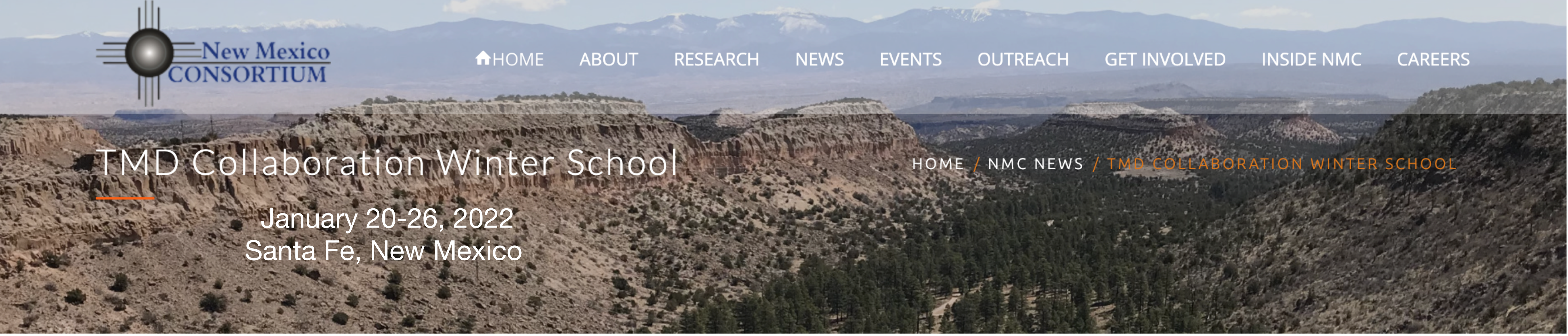
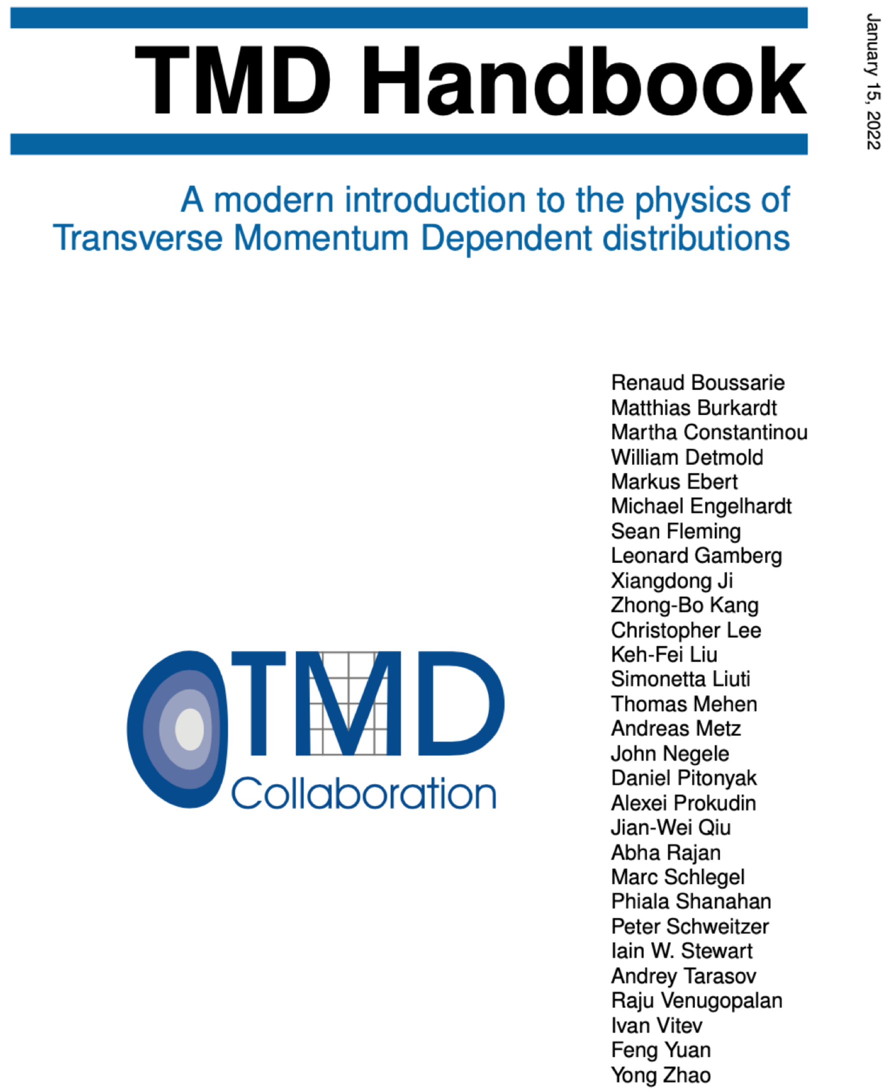


TMD Collaboration Winter School

January 20-26, 2022
Santa Fe, New Mexico

Introduction to QCD

- **Lec. 1: Fundamentals of QCD**
- **Lec. 2: QCD for cross sections with identified hadrons, & hadron structure**
- **Lec. 3: QCD for observables with polarization, & role/power of lattice QCD**



TMD Handbook

A modern introduction to the physics of Transverse Momentum Dependent distributions

January 15, 2022

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Polarization and spin asymmetry

Explore new QCD dynamics – vary the spin orientation

☐ **Cross section:**

Scattering amplitude square – Probability – Positive definite

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

☐ **Spin-averaged cross section:**

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$ – Positive definite

☐ **Asymmetries or difference of cross sections:**

– Not necessary positive!

- both beams polarized A_{LL}, A_{TT}, A_{LT}

$$A_{LL} = \frac{[\sigma(+,+) - \sigma(+,-)] - [\sigma(-,+) - \sigma(-,-)]}{[\sigma(+,+) + \sigma(+,-)] + [\sigma(-,+) + \sigma(-,-)]} \quad \text{for } \sigma(s_1, s_2)$$

- one beam polarized A_L, A_N

$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s) \qquad A_N = \frac{\sigma(Q,\vec{s}_T) - \sigma(Q,-\vec{s}_T)}{\sigma(Q,\vec{s}_T) + \sigma(Q,-\vec{s}_T)}$$

Chance to see quantum interference directly

Two roles of the proton spin program

☐ **Proton is a composite particle:**

Spin is a consequence of internal dynamics of the bound state

For example, the nucleon-nucleon interaction and shell structure determines the observed nuclear spin states

➡ Decomposition of proton spin in terms of quark and gluon d.o.f.

helps understand the dynamics of a fundamental QCD bound state

– Nucleon is a building block all hadronic matter

(> 95% mass of all visible matter)

☐ **Use the spin as a tool – asymmetries:**

Cross section is a probability – classically measured

Spin asymmetry – the difference of two cross sections
involving two different spin states

Asymmetry could be a pure quantum effect!

Spin of a composite particle

- ❑ **Spin:**
  - ✧ Pauli (1924): two-valued quantum degree of freedom of electron
  - ✧ Pauli/Dirac: $S = \hbar\sqrt{s(s+1)}$ (fundamental constant ħ)
  - ✧ Composite particle = Total angular momentum when it is at rest
- ❑ **Spin of a nucleus:**
  - ✧ Nuclear binding: 8 MeV/nucleon << mass of nucleon
  - ✧ Nucleon number is fixed inside a given nucleus
  - ✧ Spin of a nucleus = sum of the valence nucleon spin
- ❑ **Spin of a nucleon – Naïve Quark Model:**
  - ✧ If the probing energy << mass of constituent quark
  - ✧ Nucleon is made of three constituent (valence) quark
  - ✧ Spin of a nucleon = sum of the constituent quark spin

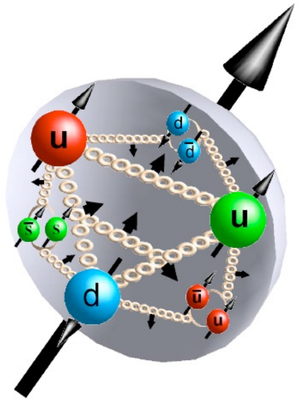
State: $$|p\uparrow\rangle = \sqrt{\frac{1}{18}}\left[u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow - 2u\uparrow u\uparrow d\downarrow + \text{perm.}\right]$$

Spin: $$S_p \equiv \langle p\uparrow|S|p\uparrow\rangle = \frac{1}{2}, \qquad S = \sum_i S_i$$

Carried by valence quarks

Spin of a composite particle

☐ **Spin of a nucleon – QCD:**



- ✧ Current quark mass << energy exchange of the collision
- ✧ Number of quarks and gluons depends on the probing energy

☐ **Angular momentum of a proton at rest:**

$$S = \sum_f \langle P, S_z = 1/2 |\, \hat{J}_f^z \,| P, S_z = 1/2 \rangle = \frac{1}{2}$$

☐ **QCD Angular momentum operator:**

Energy-momentum tensor

$$J^i_{\rm QCD} = \frac{1}{2}\,\epsilon^{ijk} \int d^3x\ M^{0jk}_{\rm QCD} \quad \Longleftarrow \quad M^{\alpha\mu\nu}_{\rm QCD} = T^{\alpha\nu}_{\rm QCD}\, x^\mu - T^{\alpha\mu}_{\rm QCD}\, x^\nu$$

Angular momentum density

- ✧ Quark angular momentum operator:

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma}\gamma_5 \psi_q + \psi_q^\dagger (\vec{x} \times (-i\vec{D}))\, \psi_q\right]$$

- ✧ Gluon angular momentum operator:

$$\vec{J}_g = \int d^3x \left[\vec{x} \times (\vec{E} \times \vec{B})\right]$$

Need to have the matrix elements of these partonic operators measured independently

Current understanding for Proton Spin

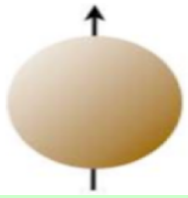
See lectures by Gao

❑ **The sum rule:**

$$S(\mu) = \sum_f \langle P,S|\hat{J}^z_f(\mu)|P,S\rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)$$

- **Infinite possibilities of decompositions – connection to observables?**
- **Intrinsic properties + dynamical motion and interactions**

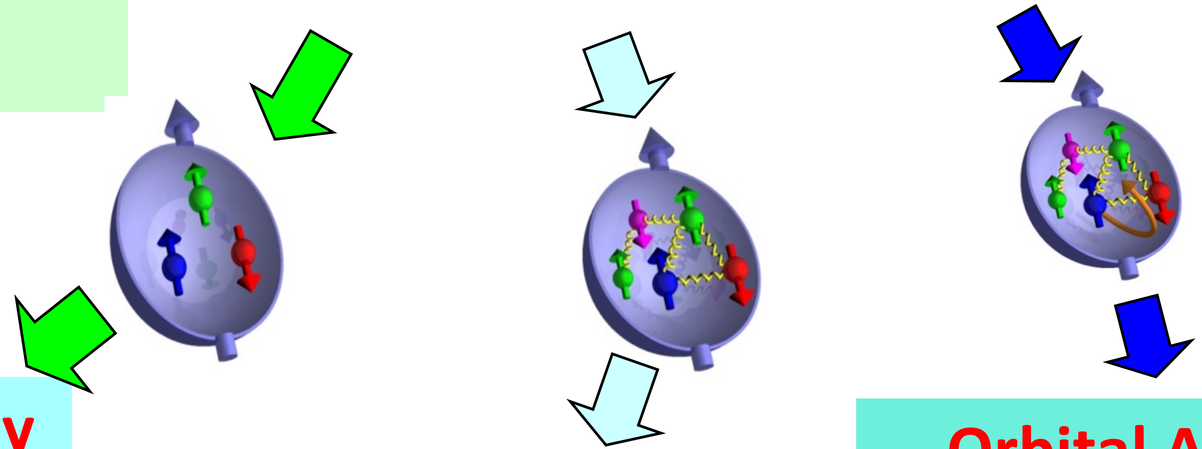
❑ **An incomplete story:**



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

Jaffe-Manohar, 90
Ji, 96, …

Proton Spin



Quark helicity
Best known

$$\frac{1}{2}\int dx\left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$
$$\sim 30\%$$

Sea quarks?

Gluon helicity
Start to know

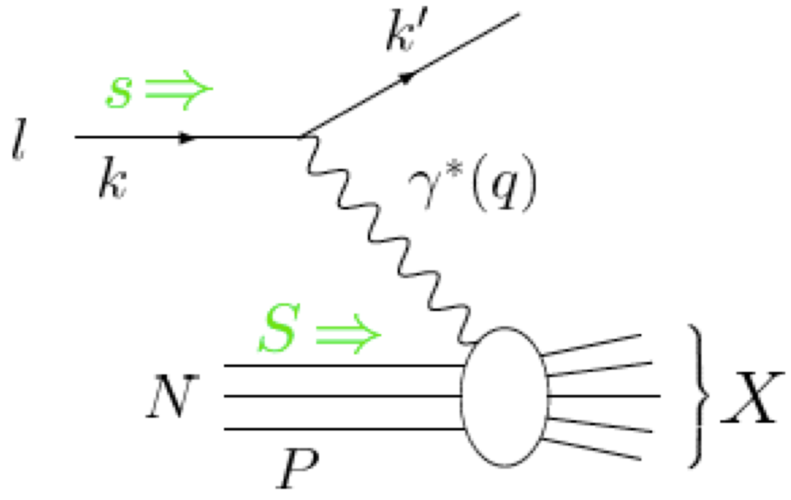
$$\Delta G = \int dx \Delta g(x)$$
$$\sim 40\% (\text{with RHIC data})$$

Orbital Angular Momentum
of quarks and gluons
Little known

Net effect of partons' transverse motion?

Polarized deep inelastic scattering

❑ DIS with polarized beam(s):



"Resolution" $Q \equiv \sqrt{-q^2}$

$$\frac{\hbar}{Q} = \frac{2\times 10^{-16}\text{m}}{Q/\text{GeV}} \lesssim 10^{-16}\text{m} = 1/10\text{fm}$$

"Inelasticity" – known as Bjorken variable

$$x_B = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2 + M_X^2 - m^2}$$

✧ Recall – from lecture 2:

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x_B, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x_B, Q^2\right)$$

$$+iM_p \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{p.q} g_1\left(x_B, Q^2\right) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2} g_2\left(x_B, Q^2\right)\right]$$

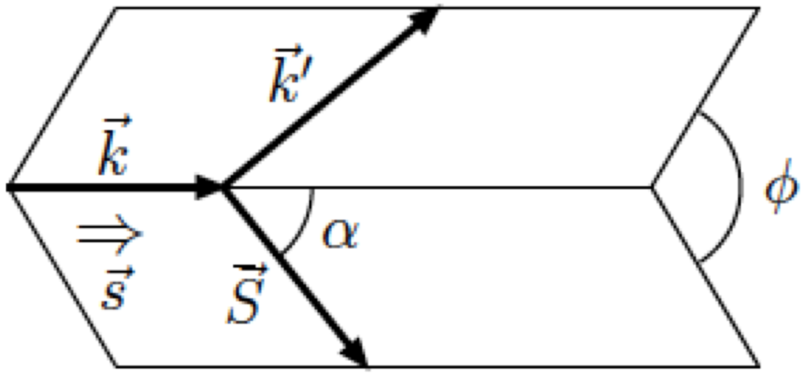
✧ Polarized structure functions:

$$g_1(x_B, Q^2),\ \ g_2(x_B, Q^2)$$

Polarized deep inelastic scattering

❑**Extract the polarized structure functions:**

$$\mathcal{W}^{\mu\nu}(P,q,S) \; - \; \mathcal{W}^{\mu\nu}(P,q,-S)$$



- ✧ **Define:** $\angle(\hat{k},\hat{S}) = \alpha$ **,**
  and lepton helicity λ

- ✧ **Difference in cross sections with hadron spin flipped**

$$\frac{d\sigma^{(\alpha)}}{dx\,dy\,d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx\,dy\,d\phi} = \frac{\lambda\; e^4}{4\pi^2 Q^2} \times$$

$$\times\left\{\cos\alpha\left\{\left[1-\frac{y}{2}-\frac{m^2x^2y^2}{Q^2}\right] g_1(x,Q^2) \; - \; \frac{2m^2x^2y}{Q^2} g_2(x,Q^2)\right\}\right.$$

$$\left. - \sin\alpha\cos\phi\,\frac{2mx}{Q}\sqrt{\left(1-y-\frac{m^2x^2y^2}{Q^2}\right)}\;\left(\frac{y}{2}\,g_1(x,Q^2) \; + \; g_2(x,Q^2)\right)\right\}$$

- ✧ **Spin orientation:**

$$\alpha \;=\; 0 \;:\; \Rightarrow \quad g_1$$

$$\alpha \;=\; \pi/2 : \; \Rightarrow \quad y\,g_1 \;+\; 2\,g_2$$, suppressed m/Q

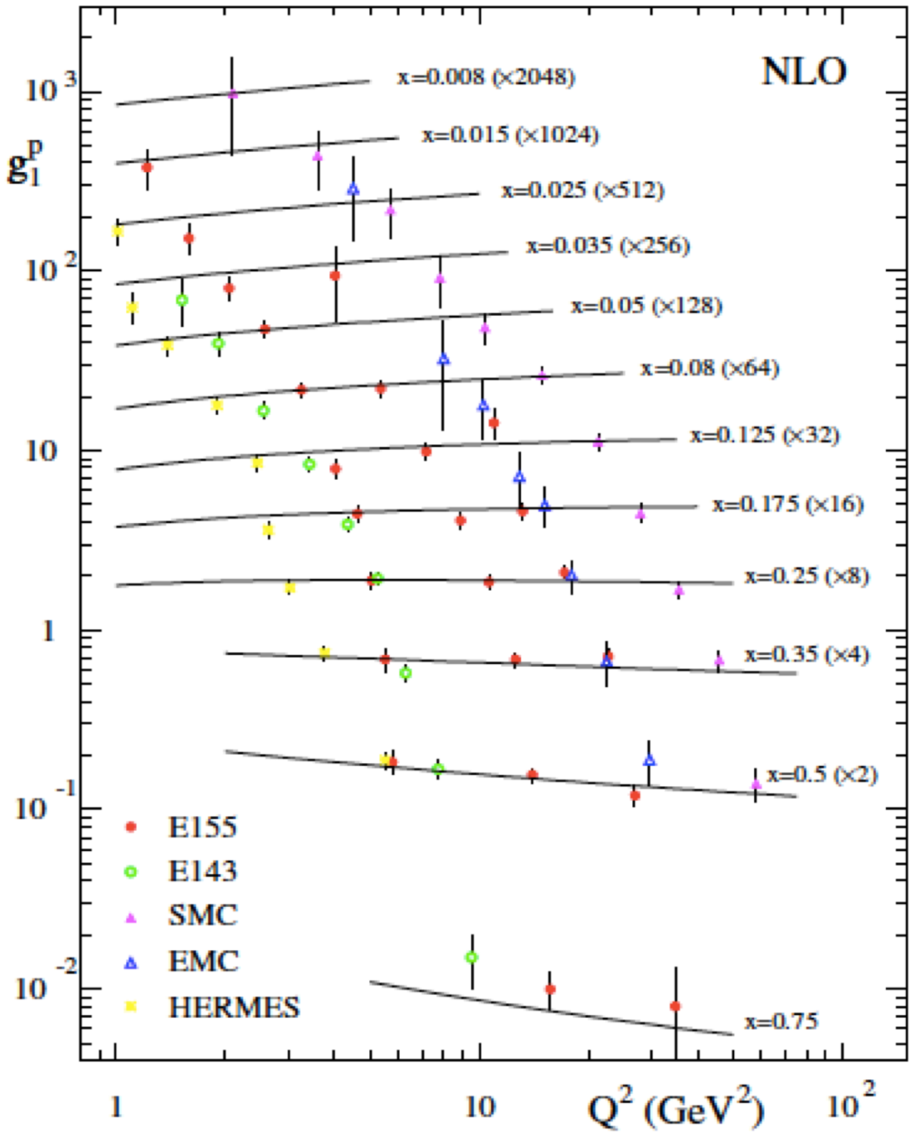
Polarized deep inelastic scattering

❑ **Spin asymmetries – measured experimentally:**

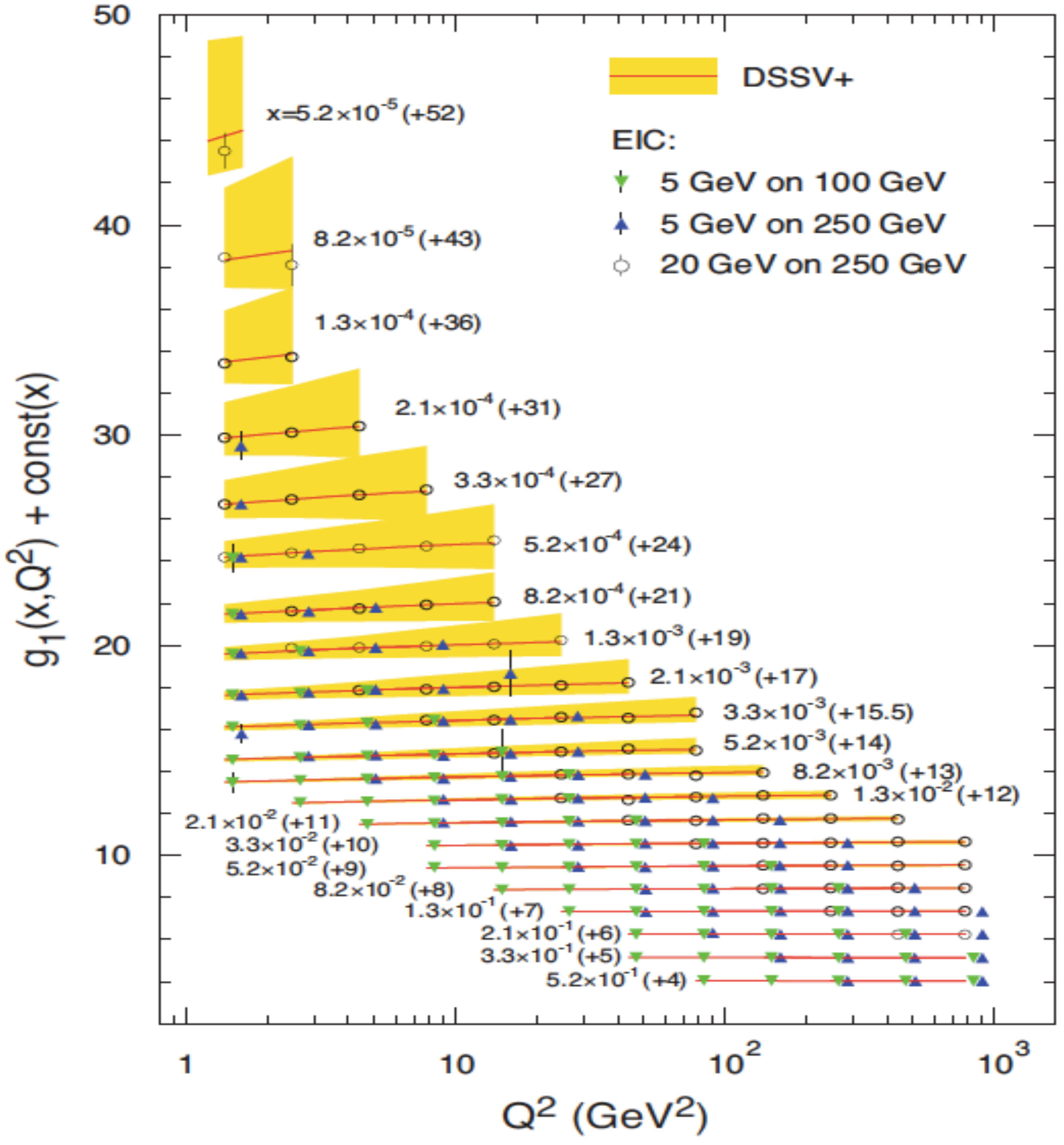
✧**Longitudinal polarization –** $\alpha = 0$

Known function

$$A_{\parallel} = \frac{d\sigma^{(\rightarrow\Leftarrow)} - d\sigma^{(\rightarrow\Rightarrow)}}{d\sigma^{(\rightarrow\Leftarrow)} + d\sigma^{(\rightarrow\Rightarrow)}} = D(y)\,\frac{g_1(x,Q^2)}{F_1(x,Q^2)} \equiv D(y)\,A_1(x,Q^2)$$

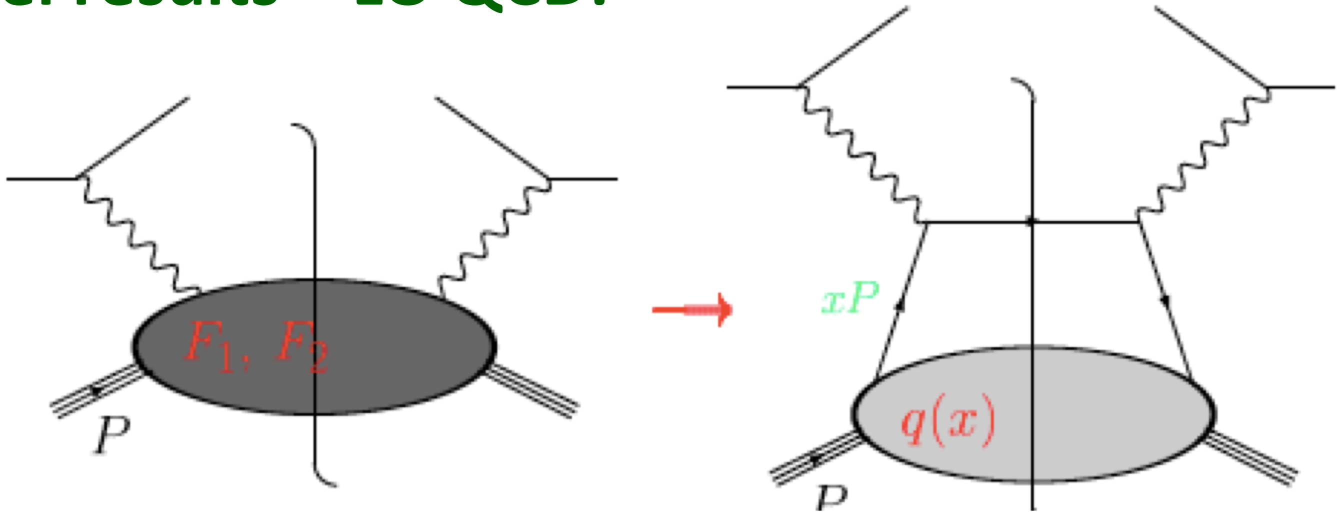


Polarized DIS at EIC



Polarized deep inelastic scattering

❑ Parton model results – LO QCD:



✧ Structure functions:

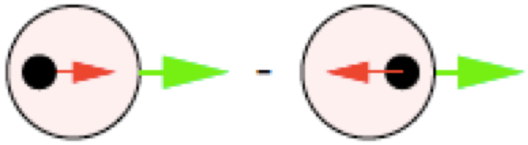
$$F_1(x) = \frac{1}{2}\sum_q e_q^2\,[\,q(x) + \bar{q}(x)\,]$$

$$g_1(x) = \frac{1}{2}\sum e_q^2\,[\,\Delta q(x) + \Delta\bar{q}(x)\,]$$

$$g_1 = \frac{1}{2}\left[\frac{4}{9}\left(\Delta u + \Delta\bar{u}\right) + \frac{1}{9}\left(\Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)\right]$$

✧ Polarized quark distribution:

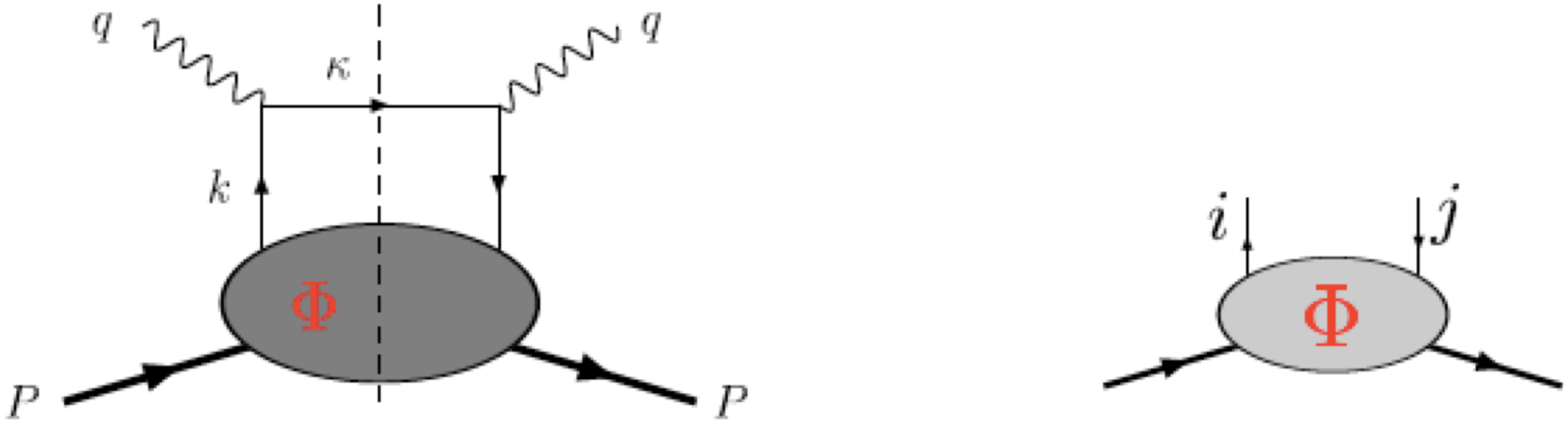
$$\boxed{\Delta f(\xi) \equiv f^{+}(\xi) - f^{-}(\xi)}$$



Information on nucleon's spin structure

Polarized deep inelastic scattering

❑ Systematics polarized PDFs – LO QCD:



✧ Two-quark correlator:

$$\Phi_{ij}(k,P,S) = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3\,2E_X} (2\pi)^4\,\delta^4(P-k-P_X)\,\langle PS|\bar{\psi}_j(0)|X\rangle\,\langle X|\psi_i(0)|PS\rangle$$

$$= \int d^4z\; e^{ik\cdot z}\,\langle PS\,|\,\bar{\psi}_j(0)\,\psi_i(z)\,|\,PS\,\rangle$$

✧ Hadronic tensor (one –flavor):

$$\mathcal{W}^{\mu\nu} = e^2 \int \frac{d^4k}{(2\pi)^4}\,\delta\left((k+q)^2\right)\;\mathrm{Tr}\left[\,\Phi\,\gamma^\mu(\not{k}+\not{q})\gamma^\nu\,\right]$$

Polarized deep inelastic scattering

✧ **General expansion of $\phi(x)$:**

must have general expansion in terms of $\not{P}$, $\not{n}$, $\not{s}$ etc.

$$\phi(x) = \frac{1}{2}\left[q(x)\gamma \cdot P + s_{\|}\Delta q(x)\gamma_5\gamma \cdot P + \delta q(x)\gamma \cdot P\gamma_5\gamma \cdot S_{\perp}\right]$$

✧ **3-leading power quark parton distribution:**

$$q\,(x) \;=\; \frac{1}{4\pi}\int dz^-\,\mathrm{e}^{iz^-xP^+}\,\langle P,S|\bar{\psi}(0)\,\gamma^+\,\psi\,(0,z^-,\mathbf{0}_{\perp})\,|P,S\rangle$$

$$\Delta q\,(x) \;=\; \frac{1}{4\pi}\int dz^-\,\mathrm{e}^{iz^-xP^+}\,\langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_5\,\psi\,(0,z^-,\mathbf{0}_{\perp})\,|P,S\rangle$$

$$\delta q\,(x) \;=\; \frac{1}{4\pi}\int dz^-\,\mathrm{e}^{iz^-xP^+}\,\langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_{\perp}\,\gamma_5\,\psi\,(0,z^-,\mathbf{0}_{\perp})\,|P,S\rangle$$

“unpolarized” – “longitudinally polarized” – “transversity”

Polarized deep inelastic scattering

❑ Physical interpretation:

Notation in handbook (p14):

$$q(x) \to f_{q/h}(\xi)$$
$$\Delta q(x) \to \Delta f_{q/h}(\xi)$$
$$\delta q(x) \to \delta f_{q/h}(\xi)$$

$$q(x) = \frac{1}{2}\sum_X \delta\left(P_X^+ - (1-x)P^+\right) \times \left[\left|\langle X|\mathcal{P}^+\psi_+(0)|P,\lambda=\frac{1}{2}\rangle\right|^2 + \left|\langle X|\mathcal{P}^-\psi_+(0)|P,\lambda=\frac{1}{2}\rangle\right|^2\right]$$

$$\Delta q(x) = \frac{1}{2}\sum_X \delta\left(P_X^+ - (1-x)P^+\right) \times \left[\left|\langle X|\mathcal{P}^+\psi_+(0)|P,\lambda=\frac{1}{2}\rangle\right|^2 - \left|\langle X|\mathcal{P}^-\psi_+(0)|P,\lambda=\frac{1}{2}\rangle\right|^2\right]$$

$$\delta q(x) = \frac{1}{2}\sum_X \delta(P_X^+ - (1-x)P^+) \times \left[\left|\langle X|\mathcal{P}^\uparrow\psi_+(0)|P,S_\perp=\frac{1}{2}\rangle\right|^2 - \left|\langle X|\mathcal{P}^\downarrow\psi_+(0)|P,S_\perp=\frac{1}{2}\rangle\right|^2\right]$$

Spin projection: $\mathcal{P}^\pm = \frac{1\pm\gamma_5}{2}$ **and** $\mathcal{P}^{\uparrow\downarrow} = \frac{1\pm\gamma_\perp\gamma_5}{2}$

Basics for spin observables

- ☐ **Factorized cross section:**

$$\sigma_{h(p)}(Q,s) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

$$e.g.\ \mathcal{O}(\psi,A^\mu)=\overline{\psi}(0)\,\hat{\Gamma}\,\psi(y^-) \quad \text{with } \hat{\Gamma}=I,\gamma_5,\gamma^\mu,\gamma_5\gamma^\mu,\sigma^{\mu\nu}$$

- ☐ **Parity and Time-reversal invariance:**

$$\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \langle p,-\vec{s}|\mathcal{PT}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle$$

- ☐ **IF:** $\langle p,-\vec{s}|\mathcal{PT}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

or $\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

Operators lead to the "+" sign ➡ spin-averaged cross sections

Operators lead to the "-" sign ➡ spin asymmetries

- ☐ **Example:**

$$\mathcal{O}(\psi,A^\mu)=\overline{\psi}(0)\,\gamma^+\,\psi(y^-) \ \Rightarrow\ q(x)$$

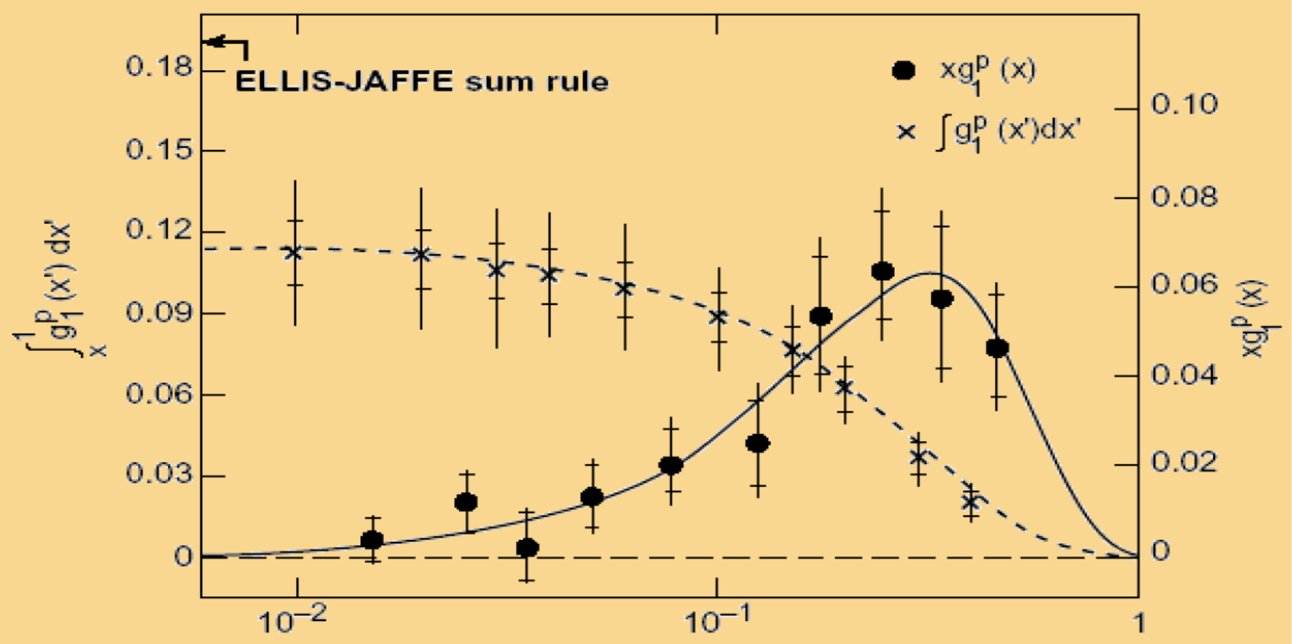
$$\mathcal{O}(\psi,A^\mu)=\overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y^-) \ \Rightarrow\ \Delta q(x)$$

$$\mathcal{O}(\psi,A^\mu)=\overline{\psi}(0)\,\gamma^+\gamma^\perp\gamma_5\,\psi(y^-) \ \Rightarrow\ \delta q(x)\to h(x)$$

$$\mathcal{O}(\psi,A^\mu)=\frac{1}{xp^+}F^{+\alpha}(0)[-i\varepsilon_{\alpha\beta}]F^{+\beta}(y^-) \Rightarrow\ \Delta g(x)$$

Proton “spin crisis” – excited the field

- **EMC (European Muon Collaboration ’87) – “the Plot”:**



$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right]$$
$$+\,\mathcal{O}(\alpha_s) + \mathcal{O}\left(1/Q\right)$$

- ✧ **Combined with earlier SLAC data:** $$\int_0^1 g_1^p(x)dx = 0.126 \pm 0.018$$

- ✧ **Combined with:** $g_A^3 = \Delta u - \Delta d$ **and** $g_A^8 = \Delta u + \Delta d - 2\Delta s$
  from low energy neutron & hyperon β decay

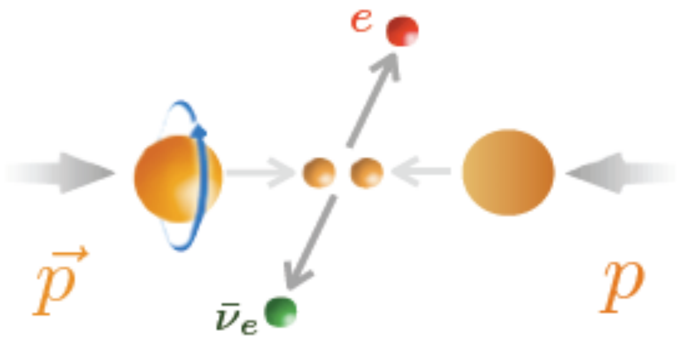
$$\Delta\Sigma = \sum_q\left[\Delta q + \Delta\bar{q}\right] = 0.12 \pm 0.17$$

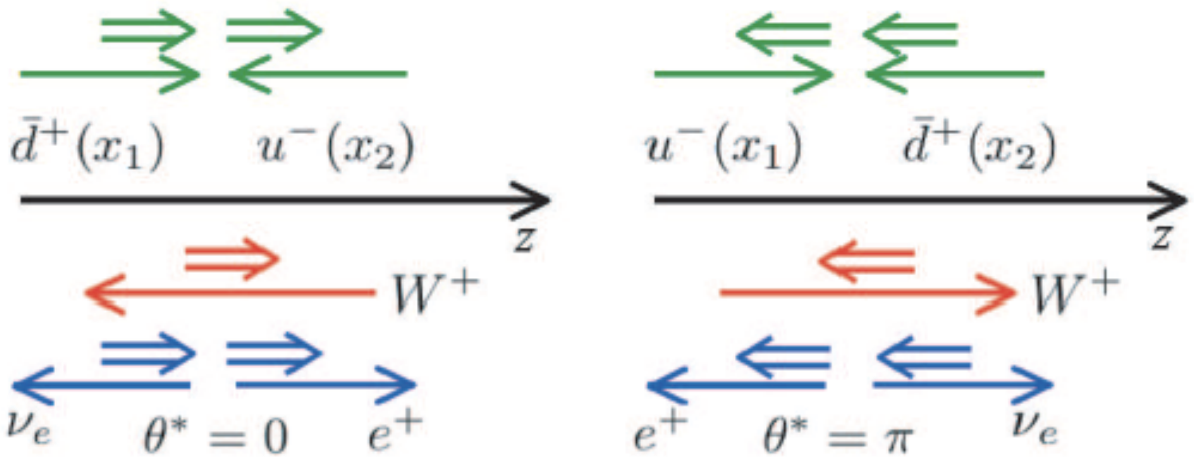
- **“Spin crisis” or puzzle:**
  - ✧ **Strange sea polarization is sizable & negative**
  - ✧ **Very little of the proton spin is carried by quarks**

New era of spin physics

Determination of Δq and $\Delta\bar{q}$

❑ W's are left-handed:





❑ Flavor separation:

Lowest order:

$$A_L^{W^+} = -\frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$$

$$x_1 = \frac{M_W}{\sqrt{s}} e^{y_W}, \quad x_2 = \frac{M_W}{\sqrt{s}} e^{-y_W}$$

Forward W$^+$ (backward e$^+$):

$$A_L^{W^+} \approx -\frac{\Delta u(x_1)}{u(x_1)} < 0$$

Backward W$^+$ (forward e$^+$):

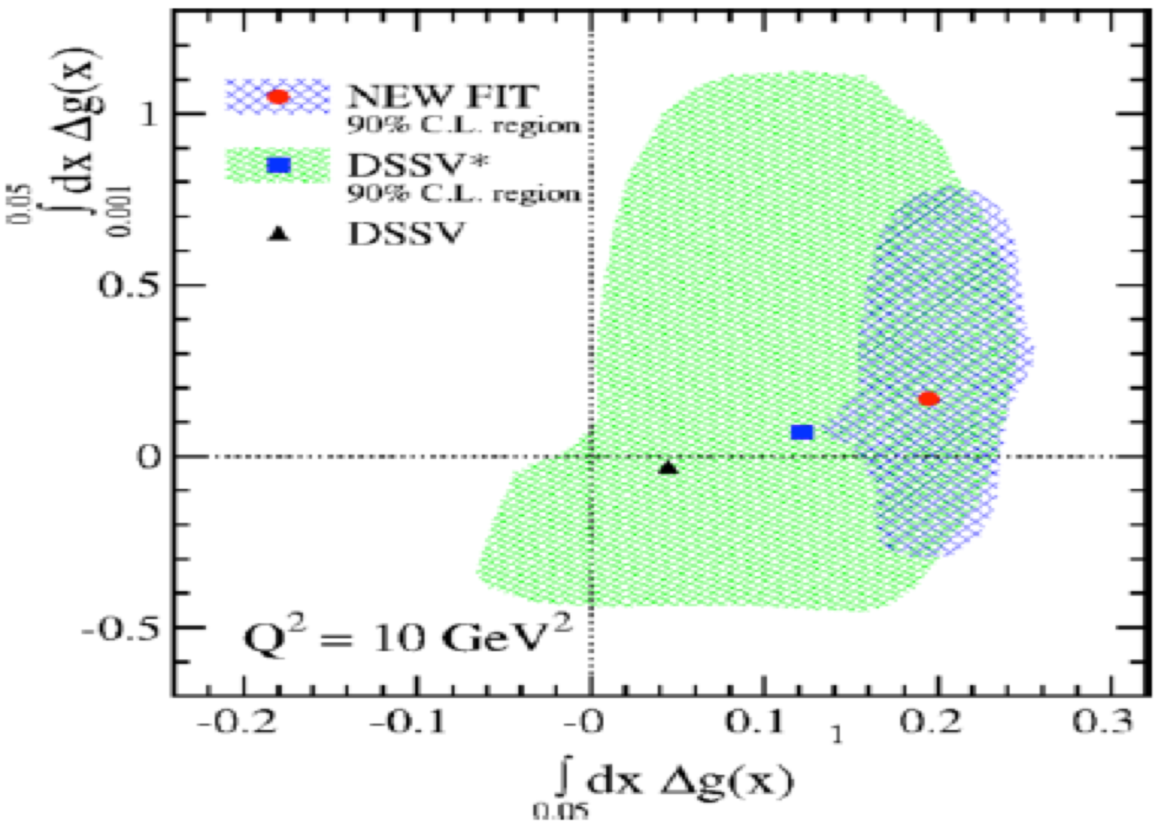
$$A_L^{W^+} \approx -\frac{\Delta\bar{d}(x_2)}{\bar{d}(x_2)} < 0$$

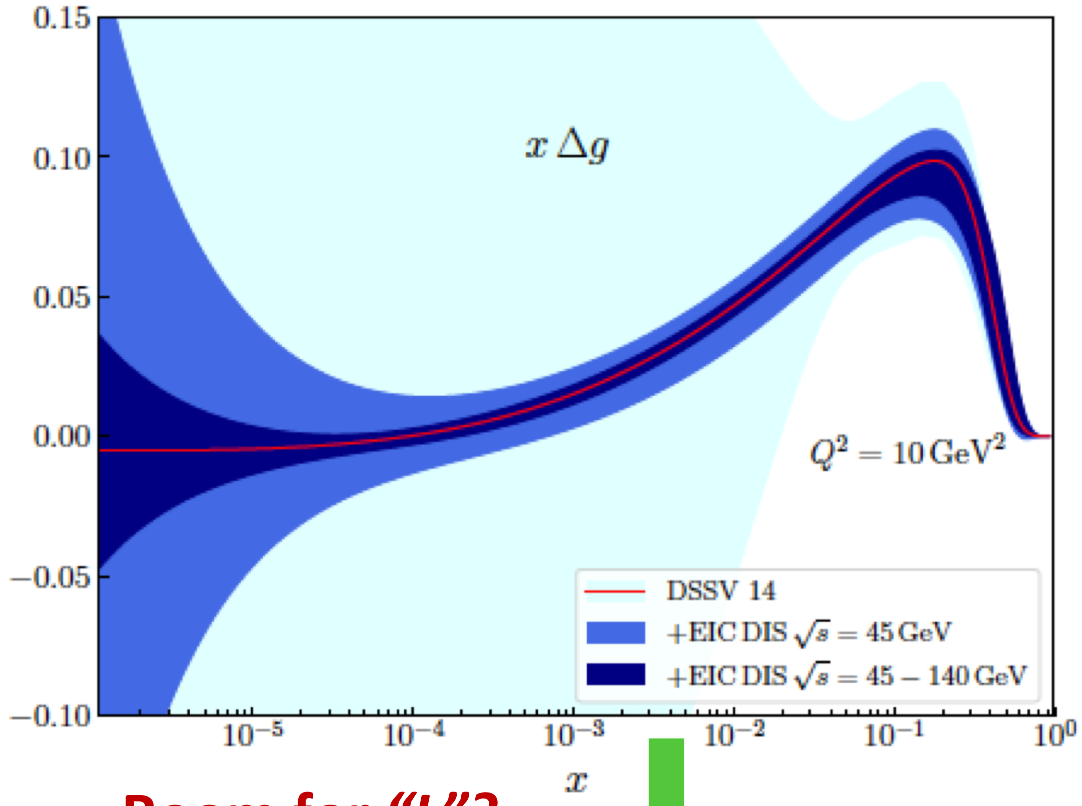
❑ Complications:

High order, W's p_T-distribution at low p_T

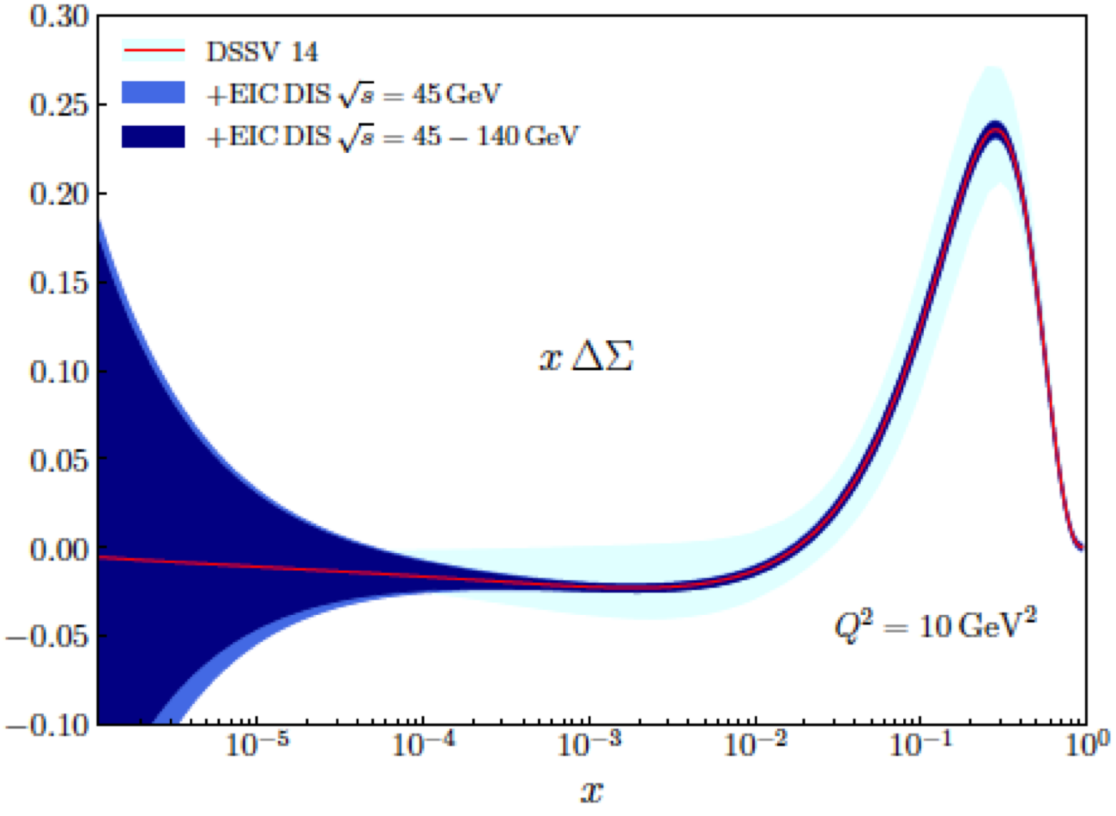
What the EIC can do – EIC Yellow Report?

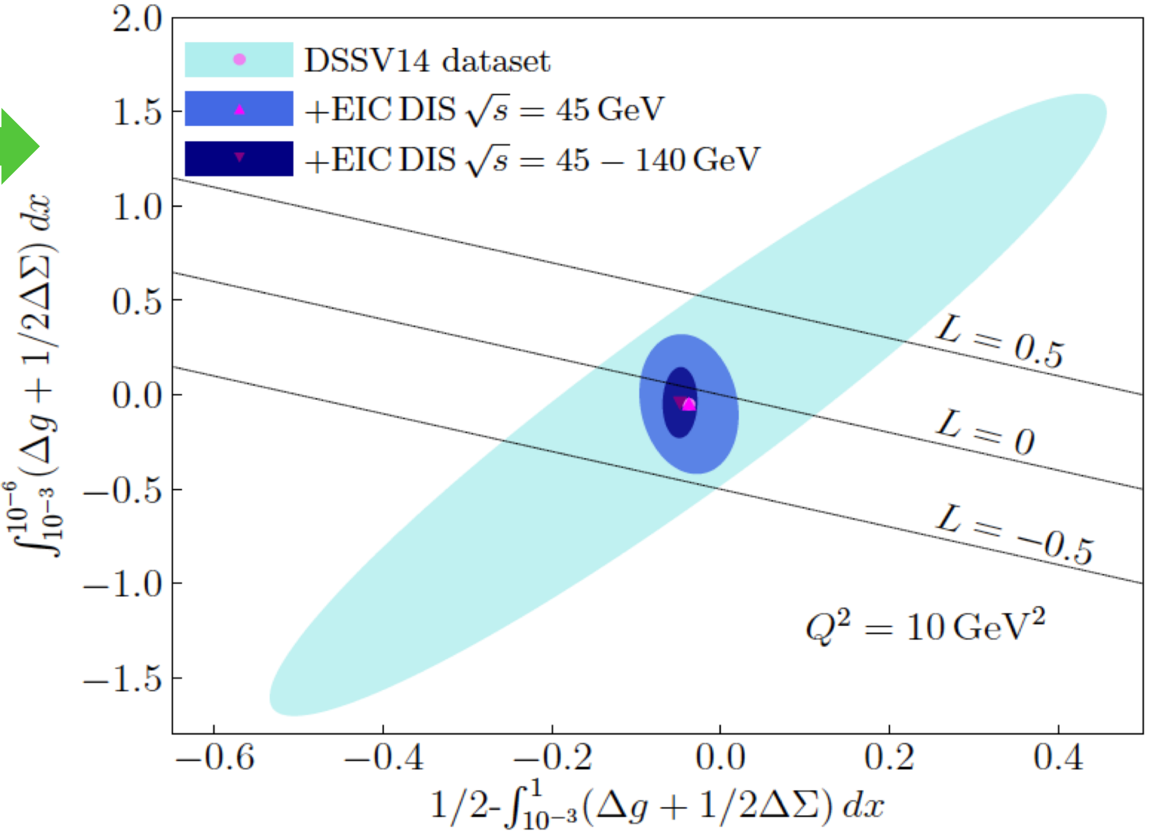
See lectures by Gao





Room for *"L"*?

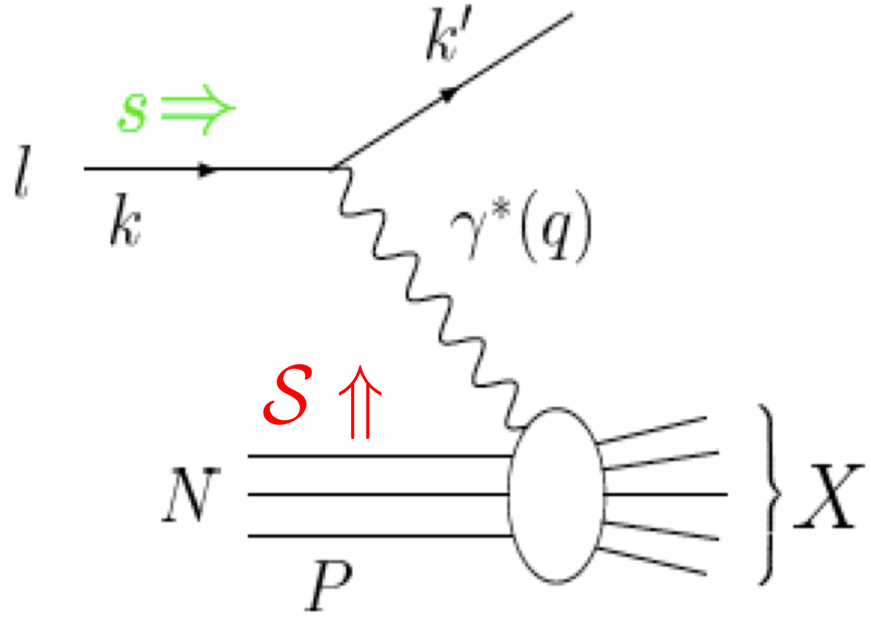




Transverse spin phenomena in QCD

- ☐ 40 years ago, Profs. Christ and Lee proposed to use A_N of inclusive DIS to test the Time-Reversal invariance

N. Christ and T.D. Lee, Phys. Rev. 143, 1310 (1966)



Single Transverse-Spin Asymmetry (SSA)

$$A(\ell,\vec{s}) \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s})-\sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s})+\sigma(\ell,-\vec{s})}$$

They predicted:

In the approximation of one-photon exchange, A_N of inclusive DIS vanishes if Time-Reversal is invariant for EM and Strong interactions

A_N for inclusive DIS

- **DIS cross section:**

$$\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, W_{\mu\nu}(\vec{s}_\perp)$$

- **Leptionic tensor is symmetric:**

$$L^{\mu\nu} = L^{\nu\mu}$$

- **Hadronic tensor:**

$$W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp |\, j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle$$

- **Polarized cross section:**

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)\right]$$

- **Vanishing single spin asymmetry:**

$$\boxed{\begin{aligned} A_N = 0 \quad \Leftrightarrow \quad & \langle P, \vec{s}_\perp |\, j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle \\ & \overset{?}{=} \langle P, -\vec{s}_\perp |\, j^\dagger_\nu(0)\, j_\mu(y)\, | P, -\vec{s}_\perp \rangle \end{aligned}}$$

A_N for inclusive DIS

- ☐ **Define two quantum states:**

$$\langle\beta| \equiv \langle P, \vec{s}_\perp | \, j_\mu^\dagger(0)\, j_\nu(y) \qquad\qquad |\alpha\rangle \equiv |P, \vec{s}_\perp\rangle$$

- ☐ **Time-reversed states:**

$$|\alpha_T\rangle = V_T\,|P, \vec{s}_\perp\rangle = |-P, -\vec{s}_\perp\rangle$$

$$\begin{aligned} |\beta_T\rangle &= V_T\left[j_\mu^\dagger(0)\, j_\nu(y)\right]^\dagger |P, \vec{s}_\perp\rangle \\ &= \left(V_T j_\nu^\dagger(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right)|-P, -\vec{s}_\perp\rangle \end{aligned}$$

- ☐ **Time-reversal invariance:**

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|\, V_T^\dagger\, V_T\,|\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

$$\boxed{\begin{aligned} &\langle -P, -\vec{s}_\perp|\,\left(V_T j_\nu^\dagger(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right)|-P, -\vec{s}_\perp\rangle \\ &= \langle P, \vec{s}_\perp|\, j_\mu^\dagger(0)\, j_\nu(y)\,|P, \vec{s}_\perp\rangle \end{aligned}}$$

A_N for inclusive DIS

- ☐ **Parity invariance:**

$$\boxed{1 = U_P^{-1} U_P = U_P^{\dagger} U_P}$$

$$\langle -P, -\vec{s}_\perp | \left(V_T j_\nu^{\dagger}(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right) | -P, -\vec{s}_\perp \rangle$$

$$\Downarrow$$

$$\langle P, -\vec{s}_\perp | \left(U_P V_T j_\nu^{\dagger}(y) V_T^{-1} U_P^{-1}\right)\left(U_P V_T j_\mu(0) V_T^{-1} U_P^{-1}\right) | P, -\vec{s}_\perp \rangle$$

$$\Downarrow$$

$$\langle P, -\vec{s}_\perp | \, j_\nu^{\dagger}(-y) \, j_\mu(0) \, | P, -\vec{s}_\perp \rangle$$

Translation invariance: →

$$\Downarrow$$

$$\boxed{\begin{aligned} &\langle P, -\vec{s}_\perp | \, j_\nu^{\dagger}(0) \, j_\mu(y) \, | P, -\vec{s}_\perp \rangle \\ &= \langle P, \vec{s}_\perp | \, j_\mu^{\dagger}(0) \, j_\nu(y) \, | P, \vec{s}_\perp \rangle \end{aligned}}$$

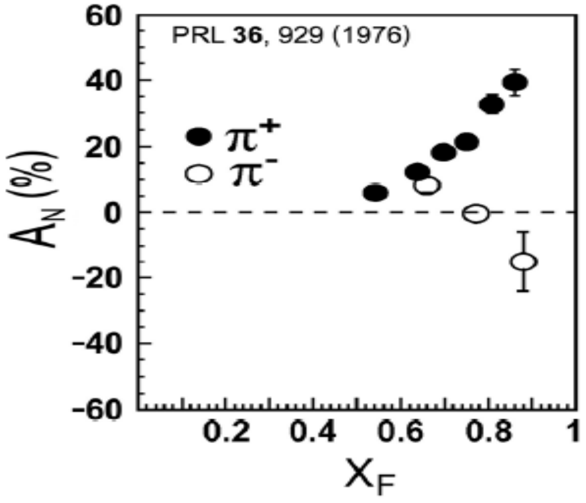
- ☐ **Polarized cross section:**

$$\boxed{\begin{aligned} \Delta\sigma(\vec{s}_\perp) &\propto L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp) \right] \\ &= L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\nu\mu}(\vec{s}_\perp) \right] = 0 \end{aligned}}$$

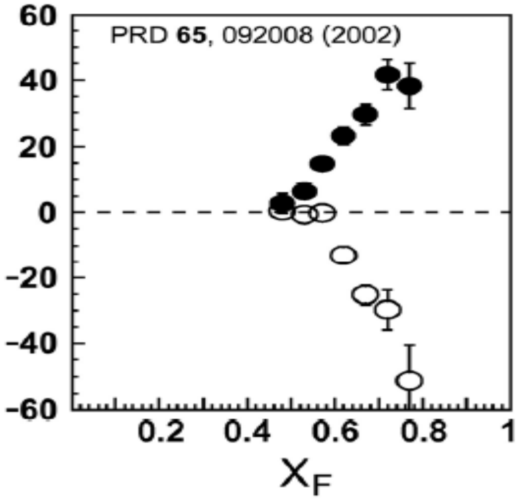
A_N in hadronic collisions

- ☐ A_N - consistently observed for over 40 years!

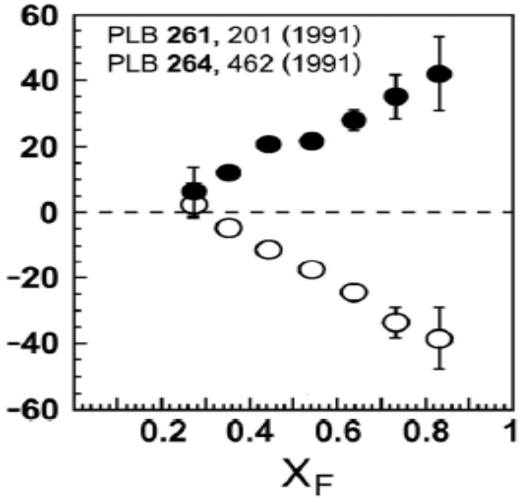
ANL – 4.9 GeV



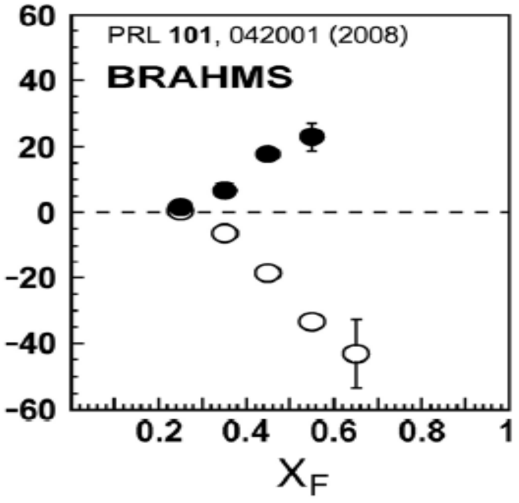
BNL – 6.6 GeV



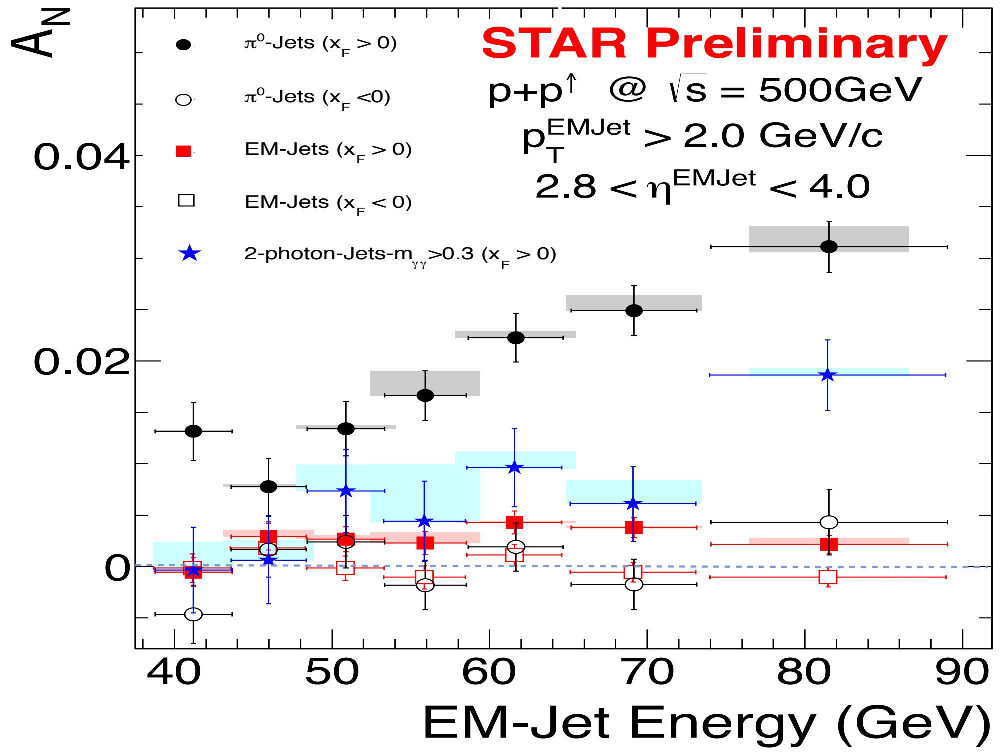
FNAL – 20 GeV

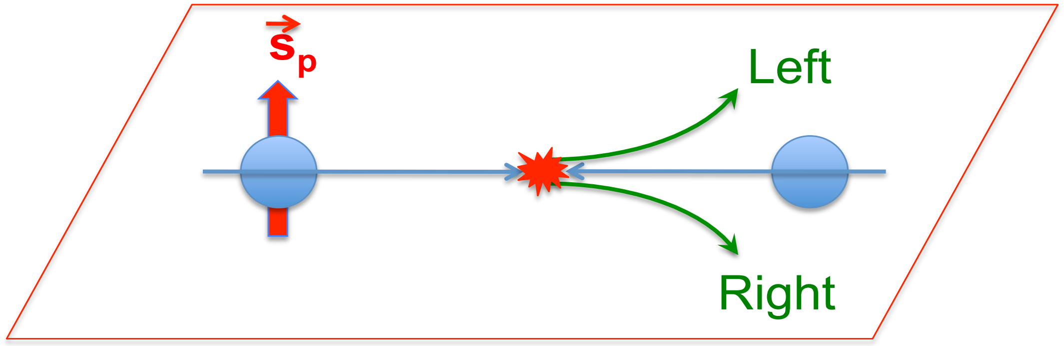


BNL – 62.4 GeV



- ☐ Survived the highest RHIC energy:





$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

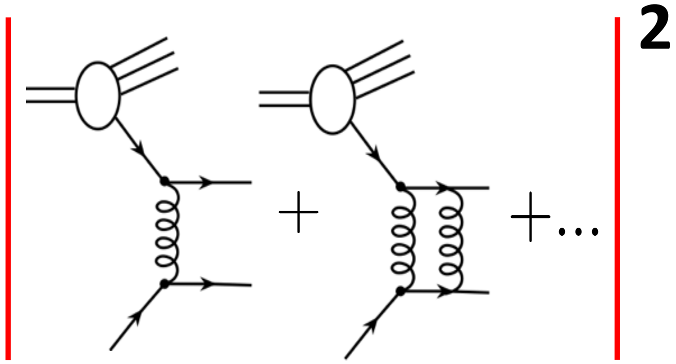
Do we understand this?

A_N in hadronic collisions

☐ **Early attempt:**

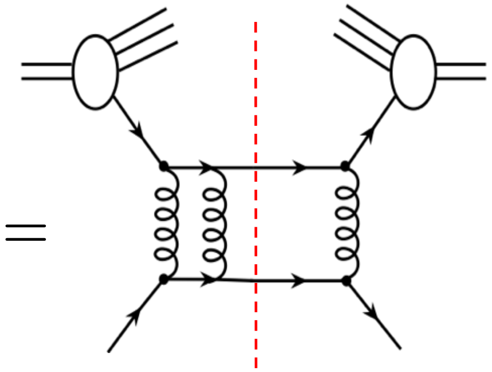
Kane, Pumplin, Repko, PRL, 1978

Cross section:



$$\sigma_{AB}(p_T,\vec{s}) \propto \left| \quad + \quad + \ldots \right|^2$$

Asymmetry:



$$\sigma_{AB}(p_T,\vec{s}) - \sigma_{AB}(p_T,-\vec{s}) = \quad \propto \alpha_s \frac{m_q}{p_T}$$

Too small to explain available data!

☐ **What do we need?** $A_N \propto i\vec{s}_p \cdot (\vec{p}_h \times \vec{p}_T) \Rightarrow i\epsilon^{\mu\nu\alpha\beta} p_{h\mu} s_\nu p_\alpha p'_{h\beta}$

Need a phase, a spin flip, enough vectors

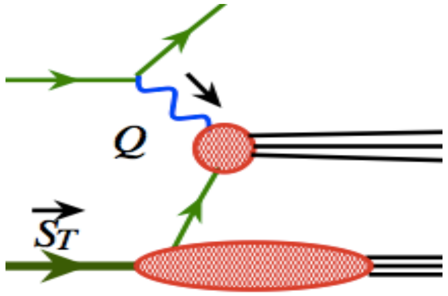
☐ **Vanish without parton's transverse motion:**

➡ **A direct probe for parton's transverse motion,**

Spin-orbital correlation, QCD quantum interference

Current understanding of A_N

☐ **Symmetry plays important role:**



Inclusive DIS
Single scale
Q

Parity
Time-reversal

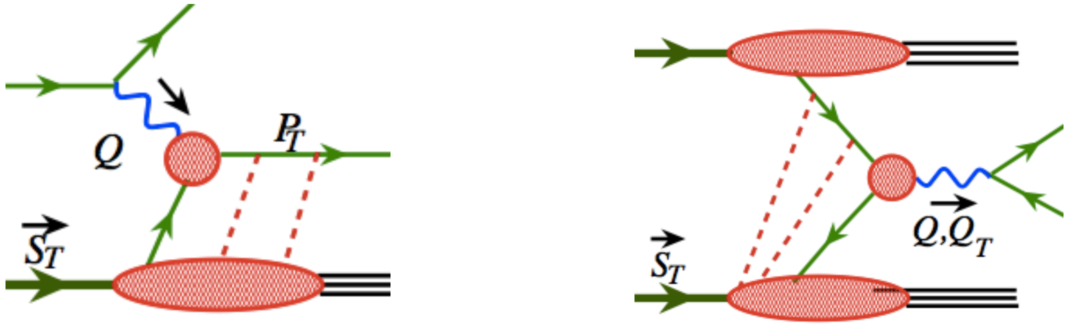
$\Rightarrow$ $A_N = 0$

☐ **One scale observables – $Q \gg \Lambda_{QCD}$:**

SIDIS: $Q \sim P_T$ DY: $Q \sim P_T$; Jet, Particle: P_T

Collinear factorization
Twist-3 distributions

☐ **Two scales observables – $Q_1 \gg Q_2 \sim \Lambda_{QCD}$:**

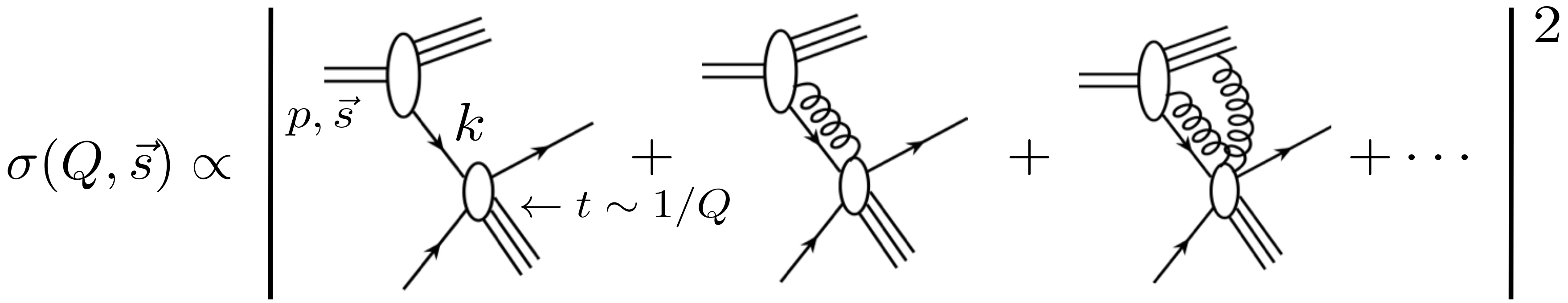


SIDIS: $Q \gg P_T$ DY: $Q \gg P_T$ or $Q \ll P_T$

TMD factorization
TMD distributions

How collinear factorization generates A_N?

☐ Collinear factorization beyond leading power:



$$\left(\frac{\langle k_\perp \rangle}{Q}\right)^n$$ **– Expansion**

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

Too large to compete!

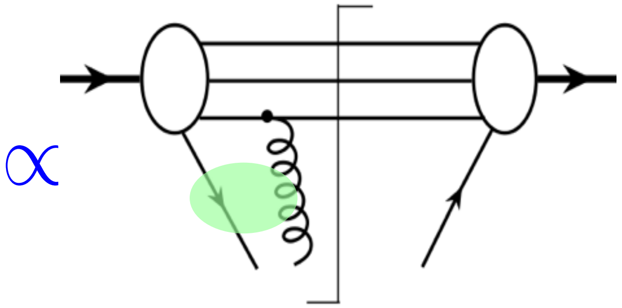
Three-parton correlation

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

☐ Single transverse spin asymmetry:

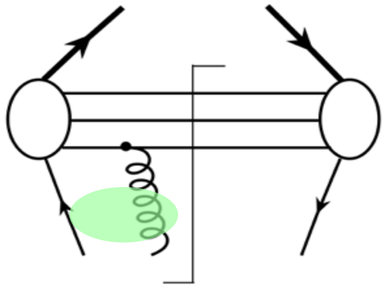
$$\Delta\sigma(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z) + \ldots$$

$$T^{(3)}(x,x) \propto$$



Qiu, Sterman, 1991, …

$$D^{(3)}(z,z) \propto$$



Kang, Yuan, Zhou, 2010

Integrated information on parton's transverse motion!

Needed Phase: **Integration of "dx" using unpinched poles**

Twist-3 distributions relevant to A$_N$

❑ **Twist-2 distributions:**

- **Unpolarized PDFs:**
- **Polarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

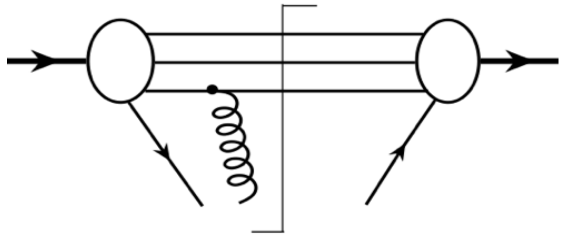
$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

$$\Delta q(x) \propto \langle P, S_{||}|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P, S_{||}\rangle$$

$$\Delta G(x) \propto \langle P, S_{||}|F^{+\mu}(0)F^{+\nu}(y)|P, S_{||}\rangle(i\epsilon_{\perp\mu\nu})$$

❑ **Two-sets Twist-3 correlation functions:**

No probability interpretation!



$$\widetilde{T}_{q,F} = \int\frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma^{\ +}(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

Kang, Qiu, 2009

$$\widetilde{T}_{G,F}^{(f,d)} = \int\frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\frac{1}{P+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma^{\ +}(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int\frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\left[i\, s_T^\sigma\, F_\sigma^{\ +}(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int\frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\frac{1}{P+}\langle P, s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma^{\ +}(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!

❑ **Twist-3 fragmentation functions:**

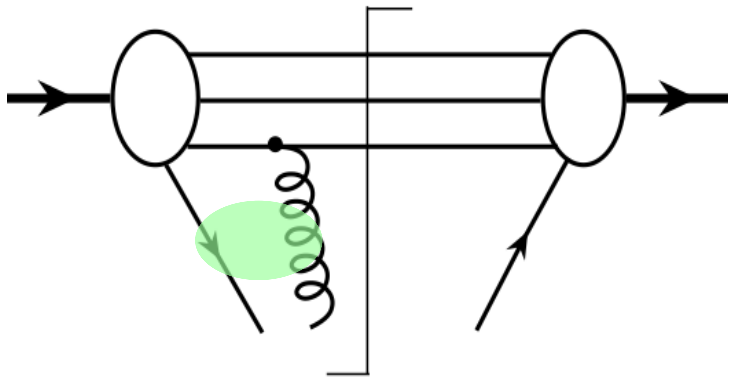
See Kang, Yuan, Zhou, 2010, Kang 2010

“Interpretation” of twist-3 correlation functions

❑ Measurement of direct QCD quantum interference:

Qiu, Sterman, 1991, …

$$T^{(3)}(x,x,S_\perp) \;\propto$$



Interference between a single active parton state and an active two-parton composite state

❑ “Expectation value” of QCD operators:

$$\langle P,s|\overline{\psi}(0)\gamma^+\psi(y^-)|P,s\rangle \;\longrightarrow\; \langle P,s|\overline{\psi}(0)\gamma^+\left[\epsilon_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta^{\ +}(y_2^-)\right]\psi(y^-)|P,s\rangle$$

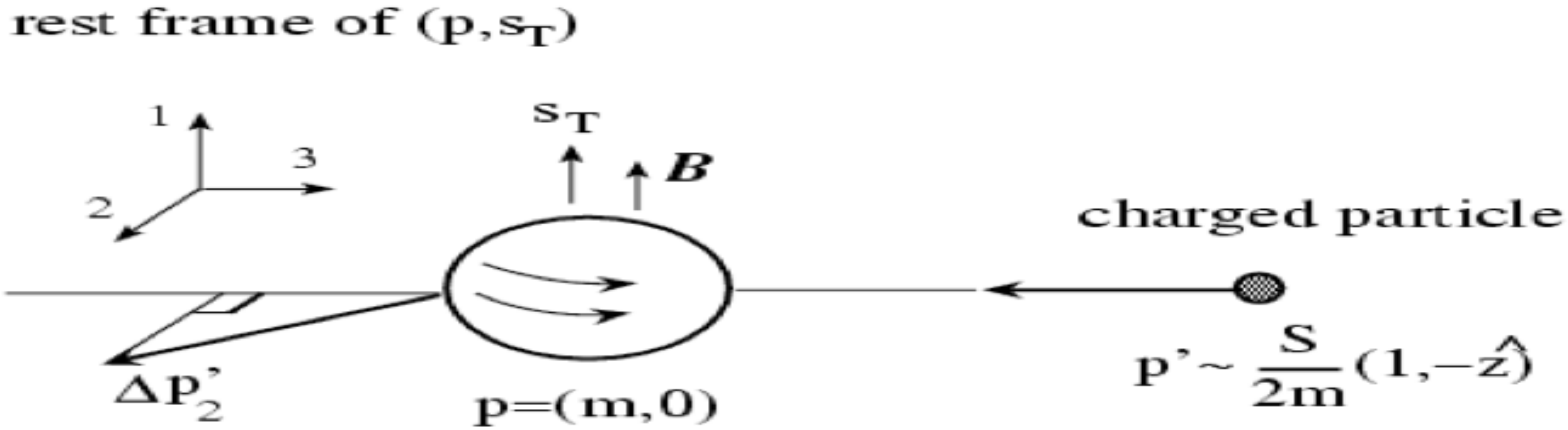
$$\langle P,s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P,s\rangle \;\longrightarrow\; \langle P,s|\overline{\psi}(0)\gamma^+\left[i\,g_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta^{\ +}(y_2^-)\right]\psi(y^-)|P,s\rangle$$

How to interpret the “expectation value” of the operators in RED?

A simple example

Qiu, Sterman, 1998

- **The operator in Red – a classical Abelian case:**



- **Change of transverse momentum:**

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3B_1 = ev_3F_{23}$$

- **In the c.m. frame:**

$$(m,\vec{0}) \to \bar{n} = (1,0,0_T), \quad (1,-\hat{z}) \to n = (0,1,0_T)$$

$$\Longrightarrow \frac{d}{dt}p_2' = e\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^{+}$$

- **The total change:**

$$\boxed{\Delta p_2' = e\int dy^-\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^{+}(y^-)}$$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Test QCD at twist-3 level

❑ Scaling violation – “DGLAP” evolution:

Kang, Qiu, 2009

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \underbrace{\begin{pmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{pmatrix}}_{(x,\, x+x_2,\, \mu,\, s_T)} = \underbrace{\begin{pmatrix} K_{qq} & K_{q\Delta q} & K^{(f)}_{qG} & K^{(d)}_{qG} & K^{(f)}_{q\Delta G} & K^{(d)}_{q\Delta G} \\ K_{\Delta qq} & K_{\Delta q\Delta q} & K^{(f)}_{\Delta qG} & K^{(d)}_{\Delta qG} & K^{(f)}_{\Delta q\Delta G} & K^{(d)}_{\Delta q\Delta G} \\ K^{(f)}_{Gq} & K^{(f)}_{G\Delta q} & K^{(ff)}_{GG} & K^{(fd)}_{GG} & K^{(ff)}_{G\Delta G} & K^{(fd)}_{G\Delta G} \\ K^{(d)}_{Gq} & K^{(d)}_{G\Delta q} & K^{(df)}_{GG} & K^{(dd)}_{GG} & K^{(df)}_{G\Delta G} & K^{(dd)}_{G\Delta G} \\ K^{(f)}_{\Delta Gq} & K^{(f)}_{\Delta G\Delta q} & K^{(ff)}_{\Delta GG} & K^{(fd)}_{\Delta GG} & K^{(ff)}_{\Delta G\Delta G} & K^{(fd)}_{\Delta G\Delta G} \\ K^{(d)}_{\Delta Gq} & K^{(d)}_{\Delta G\Delta q} & K^{(df)}_{\Delta GG} & K^{(dd)}_{\Delta GG} & K^{(df)}_{\Delta G\Delta G} & K^{(dd)}_{\Delta G\Delta G} \end{pmatrix}}_{(\xi,\, \xi+\xi_2;\, x,\, x+x_2,\, \alpha_s)} \underbrace{\otimes \begin{pmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{pmatrix}}_{\int d\xi \int d\xi_2}$$

❑ Evolution equation – consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

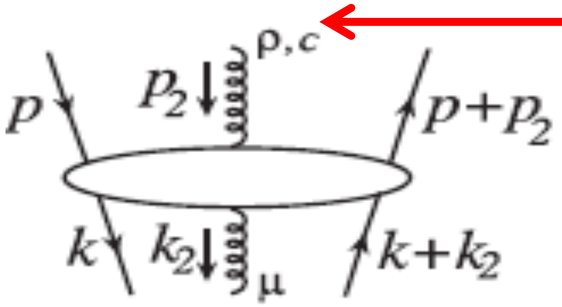
DGLAP for f_2: $\dfrac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

Evolution for f_3: $\dfrac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\dfrac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)}\right) \otimes f_3$

Evolution kernels – an example

Kang, Qiu, 2009

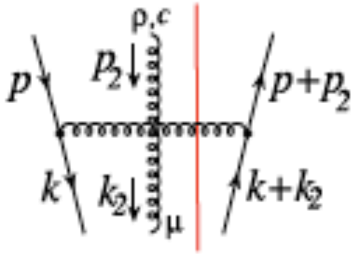
❑ Quark to quark:



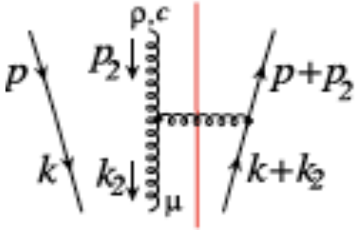
$$\mathcal{P}^{(\mathrm{LC})}_{q,F} = \frac{1}{2}\gamma \cdot P\left(\frac{-1}{\xi_2}\right)(i\epsilon^{s_T\rho n\bar{n}})\tilde{\mathcal{C}}_q$$

$$\mathcal{V}^{\mathrm{LC}}_{q,F} = \frac{\gamma^+}{2P^+}\delta\left(x - \frac{k^+}{P^+}\right)x_2\delta\left(x_2 - \frac{k_2^+}{P^+}\right)(i\epsilon^{s_T\sigma n\bar{n}}[-g_{\sigma\mu}]\mathcal{C}_q$$

❑ Feynman diagram calculation:

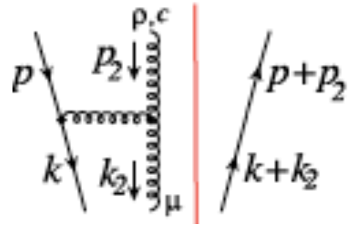


$$\int d\xi \int d\xi_2\ \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\ \delta(\xi_2)\frac{1}{\xi}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[C_F - \frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1+z^2}{1-z}\right)$$



$$\int d\xi \int d\xi_2\ \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\ \delta(\xi - x)\frac{1}{\xi_2}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1}{2}\frac{2x+\xi_2}{x+\xi_2}\right)$$

$$-\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\ \mathcal{T}_{q,F}(x,x)$$



$$\int d\xi \int d\xi_2\ \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\ \delta(\xi+\xi_2 - x)\frac{1}{\xi}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1}{2}\frac{1+z}{1-z}\right)$$

$$-\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\ \mathcal{T}_{q,F}(x,x)$$

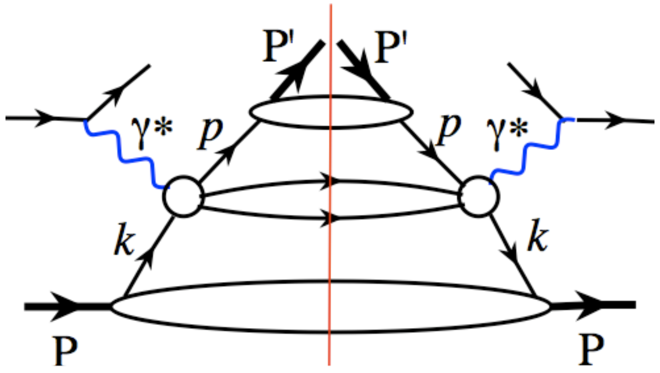
+ Virtual loop diagrams

How TMD factorization generates A_N?

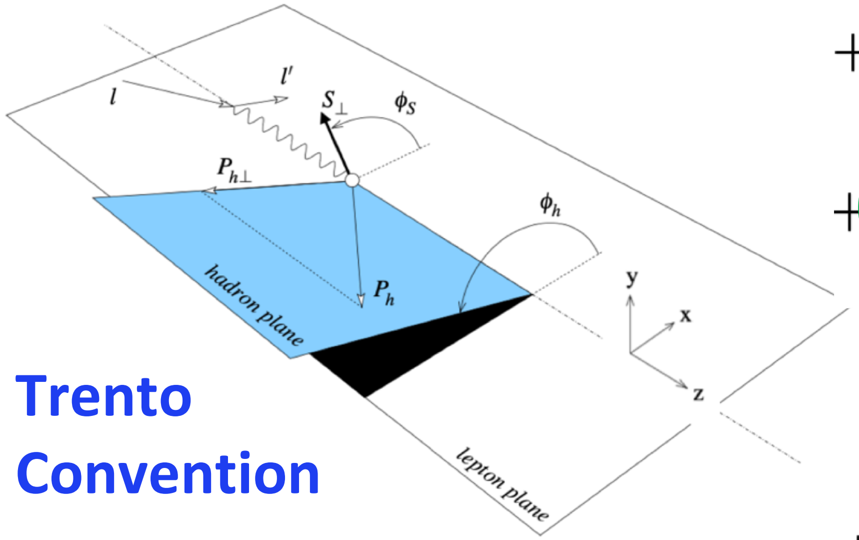
See lectures by Stewart

❑ SIDIS – “one-photon approximation”:

- 18 Structure functions
- A_N = at least one of 6 F_{UT} structure functions is finite!



$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = {} & \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon\,F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
& +\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
& +S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
& +S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
& +\boxed{|\boldsymbol{S}_\perp|}\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
& \quad +\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad +\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
& +|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
& \quad +\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$



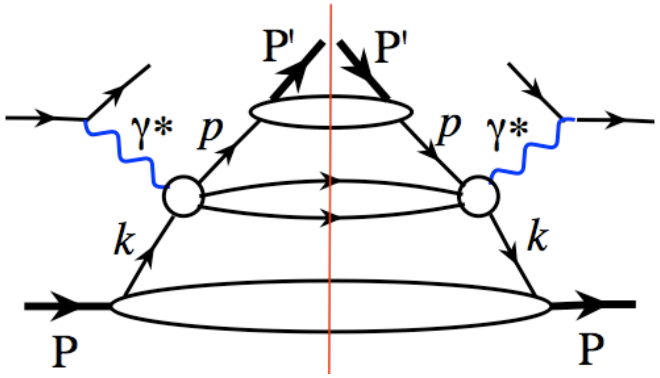
Trento Convention

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2}$$

How TMD factorization generates A_N?

See lectures by Stewart

❑ TMD factorization for SIDIS:



In the photon-hadron frame, 8 of 18 structure functions can be factorized in terms of convolution of TMDs at leading power

- **Unpolarized:**

$$F_{UU,T} = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) f^a(x, p_T^2)\, D^a(z, k_T^2)$$

- **Transverse Single-Spin Asymmetry – Sivers:**

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right] \qquad\qquad \hat{\boldsymbol{h}} = \frac{\boldsymbol{P}_{h\perp}}{|\boldsymbol{P}_{h\perp}|}$$

- **Transverse Single-Spin Asymmetry – Collins:**

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right]$$

With:

$$\mathcal{C}[w\, f\, D] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

Orbital angular momentum

See lectures by Burkardt

OAM: Correlation between parton's position and its motion – in an averaged (or probability) sense

- **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

- **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

- **Difference between them:**
  - generated by a "torque" of color Lorentz force

Hatta, Yoshida, Burkardt, Meissner, Metz, Schlegel, …

$$\mathcal{L}_q^3 - L_q^3 \propto \int \frac{dy^- d^2y_T}{(2\pi)^3} \langle P'|\overline{\psi}_q(0)\frac{\gamma^+}{2}\int_{y^-}^{\infty} dz^- \Phi(0,z^-) \times \underbrace{\sum_{i,j=1,2}\left[\epsilon^{3ij} y_T^i F^{+j}(z^-)\right]}_{\text{“Chromodynamic torque”}} \Phi(z^-,y)\psi(y)|P\rangle_{y^+=0}$$

Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of g_2

Partonic motion seen by a hard probe – GTMD

See lectures by Burkardt

☐ **Fully unintegrated distribution:**

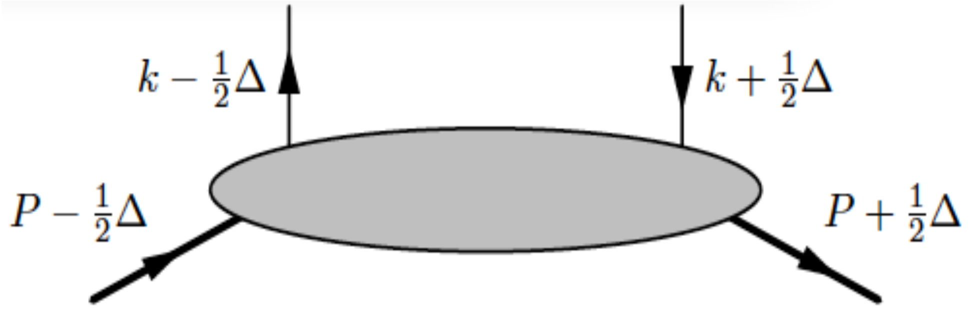
Meissner, Metz, Schiegel, 2009

$$W^{[\Gamma]}_{\lambda\lambda'}(P,k,\Delta,N;\eta) = \frac{1}{2}\int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z}\, \langle p',\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\,\Gamma\,\mathcal{W}(-\tfrac{1}{2}z,\tfrac{1}{2}z\,|\,n)\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle$$

– not factorizable in general

☐ **Generalized TMDs – hard probe:**

$$\mathcal{W}(x,k_T,\Delta)_\Gamma = \int dk^2\, W(P,k,\Delta)_\Gamma$$

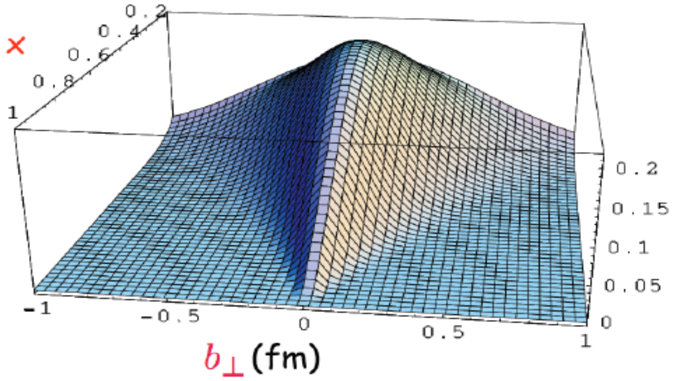


– could be factorized assuming on-shell parton for the hard probe

☐ **Wigner function:**

Belitsky, Ji, Yuan

$$W(x,k_T,b) \propto \int d^3\Delta\, e^{i\vec{b}\cdot\vec{\Delta}}\, \mathcal{W}(x,k_T,\Delta)_{\Gamma=\gamma^+}$$



Burkardt, 2002

☐ **Connection to all other known distributions:**

$W(x,k_T,b) \Rightarrow$ **Tomographic image of nucleon**

$$q(x,b_\perp) = \int d^2k_T\, db^-\, W(x,k_T,b)_{\gamma^+}$$

$\mathcal{W}(x,k_T,\Delta)_\Gamma$

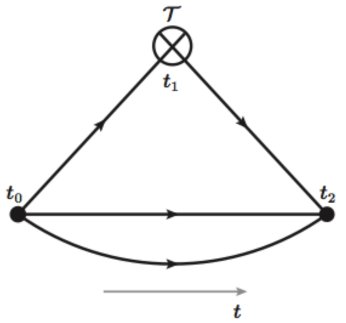
$\Rightarrow$ **TMDs** $(\Delta=0)$, **GPDs** $(\int d^2k_T)$, **PDFs** $(\Delta=0, \int d^2k_T)$

Nucleon spin and OAM from lattice QCD

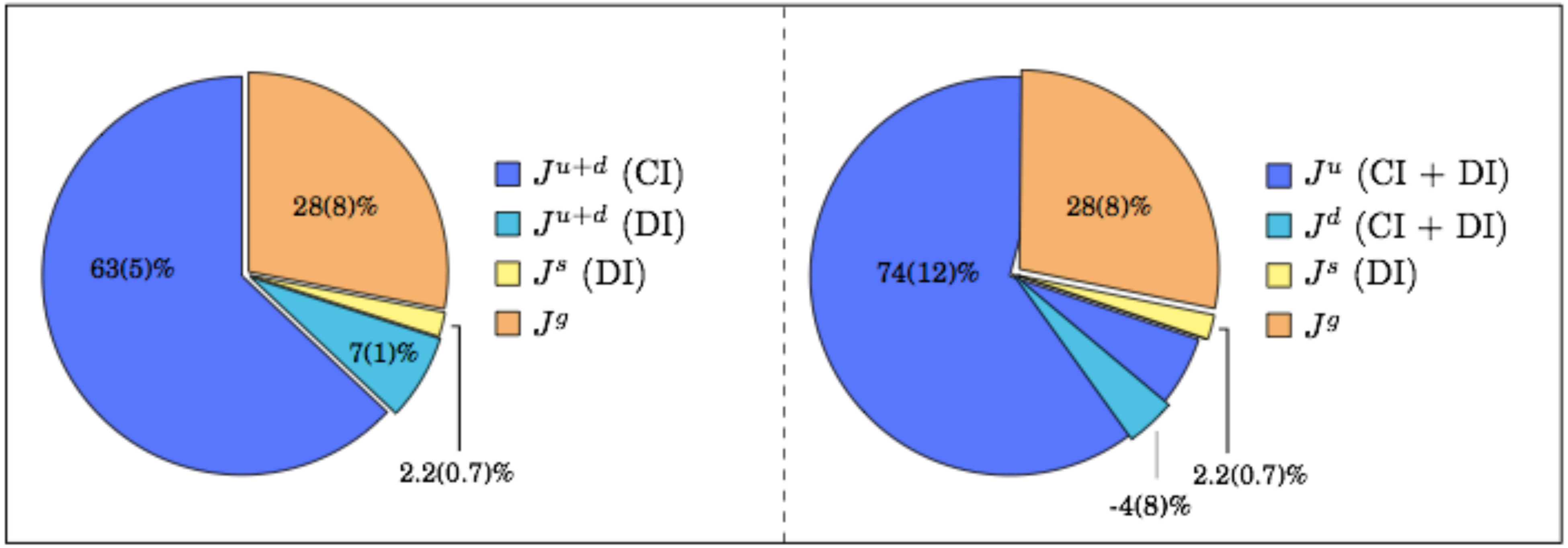
See lectures by Shanahan

- ❑ **QCD Collaboration:**

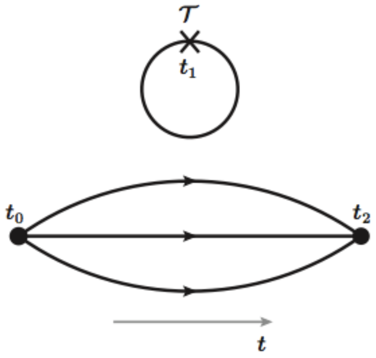
Deka *et al. Phys.Rev.D91 (2015) 014505*



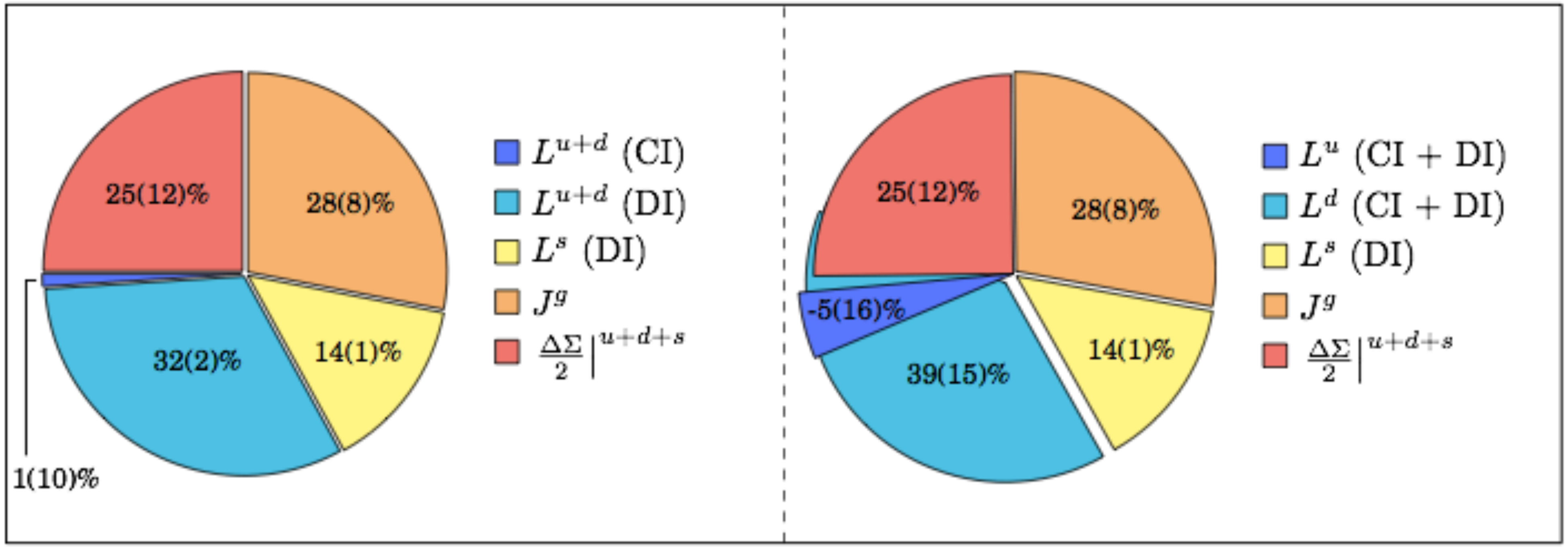
Connected Interaction (CI)



(b)



Disconnected Interaction (DI)



(c)

Lattice QCD calculation of hadron structure

See lectures by Ji

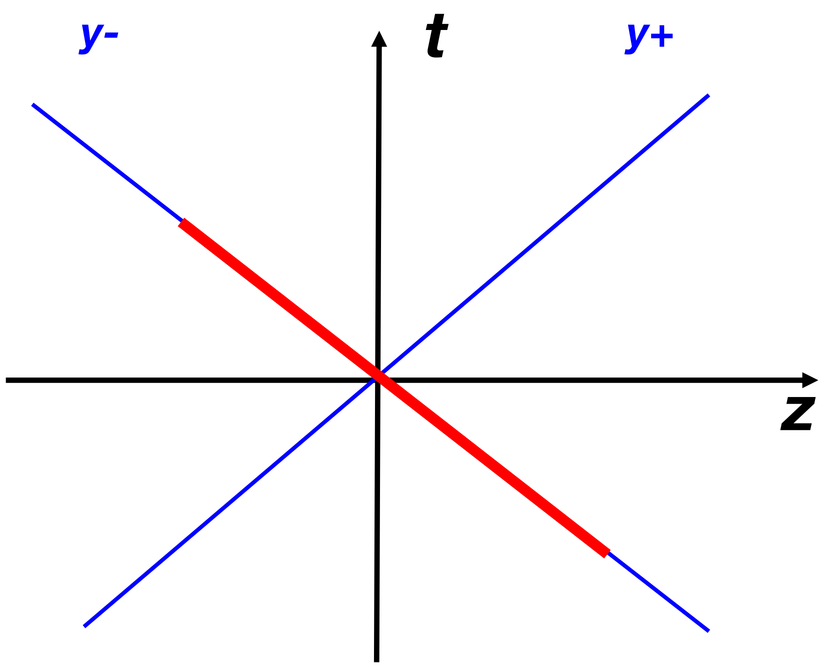
☐ **Hadron structure measured with a hard probe:**

Probing operators living on the "light-Cone",

$$q(x) \propto \mathrm{F.T.}\langle P|\overline{\psi}_q(-y^-)\Gamma\Phi\psi_q(y^-)|P\rangle|_{y^+=0,y_\perp=0_\perp}$$

PDFs are boost invariant with twist-2 operators

Such matrix elements are non-perturbative, and Cannot be calculated by lattice QCD directly, because of its Euclidean space formulation



Ji, arXiv:1305.1539

☐ **Quasi-PDFs approach:**

$$\tilde{q}(\tilde{x}, P_z) \propto \mathrm{F.T.}\langle P|\overline{\psi}_q(-z)\Gamma\Phi\psi_q(z)|P\rangle|_{y^0=0,y_\perp=0_\perp}$$

$$\to q(x) \quad \text{as } P_z \to \infty$$

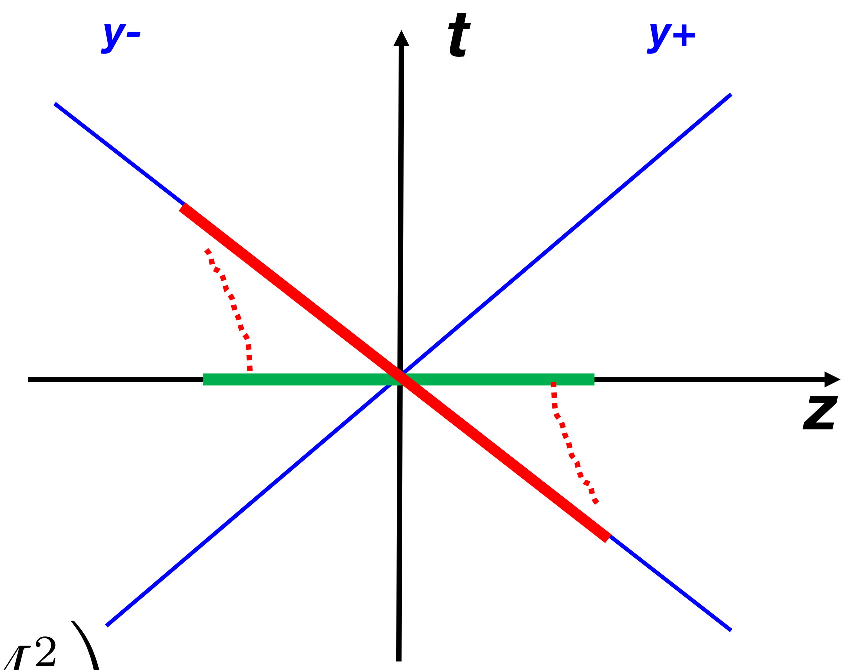
Calculable in LQCD

Quasi-PDFs are NOT boost invariant, not by twist-2 operators

In Lattice QCD calculation, difficult to take $P_z \to \infty$

Matching - Formulated in LaMET:

$$\tilde{q}(\tilde{x}, P_z) = \int_x^1 \frac{dx}{x} Z\left(\frac{\tilde{x}}{x}, \frac{\mu}{P_z}\right) q(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\tilde{x}^2(1-\tilde{x})P_z^2}, \frac{M^2}{P_z^2}\right)$$



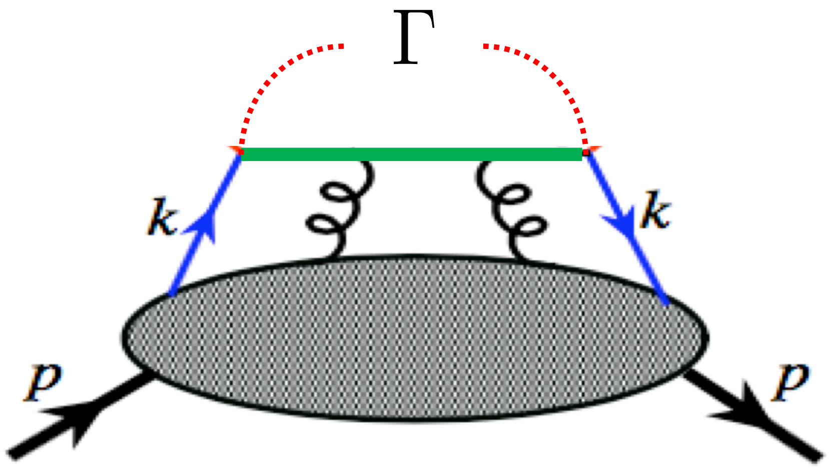
Approach to light-cone under a boost $P_z \to \infty$

Extracting PDFs requires solving the inverse problem

Lattice QCD calculation of hadron structure

❑ **Space-like parton correlation functions (PCFs):**

$$\langle P|\overline{\psi}_q(-z)\Gamma\Phi\psi_q(z)|P\rangle|_{y^0=0,y_\perp=0_\perp} \qquad \langle P|F^{\alpha\beta}(-z)\Phi F^{\mu\nu}(z)|P\rangle|_{y^0=0,y_\perp=0_\perp}$$



Unlike measured cross section,

- They are not physically measured observable
- Their value depend on UV renormalization
- They have UV power divergence
- They are multiplicatively renormalizable

❑ **Renormalization of space-like PCFs:**

UV divergence is a property of the operator, not the state

$$\langle P|\mathcal{O}(z)|P\rangle_{\rm Ren} \equiv \frac{\langle P|\mathcal{O}(z)|P\rangle}{\langle \mathrm{RS}|\mathcal{O}(z)|\mathrm{RS}\rangle}$$

Renormalization scheme = different choice of the state $|\mathrm{RS}\rangle$

RI-MOM for quasi-PDFs: $|\mathrm{RS}\rangle = \text{An off-shell parton state}$

Pseudo-PDFs: $|\mathrm{RS}\rangle = |P_z = 0\rangle$

Vacuum-state: $|\mathrm{RS}\rangle = |\Omega\rangle$

arXiv:1705.11193
arXiv:1709.04933

arXiv:1706.05373

arXiv:1810.00048
arXiv:2006.12370

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Lattice QCD calculation of hadron structure

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

☐ Short-distance factorization approach:

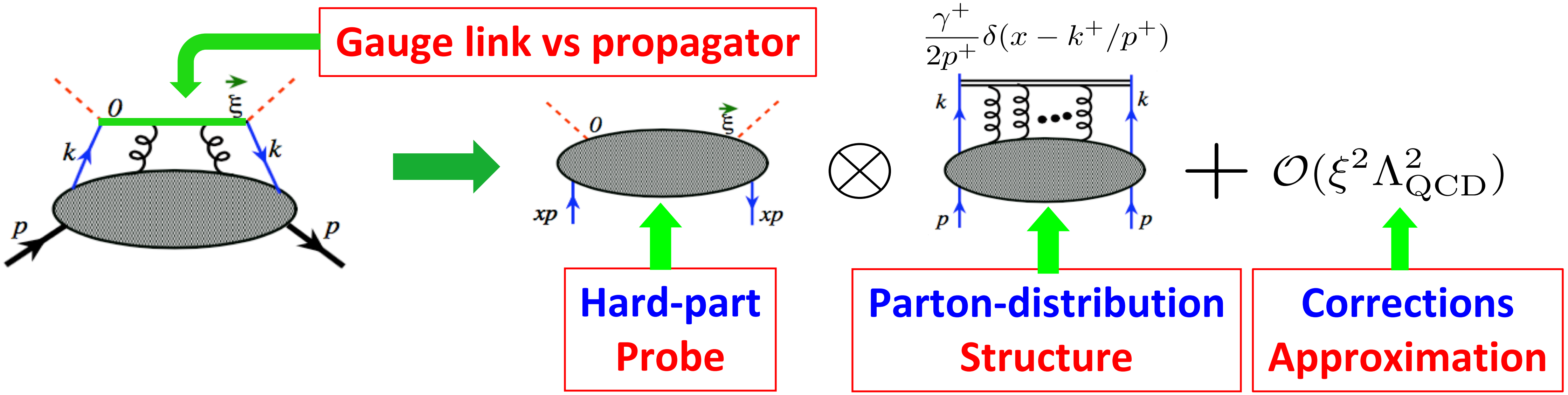
- **Single hadron matrix element:** $\sigma_n(\omega,\xi^2,P^2)=\langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle$

  with Ioffe time: $\omega\equiv P\cdot\xi,\ \xi^2\neq 0,\ \text{and}\ \xi_0=0;$

- **Two-parton correlator:** $\mathcal{O}_q(\xi)=Z_q^{-1}(\xi^2)\overline{\psi}_q(\xi)\,\gamma\cdot\xi\,\Phi(\xi,0)\,\psi_q(0)$

  Same operator for quasi-PDFs $\Phi(\xi,0)=\mathcal{P}e^{-ig\int_0^1\xi\cdot A(\lambda\xi)\,d\lambda}$

- **Two-current correlator:** $\mathcal{O}_{j_1j_2}(\xi)\equiv\xi^{d_{j_1}+d_{j_2}-2}\,Z_{j_1}^{-1}\,Z_{j_2}^{-1}j_1(\xi)\,j_2(0)$



Gauge link vs propagator

Hard-part Probe

Parton-distribution Structure

Corrections Approximation

$$\longrightarrow\quad \sigma_n(\omega,\xi^2,P^2)=\sum_a\int_{-1}^{1}\frac{dx}{x}f_a(x,\mu^2)\times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2)+O(\xi^2\Lambda^2_{\rm QCD})$$

Extracting PDFs requires solving the inverse problem

Observables with identified hadrons – Phenomenology

See Chapter 5

THEORY

GPD

PDF

Hadron Structure

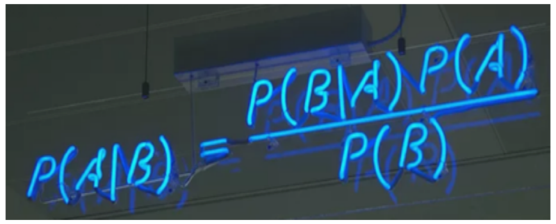
TMD PDF

Factorization

FF

Hadronization

TMD FF



DATA

USQCD

Extended Twisted Mass Collaboration

Bayesian Inference

Lat.

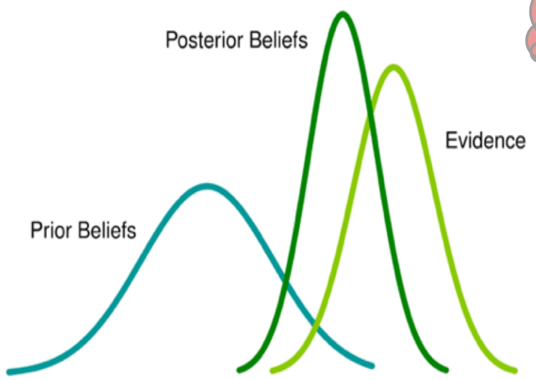
Exp.

CERN

Jefferson Lab

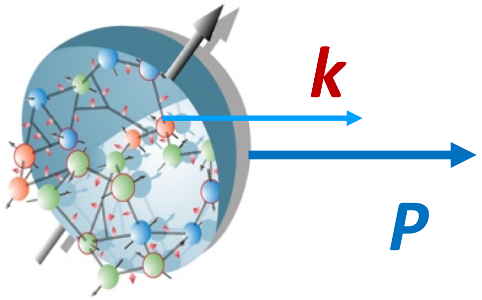
KEK

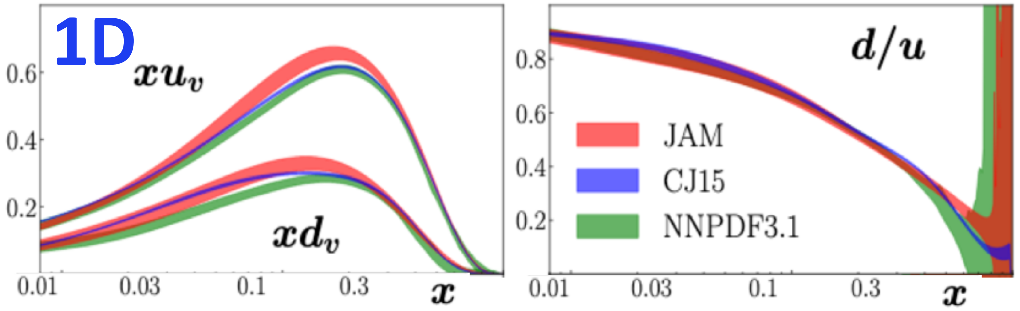
BROOKHAVEN NATIONAL LABORATORY



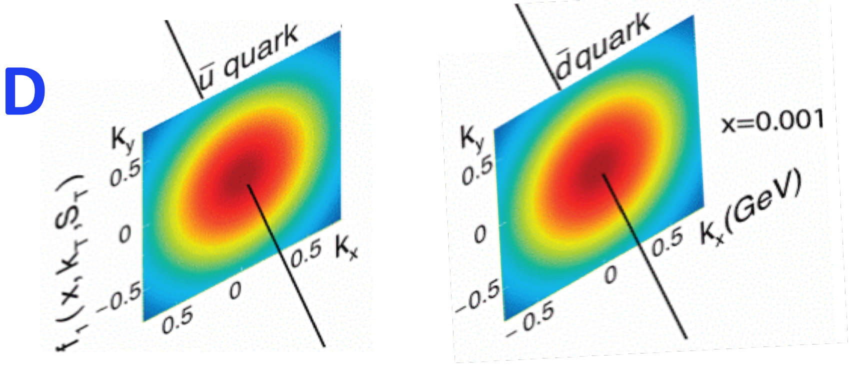
AI/ML
Synergy with EXP/COMP

GOAL





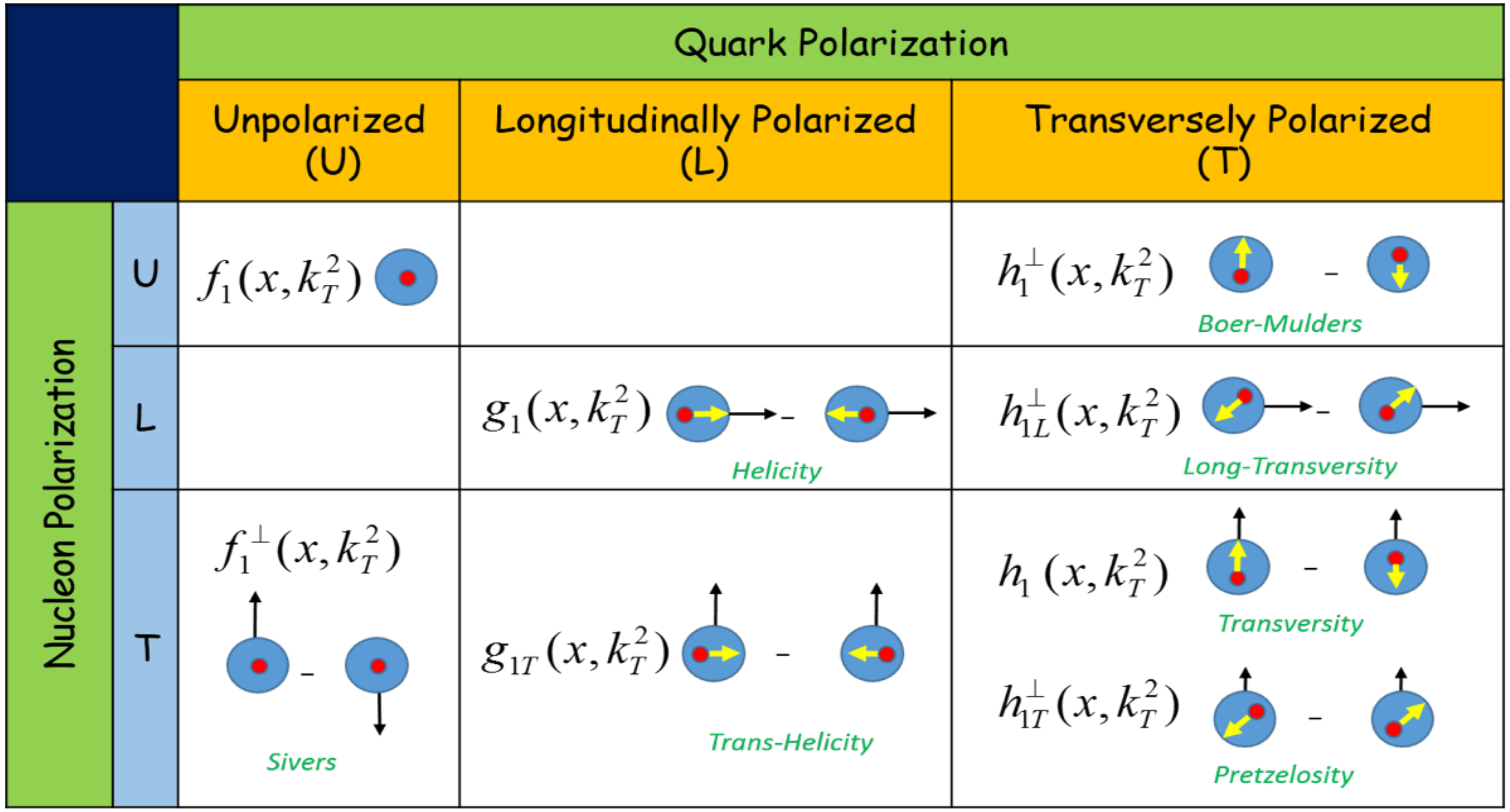
3D



Transverse momentum dependent PDFs (TMDs)

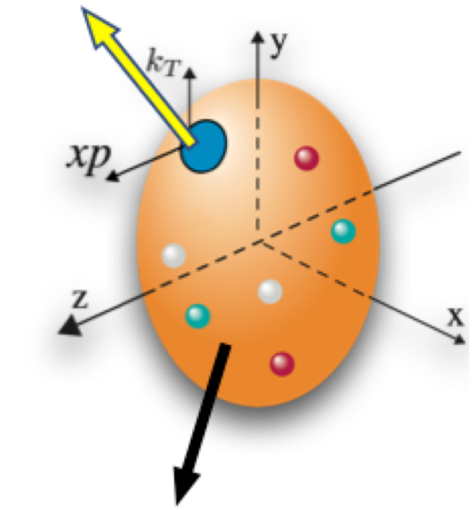
see lectures by Stewart

☐ Quark TMDs with polarization:



		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$		$h_1^\perp(x,k_T^2)$ - Boer-Mulders
	L		$g_1(x,k_T^2)$ - Helicity	$h_{1L}^\perp(x,k_T^2)$ - Long-Transversity
	T	$f_1^\perp(x,k_T^2)$ - Sivers	$g_{1T}(x,k_T^2)$ - Trans-Helicity	$h_1\ (x,k_T^2)$ - Transversity $h_{1T}^\perp(x,k_T^2)$ - Pretzelosity

Quark Polarization



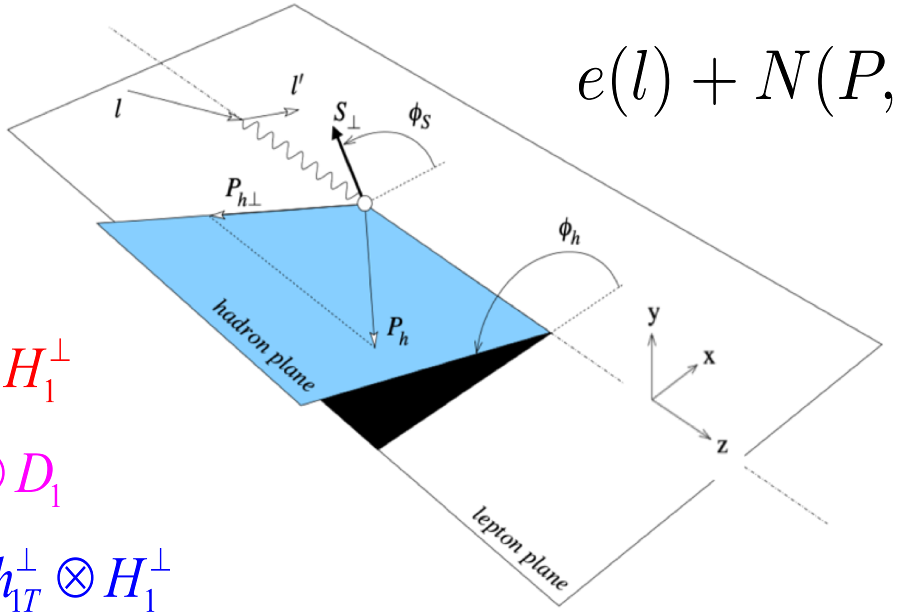
Nucleon Polarization

Analogous tables for:

- **Gluons** $f_1 \to f_1^g$ etc
- **Fragmentation functions**
- **Nuclear targets** $S \neq \frac{1}{2}$

☐ Semi-Inclusive DIS (SIDIS):

$$A_{UT} = \frac{1}{P}\frac{\sigma_{lN(\uparrow)} - \sigma_{lN(\downarrow)}}{\sigma_{lN(\uparrow)} + \sigma_{lN(\downarrow)}}$$



$$e(l) + N(P,\uparrow) \to e(l') + h(P_h) + X$$

Photon-hadron frame

$$A_{UT}^{Collins} \propto \left\langle \sin(\phi_h + \phi_S) \right\rangle_{UT} \propto h_1 \otimes H_1^\perp$$

$$A_{UT}^{Sivers} \propto \left\langle \sin(\phi_h - \phi_S) \right\rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \left\langle \sin(3\phi_h - \phi_S) \right\rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$

Two planes

Leptonic plane

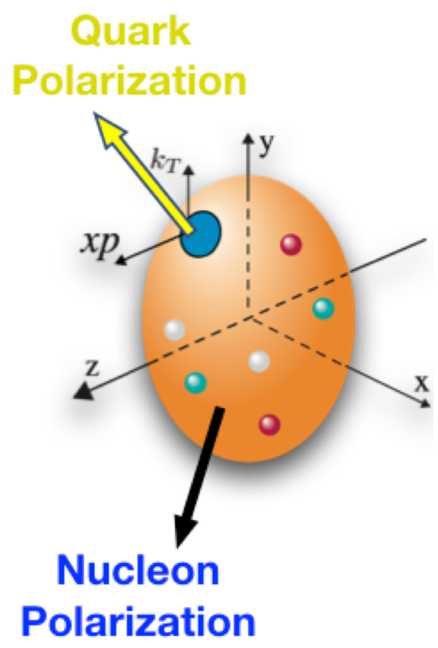
Hadronic plane

What can we learn from TMDs?

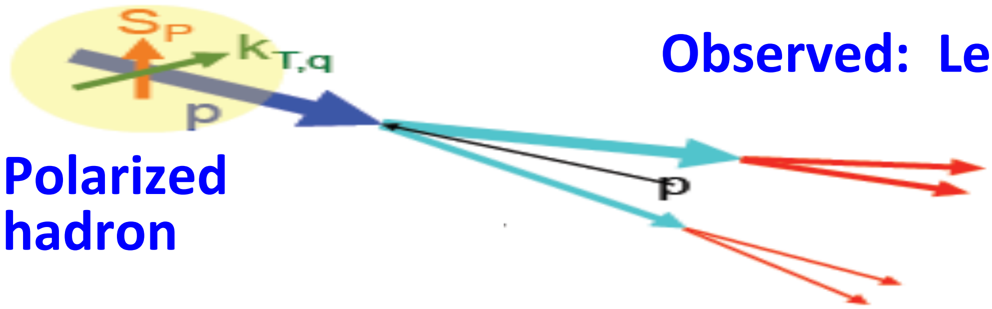
see lectures by Stewart

❑ Intrinsic & confined parton motion:

- ✧ Fundamental information sensitive to how partons are bound together
- ✧ Responsible for dynamical contribution to emergent hadron properties, such as spin, mass, ..



❑ Quantum correlation between hadron spin and parton motion:



Polarized hadron

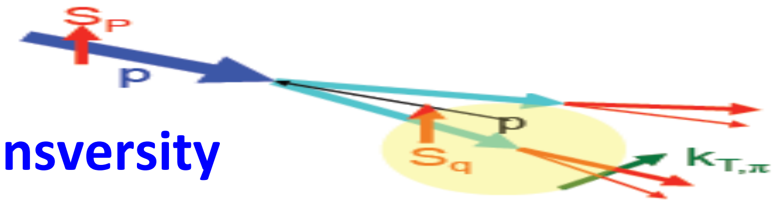
Observed: Left-Right Asymmetry

✧ Sivers effect – Sivers function

Hadron spin influences parton's transverse motion

❑ Quantum correlation between parton's spin and its hadronization:

Same Left-Right Asymmetry



Transversity

✧ Collins effect – Collins function

Parton's transverse polarization influences its hadronization

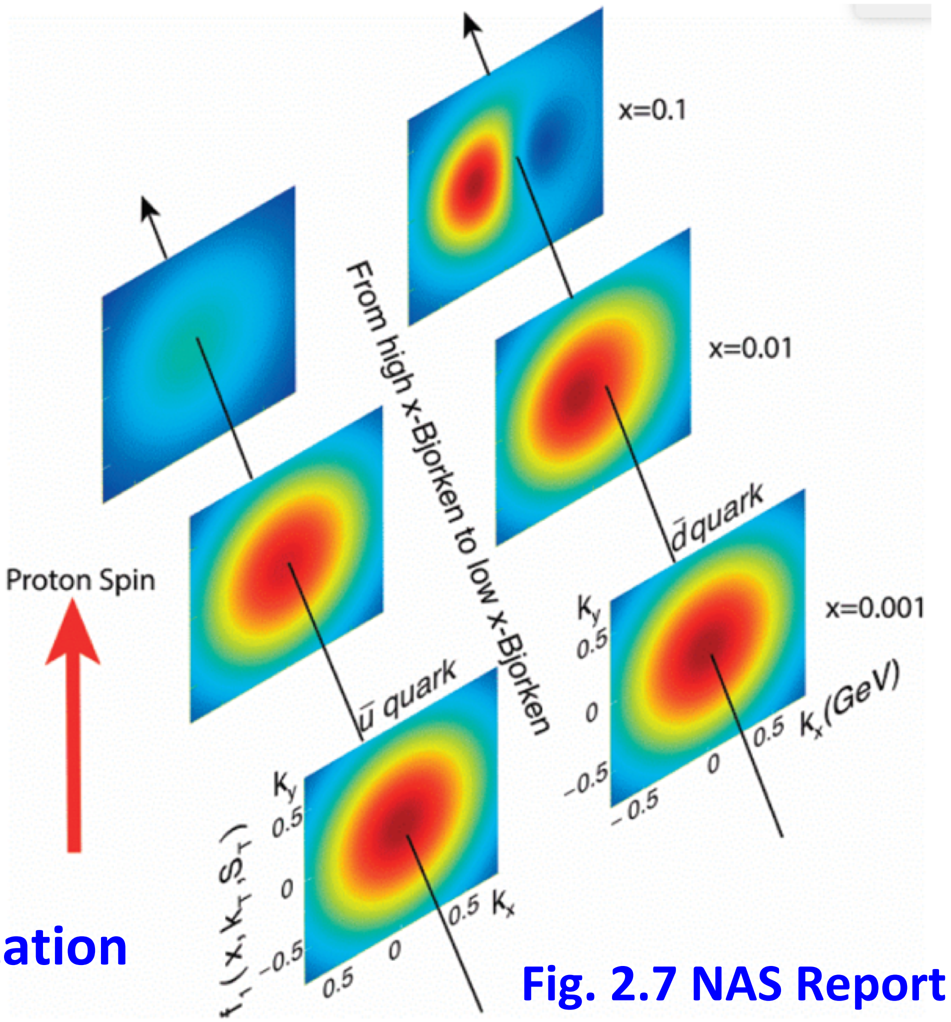
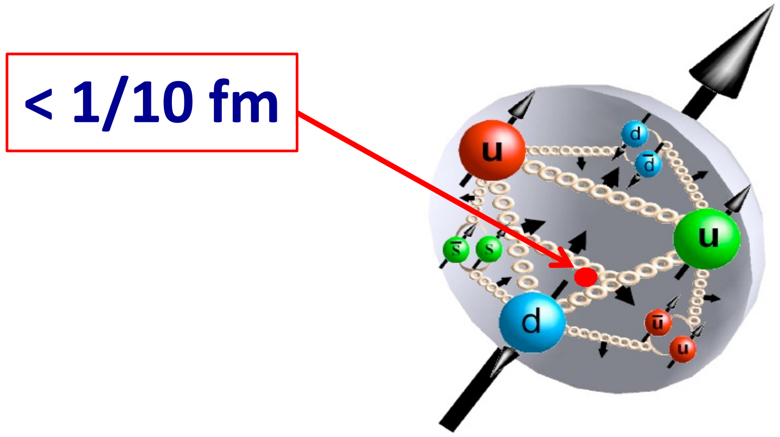


Fig. 2.7 NAS Report

Summary

☐ QCD has been extremely successful in interpreting and predicting high energy experimental data!



☐ But, we still do not know much about hadron structure – The emerging phenomena of QCD!

☐ Nuclear Femtography – QCD at a Fermi scale requires two-scale probes. Major advance in both measurement and factorization of two-scale observables!

☐ Lepton-Hadron facility, such as EIC, is ideal for two-scale observables

☐ TMDs and GPDs, accessible by high energy scattering with polarized beams, encode important information on hadron's 3D structure – distributions as well as motions of quarks and gluons

Thank you! Enjoy the rest of the school!

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