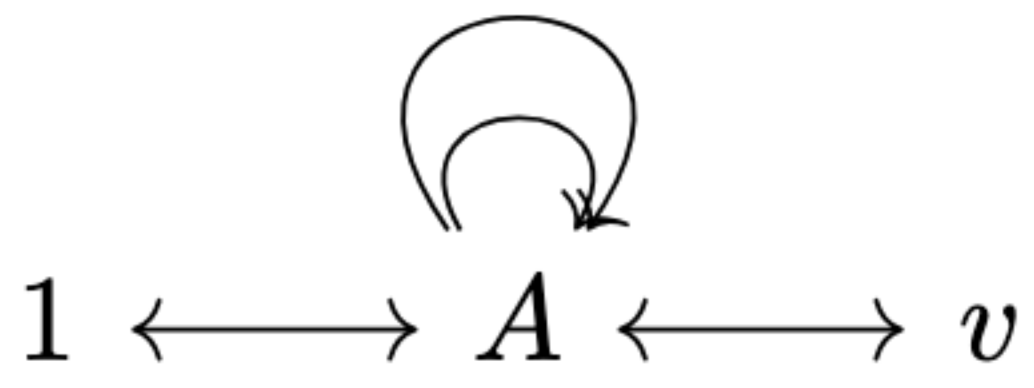
IR TQFT and non-invertible lines for SU(2)+5

- **Coset CFT:** $\dfrac{SO(5)_1}{SU(2)_{10}}$ is a TQFT

- Bosonic version: $\dfrac{Spin(5)_1}{SU(2)_{10}}$ in a single universe has topological lines 1, v, A, each corresponding to vacuum if symmetry spontaneously broken [Cordóva, García-Sepúlveda, and Holfester 2024]

- Fusion category

	1	**V**	**A**
1	1	V	A
V	V	1	A
A	A	A	1+v+2A

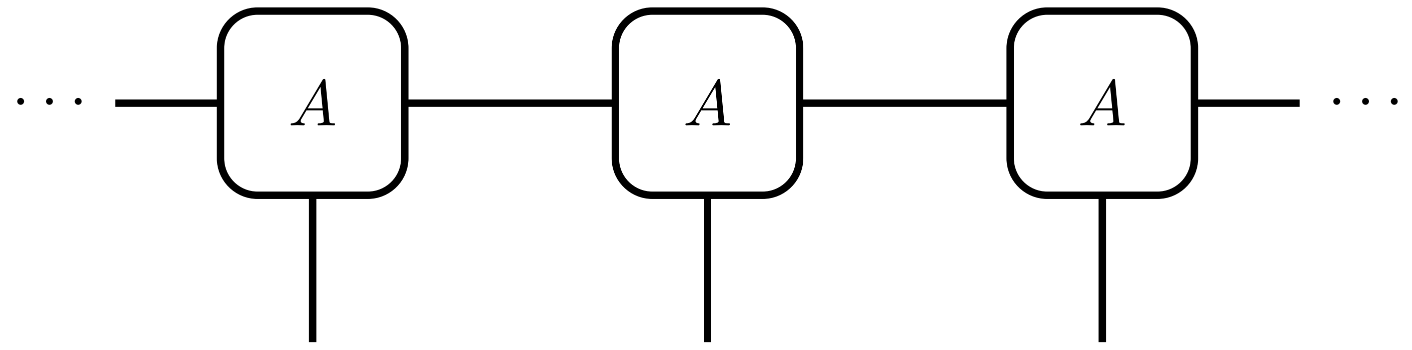


- Fermionic version: $1 \oplus v$, A are vacua and lines

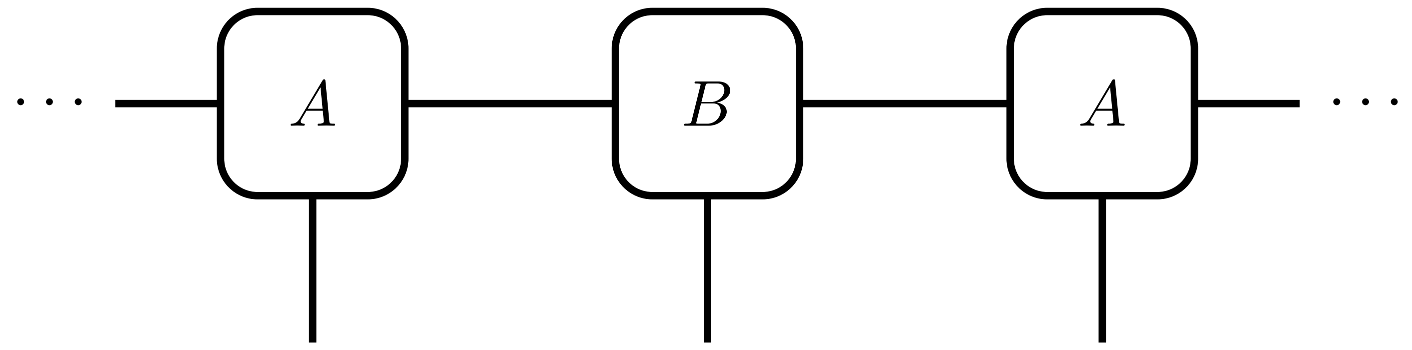
Quasiparticle ansatz

[Haegeman, Michalakis, Nachtergaele, Osborne, Schuch, Verstraete, 2013[

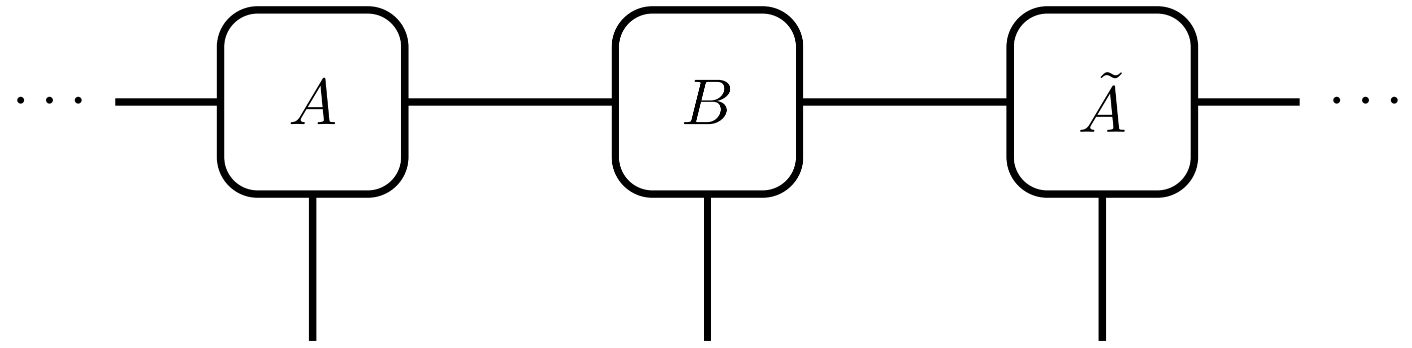
Vacuum:



Particle:

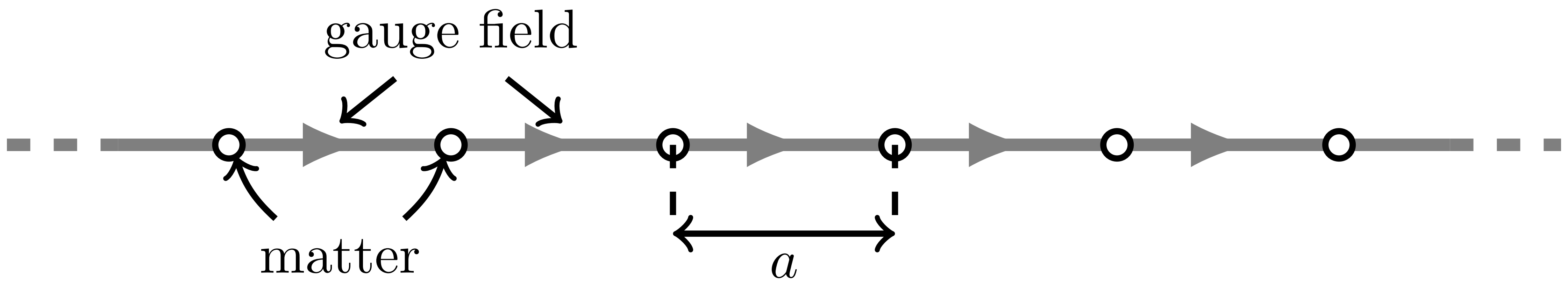


Soliton:



Infinite Hamiltonian Lattice for Gauge Theories

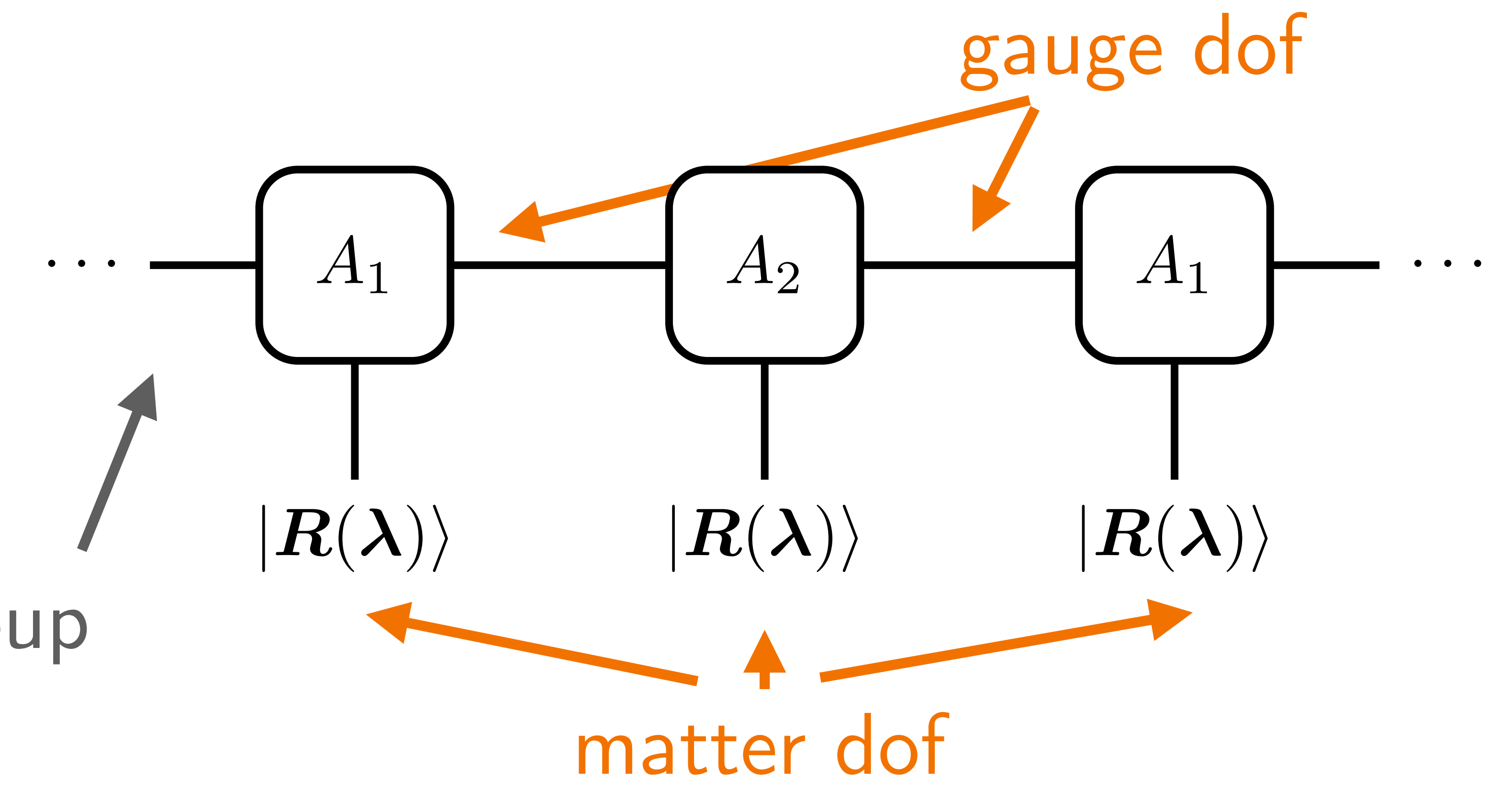
- Hamiltonian lattice:



Discretized space, Continuous time

- Infinite Symmetric Matrix Product State Ansatz: $|\psi\rangle \approx$

  [Vidal 2007, Pérez-García et al, 2008, Sanz et al, 2009, ...]



A decomposes into invariant tensors of the gauge group

- Act on infinite MPS with Hamiltonian as **link-enhanced matrix product operator** [Dempsey, AMG, Pufu, Søgaard 2025] and find groundstate with iterative optimizing algorithms such as VUMPS [Zauner-Stauber et al, 2017]

$H|\psi\rangle =$