QUANTUM MODELS WITH THE YANG-LEE PHASE TRANSITION

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Based on the upcoming work with Grisha Tarnopolsky (CMU):

OUTLINE

- A Yang-Lee recipe
- Schwinger model
- 3-State Quantum Clock model
- Antiferromagnetic Yang-Lee model
- Blume Capel model
- Conclusions and Future directions

A Yang-Lee Recipe

$$H_{\mathrm{YL}} = -J\sum_{j=1}^{N} Z_j Z_{j+1} - h_x \sum_{j=1}^{N} X_j - i h_z \sum_{j=1}^{N} Z_j$$

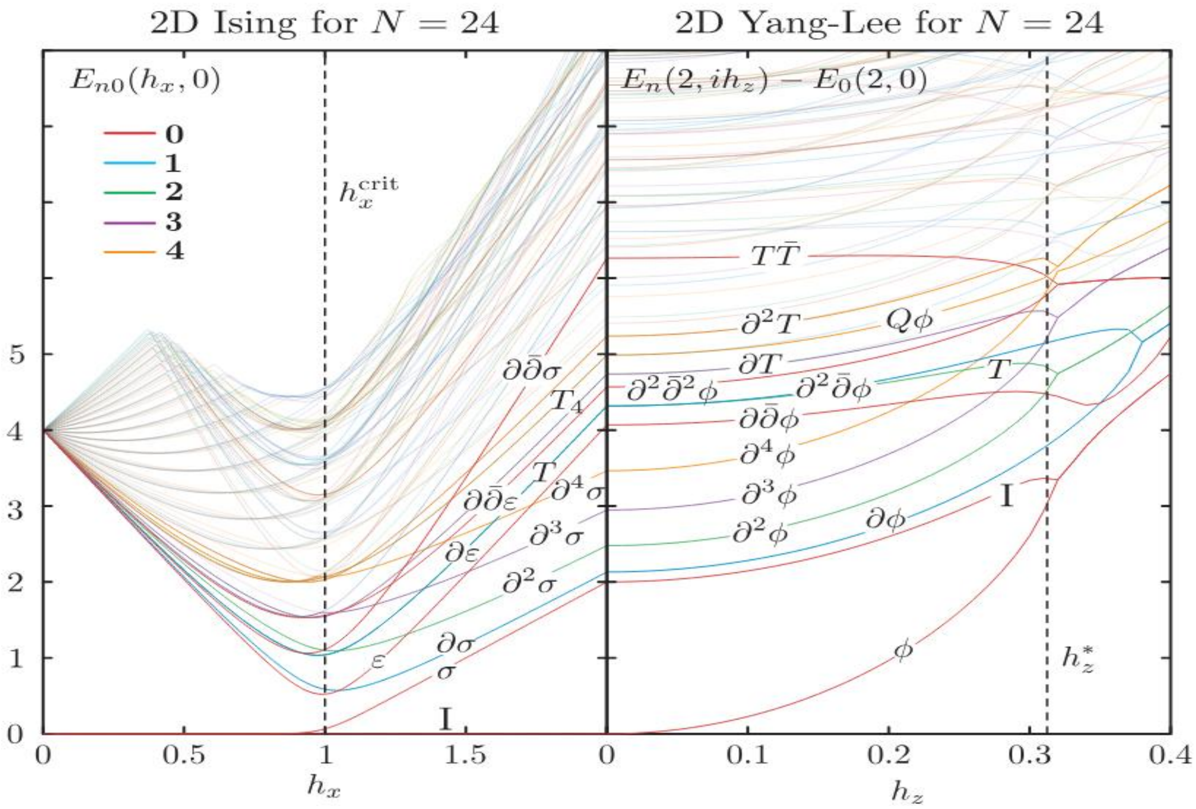
[von Gehlen ‘91; ’94; Uzelac, Julien ’81]

- Tune to ‘paramagnetic’ phase and turn on a term that:
  - Breaks $\mathbb{Z}_2$ symmetry: $Z_j \to -Z_j$
  - Breaks Hermicity
  - Preserves $\mathbb{Z}_2\mathcal{T} = \mathcal{PT}$ symmetry: $Z_j \to -Z_j,\ i \to -i$,

  where $\mathcal{T}$ is Time-reversal symmetry (complex conjugation)

[Castro-Alvaredo, Fring ‘09]

- From the energy levels of H_{YL} we can see the spectrum flow from Ising to YL:



Massive Schwinger model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{e\theta}{4\pi}\epsilon^{\mu\nu}F_{\mu\nu} + \bar{\Psi}(i\partial\!\!\!/ - eA\!\!\!/ - m)\Psi$$

[Schwinger '62; Casher, Kogut, Susskind '73; Coleman, Jackiw, Susskind '75; Coleman '76]

- Using Bosonization, we rewrite the Lagrangian as

$$\mathcal{L} = \frac{1}{2}F_{01}^2 + e\frac{\phi}{\sqrt{\pi}}F_{01} + \frac{1}{2}(\partial_\mu\phi)^2 + cm^2\cos\left(2\sqrt{\pi}\phi - \theta\right)$$

where $:\bar{\Psi}\Psi: = -cmN_m\cos\left(2\sqrt{\pi}\phi\right), \quad c = \frac{e^\gamma}{2\pi} \simeq 0.283$

[Coleman '75, '76]

- Full Hamiltonian on a staggered lattice:

$$H_S = \frac{e^2 N a}{2}\left(\mathcal{E} + \frac{\theta}{2\pi}\right)^2 - \frac{e^2 a}{4N}\sum_{n,n'=0}^{N-1} |n-n'|\left(N - |n-n'|\right) Q_n Q_{n'}$$

$$- \frac{i}{2a}\sum_{n=0}^{N-1}\left(c_n^\dagger e^{i\varphi_n} c_{n+1} - c_{n+1}^\dagger e^{-i\varphi_n} c_n\right) + m_{\text{lat}}\sum_{n=0}^{N-1}(-1)^n c_n^\dagger c_n,$$

with $Q_n = c_n^\dagger c_n - \delta_{n,\text{odd}}$ and $m_{\text{lat}} = m - e^2 a/8$.

[Banks, Kogut, Susskind '75; Dempsey, Klevanov, Pufu, Zan '23]

- $\mathbb{Z}_2 = \mathcal{C}$ symmetry:

$$\Psi \to \gamma^5 \Psi^*, \quad \phi \to -\phi, \quad A_\mu \to -A_\mu,$$

$$\mathcal{E} \to -\mathcal{E} - \frac{\theta}{\pi}, \quad e^{i\varphi_n} \to e^{-i\varphi_{n+1}}$$

$$c_n \to c_{n+1}^\dagger, \quad c_n^\dagger \to c_{n+1}$$

- Ising phase transition at $\theta = \pi$ and $m_c/e = 0.333561(4)$

[Byrnes, Sriganesh, Bursill, Hamer '02; Byrnes '03; Ohata '23; Dempsey, Klebanov, Pufu, Sogaard, Zan '24, EAC, Tarnopolsky, Xin '25]

- Consider the imaginary pseudo-scalar mass operator:

$$i(i\bar{\Psi}\gamma^5\Psi) \longrightarrow i\sin\phi$$

- ✓ $\mathbb{Z}_2$-odd and $\mathcal{T}$-odd
- ✓ Not Hermitian
- ✓ $\mathbb{Z}_2\mathcal{T} = \mathcal{PT}$-even

} Possible Yang-Lee deformation

- On the lattice, we add this term to the Schwinger Hamiltonian:

$$H_{\mathrm{PTS}} = H_{\mathrm{S}} - i m_5 \sum_{n=0}^{N-1} i(-1)^n \left(c_n^\dagger e^{i\varphi_n} c_{n+1} - c_{n+1}^\dagger e^{-i\varphi_n} c_n \right)$$

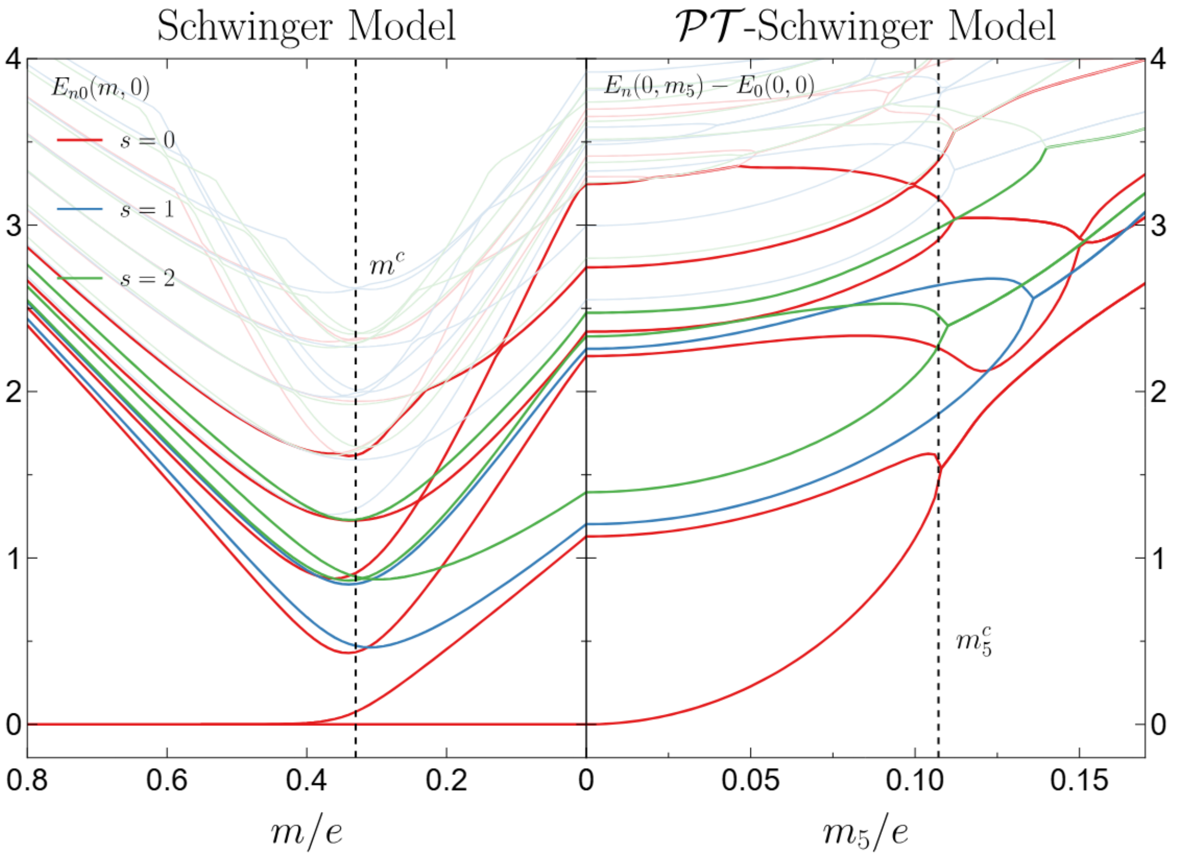
[Susskind '77]

- Look for critical point using Finite Size Scaling (FSS)

$$r_N = \frac{(N+2)\Delta E(N+2)}{N\Delta E(N)} = 1$$

[Hamer, Barber '81]

- Spectrum for $x = 1$, $\theta = \pi$, $\ell_{max} = 5$, and system size $N = 28$ where $x = 1/(e^2a^2)$



- First scaling dimension: $\Delta_\phi = -0.40016$
- YL scaling dimension: $\Delta_\phi = -0.4$
- Effective central charge: $\tilde{c} = 0.39848$
- YL effective central charge: $\tilde{c} = 0.4$
- Virtually same spectrum as for the usual spin ½ chain.
- Clear case of the Universality Hypothesis

3-State Quantum Clock model

- Consider the $\mathbb{Z}_3$-Quantum Clock model (CM):

$$H_{\rm CM} = -J\sum_{n=0}^{N-1}(\sigma_n^\dagger\sigma_{n+1} + \sigma_n\sigma_{n+1}^\dagger) - f\sum_{n=0}^{N-1}(\tau_n + \tau_n^\dagger)$$

[Ostlund '81; Jin, et all '02; Ortiz, et all '12; Fendley '12]

where

$$\sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad \tau = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \omega = \exp(2\pi i/3) \qquad \begin{array}{l} \tau^3 = \sigma^3 = 1, \\ \sigma\tau = \omega\tau\sigma, \end{array}$$

- $\mathbb{Z}_3 = \mathcal{Q}$ symmetry: $\begin{array}{l} \tau_n \to \tau_n \\ \sigma_n \to \omega\sigma_n \end{array}$, $\mathbb{Z}_2 = \mathcal{C}$ symmetry: $\begin{array}{l} \sigma_n \to \sigma_n^\dagger \\ \tau_n \to \tau_n^\dagger \end{array}$

- Time reversal symmetry $\mathcal{T}$: $\sigma_n \to \sigma_n^*, \quad \tau_n \to \tau_n^*, \quad i \to -i$

- 3-State Potts phase transition at $J = f$
- Consider the operator:

$$V = \sum_{n=0}^{N-1} \sigma_n$$

- ✓ Breaks $\mathbb{Z}_2$ (and $\mathbb{Z}_3$)
- ✓ Not Hermitian
- ✓ $\mathbb{Z}_2\mathcal{T} = \mathcal{PT}$ symmetric

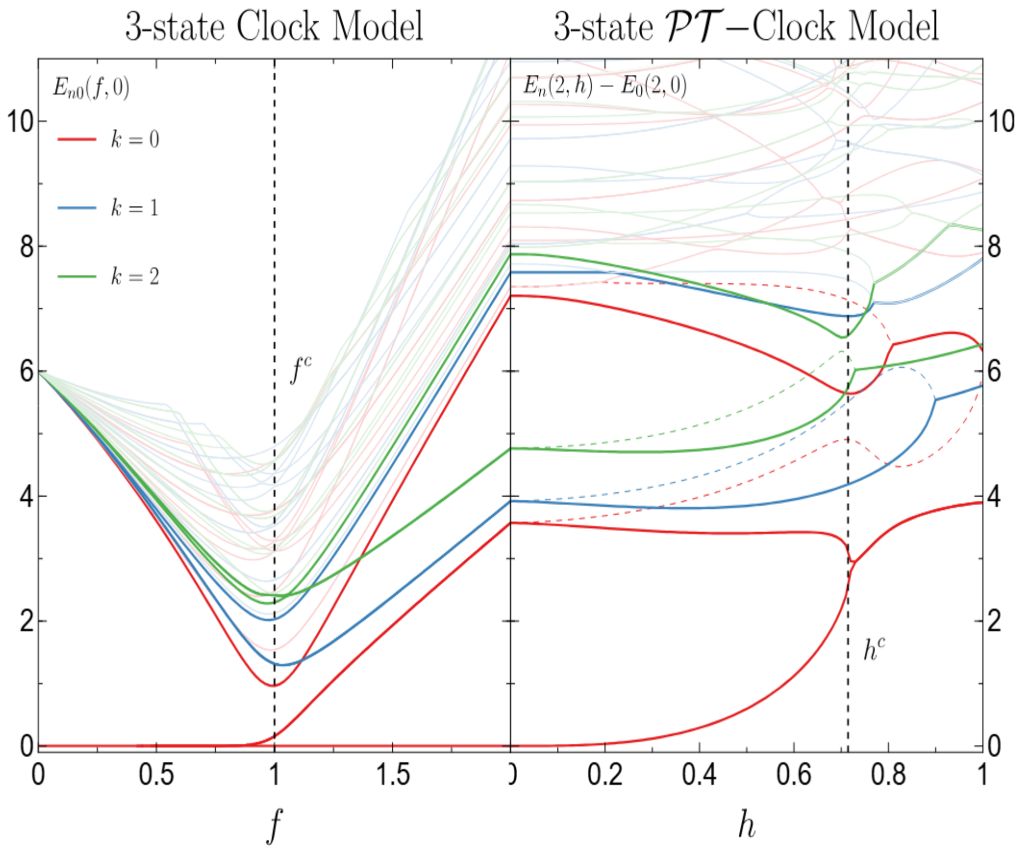
} Possible Yang-Lee deformation

- We add this term to the CM Hamiltonian: $H_{\text{PTCM}} = H_{\text{CM}} + hV$
- Look for critical point using Finite Size Scaling (FSS)

$$r_N = \frac{(N+1)\Delta E(N+1)}{N\Delta E(N)} = 1$$

[Hamer, Barber '81]

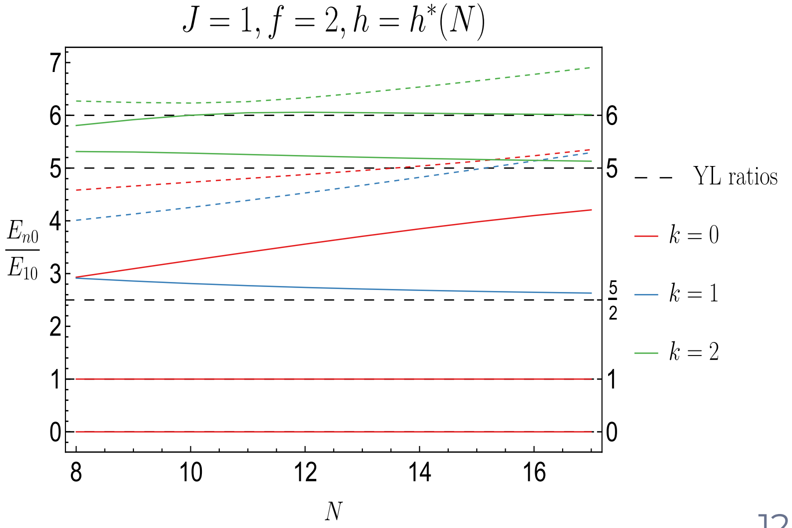
Spectrum for $J = 1$ and system size $N = 14$



- First scaling dimension $\Delta_\phi = -0.40099$
- YL scaling dimension: $\Delta_\phi = -0.4$

- Effective central charge: $\tilde{c} = 0.40864$
- YL effective central charge: $\tilde{c} = 0.4$

- Are dashed states massive states?



- Consider a GL description of CM:

$$S = \int d^d x \left[|\partial\Phi|^2 + \mu^2 |\Phi|^2 + g\left(\Phi^3 + \Phi^{*\,3}\right) + \lambda|\Phi|^4 \right]$$

[Amit, Roginsky '79; Zinati, Codello '18; Whitsitt, et all '18]

- Deform it by a linear term in Φ

$$\tilde{S} = \int d^d x \left[|\partial\Phi|^2 + \mu^2 |\Phi|^2 + g\left(\Phi^3 + \Phi^{*\,3}\right) + \lambda|\Phi|^4 + h\Phi \right]$$

- Write $\Phi = \phi_1 + i\phi_2$. Since ϕ_2 is $\mathcal{PT}$ symmetric and couples quadratically with ϕ_1, we integrate out ϕ_1 :

$$\tilde{S}_{eff} = \int d^d x \left[(\partial_\mu \phi_2)^2 + \tilde{\mu}^2 \phi_2^2 + \tilde{\lambda}\phi_2^4 + ih\phi_2 + \dots \right]$$

- Describes YL phase transition. [Fisher '78]

Conclusions & Future directions

- The YL recipe is not a general procedure to look for YL models but gives a starting point. If YL is present, it satisfies the conditions.
- Massive Schwinger model has a YL phase transition if we include an imaginary pseudo-scalar mass.
- 3-State Clock model seems to have massive + massless states. Massless ones describe a YL critical point.
- One can study the 3D version of the CM model using Fuzzy Sphere regularization and verify the 2D results (work in progress).

Thank you for your attention!