Challenges in Understanding ~~nuclear and~~ hadron physics from QCD

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Symposium on QCD and Nuclei, LNS/MIT, Oct 10,2021

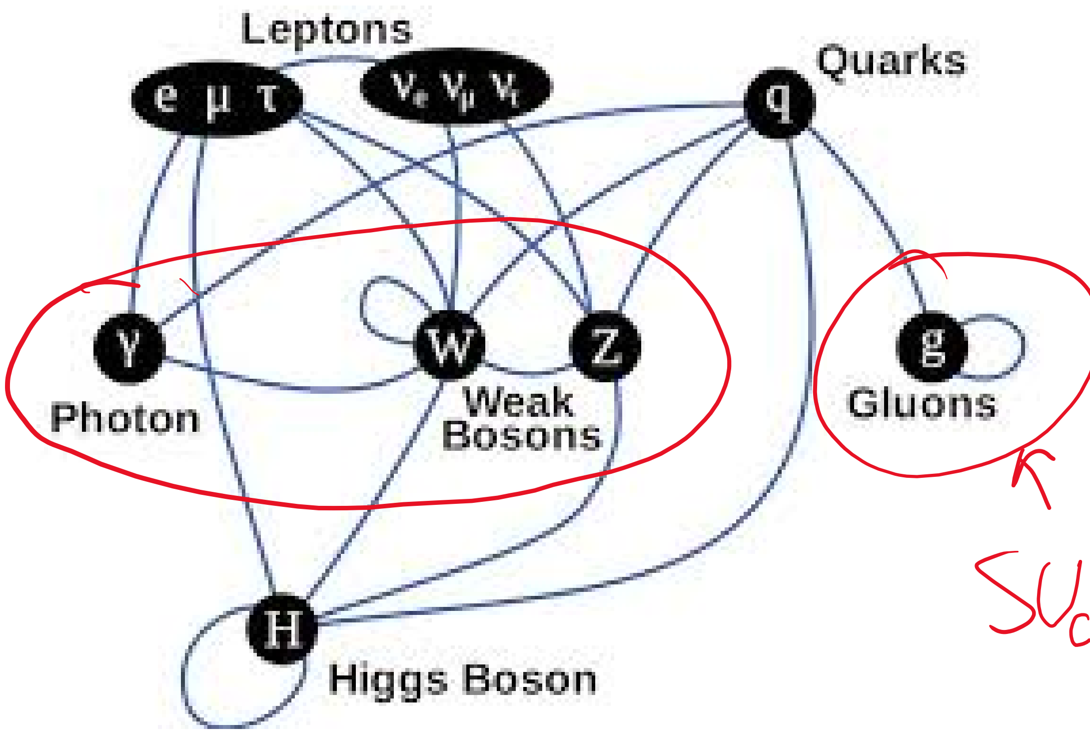
Outline

- The standard model & successes
- Challenges of non-perturbative QCD
- Bottom-up strategy: how is the whole made of individual parts?
  - Mass of the proton and quantum anomalous energy
  - Spin structure of the proton
  - Momentum-current multiples and gravitational fields near a nucleon
- Outlook

The standard model & successes

The Standard Model

- U(1)xSU(2)xSU(3): Quantum Field Theories



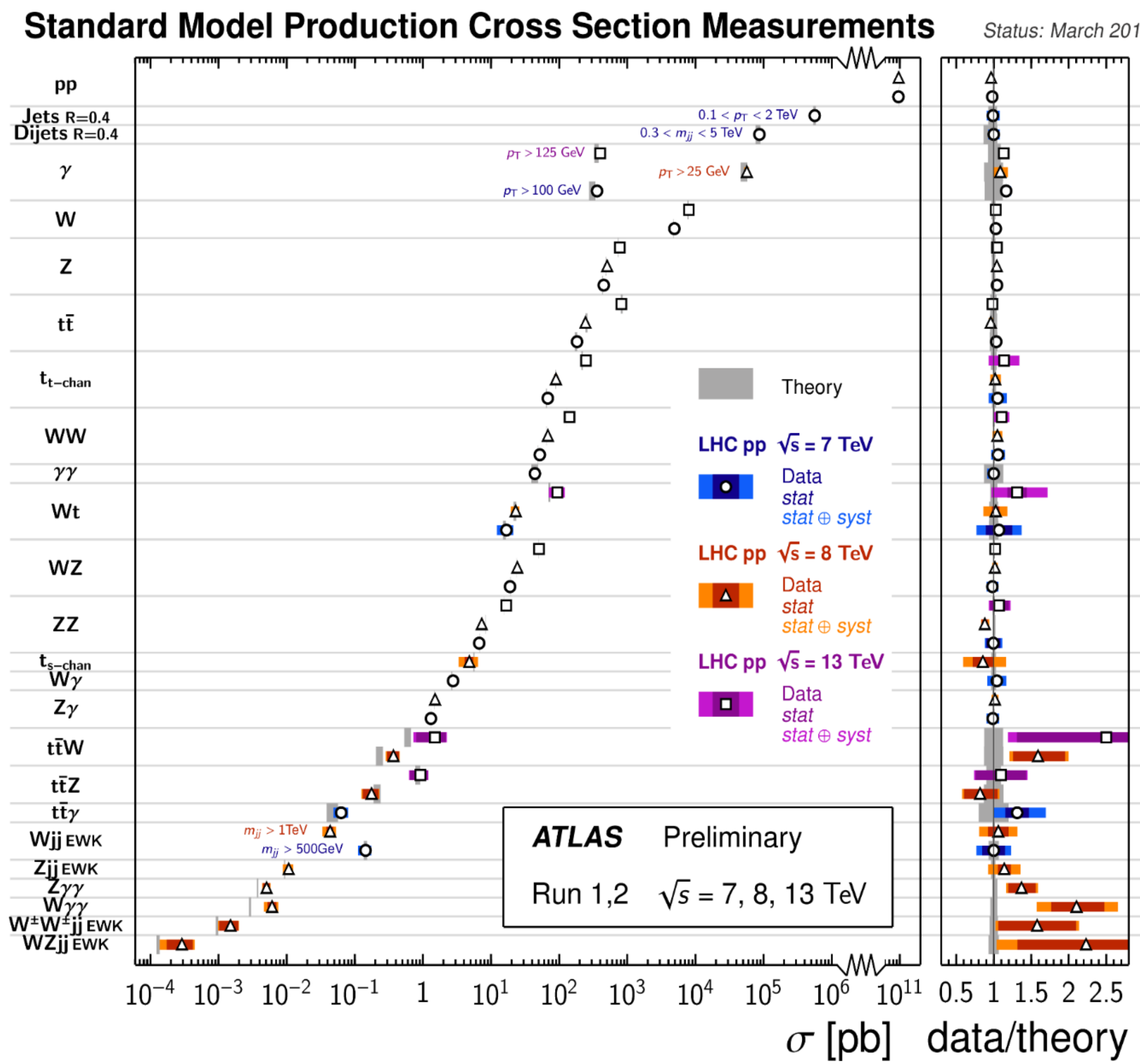
$U_Y(1) \times SU_L(2)$

$SU_c(3)$

Standard model successes:

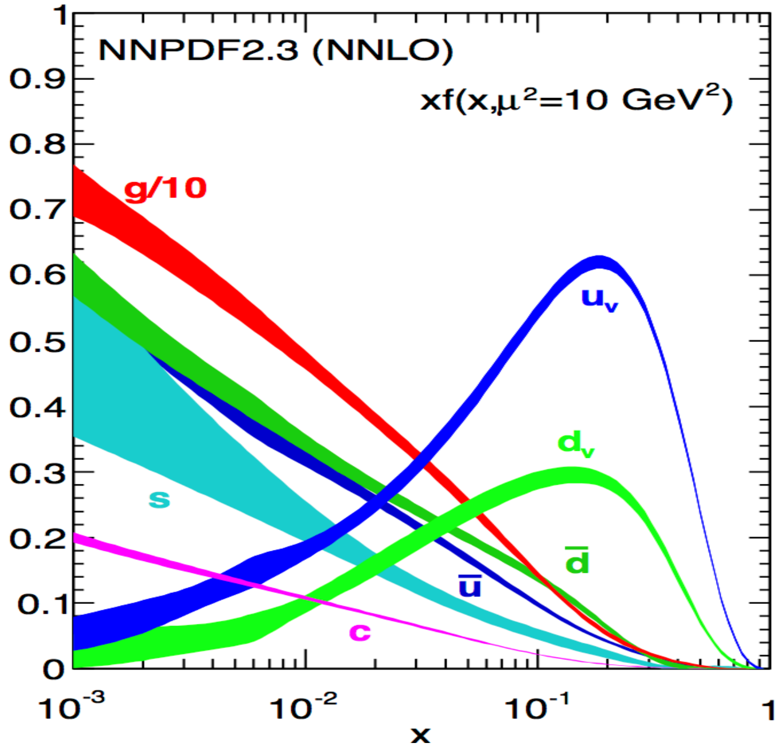
- Hugely successful in explaining many of the experimental data from LHC
  - Electroweak processes
  - High-energy QCD processes

Perturbation theory works!　(LHC)



Phenomenological PDFs:

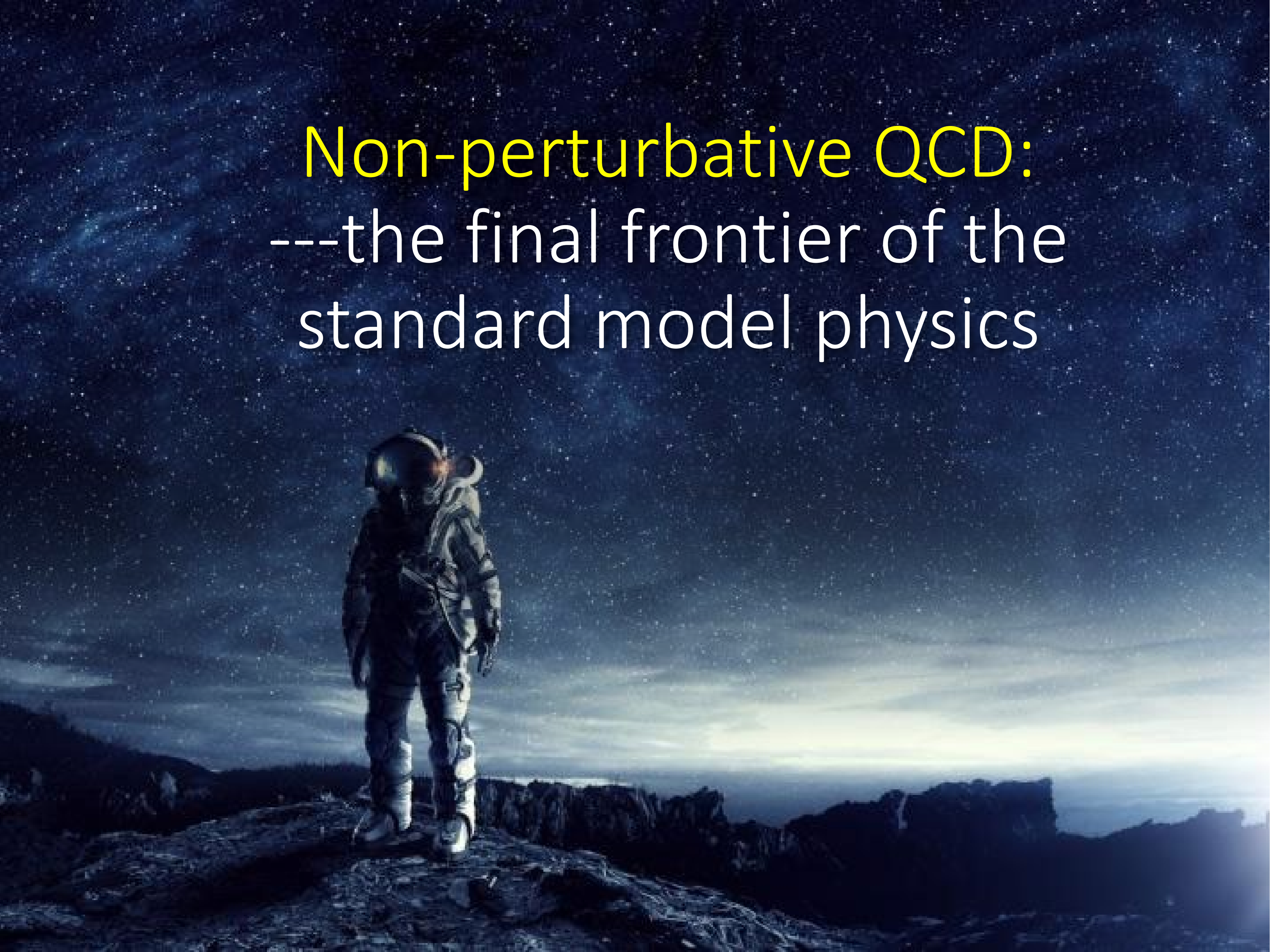
- Extracted from experimental data (50 yrs)



J. Gao, et al, Phys. Rept. 742 (2018) 1-121

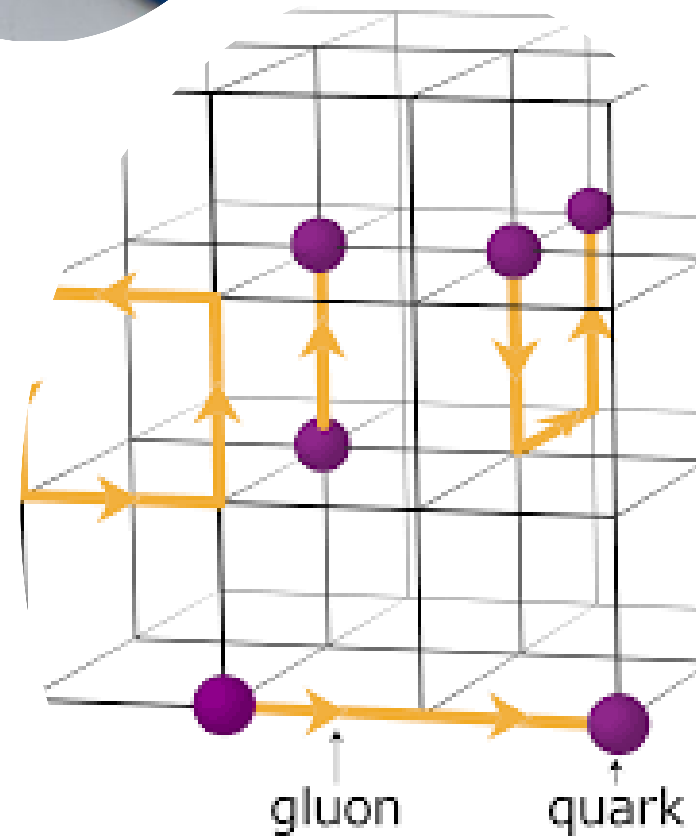
- Not a calculation from non-pert QCD!

Non-perturbative QCD:
---the final frontier of the standard model physics



We can calculate!

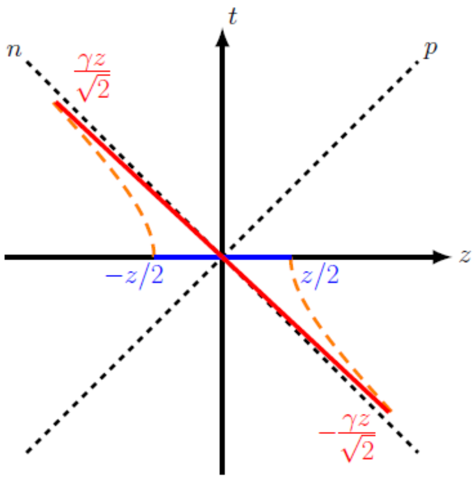
- Wilson's lattice field theory
- Only UV-regularization that has been successfully implement non-perturbatively
- Using Feynman path integral formulation of quantum mechanics
- Fantastic progress!

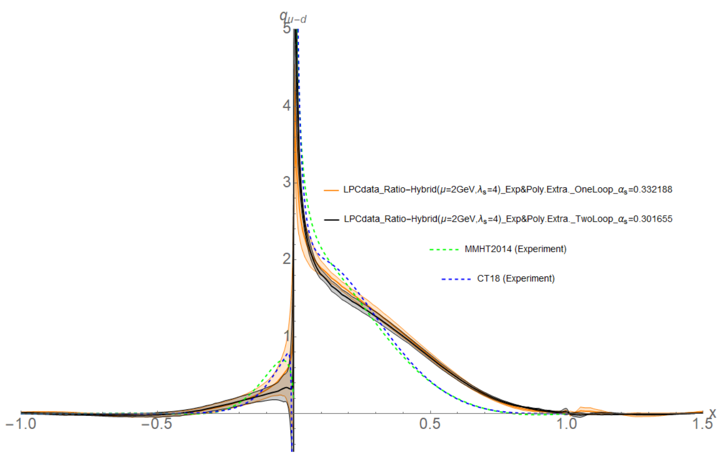


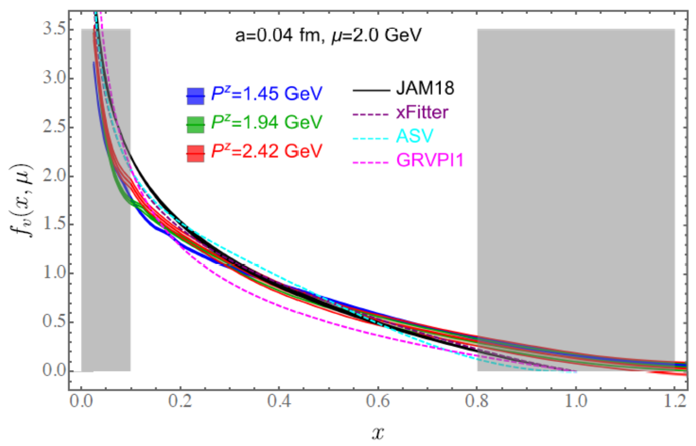
Large-momentum effective theory

- Parton physics (light-cone correlations) from Euclidean lattice

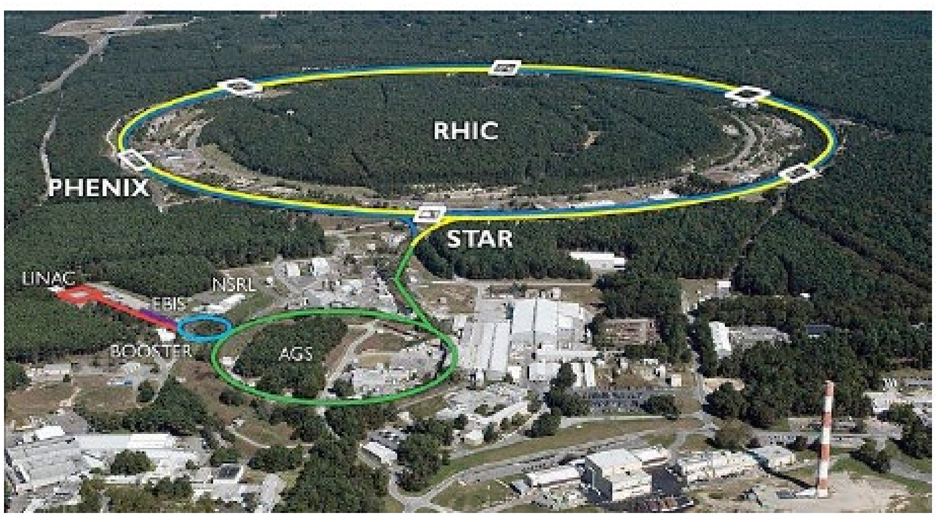
  Ji, Liu, Liu, Zhang, Zhao, Review of Modern Physics, 93 (2021) 3

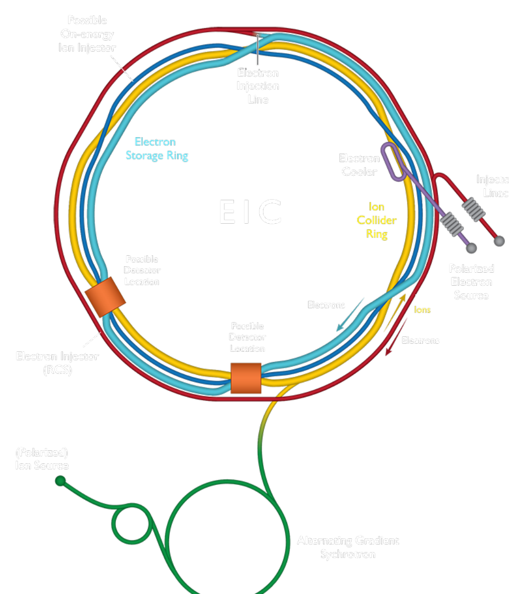






We can do measurements!





But, we don't have much understanding!

- How does non-perturbative QCD works?

  We may not have the right mathematics.

https://www.quantamagazine.org/the-mystery-at-the-heart-of-physics-that-only-math-can-solve-20210610/

SERIES MATH MEETS QFT

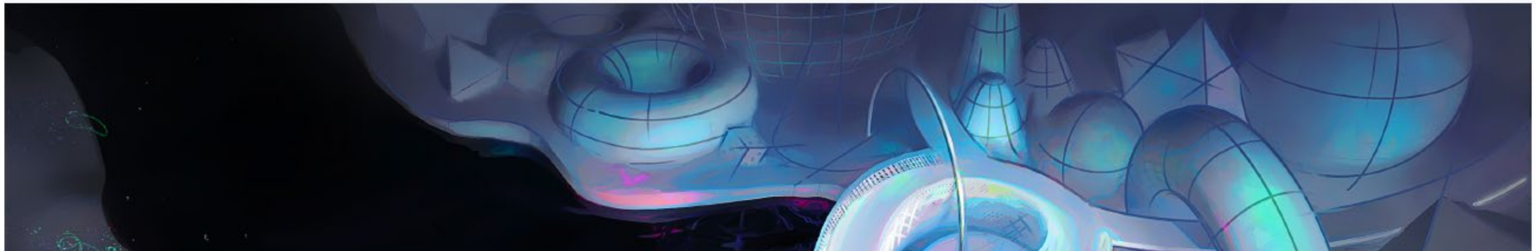
The Mystery at the Heart of Physics That Only Math Can Solve

By KEVIN HARTNETT

June 10, 2021

The accelerating effort to understand the mathematics of quantum field theory will have profound consequences for both math and physics.

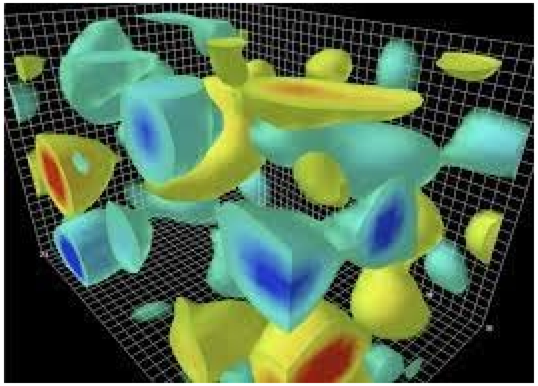
81



Practical approaches

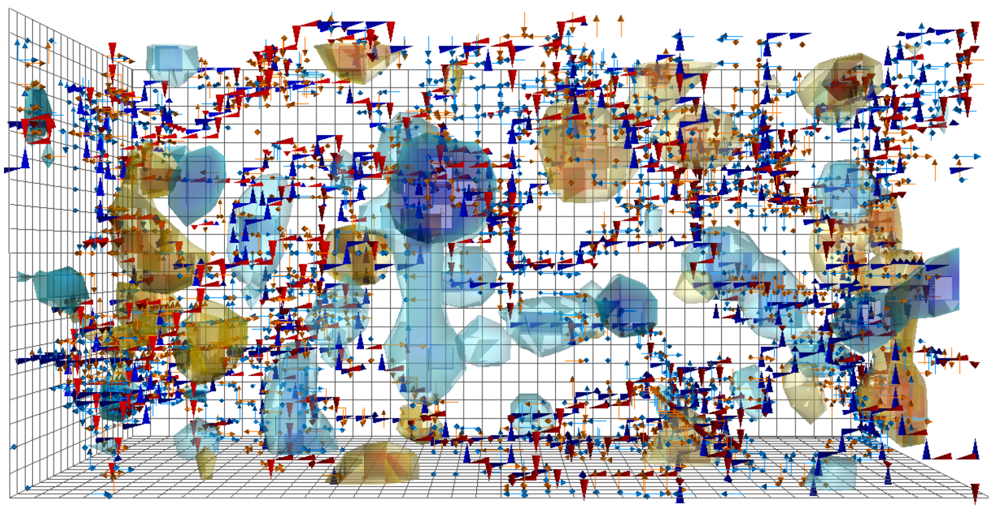
- Top down:
  - To understand the important gauge field configurations that goes into the Feynman path integrals.
  - Large N_C, AdS/CFT duality, ….
- Bottom up:

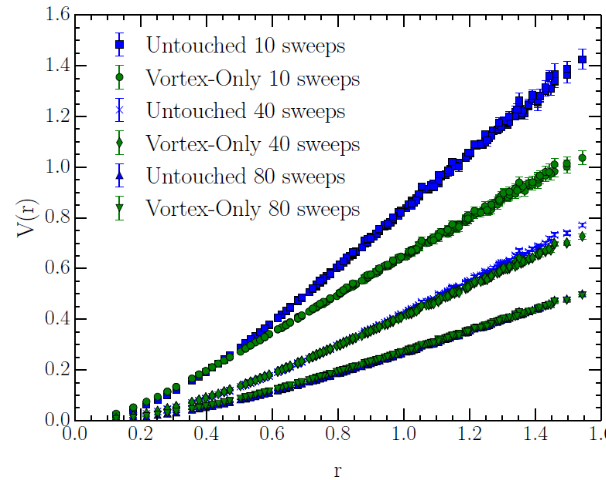
  To break up the physics observables into more extensive quantities.



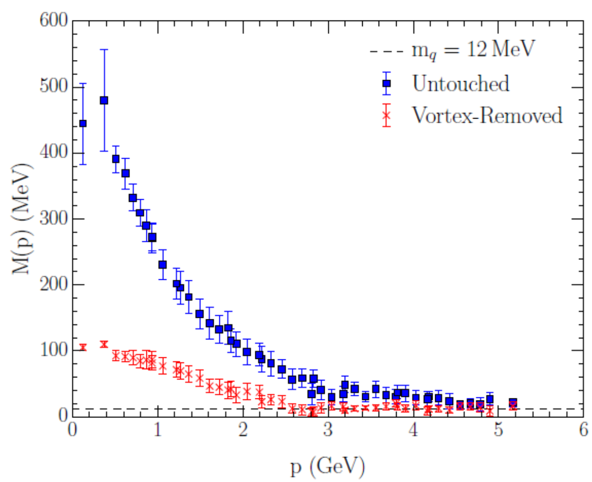
Negele et al, 1990’s
Shuryak et al, 1990’s
Leinweber et al, 2000’s
….

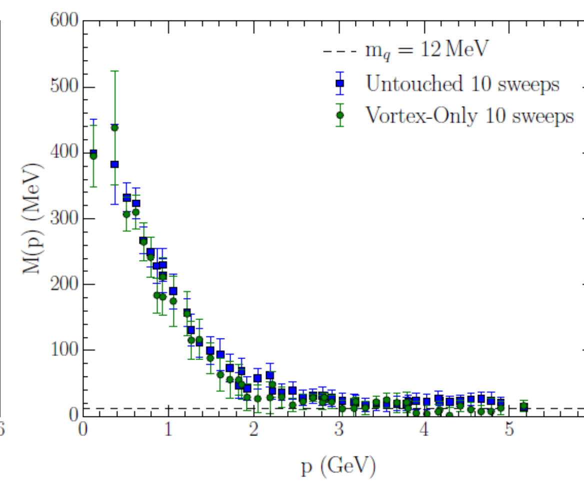
Center-vertices: confinement & chiral symmetry breaking





Leinweber et al. *J.Phys.G* 44 (2017) 12, 125002





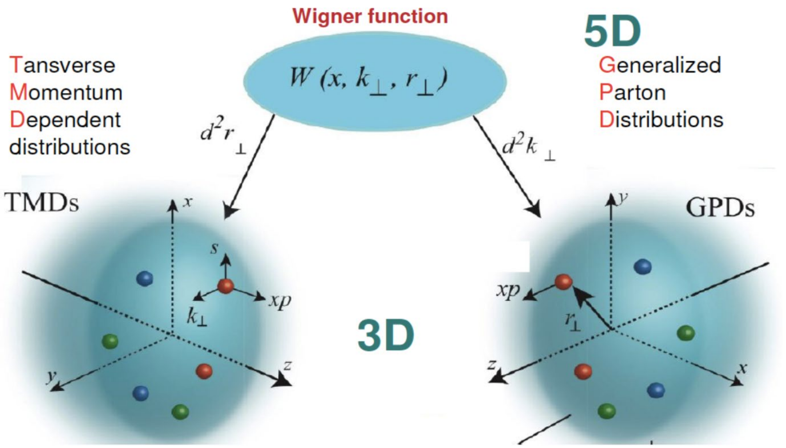
Bottom-up strategy: how is the whole made from individual parts?

Methodology

- Try to understand the details of the quark and gluon distributions that give rises to the global properties of hadrons and nuclei.
- Sum rules for mass and spin
- Form factors: spatial distributions
- PDFs: momentum-space distributions
- Wigner distributions

  GPDs & TMDs:

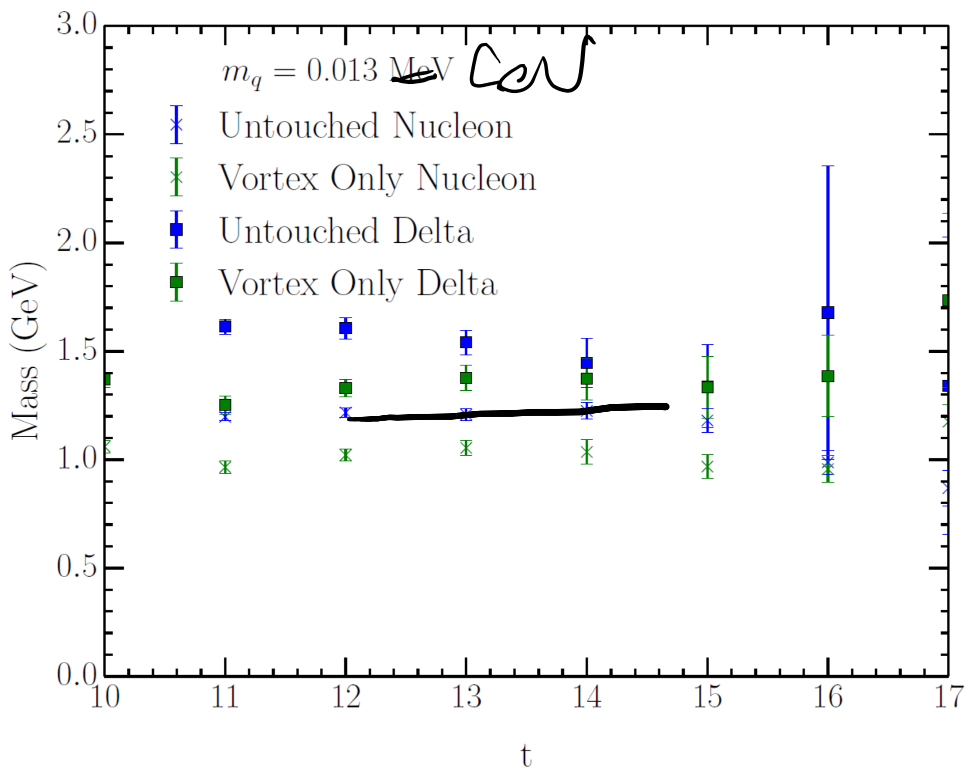
  3D structure of the nucleon



The nucleon mass

The nucleon & Δ masses (D. Leinweber et al, 2017)

- Lattice QCD: $M = E_{\vec{P}=0} = \langle P | H_{QCD} | P \rangle_{\vec{P}=0}$



Mass as internal energy

- Internal mass as a store of energy

$$\mathrm{M}c^2 = \langle N | \hat{H}_{QCD} | N \rangle |_{\vec{P}=0}$$

- For any relativistic system, the Hamiltonian can be separated into two terms (Ji, PRL,1995),

$$\hat{H}_{QCD} = \hat{H}_T + \hat{H}_S$$

This separation is a fundamental property of special relativity and both parts are scheme independent & scale invariant

Tensor and scalar energies

- Tensor energy

$$E_T = \langle H_T \rangle$$

is related to the usual kinetic and potential energy sources.

- Scalar energy

$$E_S = \langle H_S \rangle$$

is related to related to scale-breaking properties of the theory ($\partial^\mu j_{D\mu} \sim H_s$), such as

  - Quark mass m_q
  - Trace anomaly (quantum breaking of scale symmetry).

Relativistic virial theorem

- As an important feature of relativity, one can show

  $$E_T = 3E_S \qquad \text{(virial theorem)}$$

  3 is the dimension of space.
- Scalar energy sets the scale of the tensor energy (kinetic and potential energies of the system).
- In non-relativistic limit of QED & gravity, it reduces

  $$\langle V \rangle = -2\langle T \rangle$$

  kinetic energy sets the scale for potential energy!

Scalar energy in QCD

- What is the scalar Hamiltonian in QCD?

$$H_S = H_m + H_a$$

$$H_m = \frac{1}{4}\int d^3x\, m\bar{\psi}\psi$$

$$H_a = \frac{1}{4}\int d^3\vec{x}\left(\frac{\beta(g_0)}{2g_0}F^2 + m_0\gamma_m\bar{\psi}\psi\right)$$

- Both H_m and H_a are scale invariant. H_a depends on the running of couplings. Also be derived from dilatation in time. X. Ji & Y. Z. Liu: 2101.04483
- In the massless limit, $H_s \sim H_a \sim \beta F^2$

Quantum Anomalous Energy (QAE)

- There is an anomalous scalar contribution to the nucleon mass

$$E_a \sim \langle N|F^2|N\rangle \sim \frac{1}{4} m_N c^2$$

- This particular contribution comes from the scalar response to the presence of the quarks (Similar to MIT bag constant). Its contribution is also similar to Higgs mechanism in electroweak theory, with

$$\phi = F^2 - \langle 0|F^2|0\rangle$$

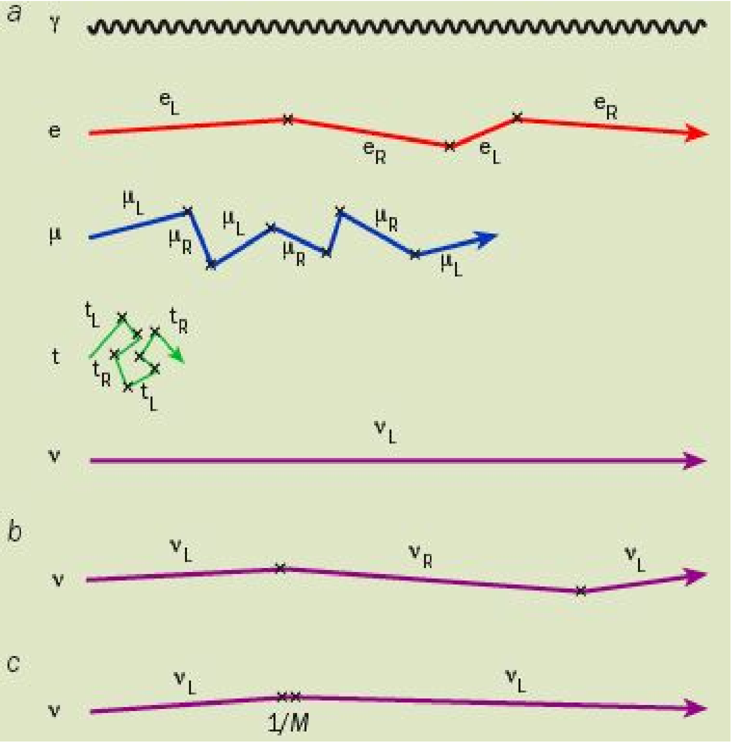
as a dynamical Higgs field.

X. Ji & Y. Liu, e-Print: 2101.04483

Masses of electrons and leptons: Higgs mechanism

- There is a scalar field H, which interacts with the fermions $g\bar{\psi}\psi H$. H acquires an expectation value in the vacuum after SSB, $\langle H \rangle = v = 246$ GeV, hence the fermion mass,

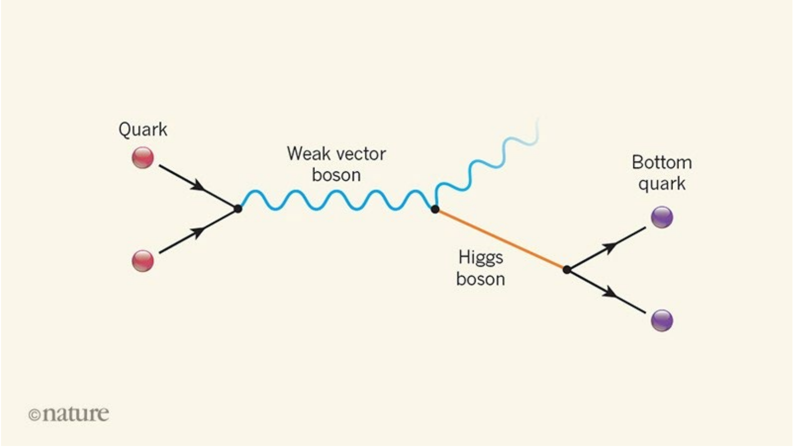
m= gv

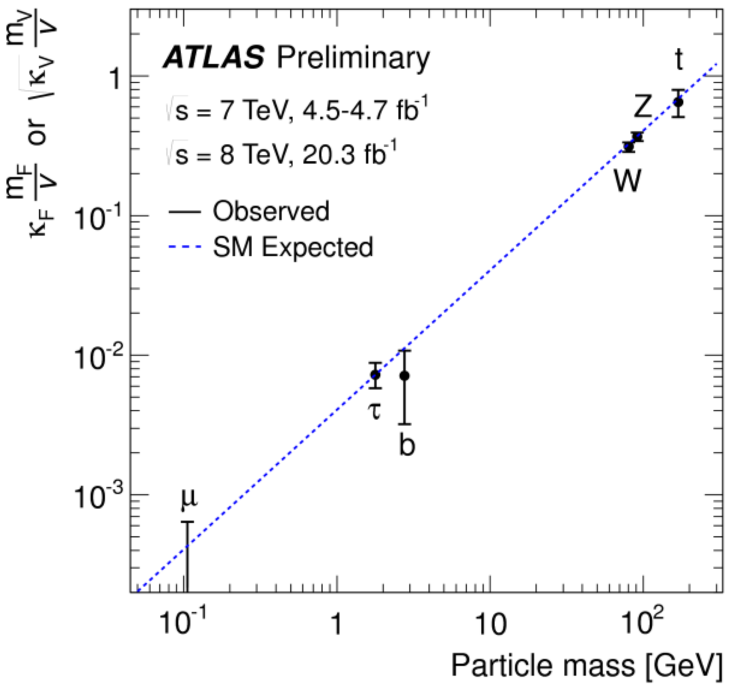


Test of Higgs mechanism

- Dynamical Higgs-boson-fermion coupling , g=m/v

$$g\,\bar{\psi}\psi h$$

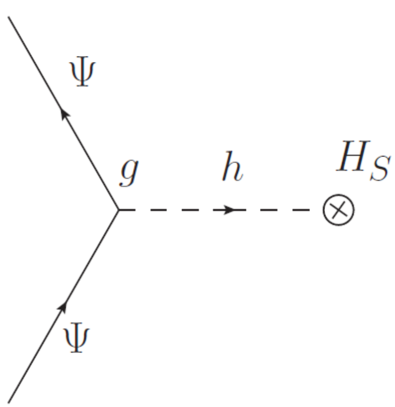


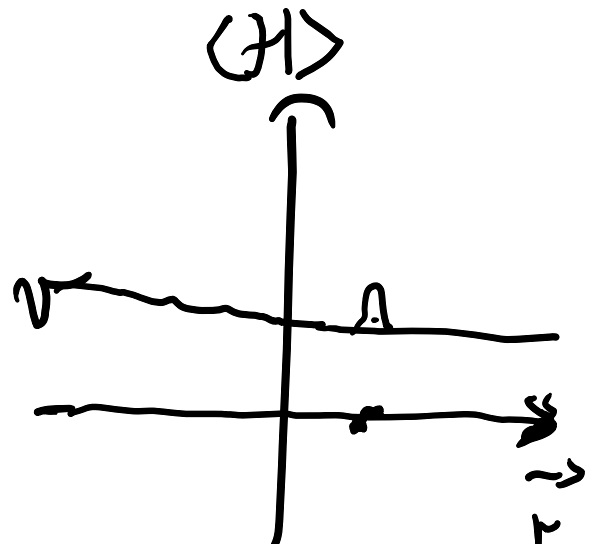


Dynamical picture of the fermion mass

- Part of the fermion mass comes from the dynamical excitation of the higgs field in the presence of fermion

$$m_f \sim \langle f|H_S|f\rangle \sim \langle f|h|f\rangle$$



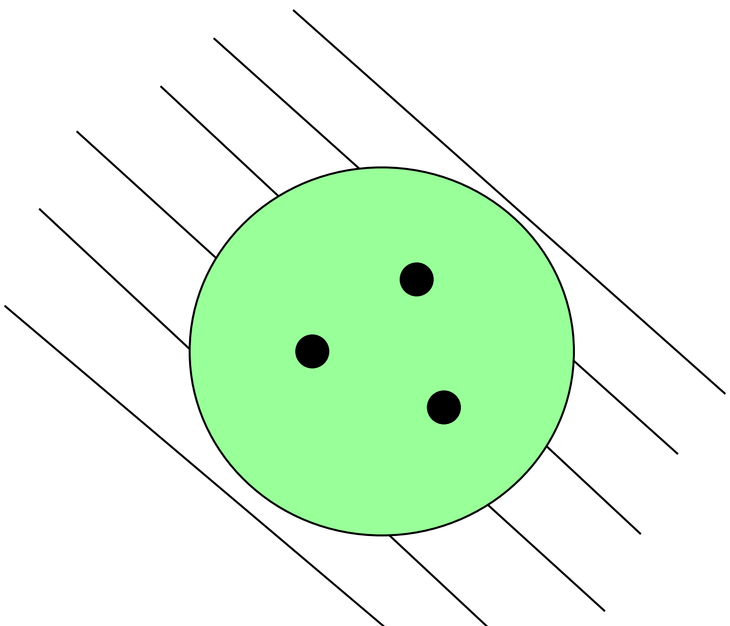


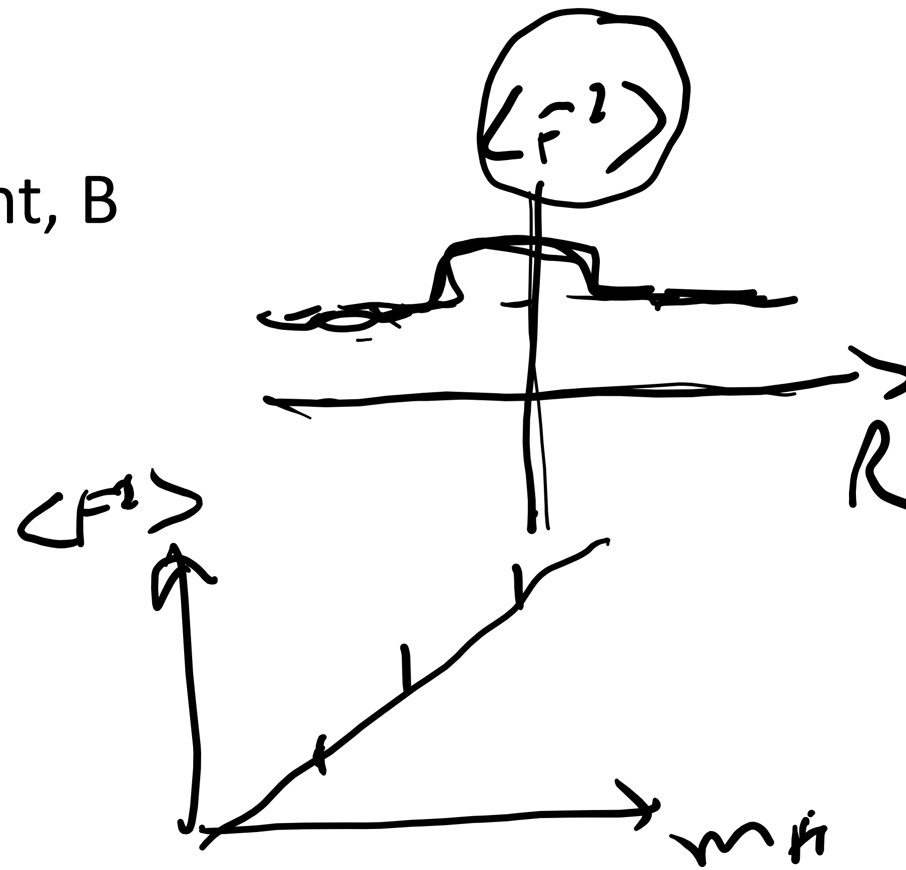
Static response of scalar field F^2

- Directly measure the static response of the F^2 in the nucleon or any other particles

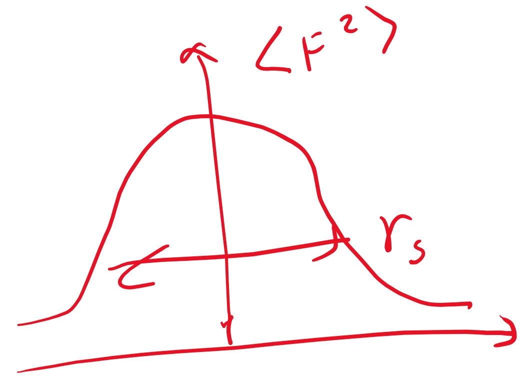
$$\langle H | F^2 | H \rangle$$

Similar to MIT bag constant, B





Scalar form factor & the MIT Bag



- Scalar field distribution:

$$\langle P' \left| T^{\mu}_{\mu} \right| P \rangle = \bar{u}\left(P'\right) u(P) G_s(Q^2) \ ,$$

where,

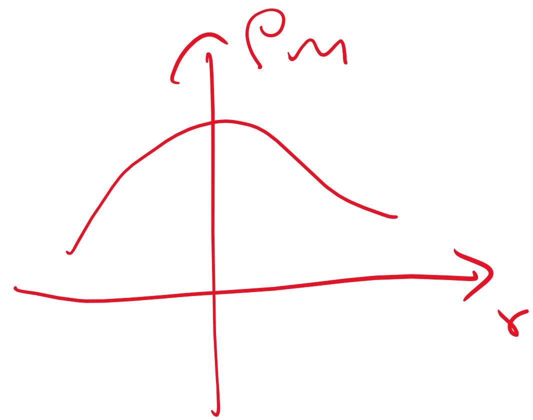
$$G_s(Q^2) = \left[M A(Q^2) - B(Q^2)\frac{Q^2}{4M} + C(Q^2)\frac{3Q^2}{M} \right]$$

- A & B & C are form factors of QCD energy-momentum tensor (EMT)

$$\langle P' \left| T^{\mu\nu} \right| P \rangle = \bar{u}\left(P'\right) \Big[A\left(Q^2\right) \gamma^{(\mu} \bar{P}^{\nu)}$$
$$+ B\left(Q^2\right) \bar{P}^{(\mu} i\sigma^{\nu)\alpha} q_\alpha / 2M$$
$$+ C\left(Q^2\right) \left(q^\mu q^\nu - g^{\mu\nu} q^2\right) / M \Big] u(P) \ ,$$

Mass distribution and radius

- Mass distribution can be obtained from the EMT of the QCD, $T^{\mu\nu}$



- Mass form factor

$$\langle P' \left| T^{00} \right| P \rangle = \bar{u}\left(P'\right) u(P) G_m(Q^2) \; .$$

where

$$G_m(Q^2) = \left[M A\left(Q^2\right) - B(Q^2)\frac{Q^2}{4M} + C(Q^2)\frac{Q^2}{M} \right]$$

- Mass radius

$$\langle r^2 \rangle_{s,m} = -6 \frac{dG_{s,m}(Q^2)}{dQ^2} \; ,$$

$$\boxed{\langle r^2 \rangle_s - \langle r^2 \rangle_m = -12 \frac{C(0)}{M^2}}$$

$$\langle r^2 \rangle_s = -6 \frac{dA(Q^2)}{dQ^2} - 18 \frac{C(0)}{M^2}$$

$$\langle r^2 \rangle_m = -6 \frac{dA(Q^2)}{dQ^2} - 6 \frac{C(0)}{M^2} \; ,$$

$C(0) < 0$?

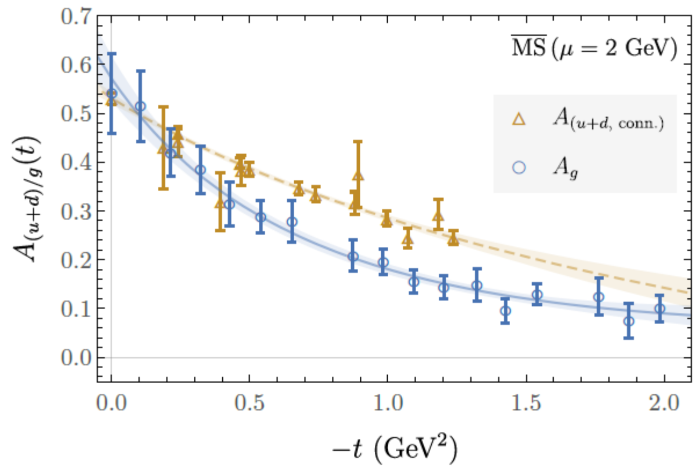
Lattice calculations

- Radius from A-FF:

Hagler, Negele et al (2008)

Shanahan et al (2018)

$$\langle r^2 \rangle_A = (0.5\ fm)^2$$

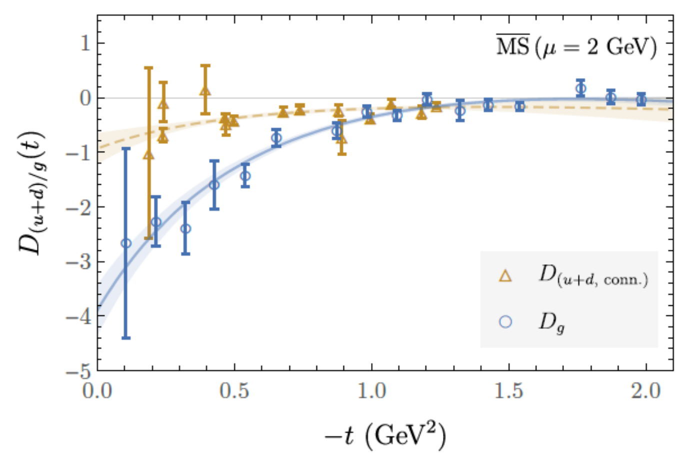


- C-FF contribution

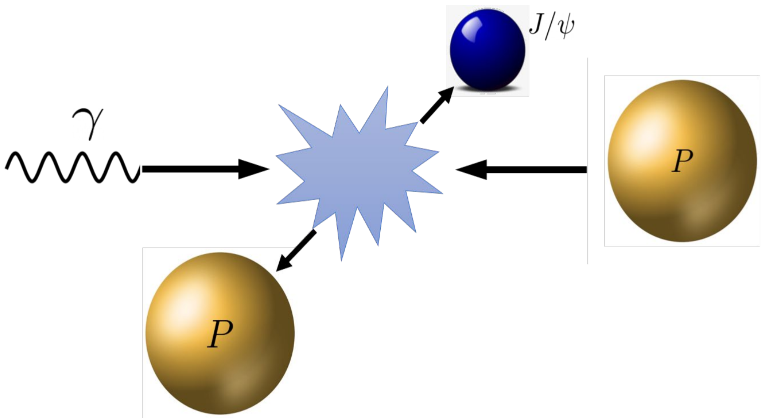
D = - 5.0

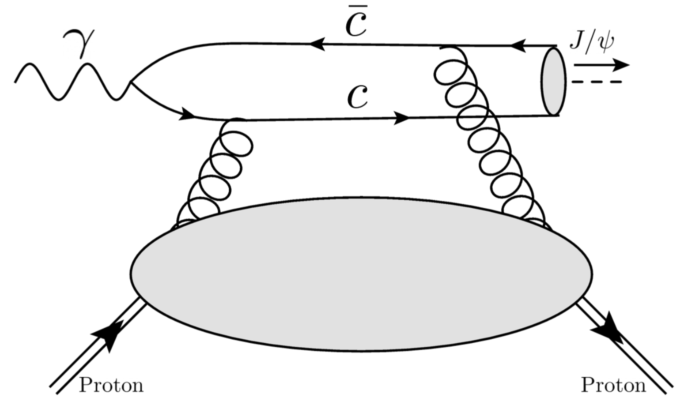
$$\langle r^2 \rangle_s = (1.1\ fm)^2$$

$$\langle r^2 \rangle_m = (0.75\ fm)^2$$



Heavy-quarkonium production





$$\frac{d\sigma}{dt} = \frac{e^2 e_Q^2}{16\pi \left(W^2 - M_N^2\right)^2} \frac{1}{2} \sum_{\text{polarization}} \left|\mathcal{M}(\varepsilon_V, \varepsilon)\right|^2 \ ,$$

$$= \frac{\alpha_{\rm EM} e_Q^2}{4\left(W^2 - M_N^2\right)^2} \frac{(16\pi\alpha_S)^2}{3M_V^3} |\psi_{\rm NR}(0)|^2 |G(t,\xi)|^2 \ ,$$

$$G(t,\xi) = \frac{1}{2\xi^2 (\bar{P}^+)^2} \left\langle P' \left| T_g^{++} \right| P \right\rangle \ ,$$

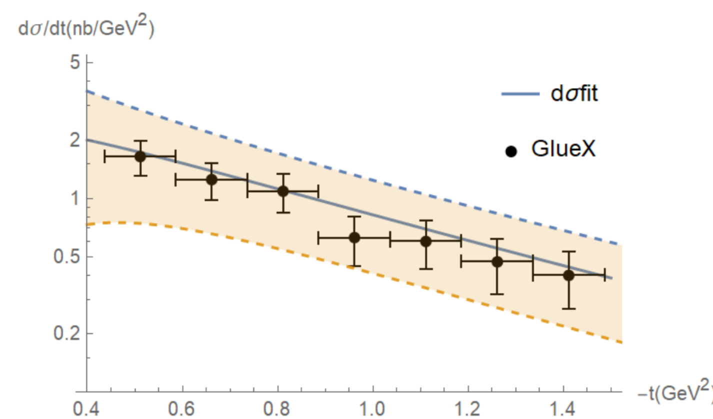


FIG. 7: Fit differential cross section for J/ψ production compared with the differential cross section at $W = 4.58$ GeV measured at GlueX [36].

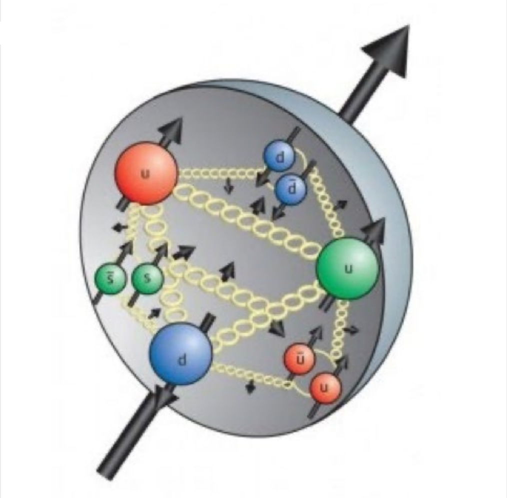
Meziani et al, Hall-C J/psi data

The proton spin structure

X. Ji, F. Yuan & Y. Zhao,

Nature Review Phys. 3 (2021) 1, 27

K. Shiells, CFNS talk, Oct, 7, 2021

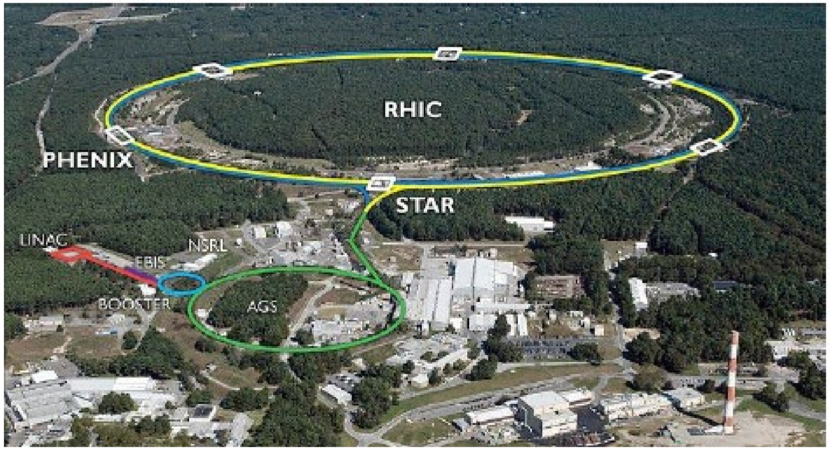


Jaffe-Manohar sum rule

- For longitudinal spin, in the infinite-momentum frame.
  - Jaffe & Manohar, 1990

$$\frac{1}{2} = \frac{1}{2}\ \Delta\Sigma + \Delta G + \ell_q^z + \ell_g^z$$

- Measuring ΔG in experiment has been a great drive for spin physics program for many years.



Understanding ΔG

- There is no gauge-invariant local axial vector corresponding to the gluon spin.
- There is a gauge-dependent candidate for ΔG

$$\vec{S}_g = \overrightarrow{E_a} \times \vec{A}_a$$

- In the infinite-momentum limit, the gauge-dependent part of the above operator vanishes

X. Ji, J. Zhang & Y. Zhao Phys.Rev.Lett. 111 (2013)

- This provides a recipe for lattice QCD calculation

First calculation (Yang et al, PRL (2017))

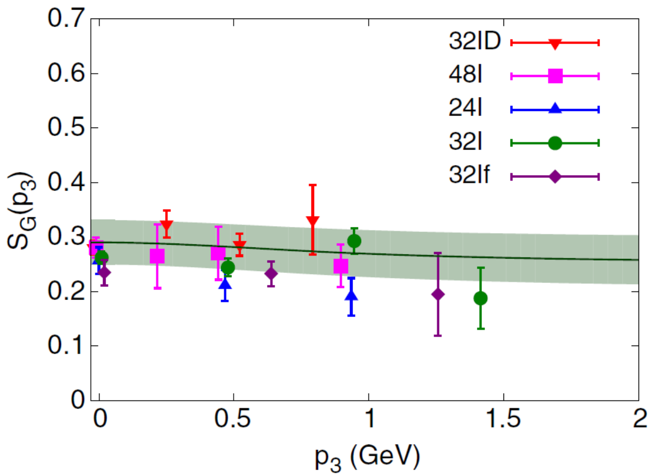
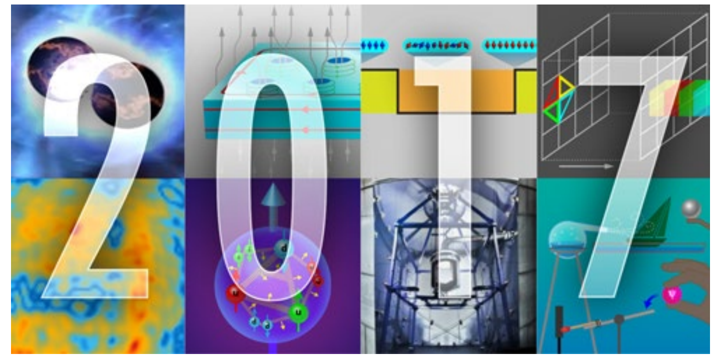


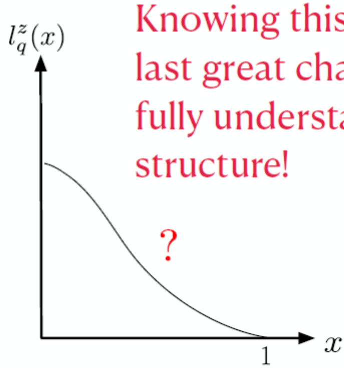
FIG. 4. The results extrapolated to the physical pion mass as a function of the absolute value of $\vec{p} = (0, 0, p_3)$, on all the five ensembles. All the results have been converted to $\overline{\mathrm{MS}}$ at $\mu^2 = 10\ \mathrm{GeV}^2$. The data on several ensembles are shifted horizontally to enhance the legibility. The green band shows the frame dependence of the global fit [with the empirical form in Eq. (11)] of the results.



Gluons Provide Half of the Proton's Spin
The gluons that bind quarks together in nucleons provide a considerable chunk of the proton's total spin. That was the conclusion reached by Yi-Bo Yang from the University of Kentucky, Lexington, and colleagues (see Viewpoint: [Spinning Gluons in the Proton]()). By running state-of-the-art computer simulations of quark-gluon dynamics on a so-called spacetime lattice, the researchers found that 50% of the proton's spin comes from

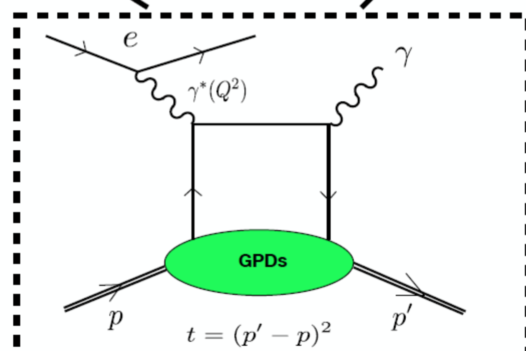
Parton OAM & twist-3 GPD

$$l_q^z = \int dx\, l_q^z(x) \longrightarrow l_q^z(x) = \int dy \left[G_{q,D,3}(x,y) + \mathcal{P}\frac{1}{y-x} G_{q,F,3}(x,y) \right]$$



OAM density

Knowing this curve will be the last great challenge in fully understanding proton spin structure!



Twist-3 GPDs extractable from DVCS & DVMP

A frame-independent sum rule

- Frame-independent longitudinal spin sum rule

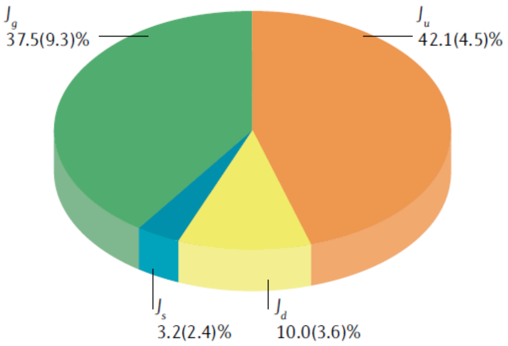
$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Delta\Sigma + L_q^z + \mathrm{J_g}$$

- J_q & J_g are related to the EMT form factor

$$J_{q,g} = \frac{1}{2}[A_{q,g}(0) + B_{q,g}(0)]$$

- They can be calculated using the standard lattice QCD approach

ETMC collaboration

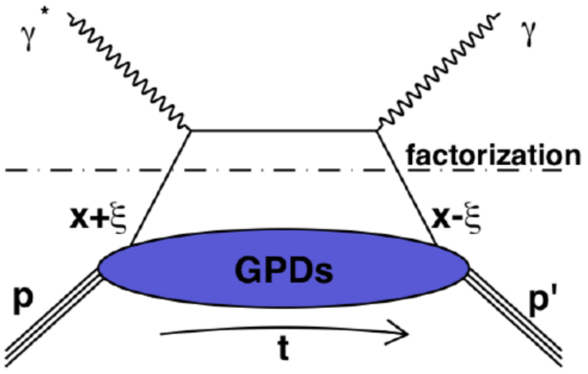


GPD sum rule and DVCS etc

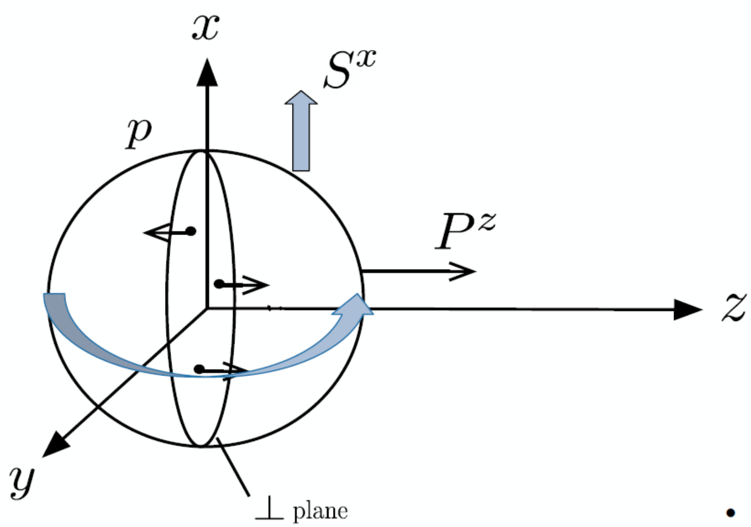
- GPDs were introduced to extract the form factors of EMT through the sum rules

$$\int_{-1}^{1} dx \; xH(x,\xi,t) = A(t) + \xi^2 \; C(t) \; ,$$
$$\int_{-1}^{1} dx \; xE(x,\xi,t) = B(t) - \xi^2 \; C(t) \; .$$

- To extract H & E from DVCS and similar processes model-independently are extremely challenging



Transversely Polarized Proton:



$$J^x = \sum_i r_i^y p_i^z - r_i^z p_i^y$$

Kinematics: $\sim \gamma$ $\sim \gamma^{-1}$

Tw-2 Tw-4

$$\gamma = (1-\beta^2)^{-1/2}$$

$$[J^{x,y}, K^z] \neq 0$$

- This means that J^x is P^z (frame)-dependent 

- Less studied than Longitudinal case because:
  1. It is frame-dependent with non-trivial boost properties
  2. A key issue is separating intrinsic contributions from CM ones
- This has led to some controversy in previous works

Transverse Polarization Sum Rules:

- Let's look again at transverse AM, but **split in terms of its 2 contributions**:

$$\xi^y T^{0z}(\xi) \qquad \xi^z T^{0y}(\xi)$$

Note: equal in rest frame by rotational symmetry

$$J^x = \boxed{\left(\gamma - \frac{1}{2\gamma}\right) J^{x(2)}} + \boxed{\frac{1}{2\gamma} J^{x(3)}} = \gamma \frac{\hbar}{2}$$

$J^{x(2)}$ & $J^{x(3)}$ ~ proton matrix elements of q and g fields

- Solution: $J^{x(2)} = J^{x(3)} = \frac{\hbar}{2}$

Twist-2 Ji Sum Rule

Novel Twist-3 Sum Rule

Spin sum rules are γ-independent!

2 Sum Rules:

$$J_q^{x(2)} + J_g^{x(2)} = \frac{\hbar}{2} \quad \text{Twist-2}$$

$$J_q^{x(3)} + J_g^{x(3)} = \frac{\hbar}{2} \quad \text{Twist-3}$$

Simple parton sum rule for transverse spin

- E and H are twist-two densities
- Spin sum rule as an infinite momentum frame simple parton sum rule for transverse spin

Twist-2 Sum Rule:

$$\boxed{J_q^{x(2)} + J_g^{x(2)} = \frac{\hbar}{2}}$$

$$J_q^{x(2)}(x) = \frac{x}{2}\Big(H_q(x) + E_q(x)\Big)$$

$$J_g^{x(2)}(x) = \frac{x}{2}\Big(H_g(x) + E_g(x)\Big)$$

Twist-3 transverse spin sum rule

- Rotated version of the Jaffe Manohar Longitudinal spin sum rule

X. Ji, Y. Guo & K. Shiells, Nuc. Phys. B 969, 115440 (2021)

Transverse Polarization: $$\frac{1}{2}\Delta q_T + \Delta G_T + l_q^{x(3)} + l_g^{x(3)} = \frac{\hbar}{2}$$

Δq_T , ΔG_T Involve measurable PDFs in DIS and correspond to spin

$l_q^{x(3)}$, $l_g^{x(3)}$ Involve twist-3 GPDs and correspond to canonical OAM

- Δq_T is related to g_2 structure function
- ΔG_T is a twist-three, spin-dependent gluon distribution
- Twist-3 OAM are a challenge to measure

Experimental Roadmap for spin sums

	Longitudinal	Transverse
Frame-independent sum rule:	$\frac{1}{2}\Delta q + L_q^i + J_g^i = \frac{\hbar}{2}$	$\rightarrow J_q^x + J_g^x = \frac{\hbar}{2}$ $\sim \int dx\, x$ moments of: $H_{q,g}(x), E_{q,g}(x)$ 4 twist-2 GPDs extractable from DVCS & DVMP!
IMF sum rule:	$\frac{1}{2}\Delta q + l_q^z + \Delta G + l_g^z = \frac{\hbar}{2}$	$\frac{1}{2}\Delta q_T + l_q^x + \Delta G_T + l_g^x = \frac{\hbar}{2}$

$\int dx g_1(x)$

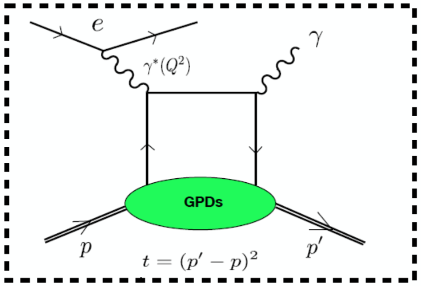
Quark helicity distributions probed in SIDIS

$\int dx \Delta G(x)$

Gluon helicity distribution probed in p-p collisions at RHIC

$\int dx dy \mathcal{F}^{(3)}_{\mathrm{GPD}}(x, y)$

Twist-3 OAM quark and Gluon GPDs extractable From DVCS & DVMP

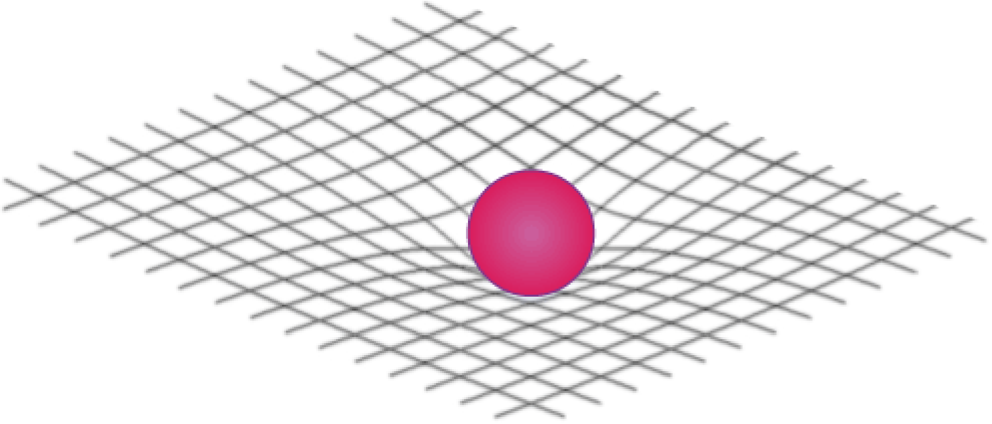


Less experimental constraints

The gravitational properties of hadrons

Energy-momentum tensor as a source of gravity

- Space-time is distorted near a hadron. EMT is the source of gravity or space-time distortion $g^{\mu\nu}$



$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Gravity from a nucleon

- The gravity from a single nucleon is extremely week.

$$g^{\mu\nu} = \eta^{\mu\nu}(\text{mink.}) + h^{\mu\nu}(\text{pert.})$$

$$\Box \bar{h}^{\mu\nu} = \frac{16\pi G}{c^4} T^{\mu\nu}$$

- Apart from dark matter and dark energy, the dominant gravity comes from the nucleons in stars.
- Far away from the nucleon, only the mass and spin matters

$$\bar{h}^{00} = -\frac{2GM}{r} \, ,$$

$$\bar{h}^{0i} = \frac{2G(\vec{S} \times r)^i}{r^3}$$

Near source gravity

- Near the nucleon, the momentum currents T^{ij} are an important source of spacetime perturbation.
- For a static source, the momentum conservation

$$\partial_i T^{ij} = 0$$

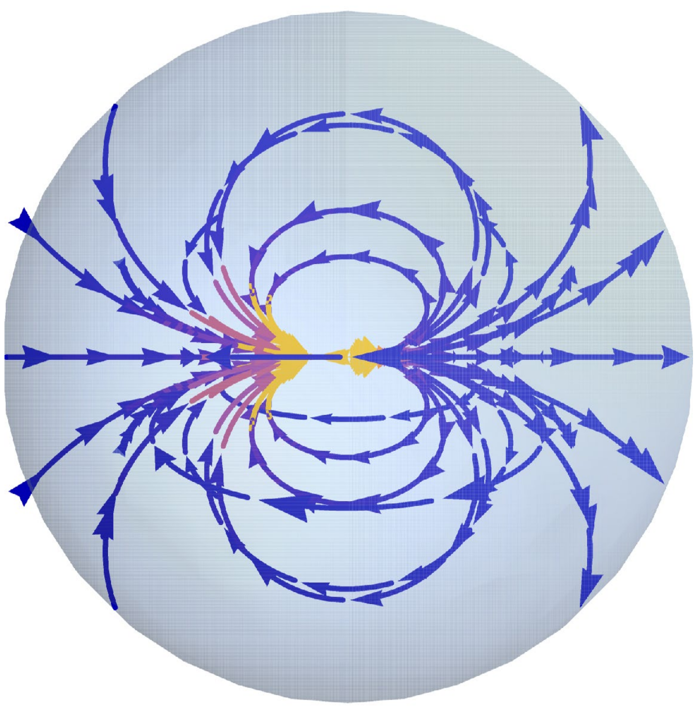
leads to

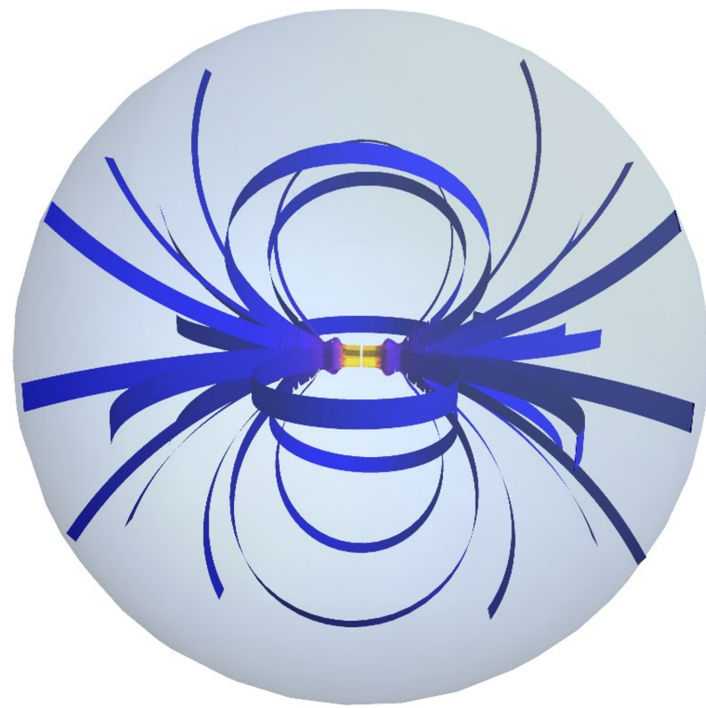
$$\int d^3\vec{r}\, T^{ij} = 0 \qquad \text{(Laue condition)}$$

- The leading multiple of momentum currents come from small q expansion

$$T^{ij}(q) = q^i q^j \epsilon^{ikm} \epsilon^{jln} \tau^{mn} + O(q^3)$$

Conserved momentum currents





Scalar & tensor moments

- Leading MC moments

$$\tau^{ij} = -\delta^{ij}\tau + \hat{\tau}^{ij}$$

- Scalar monopole radius (exist for spin 0 and ½)

$$\tau = \frac{1}{6}\int d^3\vec{r}r^2 T^{ii}(r) \ .$$

- Tensor quadrupole (exist only for spin=1 or higher)

$$\hat{\tau}^{ij} = -\frac{1}{6}\int d^3\vec{r}\epsilon^{ilm}\epsilon^{jkn}r^l r^k \hat{T}^{mn}(\vec{r})$$

Space time perturbation

- Metric perturbation produced by moments of the moment currents,

$$\bar{h}^{ij}_{\text{monopole}} = -\frac{4G\tau}{c^4 r^5}(\delta^{ij} r^2 - 3r^i r^j) \ ,$$
$$\bar{h}^{ij}_{\text{tensor}} = -\frac{4G\delta^{ij}}{r^5}\hat{\tau}^{kl}(3r^k r^l - \delta^{kl} r^2) + \frac{4G}{r^5}\hat{\tau}^{ik}(3r^k r^j - \delta^{kj} r^2) + (i \leftrightarrow j)$$

- Fall as $\frac{1}{r^3}$ only comparable to the mass effect near the nucleon surface.

Monopole and tensor moments

- For pion and nucleon, the monopole moments can be extracted from form factor C,

$$T^{ij}(\vec{r}) = (\nabla^2\delta^{ij} - \nabla^i\nabla^j)C(|\vec{r}|) \ .$$

- C(r) form factor can be measured from DVCS

  Burkert, Elouadrhiri, Girod, Nature 557 (2018)

- τ =C(q=0) has been called D-term in the literature, can have positive or negative sign, unrelated to the stability of the system.

Outlook

- The need to understand non-perturbative QCD will continue to drive experiments and lattice QCD calculations
- We need to find a language to describe the QCD physics, or even better ways to understand the lattice QCD results.
- Much can be learned about the hadron structure by the bottom-up approaches.