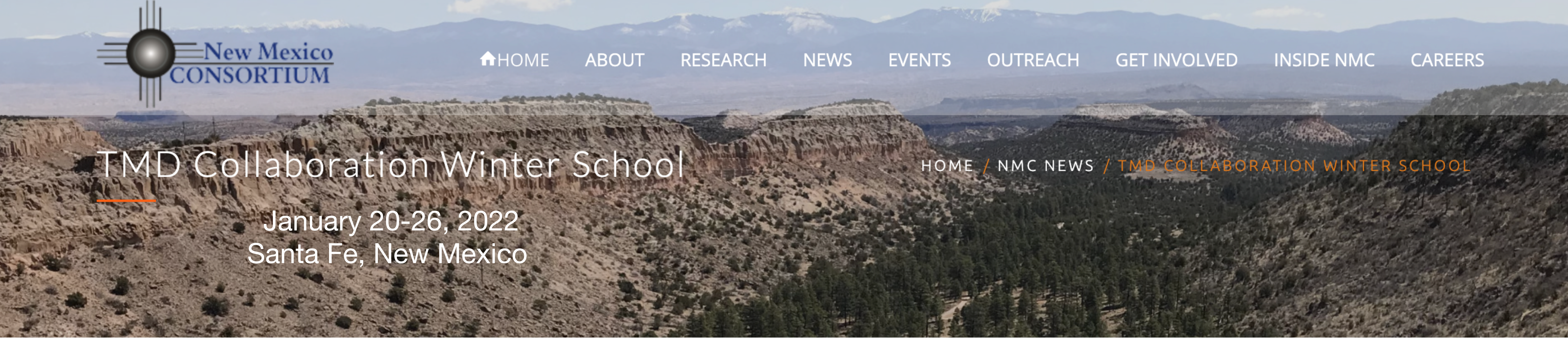

Introduction to QCD

- **Lec. 1: Fundamentals of QCD**
- **Lec. 2: QCD for cross sections with identified hadrons, & hadron structure**
- **Lec. 3: QCD for observables with polarization, & role/power of lattice QCD**

TMD Handbook

A modern introduction to the physics of Transverse Momentum Dependent distributions

January 15, 2022

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The harder question

- ❑ **Question:**

  How to test QCD in a reaction with identified hadron(s)?

  – to probe the quark-gluon structure of the hadron

- ❑ **Facts:**

  Hadronic scale ~ 1/fm ~ Λ_{QCD} is non-perturbative

  Cross section involving identified hadron(s) is not IR safe and is NOT perturbatively calculable!

- ❑ **Solution – Factorization – Approximation:**

  ✧ **Isolate the calculable dynamics of quarks and gluons**

  ✧ **Connect quarks and gluons to hadrons via non-perturbative but universal distribution functions**

  – provide information on the partonic structure of the hadron

An Instructive Exercise for Factorization

❑ **Consider a cross section:**

$$\sigma(Q^2, m^2) = \sigma_0 \left[1 + \alpha_s I(Q^2, m^2) + \mathcal{O}(\alpha_s^2)\right]$$

❑ **Leading order quantum correction:**

$$I(Q^2, m^2) = \int_0^\infty dk^2 \, \frac{1}{k^2 + m^2} \, \frac{Q^2}{Q^2 + k^2}$$

❑ **Leading power contribution in O(m²/Q²):**

$$I(Q^2, m^2) = \int_{k^2 \ll Q^2} dk^2 \, \frac{1}{k^2 + m^2} + \int_{k^2 \sim Q^2} dk^2 \, \frac{1}{k^2} \, \frac{Q^2}{Q^2 + k^2} + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

❑ **Leading power contribution to the cross section:**

$$\sigma(Q^2, m^2) = \left[1 + \alpha_s \int_{k^2 \ll Q^2} dk^2 \, \frac{1}{k^2 + m^2}\right] \left[1 + \alpha_s \int_{k^2 \sim Q^2} dk^2 \, \frac{1}{k^2} \, \frac{Q^2}{Q^2 + k^2}\right]$$

$$+ \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

$$\equiv \phi \otimes \hat{\sigma} + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

Long-distance distribution (ϕ)

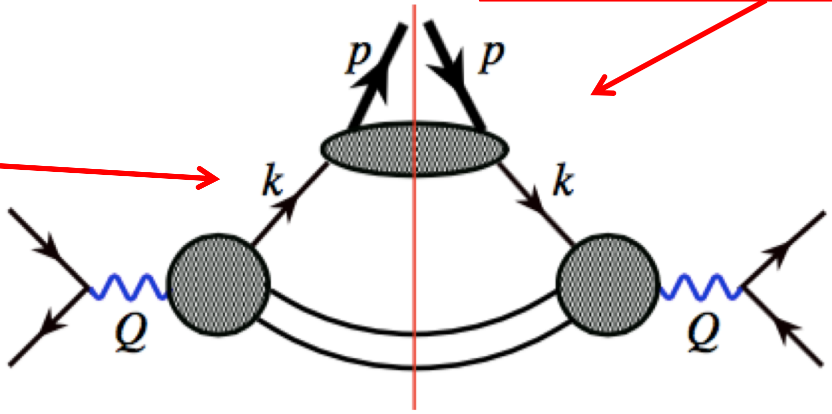
Short-distance hard part ($\hat{\sigma}$)

Observables with ONE identified hadron

- **Creation of an identified hadron:**

Non-perturbative!

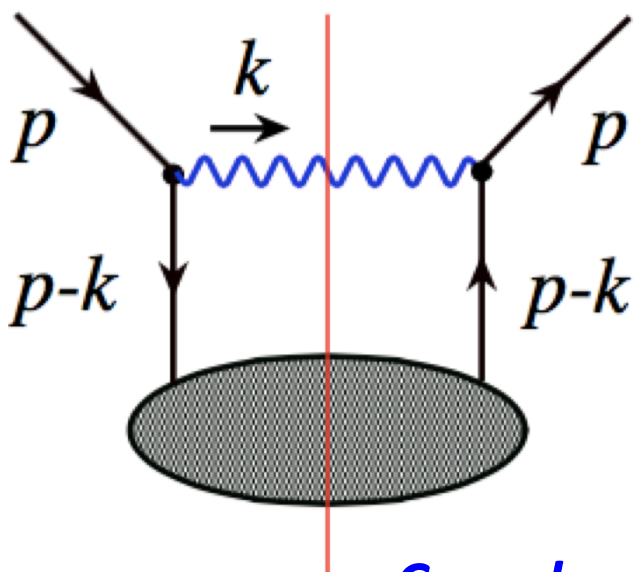
Not necessary to be dominated by one parton, which is always virtual!



– in a "cut-diagram" notation

- **"Square" of the diagram with a *"unobserved gluon"*:**

"Cut-line" – final-state



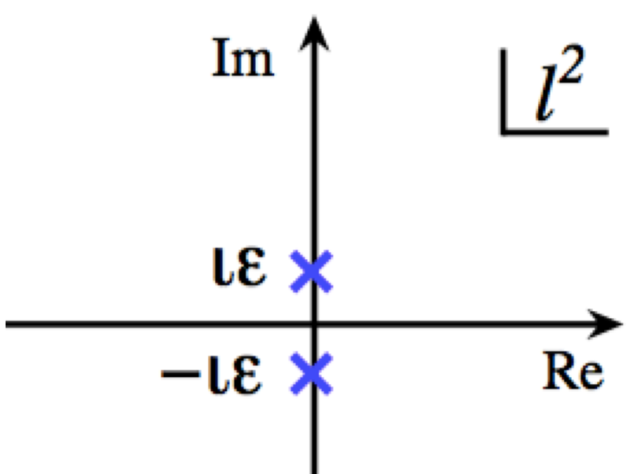
Amplitude

Complex conjugate of the Amplitude

$$\propto \int \mathcal{T}(p-k,Q)\,\frac{1}{(p-k)^2+i\epsilon}\,\frac{1}{(p-k)^2-i\epsilon}\,d^4k\,\delta(k^2)_+$$

$$\propto \int \mathcal{T}(l,Q)\,\frac{1}{l^2+i\epsilon}\,\frac{1}{l^2-i\epsilon}\,dl^2$$

$$\Rightarrow \infty$$



Pinch singularity & pinch surface

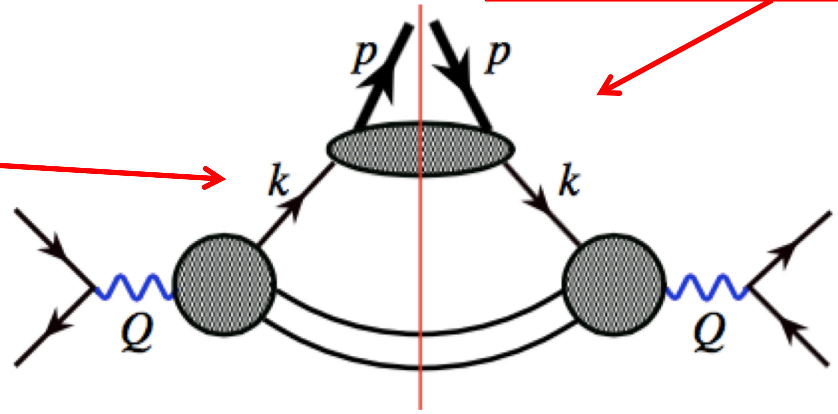
Two parts connected by a "classical" parton

Observables with ONE identified hadron

❑ Creation of an identified hadron:

Non-perturbative!

Not necessary to be dominated by one parton, which is always virtual!



– in a "cut-diagram" notation

❑ On-shell approximation:

$$\sigma_{e^+e^-\to h(p)X} \approx \sum_f \int \frac{d^4k}{(2\pi)^4} \mathcal{H}_{e^+e^-\to f(k)}(Q,k;\sqrt{S})\, \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \ldots$$

On-shell $\hat{k}^2 = 0$

$$\approx \sum_f \int \frac{d^4k}{(2\pi)^4} \mathcal{H}_{e^+e^-\to f(k)}(Q,\hat{k};\sqrt{S})\, \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \mathcal{O}(\frac{\langle k^2\rangle}{Q^2}) + \ldots$$

Collinear

$$1 = \int dz\, \delta(z - \frac{p^+}{k^+})$$

$$\approx \sum_f \int dz\, \mathcal{H}_{e^+e^-\to f(k)}(Q,\frac{p}{z};\sqrt{S}) \int \frac{d^4k}{(2\pi)^4} \delta(z - \frac{p\cdot n}{k\cdot n}) \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \ldots$$

$$\approx \sum_f \int dz\, \hat{\sigma}_{e^+e^-\to f(k)}(Q,z;\sqrt{S})\; D_{f(k)\to h(p)X}(z,p;\Lambda_{\rm QCD}) + \ldots$$

Hard collision to produce an on-shell parton – Perturbatively calculable!

FF: Probability for the parton to become the observed hadron – Non-perturbative, universal!

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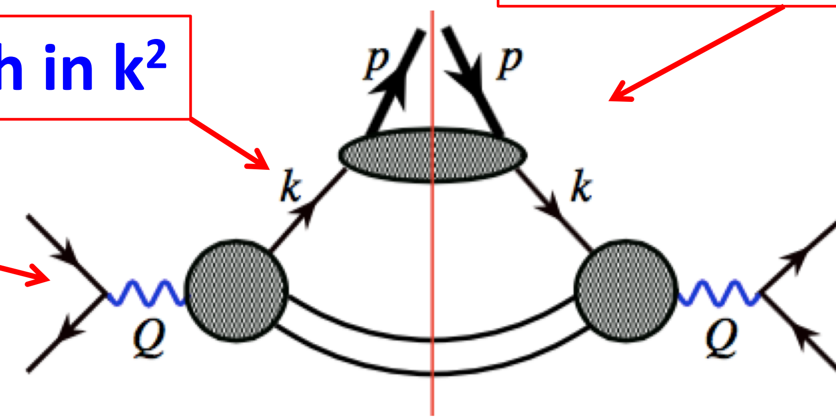
Observables with identified hadron(s)

- **Creation of an identified hadron:**

Non-perturbative!

Pinch in k^2

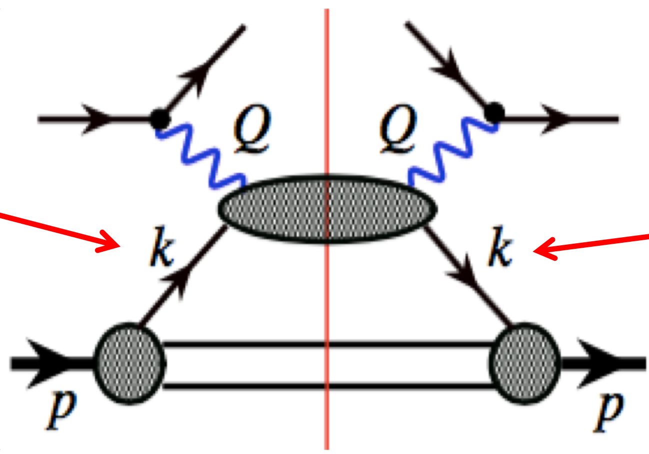
Perturbative!



– in a "cut-diagram" notation

- **Identified initial hadron:**

Pinch in k^2

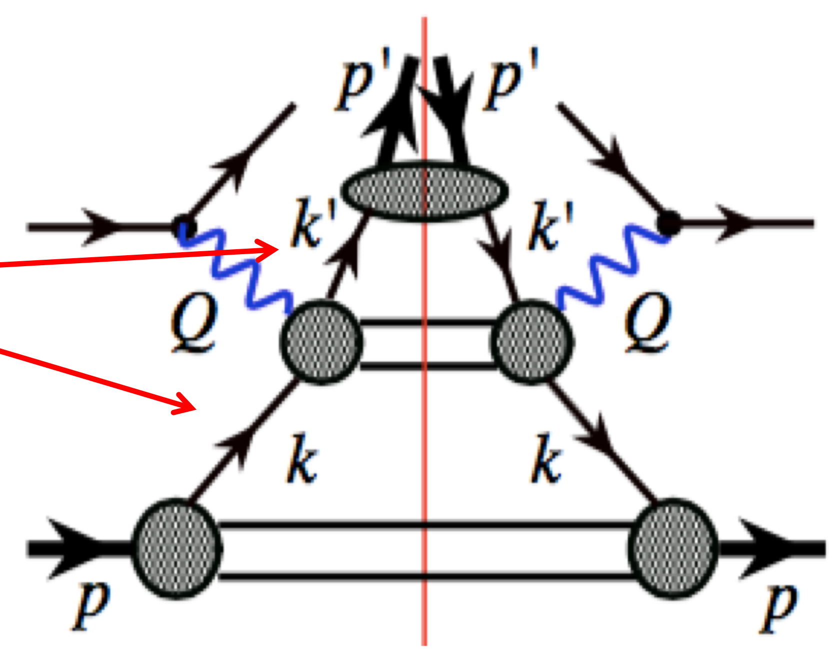


Perturbative!

Non-perturbative!

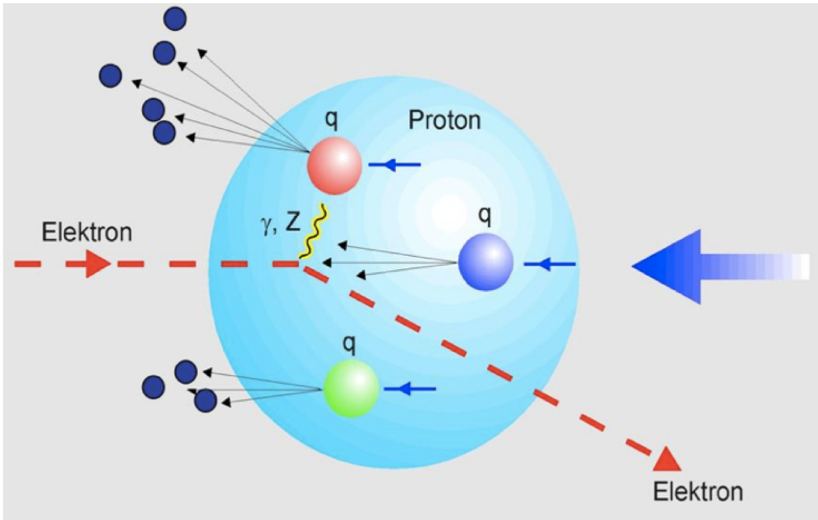
- **Identified initial + created hadron(s):**

Pinch in both k^2 and k'^2



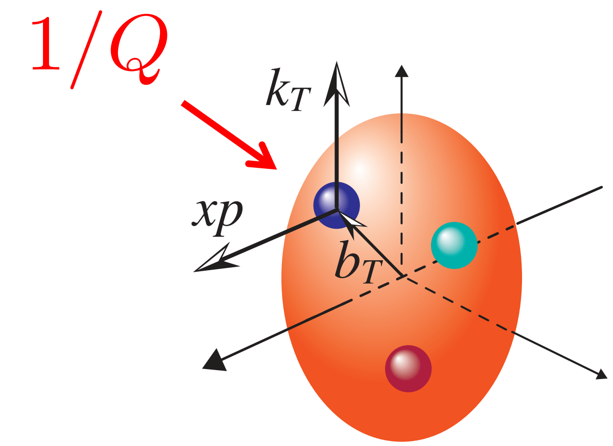
Quantum interference between dynamics at the HARD and hadronic scales is powerly suppressed!

Inclusive lepton-hadron DIS – one hadron



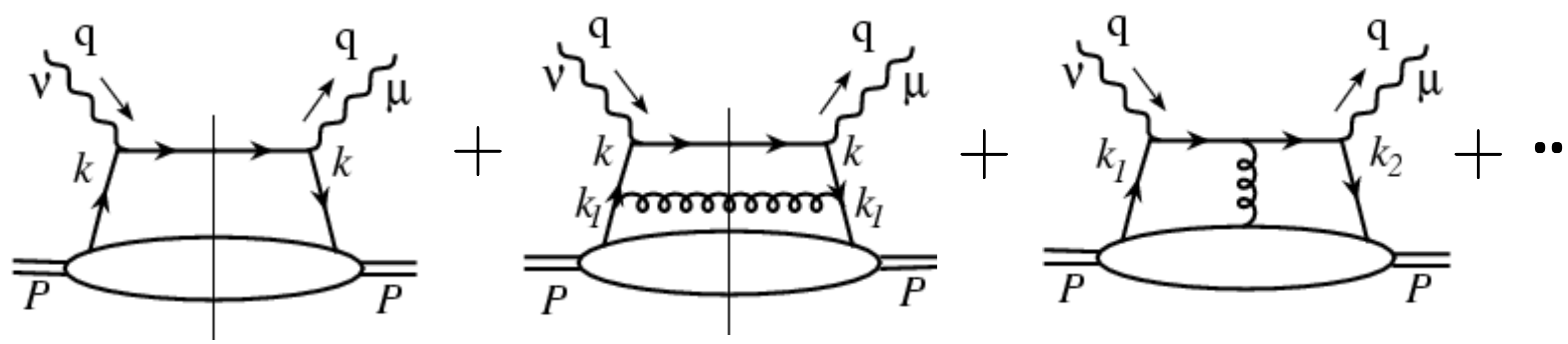
$$\sigma^{\mathrm{DIS}}_{\ell p \to \ell' X(\mathrm{everything})}$$

Identified initial-state hadron-proton!



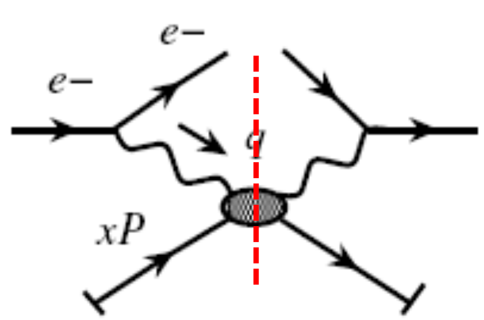
- ❑ **DIS cross section is infrared divergent, and nonperturbative!**

$$\sigma^{\mathrm{DIS}}_{\ell p \to \ell' X(\mathrm{everything})} \propto$$

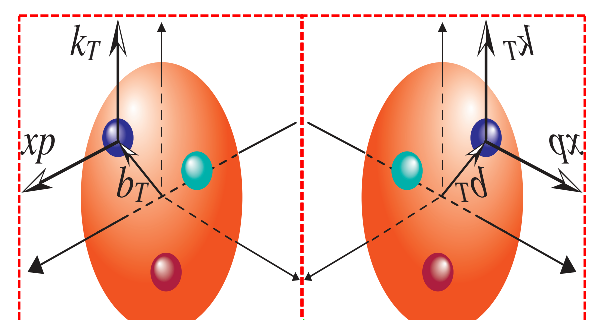


- ❑ **QCD factorization (approximation!)**

$$\sigma^{\mathrm{DIS}}_{\ell p \to \ell' X(\mathrm{everything})} =$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$

Physical Observable	Controllable Probe	Quantum Probabilities Structure	Color entanglement Approximation

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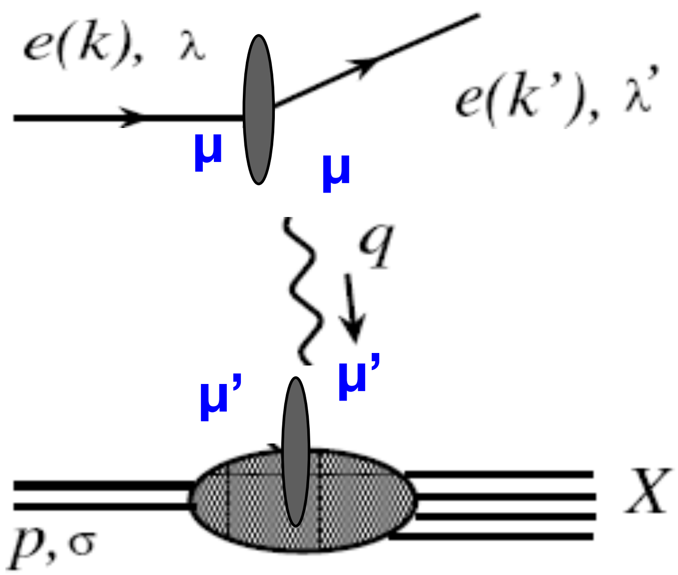
Inclusive lepton-hadron DIS – one hadron

- Scattering amplitude:

$$\mathrm{M}(\lambda,\lambda';\sigma,q) = \bar{u}_{\lambda'}(k')\left[-ie\gamma_{\mu}\right]u_{\lambda}(k)$$
$$* \left(\frac{i}{q^2}\right)\left(-g^{\mu\mu'}\right)$$
$$* \left\langle X\left|eJ^{em}_{\mu'}(0)\right|p,\sigma\right\rangle$$



- Cross section:

$$d\sigma^{\mathrm{DIS}} = \frac{1}{2s}\left(\frac{1}{2}\right)^2 \sum_X \sum_{\lambda,\lambda',\sigma} \left|\mathrm{M}(\lambda,\lambda';\sigma,q)\right|^2 \left[\prod_{i=1}^{X}\frac{d^3 l_i}{(2\pi)^3 2E_i}\right]\frac{d^3k'}{(2\pi)^3 2E'}(2\pi)^4\delta^4\left(\sum_{i=1}^{X} l_i + k' - p - k\right)$$

$$E'\frac{d\sigma^{\mathrm{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)$$

- Leptonic tensor:

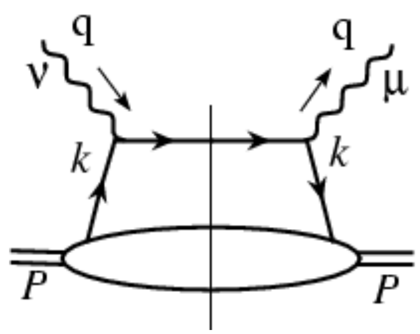
– known from QED:

$$L^{\mu\nu}(k,k') = \frac{e^2}{2\pi^2}\left(k^{\mu}k'^{\nu} + k^{\nu}k'^{\mu} - k\cdot k' g^{\mu\nu}\right)$$

DIS structure functions

❑ Hadronic tensor:

$$W_{\mu\nu}(q,p,S)=\frac{1}{4\pi}\int d^4z\,\mathrm{e}^{iq\cdot z}\,\langle p,S|J_\mu^\dagger(z)J_\nu(0)|p,S\rangle$$



❑ Symmetries:

- ✧ **Parity invariance (EM current)** ⟶ $W_{\mu\nu}=W_{\nu\mu}$ sysmetric for spin avg.
- ✧ **Time-reversal invariance** ⟶ $W_{\mu\nu}=W_{\mu\nu}^*$ real
- ✧ **Current conservation** ⟶ $q^\mu W_{\mu\nu}=q^\nu W_{\mu\nu}=0$

$$W_{\mu\nu}=-\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right)+\frac{1}{p\cdot q}\left(p_\mu-q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu-q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1\left(x_B,Q^2\right)+\frac{(p.q)S_\sigma-(S.q)p_\sigma}{(p.q)^2}g_2\left(x_B,Q^2\right)\right]$$

$$Q^2=-q^2$$

$$x_B=\frac{Q^2}{2p\cdot q}$$

❑ Structure functions – infrared sensitive:

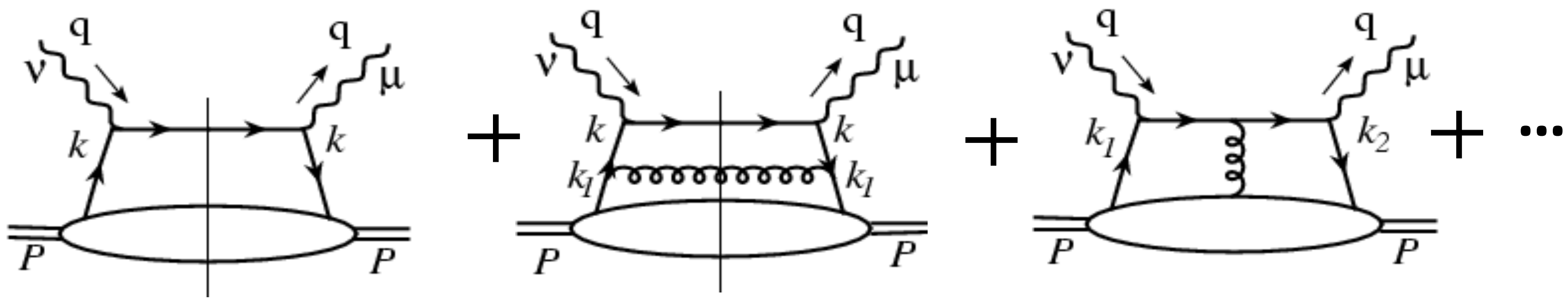
$$F_1\left(x_B,Q^2\right),F_2\left(x_B,Q^2\right),g_1\left(x_B,Q^2\right),g_2\left(x_B,Q^2\right)$$

No QCD parton dynamics used in above derivation!

Long-lived parton states

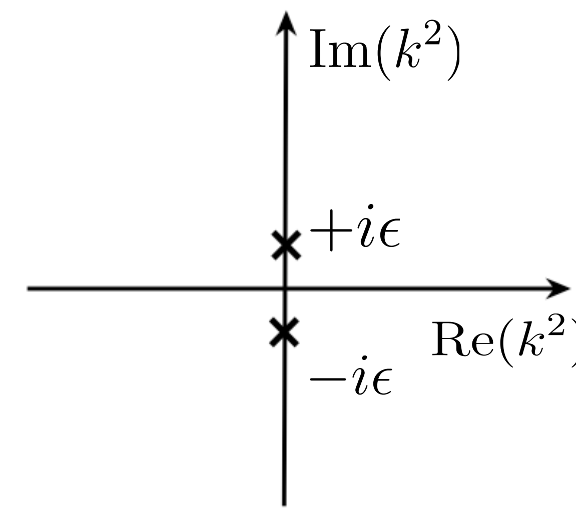
❑ Feynman diagram representation of the hadronic tensor:

$$W^{\mu\nu} \propto$$



❑ Perturbative pinched poles:

$$\int d^4k\ \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) \Rightarrow \infty \ \text{perturbatively}$$



❑ Perturbative factorization:

Light-cone coordinate:

$$v^\mu = (v^+, v^-, v^\perp),\ v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3)$$

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

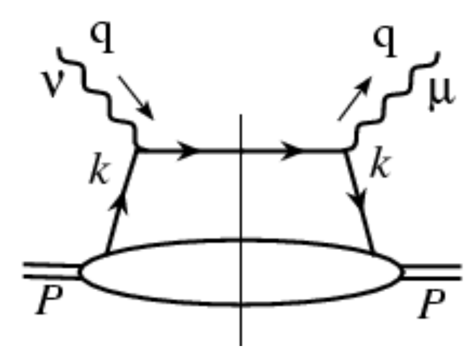
$$\int \frac{dx}{x} d^2k_T\ \mathrm{H}(Q,k^2=0) \int dk^2 \left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) + \mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right)$$

Short-distance

Nonperturbative matrix element

Collinear factorization – further approximation

- **Collinear approximation, if** $Q \sim xp\cdot n \gg k_T, \sqrt{k^2}$



  - **Lowest order:** $\delta((k+q)^2) = \frac{1}{2P\cdot q}\delta(x-\xi) = \frac{1}{2P\cdot q}\delta\left(x-\frac{k^+}{P^+}\right)$

$$W^{\mu\nu}_{\gamma^* p} = \sum_f \int \frac{d^4k}{(2\pi)^4} \sum_{ij} (\gamma^\mu \gamma\cdot(k+q)\gamma^\nu)_{ij}\,(2\pi)\delta((k+q)^2) \int d^4y e^{iky} \langle p|\overline{\psi}_j(0)\psi_i(y)|p\rangle + \ldots$$

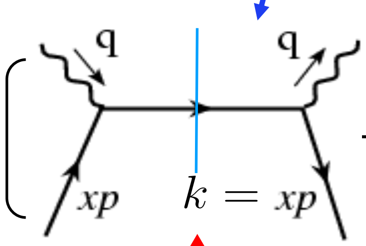
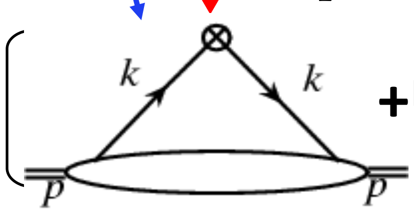
$$\equiv \sum_f \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,k)\mathcal{F}_{f/p}(k,p)\right] + \ldots$$

$$\approx \sum_f \int dx\ \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,k\approx xp)\int \frac{d^4k}{(2\pi)^4}\delta(x-\frac{k\cdot n}{p\cdot n})\mathcal{F}_{f/p}(k,p)\right] + \mathcal{O}(\frac{\langle k^2\rangle}{Q^2},\frac{\langle k_T^2\rangle}{Q^2}) + \ldots$$

$$\approx \sum_f \int \frac{dx}{x}\ \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,xp)\frac{1}{2}\gamma\cdot(xp)\right] \int \frac{d^4k}{(2\pi)^4}\delta(x-\frac{k\cdot n}{p\cdot n})\mathrm{Tr}\left[\frac{\gamma\cdot n}{2p\cdot n}\mathcal{F}_{f/p}(k,p)\right] + \ldots$$

$$\approx \sum_f \int \frac{dx}{x}\ \widehat{W}^{\mu\nu}_{\gamma^* f}(x,Q^2/\mu^2)\ \phi_{f/p}(x,\mu^2) + \ldots$$

$$\int \frac{dx}{x} \qquad \frac{\gamma\cdot n}{2p\cdot n}\delta\left(x-\frac{k\cdot n}{p\cdot n}\right)\frac{d^4k}{(2\pi)^4}$$

$\approx$  $+O\left(\frac{k_T^2}{Q^2}\right)$ $\otimes$  **+UVCT(μ)**

$\frac{1}{2}\gamma\cdot(xp)$ $\qquad \widehat{W}^{\mu\nu}_{\gamma^* f}(x,Q^2/\mu^2) = \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,xp)\frac{1}{2}\gamma\cdot(xp)\right]$

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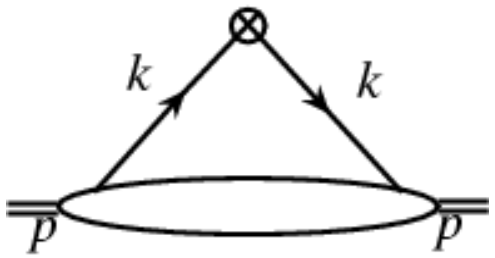
Parton distribution functions (PDFs)

❑ **PDFs as matrix elements of two parton fields – twist 2 operators:**

– combine the amplitude & its complex-conjugate



$$\phi_{q/h}(x,\mu^2) = \int \frac{p^+ dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0)\,\frac{\gamma^+}{2p^+}\,\psi_q(y^-)|h(p)\rangle$$

Note: $\phi_{q/h} = f_{q/h}$ **in Handbook**

$|h(p)\rangle$ **can be a hadron, or a nucleus, or a parton state!**

Twist = Dim. of the operator – its spin

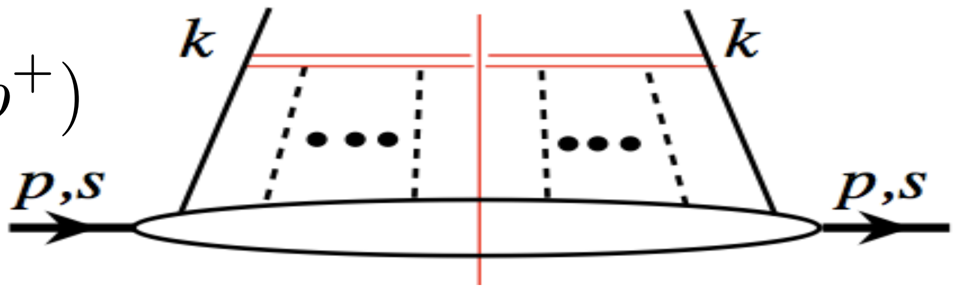
But, it is NOT gauge invariant! $\psi(x) \to e^{i\alpha_a(x)t_a}\psi(x)$ $\quad \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\alpha_a(x)t_a}$

– need a gauge link:

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0)\left[\mathcal{P}e^{-ig\int_0^{y^-} d\eta^- A^+(\eta^-)}\right]\frac{\gamma^+}{2p^+}\,\psi_q(y^-)|h(p)\rangle\, \mathcal{Z}_{\mathcal{O}}(\mu^2)$$

– corresponding diagram in momentum space:

$$\int \frac{d^4k}{(2\pi)^4}\,\delta(x - k^+/p^+)$$



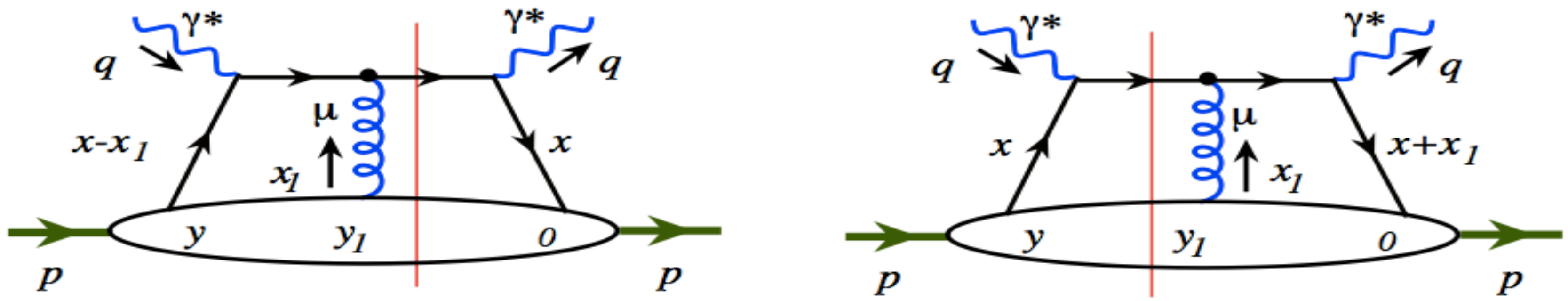
$$+\,\mathrm{UVCT}(\mu^2)$$

μ-dependence

Universality – process independence – predictive power

Gauge link – 1st order in coupling "g"

- **Longitudinal gluon:**



- **Left diagram:**

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1 p^+ (y_1^- - y^-)} n\cdot A^a(y_1^-)\right] \mathcal{M}(-igt^a)\frac{\gamma\cdot p}{p^+}\frac{i((x-x_1-x_B)\gamma\cdot p + (Q^2/2x_B p^+)\gamma\cdot n)}{(x-x_1-x_B)Q^2/x_B + i\epsilon}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1\, \frac{1}{-x_1+i\epsilon}\, \mathrm{e}^{ix_1 p^+(y_1^- - y^-)}\right] \mathcal{M} = -ig\int_{y^-}^{\infty} dy_1^-\, n\cdot A(y_1^-)\mathcal{M}$$

- **Right diagram:**

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1 p^+ y_1^-} n\cdot A^a(y_1^-)\right] \frac{-i((x+x_1-x_B)\gamma\cdot p + (Q^2/2x_B p^+)\gamma\cdot n)}{(x+x_1-x_B)Q^2/x_B - i\epsilon}(+igt^a)\,\frac{\gamma\cdot p}{p^+}\,\mathcal{M}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1\, \frac{1}{x_1-i\epsilon}\, \mathrm{e}^{ix_1 p^+ y_1^-}\right] \mathcal{M} \;\; = ig\int_0^{\infty} dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}$$

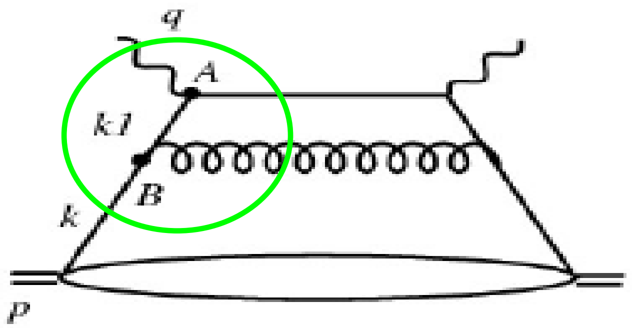
- **Total contribution:**

$$-ig\left[\int_0^{\infty} - \int_{y^-}^{\infty}\right] dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}_{\mathrm{LO}}$$

O(g)-term of the gauge link!

QCD high order corrections

- **NLO partonic diagram to structure functions:**



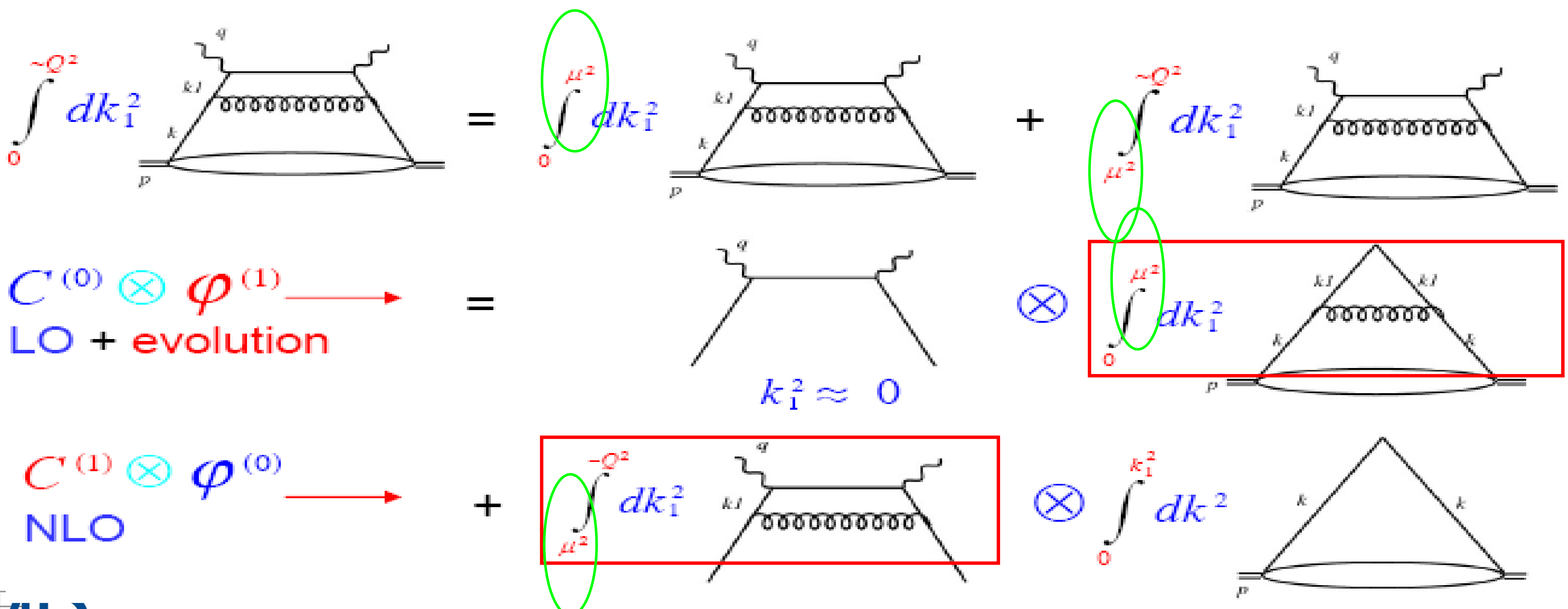
$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2}$$

Dominated by

$$\begin{cases} k_1^2 \sim 0 \\ t_{AB} \to \infty \end{cases}$$

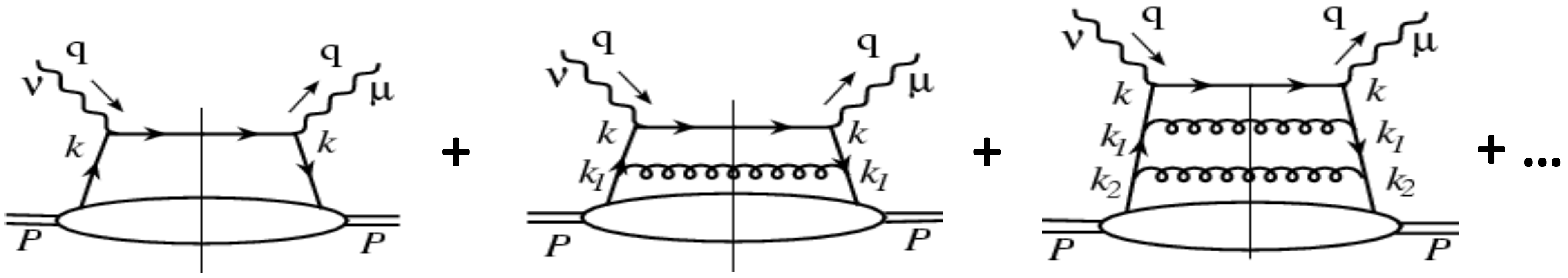
Diagram has both long- and short-distance physics

- **Factorization, separation of short- from long-distance:**

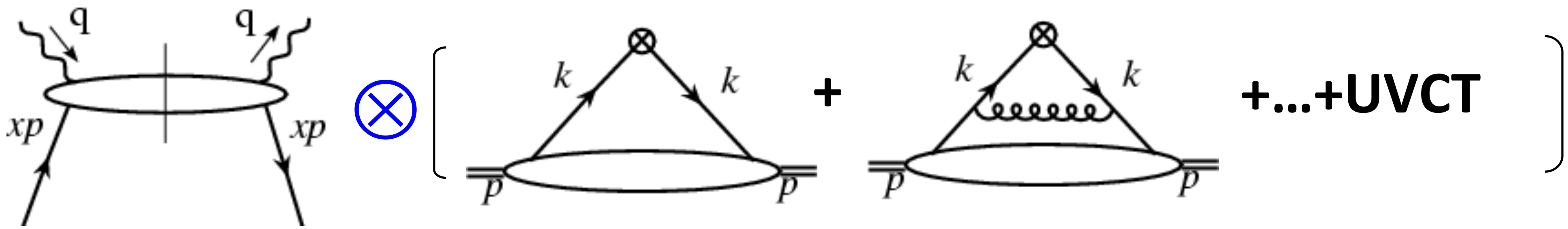


QCD leading power factorization

- **QCD corrections: pinch singularities in** $\int d^4 k_i$



- **Logarithmic contributions into parton distributions:**



$$F_2(x_B, Q^2) = \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes f(x, \mu_F^2) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$

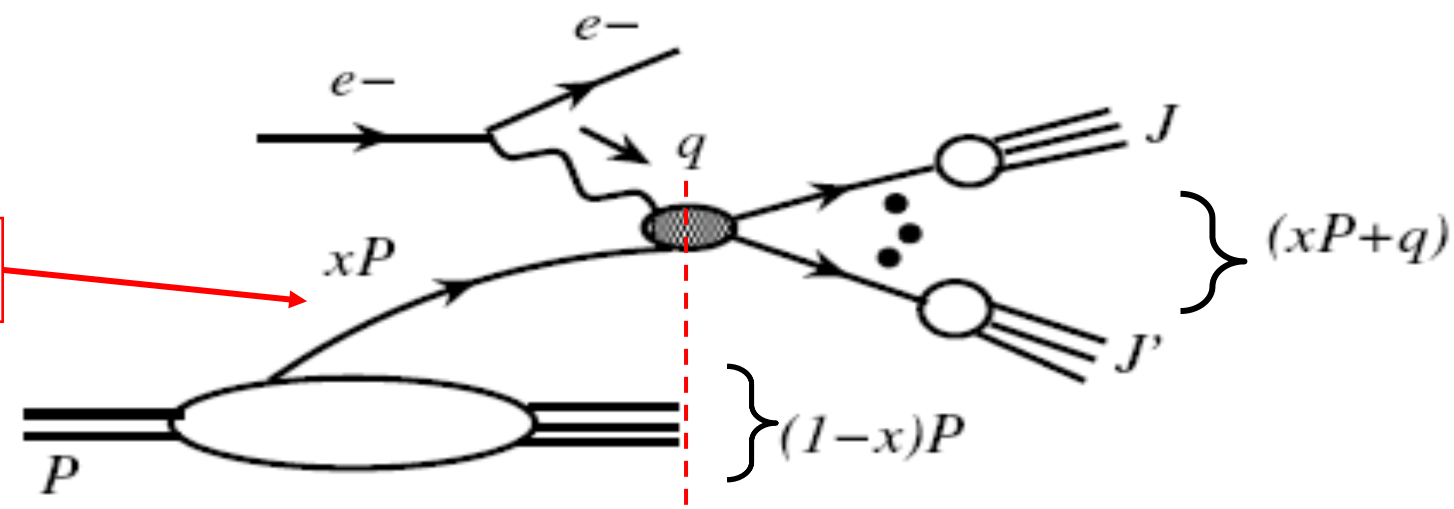
- **Factorization scale:** μ_F^2

To separate the collinear from non-collinear contribution

Recall: renormalization scale to separate local from non-local contribution

Picture of factorization for DIS

❑ **Time evolution:**

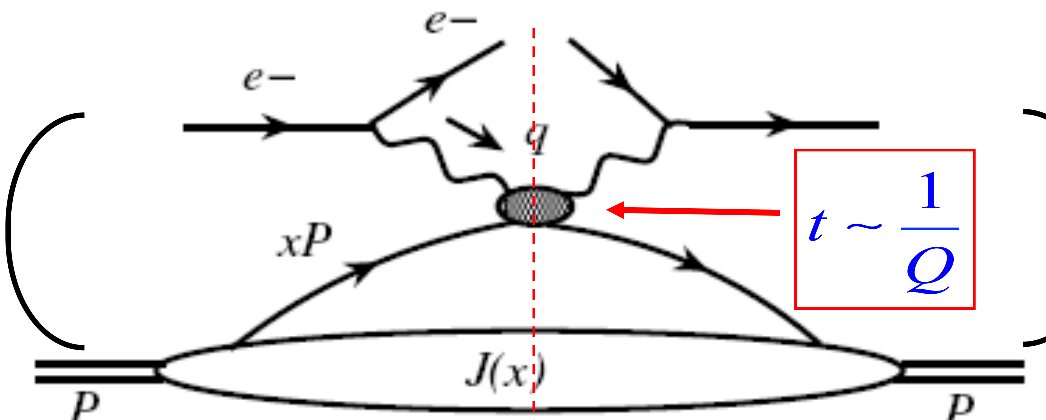


Long-lived parton state

Time: "Past" Now "Future"

❑ **Unitarity – summing over all hard jets:**

$$\sigma_{\text{tot}}^{\text{DIS}} \propto \text{Im}\left(\quad \right)$$



$t \sim \dfrac{1}{Q}$

$t \sim R$

Not IR safe

Interaction between the "past" and "now" are suppressed!

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How to calculate the perturbative parts?

- Use DIS structure function F_2 as an example:

$$F_{2h}(x_B,Q^2)=\sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu^2\right)+O\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

✧ **Apply the factorized formula to a parton state:** $h\to q$

Feynman diagrams → $$F_{2q}(x_B,Q^2)=\sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu^2\right)$$ ← **Feynman diagrams**

✧ **Express both SFs and PDFs in terms of powers of α_s:**

0th order:

$$F^{(0)}_{2q}(x_B,Q^2)=C^{(0)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(0)}_{q/q}\left(x,\mu^2\right)$$

➡ $$C^{(0)}_q(x)=F^{(0)}_{2q}(x)$$ $$\varphi^{(0)}_{q/q}\left(x\right)=\delta_{qq}\delta\left(1-x\right)$$

1th order:

$$F^{(1)}_{2q}(x_B,Q^2)=C^{(1)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(0)}_{q/q}\left(x,\mu^2\right)+C^{(0)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)$$

➡ $$C^{(1)}_q(x,Q^2/\mu^2)=F^{(1)}_{2q}(x,Q^2)-F^{(0)}_{2q}(x,Q^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)$$

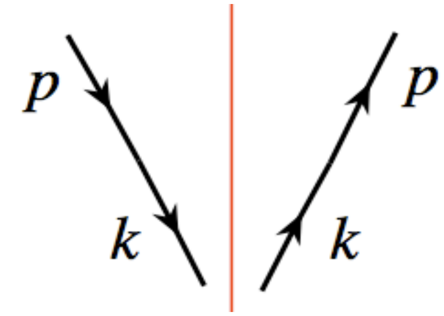
PDFs of a parton

❑ Change the state without changing the operator:

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\overline{\psi}_q(0)\frac{\gamma^+}{2}\, U^n_{[0,y^-]}\, \psi_2(y^-)|h(p)\rangle$$

$|h(p)\rangle \;\Rightarrow\; |\text{parton}(p)\rangle$ ➡ $\phi_{f/q}(x,\mu^2)$ **– given by Feynman diagrams**

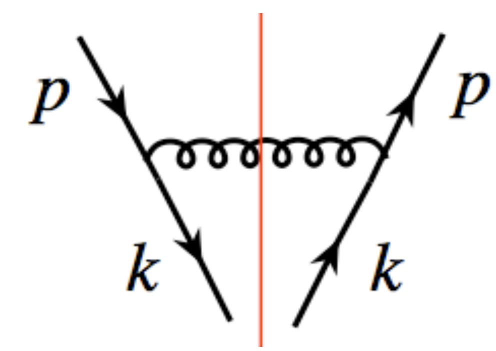
❑ Lowest order quark distribution:



✧ From the operator definition:

$$\begin{aligned}\phi^{(0)}_{q'/q}(x) &= \delta_{qq'} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\left(\frac{1}{2}\gamma\cdot p\right)\left(\frac{\gamma^+}{2p^+}\right)\right] \delta\left(x - \frac{k^+}{p^+}\right) (2\pi)^4 \delta^4(p-k) \\ &= \delta_{qq'}\; \delta(1-x)\end{aligned}$$

❑ Leading order in α_s quark distribution:



$+\ldots$

✧ Expand to $(g_s)^2$ – logarithmic divergent:

$$\phi^{(1)}_{q/q}(x) = C_F \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2} \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(1-x)\right] + \mathrm{UVCT}$$

UV and CO divergence

Choice of regularization

Partonic cross sections

❑ Projection operators for SFs:

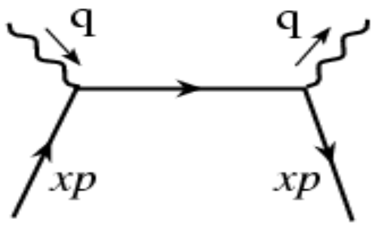
$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x, Q^2\right)$$

$$F_1(x, Q^2) = \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x, Q^2)$$

$$F_2(x, Q^2) = x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x, Q^2)$$

❑ 0^{th} order:

$$F_{2q}^{(0)}(x) = x g^{\mu\nu} W_{\mu\nu,q}^{(0)} = x g^{\mu\nu} \left\{ \frac{1}{4\pi} \right.$$



$$\left. \right\}$$

$$= \left(x g^{\mu\nu}\right) \frac{e_q^2}{4\pi} \mathrm{Tr}\left[\frac{1}{2}\gamma\cdot p \gamma_\mu \gamma\cdot\left(p+q\right)\gamma_\nu\right] 2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x \delta(1-x)$$

$$C_q^{(0)}(x) = e_q^2 x \delta(1-x)$$

NLO coefficient function – complete example

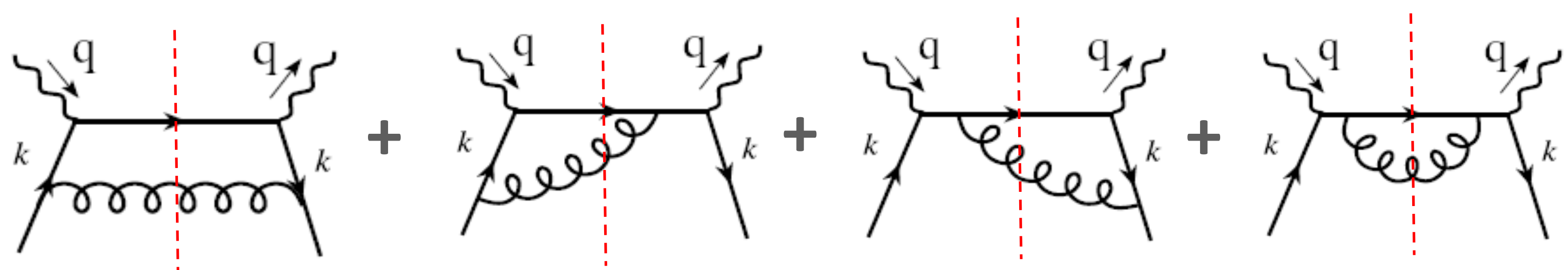
$$C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2) \otimes \varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

- **Projection operators in n-dimension:** $g_{\mu\nu} g^{\mu\nu} = n \equiv 4 - 2\varepsilon$

$$\left(1-\varepsilon\right) F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}$$

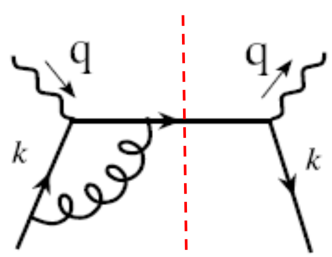
- **Feynman diagrams:**

$$W_{\mu\nu,q}^{(1)}$$

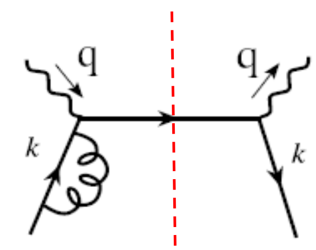


} **Real**

Virtual { +



+ c.c. [+



+ c.c. + UV CT]

- **Calculation:**

$$-g^{\mu\nu} W_{\mu\nu,q}^{(1)} \quad \text{and} \quad p^\mu p^\nu W_{\mu\nu,q}^{(1)}$$

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Contribution from the trace of $W_{\mu\nu}$

❑ **Lowest order in n-dimension:**

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

❑ **NLO virtual contribution:**

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

❑ **NLO real contribution:**

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

Contribution from the trace of $W_{\mu\nu}$

❑ The "+" distribution:

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ell n(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx \frac{f(x)-f(1)}{1-x} + \ell n(1-z) f(1)$$

❑ One loop contribution to the trace of $W_{\mu\nu}$:

$$-g^{\mu\nu} W^{(1)}_{\mu\nu,q} = e_q^2 (1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\left\{-\frac{1}{\varepsilon} P_{qq}(x) + P_{qq}(x)\ell n\left(\frac{Q^2}{\mu^2(4\pi e^{-\gamma_E})}\right)\right.$$

$$+ C_F\left[\left(1+x^2\right)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x)\right.$$

$$\left.\left. + 3 - x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

❑ Splitting function:

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

One-loop contribution to partonic F2 and quark-PDF:

- **One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:**

$$p^\mu p^\nu W^{(1)V}_{\mu\nu,q} = 0 \qquad\qquad p^\mu p^\nu W^{(1)R}_{\mu\nu,q} = e_q^2 C_F \frac{\alpha_s}{2\pi}\frac{Q^2}{4x}$$

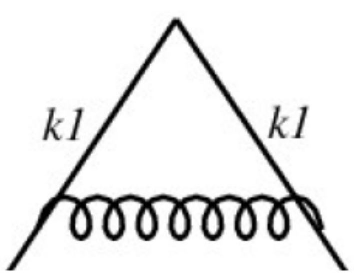
- **One loop contribution to F_2 of a quark:**

$$F^{(1)}_{2q}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{\left(-\frac{1}{\varepsilon}\right)_{\text{CO}} P_{qq}(x)\left(1+\varepsilon \ell n(4\pi e^{-\gamma_E})\right) + P_{qq}(x)\ell n\left(\frac{Q^2}{\mu^2}\right)\right.$$
$$\left. + C_F\left[(1+x^2)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

$$\Rightarrow \quad \infty \quad \text{as} \quad \varepsilon \to 0$$

- **One loop contribution to quark PDF of a quark:**

$$\varphi^{(1)}_{q/q}(x,\mu^2) = \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\text{UV}} + \left(-\frac{1}{\varepsilon}\right)_{\text{CO}}\right\} + \text{UV-CT}$$



– in the dimensional regularization

Different UV-CT = different factorization scheme!

NLO coefficient function for inclusive DIS:

❑ Common UV-CT terms:

✧ **MS scheme:**

$$\text{UV-CT}\big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}$$

✧ **$\overline{\text{MS}}$ scheme:**

$$\text{UV-CT}\big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}} \left(1+\varepsilon \ell n(4\pi e^{-\gamma_E})\right)$$

✧ **DIS scheme: choose a UV-CT, such that** $C_q^{(1)}(x, Q^2/\mu^2)\big|_{\text{DIS}} = 0$

❑ One loop coefficient function:

$$\boxed{C_q^{(1)}(x, Q^2/\mu^2) = F_{2q}^{(1)}(x, Q^2) - F_{2q}^{(0)}(x, Q^2) \otimes \varphi_{q/q}^{(1)}\left(x, \mu^2\right)}$$

$$C_q^{(1)}(x, Q^2/\mu^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{ P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right)\right.$$

$$\left. + C_F\left[(1+x^2)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

Renormalization group improvement

- Physical cross sections should not depend on the factorization scale

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0 \qquad F_2(x_B, Q^2) = \sum_f C_f(x_B/x, Q^2/\mu_F^2, \alpha_s) \otimes \phi_f(x, \mu_F^2)$$

➡ Evolution (differential-integral) equation for PDFs

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right)\right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- PDFs and coefficient functions share the same logarithms

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\mathrm{QCD}}^2\right)$

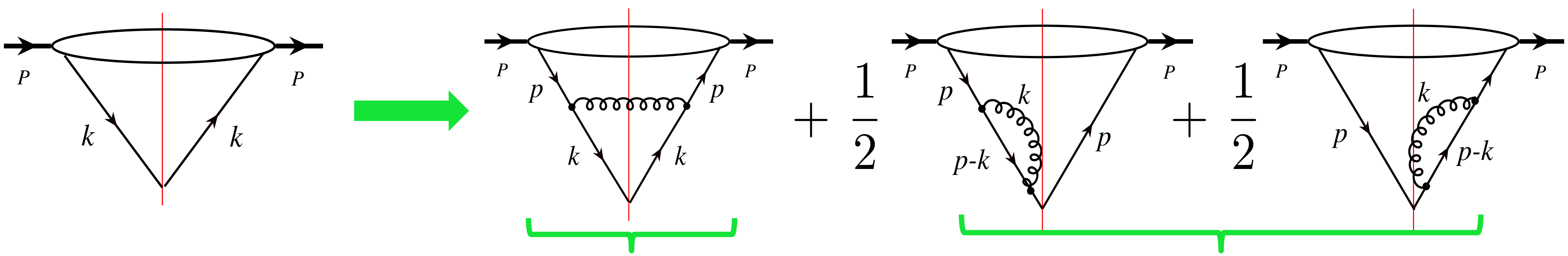
Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

➡ DGLAP evolution equation:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

Different way to calculate the evolution kernels

- ❑ **Evolution kernels are process independent**
  - ✧ **Parton distribution functions are universal**
  - ✧ **Could be derived in many different ways**
- ❑ **Extract from calculating parton PDFs' scale dependence**



Collins, Qiu, 1989

$$Q^2\frac{d}{dQ^2}q_i(x,Q^2)=\frac{\alpha_s}{2\pi}\int_x^1\frac{dx_1}{x_1}q_i(x_1,Q^2)\gamma_{qq}\left(\frac{x}{x_1}\right)-\frac{\alpha_s}{2\pi}q_i(x,Q^2)\int_0^1 dz\,\gamma_{qq}(z)$$

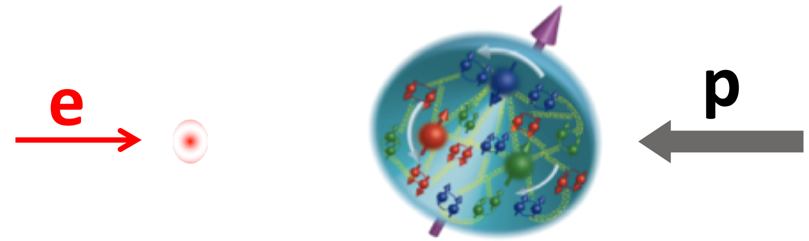
Change "Gain" "Loss"

- ✧ **Same is true for gluon evolution, and mixing flavor terms**

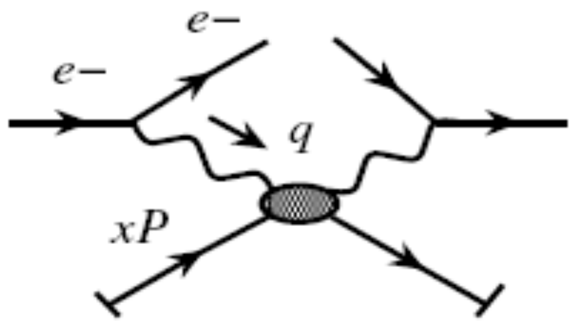
- ❑ **One can also extract the kernels from the CO divergence of partonic cross sections, anomalous dimension of the operator, …**

From one hadron to two hadrons

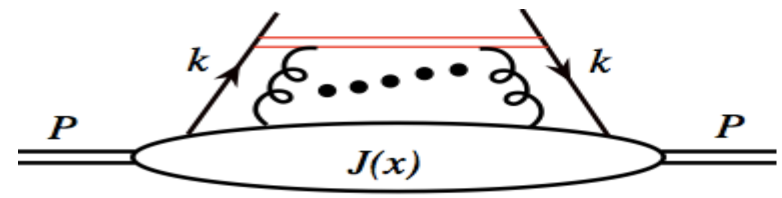
- One hadron:



$$\sigma_{\text{tot}}^{\text{DIS}} \sim$$



$$\otimes$$

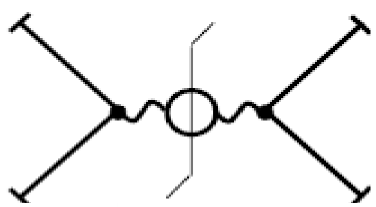


$$+\, O\left(\frac{1}{QR}\right)$$

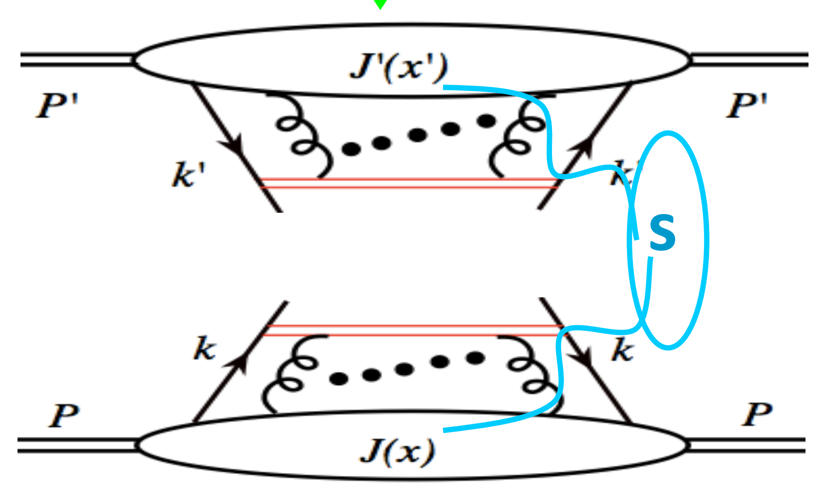
Hard-part	Parton-distribution	Power corrections
Probe	Structure	Approximation

- Two hadrons:

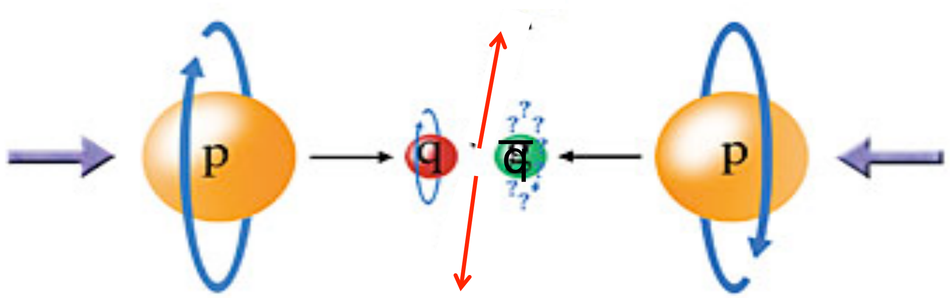
$$\sigma_{\text{tot}}^{\text{DY}} \sim$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$



Predictive power:

Universal Parton Distributions

Drell-Yan process – two hadrons

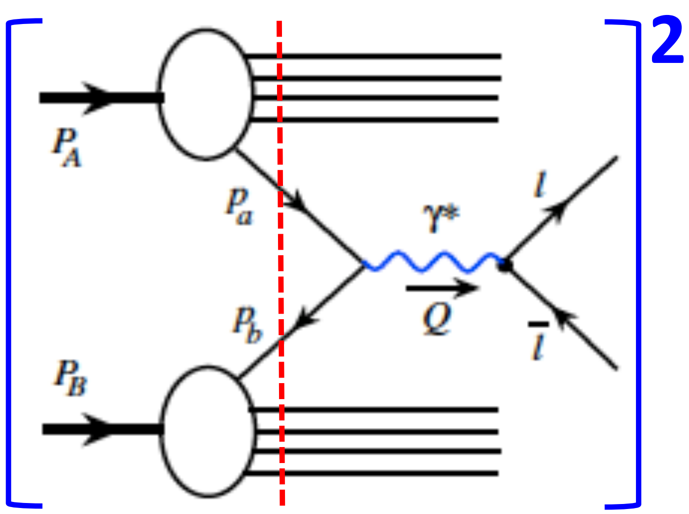
❑ Drell-Yan mechanism:

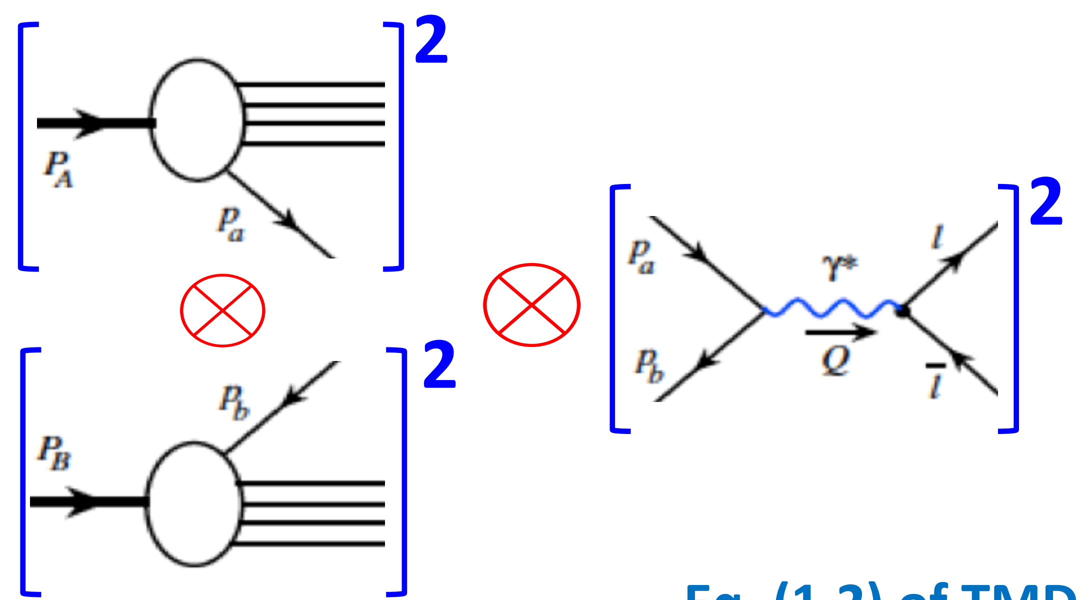
S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X \quad \textbf{with} \quad q^2 \equiv Q^2 \gg \Lambda_{\rm QCD}^2 \sim 1/{\rm fm}^2$$

Lepton pair – from decay of a virtual photon, or in general,
a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

❑ Original Drell-Yan formula:





Eq. (1.2) of TMD handbook

$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2 dy} = \frac{4\pi\alpha_{em}^2}{3Q^4} \sum_{p,\bar{p}} x_A \phi_{p/A}(x_A)\, x_B \phi_{\bar{p}/B}(x_B)$$

No color yet!

Rapidity: $y = \frac{1}{2}\ln(x_A/x_B)$

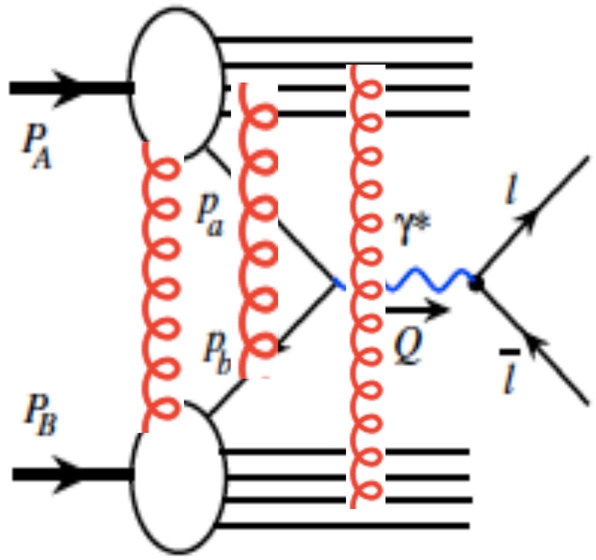
$$x_A = \frac{Q}{\sqrt{S}} e^{y} \quad x_B = \frac{Q}{\sqrt{S}} e^{-y}$$

Right shape – But – not normalization

Drell-Yan process in QCD – factorization

❑ **Beyond the lowest order:**

Collins, Soper and Sterman, Review in QCD, edited by AH Mueller 1989

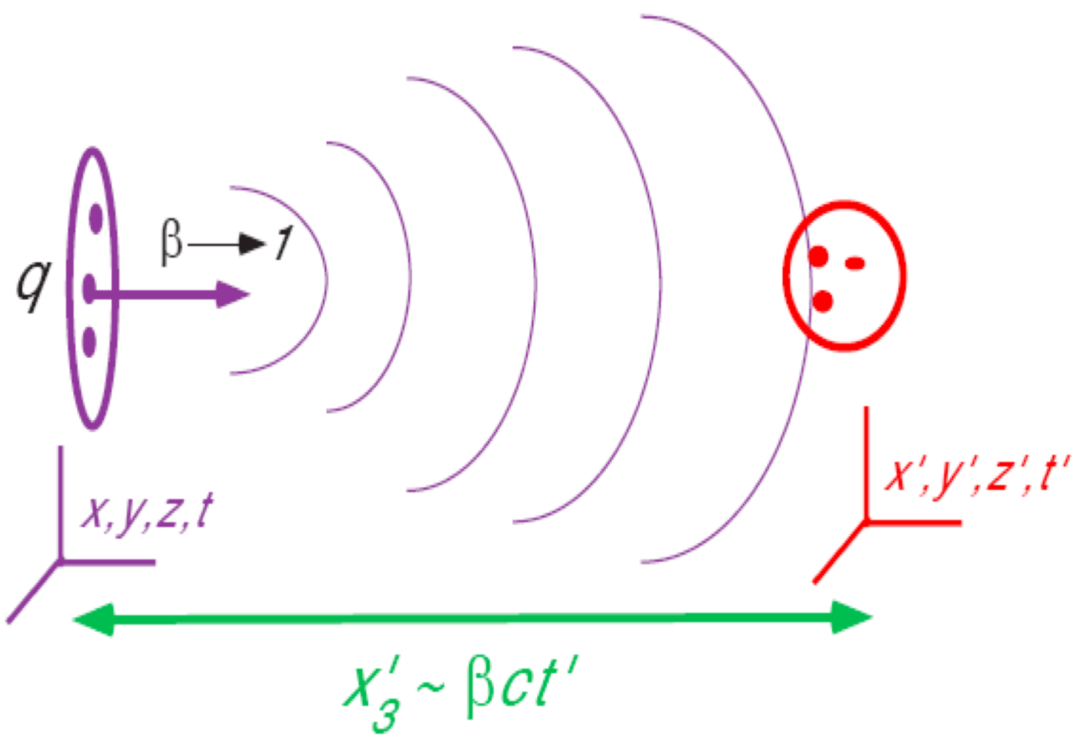


✧ Soft-gluon interaction takes place all the time

✧ Long-range gluon interaction before the hard collision

➡ Break the Universality of PDFs
Loss the predictive power

❑ **Factorization – power suppression of soft gluon interaction:**



x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$ $\Rightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$ $\Rightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

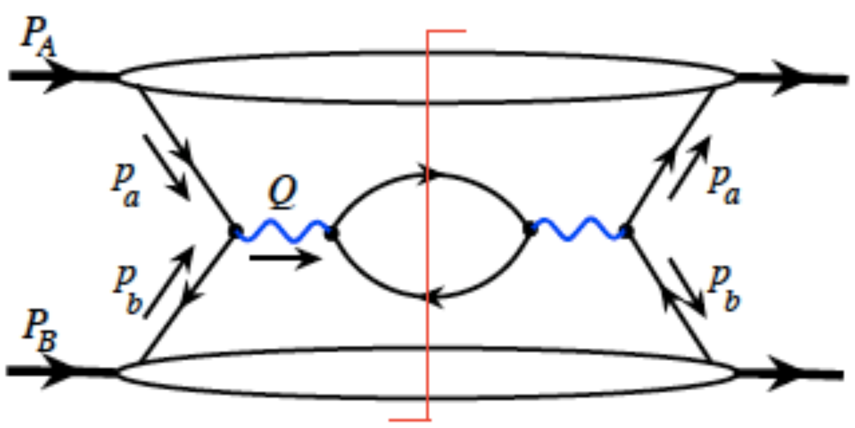
TMD Collaboration

See Chapter 3 of TMD handbook

Collins, Soper, Sterman, 1988

- Factorization – approximation:
  - ✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ $1/\Lambda_{QCD}$) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \rightarrow \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

  - ✧ Maintain the universality of PDFs:

    Long-range soft gluon interaction has to be power suppressed

  - ✧ Infrared safe of partonic parts:

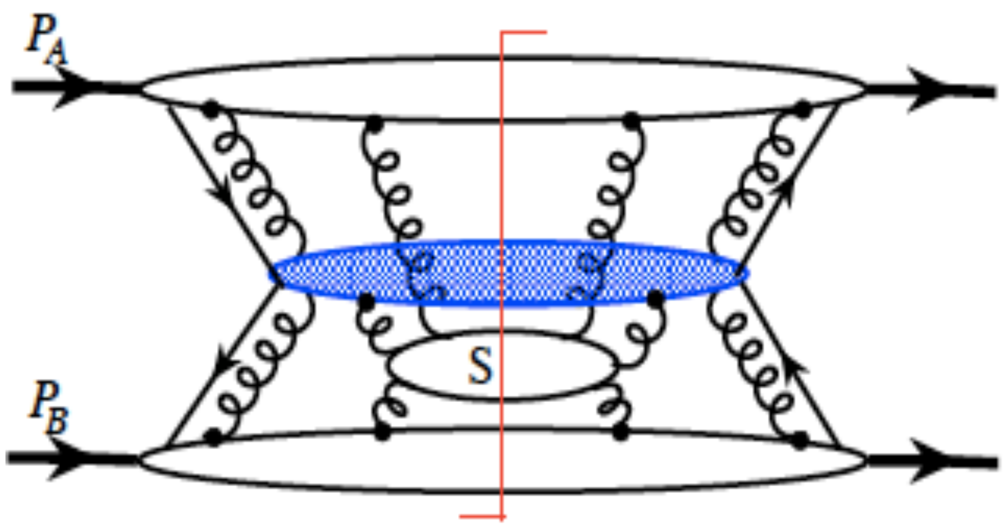
    Cancelation of IR behavior

    Absorb all CO divergences into PDFs

$$\begin{aligned} \text{on-shell:} \quad & p_a^2,\ p_b^2 \ll Q^2; \\ \text{collinear:} \quad & p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ \text{higher-power:} \quad & p_a^- \ll q^-; \quad \text{and} \\ & p_b^+ \ll q^+ \end{aligned}$$

Drell-Yan process in QCD – factorization

❑ **Leading singular integration regions (pinch surface):**



Hard: all lines off-shell by Q

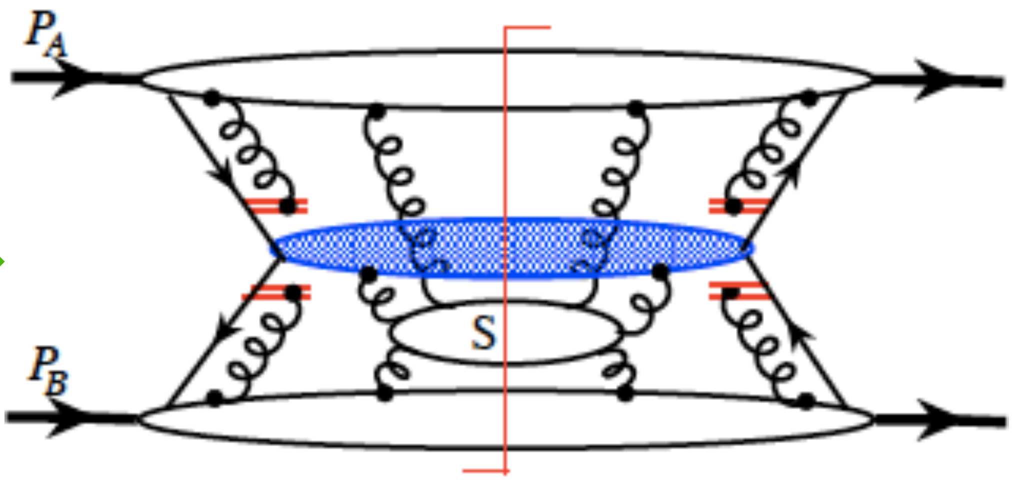
Collinear:
✧ lines collinear to A and B
✧ One "physical parton" per hadron

Soft: all components are soft

❑ **Collinear gluons:**

✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$

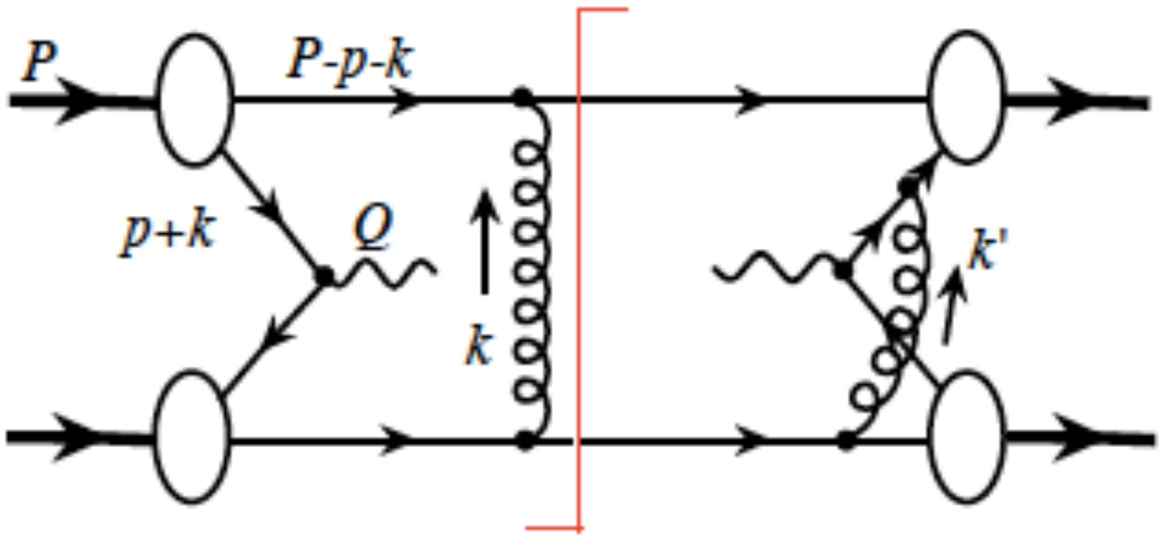
✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Drell-Yan process in QCD – factorization

❑ Trouble with soft gluons:



$$(xp + k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p - k)^2 + i\epsilon \propto k^- - i\epsilon$$

✧ Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B

✧ The soft gluon approximations (with the eikonal lines) need $k^{\pm}$ not too small. But, $k^{\pm}$ could be trapped in "too small" region due to the pinch from spectator interaction: $k^{\pm} \sim M^2/Q \ll k_{\perp} \sim M$

Need to show that soft-gluon interactions are power suppressed

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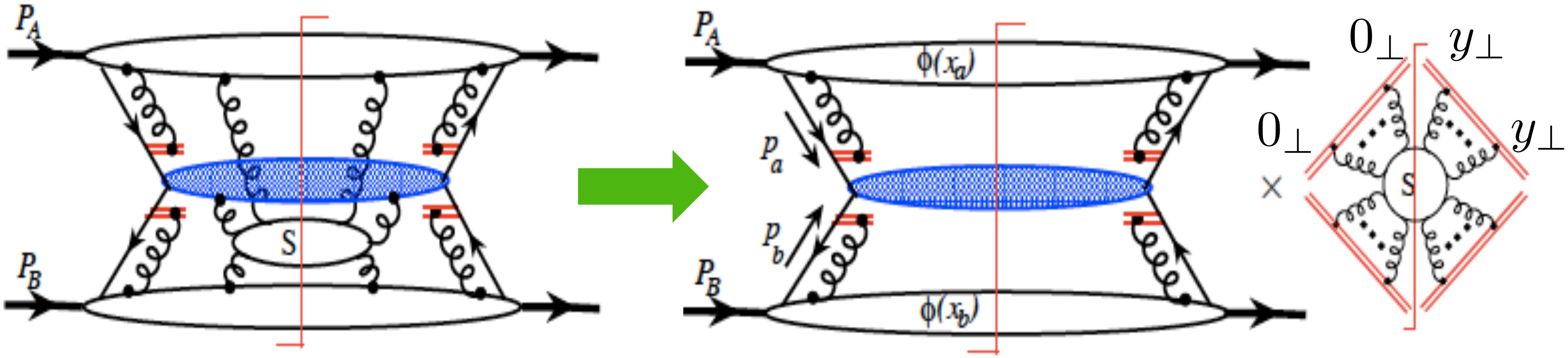
Drell-Yan process in QCD – factorization

❑ **Most difficult part of factorization:**



✧ **Sum over all final states to remove all poles in one-half plane**

– no more pinch poles

✧ **Deform the $k^{\pm}$ integration out of the trapped soft region**

✧ **Eikonal approximation ➡ soft gluons to eikonal lines**

– gauge links

✧ **Collinear factorization: Unitarity ➡ soft factor = 1**

All identified leading integration regions are factorizable!

Factorized Drell-Yan cross section

See lectures by Stewart

❑ Collinear factorization – single hard scale:

Eq.(2.5) of TMD handbook

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a,\mu) \int dx_b\, f_{b/B}(x_b,\mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a,x_b,\alpha_s(\mu),\mu) + \mathcal{O}(1/Q)$$

for $q_\perp \sim Q$ or $q_\perp$ integrated Drell-Yan cross sections: $d^4q = dQ^2\, dy\, d^2q_T$

❑ TMD factorization ($q_\perp \sim Q$):

Eq.(2.6) of TMD handbook

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp}) + \mathcal{O}(q_\perp/Q)$$

$$x_A = \frac{Q}{\sqrt{s}}\, e^{y} \qquad x_B = \frac{Q}{\sqrt{s}}\, e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

❑ Spin dependence:

The factorization arguments at the leading power are independent of the spin states of the colliding hadrons

➡ Same formula with polarized PDFs for γ*,W/Z, H^0…

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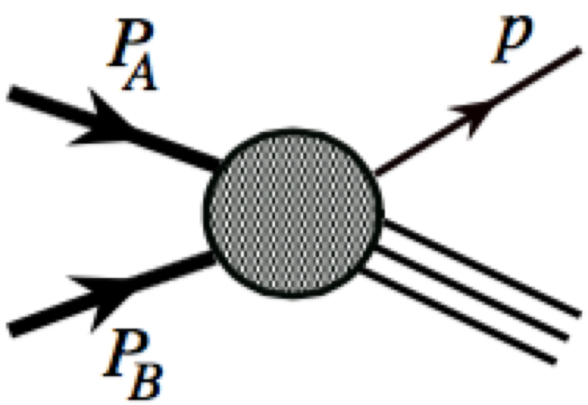
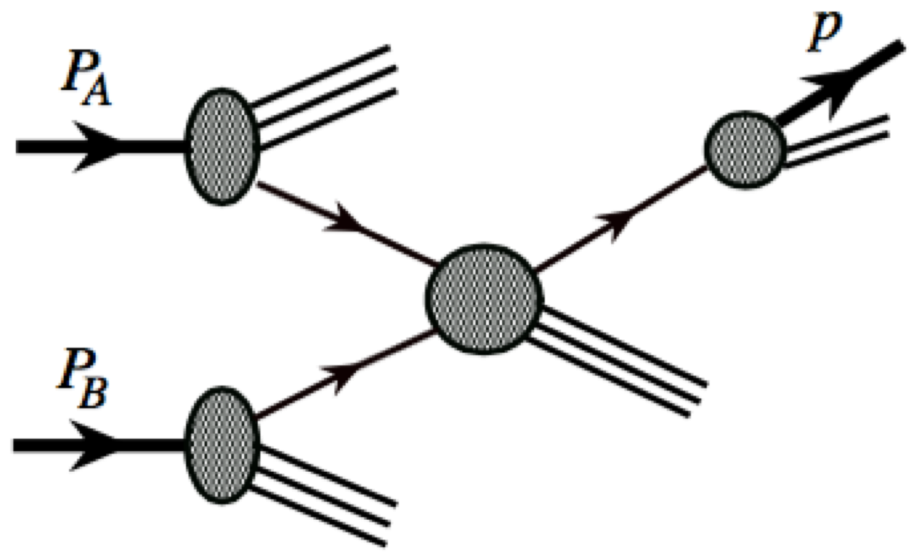
Factorization for more than two hadrons

❑ Factorization for high p_T single hadron:

Nayak, Qiu, Sterman, 2006





$\gamma, W/Z, \ell(s), \text{jet(s)}$
$B, D, \Upsilon, J/\psi, \pi, \ldots$

+ O (1/P_T^2)

$p_T \gg m \gtrsim \Lambda_{\rm QCD}$

$$\frac{d\sigma_{AB\to C+X}(p_A, p_B, p)}{dy\,dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x,\mu_F^2) \otimes \phi_{B\to b}(x',\mu_F^2)$$

$$\otimes \frac{d\hat{\sigma}_{ab\to c+X}(x, x', z, y, p_T^2 \mu_F^2)}{dy\,dp_T^2} \otimes D_{c\to C}(z,\mu_F^2)$$

✧ **Fragmentation function:** $D_{c\to C}(z, \mu_F^2)$

✧ **Choice of the scales:** $\mu_{\rm Fac}^2 \approx \mu_{\rm ren}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

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Predictive power of QCD factorization

❑ Universality of non-perturbative hadron structure:

- lepton-hadron reactions (COMPASS, JLab, **EIC**)

$$\sigma^{\rm EXP}_{l+P\to l+X} = C_{l+k\to l+X} \otimes {\rm PDF}_P + \mathcal{O}(Q_s^2/Q^2)$$

$$\sigma^{\rm EXP}_{l+P\to l+H+X} = C_{l+k\to l+k+X} \otimes {\rm PDF}_P \otimes {\rm FF}_H + \mathcal{O}(Q_s^2/Q^2)$$

- hadron-hadron reactions (LHC)

$$\sigma^{\rm EXP}_{P+P\to l+\bar{l}+X} = C_{k+k\to l+\bar{l}+X} \otimes {\rm PDF}_P \otimes {\rm PDF}_P + \mathcal{O}(Q_s^2/Q^2)$$

- lepton-lepton reactions (Belle)

$$\sigma^{\rm EXP}_{l+\bar{l}\to H+X} = C_{l+\bar{l}\to k+X} \otimes {\rm FF}_H + \mathcal{O}(Q_s^2/Q^2)$$

❑ Hadron structure = Theory + Experiment + Phenomenology:

- **Factorization – Identify "Good" observables (Theory)**
- **Measurement – Get "Reliable" data (Experiment)**
- **Global analysis – Extract "Universal" structure information (Phenomenology) by solving an inverse problem**

QCD global analysis of experimental data

- It is an inverse problem:

Flavors: u, d, s, …; g
Spin (quark, gluon hadron)
Momentum/position
(Before/after the collision)

Experimental data

QCD Factorization Theorem

Future EIC Jefferson Lab KEK CERN BROOKHAVEN NATIONAL LABORATORY DESY Fermilab

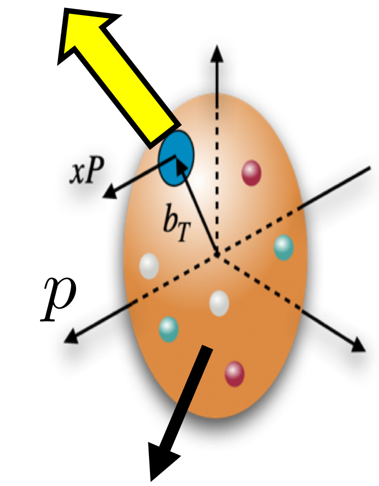
$= \int \cdots \int$ Analytical Coefficient functions derived from QCD perturbation theory $\otimes$ Quantum Correlation Functions of quarks and gluons **Unknown** $+$ Systematic **error** from theory

Bayesian inference
Theory/Phenomenology
AI/ML, Computation

Extract QCFs

PDF_P FF_H, …

Quark Polarization



Nucleon Polarization

- Input for QCD Global analysis/fitting:

PDFs, FFs at an input scale:

Fitting paramters

$$\phi_{f/h}(x, \mu_0^2, \{\alpha_j\})$$

Input scale ~ GeV

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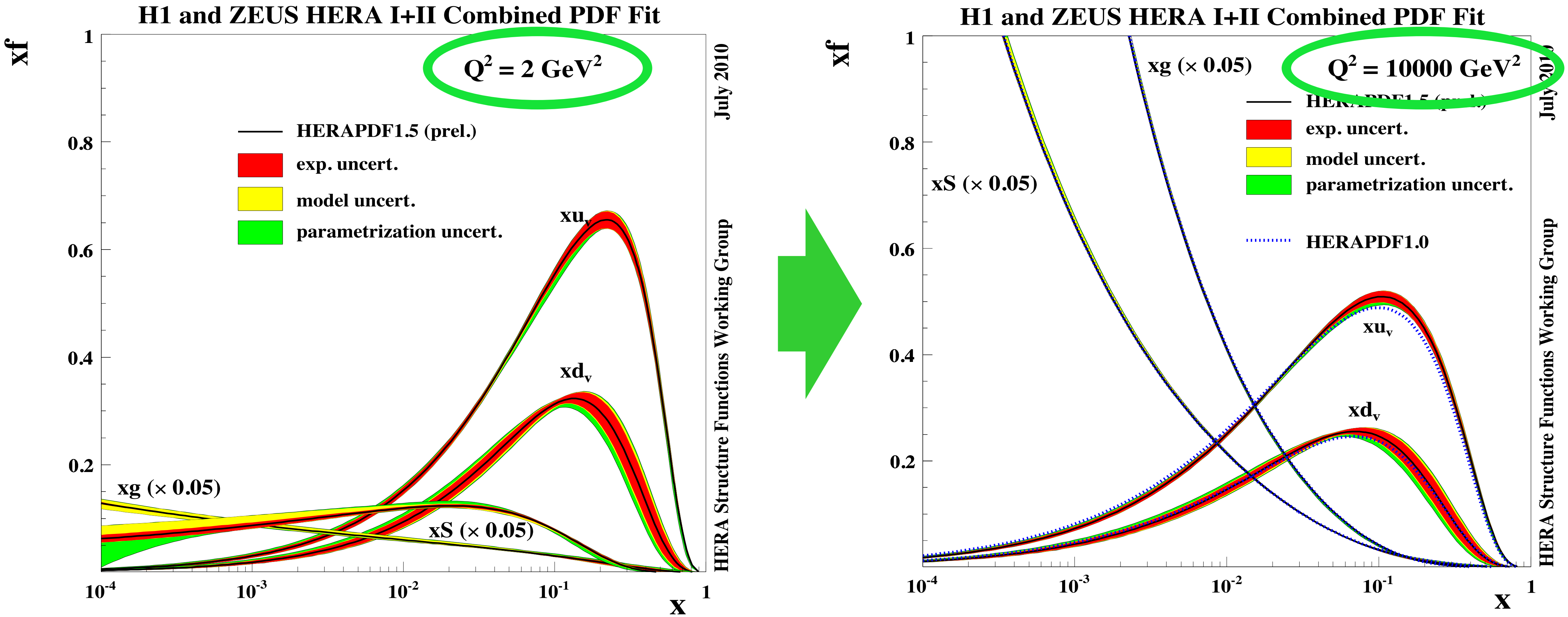
PDFs from DIS

❑ Q2-dependence is a prediction of pQCD calculation:



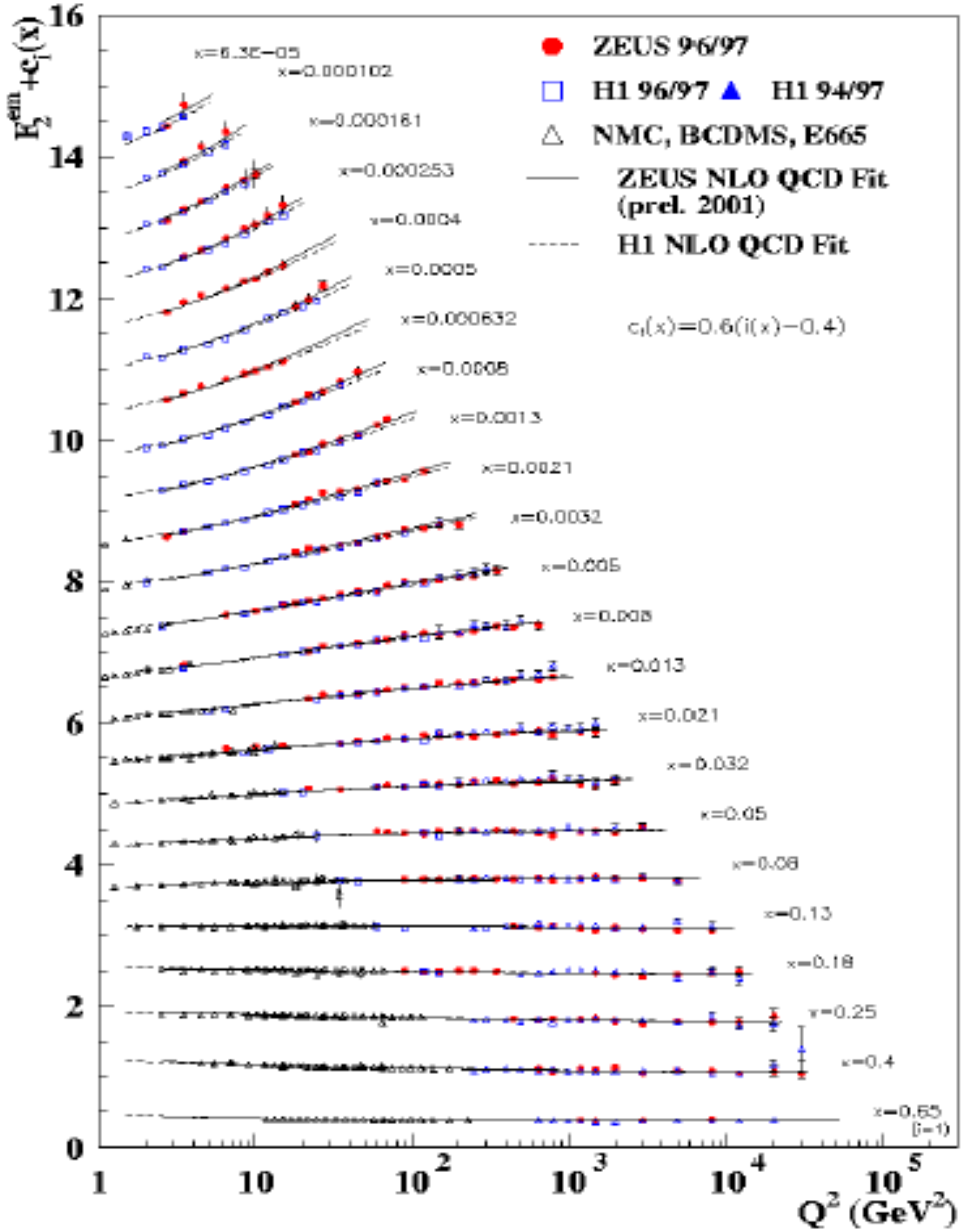
❑ Physics interpretation of PDFs:

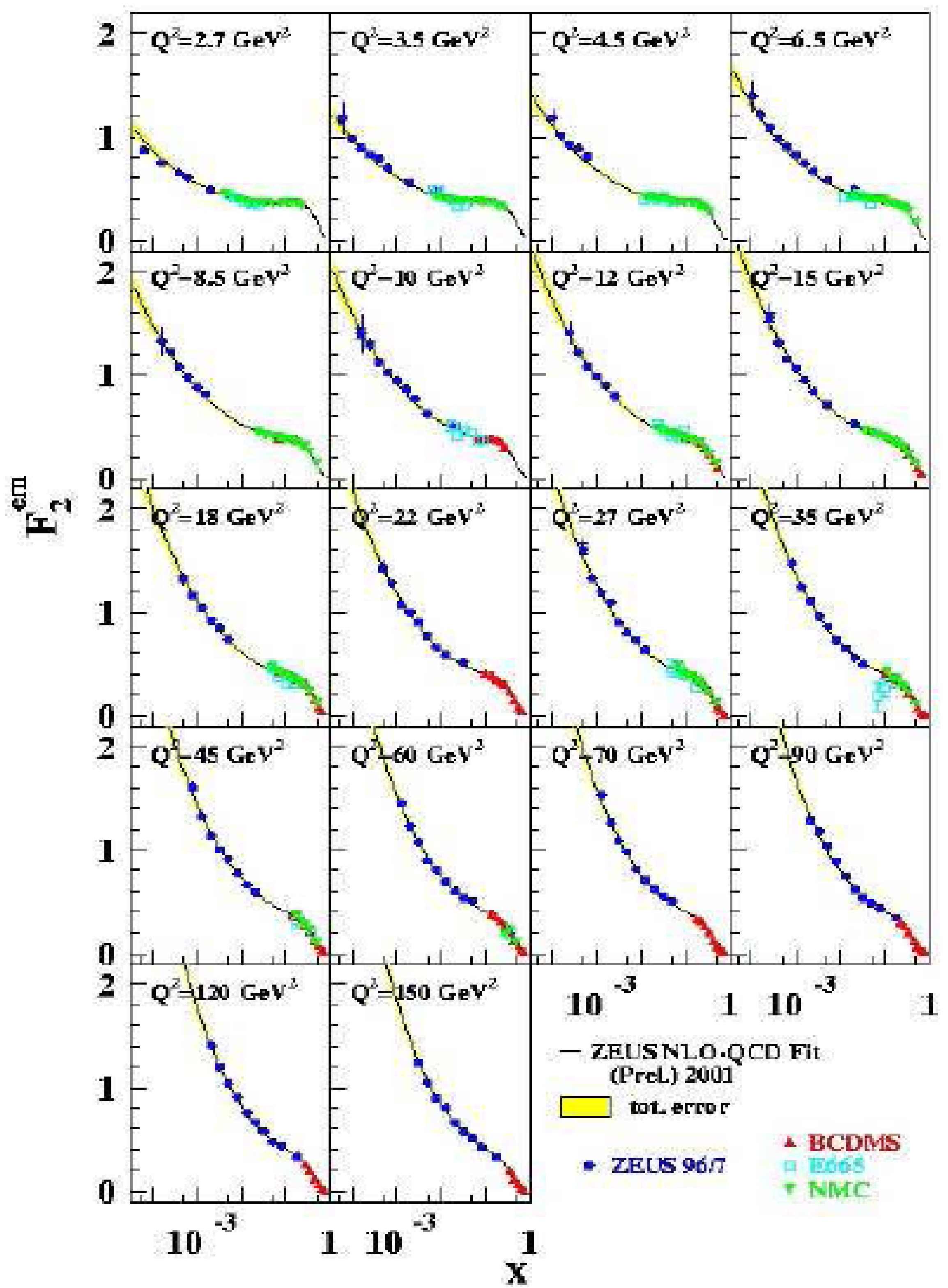
$f(x, Q^2)$: **Probability density to find a parton of flavor "f" carrying momentum fraction "x", probed at a scale of "Q²"**

✧**Number of partons:** $$\int_0^1 dx\, u_v(x,Q^2) = 2, \quad \int_0^1 dx\, d_v(x,Q^2) = 1$$

✧**Momentum fraction:** $$\langle x(Q^2)\rangle_f = \int_0^1 dx\, x\, f(x,Q^2) \;\Rightarrow\; \sum_f \langle x(Q^2)\rangle = 1$$

Scaling and scaling violation





Q^2-dependence is a prediction of pQCD calculation

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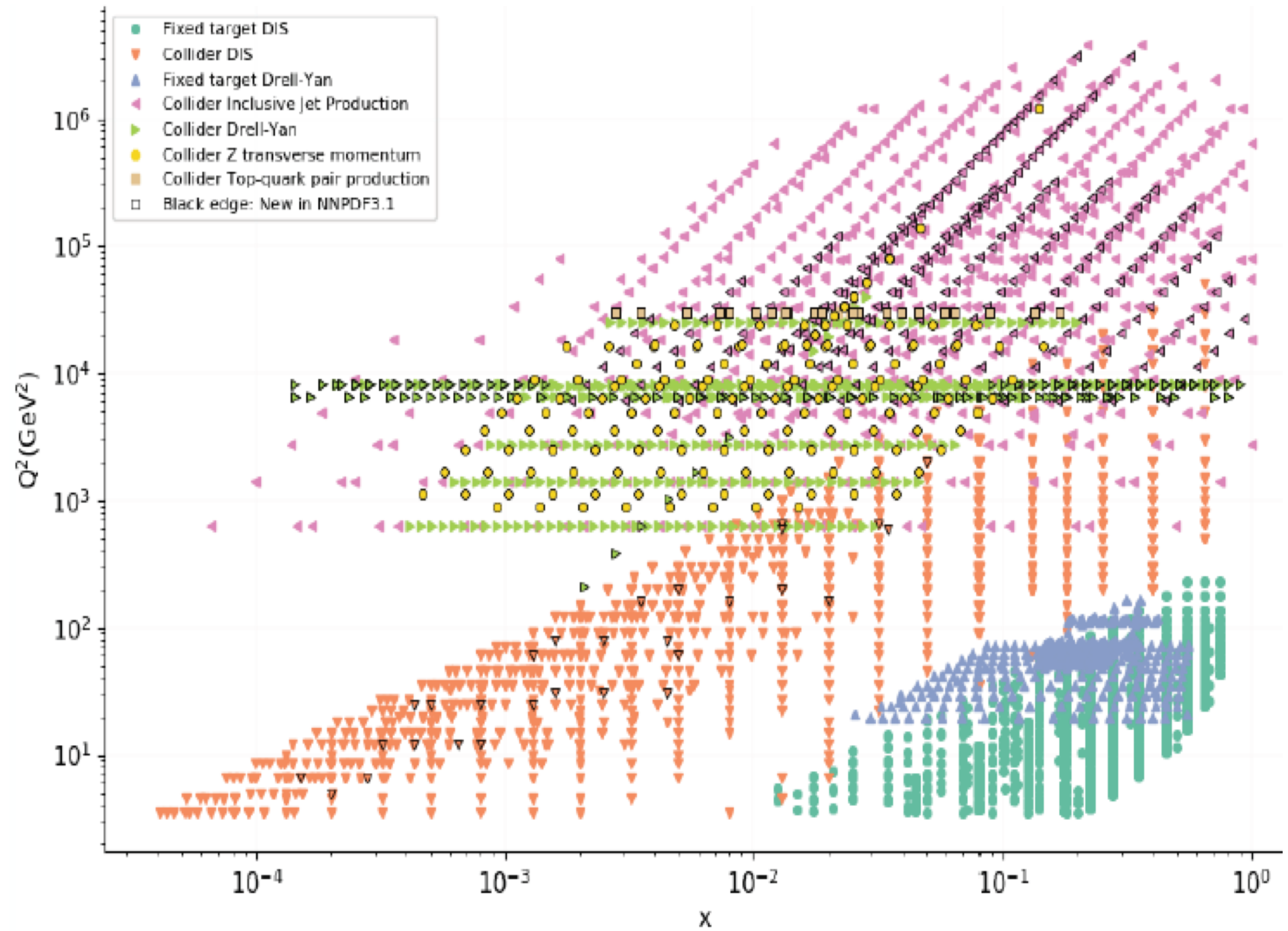
QCD factorization works to the precision

❑ Data sets for Global Fits:

	Process	Subprocess	Partons	x range
	$\ell^\pm\{p,n\} \to \ell^\pm + X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
	$\ell^\pm n/p \to \ell^\pm + X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
	$pp \to \mu^+\mu^- + X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
Fixed Target	$pn/pp \to \mu^+\mu^- + X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
	$\nu(\bar{\nu})N \to \mu^-(\mu^+) + X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
	$\nu N \to \mu^-\mu^+ + X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
	$\bar{\nu} N \to \mu^+\mu^- + X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
	$e^\pm p \to e^\pm + X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
	$e^+ p \to \bar{\nu} + X$	$W^+\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
Collider DIS	$e^\pm p \to e^\pm c\bar{c} + X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^\pm p \to e^\pm b\bar{b} + X$	$\gamma^* b \to b, \gamma^* g \to b\bar{b}$	b, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
	$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
Tevatron	$p\bar{p} \to (W^\pm \to \ell^\pm\nu) + X$	$ud \to W^+, \bar{u}\bar{d} \to W^-$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
	$p\bar{p} \to (Z \to \ell^+\ell^-) + X$	$uu, dd \to Z$	u, d	$x \gtrsim 0.05$
	$p\bar{p} \to t\bar{t} + X$	$qq \to t\bar{t}$	q	$x \gtrsim 0.1$
	$pp \to \text{jet} + X$	$gg, qg, q\bar{q} \to 2j$	g, q	$0.001 \lesssim x \lesssim 0.5$
	$pp \to (W^\pm \to \ell^\pm\nu) + X$	$u\bar{d} \to W^+, d\bar{u} \to W^-$	$u, d, \bar{u}, \bar{d}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X$	$q\bar{q} \to Z$	$q, \bar{q}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X, p_\perp$	$gq(\bar{q}) \to Zq(\bar{q})$	$g, q, \bar{q}$	$x \gtrsim 0.01$
	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, Low mass	$q\bar{q} \to \gamma^*$	$q, \bar{q}, g$	$x \gtrsim 10^{-4}$
LHC	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, High mass	$q\bar{q} \to \gamma^*$	$\bar{q}$	$x \gtrsim 0.1$
	$pp \to W^+\bar{c}, W^-c$	$sg \to W^+c, \bar{s}g \to W^-\bar{c}$	$s, \bar{s}$	$x \sim 0.01$
	$pp \to t\bar{t} + X$	$gg \to t\bar{t}$	g	$x \gtrsim 0.01$
	$pp \to D, B + X$	$gg \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to J/\psi, \Upsilon + pp$	$\gamma^*(gg) \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to \gamma + X$	$gq(\bar{q}) \to \gamma q(\bar{q})$	g	$x \gtrsim 0.005$

❑ Kinematic Coverage:

NNPDF3.1



❑ Fit Quality:

$\chi^2/\text{dof} \sim 1 \Rightarrow$ Non-trivial check of QCD

	LO	NLO	NNLO
All data sets	3706 / 2763	3267 / 2996	2717 / 2663

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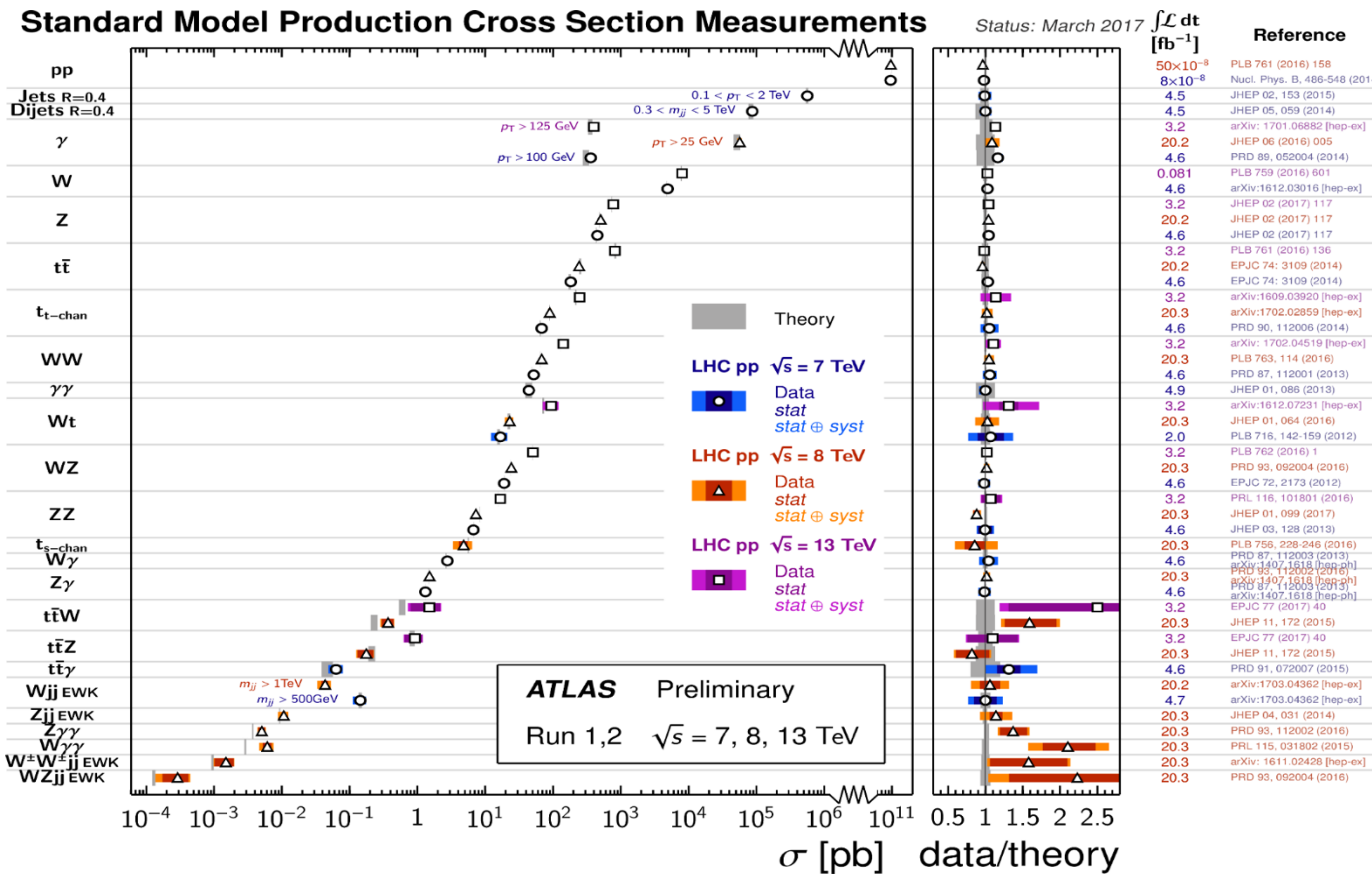
Unprecedent Success of QCD and Standard Model



SM: Electroweak processes + QCD perturbation theory + PDFs works!

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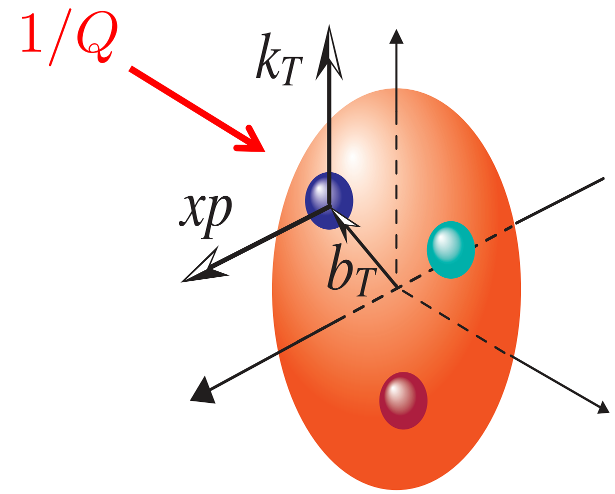
Probes for 3D hadron structure

❑ Single scale hard probe is too "localized":



- It pins down the particle nature of quarks and gluons
- But, not very sensitive to the detailed structure of hadron ~ fm
- Transverse confined motion: k_T ~ 1/fm << Q
- Transverse spatial position: b_T ~ fm >> 1/Q

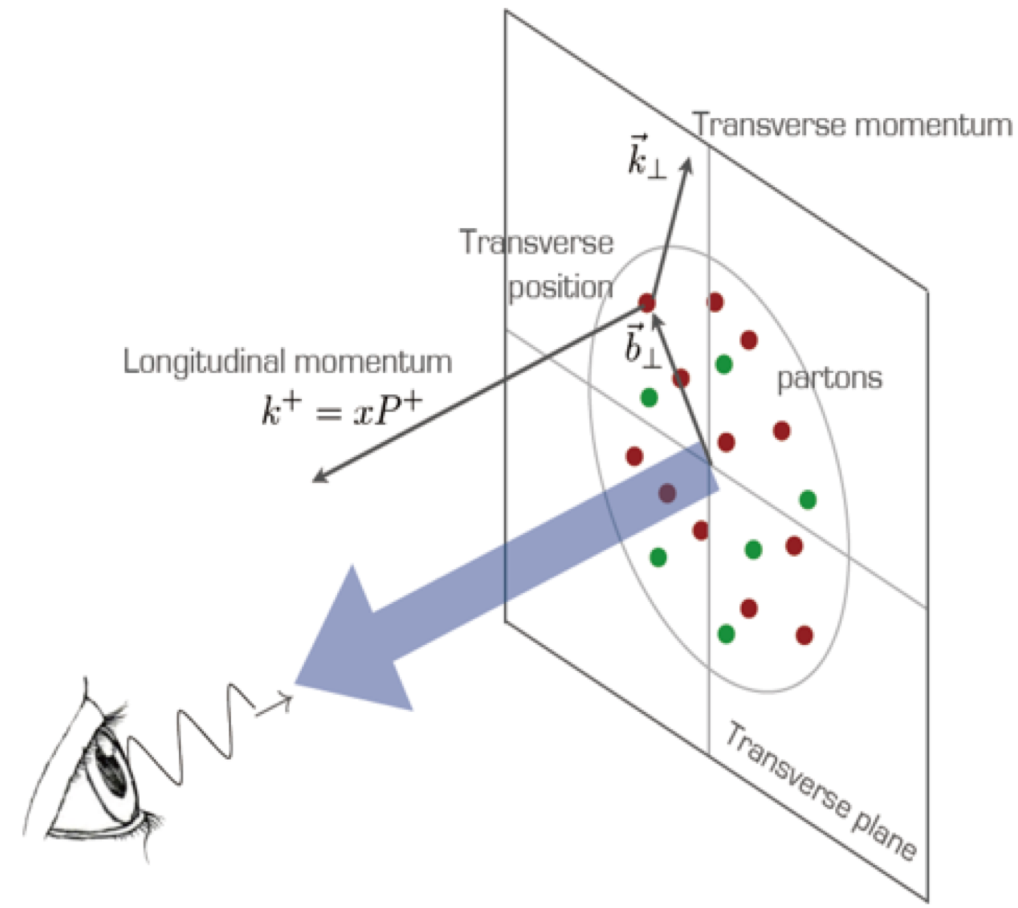
❑ Need new type of "Hard Probes" – Physical observables with TWO Scales:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

Hard scale: Q_1 To localize the probe particle nature of quarks/gluons

"Soft" scale: Q_2 could be more sensitive to the hadron structure ~ 1/fm

Hit the hadron "very hard" without breaking it, clean information on the structure!



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See lectures by Stewart

From one-scale to two-scale observables

See lectures by Stewart

❑ Drell-Yan process in hadron-hadron collisions:

The process: $\sigma^{\rm EXP}_{P+P\to l+\bar{l}+X} = C_{k+k\to l+\bar{l}+X} \otimes \mathrm{PDF}_P \otimes \mathrm{PDF}_P + \mathcal{O}(Q_s^2/Q^2)$

One-scale case: $$\frac{d\sigma^{\rm DY}_{hh'}(p_A,p_B,q)}{dQ^2} = \sum_{f,f'} \int_0^1 dx \int_0^1 dx'\, \phi_f(x)\, \frac{d\hat{\sigma}^{\rm el}_{ff'}(xp_A,x'p_B,q)}{dQ^2}\, \phi_{f'}(x')$$

Hard scale – invariant mass of the lepton-pair: $Q^2 \equiv q^2 = (l+\bar{l})^2 \gg \Lambda^2_{\rm QCD} \sim 1/R_h^2$

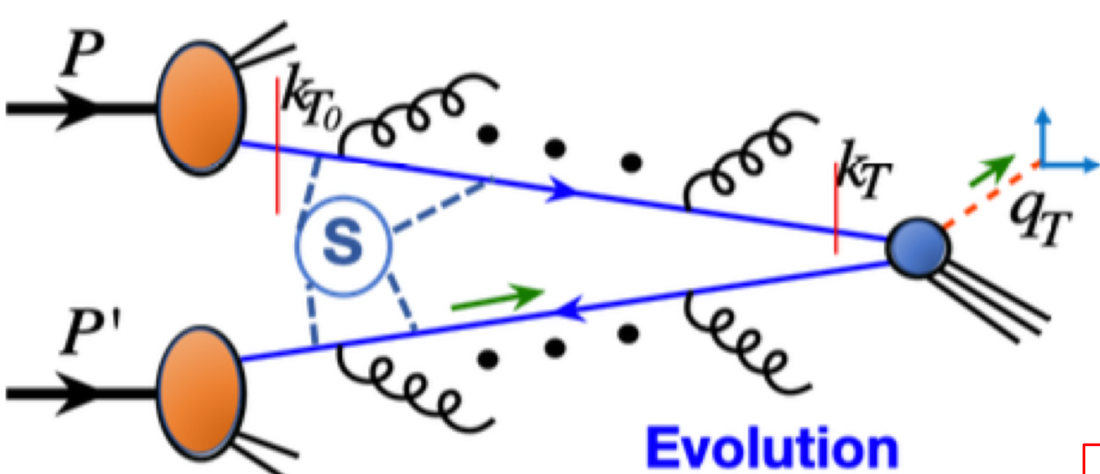
Two-scale case:

TMDs

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A,k_{a\perp}) \mathcal{F}_{b/B}(x_B,k_{b\perp}) \mathcal{S}(k_{s\perp})$$

Hard scale: Q^2 **Soft scale:** q_T^2 **when** $Q^2 \gg q_T^2$ $\qquad d^4q = dy\, dQ^2 dq_T^2 d\phi_q$

❑ Confined motion vs. collision effects:



Collision induced shower

QCD Evolution – could be non-perturbative!

TMDs: $\mathcal{F}(x,k_T) \neq \mathcal{F}(x,k_{T_0})$

Structure + Collision effect

Confined motion

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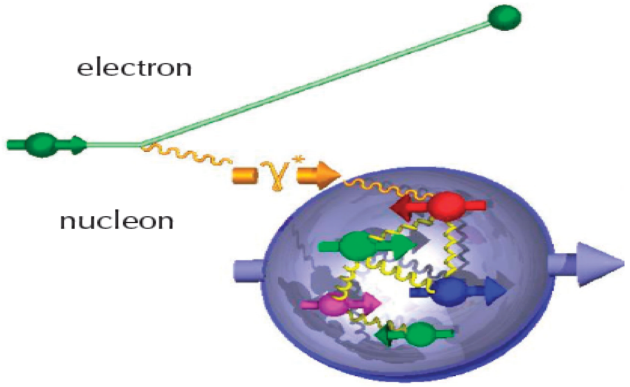
Why a lepton-hadron facility, like EIC, is special?

❑ The new generation of “Rutherford” experiment:



✧ A controlled “probe” – virtual photon

✧ Can either break or not break the hadron

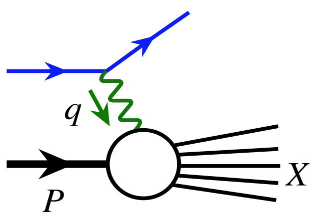
One facility covers all!

(JLab, COMPASS, EIC, …)

✧ Inclusive events: e+p/A → e’+X

Detect only the scattered lepton in the detector

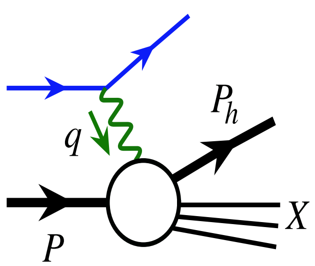
(Modern Rutherford experiment!)



✧ Semi-Inclusive events: e+p/A → e’+h(p,K,p,jet)+X

Detect the scattered lepton in coincidence with identified hadrons/jets

(Initial hadron is broken – confined motion! – cleaner than h-h collisions)



✧ Exclusive events: e+p/A → e’+ p’/A’+ h(p,K,p,jet)

Detect every things including scattered proton/nucleus (or its fragments)

(Initial hadron is NOT broken – tomography!

– almost impossible for h-h collisions)