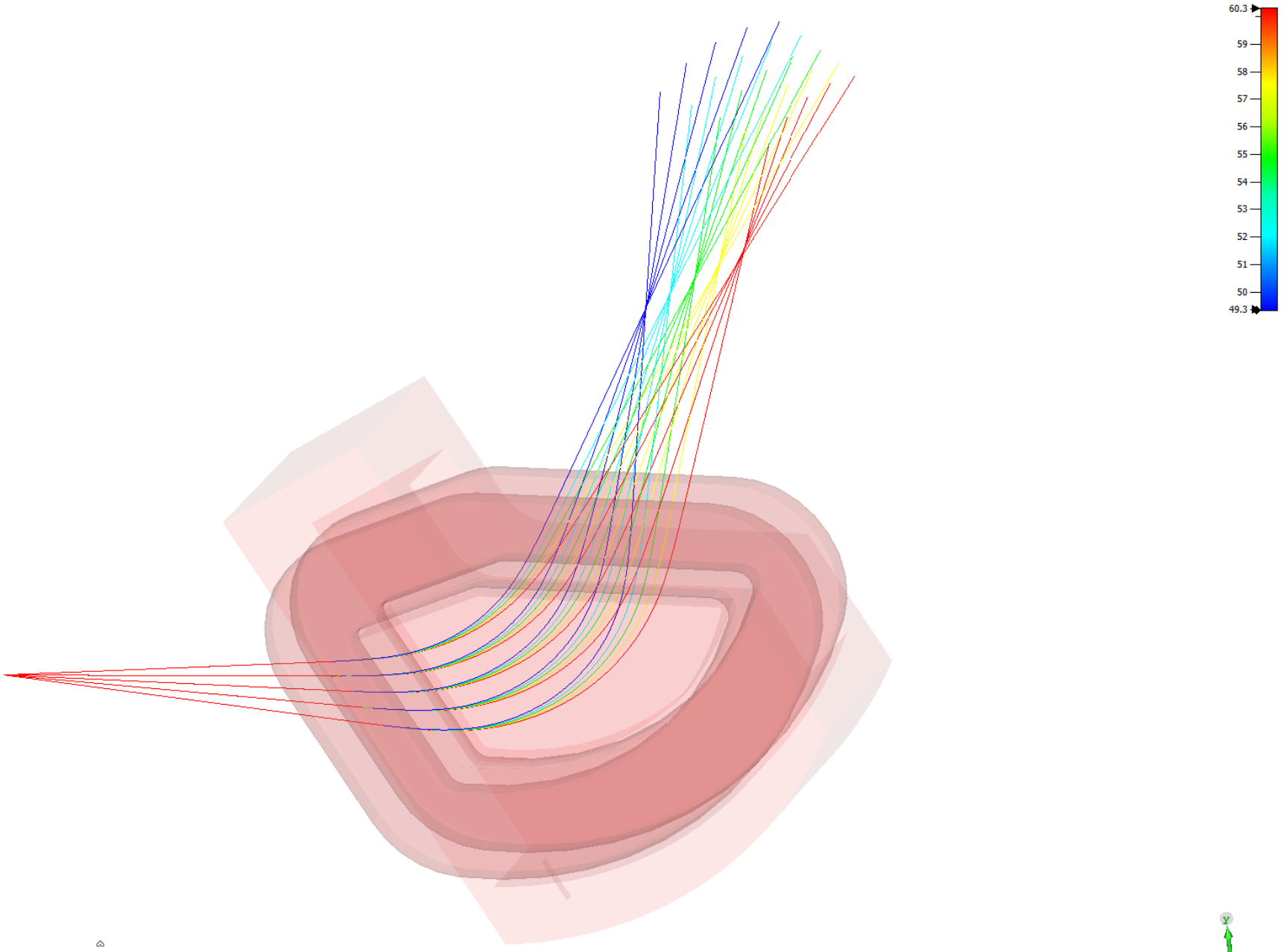


Trajectories

Type	Normed Momentum
Sample	101/101
Time	4.97285 ns
Maximum (Sample)	60.2741
Maximum	60.2741

Design parameter (guessed from CAD files...)

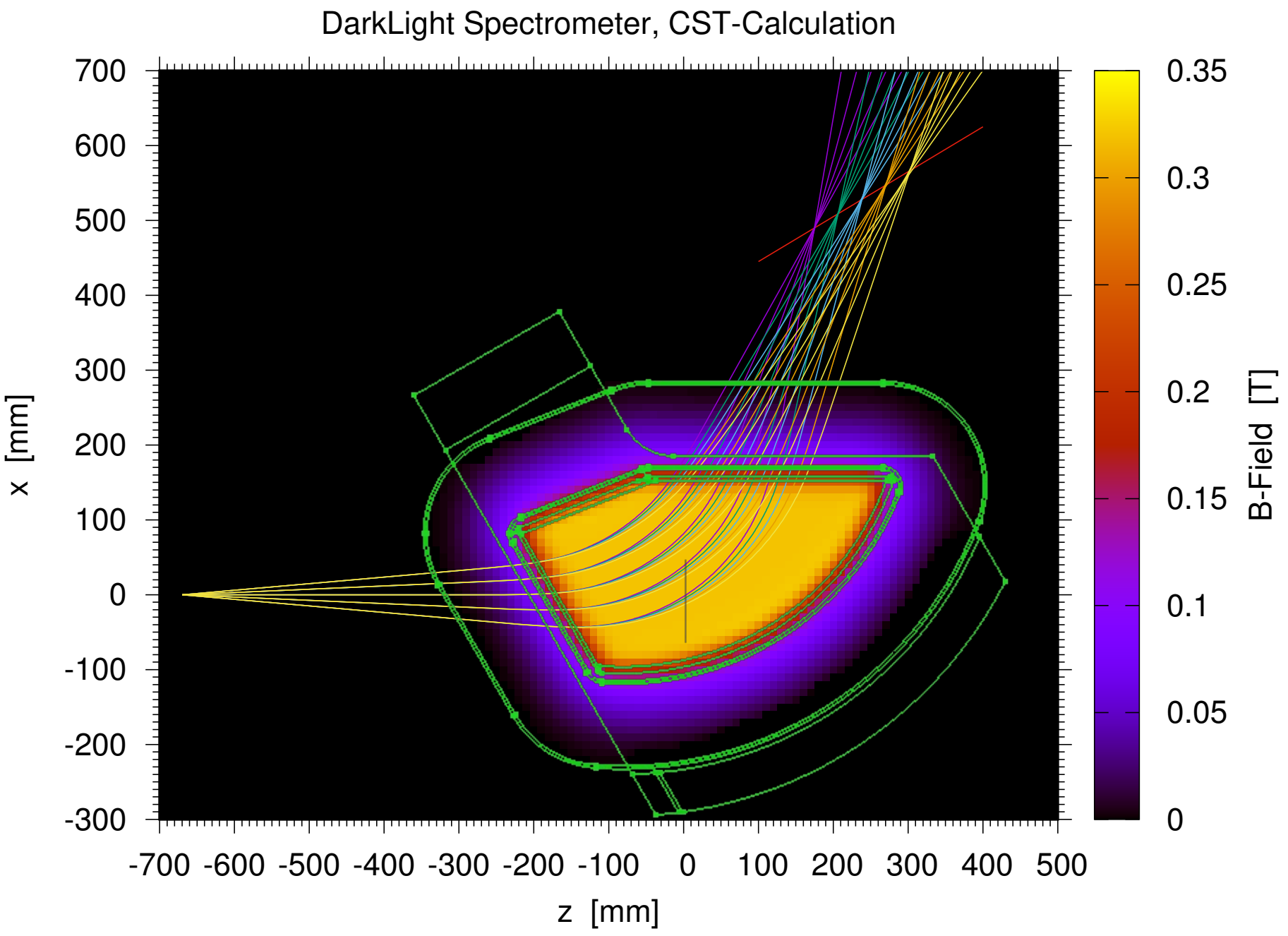
Design Momentum	p_{cent}	$=$	28 MeV/c
Bending Radius	R	$=$	300 mm
Bending Angle	θ	$=$	60 °
Entrance Angle	θ_{in}	$=$	30 °
Exit Angle	θ_{out}	$=$	30 °
Gap	a	$=$	40 mm

$\Rightarrow$ Magnetic Field $$|B| = \frac{p}{0.29979\,R}\,\frac{\mathrm{T\,m}}{\mathrm{MeV}} = 0.311\,\mathrm{T}$$

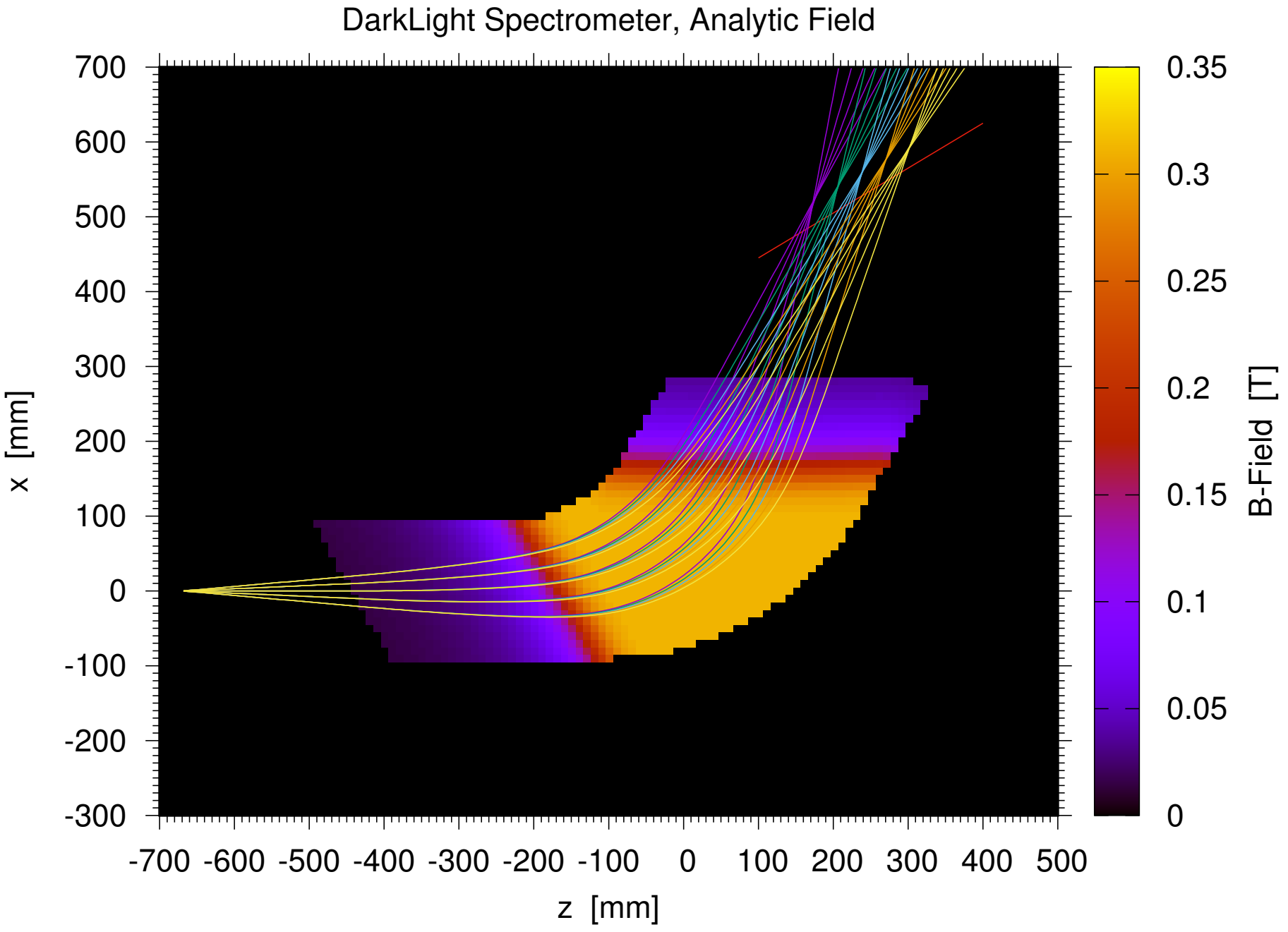
Current $$n \cdot I = \frac{a\,B}{\mu_0} = 9910\,\mathrm{A{\cdot}Turn}$$

Entrance Focal Length $$F_{in} = \frac{R}{\tan\theta_{in}} = 519.6\,\mathrm{mm}$$ $\leftarrow$ assumed target position

CST Calculation



Analytic Field Calculation (Fringe field: Rogowski-Profile)



Magnetic Spectrometers: Resolution Estimate

Optical mapping (assuming $x = 0$):

$$(x',\ \theta') = f\,(p,\ \theta)$$

Linearization (first order in p, θ)

$$(x',\ \theta') = (x'_0,\ \theta'_0) + \underbrace{\begin{pmatrix} \partial_p x' & \partial_\theta \theta' \\ \partial_p x' & \partial_\theta \theta' \end{pmatrix}}_{=:\ J} \cdot \begin{pmatrix} p \\ \theta \end{pmatrix} + O(p^2,\ p\theta,\ \theta^2)$$

Invert, choose coordinate system with $x'_0 = \theta'_0 = 0$

$$(p,\ \theta) \approx J^{-1} \cdot \begin{pmatrix} x' \\ \theta' \end{pmatrix}$$

Error Propagation (at one point $(p,\ \theta)$)

$$V = \begin{pmatrix} \sigma_p^2 & \sigma_{p\theta} \\ \sigma_{\theta p} & \sigma_\theta^2 \end{pmatrix} \approx J^{-1} \underbrace{\begin{pmatrix} \sigma_{x'}^2 & \sigma_{x'\theta'} \\ \sigma_{\theta' x'} & \sigma_{\theta'}^2 \end{pmatrix}}_{V'\text{: detector resolution}} \left(J^{-1}\right)^T$$

Set $\sigma_{x'} = \sqrt{\sigma^2_{x'\ \mathrm{detector}} + M^2 \sigma_x^2}$ to account for magnification and beam spot size.

- Tracking to calculate $(x', \theta') = f(p, \theta)$
- Numerical derivatives, e.g. $(\partial_p x', \partial_p \theta') = \frac{f(p+\varepsilon,\ \theta) - f(p-\varepsilon,\ \theta)}{2\varepsilon} \quad \Rightarrow \quad J \quad \Rightarrow \quad J^{-1}$
- Resolutions as $\sqrt{\mathrm{diag}(V)}$, covariances $\sigma_{p\theta}$ are negligible for focussing spectrometer
- Numerical challenge: numerical derivative of a numerical track

DarkLight Spectrometer Parameter

- Magnification $M = -0.66811$
- Imaging:

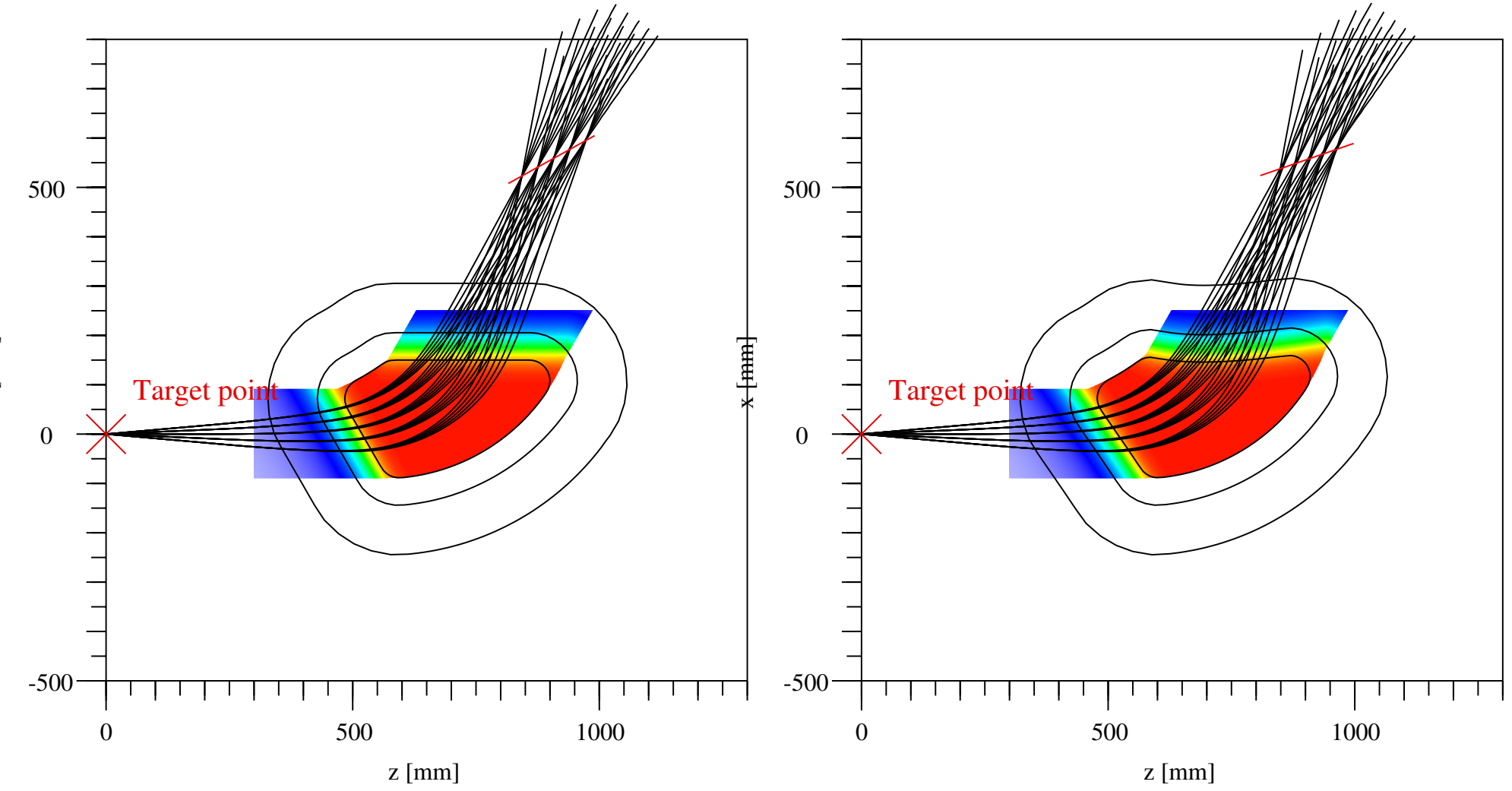
$$J = \begin{pmatrix} \partial_z p & \partial_\phi p & \partial_\theta p & \partial_y p \\ \partial_z \phi & \partial_\phi \phi & \partial_\theta \phi & \partial_y \phi \\ \partial_z \theta & \partial_\phi \theta & \partial_\theta \theta & \partial_y \theta \\ \partial_z y & \partial_\phi y & \partial_\theta y & \partial_y y \end{pmatrix} = \begin{pmatrix} 0.00122 & -0.00385 & 0.00000 & 0.00000 \\ 0.03359 & 0.70254 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.83534 & 0.06338 \\ 0.00000 & 0.00000 & 15.03599 & 0.04806 \end{pmatrix}$$

- Resolutions:

Assumed Detector Resolutions:	beam size	=	1.0000 mm	
	x(disp)	=	0.10000 mm	
	y	=	0.10000 mm	
	disp	=	3.49066 mrad	= 0.2°
	non-d	=	3.49066 mrad	= 0.2°
Target Resolutions:	$\mathrm{d}p/p$	=	0.00113	
	$\mathrm{d}\phi$	=	2.48410 mrad	= 0.14°
	$\mathrm{d}\theta$	=	1.47463 mrad	= 0.0844°
	$\mathrm{d}y$	=	1.50363 mm	

Numerical Optimization

Polynomials as entrance/exit shape:



⇒ Numerical optimization for fine-tuning of optical properties

Summary

What I can provide

- Check of your FEM calculations, including tracking (easy!)
- Check of resolutions/optics
- Numerical optimization (really neccessary?)
- ...