



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
MIT CENTER FOR THEORETICAL
PHYSICS – A LEINWEBER INSTITUTE

FWF Österreichischer
Wissenschaftsfonds

SUBMITting QCD: Simulating how a plasma of quarks and gluons thermalizes and how jets lose energy

How to obtain the numerical results for
Phys.Rev.D 113 (2026) 11, 114009 [Barata, Boguslavski, FL, Sadofyev]

Florian Lindenbauer (MIT Center for Theoretical Physics – a Leinweber Institute)

October 6, 2026, SubMIT Users Meeting



Outline

- 1 Introduction: Heavy-ion collisions
- 2 (Simulating) QCD kinetic theory (on SubMIT)
- 3 Jets in heavy-ion collisions: Obtaining the gluon emission spectrum on SubMIT
- 4 Numerical results for the spectrum
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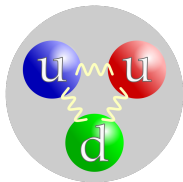
Standard model and QCD

[Cush, Wikipedia]

[Arpad Horvath, Wikipedia]

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi$$



At “room” temperature
quarks **confined** into hadrons

Standard Model of Elementary Particles

three generations of matter (fermions)				interactions / force carriers (bosons)	
	I	II	III		
mass	=2.2 MeV/c ²	=1.28 GeV/c ²	=173.1 GeV/c ²	0	=125.11 GeV/c ²
charge	2/3	2/3	2/3	0	0
spin	1/2	1/2	1/2	1	0
QUARKS	 u up	 c charm	 t top	 g gluon	 H higgs
	 d down	 s strange	 b bottom	 γ photon	
	 e electron	 μ muon	 τ tau	 Z Z boson	
LEPTONS	 ν_e electron neutrino	 ν_μ muon neutrino	 ν_τ tau neutrino	 W W boson	
					SCALAR BOSONS
					GAUGE BOSONS VECTOR BOSONS

SCALAR BOSONS

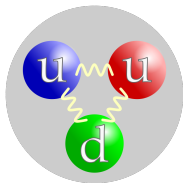
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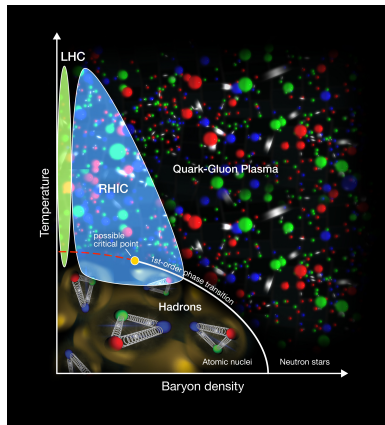
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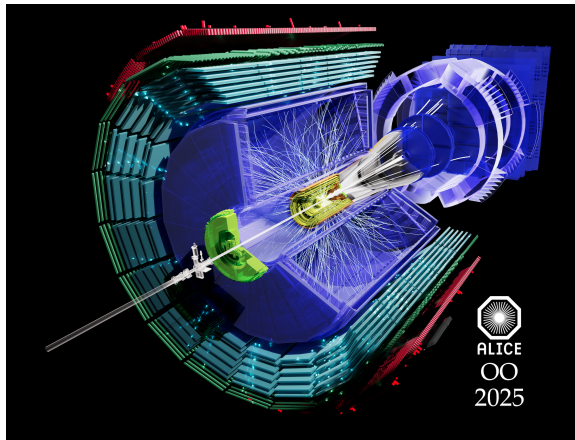
At “room” temperature
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[BNL]

Heavy-ion collisions and the quark-gluon plasma

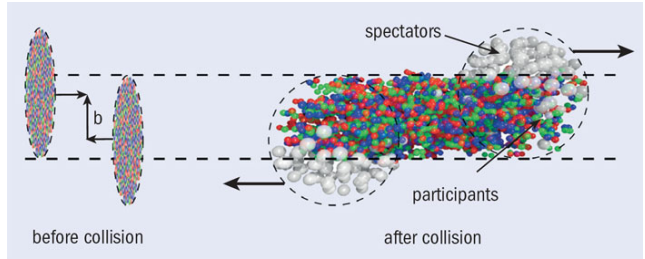
- Study high-temperature properties of the strong interaction (QCD)
- Collision of atomic nuclei at LHC and RHIC
- Creates high-temperature QCD matter = Quark-Gluon plasma (QGP)



[ALICE collaboration]

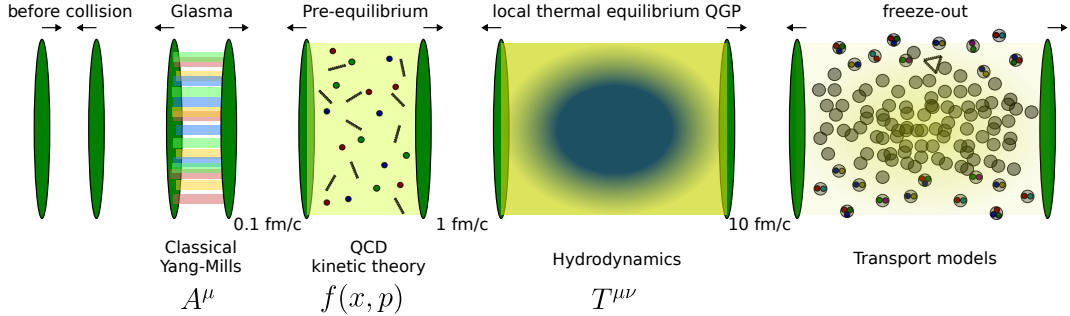
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[Alberica Toia 2013, CERN COURIER]

Weak-coupling picture of thermalization

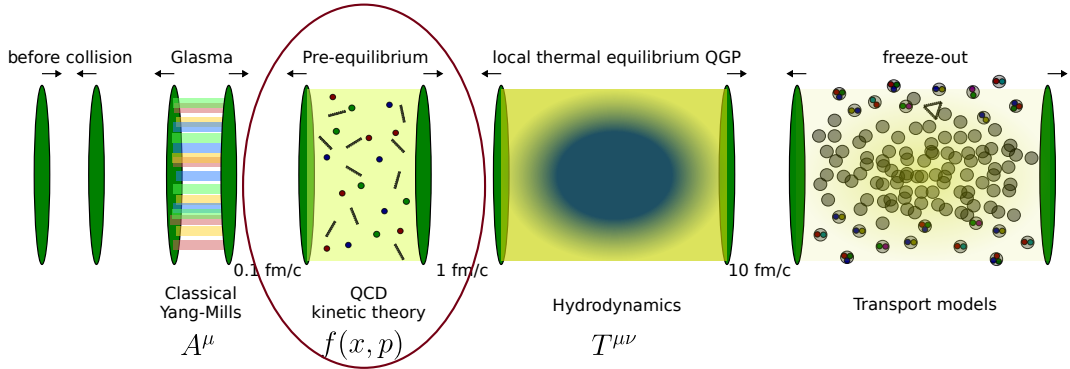


Interested in pre-equilibrium stages (“Initial stages”)

→ **QCD out of equilibrium** → Use **QCD kinetic theory** to simulate plasma evolution

[Rev.Mod.Phys. 93 (2021) [Berges, Heller, Mazeliauskas, Venugopalan]]

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Effective kinetic theory (EKT) description of the QGP

- Gluons with **distribution function** $f(t, \mathbf{x}, \mathbf{p})$ $\leftarrow$ How many particles/gluons

¹[JHEP 01 (2003) [Arnold, Moore, Yaffe], Int.J.Mod.Phys.E 16 (2007) [Arnold]]

Effective kinetic theory (EKT) description of the QGP

- Gluons with **distribution function** $f(t, \mathbf{x}, \mathbf{p}) \leftarrow$ How many particles/gluons
- Time evolution described by **Boltzmann equation** at leading-order¹

$$(\partial_t + \mathbf{v} \cdot \nabla) f = \underbrace{\left| \text{diagram 1} \right|^2 + \left| \text{diagram 2} \right|^2}_{\text{Collision term}}$$

The diagram shows the collision term of the Boltzmann equation for gluons. It consists of two terms, each squared and separated by a plus sign. The first term is a red Feynman diagram representing a gluon-gluon collision, with two incoming gluon lines (red lines with a wavy line in the middle) and two outgoing gluon lines. The second term is a red Feynman diagram representing a gluon-gluon collision, with two incoming gluon lines (red lines with a wavy line in the middle) and two outgoing gluon lines, with a blue rectangular box in the middle of the collision region.

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- ‘Stable’ particles between interactions,
mean free path $\gg$ duration of scattering processes, valid for weak coupling

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- 'Stable' particles between interactions, mean free path $\gg$ duration of scattering processes, valid for weak coupling
- Consider 3 types of plasmas:



Isotropic
Initially overocc.



Isotropic
Initially underocc.



Bjorken
expanding

¹[JHEP 01 (2003) [Arnold, Moore, Yaffe], Int.J.Mod.Phys.E 16 (2007) [Arnold]]

Bjorken expanding plasmas

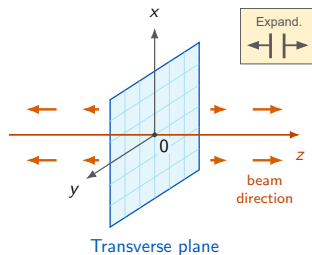
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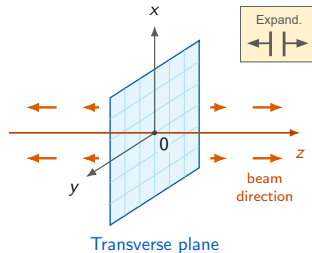
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- Homogeneous & isotropic systems: $\mathbf{v} \cdot \nabla f = 0$
- Boost-invariant Bjorken expansion:
 - Homogeneous in transverse direction
 - f only function of momentum and position rapidity difference, $f(\tau, p, y - \eta)$.



Bjorken expanding plasmas

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→ Then, at $z = \eta = 0$, [Phys.Lett.B 475 \(2000\) \[Mueller\]](#)

$$\mathbf{v} \cdot \nabla f(\tau, \mathbf{p}, z) = -\frac{p_z}{\tau} \frac{\partial}{\partial p_z} f(\tau, p, p_z)$$

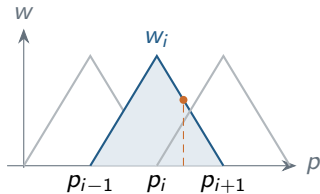
- $1/\tau \rightarrow$ Expansion becomes weaker over time!
- no explicit z-dependence in f

Discretization: Store moments of the distribution

- Axial symmetry: $f(\tau, \mathbf{p}) = f(\tau, p, \mu)$, with $\mu = \cos \theta_p$
- Store **weighted particle numbers** on a two-dimensional grid

$$W_{ij}(\mathbf{p}) = w_i(p) \tilde{w}_j(\mu),$$

$$n_{ij} = \lambda \int \frac{d^3 p}{(2\pi)^3} f(\mathbf{p}) W_{ij}(\mathbf{p}).$$



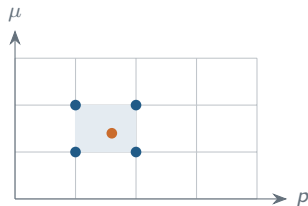
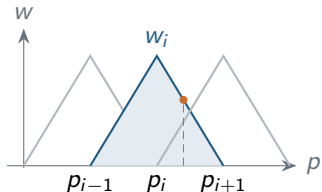
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- Linear **wedge functions**: each momentum contributes to nearby nodes



Tensor-product wedges
deposit on at most four nodes

Phys.Rev.D 89 (2014) [Abraao York, Kurkela, Lu, Moore]

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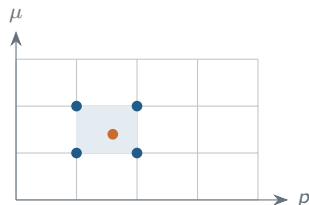
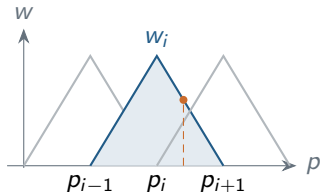
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- Linear **wedge functions**: each momentum contributes to nearby nodes
- Partition of unity and linear reproduction:

$$\sum_i w_i(p) = 1, \quad \sum_i p_i w_i(p) = p.$$

Phys.Rev.D 89 (2014) [Abraao York, Kurkela, Lu, Moore]



Tensor-product wedges
deposit on at most four nodes

Discretization: Reconstruct f and compute observables

- Approximate f at the nodes by dividing out the **cell volume**

$$f_{ij} \simeq \frac{(2\pi)^3 n_{ij}}{\lambda p_i^2 \Delta V_i^p \Delta \Omega_j},$$

$$\Delta V_i^p = \int dp \, w_i(p),$$

$$\Delta \Omega_j = 2\pi \int_{-1}^1 d\mu \, \tilde{w}_j(\mu).$$

$\nu_g = 2(N_c^2 - 1)$; f is per gluon degree of freedom.

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- Observables become weighted sums

$$\lambda \int \frac{d^3 p}{(2\pi)^3} f O \simeq \sum_{ij} n_{ij} O(p_i, \mu_j).$$

- **Number and energy** are exact moments on the represented domain

$$n_{\text{phys}} = \frac{\nu_g}{\lambda} \sum_{ij} n_{ij},$$
$$\varepsilon_{\text{phys}} = \frac{\nu_g}{\lambda} \sum_{ij} p_i n_{ij},$$
$$P_L \simeq \frac{\nu_g}{\lambda} \sum_{ij} p_i \mu_j^2 n_{ij}.$$

$$\nu_g = 2(N_c^2 - 1); \text{ } f \text{ is per gluon degree of freedom.}$$

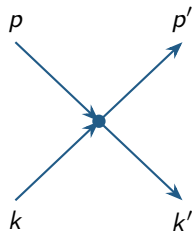
Elastic collision term: Monte Carlo integration

Most time consuming:

High-dimensional integral over phase space of elastic collision term:

$$\begin{aligned}\int d\Gamma_{\text{PS}} &= \int_{\mathbf{p}\mathbf{k}\mathbf{p}'\mathbf{k}'} (2\pi)^4 \delta^4(P + K - P' - K') \\ &= \frac{1}{2^{12}\pi^8} \int_0^\infty d\mathbf{q} \int_{-q}^q d\omega \int_{\frac{q-\omega}{2}}^\infty d\mathbf{p} \int_{\frac{q+\omega}{2}}^\infty d\mathbf{k} \int_{-1}^1 d\cos\theta_q \\ &\quad \times \int_0^{2\pi} d\phi_q \int_0^{2\pi} d\phi_{pq} \int_0^{2\pi} d\phi_{kq} .\end{aligned}$$

Frame choice: $\mathbf{q}$ -integral in “lab frame”, integral over $\mathbf{p}$ and $\mathbf{k}$ in frame in which $\mathbf{q}$ points in z -direction.

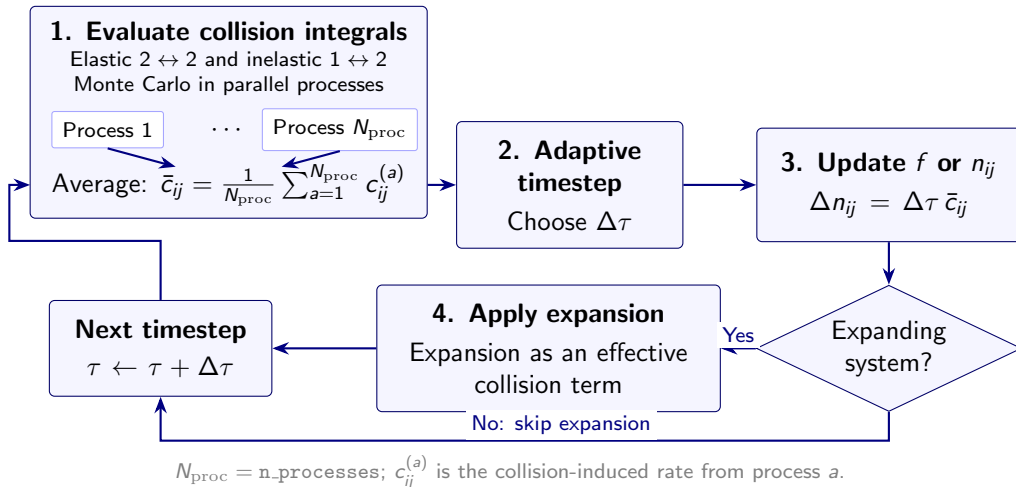


$$\mathbf{p} + \mathbf{k} = \mathbf{p}' + \mathbf{k}'$$

$$\mathbf{q} = \mathbf{p}' - \mathbf{p}$$

$$\omega = p' - p$$

Kinetic theory: Evolution at every timestep

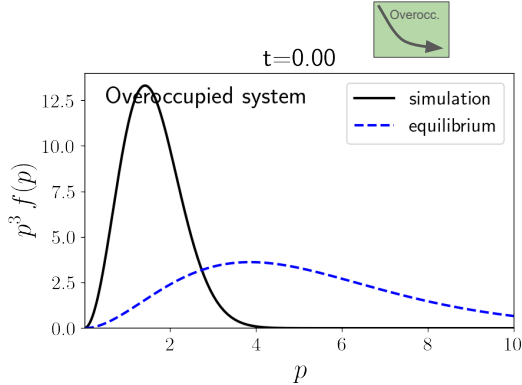


SubMIT: Memory requirement

Example of a run I recently SubMITted:

- C++
- 150 MB per process
- 16 processes
- Grid: 180×250

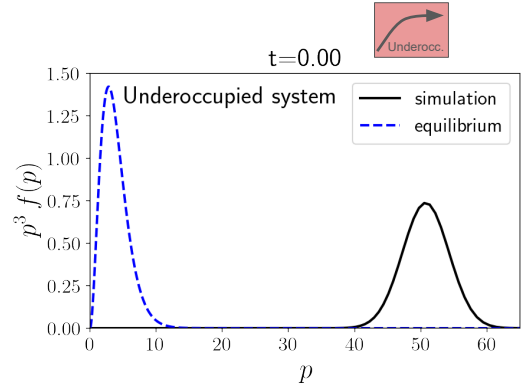
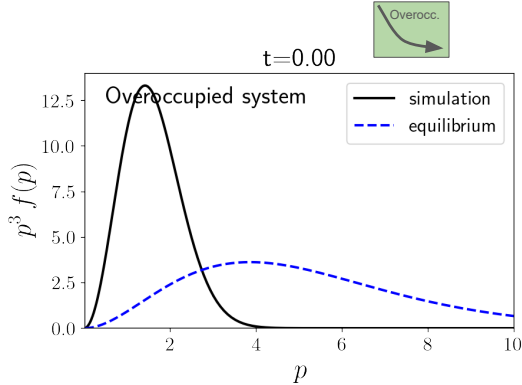
Thermalization in isotropic plasmas



- (Left): Initially over-occupied system
- (Right): Underocc. system: Soft thermal bath formed (bottom-up thermalization)

First studied in Phys.Rev.Lett. 133 (2014) [Kurkela, Lu], see also Phys.Rev.D 105 (2022) [Fu, Ghiglieri, Iqbal, Kurkela]

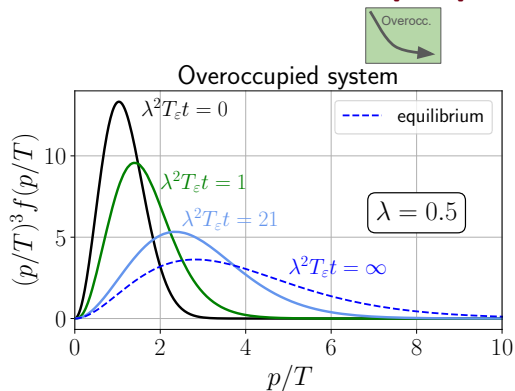
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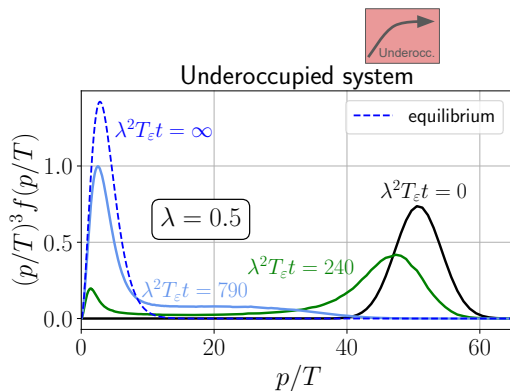
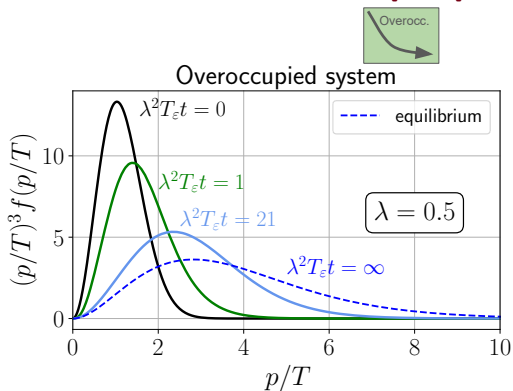
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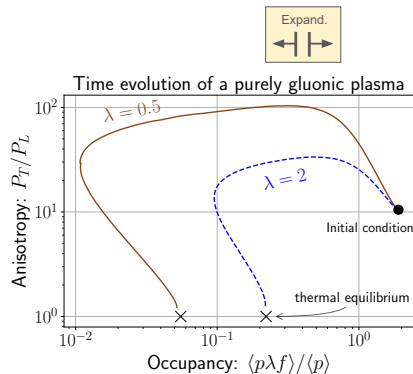
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Bottom-up thermalization in heavy-ion collisions

Initial condition², with $\lambda = g^2 N_C$:

$$f(p_\perp, p_z) = \text{"squeezed"} \quad \frac{1}{\lambda} \times \exp(-p^2) / p$$

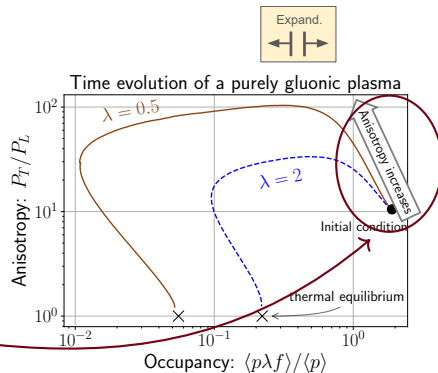
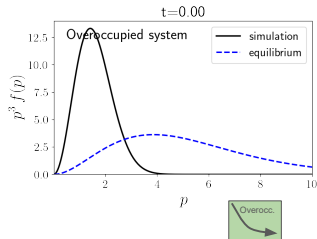


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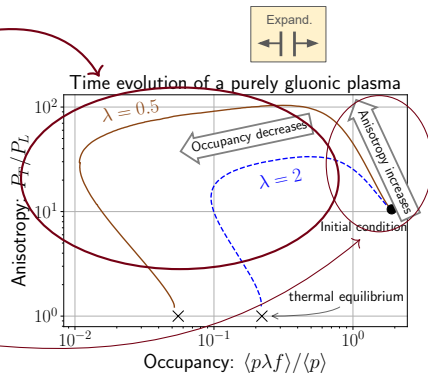
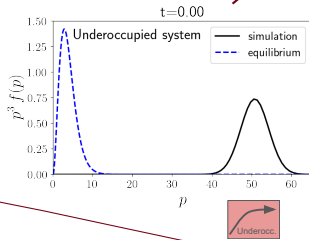
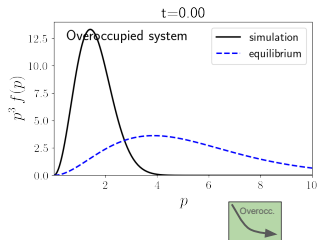


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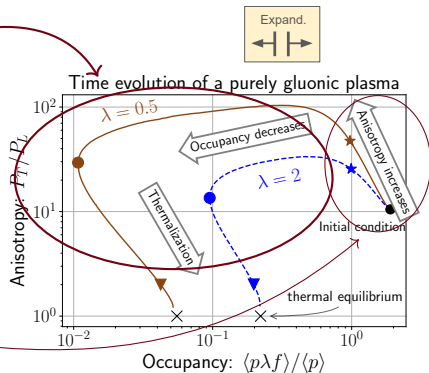
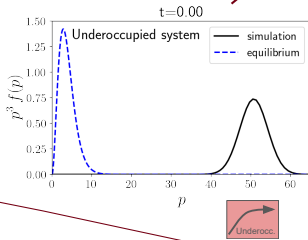
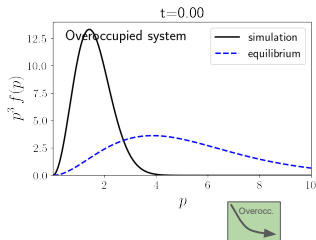


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■ **Markers** represent **different stages**

■ thermalizes at $\tau_{\text{BMSS}} = \left(\frac{\lambda}{12\pi}\right)^{-13/5} / Q_s$

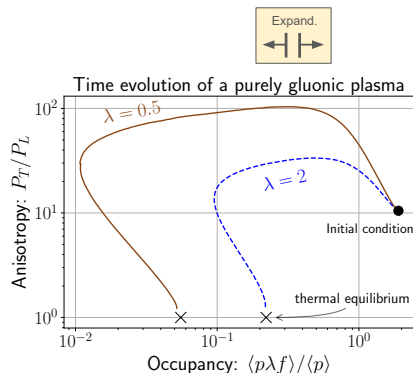
Phys.Lett.B 502 (2001) [Baier, Mueller, Schiff, Son]

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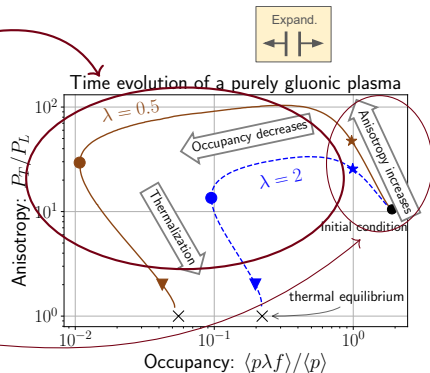
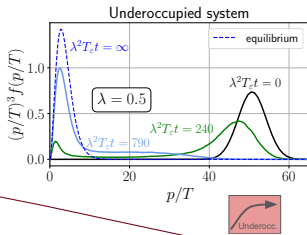
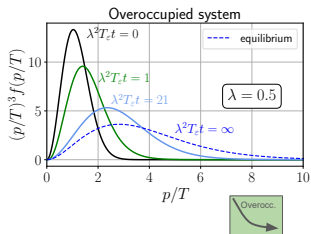


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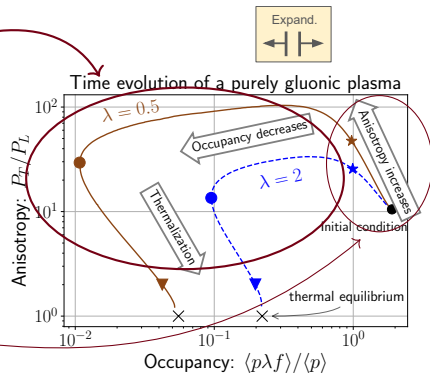
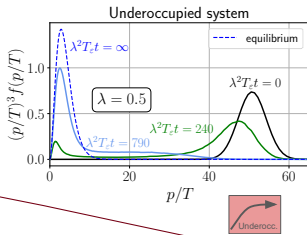
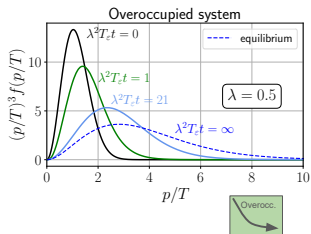


³[Phys.Rev.Lett. 115 (2015) [Kurkela, Zhu]]

Bottom-up thermalization in heavy-ion collisions

Initial condition³, with $\lambda = g^2 N_C$:

$$f(p_\perp, p_z) = \text{“squeezed” } \frac{1}{\lambda} \times \exp(-p^2) / p$$



■ **Markers** represent **different stages**

■ thermalizes at $\tau_{\text{BMSS}} = \left(\frac{\lambda}{12\pi}\right)^{-13/5} / Q_s$

Phys.Lett.B 502 (2001) [Baier, Mueller, Schiff, Son]

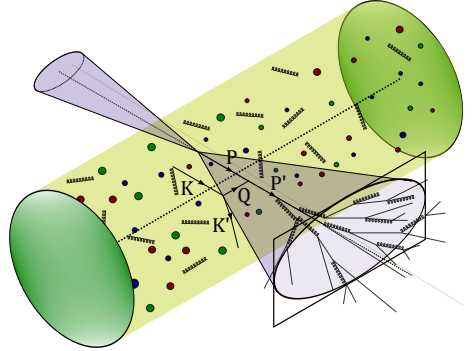
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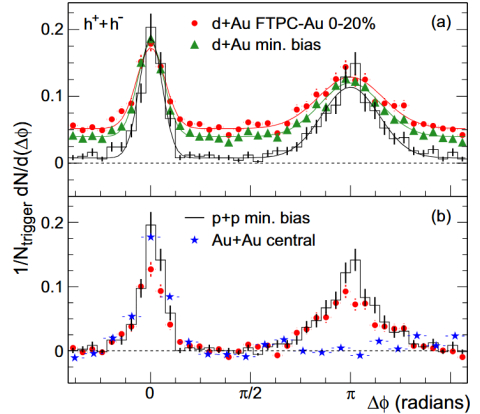
Jets in heavy-ion collisions

- **Jet** = Particle shower from **energetic parton** created in initial collision



Jets in heavy-ion collisions

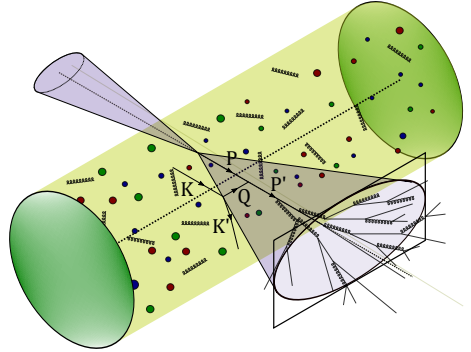
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Phys.Rev.Lett. 91 (2003) 072304 [STAR]

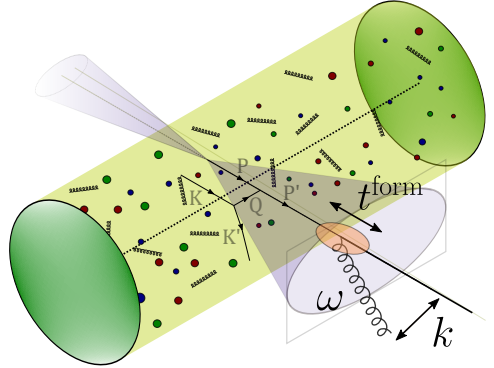
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- Imprints of **pre-equilibrium plasma**
- Many event generators use simplifying assumptions (isotropic, thermal, ...)



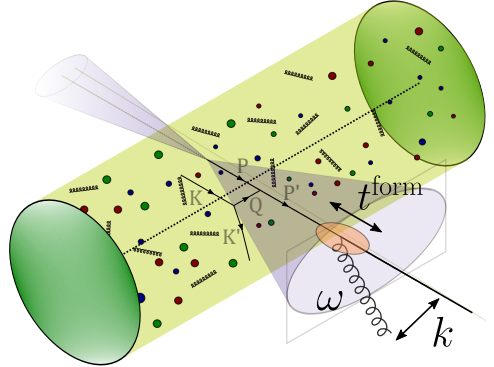
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- **Energy loss** dominated by **gluon radiation**
 - $dI/d\omega$: **Spectrum**
=Probability of emitting gluon with energy ω (relative to vacuum)



- ω : emitted gluon energy
- k : transverse momentum
- $t^{\text{form}} \sim \sqrt{\omega/\hat{q}}$: formation time

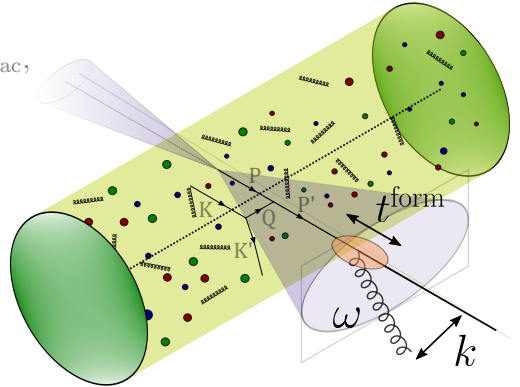
Obtaining the gluon emission spectrum

Phys.Rev.D 79 (2009) [Arnold]
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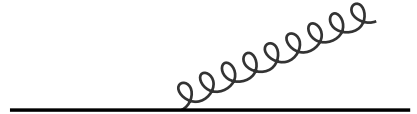


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- **Expand** perturbatively **around ‘easy’ harmonic oscillator solution**

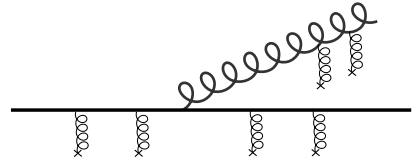
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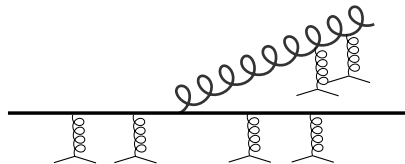
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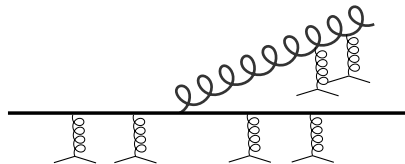
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- Medium input:

$$C(\mathbf{x}, t) = \frac{1}{4} \hat{q}_0(t) \mathbf{x}^2 \log \frac{1}{\mathbf{x}^2 \mu_*^2(t)}$$

- **Expand** perturbatively **around ‘easy’ harmonic oscillator solution**

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Spectrum more explicit

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

IOE Spectrum is obtained via

$$\frac{dI^{(0)}}{d\omega d^2k} = \frac{2\bar{\alpha}}{\pi\omega} \text{Re} \left\{ k^{-2} \left(e^{-i\frac{k^2}{2\omega\text{Cot}(L,t_0)}} - 1 \right) + \int_{t_0}^L dt \frac{\text{Cot}(t,t_0)}{\hat{P}^2(t,t_0)} e^{-\frac{k^2}{\hat{P}^2(t,t_0)}} \right\},$$

$$\begin{aligned} \frac{dI^{(1)}}{d\omega d^2k} = & \frac{\bar{\alpha}\pi}{k^4} \text{Re} \int_{t_0}^L dt_2 \left\{ \frac{i}{2\omega} \int_{t_0}^{t_2} dt_1 \frac{\hat{q}_0(t_1)}{\hat{R}_{21}^2} e^{-\frac{k^2}{\hat{R}_{21}^2}} \left[Q_s^2(L,t_2) I_a \left(\frac{k^2}{\hat{K}_{21}}, \frac{k^2 \hat{R}_{21}^2}{Q_r^2} \right) + 2C_{12} \hat{R}_{21} k^2 I_b \left(\frac{k^2}{\hat{K}_{21}}, \frac{k^2 \hat{R}_{21}^2}{Q_r^2} \right) \right] \right. \\ & \left. - \text{Cot}_{20} Q_{s0}^2(L,t_2) I_a \left(\frac{k^2}{\hat{P}^2(t_2,t_0)}, \frac{k^2}{Q_b^2} \right) \right. \\ & \left. + 2\hat{q}_0(t_2) C^2(t_2,L) I_b \left(\frac{ik^2}{2\omega C^2(t_2,L) \left(\text{Cot}_{20} - \frac{1}{C(t_2,L)S(L,t_2)} - \text{Cot}(t_2,L) \right)}; \frac{k^2}{Q_r^2 C^2(t_2,L)} \right) e^{-\frac{ik^2}{2\omega\text{Cot}(L,t_2)}} \right\} \end{aligned}$$

medium input,
obtained from $\hat{q}(\Lambda_{\perp})$

scale,
obtained self-consistently

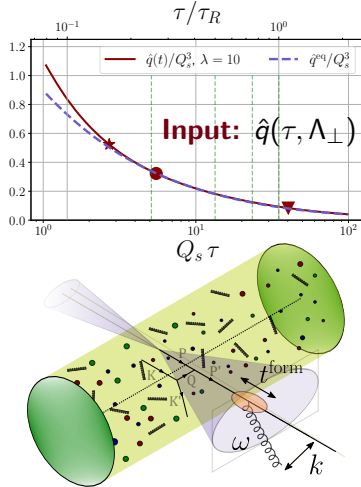
And requires solving a differential equation

$$\left[\frac{d^2}{dt^2} + \Omega^2(t) \right] F(t, t_0) = 0, \quad \Omega^2(t) = -i \frac{\hat{q}_r(t)}{\omega}, \quad \hat{q}_r(t) = \hat{q}_0(t) \log \frac{Q_r^2}{\mu_*^2(t)}$$

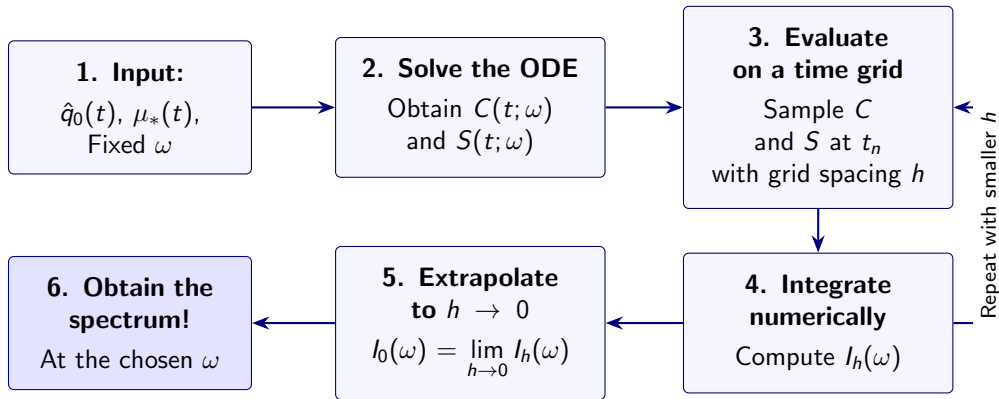
with specific boundary conditions: $S(t_0, t_0) = \partial_t C(t, t_0)_{t=t_0} = 0$,
 $\partial_t S(t, t_0)_{t=t_0} = C(t_0, t_0) = 1$

Obtaining the spectrum – Schematic

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]



From the differential equation to the spectrum



Extrapolate using several grid spacings: $h_1 > h_2 > h_3 > \dots$.

Repeat for other ω to build the full spectrum [several processes on SUBMIT!].

SubMIT

Evaluate jet spectra for different ω and different input parameters simultaneously on SubMIT.

For each evaluation:

- Python + numpy
- 500 MB per process
- 10 processes
- max time discretization: 1200

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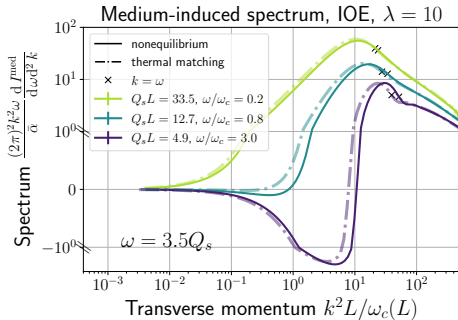
Gluon spectrum

Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]



- $\hat{q}_0(t)$, $\mu_*(t) \rightarrow$ spectrum $dI/d\omega$
- Consider different medium lengths (colors in plot: green, blue purple)
- Set $\hat{q}_0(t) = 0$ for $t > L$
- Form of spectrum dominated by ratio ω/ω_c ,

$$\omega_c = \int_{t_{min}}^{t_{min}+L} dt (t - t_{min}) \hat{q}(t)$$
- All information, difficult to analyze



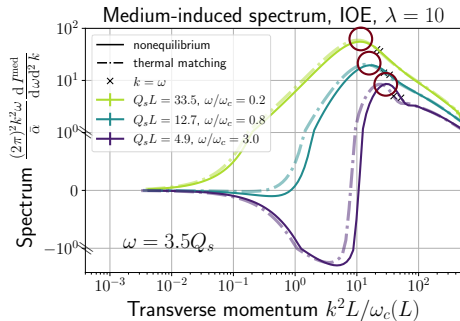
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Comparison to equilibrium

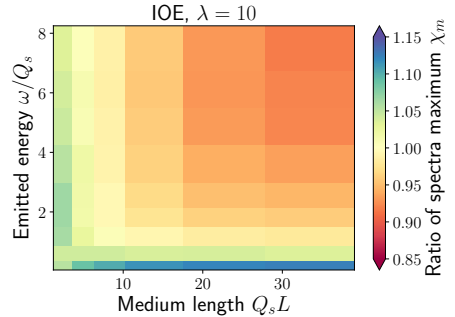
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]



- Comparison to equilibrium: Maximum of spectrum plot

$$\chi_m = \frac{\text{Maximum spectrum nonequ.}}{\text{Maximum spectrum equ.}}$$

- Clear systematics visible
- Does not approach 1 even for $L \rightarrow \infty$!



Outline

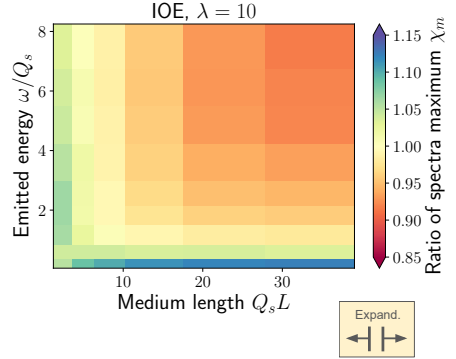
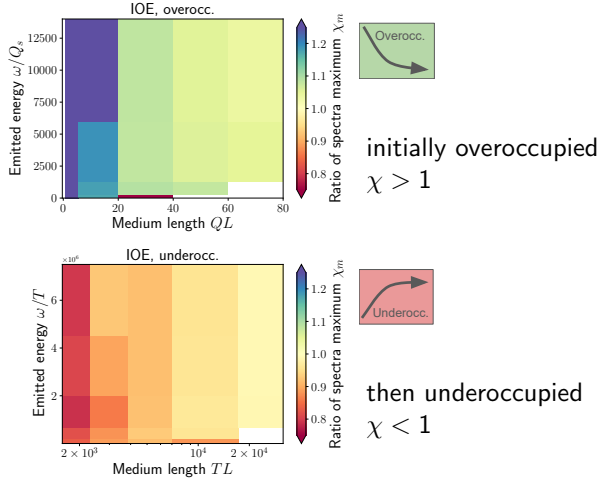
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Summary

- Simulate **QCD kinetic theory** to describe initial **nonequilibrium QCD plasma** in heavy-ion collisions
 - Computational cost from Monte Carlo evaluation of collision integrals
 - Averaging over multiple processes for increased Monte Carlo statistics
- **Gluon emission spectrum** from *improved opacity expansion*
 - 2D integral over oscillating function
 - Parallelize evaluation for different parameters

Comparison to equilibrium II

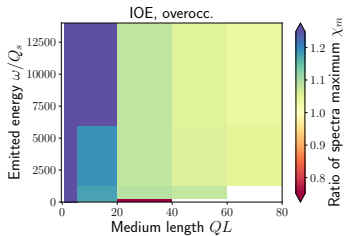
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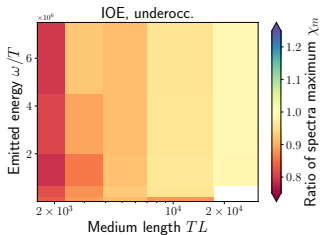
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 $\chi > 1 \rightarrow \chi < 1$
 $\rightarrow$ Can be explained with toy models

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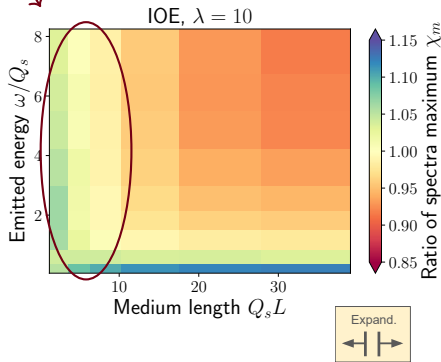
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]



initially overoccupied
 $\chi > 1$



then underoccupied
 $\chi < 1$

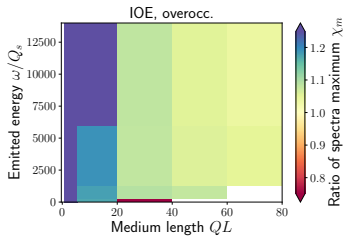


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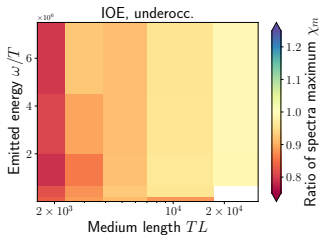
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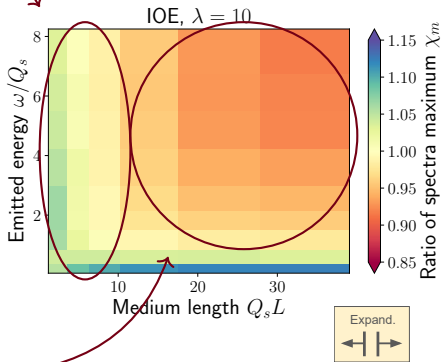
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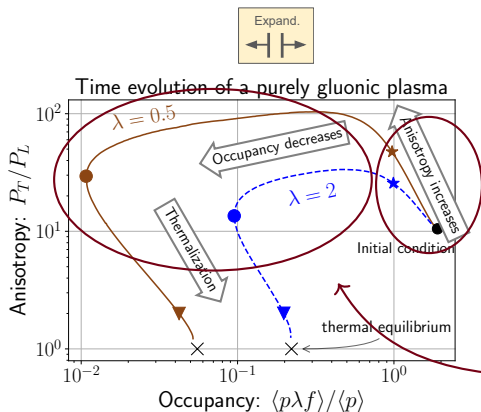
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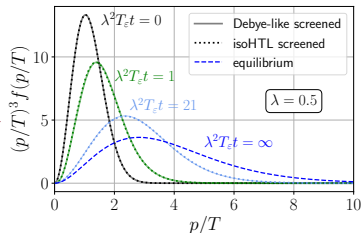
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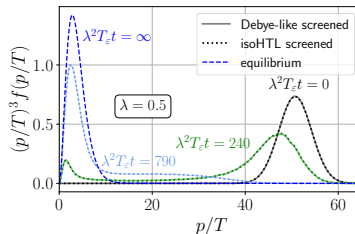
Recall: Bottom-up thermalization



Overoccupied, initial $n > n_{therm}$

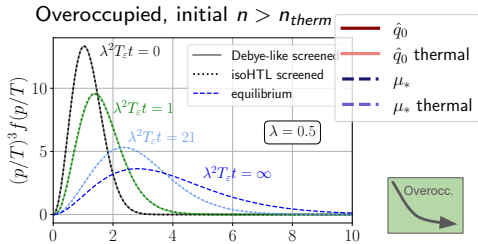
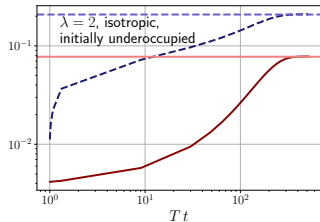
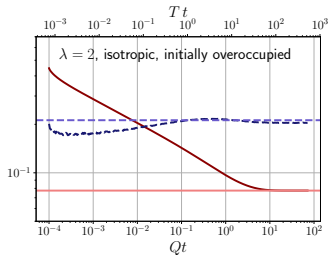


Underoccupied, initial $n < n_{therm}$

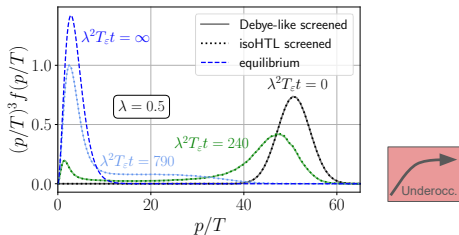


Isotropic toy models

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

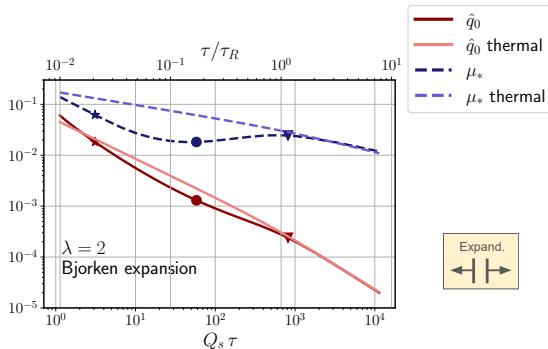
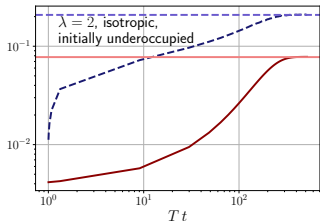
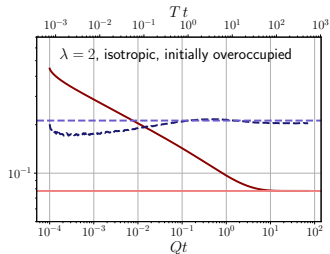


Underoccupied, initial $n < n_{therm}$



Comparison with isotropic toy models

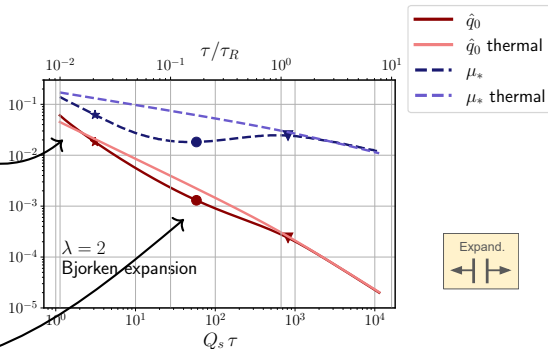
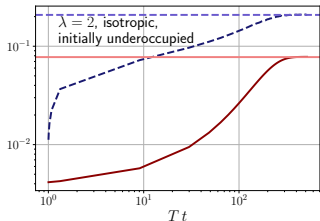
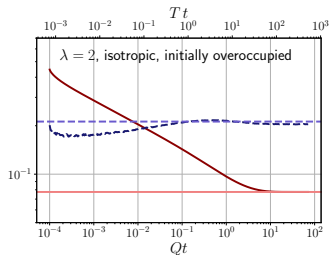
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]



- Expanding evolution is combination of over- and underoccupied evolution
- Better visible at smaller couplings!

Comparison with isotropic toy models

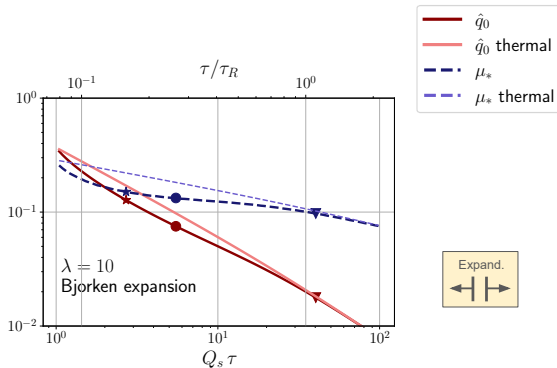
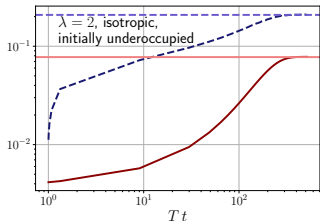
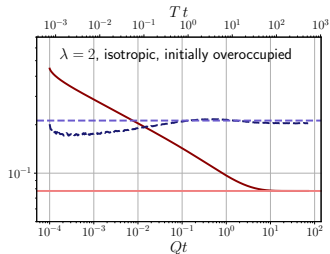
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