TMDs in SIDIS Experiments and More

Jan 20-26, 2022
2022 TMD Winter School
Santa Fe, New Mexico

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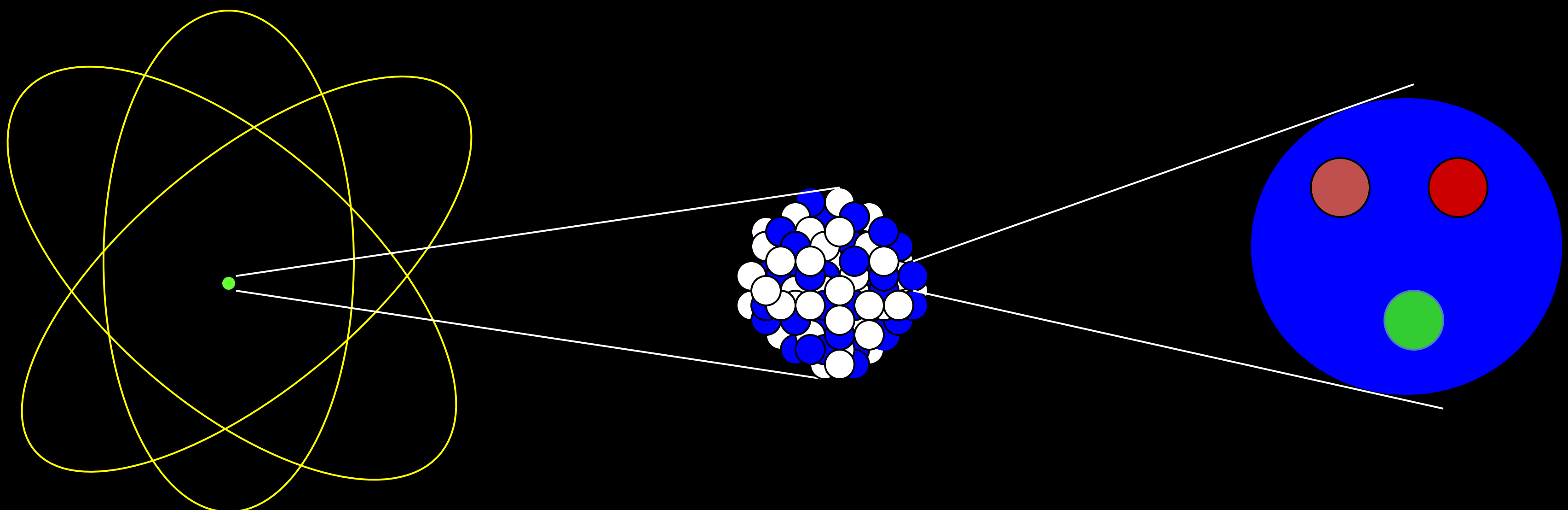
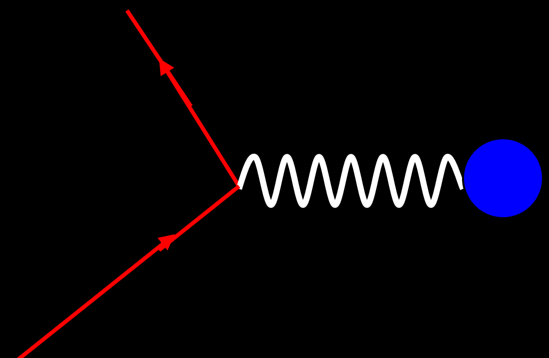
Duke University and BNL

Nuclear physics is the study of the structure of matter

– Most of the mass and energy in the universe around us comes from nuclei and nuclear reactions.
– The nucleus is a unique form of matter in that all the forces of nature are present : (strong, electromagnetic, weak).

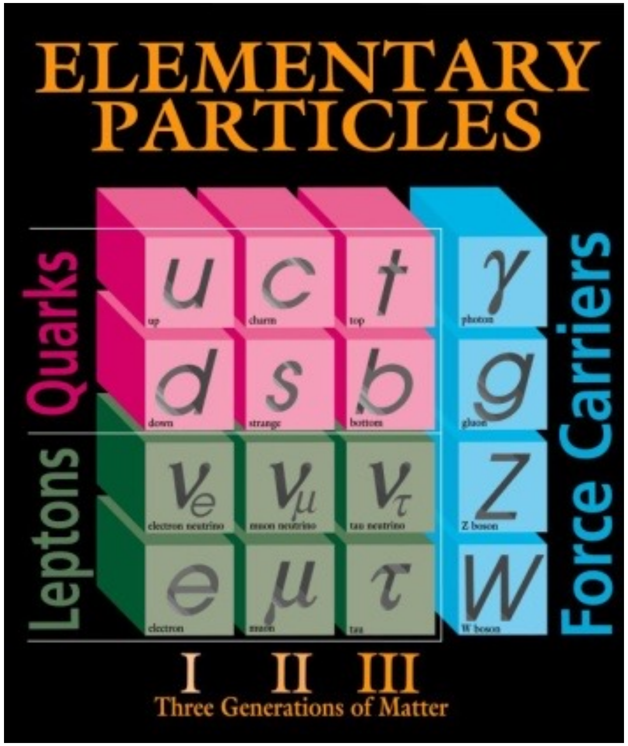


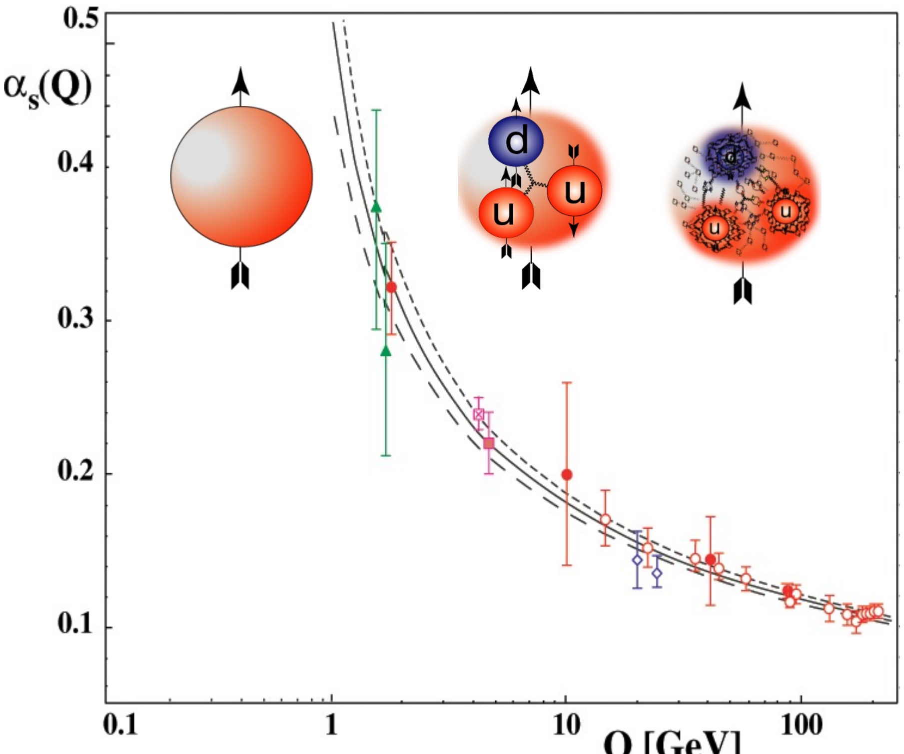
atom: 10^{-10} m   nucleus: 10^{-14} m   nucleon: 10^{-15} m



$$\lambda \approx \frac{h}{p}$$

QCD: still unsolved in non-perturbative region

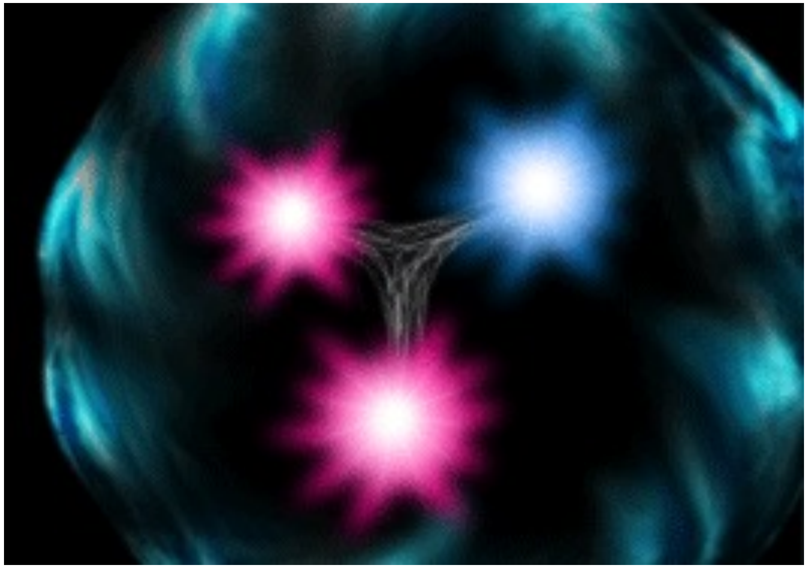


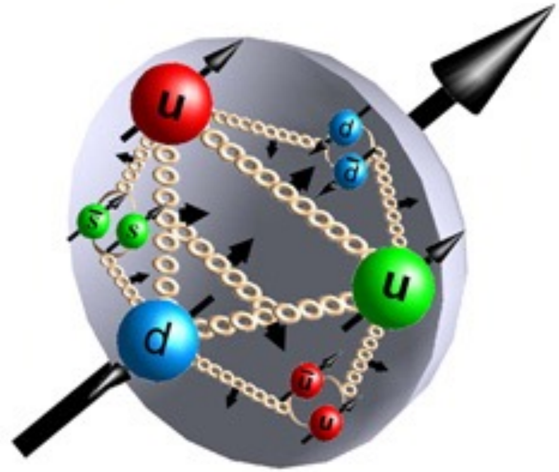


Gauge bosons: gluons (8)

- *2004 Nobel prize for ``asymptotic freedom''*
- *non-perturbative regime QCD ?????*
- *One of the top 10 challenges for physics!*
- *QCD: Important for discovering new physics beyond SM*
- *Nucleon structure is one of the most active areas*

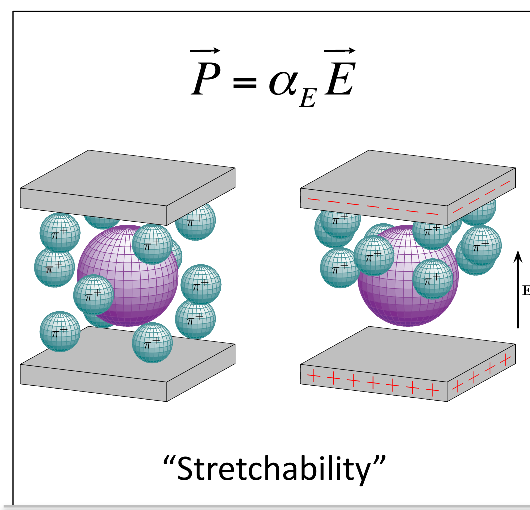
Nucleon Structure





- *Charge and magnetism (current) distribution*
- *Spin distribution*
- *Quark momentum and flavor distribution*
- *Polarizabilities*
- *Strangeness content*
- *Three-dimensional structure*
- *……*

Electric polarizability (α_E)



Magnetic polarizability (β_M)

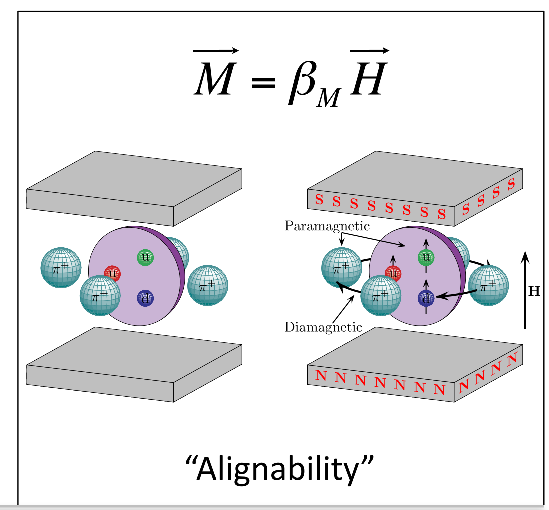


Image credit: P. Martel

Lepton scattering: powerful microscope!

- Clean probe of hadron structure
- Electron point-like particle, electron vertex is well-known from quantum electrodynamics
- One-photon exchange dominates, ***higher-order exchange diagrams are suppressed***
- ***One can vary the wave-length of the probe to view deeper inside the hadron***

Resolution $\propto h/Q$

–Q ≈ 20 MeV λ ≈ 10 fm nucleus

–Q ≈ 200 MeV λ ≈ 1 fm nucleon

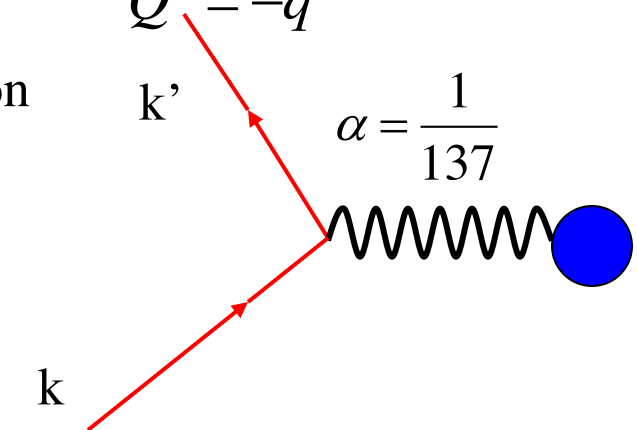
–Q≈ 2 GeV λ ≈ 0.1 fm inside nucleon

–Q≈ 20 GeV λ ≈ 0.01 fm quark

Virtual photon 4-momentum

$$q = k - k' = (\vec{q}, \omega)$$

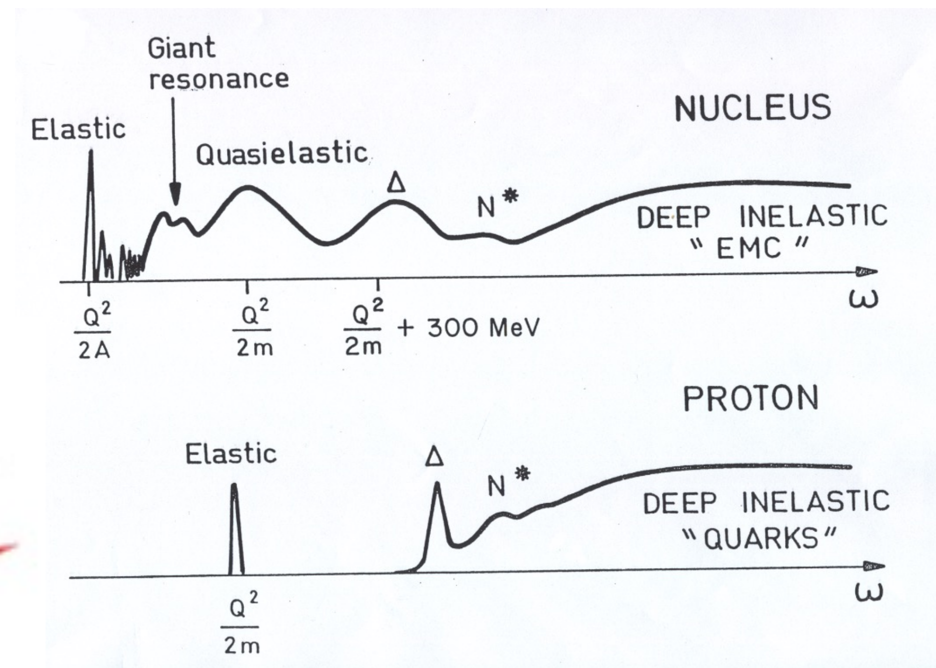
$$Q^2 = -q^2$$



Using electron scattering as example

Electron-nucleon (Nucleus) scattering

- Low Q^2 elastic scattering, $x=1=Q^2/2m\omega$
- As Q^2 increases inelastic effects dominates
- As Q^2 further increases, deep-inelastic scattering off quarks inside



Electron energy transfer

m: mass of the nucleon

What is inside the proton/neutron?

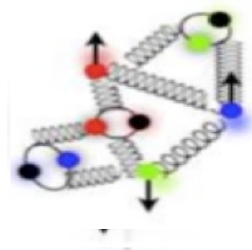
1933: Proton's magnetic moment

Nobel Prize
In Physics 1943

Otto Stern

"for … and for his discovery of the magnetic moment of the proton".

$g \neq 2$



1960: Elastic e-p scattering

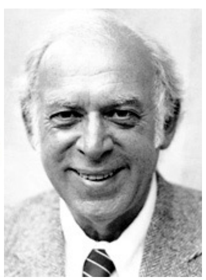
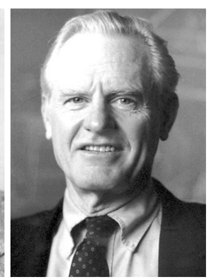
Nobel Prize
In Physics 1961

Robert Hofstadter

"for … and for his thereby achieved discoveries concerning the structure of the nucleons"

Form factors $\rightarrow$ Charge distributions

1969: Deep inelastic e-p scattering

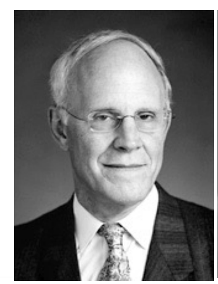

Nobel Prize in Physics 1990

Jerome I. Friedman, Henry W. Kendall, Richard E. Taylor

"for their pioneering investigations concerning deep inelastic scattering of electrons on protons …".

slide credit: Jian-Wei Qiu

1974: QCD Asymptotic Freedom

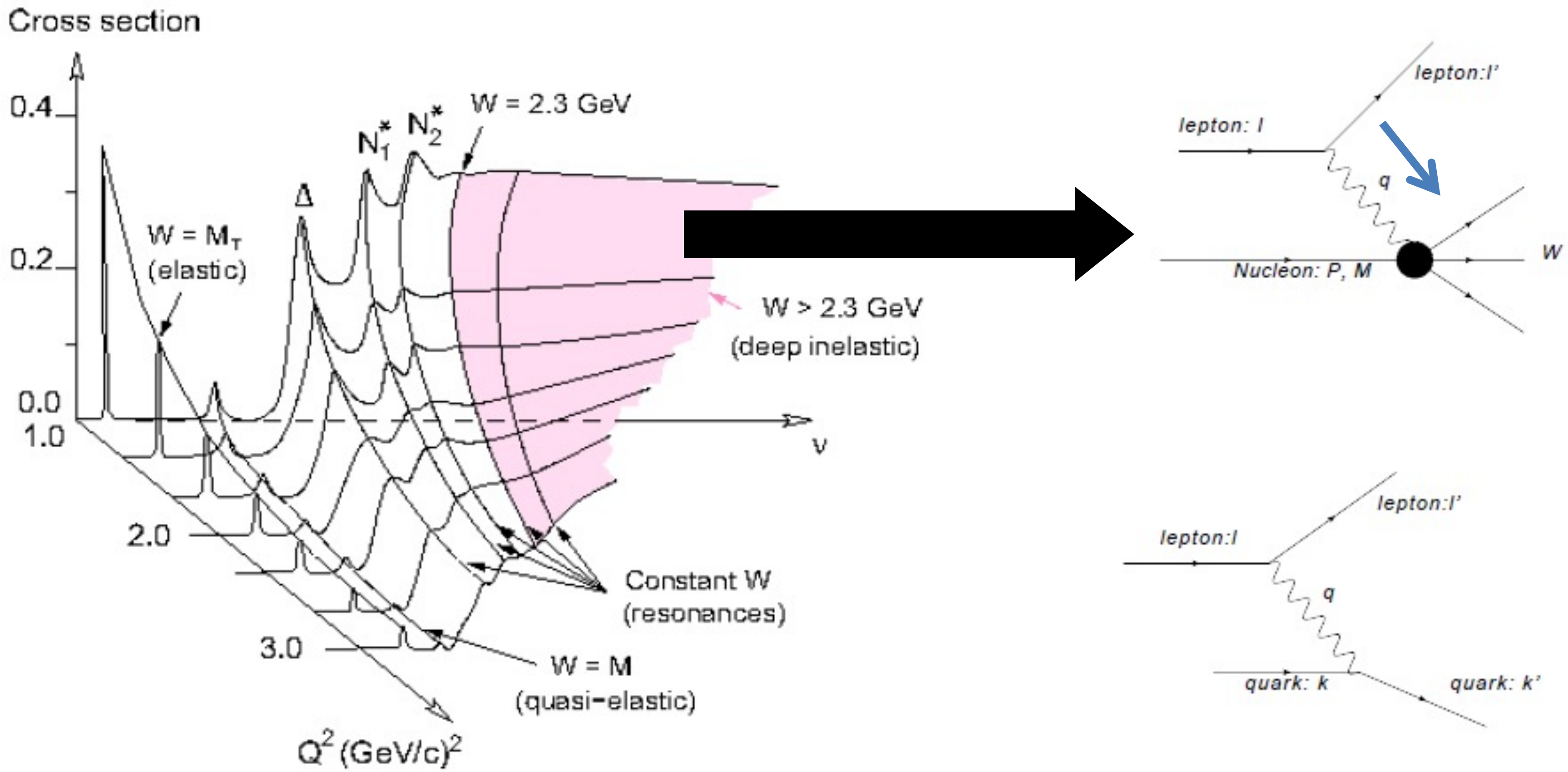


Nobel Prize in Physics 2004

David J. Gross, H. David Politzer, Frank Wilczek

"for the discovery of asymptotic freedom in the theory of the strong interaction".

Lepton Scattering ----- A powerful tool



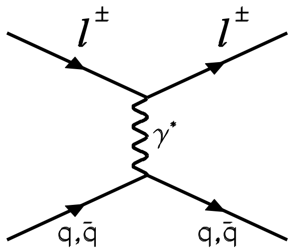
$$Q^2 = -q^2 = -(l - l')^2$$
4-momentum transfer squared: resolution.

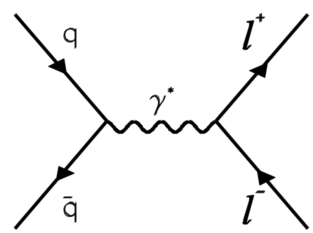
$$\nu = E_l - E_{l'}$$
Energy transfer.

$$x_{Bjorken} = \frac{Q^2}{2m\nu}$$
Longitudinal momentum fraction of parton in the light cone frame.

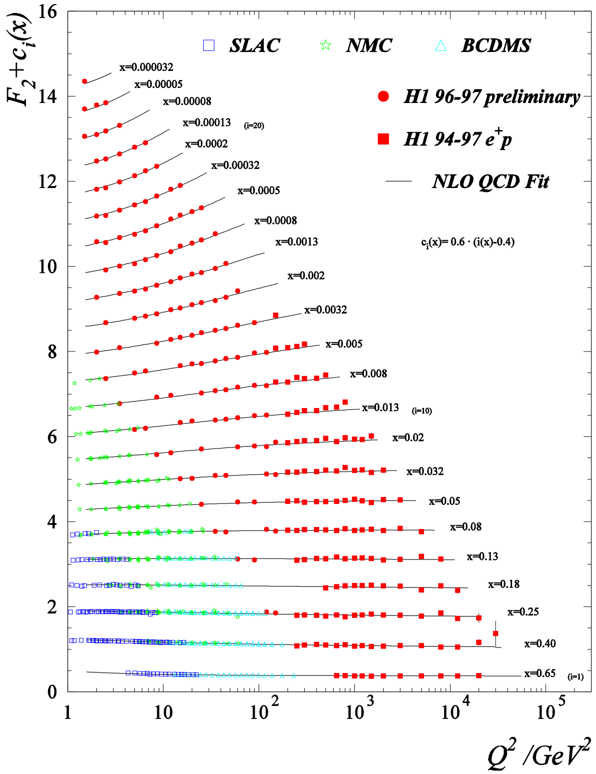
Universal Parton Distribution

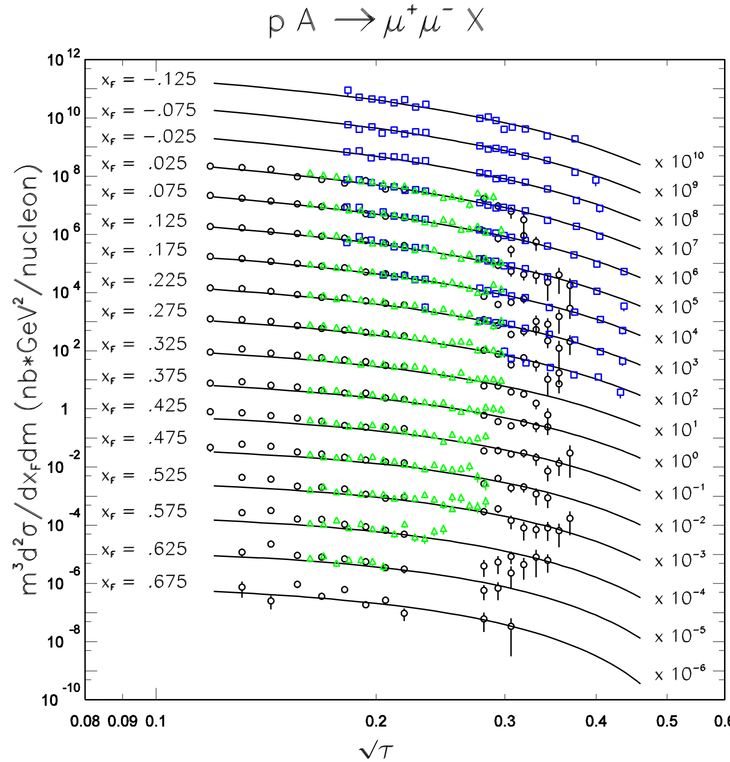
DIS





Drell-Yan



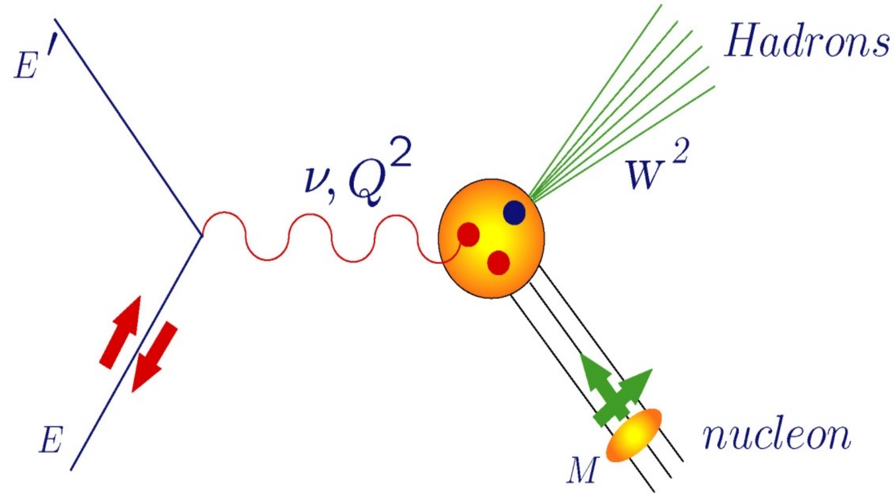


Drell-Yan and DIS cross sections are well described by Next-to-Leading Order QCD

Spin as a knob

- Spin Milestones: (Nature)
  - 1896: Zeeman effect (milestone 1)
  - 1922: Stern-Gerlach experiment (2)
  - 1925: Spinning electron (Uhlenbeck/Goudsmit)(3)
  - 1928: Dirac equation (4)
  - Quantum magnetism (5)
  - 1932: Isospin(6)
  - 1935: Proton anomalous magnetic moment
  - 1940: Spin–statistics connection(7)
  - 1946: Nuclear magnetic resonance (NMR)(8)
  - 1971: Supersymmetry(13)
  - 1973: Magnetic resonance imaging(15)
  - 1980s: "Proton spin crisis"
  - 1990: Functional MRI (19)
  - 1997: Semiconductor spintronics (23)
  - 2000s: "New breakthrough in spin physics"?

Polarized Deep Inelastic Electron Scattering



$x = \dfrac{Q^2}{2M\nu}$ Fraction of nucleon momentum carried by the struck quark

Q^2 = 4-momentum transfer of the virtual photon, ν = energy transfer, θ = scattering angle

- All information about the nucleon vertex is contained in

F_2 and F_1 the unpolarized (spin averaged) structure functions,

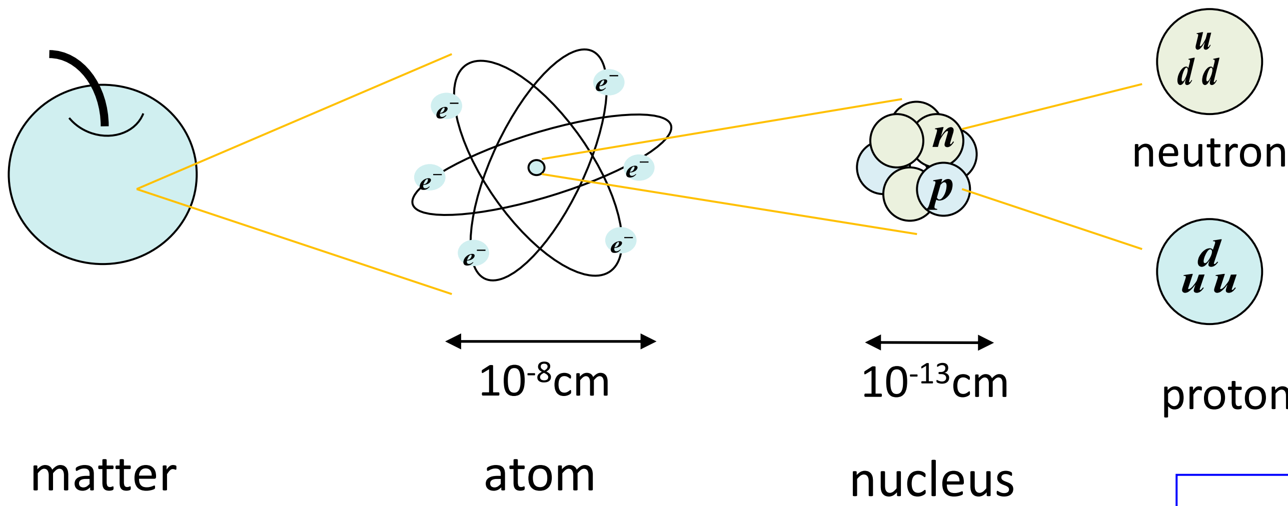
and

g_1 and g_2 the spin dependent structure functions

Brief Introduction on polarized targets and beams

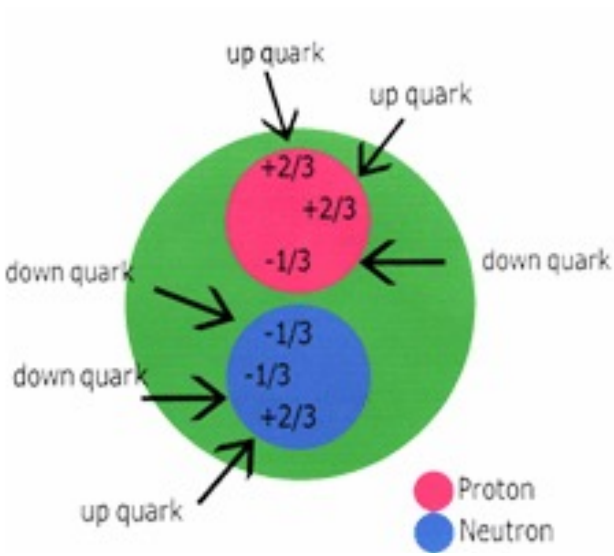
- Dynamical nuclear polarization (DNP)
- Atomic Beam Source Method (ABS)
- Optical pumping technique

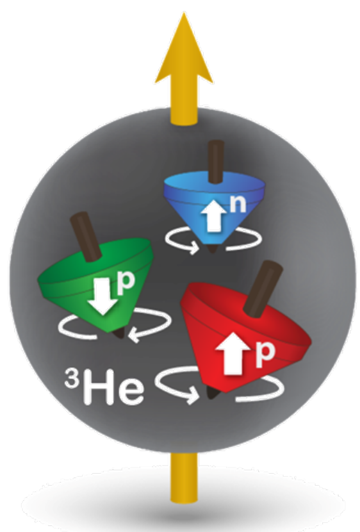
Polarized proton and “neutron” targets

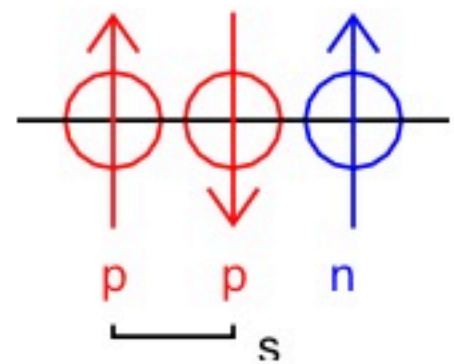


$$\mathbf{n} \rightarrow \mathbf{p} + \mathbf{e}^{-} + \bar{\mathbf{\nu}}$$

β–decay







About 88%

Dynamic Nuclear Polarization

At equilibrium the populations of the Zeeman levels will obey a Boltzmann distribution.

$$N(\uparrow)/N(\downarrow) = \exp\left[\frac{(-2\mu B)}{kT}\right]$$

T = temperature

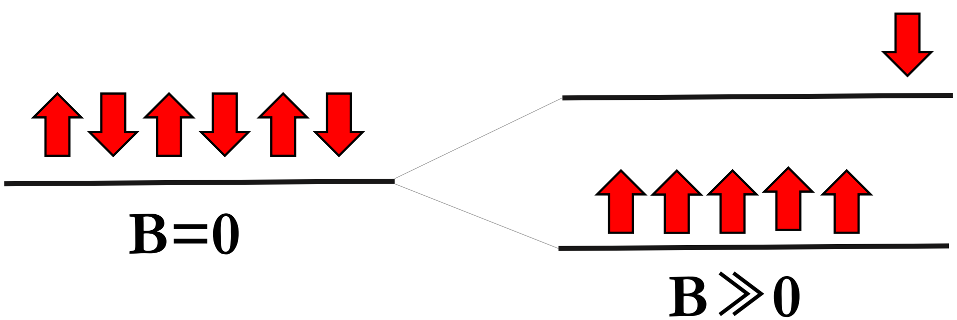
$$P_{te} = \frac{[N(\uparrow) - N(\downarrow)]}{[N(\uparrow) + N(\downarrow)]} = \tanh\left(\frac{\mu B}{kT}\right)$$

Thermal equilibrium polarization, spin ½

The polarization will approach thermal equilibrium with a *1/e* time constant called t_1, *the spin-lattice relaxation time.*

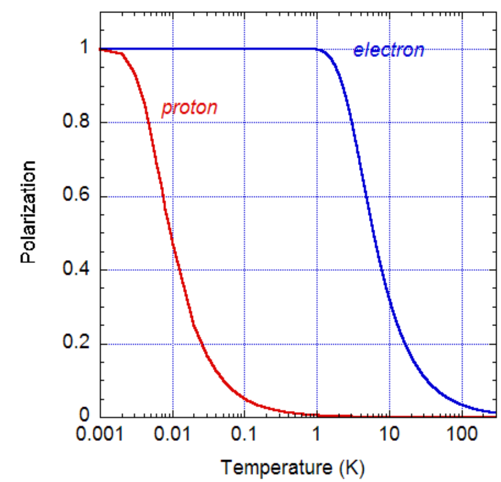
$$P(t) = P_{te}\left[1 - e^{-t/t_1}\right]$$

t_1 depends on the temperature and the magnetic field. Nuclear t1 can be VERY LONG. It's usually determined by paramagnetic impurities.



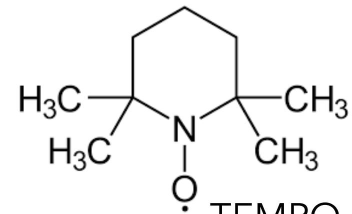
Adapted from C. Keith's Lecture at ODU

Polarization in a 5 Tesla field



Dynamic nuclear polarization

- Implant target material with paramagnetic impurities ~ 10^{19} e^- spins/cc
- Polarize the electrons in the radicals via brute force
- Use microwaves to "transfer" this polarization to nuclei

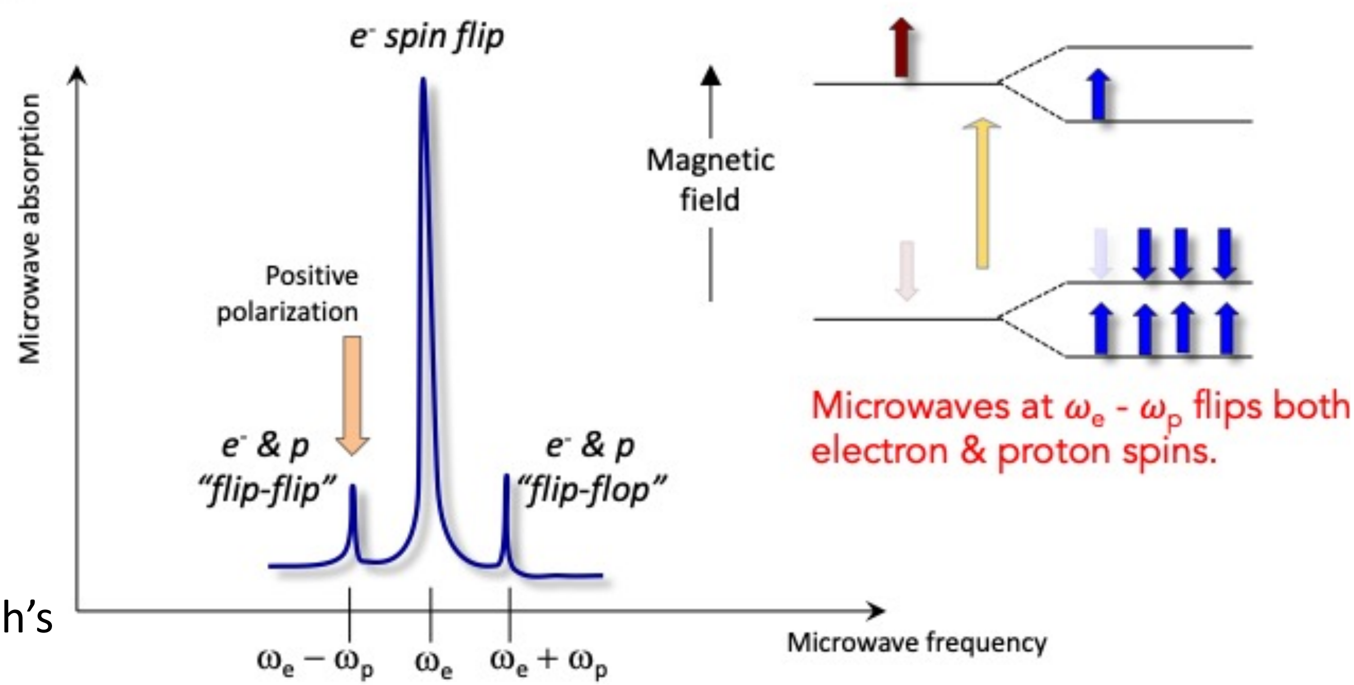


TEMPO

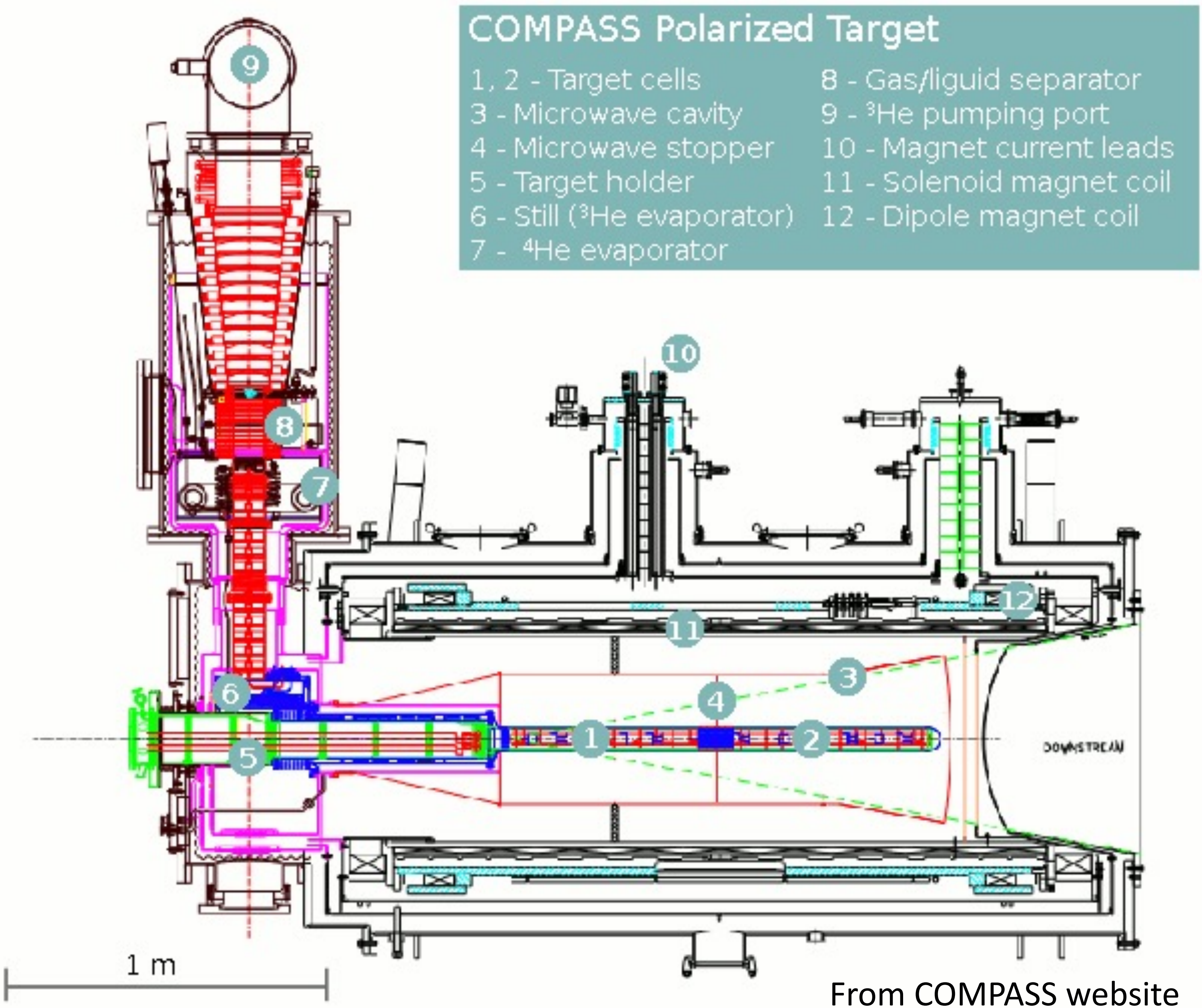
The dipole-dipole interaction between the electrons and nearby nuclear spins permits transitions in which both spins flip.

Electron Spin Resonance of a polarized solid target

- a solid dielectric with ~10^{19} cm^{-3} unpaired electrons
- low temperature
- high field



Adapted from C. Keith's Lecture at ODU

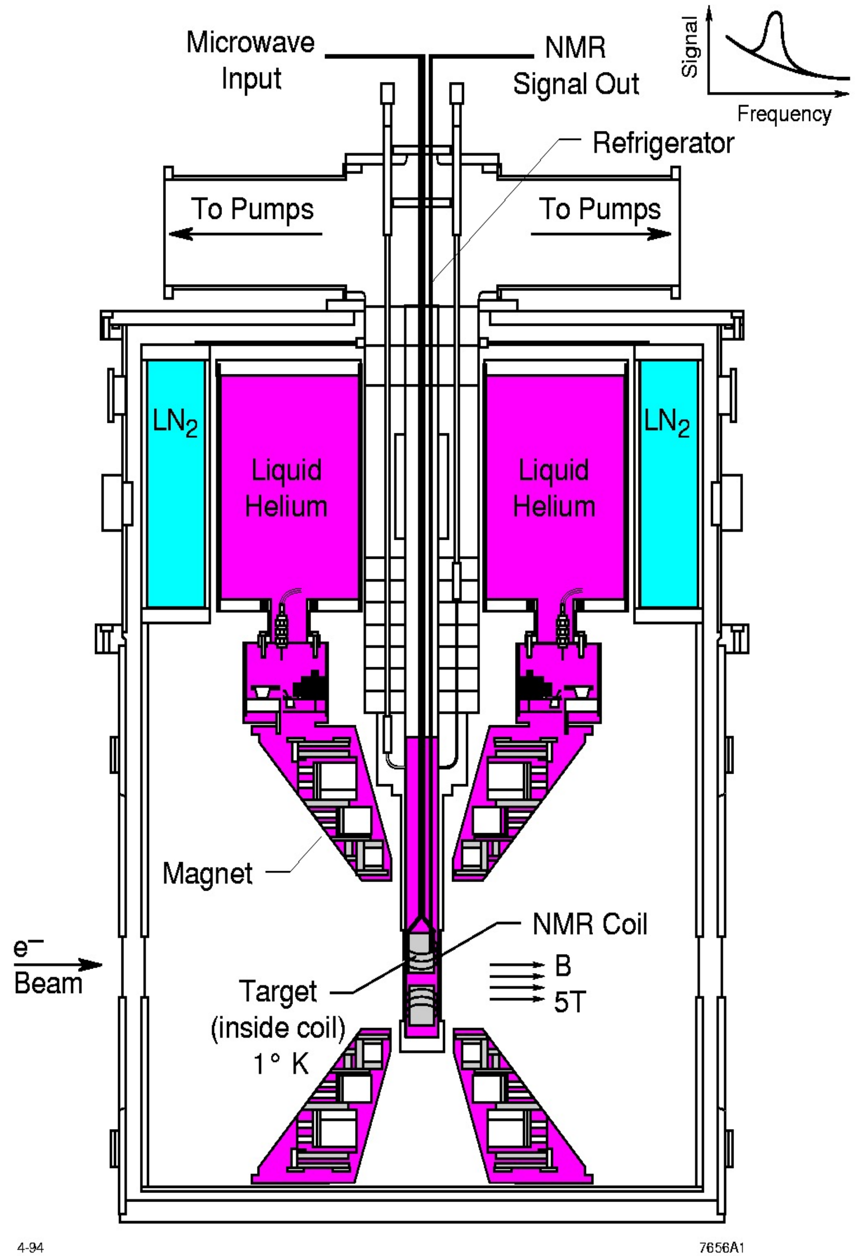


From COMPASS website

Polarized proton/deuteron target

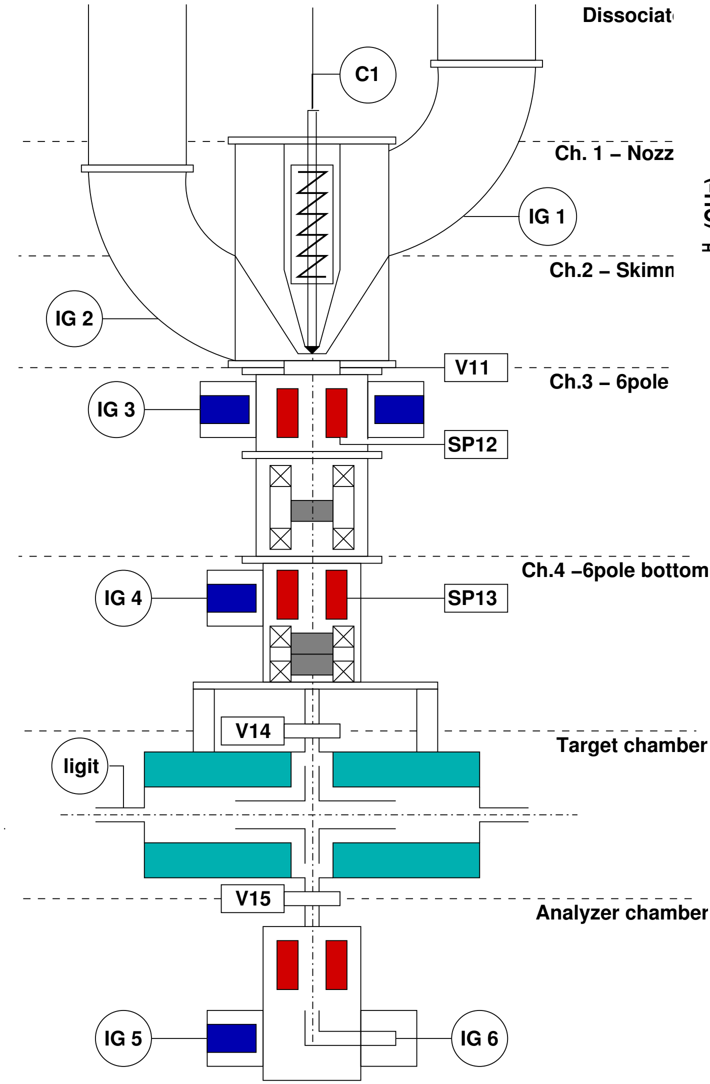
- Polarized NH_3/ND_3 targets
- Dynamical Nuclear Polarization

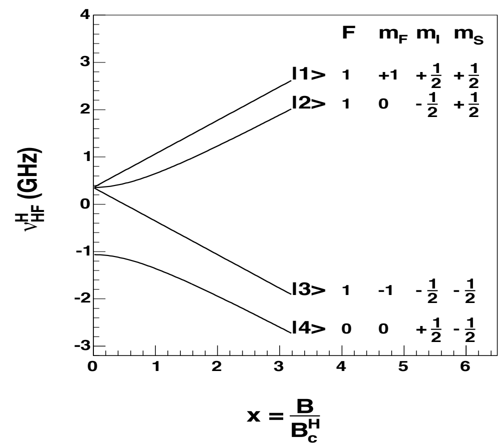
- In-beam average polarization
  70-90% for p
  30-40% for d
- Luminosity up to
  ~ 10^{35} (Hall C)
  ~ 10^{34} (Hall B)

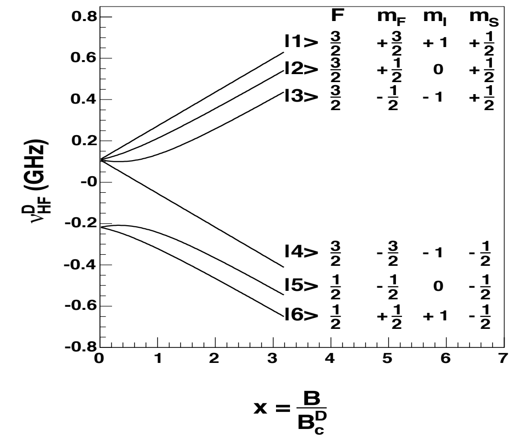


Adapted from C. Keith's Lecture at ODU

Atomic Beam Source Method for H/D



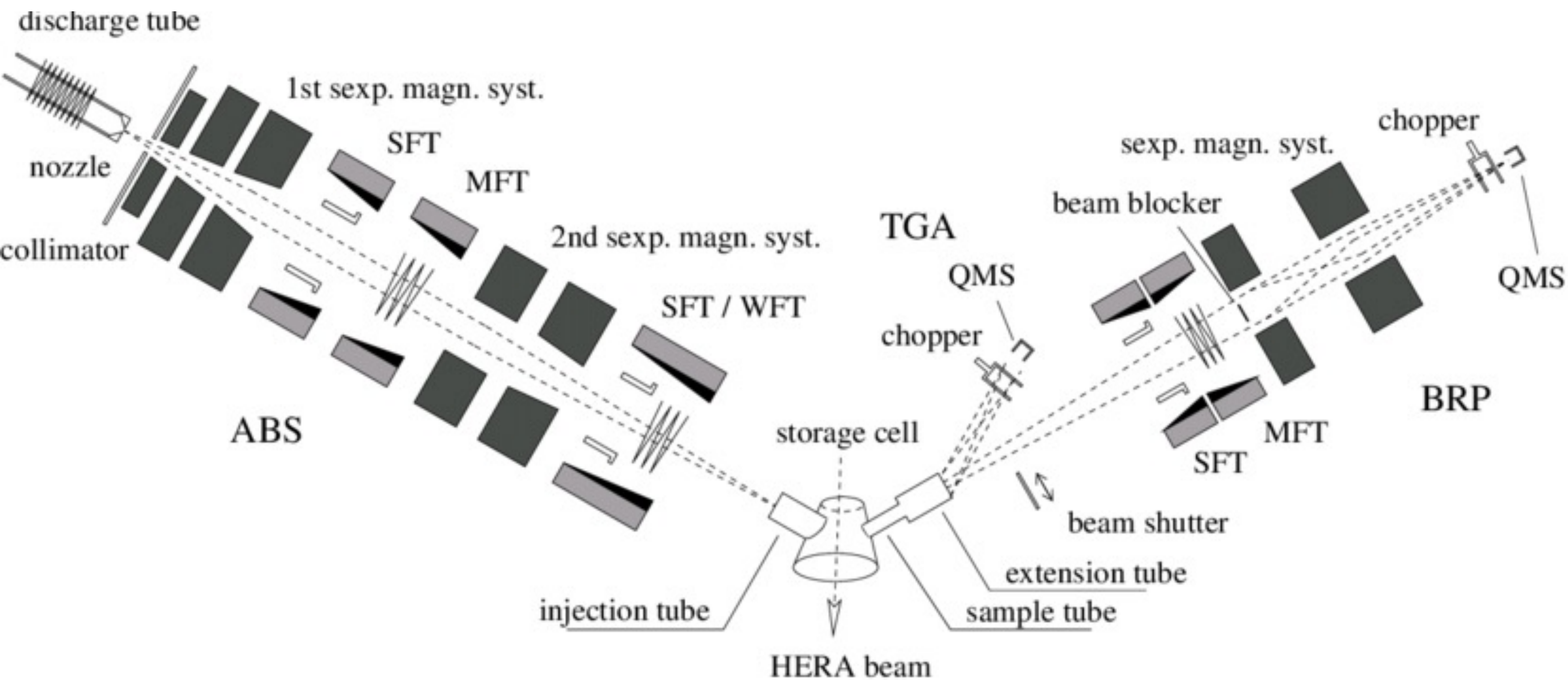




$$\begin{pmatrix}n_1\\n_2\\n_3\\n_4\end{pmatrix} \xrightarrow{6-pol} \begin{pmatrix}n_1\\n_2\\0\\0\end{pmatrix} \xrightarrow{MFT\ \ \pi_{23}} \begin{pmatrix}n_1\\0\\n_2\\0\end{pmatrix} \xrightarrow{6-pol} \begin{pmatrix}n_1\\0\\0\\0\end{pmatrix} \xrightarrow{WFT\ \ \pi_{13}} \begin{pmatrix}0\\0\\n_1\\0\end{pmatrix},$$

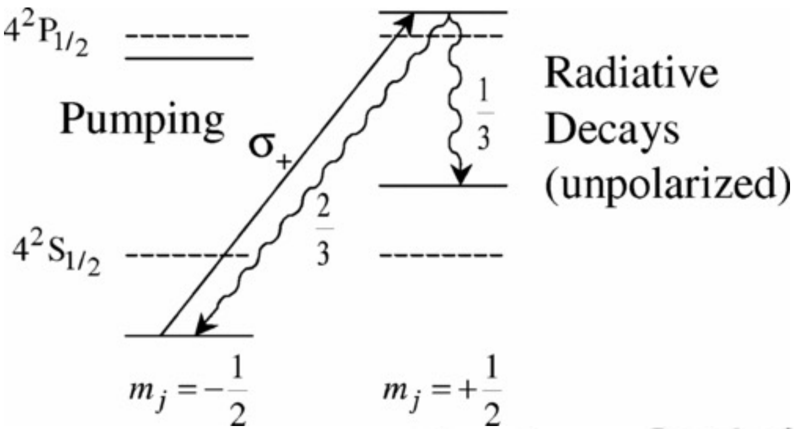
C. Crawford thesis, Ph.D. MIT (2005);
E. Steffen and W. Haeberli, Rep. Prog. Phys. **66**, 1887 (2003)

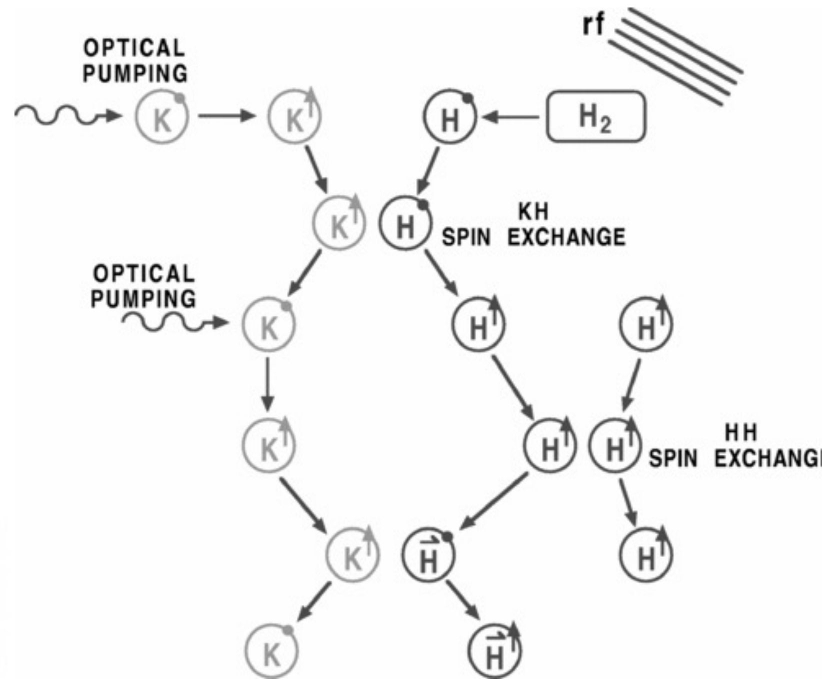
HERMES Polarized H/D target – Atomic Beam Source

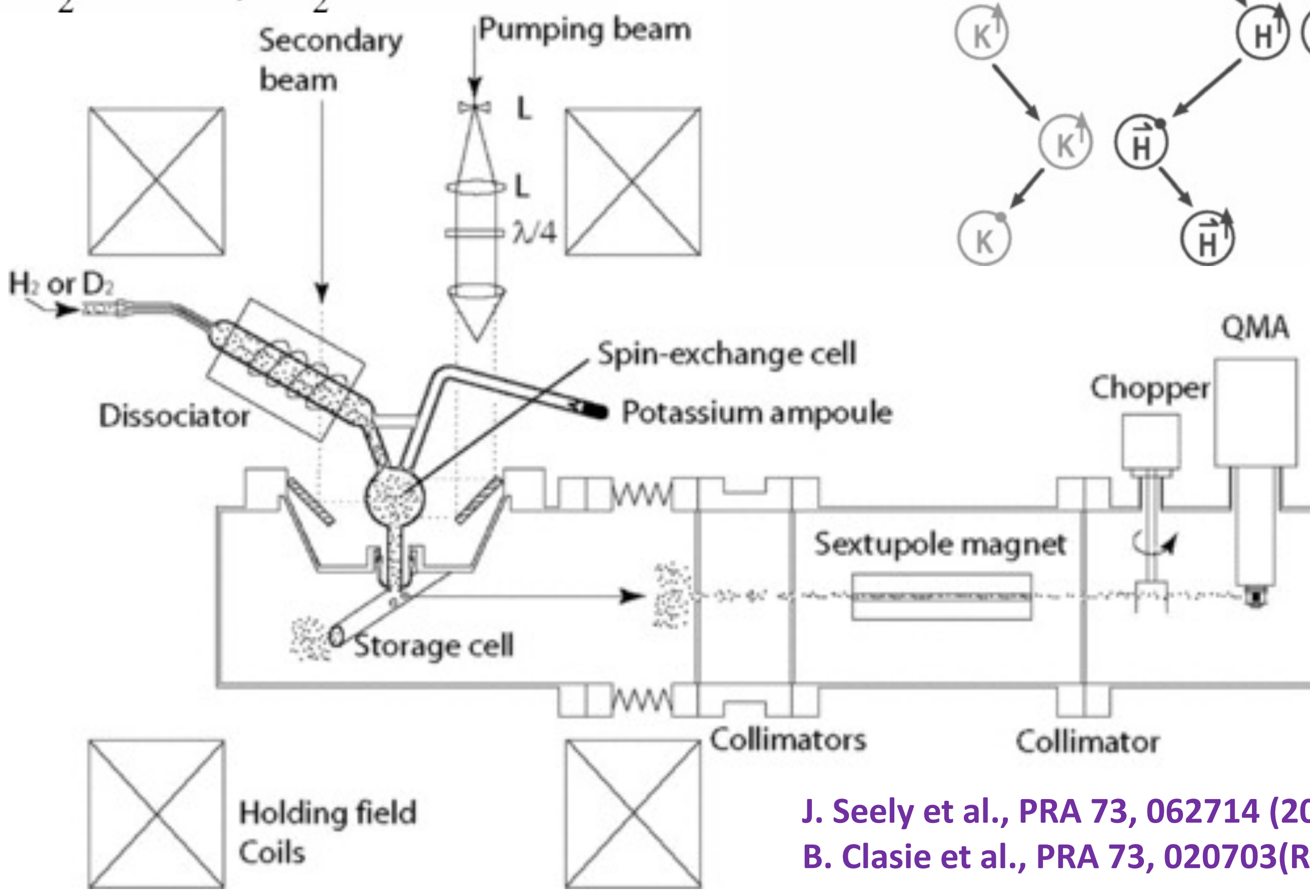


A. Airapetian *et al.*, NIMA, **540**, 68-101 (2005)

Laser-Driven Polarized H/D Target



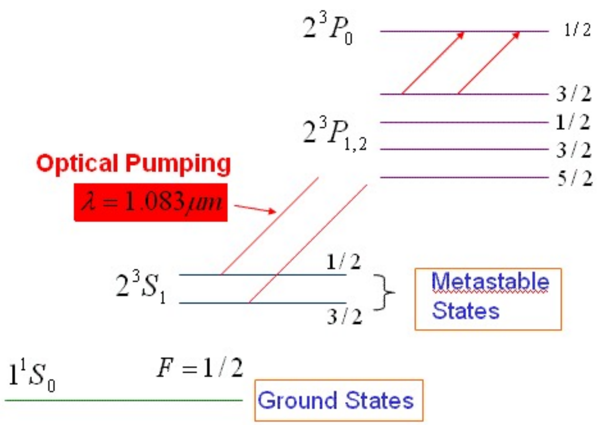




J. Seely et al., PRA 73, 062714 (2006);
B. Clasie et al., PRA 73, 020703(R) (2006)

Polarized ^{3}He Targets Pioneered at MIT-Bates

Metastability-exchange optical pumping

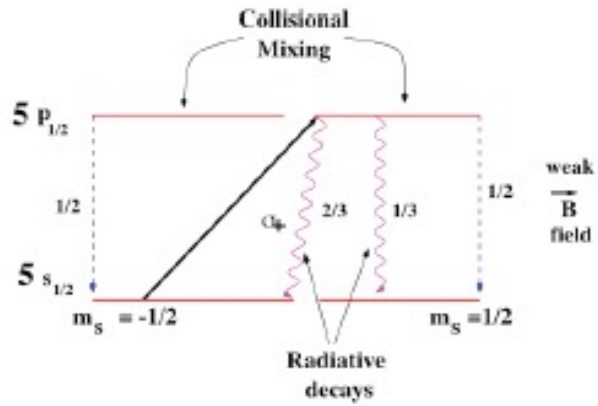


C.E.Woodward *et al.*, PRL **65**, 698 (1990)

H. Gao *et al.*, PRC **50,** R546 (1994)

J.-O. Hansen *et al.,* PRL74, 654 (1995)

Spin-exchange optical pumping



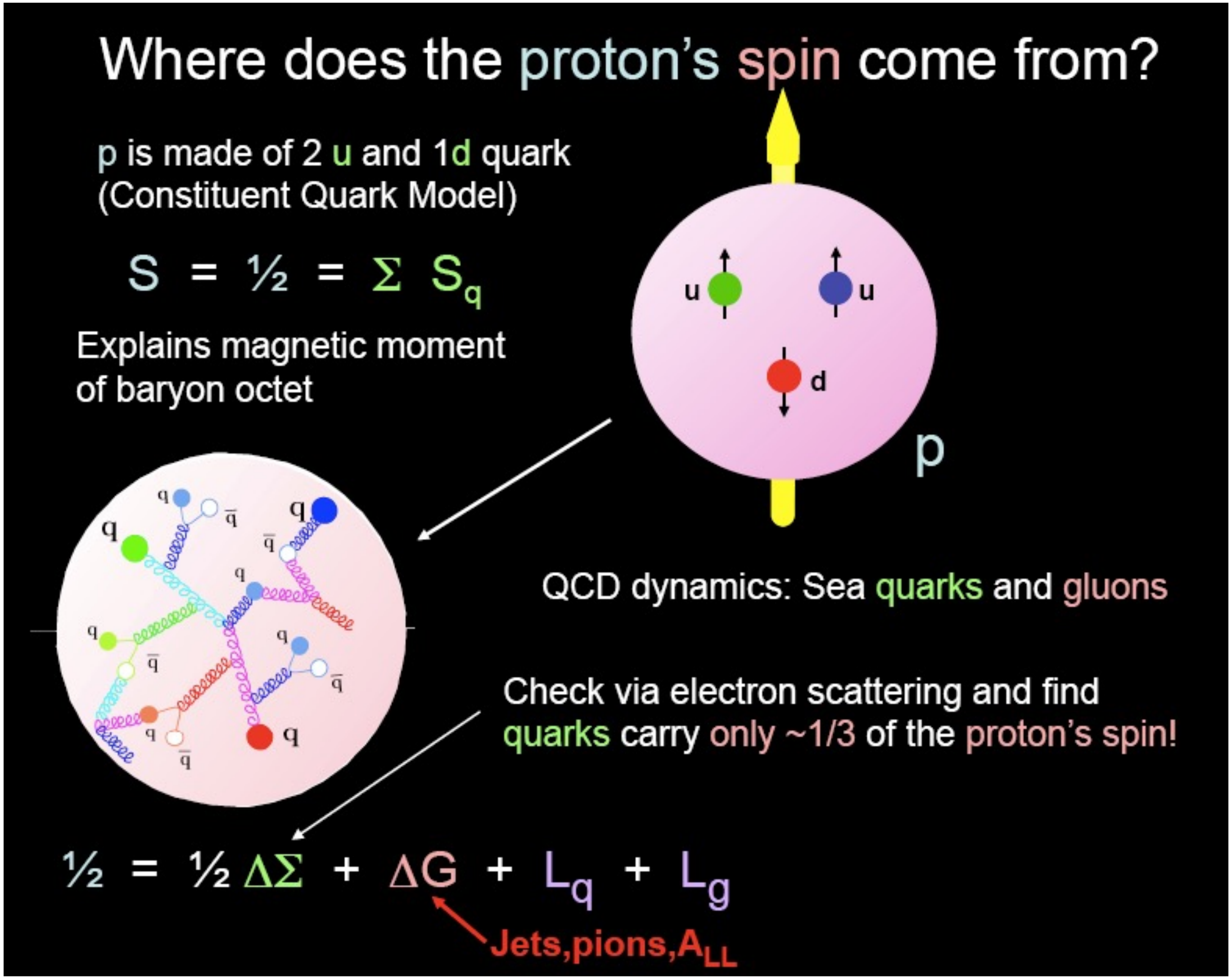
A.K.Thompson *et al.*, PRL**68**, 2901(1992)

MIT-Bates

Taken in June 93

Spin structure of the nucleon

- 1980s: "Proton spin crisis" (original EMC result from CERN)



Impressive experimental progress in QCD spin physics in the last 30+ years

- **Inclusive spin-dependent DIS**
  - CERN: EMC, SMC, COMPASS
  - SLAC: E80, E142, E143, E154, E155
  - DESY: HERMES
  - JLab: Hall A, B and C

- **Semi-inclusive DIS**
  - SMC, COMPASS
  - HERMES, JLab

- **Polarized pp collisions**
  - BNL: PHENIX & STAR
  - FNAL: POL. DY

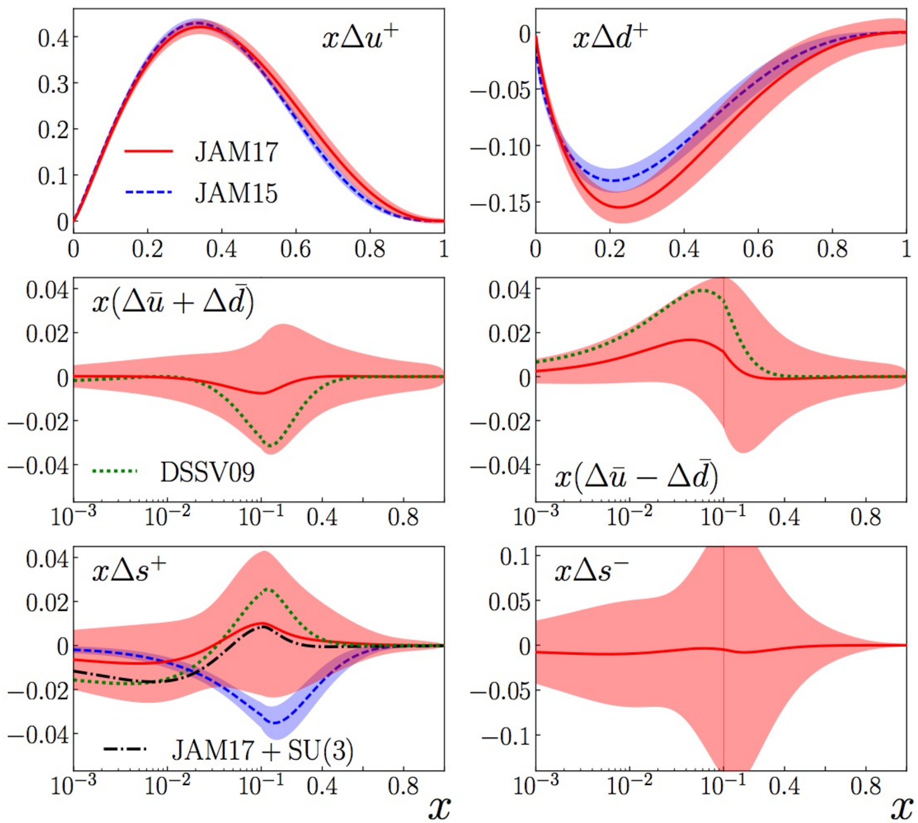
- **e+e- collisions**
  - KEK: Belle
  - BaBar
  - BESIII

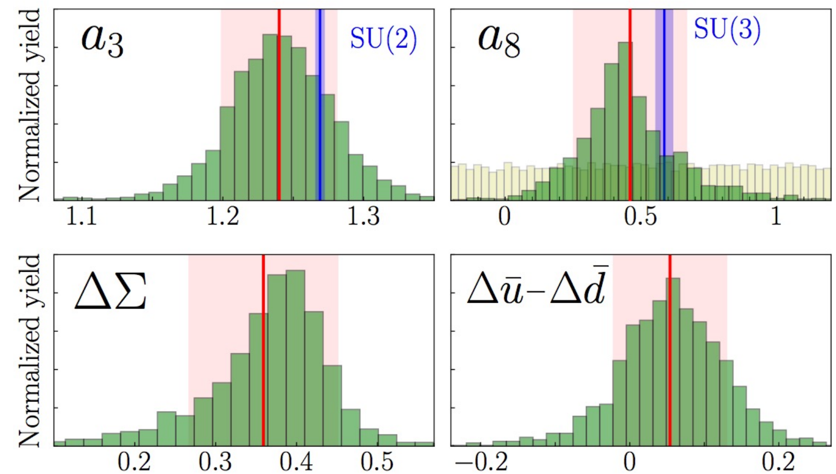
Adapted from Z. Meziani's

Ji, Yuan and Zhao, Nature Review Physics 3, 65 (2021)

Global Analysis: Polarized PDF

Global analysis of spin-dependent parton distribution functions





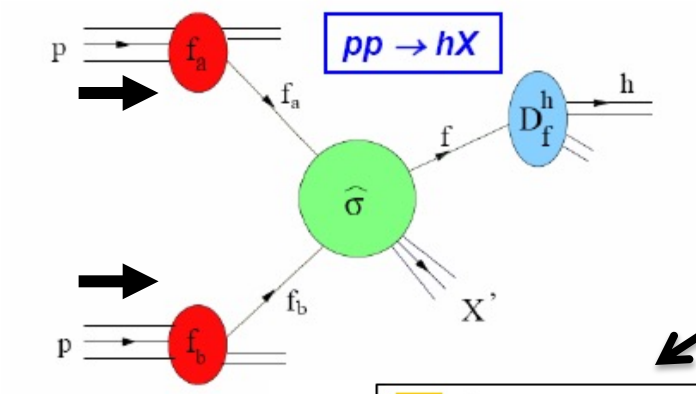
$$\Delta\Sigma = 0.36(9)$$

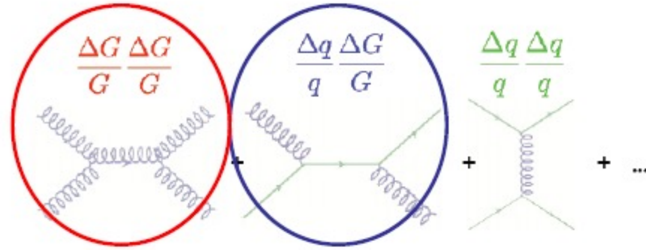
$$\Delta s^+ = -0.03(10)$$

$$(\mu^2 = 1\ \text{GeV}^2)$$

J.J. Ethier *et al.* (JAM Collaboration), Phys. Rev. Lett. 119, 132001 (2017).

Measurement of the gluon polarization Δg at RHIC



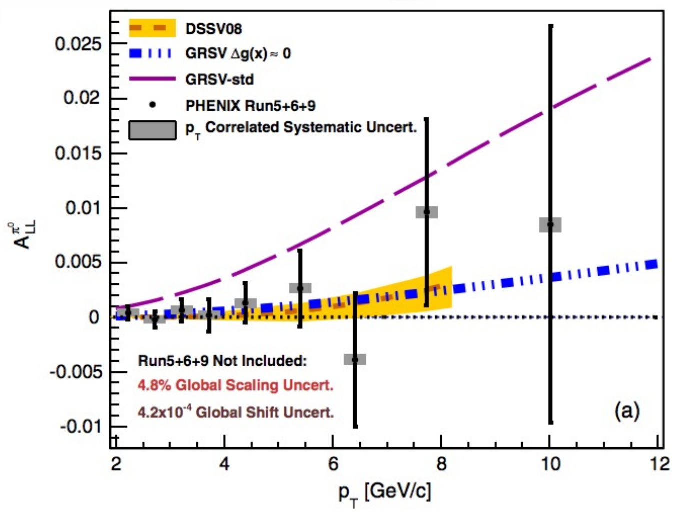


Dominates at low p_T

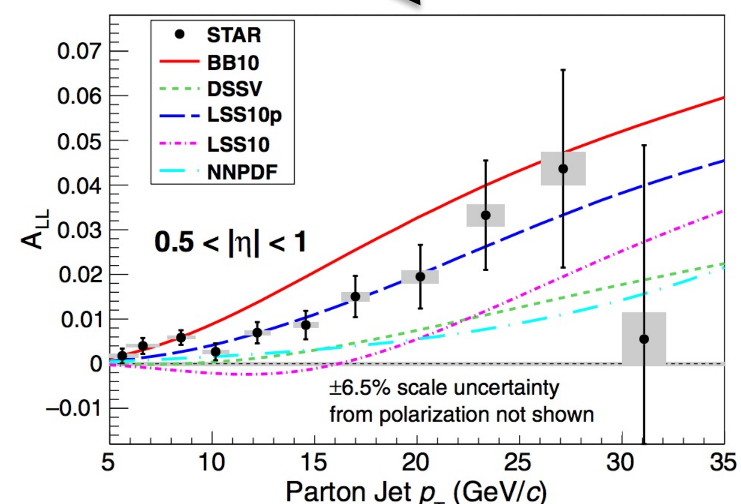
Dominates at high p_T

D. de Florian *et al*,
PRL 113 (2014) 012001

E. Nocera et al,
NPB 887 (2014) 276



Phys. Rev. D 90 (2014) 012007



Phys. Rev. Lett. 115 (2015) 092002

$\int_{0.05}^{1} dx\Delta g(x,Q^2=10GeV^2) = 0.20^{+.06}_{-.07}$ DSSV++

$\int_{0.05}^{0.2} dx\Delta g(x,Q^2=10GeV^2) = 0.17+-0.06$ NNPDFpol1.1

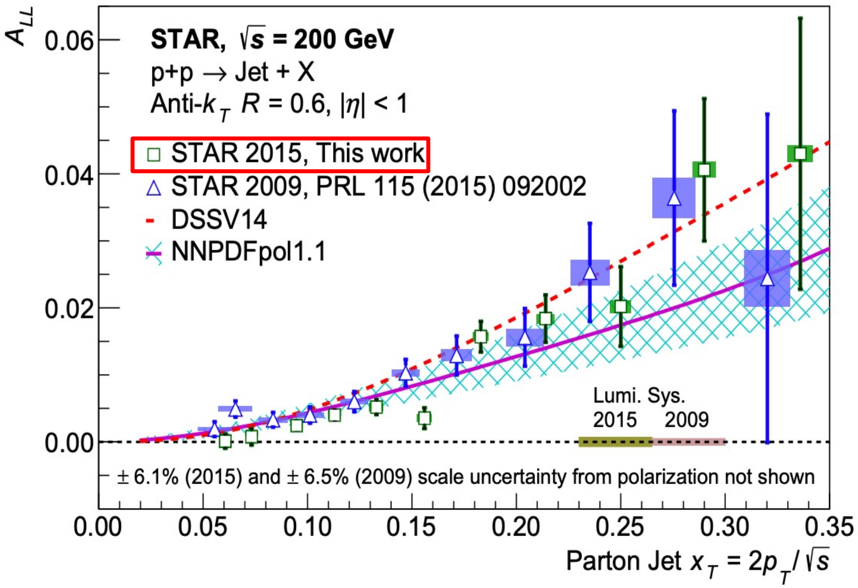
$\int_{0.001}^{0.8} dx\Delta g(x,Q^2 = 1\ GeV^2) = 0.5+-0.4$ JAM15

Surrow et al on sea quark spin
from W production at RHIC

Results from PHENIX , 40% gluon to 1/2

Helicity PDFs: ΔG

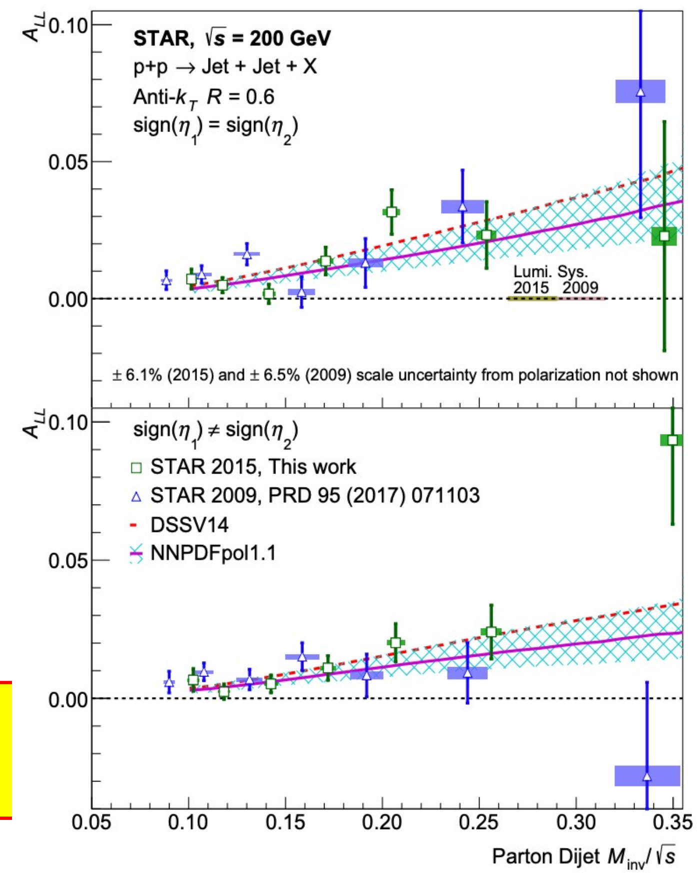
PRD 103 (2021) L091103



Newly published results:

- Largest 200 GeV longitudinally polarized pp dataset (2015); improved both statistical and systematic uncertainties
- Include jet and di-jet A_{LL} : constrain gluon polarization for x_T > 0.05

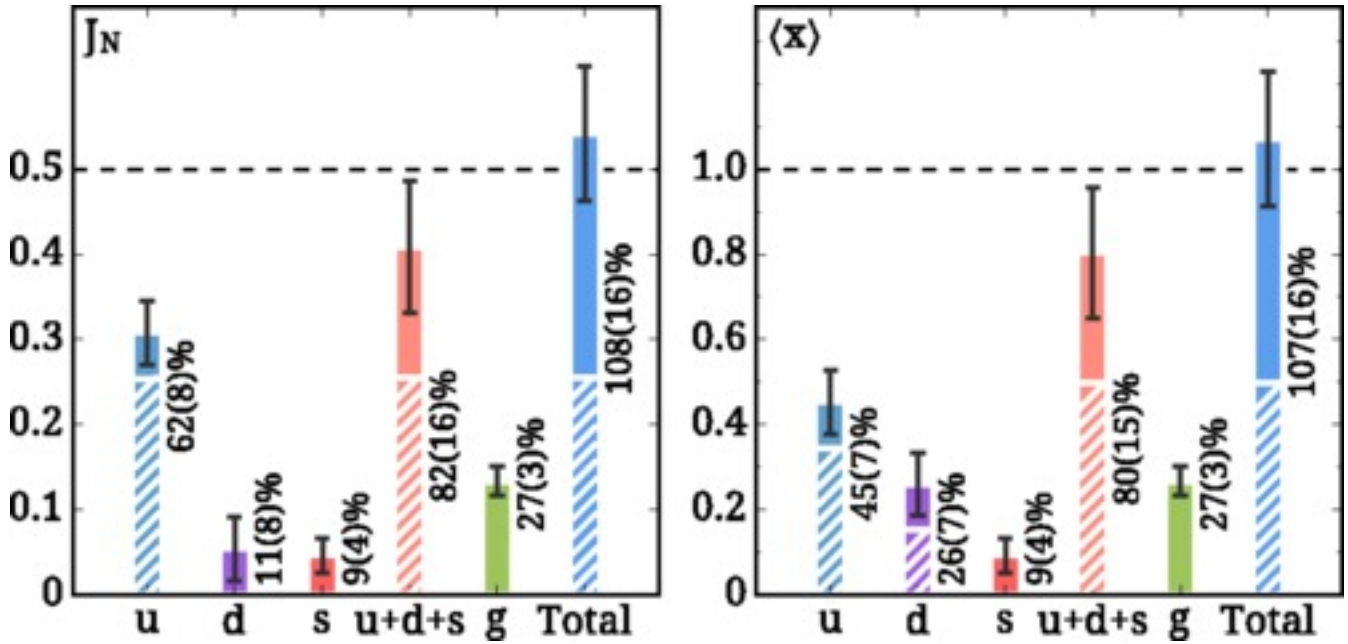
Di-jets: Much narrower ranges of initial state partonic momentum fraction tested; different topologies enhance sensitivity of the data to selected x



This result will reduce the uncertainty of gluon polarization for x_T > 0.05 if included in global fits

Proton Spin From Lattice QCD

Lattice QCD calculation of quark and gluon angular momentum contributions to proton spin



$L_{u+d+s}=0.207(64)_{stat}(45)_{syst}$

$J_g=0.133(11)_{stat}(14)_{syst}$

$J_{u+d+s}=0.408(61)_{stat}(48)_{syst}$

(MS =2 GeV)

$\mathbf{J_N=0.54(6)_{stat}(5)_{syst}}$

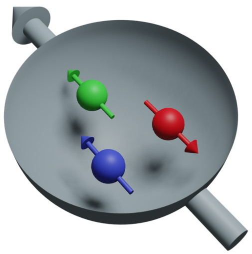
C. Alexandrou *et al.*, Phys. Rev. Lett. 119, 142002 (2017).

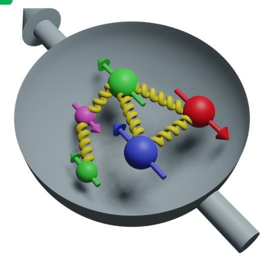
Y.-B. Yang *et al.*, Phys. Rev. Lett. 118, 102001 (2017).

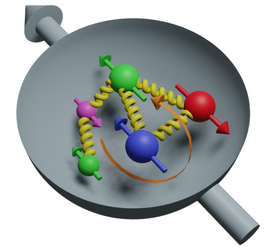
The incomplete nucleon: spin puzzle

$$\frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma}_{} + \underbrace{\Delta G}_{} + \underbrace{(L_q + L_g)}_{}$$

Jaffe-Manohar, 90
Ji, 96







Quark helicity
Best known

$$\frac{1}{2}\int dx\,(\Delta u + \Delta\overline{u} + \Delta d + \Delta\overline{d} + \Delta s + \Delta\overline{s})$$

$$\sim 30\%$$

Gluon helicity
Start to know

$$\Delta G = \int dx \Delta g(x)$$

Orbital Angular Momentum of quarks and gluons
Little known

With larger uncertainty → **~40% -- RHIC Spin data at $Q^2 = 10$ GeV^2**

Net effect of partons' transverse motion?

Lattice QCD →

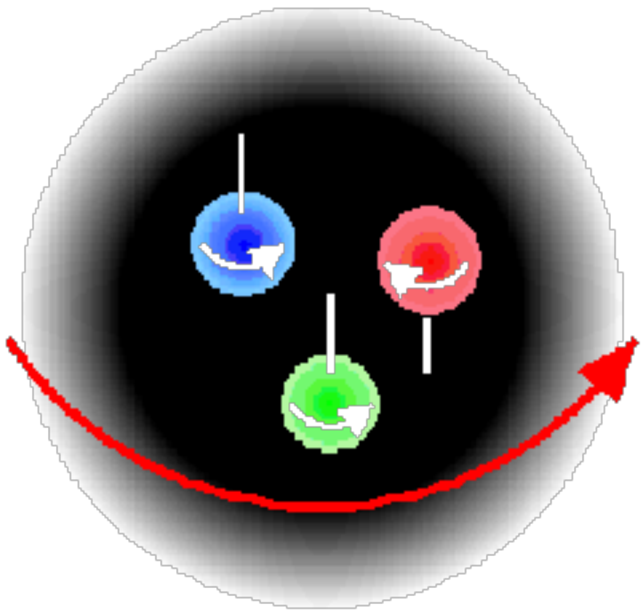
$$L_{u+d+s} = 0.207(64)_{stat}(45)_{syst}$$
$$J_{u+d+s} = 0.408(61)_{stat}(48)_{syst}$$
$$J_g = 0.133(11)_{stat}(14)_{syst}$$

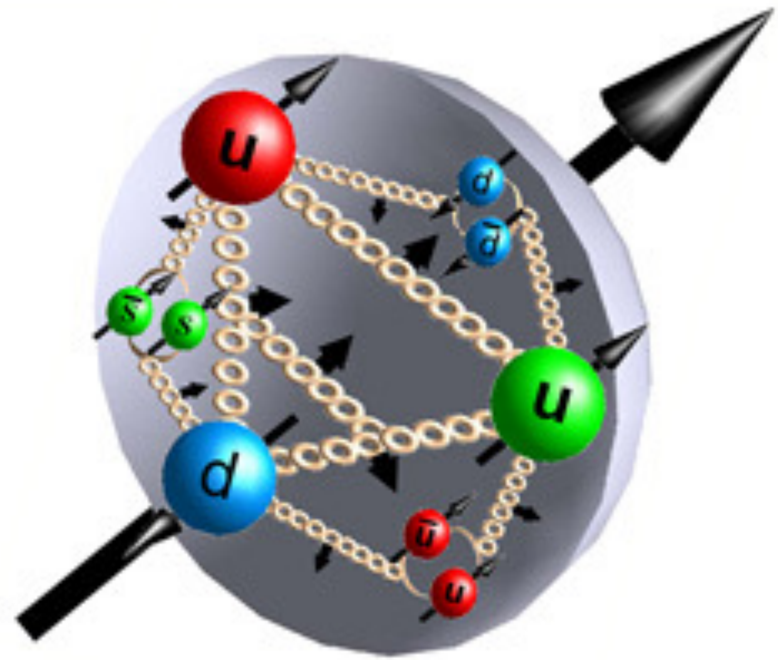
$$J_N = 0.54(6)_{stat}(5)_{syst}$$
(MS = 2 GeV)

C. Alexandrou et al., PRL 119, 142002 (2017).

The Incomplete Nucleon: Spin Puzzle

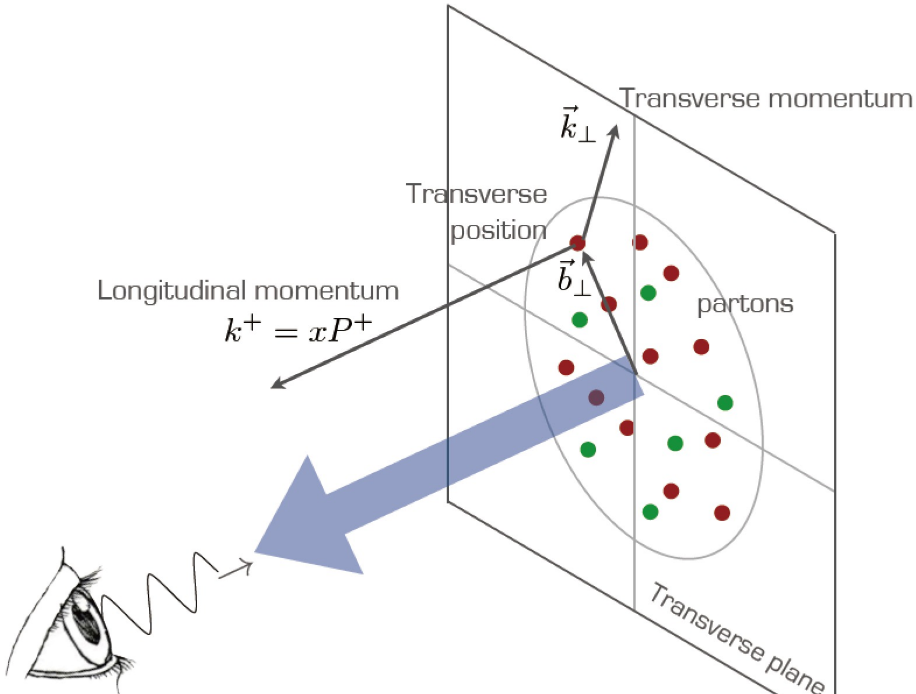
[X. Ji, 1997]

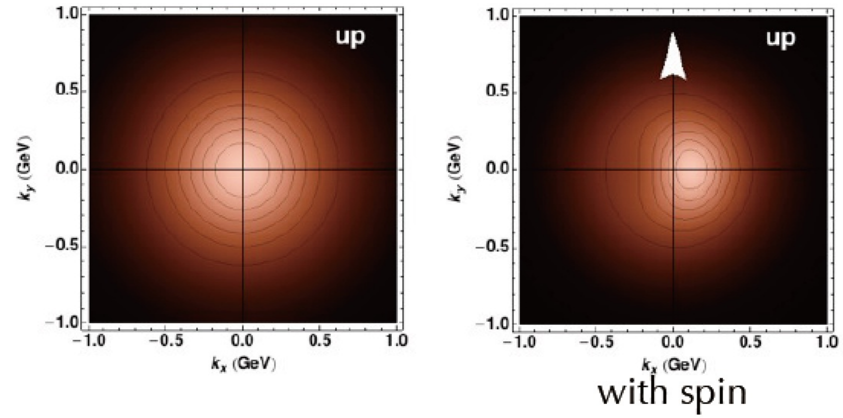


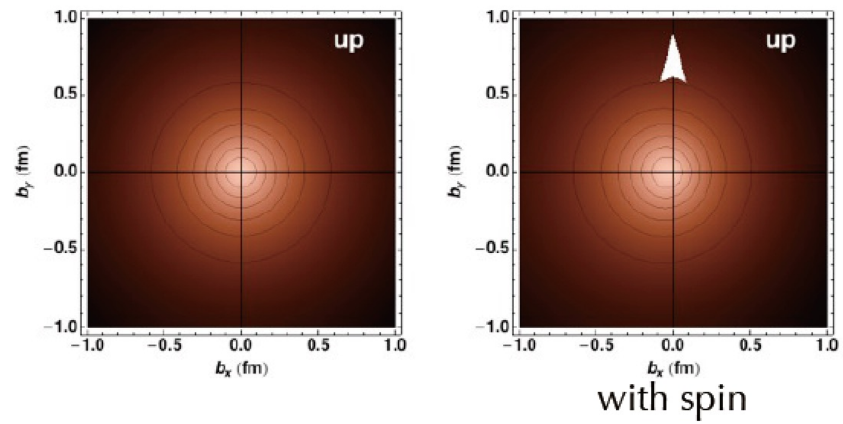


Orbital angular momentum of quarks and gluons
is important

Orbital motion - Nucleon Structure from 1D to 3D





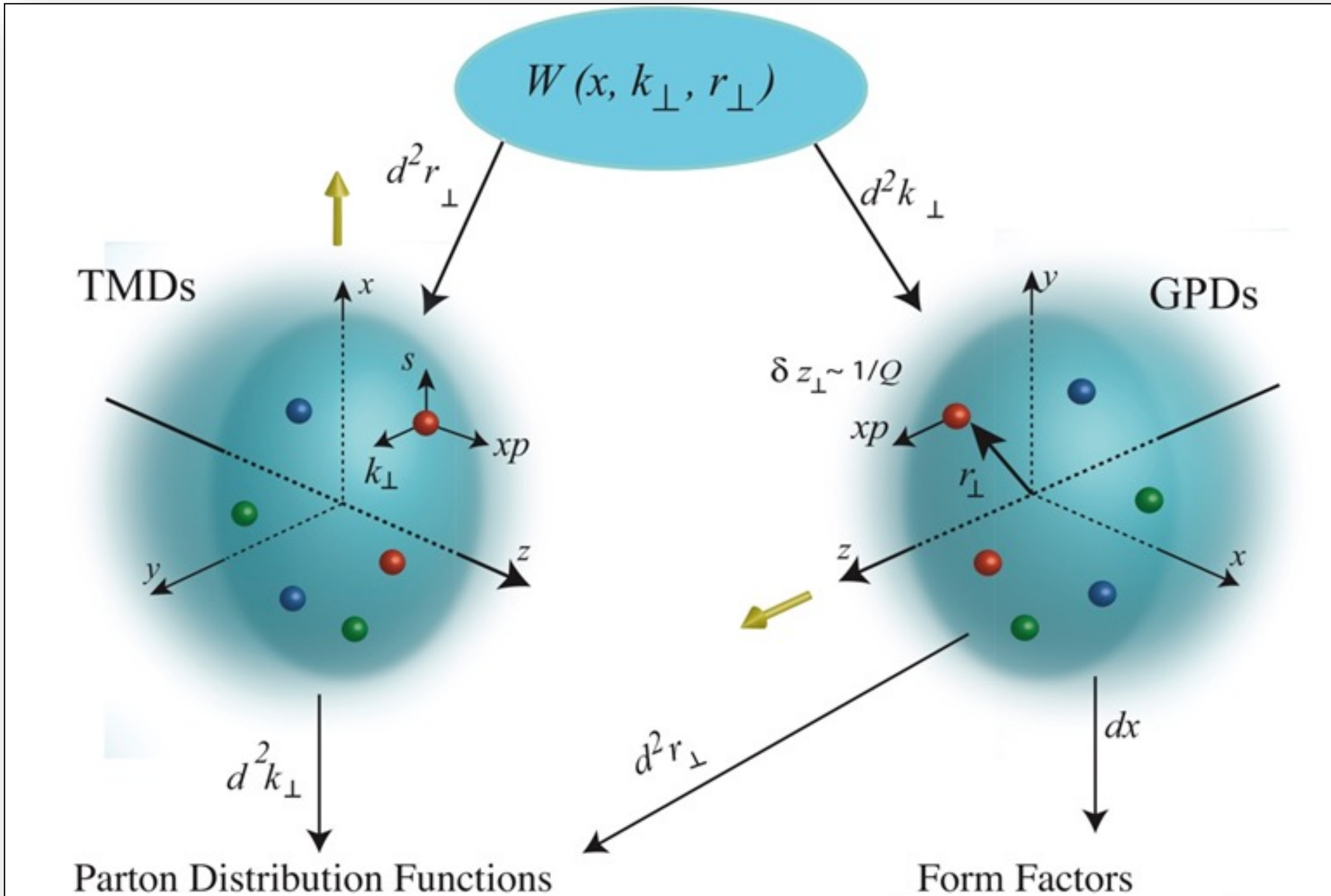


Generalized parton distribution (GPD)
Transverse momentum dependent parton distribution (TMD)

[Bacchetta's talk (2016)]

Nucleon Structure from 1D to 3D & orbital motion

5-D Wigner distribution



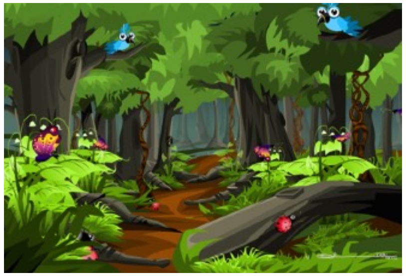
Generalized parton distribution (GPD)

Transverse momentum dependent parton distribution (TMD)

Image from J. Dudek et al., EPJA 48,187 (2012)

X.D. Ji, PRL91, 062001 (2003);
Belitsky, Ji, Yuan, PRD69,074014 (2004)

Experimental paths to GPDs

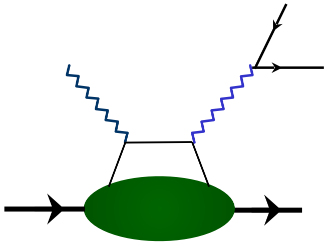


cliparts.co

Accessible in *exclusive* reactions, where all final state particles are detected.

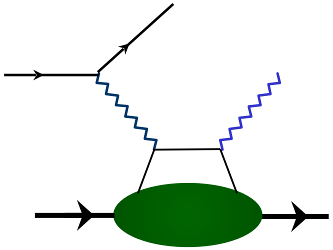
Trodden paths, or ones starting to be explored:

- Deeply Virtual Compton Scattering (DVCS)
- Deeply Virtual Meson Production (DVMP)
- Time-like Compton Scattering (TCS)
- Double DVCS



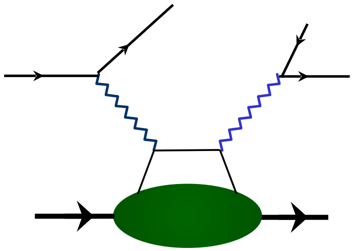
TCS

Virtual photon time-like



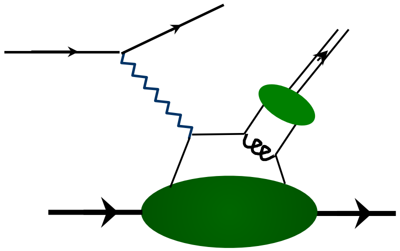
DVCS

Virtual photon space-like



DDVCS

One time-like, one space-like virtual photon

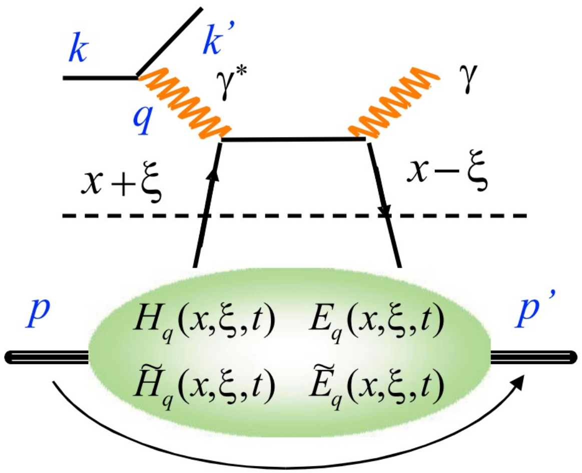


DVMP

Slide from Daria Sokhan

Access GPDs through Hard Processes

Deeply Virtual Compton Scattering (DVCS)



$t = (p' - p)^2$

Interference with Bethe-Heitler (BH) process gives access to real and imaginary part of DVCS amplitude

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I}$$

$$\mathcal{I} \propto \frac{-e_\ell}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}$$

e.g.:

$$c^{\mathcal{I}}_{1,\rm unpol.} \propto \left[F_1 \,\mathfrak{Re}\, \mathcal{H} - \frac{t}{4M_p^2} F_2 \,\mathfrak{Re}\, \mathcal{E} + \frac{x_{\rm B}}{2 - x_{\rm B}} (F_1 + F_2) \,\mathfrak{Re}\, \widetilde{\mathcal{H}} \right]$$

Access different GPDs

$$d\sigma_{LU} = \sin\phi \cdot \mathcal{I}m\{F_1\mathcal{H} + x_B(F_1 + F_2)\tilde{\mathcal{H}} - kF_2\mathcal{E}\}d\phi$$

$$d\sigma_{UL} = \sin\phi \cdot \mathcal{I}m\{F_1\tilde{\mathcal{H}} + x_B(F_1 + F_2)(\mathcal{H} + x_B/2\mathcal{E}) - x_B kF_2\tilde{\mathcal{E}} \ldots\}d\phi$$

$$d\sigma_{LL} = (A + B\cos\phi) \cdot \mathcal{R}e\{F_1\tilde{\mathcal{H}} + x_B(F_1 + F_2)(\mathcal{H} + x_B/2\mathcal{E}) \ldots\}d\phi$$

$$d\sigma_{UT} = \cos\phi \cdot \mathcal{I}m\{k(F_2\mathcal{H} - F_1\mathcal{E}) + \ldots\}d\phi$$

Alternative processes: deeply virtual meson production (DVMP), double DVCS, timelike Compton scattering (TCS)…

Quark Angular Momentum

Ji's sum rule: $$J^q = \frac{1}{2}\int_{-1}^{1} dx\, x[H^q(x,\xi,t)+E^q(x,\xi,t)] = \frac{1}{2}\Delta\Sigma + L^q$$

→ **Access to quark orbital angular momentum with GPDs**

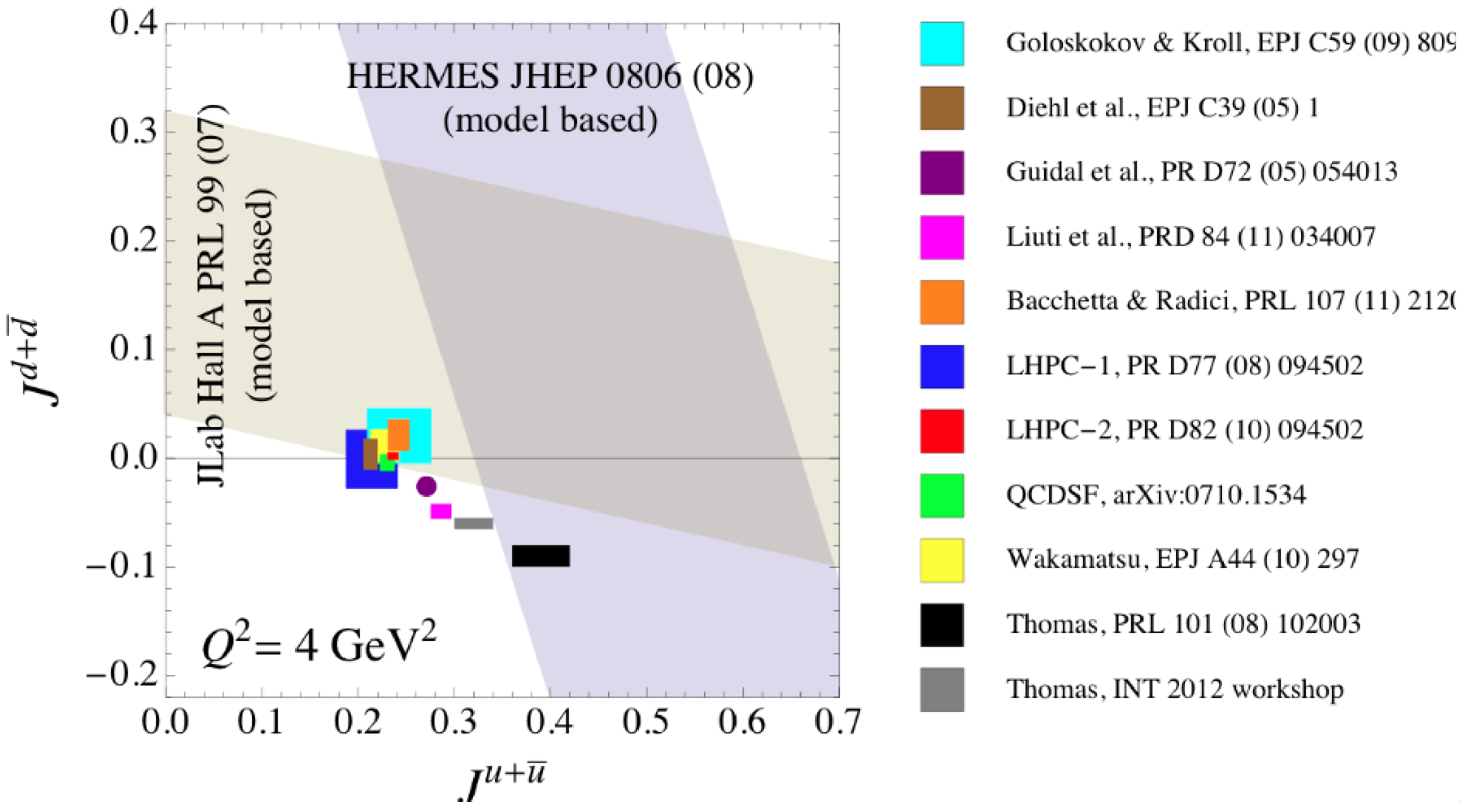


Image from J. Dudek et al., EPJA 48,187 (2012)

Leading-Twist TMD PDFs

Nucleon Spin

Quark Spin

Nucleon Polarization	Quark polarization: Unpolarized (U)	Quark polarization: Longitudinally Polarized (L)	Quark polarization: Transversely Polarized (T)
U	f_1 =		$h_1^{\perp}$ = − Boer-Mulders
L		g_1 = − Helicity	$h_{1L}^{\perp}$ = − Long-Transversity
T	$f_{1T}^{\perp}$ = − Sivers	g_{1T} = − Trans-Helicity	h_1 = − Transversity $h_{1T}^{\perp}$ = − Pretzelosity

Probed with transversely polarized targets
HERMES, COMPASS, JLab E06-010

Leading-Twist TMD PDFs

Nucleon Spin
Quark Spin

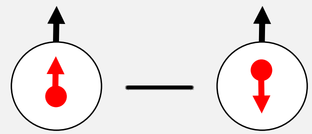
Nucleon Polarization	Quark polarization: Unpolarized (U)	Quark polarization: Longitudinally Polarized (L)	Quark polarization: Transversely Polarized (T)
U	f_1 =		$h_1^{\perp}$ = − Boer-Mulders
L		g_1 = − Helicity	$h_{1L}^{\perp}$ = − Long-Transversity
T	$f_{1T}^{\perp}$ = − Sivers	g_{1T} = − Trans-Helicity	h_1 = − Transversity $h_{1T}^{\perp}$ = − Pretzelosity

TMDs – confined motion inside the nucleon

Transversely Polarized Nucleon TMDs

→ Nucleon Spin
→ Quark Spin

Transversity

$h_{1T} =$


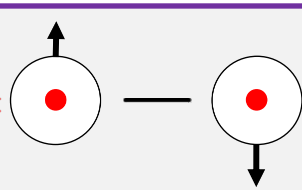
$$\mathbf{S}_T \cdot \mathbf{s}_q$$

- h_{1T} (h_1) = g_1 (no relativity)
- h_{1T} → tensor charge (lattice QCD calculations)
- Connected to nucleon beta decay and EDM

Relevant Vectors

S_T: Nucleon Spin
s_q: Quark Spin
$k_\perp$: Quark Transverse Momentum
P: Virtual photon 3-momentum (defines z-direction)

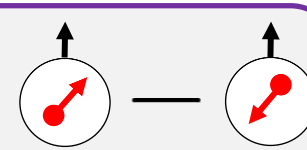
Sivers

$f_{1T}^{\perp} =$


$$\mathbf{S}_T \cdot \mathbf{k}_\perp \times \mathbf{P}$$

- Nucleon spin - quark orbital angular momentum (OAM) correlation – zero if no OAM (model dependence)

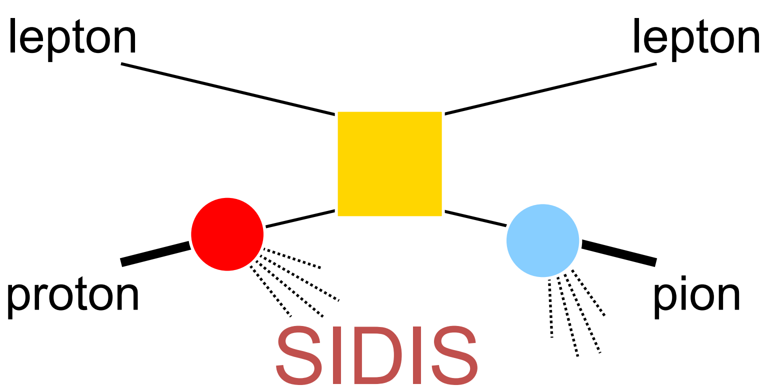
Pretzelosity

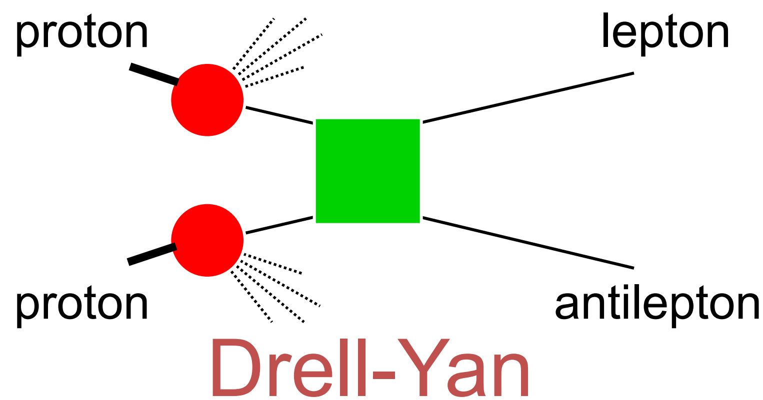
$h_{1T}^{\perp} =$


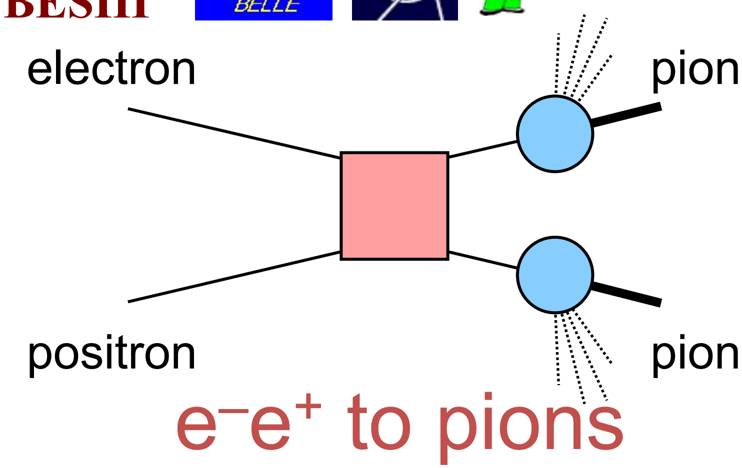
$$\mathbf{S}_T \cdot [\mathbf{k}_\perp \mathbf{k}_\perp] \cdot \mathbf{s}_{qT}$$

- Interference between components with OAM difference of 2 units (i.e., s-d, p-p) (model dependence)
- Signature for relativistic effect

Access TMDs through Hard Processes







- Partonic scattering amplitude
- Fragmentation amplitude
- Distribution amplitude

$$f_{1T}^{\perp q}(\text{SIDIS}) = -f_{1T}^{\perp q}(\text{DY})$$

$$h_1^{\perp}(\text{SIDIS}) = -h_1^{\perp}(\text{DY})$$

Fragmentation Functions

Ordinary FF → D_1

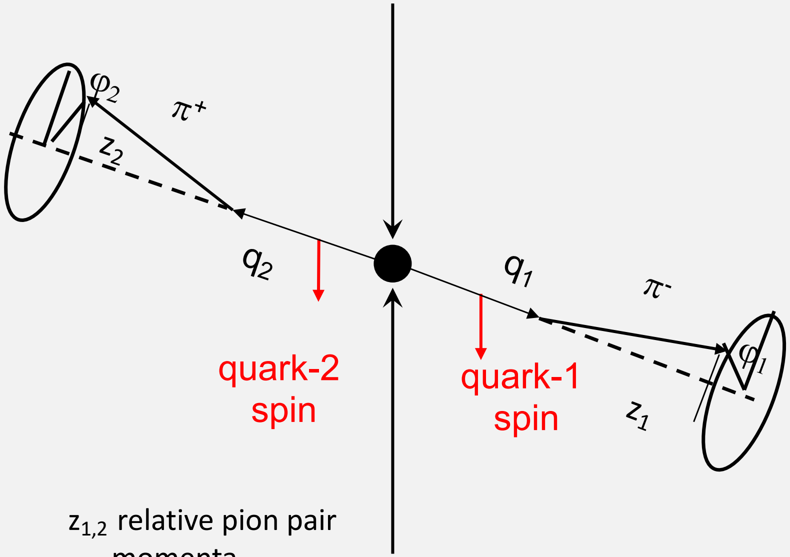
Collins FF → $H_1^{\perp}$

quark pol.

hadron pol.	U	L	T
U	D_1		$H_1^{\perp}$
L		G_1	$H_{1L}^{\perp}$
T	$D_{1T}^{\perp}$	$G_{1T}^{\perp}$	H_1 $H_{1T}^{\perp}$

A. Metz and A. Vossen, Prog. Part. Nucl. Phys., 91, 136 (2016)

COLLINS FFs IN e^+e^-



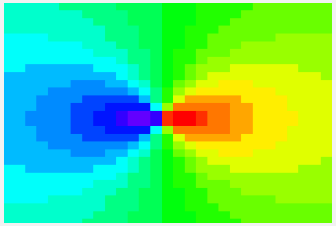
$z_{1,2}$ relative pion pair momenta

•First non-zero independent measurement of the Collins effect for pion pairs in e^+e^- annihilation by Belle Collaboration @ $\sqrt{s} \sim 10.6$ GeV (PRL 111,062002(2008), PRD 88,032011(2013)) leads to first extraction of transversity (Phys.Rev. D75 (2007) 054032) from SIDIS and e+e-

- Confirmed by BaBar @ $\sqrt{s} \sim 10.6$ GeV (PRD 90,052003 (2014); PRD 92,111101(R)(2015) for KK and Kπ)
- Measured at BESIII @ $\sqrt{s} = 3.65$ GeV (PRL 116,42001(2016))

Cross-section $e^+e^- \to (h_1 h_2)(\overline{h_1}\ \overline{h_2}) + X$

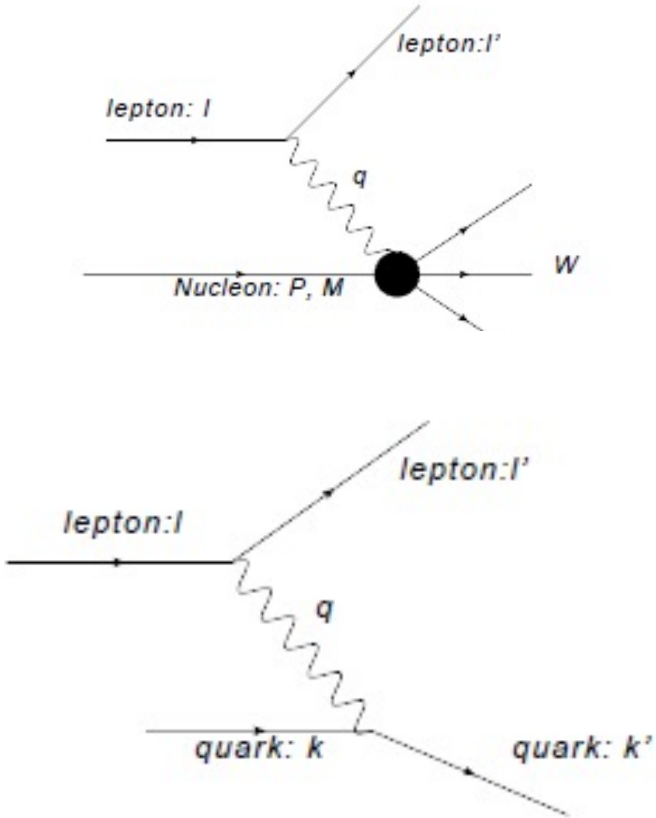
$$\propto D_1^\perp \overline{D_1^\perp} + H_1^\perp \overline{H_1^\perp} \cos(\phi_1 + \phi_2)$$

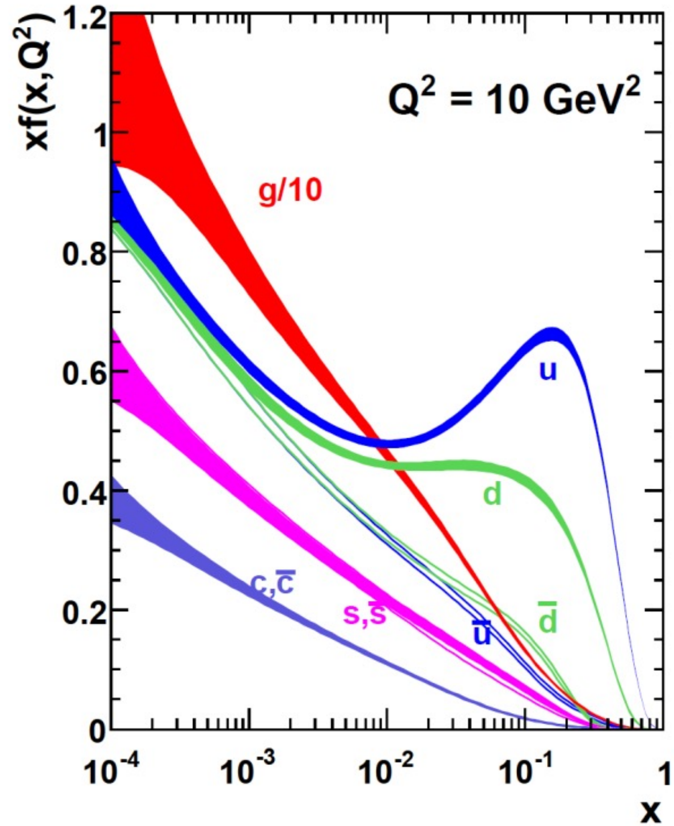


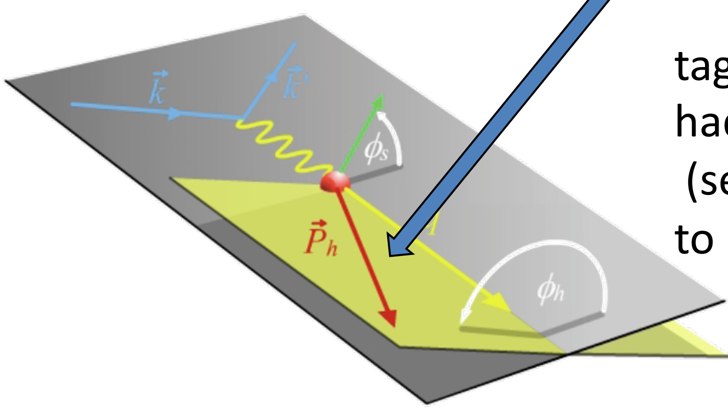
- Access spin dependence and p_T dependence (convolution or in jet) without PDF complication
- Made possible by B-factory luminosities

Workshop on Novel Probes of the Nucleon Structure in SIDIS, e+e- and pp (FF2019), chaired by Anselm Vossen and Harut Avagyan ***https://www.jlab.org/indico/event/308/***

Lepton Scattering ----- A powerful tool







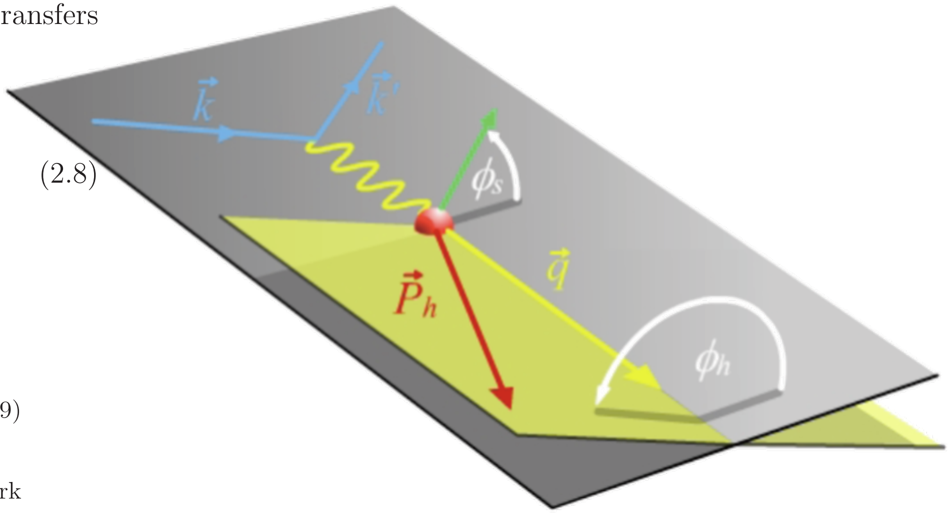
tagging the struck quark through leading hadrons
 (semi-inclusive DIS)
to image in 3-momentum space

8 New TMD PDFs
$f_1(x,k_T)$, .. $h_1(x,k_T)$

Semi-Inclusive Deep Inelastic Scattering Kinematics

- The lepton's energy loss in the nucleon rest frame, or the energy it transfers into the nucleon system:

$$\nu = E_l - E_{l'} = \frac{q \cdot P}{M} \tag{2.8}$$



- 4-momentum transfer squared of the virtual photon. [8]

$$Q^2 = -q^2 \tag{2.9}$$

- x_{bj} is the fraction of the nucleon's momentum carried by the struck quark $k = x_{bj} \cdot P$ in the parton model and in the light-cone frame.

$$x_{bj} \equiv x = \frac{Q^2}{2M\nu} \tag{2.10}$$

- The fraction of the lepton's energy transfer in the nucleon rest frame.

$$y = \frac{E_l - E_{l'}}{E_l} = \frac{q \cdot P}{l \cdot P} \tag{2.11}$$

- W is the mass of the recoiling system.

$$W = \sqrt{(P+q)^2} \tag{2.12}$$

- The center-of-mass energy squared of the lepton-nucleon system.

$$s = (l+P)^2 \tag{2.13}$$

- Transverse momentum of the detected hadron P_T:

$$P_T = \frac{\vec{q} \cdot \vec{P}_h}{|\vec{q}|} \tag{2.32}$$

- Ratio of the energy carried by the detected hadron and the energy of the virtual

$$z = \frac{P \cdot P_h}{P \cdot q}$$

- Missing Mass W':

$$W' = \sqrt{(q + P - P_h)^2}$$