Introduction to Transverse Momentum Dependendent Parton Distributions

$\leftarrow$ TMD] TMDs

Iain Stewart

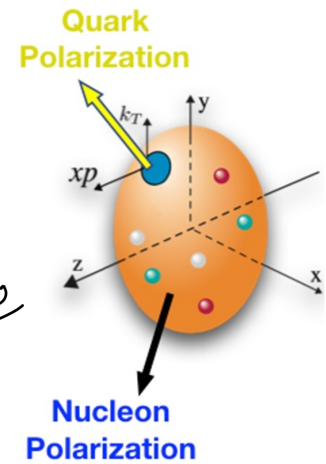
Motivation

- explore mysterys of relativistic bound particles
  - $\rightarrow$ proton

  momenta of partons $\rightarrow$ distributions $f(x, k_T)$

  spin-k_T quantum correlations $\sim k_T \cdot S_T$

  $\epsilon_T^{\alpha\beta} k_{T\alpha} S_{T\alpha}$



moving

- Precision Physics, Higgs q_T, Drell-Yan q_T

- QFT : Factorization
  Wilson Lines & Loops
  Anom Dim. in 2-dim
  Final State Interactions ...

- Improving under of Confinement & Hadronization

Peel the Onion

Handbook

① § 2.1, 2.2
② § 2.2
③ § 2.3
④ § 2.4

$\rightarrow$

Something sweet ?!

Layer ① Factorization

Observables $\leftrightarrow$ Distributions

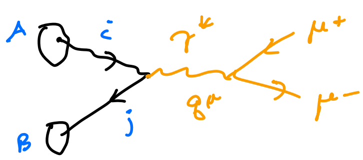
Consider Drell-Yan $\quad pp \rightarrow (\mu^+\mu^-) + X$

CM frame P_A^+, P_B^- big

$$q^\mu = (q^+, q^-, \vec{q}_T), \quad q^\pm = \frac{q^0 \pm q^z}{\sqrt{2}}$$

$$Q^2 = q^2 = 2q^+q^- - q_T^2$$

rapidity $\quad Y = \frac{1}{2}\ln\left(\frac{q^-}{q^+}\right)$



$s = 2P_A^+P_B^-$

vars $\{Q, Y, \vec{q}_T\}$

ⓐ Collinear Fact. $\int d^2 q_T$ $\quad Q^2 \gg \Lambda_{QCD}^2$

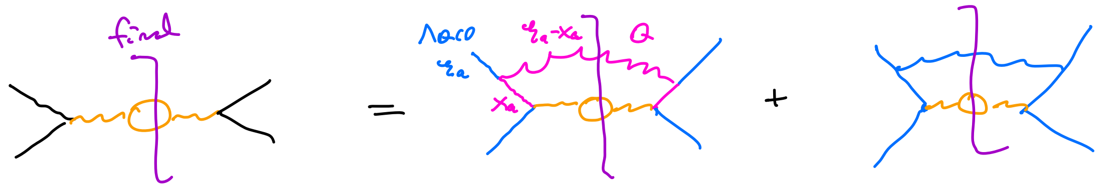
$$\frac{d\sigma}{dQ^2 dY} = \int_{x_a}^{1} d\xi_a \int_{x_b}^{1} d\xi_b \, f_{i/p}(\xi_a, \mu)\, f_{j/p}(\xi_b, \mu)\, \frac{d\hat\sigma_{ij}(\xi_{a,b}, \mu)}{dQ^2\, dY} \left[1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{Q^2}\right) \right]$$

fact. scale = renormalization scale for PDF operator

$$x_a = \frac{Q e^{Y}}{\sqrt{s}} = \frac{q^+}{P_A^+}$$

$\xi_a = x_a$ at tree level

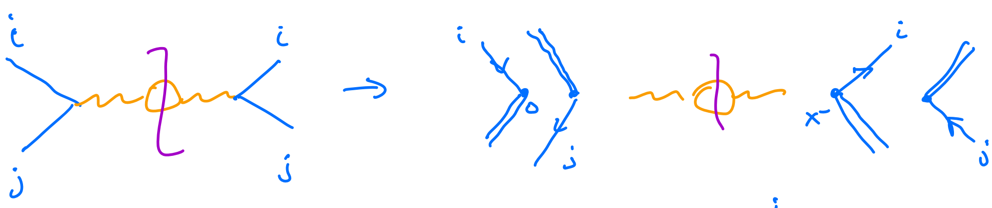
$\xi_a \geq x_a$ due gluon radiation

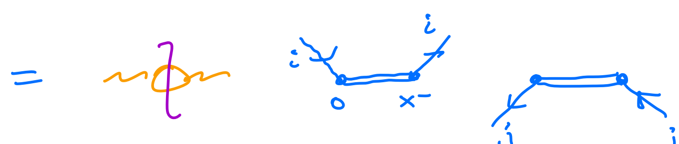


$p^2 \geq \mu^2$ $\qquad$ $p^2 \leq \mu^2$

$\hat\sigma^{(1)} f_i^{(0)} f_j^{(0)}$ $\qquad$ $\hat\sigma^{(0)} f_i^{(1)} f_j^{(0)}$

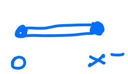
And From point of view of i's radiation, fast moving j looks like a line of color charge //





$\hat\sigma_{ij}$ $\qquad$ $f_i(\xi_a)$ $\qquad$ $f_j(\xi_b)$

$$/\!/_\gamma = W[\gamma] = \underset{\text{line}}{\text{Wilson}} = P \exp\left[-ig \int_\gamma dx^\mu A_\mu^c(x)\, t^c \right]$$

Above get straight line paths $\gamma =$ 

$$f_i^{\text{bare}}(\xi, \epsilon) = \int \frac{dx^-}{2\pi} e^{-i\xi p^+ x^-} \langle p | \underbrace{\bar\psi_i(x^-) \frac{\gamma^+}{2} W[\gamma]\, \psi_i(0)}_{\text{gauge invariant}} | p \rangle$$

(b) $q_T^2 \sim Q^2$

$$\frac{d\sigma}{dQ^2\,dY\,d^2q_T} = \int d\xi_a\, d\xi_b\; f_{i/p}(\xi_a,\mu)\, f_{j/p}(\xi_b,\mu)\, \frac{d\hat{\sigma}_{ij}(\xi_{a,b},\mu)}{dQ^2\,dY\,d^2q_T} * \left[1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{q_T^2 \text{ or } Q^2}\right)\right]$$

(c) $q_T^2 \ll Q^2$ $\boxed{\text{2 scales}}$ $q_T^2 \gg \Lambda_{QCD}^2$ or $q_T^2 \sim \Lambda_{QCD}^2$

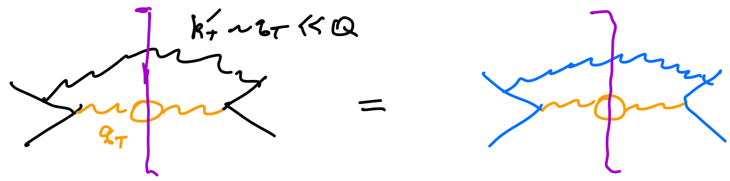
$$\frac{d\sigma}{dQ^2\,dY\,d^2q_T} = H_{i\bar{\imath}}(Q^2,\mu) \int d^2k_T\; f_{i/p}(x_a, \vec{k}_T, \mu, \dots)\, f_{\bar{\imath}/p}(x_b, \vec{q}_T - \vec{k}_T, \mu, \dots) \times \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}, \frac{\Lambda_{QCD}^2}{Q^2}\right)\right]$$

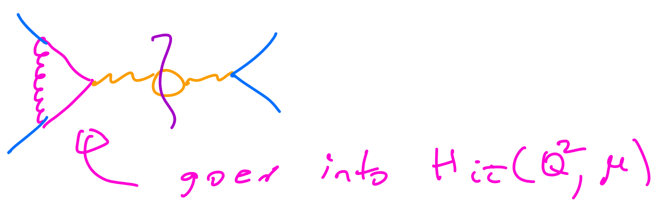
$$= \underbrace{H_{i\bar{\imath}}(Q^2,\mu)}_{\text{virtual}} \int d^2b_T\; e^{i\vec{q}_T\cdot\vec{b}_T}\; \tilde{f}_{i/p}(x_a, b_T, \mu, \dots)\, \tilde{f}_{\bar{\imath}/p}(x_b, b_T, \mu, \dots)$$

- no ξ_a, ξ_b?

  $\xi_a = x_a$

  $\xi_b = x_b$





- Compare (b) & (c) with $\Lambda_{QCD}^2 \ll q_T^2 \ll Q^2$

  TMD PDF $\leftrightarrow$ collinear PDF

$$f_i(x, k_T, \mu, \dots) = \sum_j \int_x^1 \frac{d\xi}{\xi}\, C_{ij}\left(\frac{x}{\xi}, k_T, \mu, \dots\right) f_j(\xi,\mu) \times \left[1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{k_T^2}\right)\right]$$

$$\sum_i H_{i\bar{\imath}}\, C_{ij}\, C_{\bar{\imath}\bar{\jmath}} = \frac{d^2\hat{\sigma}_{j\bar{\jmath}}(\xi_a,\xi_b,\mu)}{dQ^2\,dY\,d^2q_T} \quad \text{when taking } q_T^2 \ll Q^2$$

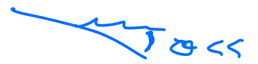
Layer ②

TMD factorization Thm

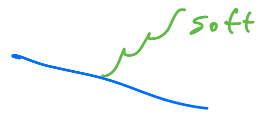
- what type of radiation is allowed ?

$$\text{collinear} \quad \sim \overset{+}{(Q}\,,\ \overset{-}{\frac{q_T^2}{Q}}\,,\ \overset{\perp}{q_T)}$$

small angle $\theta \ll 1$



$$\text{soft} \quad \sim (q_T,\ q_T,\ q_T)$$



distinguish ? by rapidity $\quad \nu =$ "rapidity cutoff"

$$Y_{\text{collinear}} \simeq \frac{1}{2}\ln\left(\frac{Q^2}{q_T^2}\right) \gg Y_{\text{soft}} \simeq \frac{1}{2}\ln(1)$$

$$q_T e^{2Y} > \nu \qquad\Big|\qquad \nu > q_T e^{Y}$$
collinear | soft

$$\frac{d\sigma}{dQ\,dY\,d^2q_T} = H_{i\bar{i}}(Q^2,\mu)\int d^2b_T\, e^{i q_T\cdot b_T}\, \tilde{B}_{i/p}\left(x_a, b_T, \mu, \frac{\zeta_a}{\nu^2}\right) \tilde{B}_{\bar{i}/p}\left(x_b, b_T, \mu, \frac{\zeta_b}{\nu^2}\right) * \tilde{S}(b_T,\mu,\nu)$$

$$= H_{i\bar{i}}\int d^2b_T\, e^{i q_T\cdot b_T}\, \tilde{f}_{i/p}(x_a, b_T, \mu, \zeta_a)\, \tilde{f}_{\bar{i}/p}(x_b, b_T, \mu, \zeta_b)$$

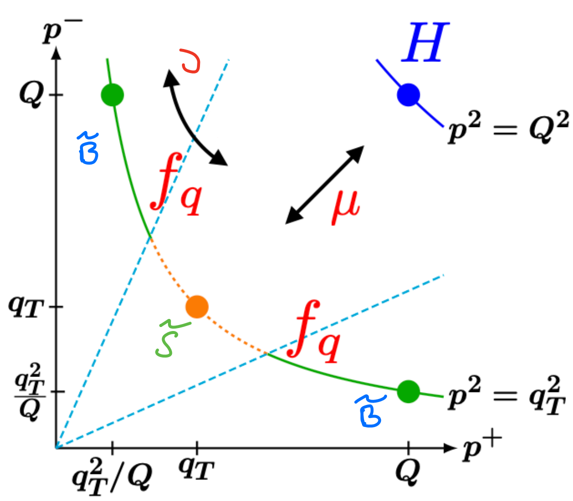
- $\tilde{f} = \tilde{B}\sqrt{\tilde{S}}$ indep. of ν

cutoff cancels here

- Collins-Soper Scales

$$\zeta_a = 2(x_a P_A^+)^2 e^{-2Y_n}$$
$$\zeta_b = 2(x_b P_B^+)^2 e^{+2Y_n}$$

$$\zeta_a \zeta_b = Q^4$$



Layer 3

Operators

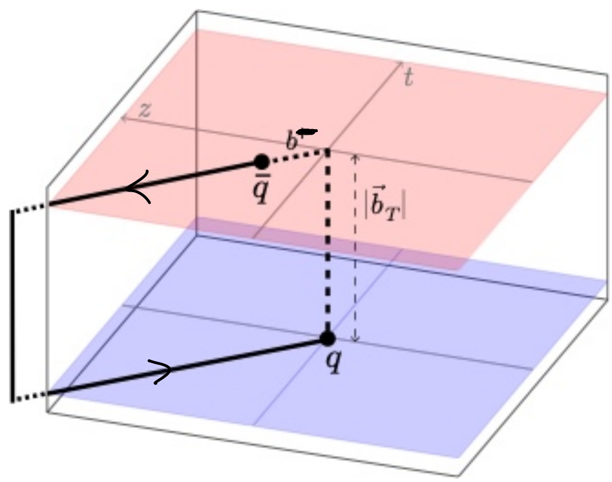
use bare $\tilde{B}$, $\tilde{S}$ dim. reg. $d = 4 - 2\epsilon$, $\mu \to \epsilon$

bare for rapidity $\nu \to \tau$? (onion)

$$\tilde{f}_{i/p}(x, b_T, \mu, y) = \lim_{\substack{\epsilon \to 0 \\ \tau \to 0}} \underbrace{Z^{i}_{uv}(\mu, y, \epsilon)}_{\text{UV counterterm}} \underbrace{\frac{\tilde{f}^{(u)}_{i/p}(x, b_T, \epsilon, \tau, x p^+)}{\tilde{S}^{\text{subt}}(b_T, \epsilon, \tau)}}_{\text{bare } \tilde{B}(x, b_T, \epsilon, \tau, x P^+)} \sqrt{\tilde{S}(b_T, \epsilon, \tau)}$$

$$f^{(u)}_{i/p} = \int \frac{db^-}{2\pi} e^{-ib^- x p^+} \langle p | \left[\bar{\psi}_i(b^\mu) \frac{\gamma^+}{2} W_{\sqsubset} \psi_i(0) \right]_\tau | p \rangle$$

- $b^\mu = (0, b^-, b_T)$
- staple shaped Wilson line
- generated by expansions that replace fields from other proton (like before)
- encode initial state interactions



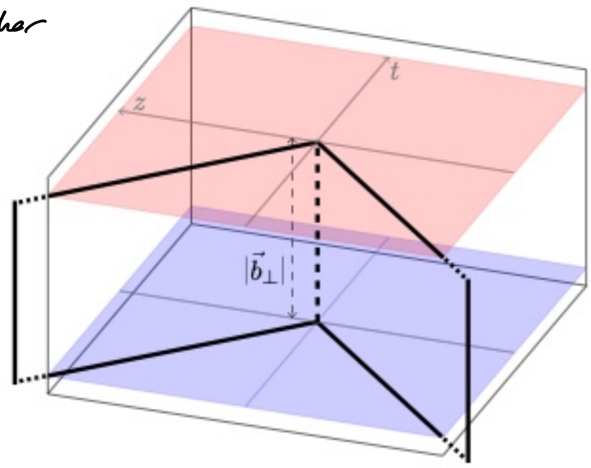
- reduces to the bare collinear PDF for $b_T \to 0$ (only bare)

Soft Fn

$$\tilde{S}(b_T, \epsilon, \tau) = \frac{1}{N_c} \langle 0 | \left[\mathrm{Tr}\, W_{\gg}(b_T) \right]_\tau | 0 \rangle$$

- ~ like 2 staples pasted together

$\longleftrightarrow$ soft approx to high energy collision



- $\tilde{S}(b_T \to 0, \epsilon, \tau) = 1$

Subtraction

$\tilde{S}^{subt} = ?$ depends on choice on τ

- remove infrared double counting between $\tilde{f}^{(u)}$ & $\tilde{S}$

- many choice for τ (§2.4)

i) common $\tilde{S}^{subt} = \tilde{S}$, $\frac{\sqrt{\tilde{S}}}{\tilde{S}} = \frac{1}{\sqrt{\tilde{S}}}$

ii) also eqs $\tilde{S}^{subt} = 1$

Layer 4 Rapidity Regulators $\rightarrow$ lecture #2