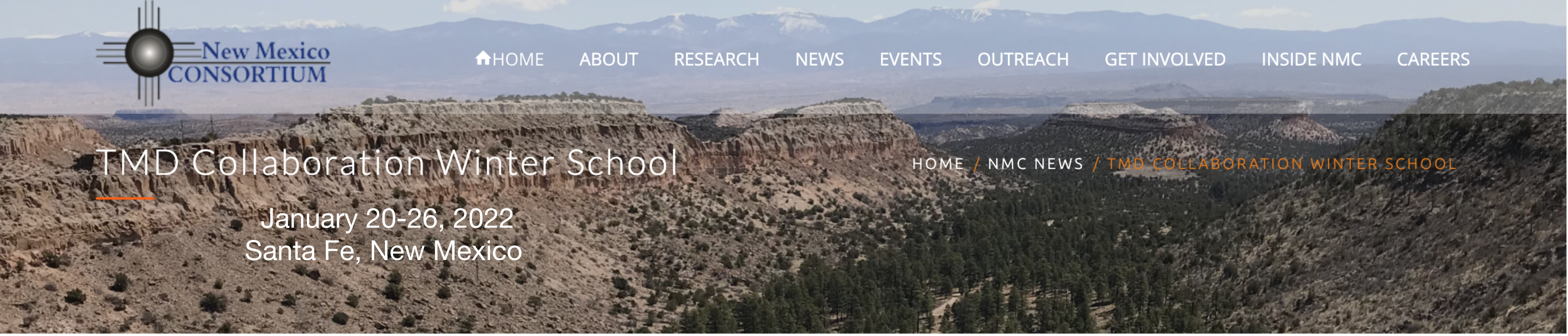

Introduction to QCD

- **Lec. 1: Fundamentals of QCD**
- **Lec. 2: QCD for cross sections with identified hadrons, & hadron structure**
- **Lec. 3: QCD for observables with polarization, & role/power of lattice QCD**

Jianwei Qiu

TMD Collaboration

TMD Handbook

January 15, 2022

A modern introduction to the physics of Transverse Momentum Dependent distributions

TMD Collaboration

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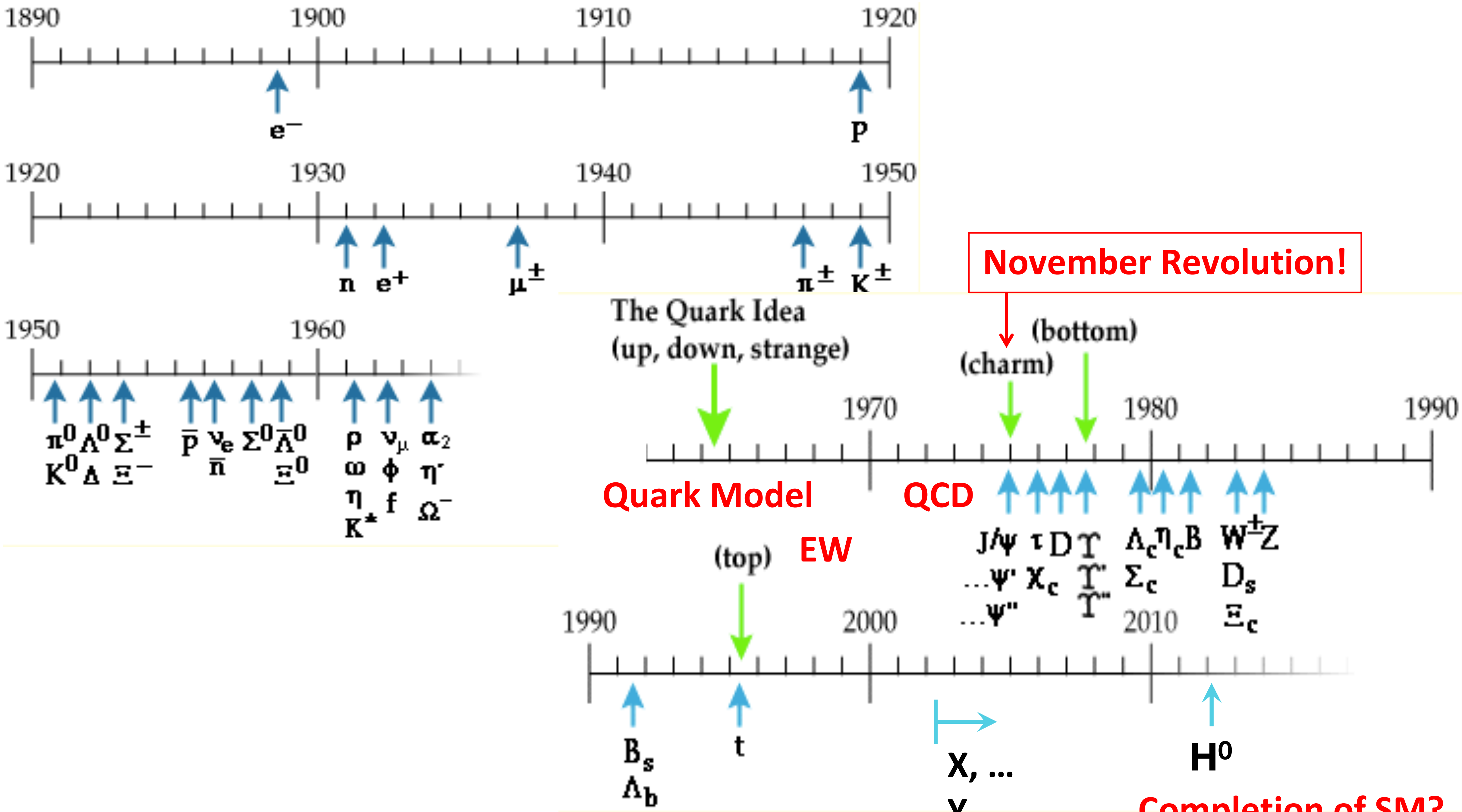
Raju Venugopalan
Ivan Vitev
Feng Yuan
Yong Zhao

New Particles, New Ideas, and New Theories:

- Early proliferation of new hadrons – "particle explosion":



X, …
Y, …
Z, …
Pentaquark, …

Another particle explosion?

How do we make sense of all of these?

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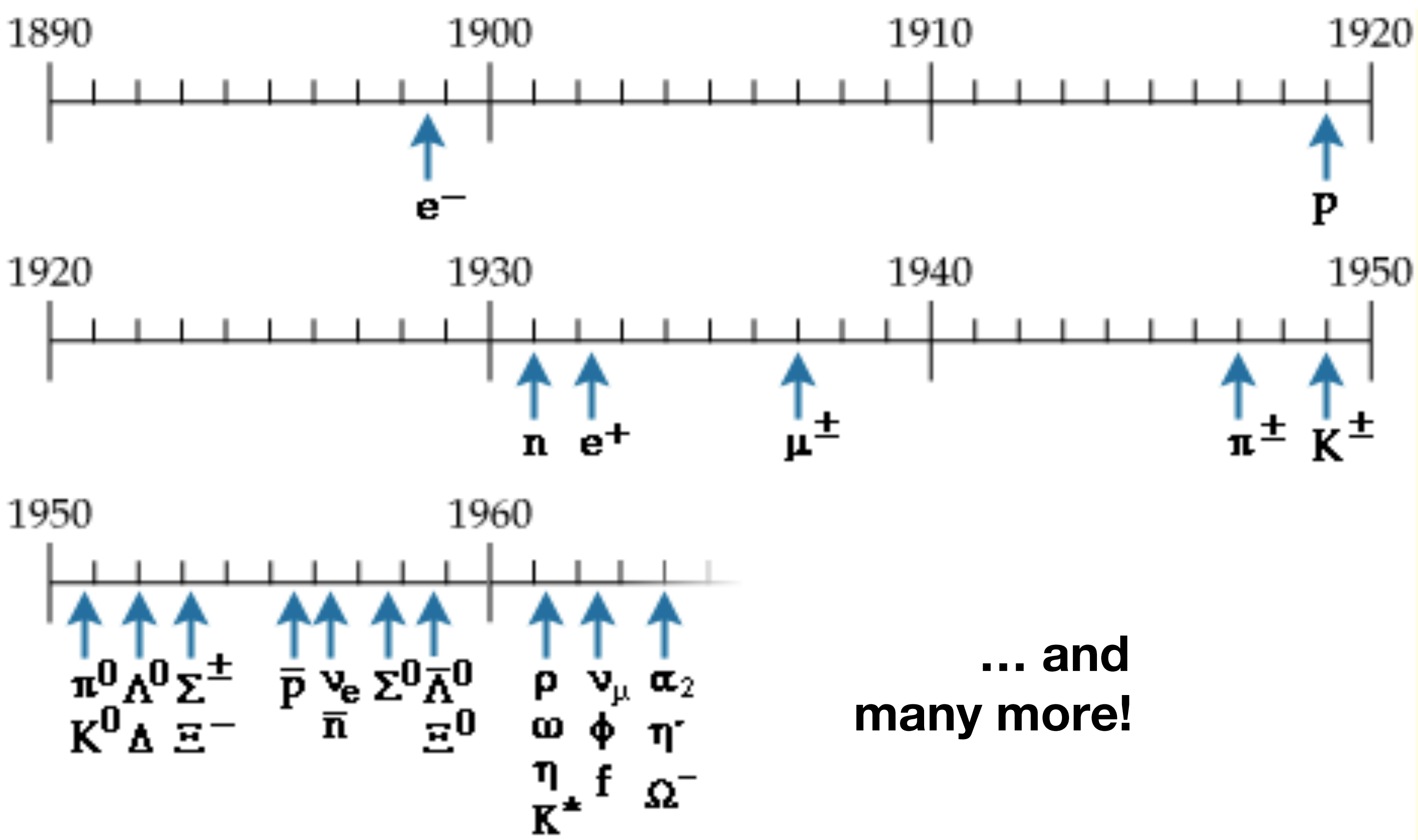
New Particles, New Ideas, and New Theories:

- ❑ **Early proliferation of new hadrons – “particle explosion”:**



- ❑ **Nucleons has internal structure!**

1933: Proton’s magnetic moment

Otto Stern

Nobel Prize 1943

$$\mu_p = g_p \left(\frac{e\hbar}{2m_p}\right)$$

$$g_p = 2.792847356(23) \neq 2!$$

$$\mu_n = -1.913 \left(\frac{e\hbar}{2m_p}\right) \neq 0!$$

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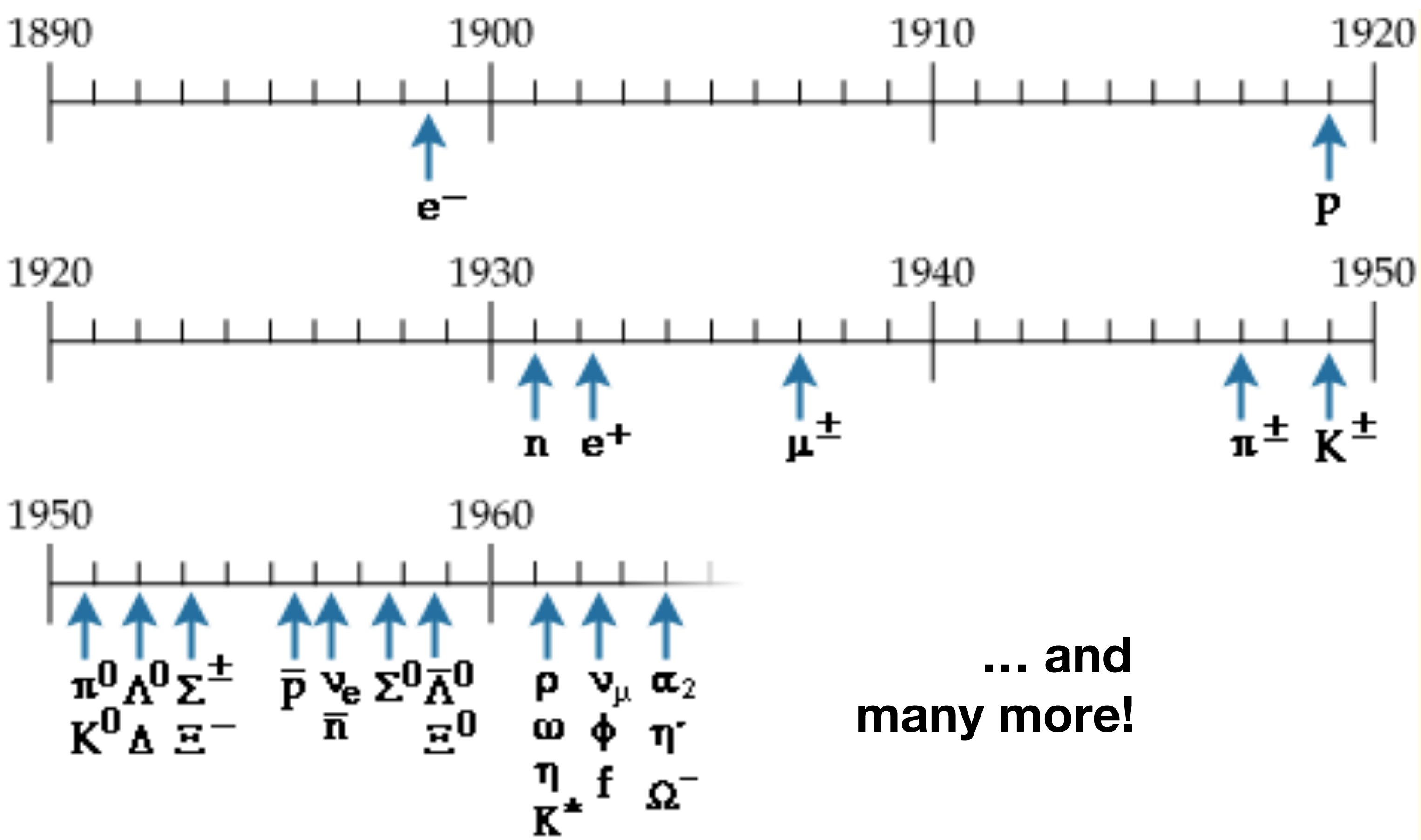
New Particles, New Ideas, and New Theories:

❑ Early proliferation of new hadrons – "particle explosion":

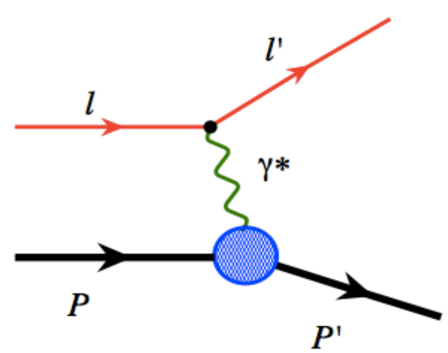


❑ Nucleons has internal structure!

1960: Elastic e-p scattering

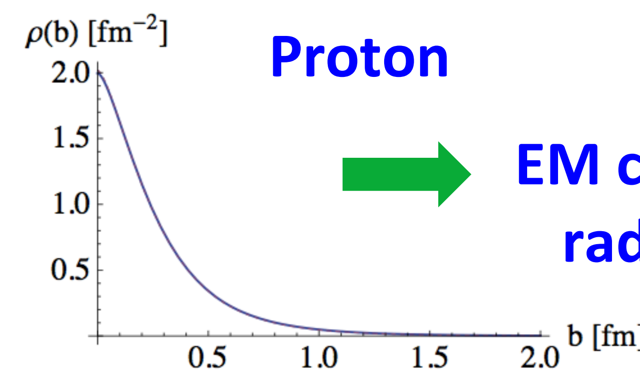
Robert Hofstadter

Nobel Prize 1961

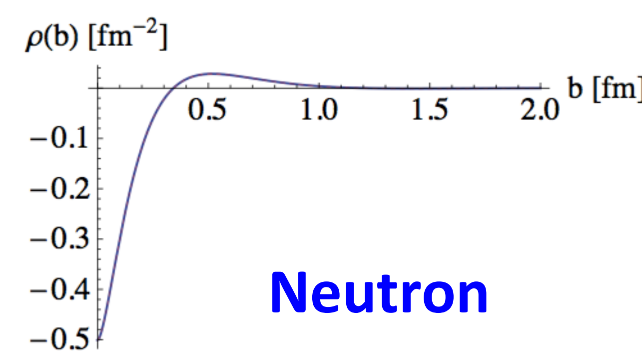


Form factors

Electric charge distribution



EM charge radius!



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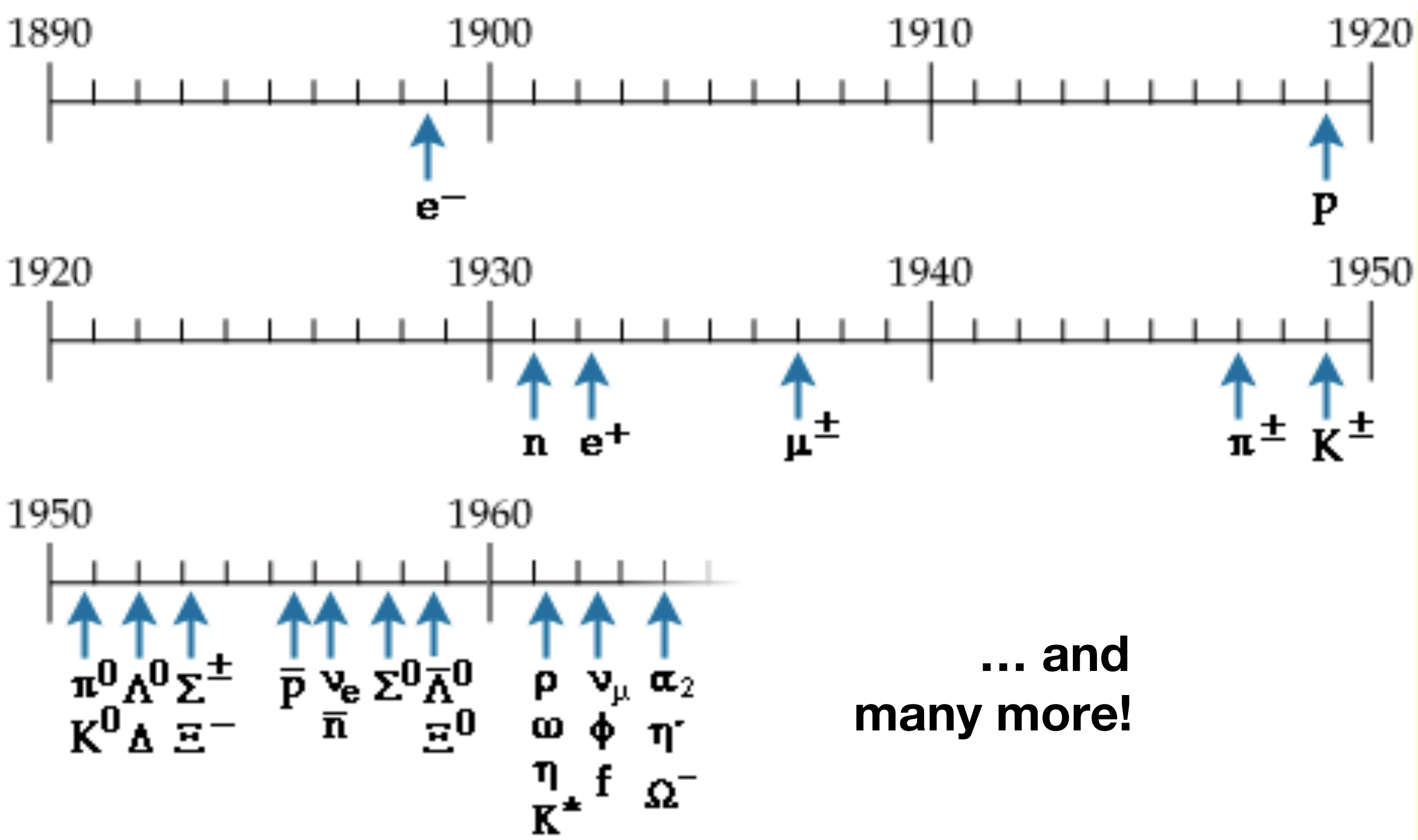
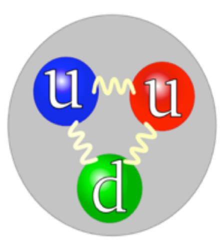
New Particles, New Ideas, and New Theories:

- ❑ **Early proliferation of new hadrons – "particle explosion":**

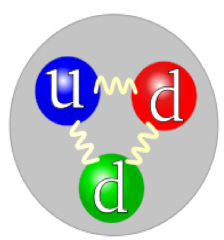


- ❑ **Nucleons has internal structure!**

Proton



Neutron



→ Quark Model

Murray Gell-Mann

Nobel Prize, 1969

The Naïve Quark Model:

Flavor SU(3) – assumption:

Physical states for u, d, s , neglecting any mass difference, are represented by 3-eigenstates of the fundamental representation of flavor SU(3)

Generators for the fundamental rep'n of SU(3) – 3x3 matrices:

$$J_i = \frac{\lambda_i}{2}$$ with $\lambda_i, i = 1, 2, ..., 8$ Gell-Mann matrices

Good quantum numbers to label the states:

$$J_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad J_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

simultaneously diagonalized

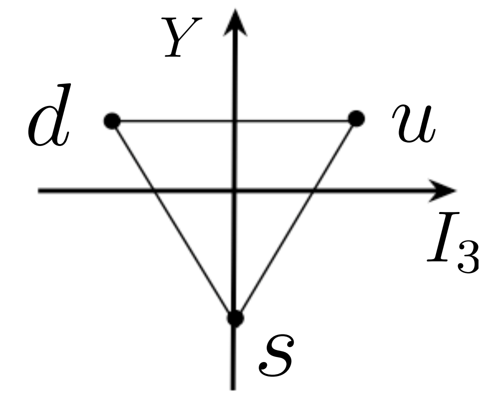
Isospin: $\hat{I}_3 \equiv J_3$, Hypercharge: $\hat{Y} \equiv \frac{2}{\sqrt{3}} J_8$

Basis vectors – Eigenstates: $|I_3, Y\rangle$

$$v^1 \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow u = |\tfrac{1}{2}, \tfrac{1}{3}\rangle \qquad v^2 \equiv \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow d = |\text{-}\tfrac{1}{2}, \tfrac{1}{3}\rangle \qquad v^3 \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow s = |0, \text{-}\tfrac{2}{3}\rangle$$

The Naïve Quark Model:

Quark states:



$$u = |\tfrac{1}{2}, \tfrac{1}{3}\rangle \quad d = |\text{-}\tfrac{1}{2}, \tfrac{1}{3}\rangle \quad s = |0, \text{-}\tfrac{2}{3}\rangle$$

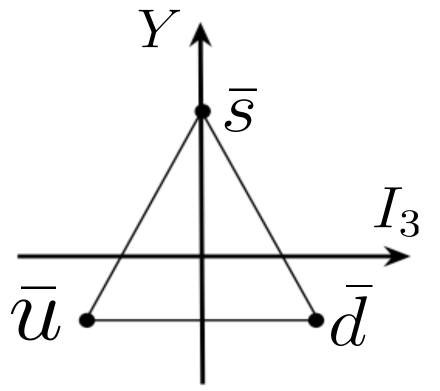
Spin: ½

Baryon #: ***B* = ⅓**

Strangeness: ***S* = *Y* – *B*** **Electric charge:** $Q \equiv I_3 + \frac{Y}{2}$

$$u\begin{cases} Q = 2/3\,e \\ s = 1/2 \\ I_3 = 1 \\ Y = 1/3 \\ B = 1/3 \\ S = 0 \end{cases} \quad d\begin{cases} Q = -1/3\,e \\ s = 1/2 \\ I_3 = -1 \\ Y = 1/3 \\ B = 1/3 \\ S = 0 \end{cases} \quad s\begin{cases} Q = -1/3\,e \\ s = 1/2 \\ I_3 = 0 \\ Y = -2/3 \\ B = 1/3 \\ S = -1 \end{cases}$$

Antiquark states: $v_i \equiv \epsilon_{ijk}\, v^j v^k$



$$\hat{I}_3 v_1 = \epsilon_{123}[(\hat{I}_3 v^2)v^3 + v^2(\hat{I}_3 v^3)] + \epsilon_{132}[(\hat{I}_3 v^3)v^2 + v^3(\hat{I}_3 v^2)] = -\frac{1}{2}v_1$$

$$\hat{Y} v_1 = \epsilon_{123}[(\hat{Y} v^2)v^3 + v^2(\hat{Y} v^3)] + \epsilon_{132}[(\hat{Y} v^3)v^2 + v^3(\hat{Y} v^2)] = -\frac{1}{3}v_1$$

$$u \longrightarrow \bar{u} = |\text{-}\tfrac{1}{2}, \text{-}\tfrac{1}{3}\rangle$$

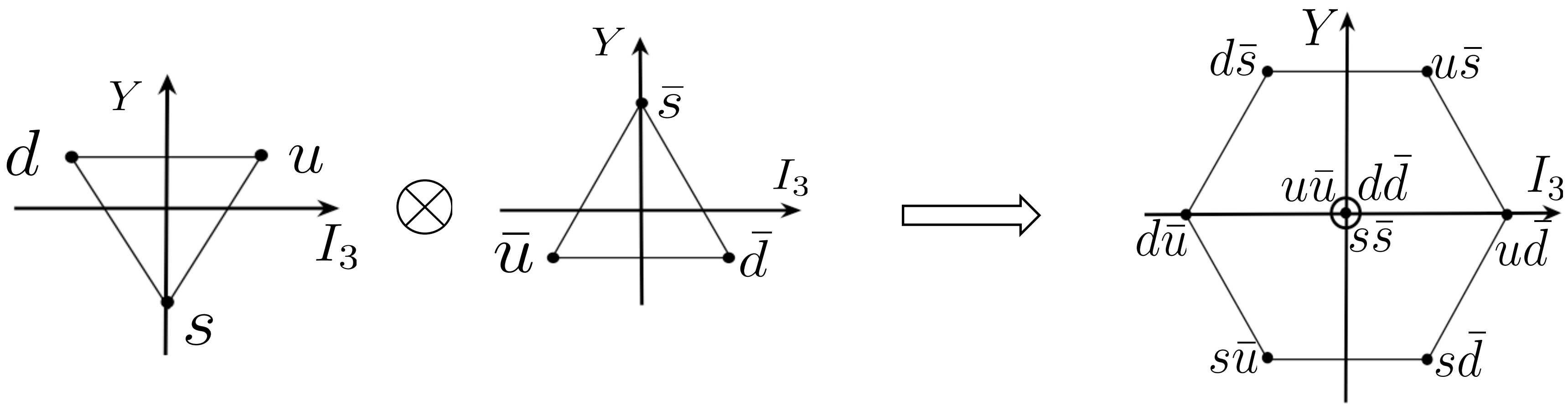
Mesons – Quark Model:

Quark-antiquark $q\bar{q}$ flavor states:

❑ Group theory says:

$$q(u,d,s) = \mathbf{3}, \quad \bar{q}(\bar{u},\bar{d},\bar{s}) = \bar{\mathbf{3}}, \quad \text{of flavor SU(3)}$$

$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$ ⟹ **1 flavor singlet + 8 flavor octet states**



There are three states with $I_3 = 0, Y = 0$: $u\bar{u}, d\bar{d}, s\bar{s}$

❑ Physical meson states (L=0, S=0):

✧ **Octet states:**

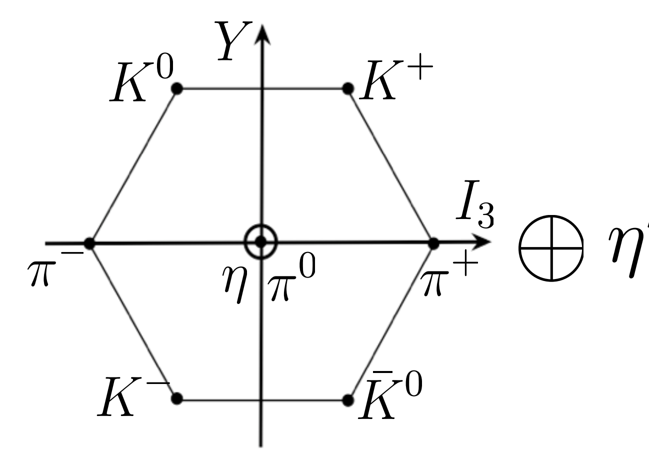
$$A = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \quad \Rightarrow \quad \pi^0$$

$$B = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \quad \Rightarrow \quad \eta_8$$

✧ **Singlet states:**

$$C = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \quad \Rightarrow \quad \eta_1$$

η_8, η_1 ⟹ η, η' ⟹

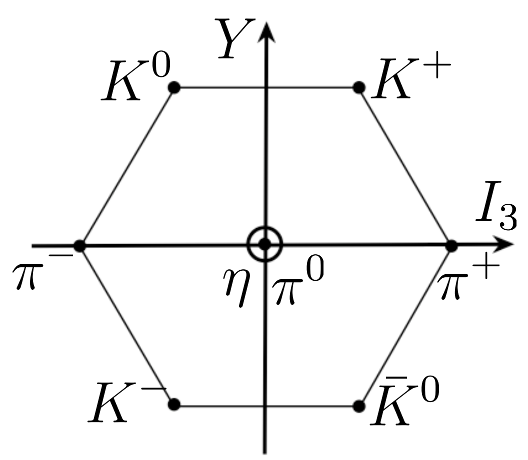


Quantum Numbers:

- **Meson states:** J^{PC}
  - **Spin of $q\bar{q}$ pair:** $\vec{S} = \vec{s}_q + \vec{s}_{\bar{q}} \;\rightarrow\; S = 0, 1$
  - **Spin of mesons:** $J = S + L$
  - **Charge conjugation:** $C = (-1)^{L+S}$

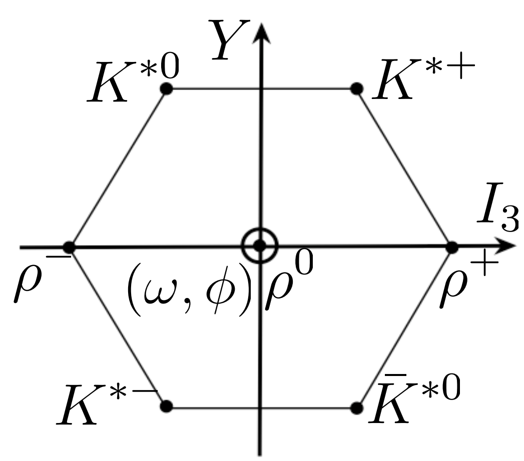
- **L=0 states:**

$J^{PC} = 0^{-+}$: ***(Y=S)***



$\oplus\, \eta'$

$J^{PC} = 1^{--}$: ***(Y=S)***



$\oplus\, \mathrm{mix}(\omega, \phi)$

Flavor singlet, spin octet

Flavor octet, spin octet

- **Color:**

No color was introduced!

Baryons – Quark Model:

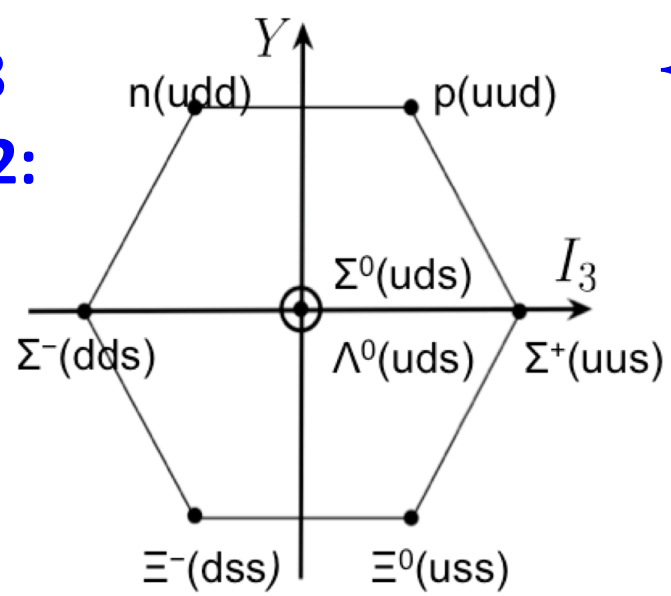
3 quark qqq **states:** $B = 1$

- **Group theory says:**
  - **Flavor:** $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_{M_S} \oplus \mathbf{8}_{M_A} \oplus \mathbf{1}_A$

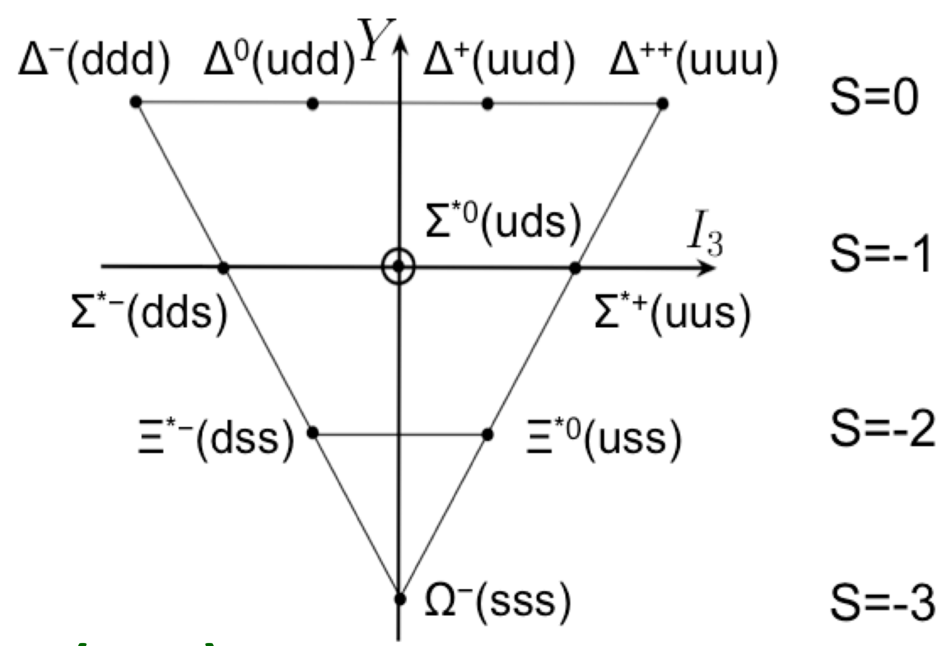
    S: symmetric in all 3 q, M_S : symmetric in 1 and 2,

    M_A : antisymmetric in 1 and 2, A : antisymmetric in all 3
  - **Spin:** $\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} = \mathbf{4}_S \oplus \mathbf{2}_{M_s} \oplus \mathbf{2}_{M_A} \Longrightarrow S = \frac{3}{2}, \frac{1}{2}, \frac{1}{2}$

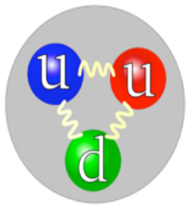
- **Physical baryon states:**
  - **Flavor-8 Spin-1/2:**



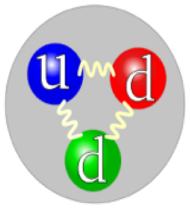
  - **Flavor-10 Spin-3/2:**



Proton



Neutron



Δ^{++}(uuu), …

Violation of Pauli exclusive principle

Need another quantum number - color!

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Color:

❑ Minimum requirements:

- ✧ Quark needs to carry at least 3 different colors
- ✧ Color part of the 3-quarks' wave function needs to antisymmetric

❑ SU(3) color:

Recall: $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_{M_S} \oplus \mathbf{8}_{M_A} \oplus \mathbf{1}_A$

Antisymmetric color singlet state:

$$\Longrightarrow \; c(\text{Red, Green, Blue})$$

$$\psi_{\text{Color}}(c_1, c_2, c_3) = \frac{1}{\sqrt{6}}[\text{RGB-GRB+RBG-BRG+GBR-BGR}]$$

❑ Baryon wave function:

$$\Psi(q_1, q_2, q_3) = \psi_{\text{Space}}(x_1, x_2, x_3) \otimes \psi_{\text{Flavor}}(f_1, f_2, f_3) \otimes \psi_{\text{Spin}}(s_1, s_2, s_3) \otimes \psi_{\text{Color}}(c_1, c_2, c_3)$$

Antisymmetric **Symmetric** **Symmetric** **Symmetric** **Antisymmetric**

A Complete Example: Proton

- **Wave function – the state:**

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}}\left[uud(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow -2\uparrow\uparrow\downarrow) + udu(\uparrow\uparrow\downarrow + \downarrow\uparrow\uparrow -2\uparrow\downarrow\uparrow) + duu(\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow -2\downarrow\uparrow\uparrow)\right]$$

- **Normalization:**

$$\langle p\uparrow|p\uparrow\rangle = \frac{1}{18}[(1+1+(-2)^2)+(1+1+(-2)^2)+(1+1+(-2)^2)] = 1$$

- **Charge:** $\hat{Q} = \sum_{i=1}^{3}\hat{Q}_i$

$$\langle p\uparrow|\hat{Q}|p\uparrow\rangle = \frac{1}{18}[(\tfrac{2}{3}+\tfrac{2}{3}-\tfrac{1}{3})(1+1+(-2)^2)+(\tfrac{2}{3}-\tfrac{1}{3}+\tfrac{2}{3})(1+1+(-2)^2) +(-\tfrac{1}{3}+\tfrac{2}{3}+\tfrac{2}{3})(1+1+(-2)^2)] = 1$$

- **Spin:** $\hat{S} = \sum_{i=1}^{3}\hat{s}_i$

$$\langle p\uparrow|\hat{S}|p\uparrow\rangle = \frac{1}{18}\{[(\tfrac{1}{2}-\tfrac{1}{2}+\tfrac{1}{2})+(-\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2})+4(\tfrac{1}{2}+\tfrac{1}{2}-\tfrac{1}{2})] +[\tfrac{1}{2}+\tfrac{1}{2}+4\tfrac{1}{2}]+[\tfrac{1}{2}+\tfrac{1}{2}+4\tfrac{1}{2}]\} = \frac{1}{2}$$

- **Magnetic moment:**

$$\mu_p = \langle p\uparrow|\sum_{i=1}^{3}\hat{\mu}_i(\hat{\sigma}_3)_i|p\uparrow\rangle = \frac{1}{3}[4\mu_u - \mu_d]$$

$$\mu_n = \frac{1}{3}[4\mu_d - \mu_u] \qquad \frac{\mu_u}{\mu_d} \approx \frac{2/3}{-1/3} = -2$$

$\Rightarrow$

$$\left(\frac{\mu_n}{\mu_p}\right)_{\rm QM} = -\frac{2}{3}$$

$$\left(\frac{\mu_n}{\mu_p}\right)_{\rm Exp} = -0.68497945(58)$$

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How to "see" Substructure of a Nucleon?

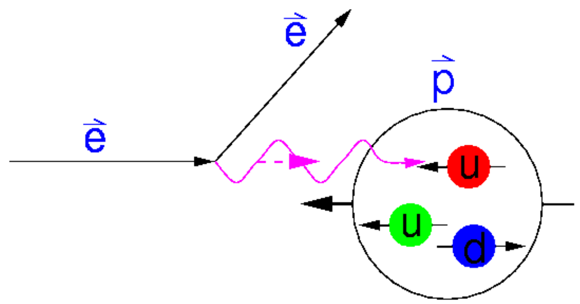
- A modern "Rutherford" experiment (over 50 years ago):

SLAC 1968: $e(l) + h(p) \to e'(l') + X$

Need a localized probe:

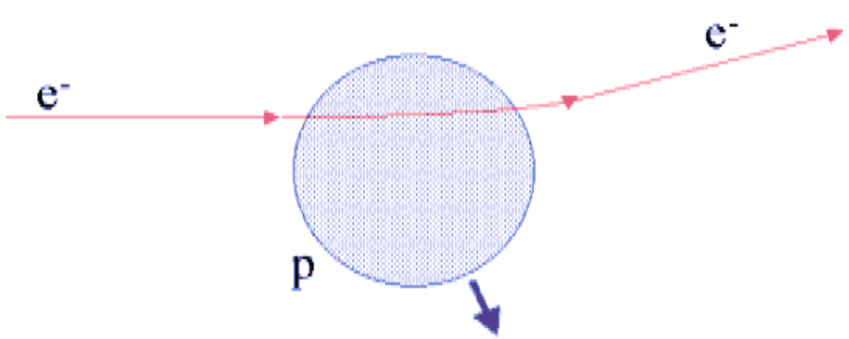
$$Q^2 = -(l - l')^2 \gg 1\ \text{fm}^{-2}$$

$$\Rightarrow \frac{1}{Q} \ll 1\ \text{fm}$$

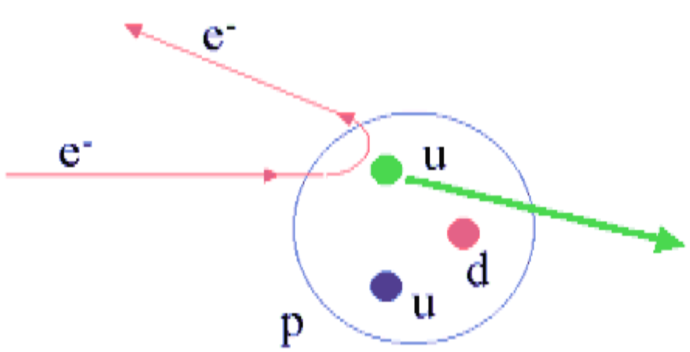


Prediction:

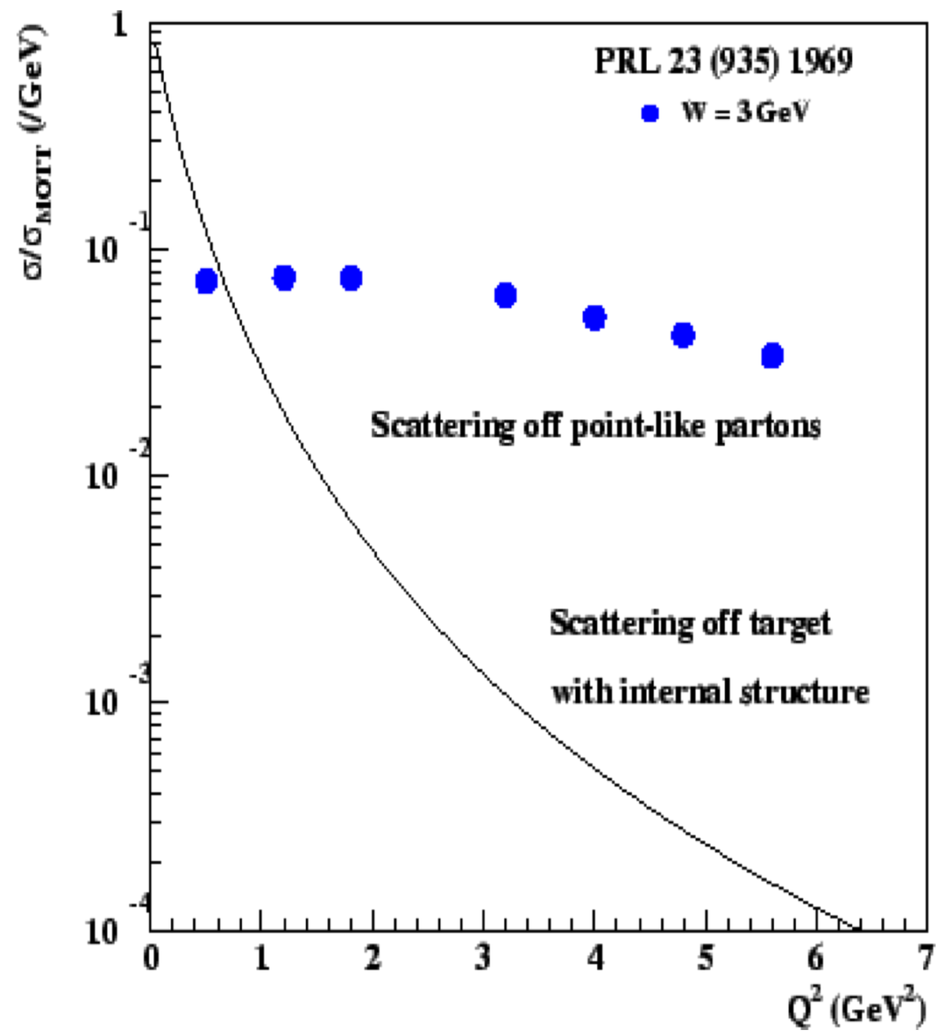
- If proton "charge cloud":



- If proton contains point charges, some of time see:



Discovery:



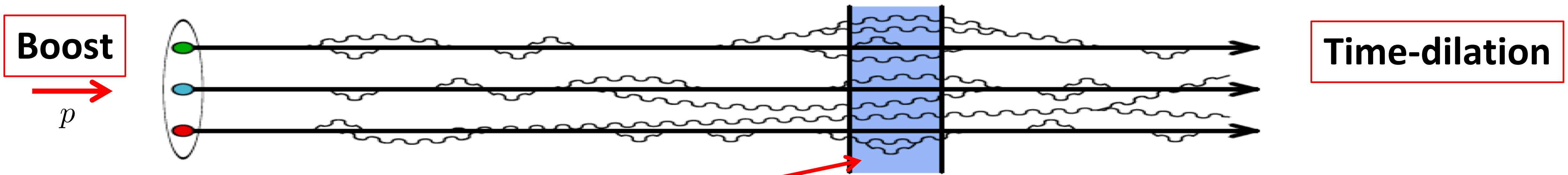
Discovery:

Partons/Quarks

How to “see” a Parton inside a Nucleon?

❑ Feynman’s parton picture and parton model:

High energy scattering with a large momentum transfer: $Q \gg 1/R \sim 1/\text{fm} \sim 200\ \text{MeV}$



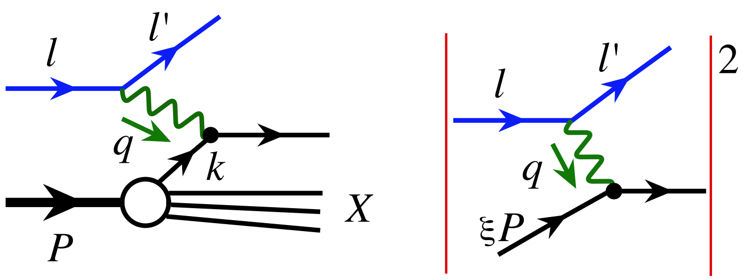
Hard probe:
t ~ 1/Q to catch the fluctuation!

$$k^\mu = (k^+, k^-, k_T) \approx (k^+, 0^-, 0_T)$$
$$k^+ = xp^+ \sim Q \gg \langle k^- \rangle, \langle k_T \rangle$$

Momentum fraction → $x = k^+/p^+$

❑ Inclusive deep inelastic scattering (DIS) – Parton Model:

$$e(l) + h(p) \to e'(l') + X$$



$$d\hat{\sigma}_{ei\to e'X} = \frac{1}{2\hat{s}} \left|\mathcal{M}_{ei\to e'X}\right|^2 \times (2\pi)^4 \delta(l + xp - l' - k) \times \frac{d^3k}{(2\pi)^3 2E_k} \frac{d^3l'}{(2\pi)^3 2E'}$$

$$E' \frac{d\sigma_{eh\to e'X}}{d^3l'} = \sum_i \int d\xi\, f_{i/h}(\xi)\, E' \frac{d\hat{\sigma}_{ei\to e'X}}{d^3l'}$$

$f_{i/h}(\xi)$: **Parton distribution function (PDF)**
Probability density to find a parton of flavor “i” within the hadron “h”, carrying hadron’s momentum fraction ξ.

$$E' \frac{d\hat{\sigma}_{ei\to e'X}}{d^3l'} \propto \delta(\xi - x)$$

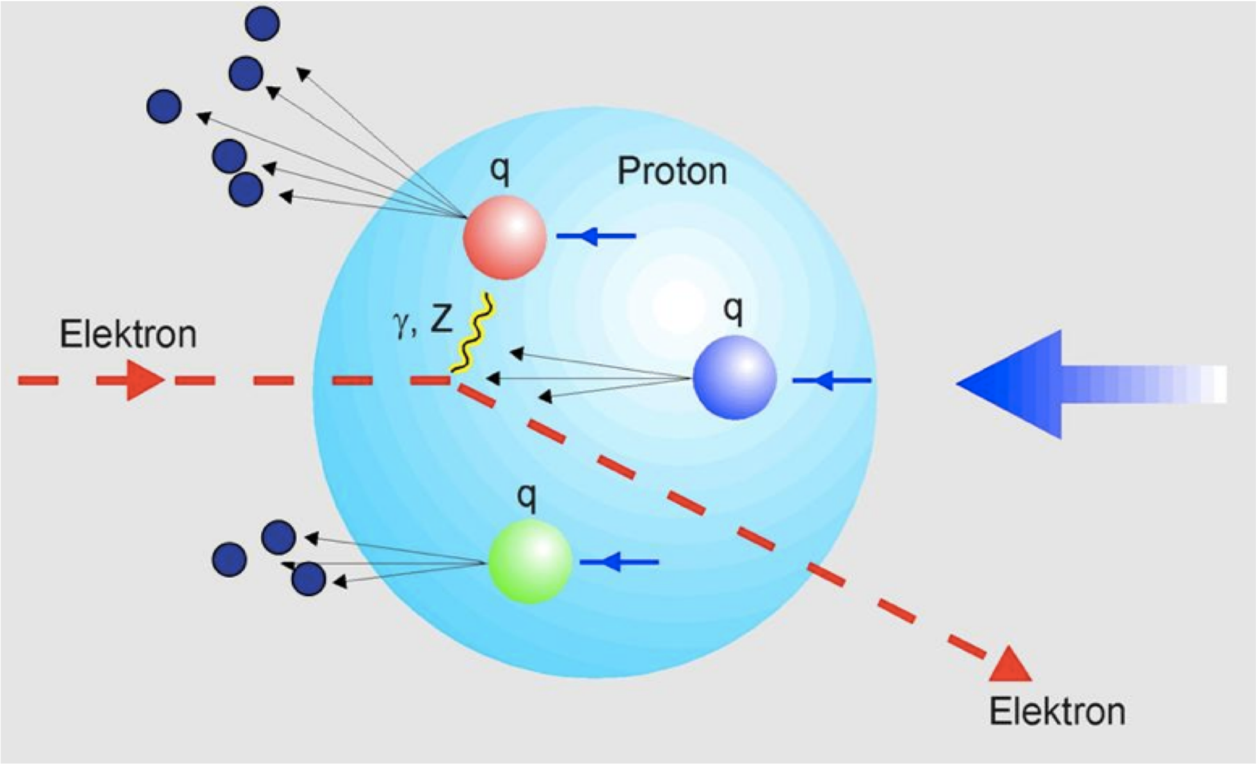
$$E' \frac{d\sigma_{eh\to e'X}}{d^3l'} \propto \sum_i e_i^2\, f_{i/h}(x)$$

TMD Handbook Eq. (1.1)

Discovery of Quarks and Emergence of QCD

❑ **Lepton-hadron inclusive DIS:**

SLAC 1968: $e(l) + h(p) \to e'(l') + X$



- **Localized probe:**

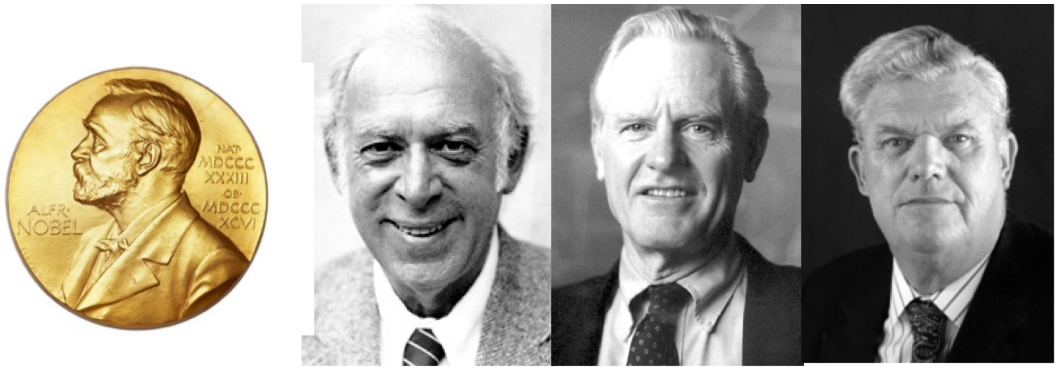
$$Q^2 = -(l - l')^2 \gg 1 \text{ fm}^{-2}$$

$$\Longrightarrow \frac{1}{Q} \ll 1 \text{ fm}$$

- **Two variables:**

$$Q^2 = 4EE' \sin^2(\theta/2)$$
$$x_B = \frac{Q^2}{2m_N \nu}$$
$$\nu = E - E'$$

➡ **Discovery of spin ½ quarks, and partonic structure!**



Nobel Prize, 1990

➡ **The birth of QCD (1973)**

– Quark Model + Yang-Mill gauge theory

Quantum Chromodynamics (QCD)

= A quantum field theory of quarks and gluons =

- **Fields:** $\psi_i^f(x)$ **Quark fields: spin-½ Dirac fermion (like electron)**

  Color triplet: $i = 1, 2, 3 = N_c$

  Flavor: $f = u, d, s, c, b, t$

  $A_{\mu,a}(x)$ **Gluon fields: spin-1 vector field (like photon)**

  Color octet: $a = 1, 2, ..., 8 = N_c^2 - 1$

- **QCD Lagrangian density:**

$$\mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[(i\partial_\mu \delta_{ij} - gA_{\mu,a}(t_a)_{ij})\gamma^\mu - m_f \delta_{ij} \right] \psi_j^f$$
$$- \frac{1}{4} \left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - gC_{abc} A_{\mu,b} A_{\nu,c} \right]^2$$
$$+ \text{gauge fixing} + \text{ghost terms}$$

- **QED – force to hold atoms together:**

$$\mathcal{L}_{QED}(\phi, A) = \sum_f \overline{\psi}^f \left[(i\partial_\mu - eA_\mu)\gamma^\mu - m_f \right] \psi^f - \frac{1}{4} \left[\partial_\mu A_\nu - \partial_\nu A_\mu \right]^2$$

QCD is much richer in dynamics than QED

Gluons are dark, but, interact with themselves, NO free quarks and gluons

Gauge Properties of QCD:

❑ Gauge Invariance:

$$\psi_i(x) \to \psi'_j(x) = U(x)_{ji}\,\psi_i(x)$$
$$A_\mu(x) \to A'_\mu(x) = U(x)\,A_\mu(x)\,U^{-1}(x) + \frac{i}{g}\left[\partial_\mu U(x)\right]U^{-1}(x)$$

where $A_\mu(x)_{ij} \equiv A_{\mu,a}(x)(t_a)_{ij}$

$$U(x)_{ij} = \left[e^{i\,\alpha_a(x)\,t_a}\right]_{ij} \quad \text{Unitary} \;\; [\det=1,\ \mathrm{SU}(3)]$$

❑ Color matrices:

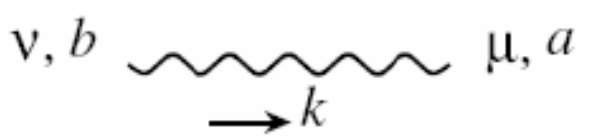
$$[\,t_a, t_b\,] = i\,C_{abc}\,t_c$$

Generators for the fundamental representation of SU3 color

❑ Gauge Fixing:

$$\mathcal{L}_{gauge} = -\frac{\lambda}{2}(\partial_\mu A_a^\mu)(\partial_\nu A_a^\nu)$$

Allow us to define the gauge field propagator:



$$G_{\mu\nu}(k)_{ab} = \frac{\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\left(1 - \frac{1}{\lambda}\right)\right]$$

with $\lambda = 1$ **the Feynman gauge**

Ghost in QCD

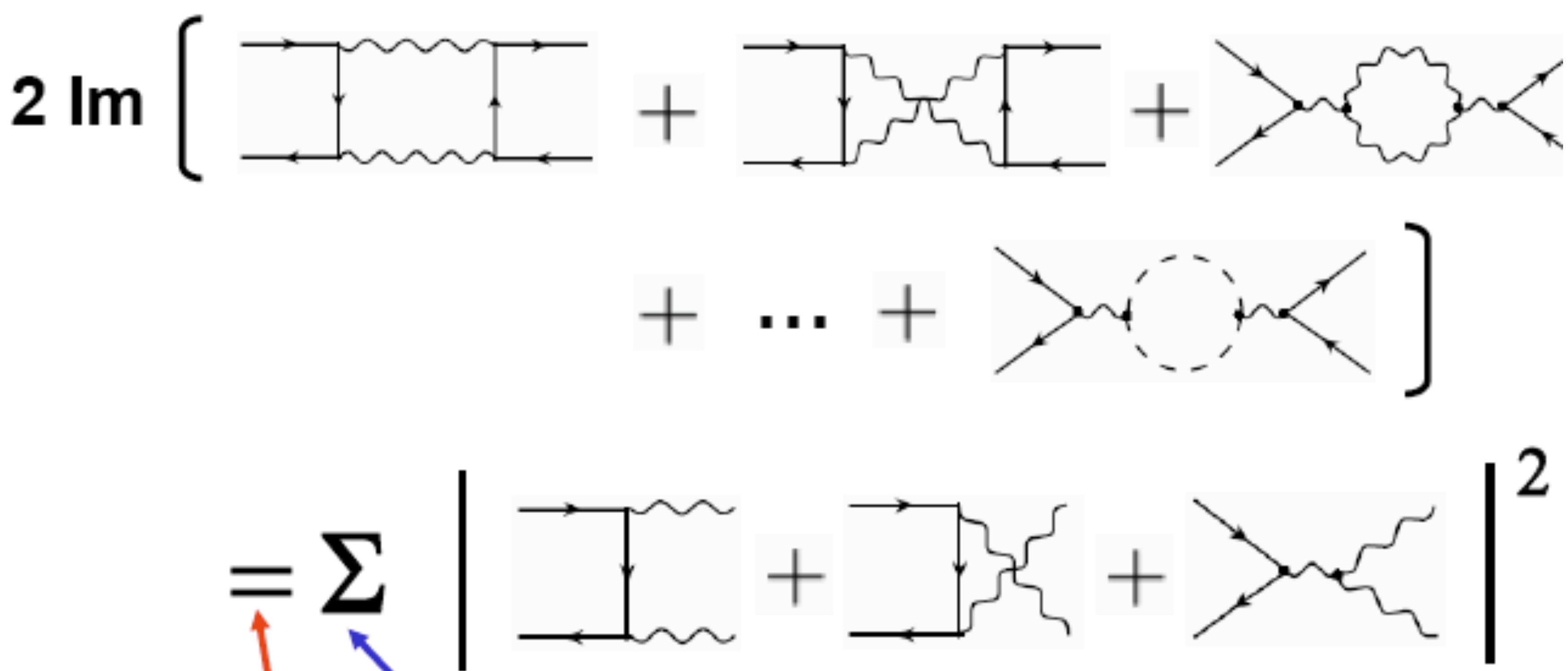
❑ Ghost:

Only needed when we are working in a covariant gauge

Ghost

$$\mathcal{L}_{ghost} = (\partial_\mu \bar{\eta}_a(x))(\partial^\mu \eta_a(x) - g\, C_{abc}\, A_b^\mu(x)\, \eta_c(x))$$

so that the optical theorem (hence the unitarity) can be respected



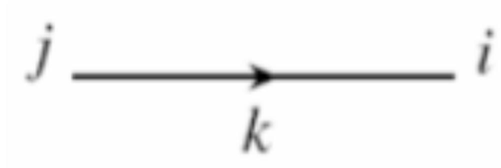
Sum over all physical polarizations

Fail without the ghost loop

Feynman Rules in QCD

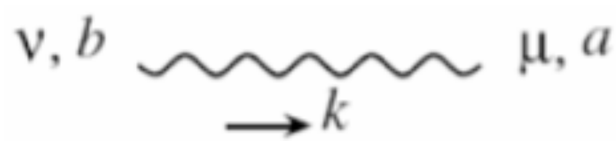
❑ Propagators:

Quark:



$$\frac{i}{\gamma\cdot k - m}\,\delta_{ij}$$

Gluon:



$$\frac{i\delta_{ab}}{k^2}\left[-g_{\mu\nu}+\frac{k_\mu k_\nu}{k^2}\left(1-\frac{1}{\lambda}\right)\right]$$

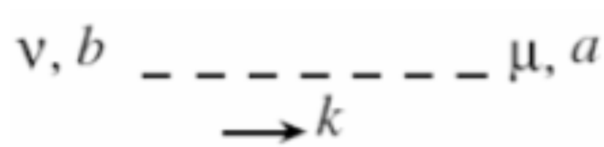
for a covariant gauge

$$\frac{i\delta_{ab}}{k^2}\left[-g_{\mu\nu}+\frac{k_\mu n_\nu + n_\mu k_\nu}{k\cdot n}\right]$$

for a light-cone gauge

$$n\cdot A(x)=0 \quad \text{with} \quad n^2=0$$

Ghost:



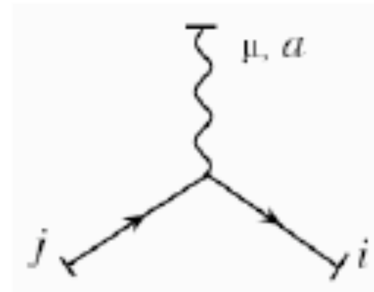
$$\frac{i\delta_{ab}}{k^2}$$

Only needed when we are working in a covariant gauge

Feynman Rules in QCD

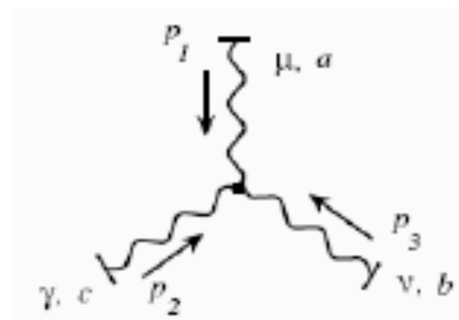
❑ Interactions:

$$-g\bar{\psi}\gamma^{\mu}A_{\mu,a}t_a\psi$$



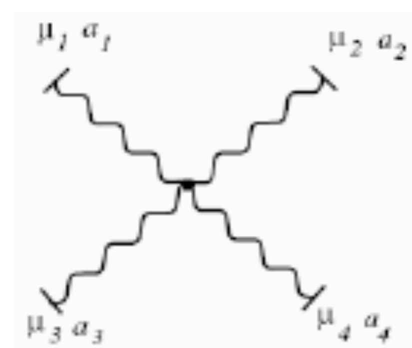
$$-ig(t_a)_{ij}\gamma_{\mu}$$

$$\frac{1}{2}gC_{abc}(\partial_{\mu}A_{\nu,a} - \partial_{\nu}A_{\mu,a})A_b^{\mu}A_c^{\nu}$$



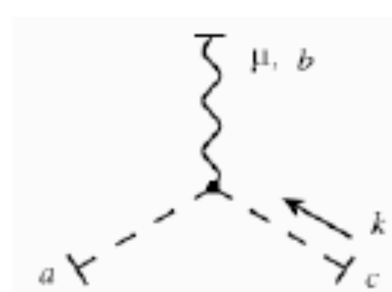
$$-gC_{abc}\left[g_{\mu\nu}(p_1 - p_2)_{\gamma} + g_{\nu\gamma}(p_2 - p_3)_{\mu} + g_{\gamma\mu}(p_3 - p_1)_{\nu}\right]$$

$$-\frac{g^2}{4}C_{abc}C_{ab'c'} * A_b^{\mu}A_c^{\nu}A_{\mu,b'}A_{\nu,c'}$$



$$-ig^2\left[C_{ea_1a_2}C_{ea_3a_4} * (g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_4}g_{\mu_2\mu_3}) + \ldots\right]$$

$$\partial_{\mu}\bar{\eta}_a\left(gC_{abc}A_b^{\mu}\right)\eta_c$$



$$gC_{abc}k_{\mu}$$

Only needed when we are working in a covariant gauge

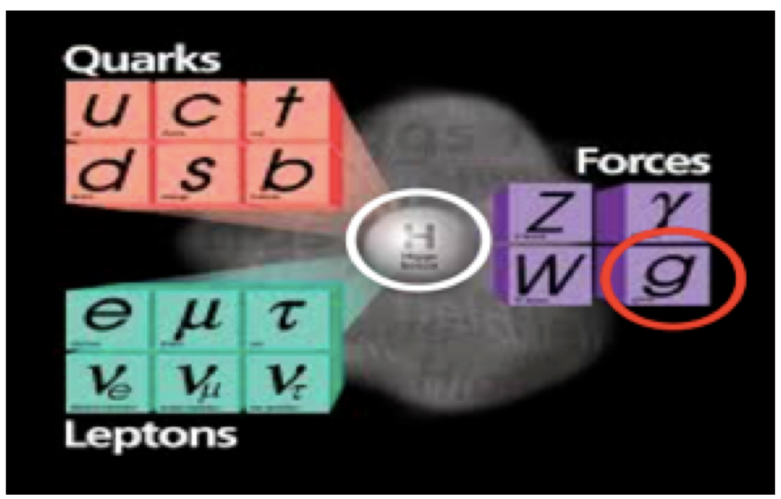
QCD is everywhere in our universe

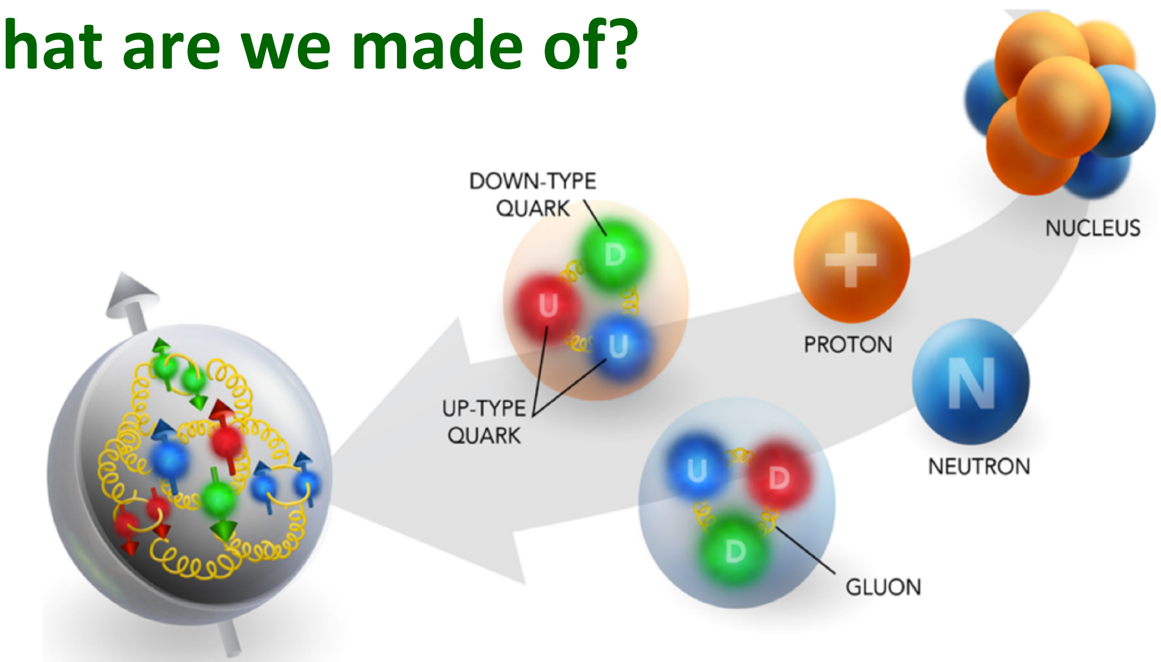
- ❑ **Understanding where did we come from?**

Global Time: →

  - ○ **QCD at high temperature, high densities, phase transition, …**
  - ○ ***Facilities – Relativistic heavy ion collisions: SPS, RHIC, the LHC, …***

- ❑ **Understanding the visible world at 3°K – what are we made of?**





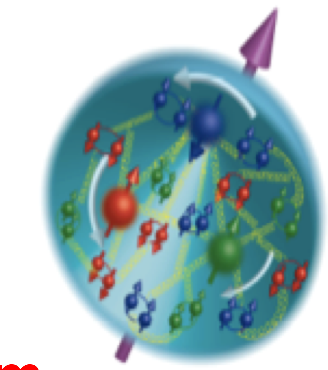
  - ○ **How to understand the emergence and properties of nucleon and nuclei (elements of the periodic table) in terms of elements of the modern periodic table?**
  - ○ **How does the glue bind us all?**

Nuclear Femtography

Search for answers to these questions at a Fermi scale!

○ ***Facilities – CEBAF, LHC, Amber, EIC, …***

QCD Color is Fully Entangled



- **QCD color confinement:**
  - *Do not see any quarks and gluons in isolation*
  - *The structure of nucleons and nuclei – emergent properties of QCD*

Color Confinement **Asymptotic freedom**

$Q\ (\mathrm{GeV})$

20 MeV (10 fm) 200 MeV (1 fm) 2 GeV (1/10 fm) ***Probing scale***

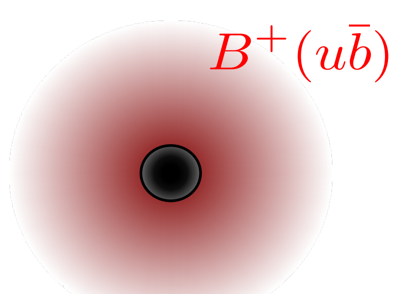
QCD at the Fermi Scale: Femto-science (0.1-10 fm)

Asymptotic regime PQCD works beautifully!

- The most interesting, rich, and complex regime of the theory!
- Emergent phenomena depend on the scale at which we probe them!

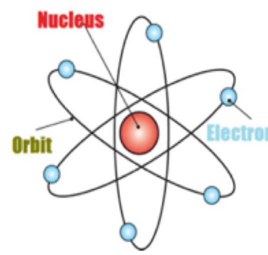
- **QCD is non-perturbative:**
  - *Any cross section/observable with identified hadron is not perturbatively calculable!*
  - *Color is fully entangled!*

B-meson



Brown-Muck

Atomic structure



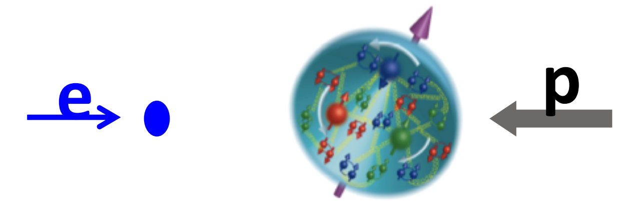
Quantum orbits

TMD Collaboration

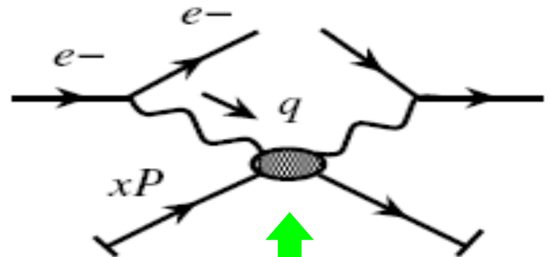
Theoretical Approaches – Approximations:

❑ Perturbative QCD Factorization:

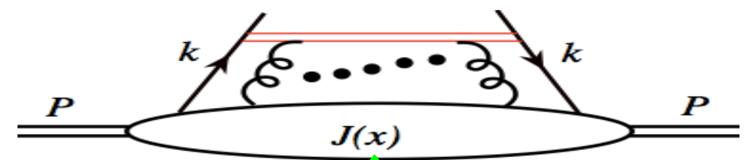
– *Approximation at Feynman diagram level*



$$\sigma_{\rm tot}^{\rm DIS} \sim$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$

Probe	Structure	Approximation
Hard-part	Parton-distribution	Power corrections

❑ Effective field theory (EFT):

– *Approximation at the Lagrangian level*

Soft-collinear effective theory (SCET), Non-relativistic QCD (NRQCD), Heavy quark EFT, chiral EFT(s), …

❑ Lattice QCD:

– *Approximation for finite lattice spacing, finite box, lightest quark masses (removable with increased computational cost)*

Hadron structure, hadron spectroscopy, nuclear structure, phase shift, …

❑ Other approaches:

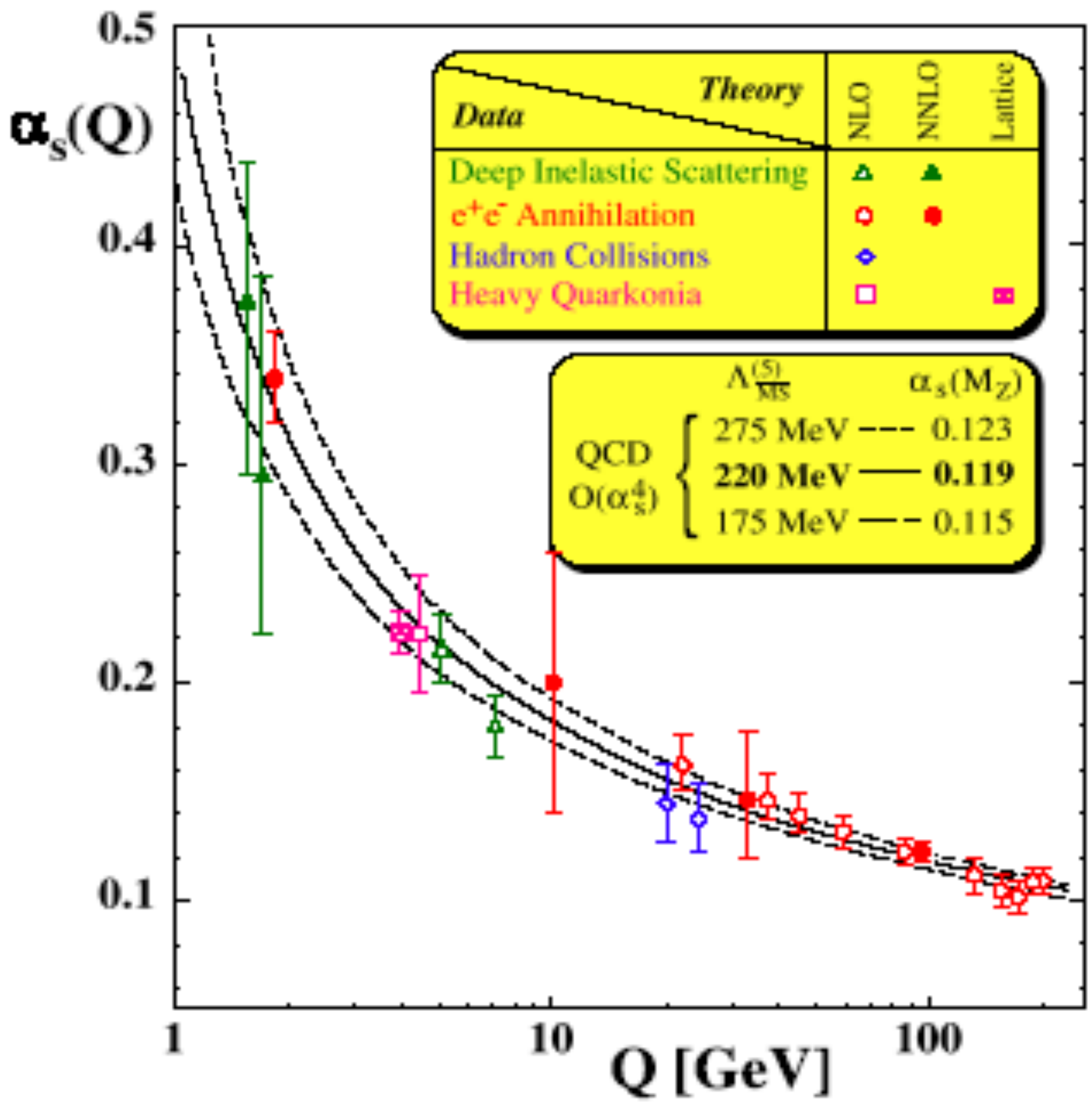
Light-cone perturbation theory, Dyson-Schwinger Equations (DSE), Constituent quark models, AdS/CFT correspondence, …

QCD Asymptotic Freedom

- **Interaction strength:**

$$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1-\frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \equiv \frac{4\pi}{-\beta_1\ln\left(\frac{\mu_2^2}{\Lambda_{QCD}^2}\right)}$$

μ_2 and μ_1 not independent



Asymptotic Freedom ⇔ antiscreening

$$\text{QCD: } \frac{\partial\alpha_s(Q^2)}{\partial\ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial\alpha_{EM}(Q^2)}{\partial\ln Q^2} = \beta(\alpha_{EM}) > 0$$

D.Gross, F.Willczek, Phys.Rev.Lett 30, (1973)
H.Politzer, Phys.Rev.Lett. 30, (1973)

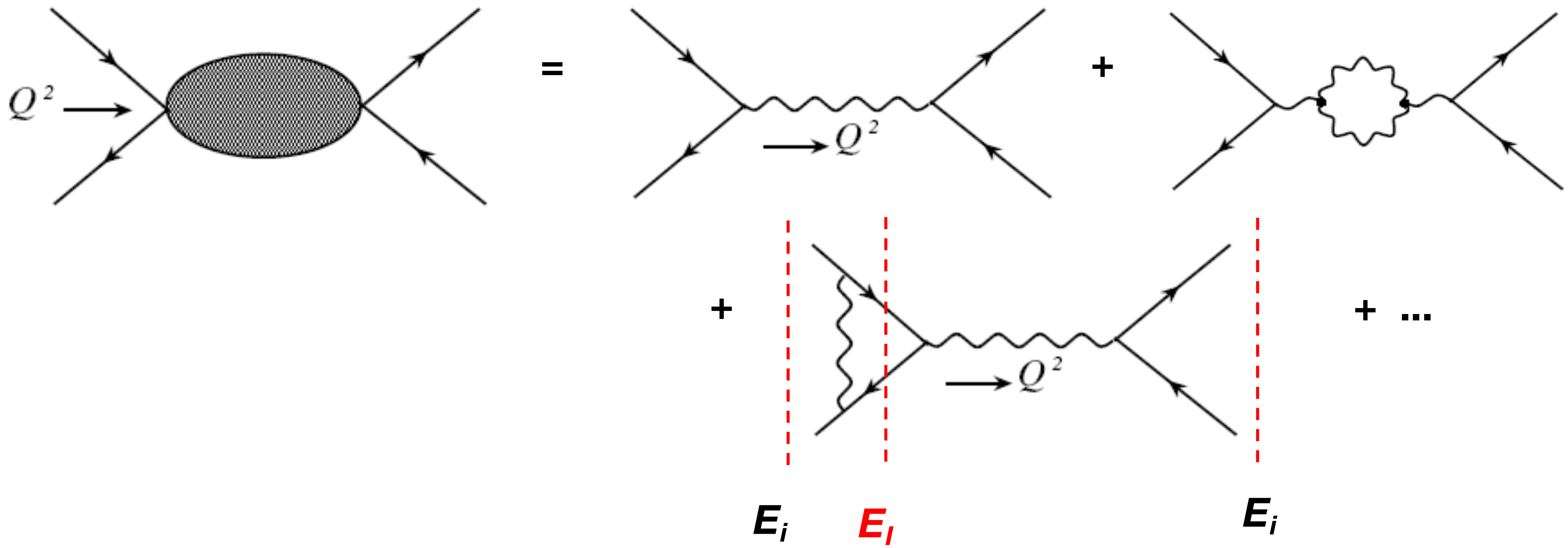
2004 Nobel Prize in Physics

Discovery of QCD Asymptotic Freedom

TMD Collaboration

Renormalization, Why need?

- **Scattering amplitude:**



$$= \int \langle PS \rangle_I \left(\frac{1}{E_i - E_I} + \ldots \right) + \ldots \Rightarrow \infty$$

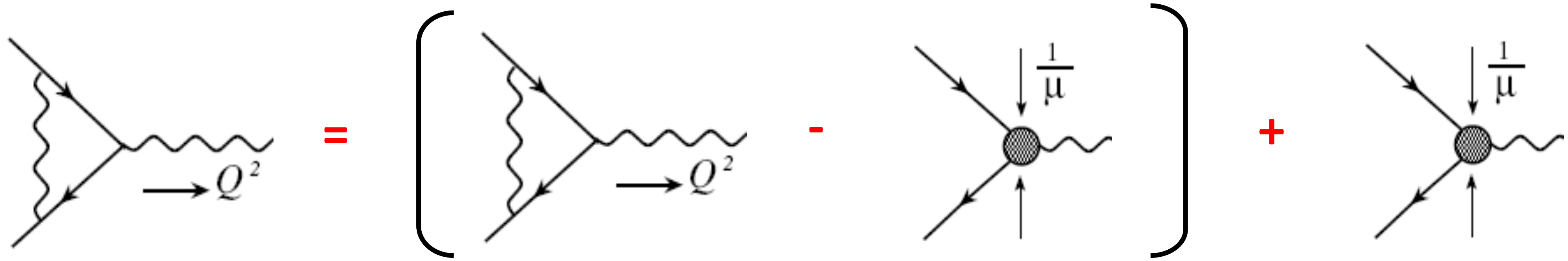
UV divergence: **result of a "sum" over states of high masses**

Uncertainty principle: **High mass states = "Local" interactions**

No experiment has an infinite resolution!

Physics of Renormalization

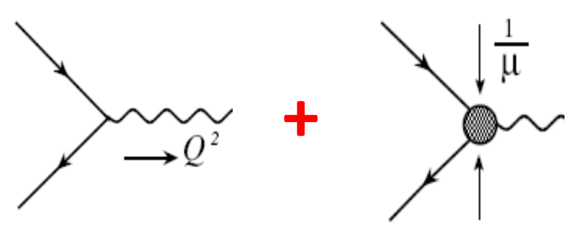
- UV divergence due to “high mass” states, not observed



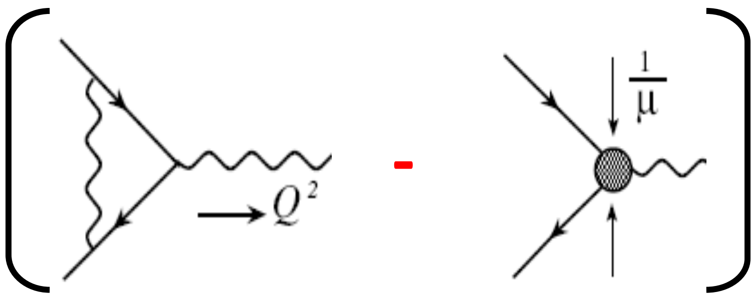
“Low mass” state

“High mass” states

- Combine the “high mass” states with LO

LO: 

$= \quad g(\mu)$ ← Renormalized coupling

NLO: 

+ ...

No UV divergence!

- Renormalization = re-parameterization of the expansion parameter in perturbation theory

Renormalization Group

- Physical quantity should not depend on the choice of renormalization scale μ ➡ renormalization group equation:

$$\mu^2 \frac{d}{d\mu^2}\, \sigma_{\rm Phy}\left(\frac{Q^2}{\mu^2}, g(\mu), \mu\right) = 0 \quad \Longrightarrow \quad \sigma_{\rm Phy}(Q^2) = \sum_n \hat{\sigma}^{(n)}(Q^2, \mu^2) \left(\frac{\alpha_s(\mu)}{2\pi}\right)^n$$

- Running coupling constant:

$$\mu \frac{\partial g(\mu)}{\partial \mu} = \beta(g) \qquad \alpha_s(\mu) = \frac{g^2(\mu)}{4\pi}$$

- QCD β function:

$$\beta(g) = \mu \frac{\partial g(\mu)}{\partial \mu} = +g^3 \frac{\beta_1}{16\pi^2} + \mathcal{O}(g^5) \qquad \beta_1 = -\frac{11}{3} N_c + \frac{4}{3}\frac{n_f}{2} < 0 \quad \text{for } n_f \leq 6$$

- QCD running coupling constant:

$$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1) \ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \Rightarrow 0 \quad \text{as } \mu_2 \to \infty \quad \text{for } \beta_1 < 0$$

Effective Quark Mass

- **Running quark mass:**

$$m(\mu_2) = m(\mu_1)\exp\left[-\int_{\mu_1}^{\mu_2}\frac{d\lambda}{\lambda}[1+\gamma_m(g(\lambda))]\right]$$

Quark mass depend on the renormalization scale!

- **QCD running quark mass:**

$$m(\mu_2) \Rightarrow 0 \qquad \text{as } \mu_2 \to \infty \qquad \text{since } \gamma_m(g(\lambda)) > 0$$

- **Choice of renormalization scale:**

$\mu \sim Q$ **for small logarithms in the perturbative coefficients**

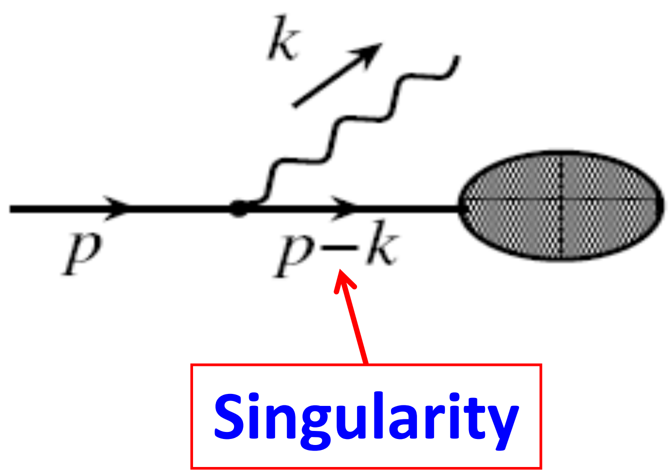
- **Light quark mass:** $m_f(\mu) \ll \Lambda_{\rm QCD}$ for $f = u, d,$ even s

QCD perturbation theory ($Q \gg \Lambda_{QCD}$) is effectively a massless theory

Infrared and collinear divergences

❑ **Consider a general diagram:**

$$p^2 = 0, \quad k^2 = 0$$ **for a massless theory**



✧ $$k^\mu \to 0 \;\Rightarrow\; (p-k)^2 \to p^2 = 0$$

⇨ **Infrared (IR) divergence**

✧ $$k^\mu \,||\, p^\mu \;\Rightarrow\; k^\mu = \lambda p^\mu \quad \text{with} \;\; 0 < \lambda < 1$$

$$\Rightarrow\; (p-k)^2 \to (1-\lambda)^2 \, p^2 = 0$$

⇨ **Collinear (CO) divergence**

IR and CO divergences are generic problems of a massless perturbation theory

Infrared Safety

❑ Infrared safety:

$$\sigma_{\rm Phy}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\frac{m^2(\mu^2)}{\mu^2}\right)\Rightarrow\hat{\sigma}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)+\mathcal{O}\left[\left(\frac{m^2(\mu^2)}{\mu^2}\right)^{\kappa}\right]$$

Infrared safe = κ > 0

Purely perturbative calculations alone
(exploiting asymptotic freedom)
are only useful for quantities that are infrared safe

❑ Cross section with identified hadron(s):

- ***Can not be calculated perturbatively!***
- ***Solution – QCD factorization:***
  - ***to isolated what can be calculated perturbatively,***
  - ***to represent the leading non-perturbative information by universal functions***
  - ***to justify the approximation to neglect other nonperturbative information, such as power corrections, …***

Foundation of QCD perturbation theory

- **Renormalization**
  - QCD is renormalizable

Nobel Prize, 1999
't Hooft, Veltman

- **Asymptotic freedom**
  - weaker interaction at a shorter distance

Nobel Prize, 2004
Gross, Politzer, Welczek

- **Infrared safety and factorization**
  - calculable short distance dynamics
  - pQCD factorization – connect the partons to physical cross sections

J. J. Sakurai Prize, 2003
Mueller, Sterman

Look for infrared safe and factorizable observables!

Physical Observables

Cross sections with identified hadron(s)
are
non-perturbative!

Hadronic scale ~ 1/fm ~ 200 MeV is not
a perturbative scale

Look for two-types physical observables:

- ❑ Purely infrared safe quantities
- ❑ Observables with identified hadron(s), but, factorizable in QCD

Fully infrared safe observables – I

Fully inclusive, without any identified hadron!

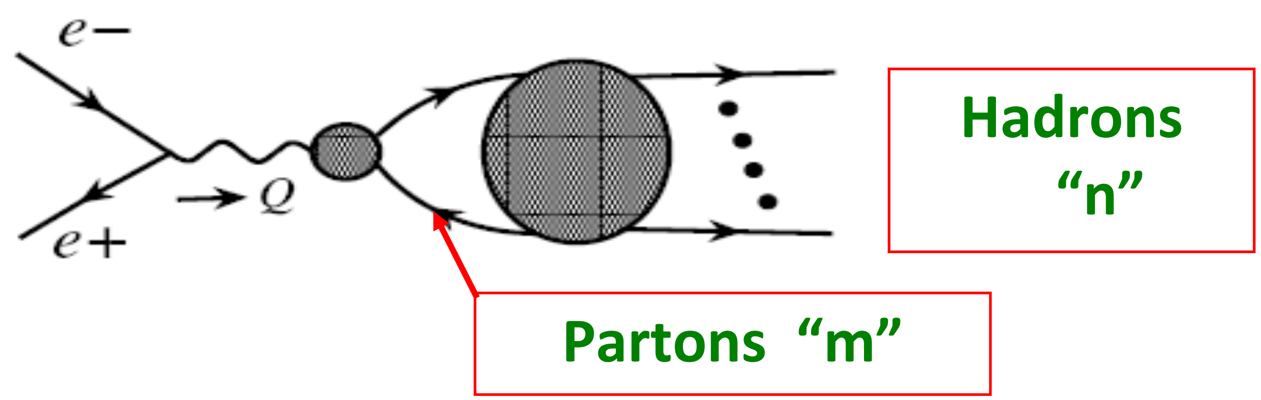
$$\sigma^{\text{total}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{total}}_{e^+e^-\to\text{partons}}$$

The simplest observable in QCD

$e^+e^- \rightarrow$ hadrons inclusive cross sections

- ❑ $e^+e^- \rightarrow$ hadron **total** cross section – not a specific hadron!

$$\sigma^{\rm tot}_{e^+e^-\to {\rm hadrons}} \propto \left| \right|^2$$



If there is no quantum interference between partons and hadrons,

$$\sigma^{\rm tot}_{e^+e^-\to {\rm hadrons}} \propto \sum_n {\rm P}_{e^+e^-\to n} = \sum_n \sum_m {\rm P}_{e^+e^-\to m} {\rm P}_{m\to n} = \sum_m {\rm P}_{e^+e^-\to m} \sum_n {\rm P}_{m\to n} \quad =1$$

$$\sigma^{\rm tot}_{e^+e^-\to {\rm partons}} \propto \sum_m {\rm P}_{e^+e^-\to m}$$

$$\Rightarrow \quad \sigma^{\rm tot}_{e^+e^-\to {\rm hadrons}} = \sigma^{\rm tot}_{e^+e^-\to {\rm partons}}$$

Finite in perturbation theory – KLN theorem

- ❑ $e^+e^- \rightarrow$ parton **total** cross section:

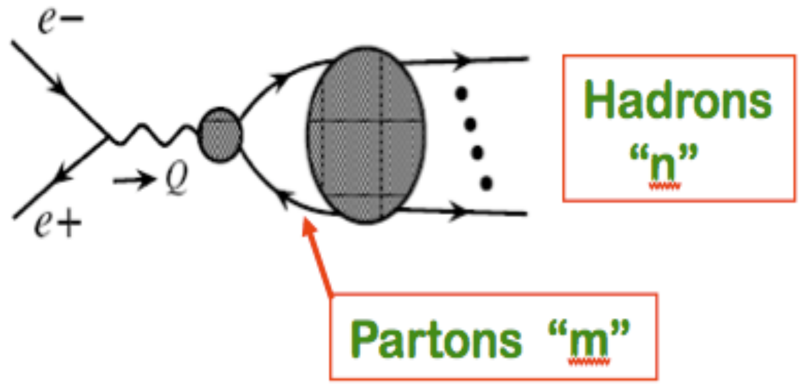
$$\sigma^{\rm tot}_{e^+e^-\to {\rm partons}}(s = Q^2) = \sum_n \sigma^{(n)}(Q^2, \mu^2) \left(\frac{\alpha_s(\mu^2)}{\pi} \right)^n$$

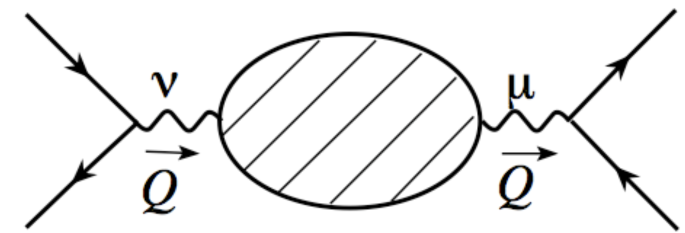
Calculable in pQCD

Infrared safety of e⁺e⁻ total cross sections

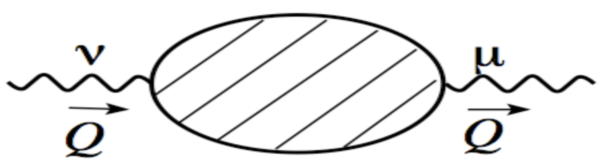
❑ **Optical theorem:**

$$\sigma^{\text{tot}}_{e^+e^-} = \frac{1}{2S}\left| \quad \right|^2 \propto \text{Im}$$





❑ **Time-like vacuum polarization:**



$$= \left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right)\Pi(Q^2)$$

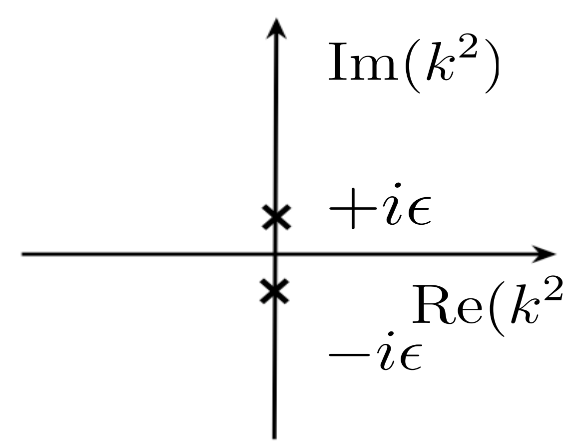
IR safety of $\sigma^{\text{tot}}_{e^+e^-\to\text{partons}}$ = **IR safety of** $\Pi(Q^2)$ **with** $Q^2 > 0$

❑ **IR safety of** $\Pi(Q^2)$**:**

If there were pinched poles in Π(Q²),

✧ **real partons moving away from each other**

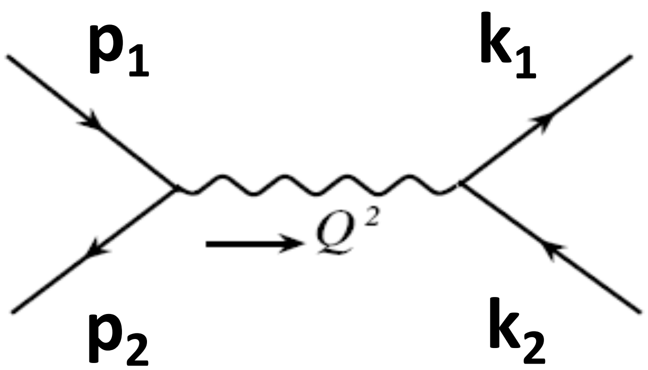
✧ **cannot be back to form the virtual photon again!**



Rest frame of the virtual photon

Lowest order (LO) perturbative calculation

- **Lowest order Feynman diagram:**



- **Invariant amplitude square:**

$$|\overline{M}_{e^+e^-\to Q\bar{Q}}|^2 = e^4 e_Q^2 N_c \frac{1}{s^2}\frac{1}{2^2}\mathrm{Tr}\left[\gamma\cdot p_2\gamma^\mu\gamma\cdot p_1\gamma^\nu\right]$$
$$\times\mathrm{Tr}\left[\left(\gamma\cdot k_1+m_Q\right)\gamma_\mu\left(\gamma\cdot k_2-m_Q\right)\gamma_\nu\right]$$
$$= e^4 e_Q^2 N_c \frac{2}{s^2}\left[(m_Q^2-t)^2+(m_Q^2-u)^2+2m_Q^2 s\right]$$

$$s=(p_1+p_2)^2$$
$$t=(p_1-k_1)^2$$
$$u=(p_2-k_1)^2$$

- **Lowest order cross section:**

$$\frac{d\sigma_{e^+e^-\to Q\bar{Q}}}{dt}=\frac{1}{16\pi s^2}|\overline{M}_{e^+e^-\to Q\bar{Q}}|^2 \quad \text{where } s=Q^2$$

$$\sigma_2^{(0)}=\sum_Q \sigma_{e^+e^-\to Q\bar{Q}}=\sum_Q e_Q^2 N_c\frac{4\pi\alpha_{em}^2}{3s}\left[1+\frac{2m_Q^2}{s}\right]\sqrt{1-\frac{4m_Q^2}{s}}$$

Threshold constraint

One of the best tests for the number of colors

TMD Collaboration

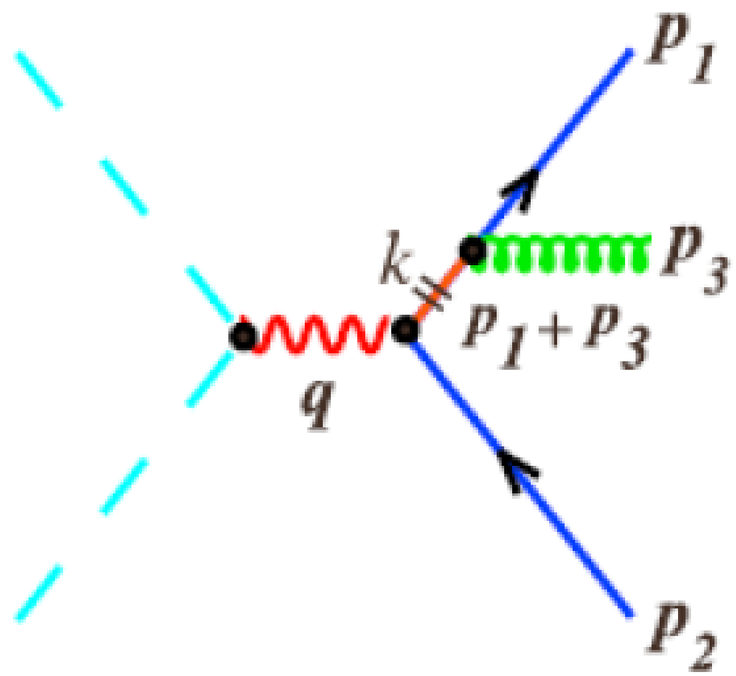
Next-to-leading order (NLO) contribution

- **Real Feynman diagram:**

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i.q}{s} \qquad \text{with } i = 1,2,3$$

$$\sum_i x_i = \frac{2\left(\sum_i p_i\right).q}{s} = 2$$

$$2(1-x_1) = x_2 x_3 (1-\cos\theta_{23}), \quad cycl.$$



+ crossing

- **Contribution to the cross section:**

$$\frac{1}{\sigma_0}\frac{d\sigma_{e^+e^-\to Q\bar{Q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

IR as $x_3 \to 0$

CO as $\theta_{13} \to 0$

$\theta_{23} \to 0$

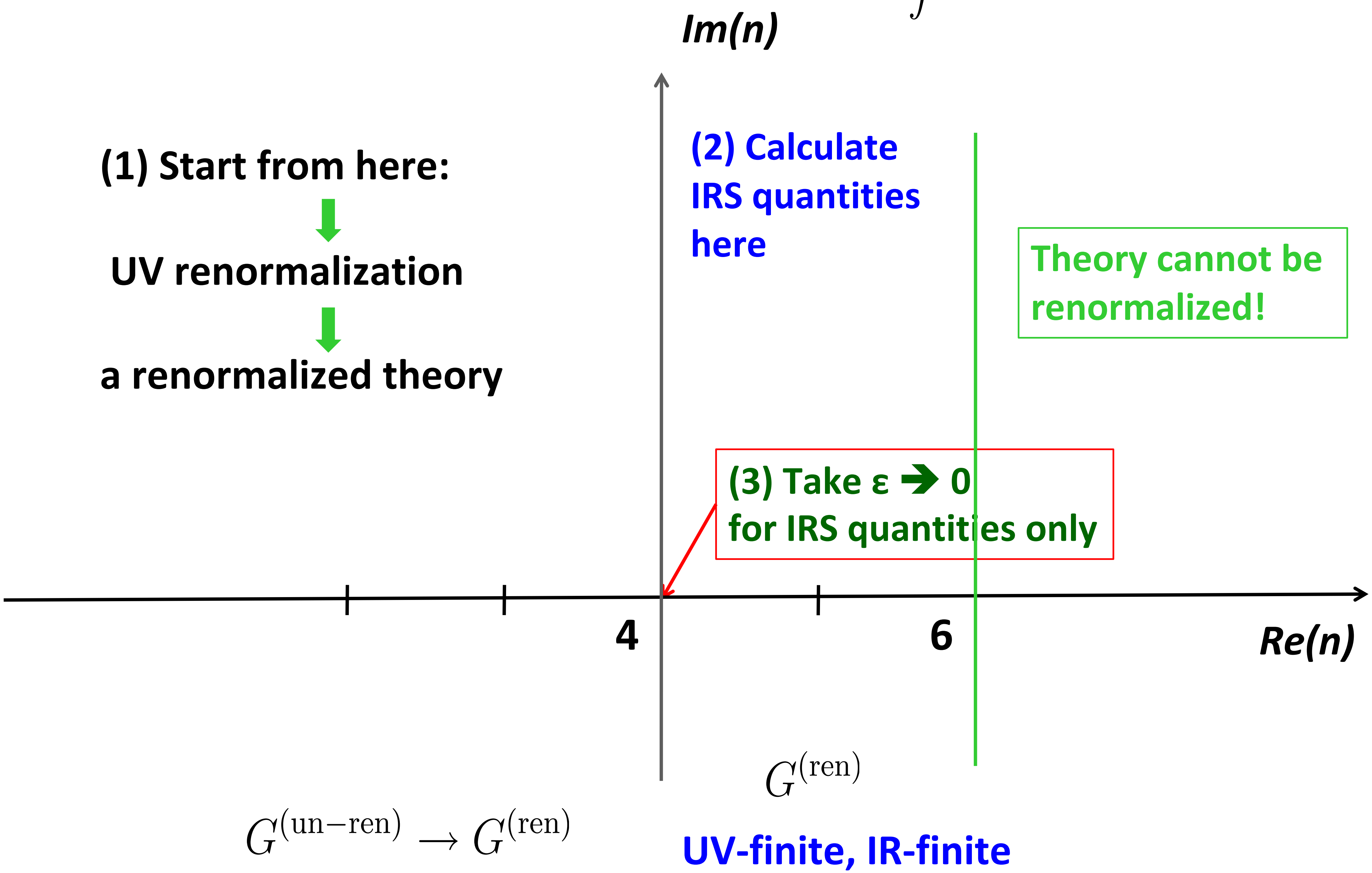
Divergent as $x_i \to 1$

Need the virtual contribution and a regulator!

How does dimensional regularization work?

- **Complex *n*-dimensional space:**

$$\int d^n k \, F(k,Q)$$



Dimensional regularization for both IR and CO

❑ NLO with a dimensional regulator:

✧ **Real:**

$$\sigma_{3,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)}\right]\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{19}{4}\right]$$

✧ **Virtual:**

$$\sigma_{2,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2\,\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\right]\left[-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}+\frac{\pi^2}{2}-4\right]$$

✧ **NLO:**

$$\sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} = \sigma_2^{(0)}\left[\frac{\alpha_s}{\pi}+O(\varepsilon)\right]$$

No ε dependence!

✧ **Total:**

$$\sigma^{\text{tot}} = \sigma_2^{(0)} + \sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} + O(\alpha_s^2) = \sigma_2^{(0)}\left[1+\frac{\alpha_s}{\pi}\right] + O(\alpha_s^2)$$

σ^{tot} is Infrared Safe!

σ^{tot} is independent of the choice of IR and CO regularization

Highest order perturbative calculations

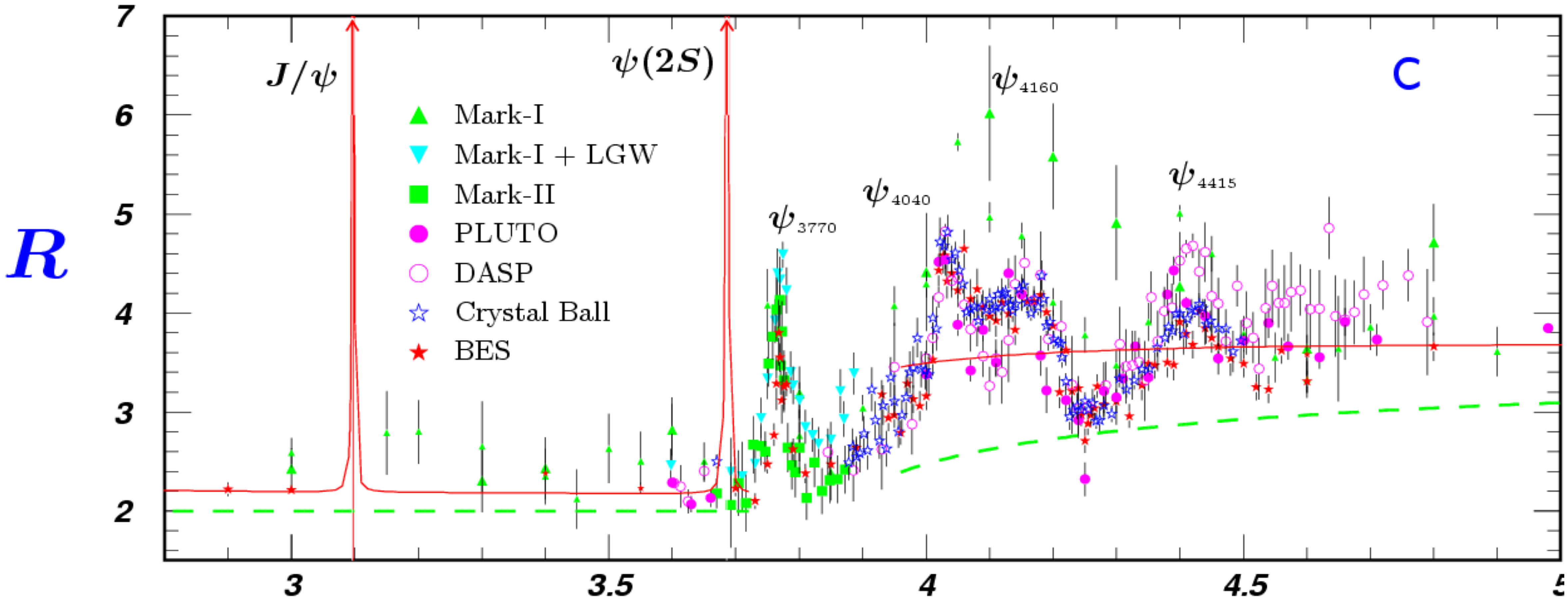
Hadronic cross section in e+e- collision

- **Normalized hadronic cross section:**

$$R_{e^+e^-}(s) \equiv \frac{\sigma_{e^+e^-\to\text{hadrons}}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}$$

$$\approx N_c \sum_{q=u,d,s} e_q^2 \left[1 + \frac{\alpha_s(s)}{\pi} + \mathcal{O}(\alpha_s^2(s))\right] \xrightarrow{N_c = 3} 2\left[1 + \frac{\alpha_s(s)}{\pi} + \ldots\right]$$

$$+ N_c \sum_{q=c,\ldots} e_q^2 \left[\left(1 + \frac{2m_q^2}{s}\right)\sqrt{1 - \frac{4m_q^2}{s}} + \mathcal{O}(\alpha_s(s))\right]$$



Go beyond the inclusive total cross section?

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Fully infrared safe observables - II

No identified hadron, but, with phase space constraints

$$\sigma^{\rm Jets}_{e^+e^-\to{\rm hadrons}} = \sigma^{\rm Jets}_{e^+e^-\to{\rm partons}}$$

Jets – "trace" or "footprint" of partons

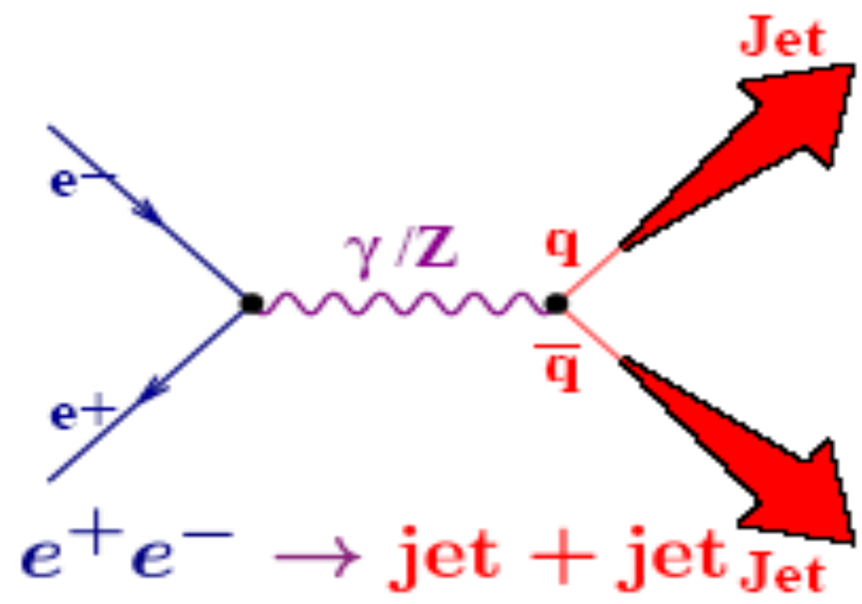
Thrust distribution in e^+e^- collisions

etc.

Jets – trace of partons

- ❑ Jets – "total" cross-section
  with a limited phase-space

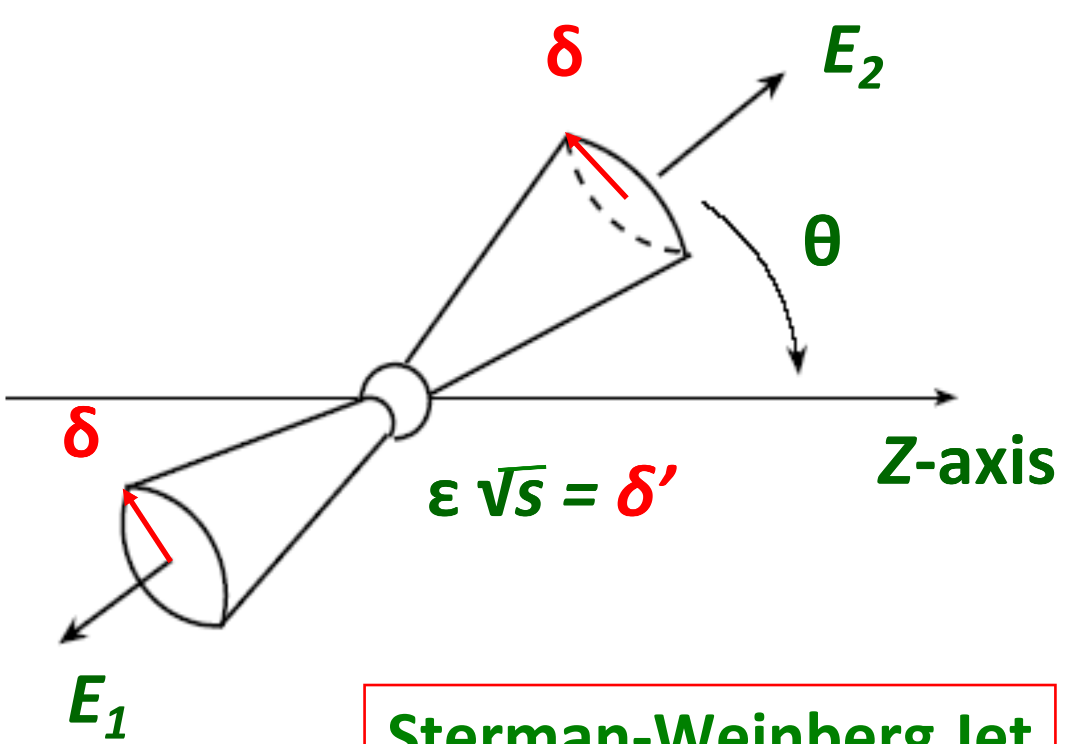
  Not any specific hadron!



- ❑ Q: will IR cancellation
  be completed?

  ✧ Leading partons are moving away from each other, carrying color!

  ✧ Soft gluon interactions should not change the direction of an energetic parton → a "jet"
  – "trace" of a parton



Sterman-Weinberg Jet

- ❑ Many Jet algorithms

Infrared safety for restricted cross sections

- **For any observable with a phase space constraint, Γ,**

$$d\sigma(\Gamma) \equiv \frac{1}{2!}\int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2}\Gamma_2(k_1,k_2)$$
$$+\frac{1}{3!}\int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3}\Gamma_3(k_1,k_2,k_3)$$
$$+\ldots$$
$$+\frac{1}{n!}\int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n}\Gamma_n(k_1,k_2,\ldots,k_n)+\ldots$$

Where $\Gamma_n(k_1,k_2,\ldots,k_n)$ are constraint functions and invariant under Interchange of n-particles

- **Conditions for IRS of dσ(Γ):**

$$\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^\mu,\lambda k_n^\mu\right)=\Gamma_n\left(k_1,k_2,\ldots,k_n^\mu\right) \quad \text{with} \quad 0\le\lambda\le 1$$

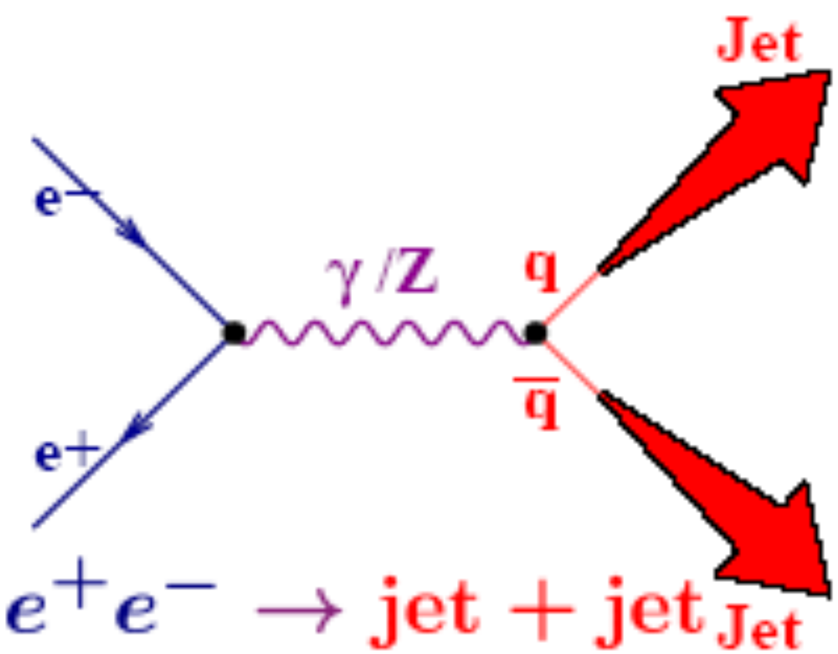
Physical meaning:

Measurement cannot distinguish a state with a zero/collinear momentum parton from a state without this parton – inclusiveness!

Special case: $\Gamma_n\left(k_1,k_2,\ldots,k_n\right)=1$ for all n $\Rightarrow$ $\sigma^{(\mathrm{tot})}$

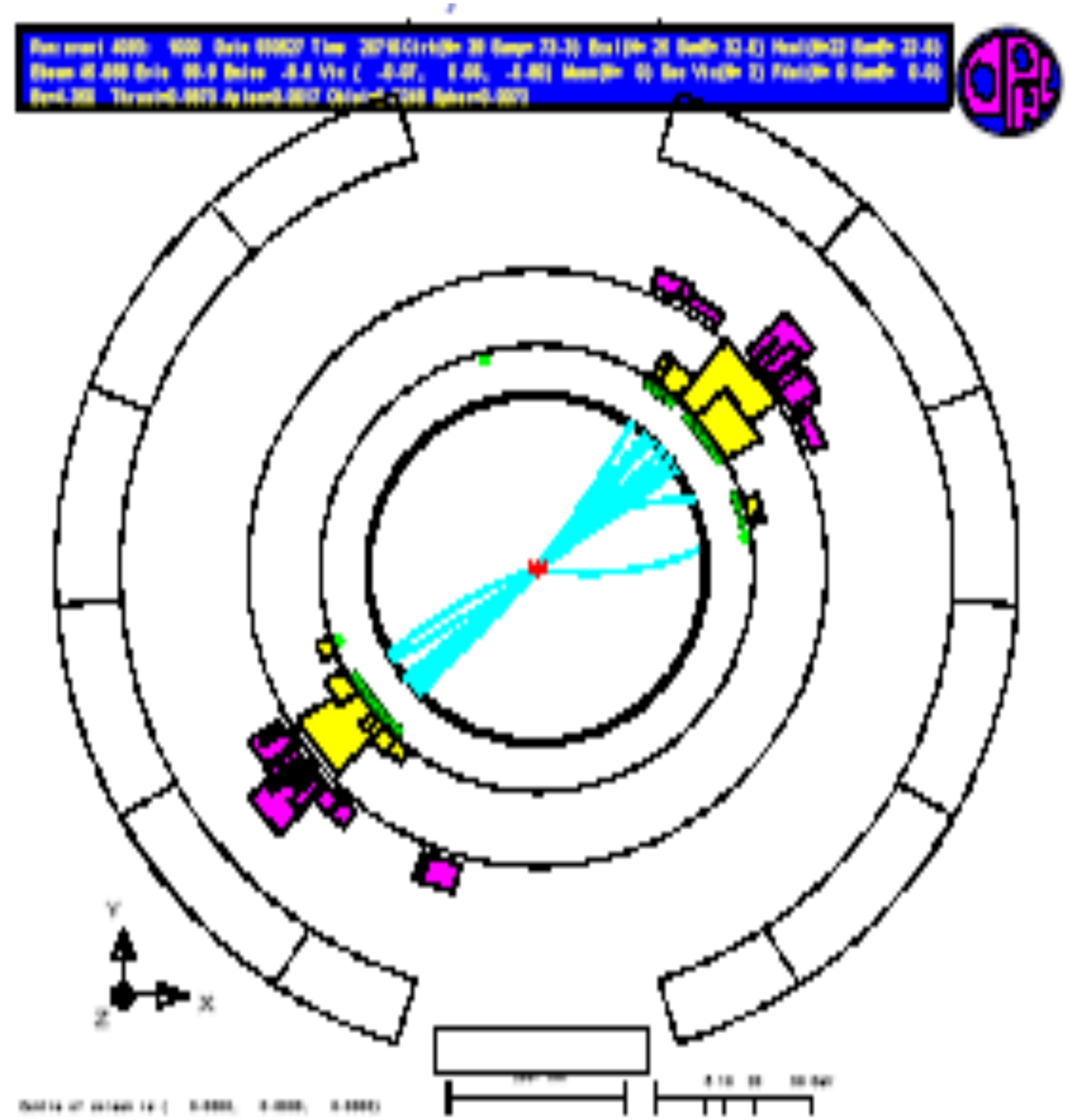
An early clean two-jet event

Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):



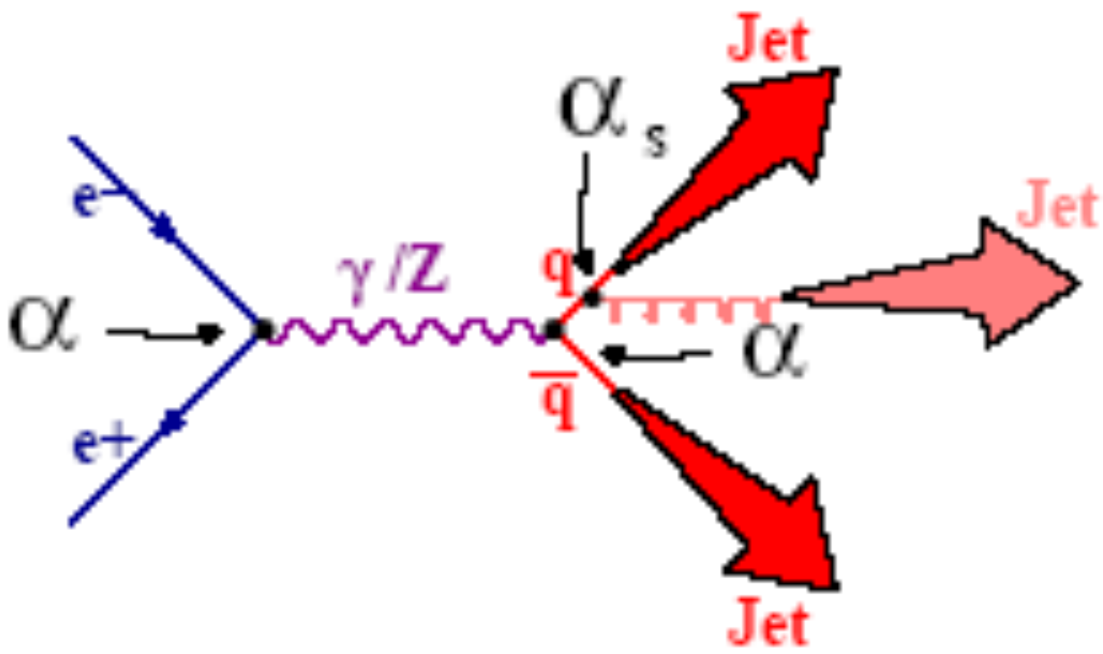
A clean trace of two partons – a pair of quark and antiquark

LEP ($\sqrt{s} = 90 - 205$ GeV)



Discovery of a gluon jet

First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

TASSO Collab., Phys. Lett. B86 (1979) 243

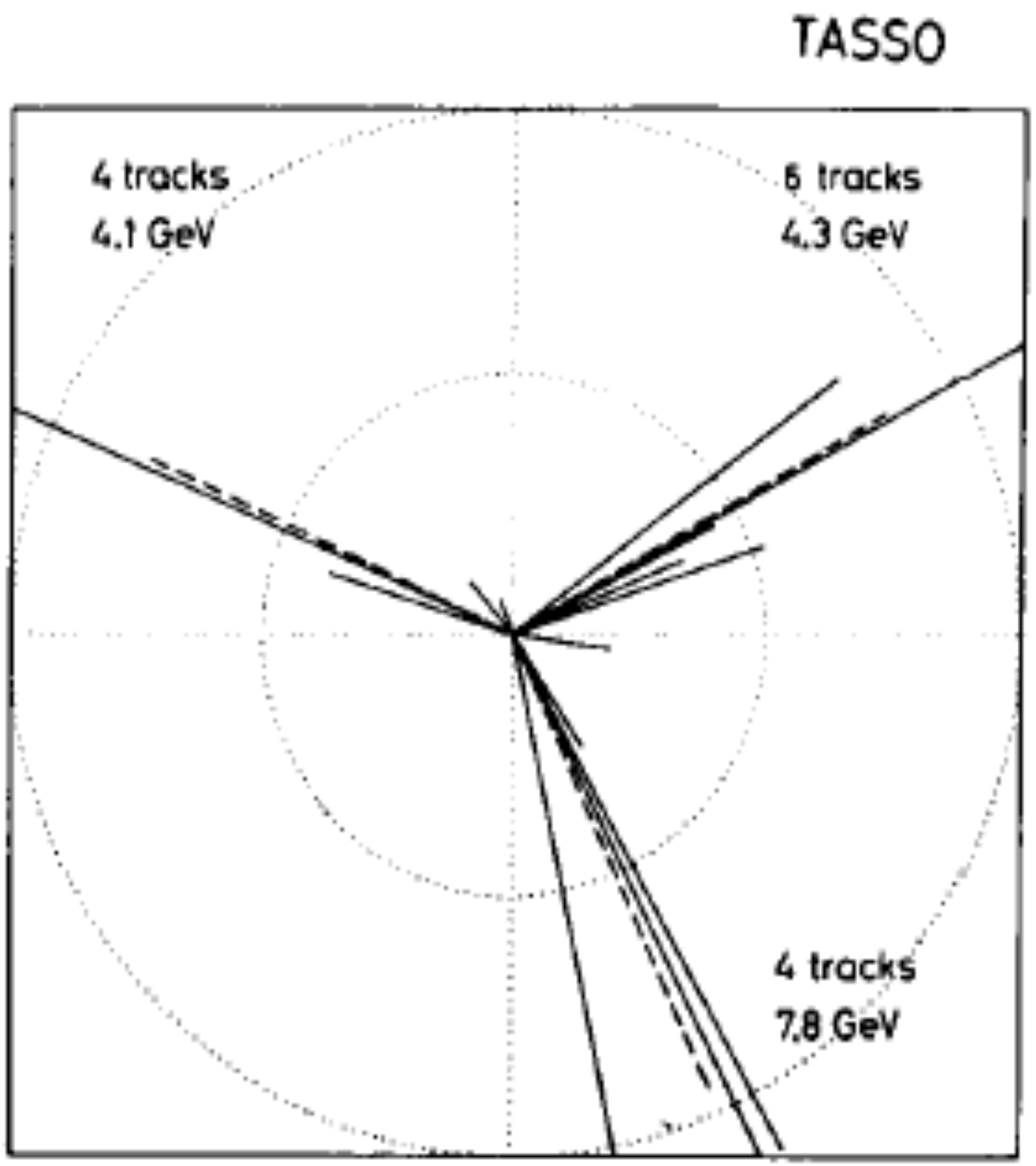
MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

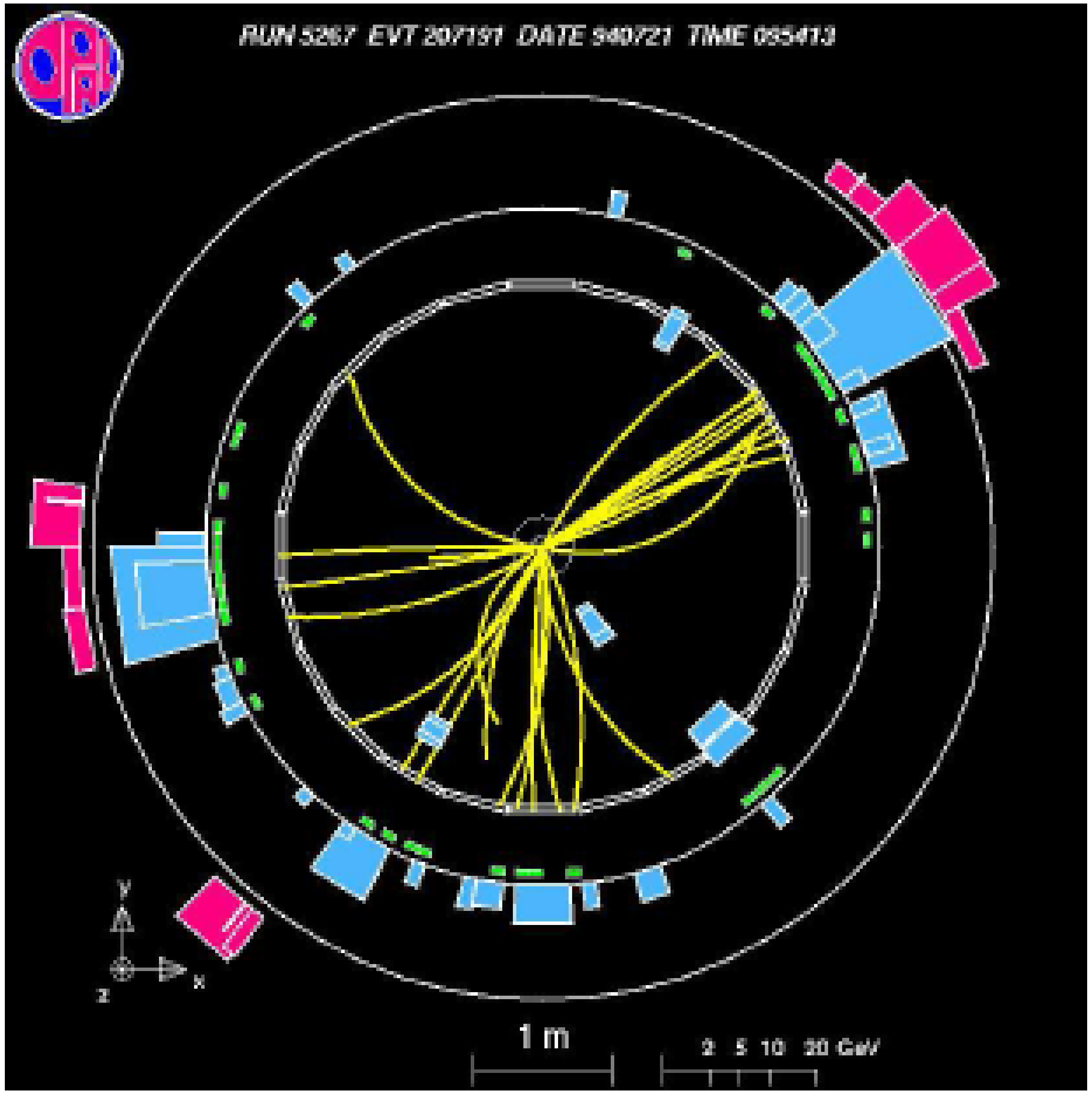
JADE Collab., Phys. Lett. B91 (1980) 142

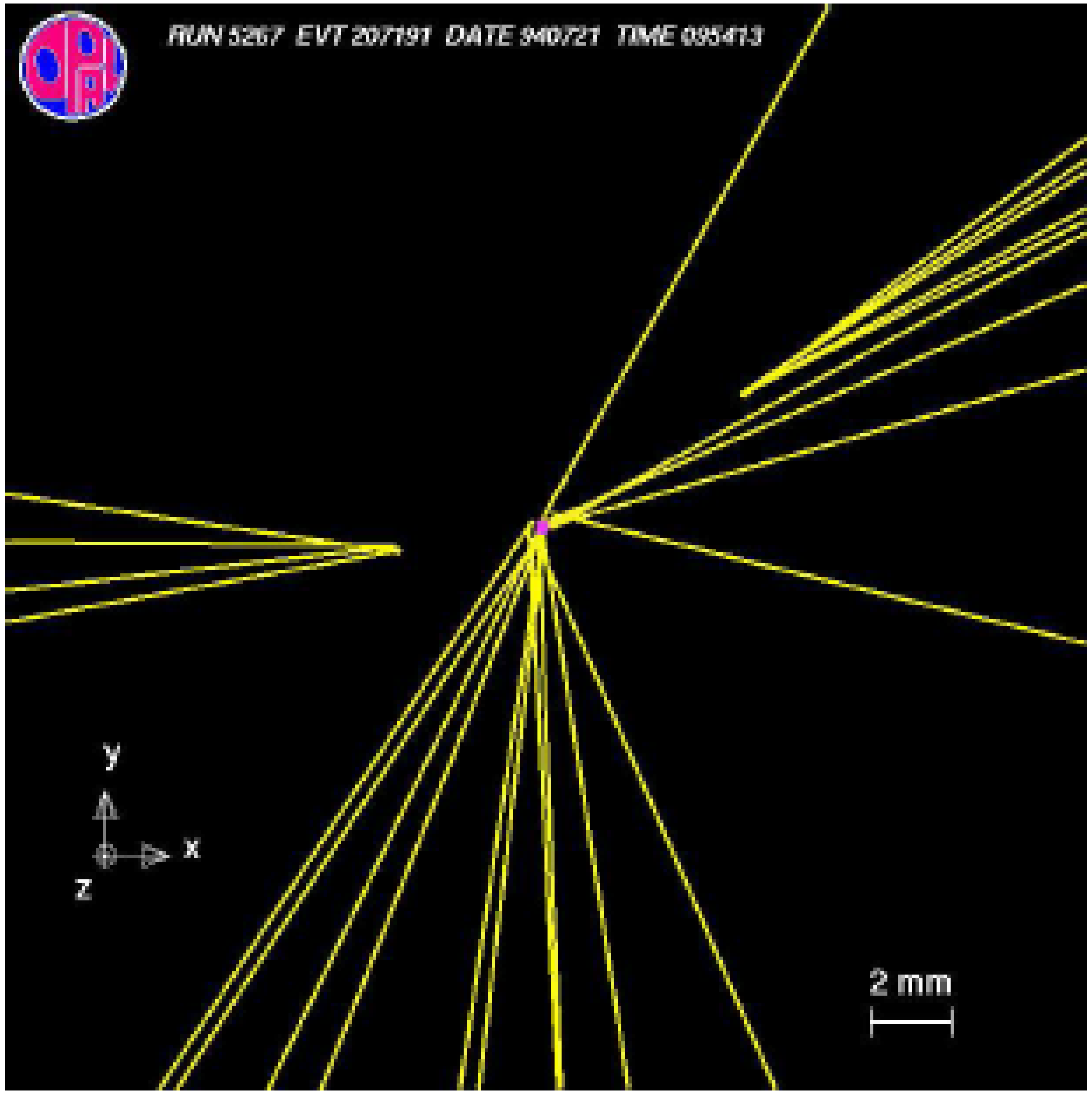
PETRA e^+e^- storage ring at DESY:

$E_{c.m.} \gtrsim 15$ GeV



Tagged three-jet event from LEP



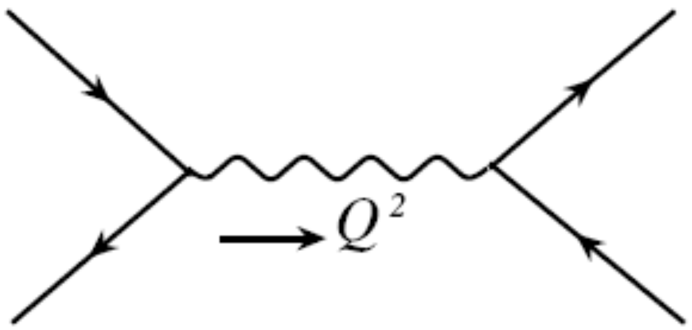


Gluon Jet

Two-jet cross section in e+e- collisions

- **Parton-Model = Born term in QCD:**

$$\sigma_{2\text{Jet}}^{(\text{PM})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$



- **Two-jet in pQCD:**

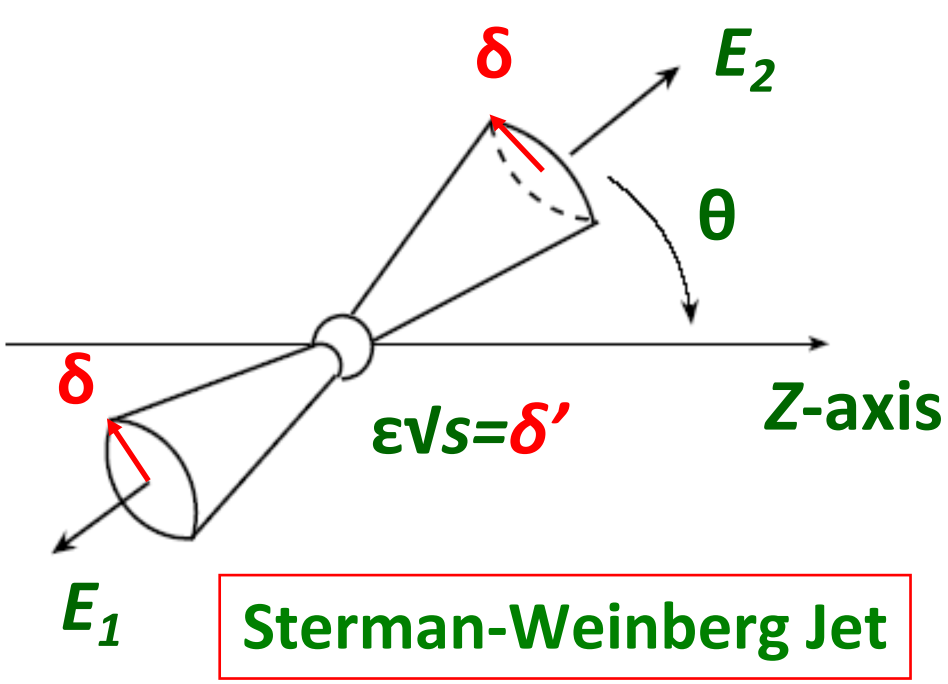
$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left(1+\sum_{n=1} C_n\left(\frac{\alpha_s}{\pi}\right)^n\right)$$

with $C_n = C_n(\delta)$

- **Sterman-Weinberg jet:**

$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$

$$\times\left[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(4\ln(\delta)\ln(\delta')+3\ln(\delta)+\frac{\pi^2}{3}+\frac{5}{2}\right)\right]$$



Sterman-Weinberg Jet

$$\sigma_{\text{total}} = \sigma_{2\text{Jet}} \qquad \text{as } Q \to \infty$$

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❑ Recombination jet algorithms (almost all e+e- colliders):

Recombination metric: $y_{ij} = \frac{M_{ij}^2}{E_{c.m.}^2}$ $\qquad M_{ij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$

for Durham k_T

- ✧ **different algorithm = different choice of M_{ij}^2 :**
- ✧ **combine the particle pair (i,j) with the smallest y_{ij}: $(i,j) \to k$**

$$e.g.\ \mathrm{E\ scheme:}\ \ p_k = p_i + p_j$$

- ✧ **iterate until all remaining pairs satisfy: $y_{ij} > y_{cut}$**

❑ Cone jet algorithms (CDF,LHC, …, colliders):

- ✧ **Cluster all particles into a cone of half angle R to form a jet:**
- ✧ **Require a minimum visible jet energy: $E_{jet} > \epsilon$** $\qquad \Delta_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$

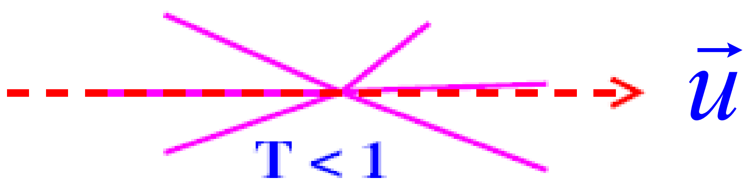
Recombination metric: $d_{ij} = \min\left(k_{T_i}^{2p}, k_{T_j}^{2p}\right) \frac{\Delta_{ij}^2}{R^2}$ $\qquad R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$

- ✧ **Classical choices: *p=1* – “*k_T* algorithm”, *p= -1* – “anti-*k_T*”, …**

Thrust distribution – event shape

- **Thrust axis:** $\vec{u}$

$$T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \max_{\vec{u}} \left(\frac{\sum_{i=1}^{n} \vec{p}_i \cdot \vec{u}}{\sum_{i=1}^{n} \left|\vec{p}_i\right|} \right)$$



- **Phase space constraint:**

$$\frac{d\sigma_{e^+e^-\to \text{hadrons}}}{dT} \quad \textbf{with} \quad \Gamma_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \delta\left(T - T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right)\right)$$

- ✧ Contribution from p=0 particles drops out the sum

- ✧ Replace two collinear particles by one particle does not change the thrust

$$|(1-\lambda)\,\vec{p}_n \cdot \vec{u}| + |\lambda\,\vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|$$

and

$$|(1-\lambda)\,\vec{p}_n| + |\lambda\,\vec{p}_n| = |\vec{p}_n|$$

The harder question

❑ Question:

How to test QCD in a reaction with identified hadron(s)?
– to probe the quark-gluon structure of the hadron

❑ Facts:

Hadronic scale ~ 1/fm ~ Λ_{QCD} is non-perturbative

Cross section involving identified hadron(s) is not IR safe and is NOT perturbatively calculable!

❑ Solution – Factorization:

✧ Isolate the calculable dynamics of quarks and gluons

✧ Connect quarks and gluons to hadrons via non-perturbative but universal distribution functions
– provide information on the partonic structure of the hadron