Intro to TMDs - Lecture #3

Iain Stewart

Review

Leading Quark TMDPDFs

Nucleon Spin   Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^{\perp}$ = − Boer-Mulders
L		g_1 = − Helicity	$h_{1L}^{\perp}$ = − Worm-gear
T	$f_{1T}^{\perp}$ = − Sivers	$g_{1T}^{\perp}$ = − Worm-gear	h_1 = − Transversity $h_{1T}^{\perp}$ = − Pretzelosity

Sign flip

$$(f_{1T}^{\perp})^{SIDIS} = -(f_{1T}^{\perp})^{DY},$$

$$(h_1^{\perp})^{SIDIS} = -(h_1^{\perp})^{DY},$$

rest equal in SIDIS & DY

Leading Quark TMDFFs

Hadron Spin   Quark Spin

Hadron Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons	D_1 = Unpolarized		$H_1^{\perp}$ = − Collins

Before we turn to ingredients for phenomenology there is one more interesting question

TMD Integral

Handbook § 2.9

$$\int d^2k_T \; f_{i/p}(x, k_T, \mu, \zeta) \overset{?}{=} f_i(x,\mu)$$

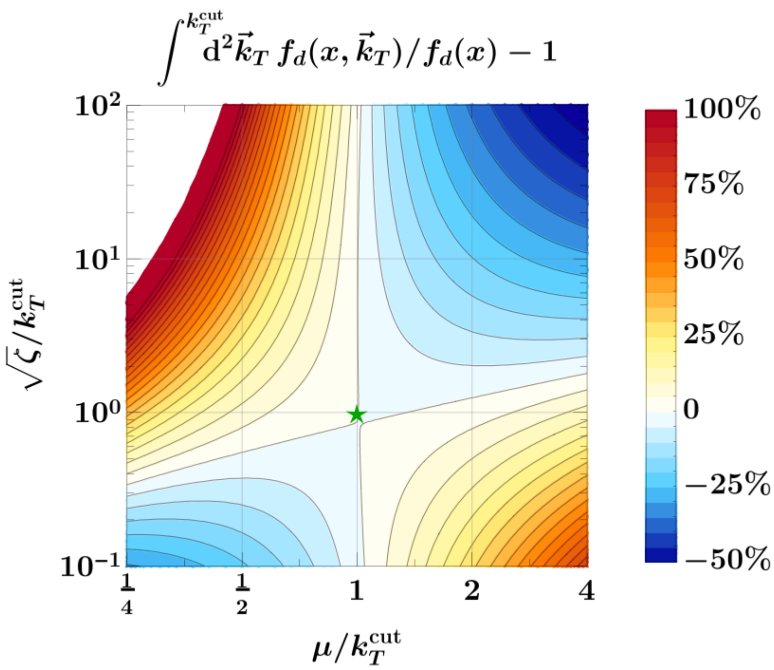
Naive : Yes

UV & RGE : No , μ & ζ dependence differs
($k_T \to \infty$) (μ, ζ) on LHS & RHS

In fact

$$\int_0^{k_T^{cut}} d^2k_T \; f_{i/p}(x, \vec{k}_T, \mu = k_T^{cut}, \zeta = (k_T^{cut})^2) \simeq f_i(x, \mu = k_T^{cut})$$

at percent level :



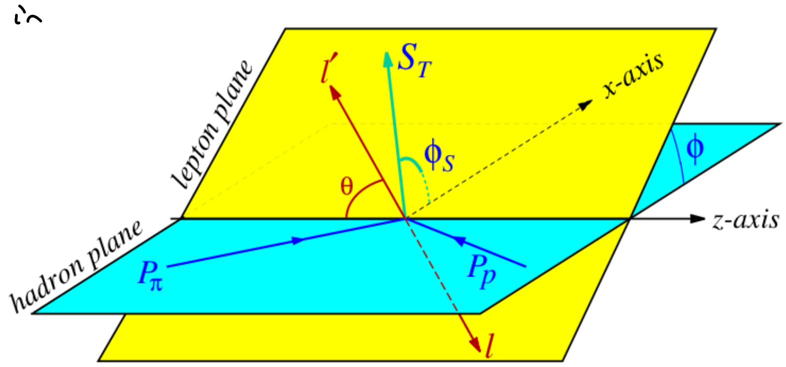
See Zhiquan Sun's talk tonight.

Polarized Drell-Yan $\pi(P_\pi) + p(P_p, S) \to \gamma^*/Z \to \ell^+\ell^- X$ (q^μ)

Angles θ, ϕ, ϕ_S defined in Collins-Soper frame

- $(\ell^+\ell^-)$ at rest
- $P_{\pi T} = P_{pT} = \frac{q_T}{2}$

Study Leading terms for $q_T \ll Q$



Structure Functions $F = F(x_\pi, x_p, q_T, Q^2)$

$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{\mathcal{F}Q^2}\Big\{\Big[(1+\cos^2\theta)F_{UU}^1 + \sin^2\theta\cos(2\phi)F_{UU}^{\cos 2\phi}\Big]$$
$$+ S_L \sin^2\theta\sin(2\phi)F_{UL}^{\sin 2\phi}$$
$$+ S_T(1-\cos^2\theta)\sin\phi_S F_{UT}^{\sin\phi_S}$$
$$+ S_T\sin^2\theta\Big[\sin(2\phi+\phi_S)F_{UT}^{\sin(2\phi+\phi_S)} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\Big]\Big\}$$

π unpol. — polarization of proton

Factorization

$$F_{UU}^1 = \mathcal{B}\big[\tilde{f}_{1,\pi}^{(0)}\ \tilde{f}_{1,p}^{(0)}\big], \quad \leftarrow \text{Unpol.}$$
$$F_{UU}^{\cos 2\phi} = M_\pi M_p\ \mathcal{B}\big[\tilde{h}_{1,\pi}^{\perp(1)}\ \tilde{h}_{1,p}^{\perp(1)}\big], \quad \leftarrow \text{Boer-Mulders}^2$$
$$F_{UL}^{\sin 2\phi} = -M_\pi M_p\ \mathcal{B}\big[\tilde{h}_{1,\pi}^{\perp(1)}\ \tilde{h}_{1L,p}^{\perp(1)}\big], \quad \leftarrow \text{'' '' \& Worm-Gear}$$
$$F_{UT}^{\sin\phi_S} = M_p\ \mathcal{B}\big[\tilde{f}_{1,\pi}^{(0)}\ \tilde{f}_{1T,p}^{\perp(1)}\big], \quad \leftarrow \text{Unpol \& Sivers}$$
$$F_{UT}^{\sin(2\phi-\phi_S)} = -M_\pi\ \mathcal{B}\big[\tilde{h}_{1,\pi}^{\perp(1)}\ \tilde{h}_{1,p}^{(0)}\big], \quad \leftarrow \text{Boer-Mulders \& Transversity}$$
$$F_{UT}^{\sin(2\phi+\phi_S)} = -\frac{M_\pi M_p^2}{4}\ \mathcal{B}\big[\tilde{h}_{1,\pi}^{\perp(1)}\ \tilde{h}_{1T,p}^{\perp(2)}\big]. \quad \leftarrow \text{Boer Mulders \& Pretzelosity}$$

$$\mathcal{B}[\tilde{f}_\pi^{(m)}\ \tilde{f}_p^{(n)}] \equiv \sum_i H_{i\bar{i}}(Q,\mu)\int_0^\infty \frac{db_T}{2\pi}\, b_T\, b_T^{m+n}\, J_{m+n}(q_T b_T)\, \tilde{f}_{i/p}^{(m)}(x_a, b_T, \mu, \zeta_a)\, \tilde{f}_{\bar{i}/\pi}^{(n)}(x_b, b_T, \mu, \zeta_b)$$

Polarized SIDIS

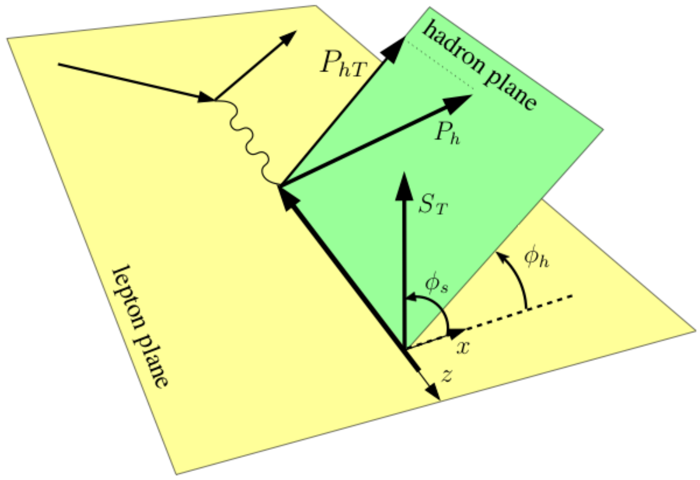
$\ell(\ell, \lambda) + p(P, S) \to \ell(\ell') + h(P_h) + X$

massless, helicity, unpol.

Frame for angles ϕ_h, ϕ_S

= Trento convention

- $\vec{q} \parallel \hat{z}$
- Leptons : x-z plane



Consider $Q \ll M_{W,Z}$

Leading Order in $q_T \ll Q$

Structure Functions

$F = F(x, z_h, P_{hT}, Q^2)$

$$\frac{d^6\sigma}{dx\,dy\,dz_h\,d\phi_S\,d\phi_h\,dP_{hT}^2} = \frac{\alpha_{em}^2}{x\,y\,Q^2}\left(1-y+\frac{1}{2}y^2\right)\Big[F_{UU,T} + \cos(2\phi_h)\,p_1\,F_{UU}^{\cos(2\phi_h)}$$
$$+ S_L \sin(2\phi_h)\,p_1\,F_{UL}^{\sin(2\phi_h)} + S_L\,\lambda\,p_2\,F_{LL}$$
$$+ S_T \sin(\phi_h-\phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)}$$
$$+ S_T \sin(\phi_h+\phi_S)\,p_1\,F_{UT}^{\sin(\phi_h+\phi_S)} + \lambda\,S_T\cos(\phi_h-\phi_S)\,p_2\,F_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+ S_T \sin(3\phi_h-\phi_S)\,p_1\,F_{UT}^{\sin(3\phi_h-\phi_S)}\Big]$$

beam pol. $\ell(\lambda)$ target pol. $p(S)$

$p_i = p_i(y)$

Factorization

$$F_{UU}(x,z_h,P_{hT},Q^2) = \mathcal{B}\left[\tilde{f}_1^{(0)}\,\tilde{D}_1^{(0)}\right],$$ ← unpol.

$$F_{UU}^{\cos 2\phi_h}(x,z_h,P_{hT},Q^2) = M_N M_h\,\mathcal{B}\left[\tilde{h}_1^{\perp(1)}\,\tilde{H}_1^{\perp(1)}\right],$$ ← Boer-Mulders * Collins

$$F_{UL}^{\sin 2\phi_h}(x,z_h,P_{hT},Q^2) = M_N M_h\,\mathcal{B}\left[\tilde{h}_{1L}^{\perp(1)}\,\tilde{H}_1^{\perp(1)}\right],$$ ← Worm-Gear * Collins

$$F_{LL}(x,z_h,P_{hT},Q^2) = \mathcal{B}\left[\tilde{g}_1^{(0)}\,\tilde{D}_1^{(0)}\right],$$ ← Helicity * Unpol.

$$F_{LT}^{\cos(\phi_h-\phi_S)}(x,z_h,P_{hT},Q^2) = M_N\,\mathcal{B}\left[\tilde{g}_{1T}^{\perp(1)}\,\tilde{D}_1^{(0)}\right],$$ ← Worm-Gear * Unpol

$$F_{UT}^{\sin(\phi_h+\phi_S)}(x,z_h,P_{hT},Q^2) = M_h\,\mathcal{B}\left[\tilde{h}_1^{(0)}\,\tilde{H}_1^{\perp(1)}\right],$$ ← Transversity * Collins

$$F_{UT}^{\sin(\phi_h-\phi_S)}(x,z_h,P_{hT},Q^2) = -M_N\,\mathcal{B}\left[\tilde{f}_{1T}^{\perp(1)}\,\tilde{D}_1^{(0)}\right],$$ ← Sivers * Unpol

$$F_{UT}^{\sin(3\phi_h-\phi_S)}(x,z_h,P_{hT},Q^2) = \frac{M_N^2 M_h}{4}\,\mathcal{B}\left[\tilde{h}_{1T}^{\perp(2)}\,\tilde{H}_1^{\perp(1)}\right],$$ ← Pretzelosity * Collins

$$\mathcal{B}[\tilde{f}^{(m)}\,\tilde{D}^{(n)}] \equiv x \sum_i e_i^2 \mathcal{H}_{ii}(Q^2,\mu) \int_0^\infty \frac{db_T}{2\pi}\, b_T\, b_T^{m+n} J_{m+n}(q_T b_T)\; \tilde{f}^{(m)}_{i/N}(x, b_T, \mu, \zeta_1)\, \tilde{D}^{(n)}_{h/i}(z_h, b_T, \mu, \zeta_2).$$

Other Processes: $e^+e^- \to h_1 + h_2 + X$ see § 2.11

→ 2 TMD FFs

Implementation

(a) Resummation $\alpha_s \ln\left(\frac{Q}{q_T}\right) \sim \alpha_s \ln(Q b_T) \sim 1$

$$\begin{aligned}\tilde{\sigma}^W(\boldsymbol{b}_T) = f_q(x_1) f_{\bar{q}}(x_2) C[\alpha_s] \exp\Big\{ &\frac{\alpha_s}{4\pi}\left(d_{12}L_b^2 + d_{11}L_b\right) \\ &+ \left(\frac{\alpha_s}{4\pi}\right)^2 \left(d_{23}L_b^3 + d_{22}L_b^2 + d_{21}L_b\right) \\ &+ \left(\frac{\alpha_s}{4\pi}\right)^3 \left(d_{34}L_b^4 + d_{33}L_b^3 + d_{32}L_b^2 + d_{31}L_b\right)\Big\} + \dots,\end{aligned}$$

LL NLL NNLL N³LL

Derived by Solving Renormalization Group Equations (RGE)

- $\mu \frac{d}{d\mu} \ln H(Q^2,\mu) = \gamma_\mu^H(Q,\mu) = 4\,\Gamma_{cusp}[\alpha_s] \ln\frac{Q}{\mu} + \gamma_\mu^H[\alpha_s]$

- $\mu \frac{d}{d\mu} \ln \tilde{f}_{i/p} = \gamma_\mu^q(\mu, \zeta) = -\Gamma_{cusp} \ln \zeta/\mu^2 + \gamma_\mu^q[\alpha_s]$

Note: always perturbative for $\mu \gg \Lambda_{QCD}$.

- $\zeta \frac{d}{d\zeta} \ln \tilde{f}_{i/p} = \frac{1}{2}\gamma_\zeta^q(\mu, b_T) \quad \left(= \frac{1}{2}\tilde{K}(b_T;\mu)\right)$

rapidity anom. dim = Collins-Soper Kernel

$$\gamma_y^q(\mu, b_T) = -2 \int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{cusp}[\alpha_s(\mu')] + \gamma_y^q[\alpha_s(1/b_T)]$$

Note : $\gamma_y^q(\mu, b_T)$ is non-perturbative for $b_T^{-1} \sim \Lambda_{QCD}$!

(b) Combine Pert. & Non-Pert

$b_T^{-1} \gg \Lambda_{QCD}$ $\quad$ $b_T^{-1} \sim \Lambda_{QCD}$

fit to TMO data

$$f_{i/h}(x, b_T, \mu, y) = f_{i/p}^{pert}(x, b^*(b_T), \mu, y) \; f^{NP*}(x, b_T)$$

$$\sum_j \int \frac{d\xi}{\xi} C_{ij}\left(\frac{x}{\xi}, b_T, \mu, y\right) f_j(\xi, \mu)$$

pert. $\quad$ Global fits

- $b^*(b_T)$ shields pert. from Landau Pole $\alpha_s(\Lambda_{QCD}) = \infty$
- f^{NP*}'s meaning depends on choice of b^*

(c) Y term (eg. DY)

$$\frac{d\sigma}{dQ\,dY\,dq_T} = \frac{d\sigma^W}{dQ\,dY\,dq_T} + \frac{d\sigma^Y}{dQ\,dY\,dq_T}$$

↑ Factorized, dominant $q_T \ll Q$

↑ $O(q_T^2/Q^2) + \cdots$ important when $q_T \sim Q$

(d) Pert. Orders $\quad$ LO , NLO (α_s) , NNLO (α_s^2) , ...

in H_{ij} , C_{ij} & Y-term

Global Fits

SV19 = Scimemi, Vladimirov (1912.06532)

Pavia19 = Bachetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici (1912.07550)

Common features:

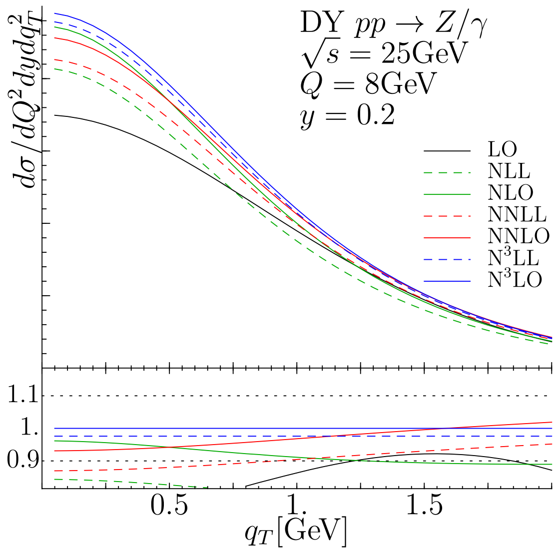
- Unpolarized Data with constraint: $q_T/Q < 0.2 - 0.25$ (4-6% power corrections)
- Perturbative accuracy: N3LL resummation + NNLO matching to PDF
- Neglect small contributions from Boer-Mulders terms (higher twist for pert. b_T)
- Common b_T dependence for all flavors

Global Fits

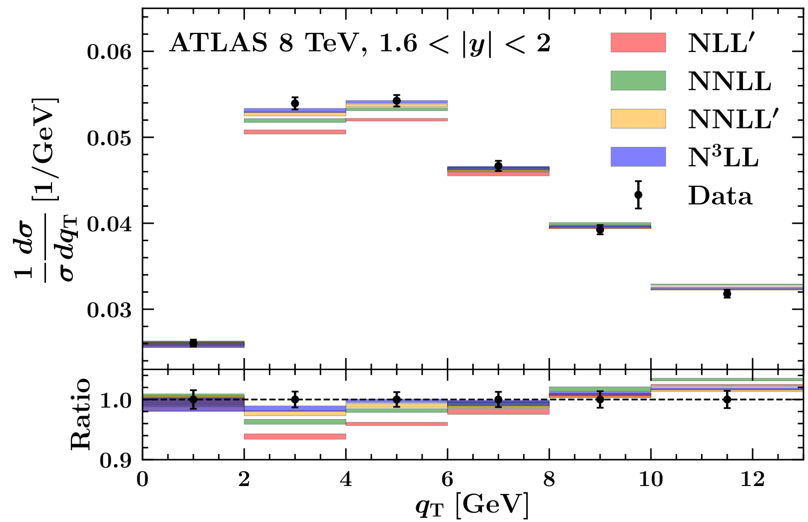
Common features:

Good Perturbative convergence:

SV19



Pavia19



Global Fits

Differences:

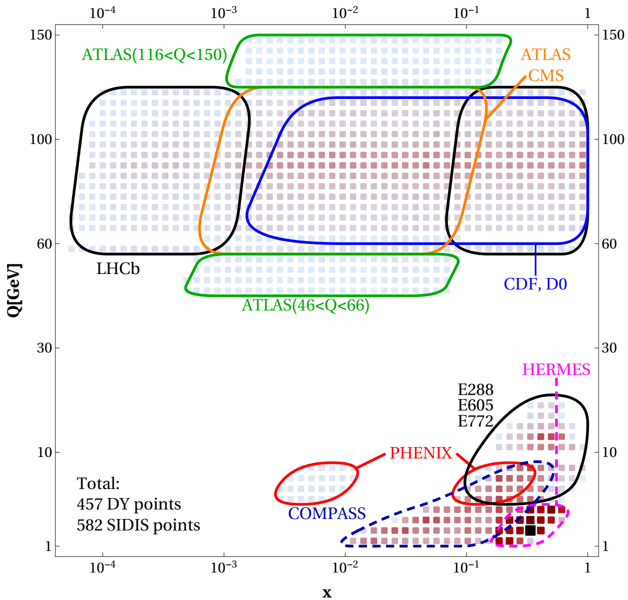
Some differences in solution of evolution equations

Datasets used

SV19

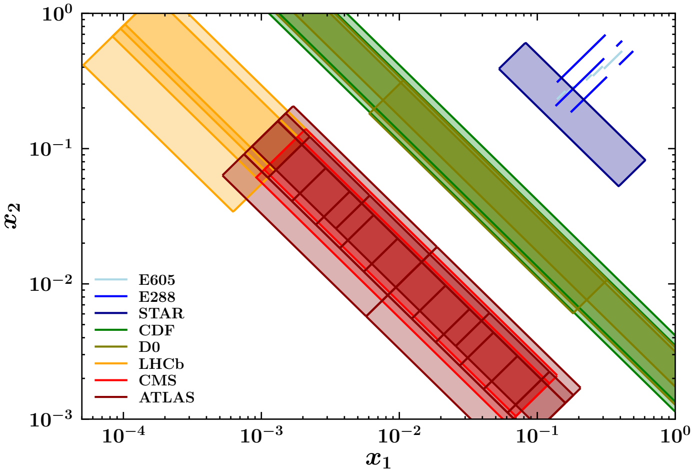
Drell-Yan (457 bins)

SIDIS (582 bins)



Pavia19

Drell-Yan (353 bins)



$$x_1 = Qe^y/\sqrt{s}, \quad x_2 = Qe^{-y}/\sqrt{s}$$

Global Fits

Differences:

Non-perturbative Models

SV19

TMDPDF: 5
TMDFF: 4
CS kernel: 2

$$f_{NP}(x,b) = \exp\left(-\frac{\lambda_1(1-x)+\lambda_2 x + x(1-x)\lambda_5}{\sqrt{1+\lambda_3 x^{\lambda_4}\boldsymbol{b}^2}}\boldsymbol{b}^2\right)$$

$$D_{NP}(x,b) = \exp\left(-\frac{\eta_1 z + \eta_2(1-z)}{\sqrt{1+\eta_3(\boldsymbol{b}/z)^2}}\frac{\boldsymbol{b}^2}{z^2}\right)\left(1+\eta_4\frac{\boldsymbol{b}^2}{z^2}\right)$$

$$\gamma_\zeta^q(\mu,b) = \gamma_\zeta^{q\,\mathrm{pert}}(\mu,b^*) - \frac{1}{2}c_0 b b^*$$

$$b^*(b) = \frac{b}{\sqrt{1+\boldsymbol{b}^2/B_{\mathrm{NP}}^2}}$$

Pavia19

TMDPDF: 7
CS kernel: 2

$$f_{\mathrm{NP}}(x,b_T) = \left[\frac{1-\lambda}{1+g_1(x)\frac{b_T^2}{4}} + \lambda\exp\left(-g_{1B}(x)\frac{b_T^2}{4}\right)\right]$$

$$g_1(x) = \frac{N_1}{x\sigma}\exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{x}{\alpha}\right)\right]$$

$$g_{1B}(x) = \frac{N_{1B}}{x\sigma_B}\exp\left[-\frac{1}{2\sigma_B^2}\ln^2\left(\frac{x}{\alpha_B}\right)\right]$$

$$\gamma_\zeta^q(\mu,b) = \gamma_\zeta^{q\,\mathrm{pert}}(\mu,b_*) - \frac{1}{2}(g_2 b_T^2 + g_{2B}b_T^4)$$

$$b_*(b_T) = b_{\max}\left(\frac{1-\exp\left(-\frac{b_T^4}{b_{\max}^4}\right)}{1-\exp\left(-\frac{b_T^4}{b_{\min}^4}\right)}\right)^{\frac{1}{4}}$$

Note: model form for b* used to split perturbative & non-perturbative parts

Global Fits

Fit Results:

SV19

$$\chi^2/N_{pt} = 1.06$$

NP-parameters			
RAD	$B_{\rm NP} = 1.93 \pm 0.22$	$c_0 = (4.27 \pm 1.05) \times 10^{-2}$	
TMDPDF	$\lambda_1 = 0.224 \pm 0.029$	$\lambda_2 = 9.24 \pm 0.46$	$\lambda_3 = 375. \pm 89.$
	$\lambda_4 = 2.15 \pm 0.19$	$\lambda_5 = -4.97 \pm 1.37$	
TMDFF	$\eta_1 = 0.233 \pm 0.018$	$\eta_2 = 0.479 \pm 0.025$	
	$\eta_3 = 0.472 \pm 0.041$	$\eta_4 = 0.511 \pm 0.040$	

Pavia19

$$\chi^2/N_{pt} = 1.02$$

Parameter	Value
g_2	0.036 ± 0.009
N_1	0.625 ± 0.282
α	0.205 ± 0.010
σ	0.370 ± 0.063
λ	0.580 ± 0.092
N_{1B}	0.044 ± 0.012
α_B	0.069 ± 0.009
σ_B	0.356 ± 0.075
g_{2B}	0.012 ± 0.003

Low and High energy data are well described

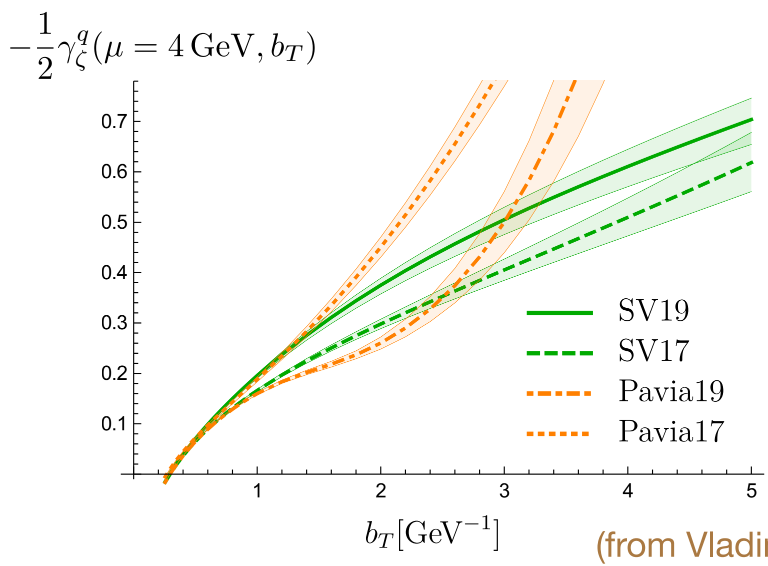
RAD parameters are less sensitive to input PDF set

Universality of RAD satisfied by DY vs. SIDIS data

Global Fits

Fit Results:

Comparison of results for CS Kernel in non-perturbative regime:



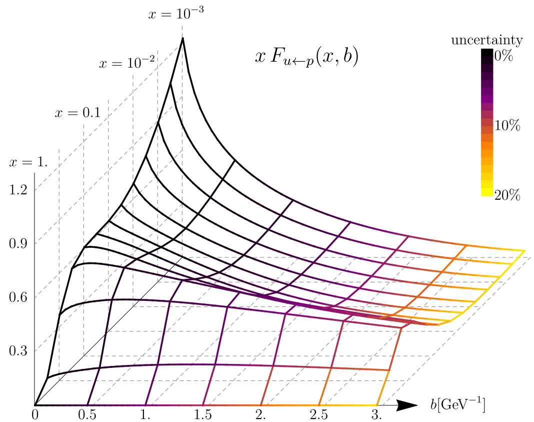
(from Vladimirov, 2003.02288)

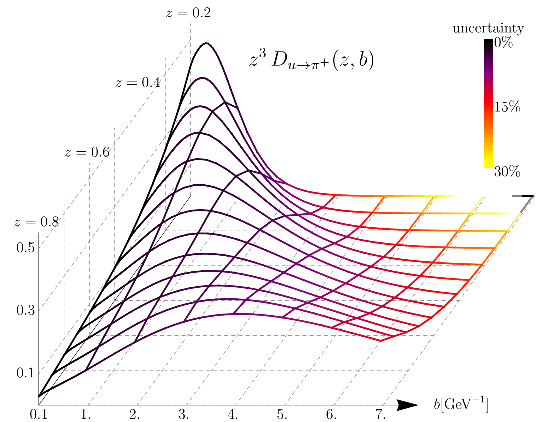
Global Fits

Fit Results:

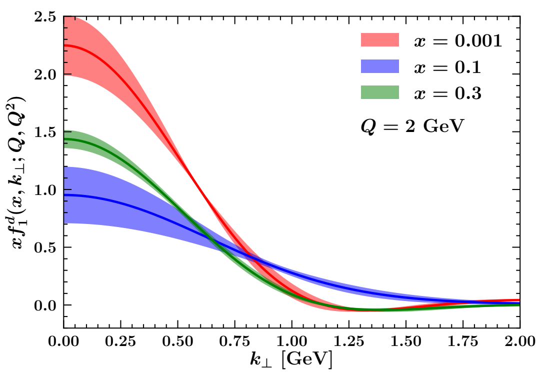
Results for intrinsic TMDPDF (& TMDFF)

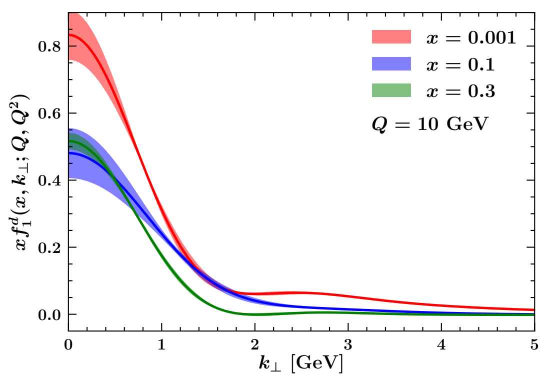
SV19





Pavia19





Quite precise determinations if we assume a given fit form.

Global Fits

Bury, Prokudin, Vladimirov (2012.05135)

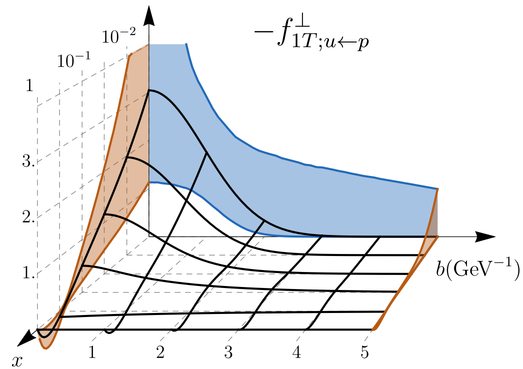
Extraction of Sivers function from global fit to SIDIS, DY, and W/Z data [76 bins: HERMES, COMPASS, Jlab (SIDIS); STAR(W/Z); COMPASS (DY)]

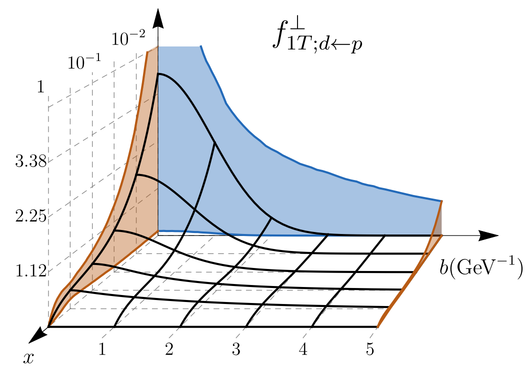
$$f_{1T}^{\perp\,\mathrm{SIDIS}} = -f_{1T}^{\perp\,\mathrm{DY}}$$

N3LL analysis following SV19

Flavor dependent parametrization (no matching)

$$f_{1T;q\leftarrow h}^{\perp}(x,b) \quad = N_q \frac{(1-x)x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q,\epsilon_q)} \;\exp\left(-\frac{r_0+x r_1}{\sqrt{1+r_2 x^2 b^2}} b^2\right)$$





Results:

Good global fit: $\chi^2/N_{pt} = 0.88$

Opposite signs for up and down Sivers functions

Data not precise enough to confirm sign flip

$$f_{1T}^{\perp\,\mathrm{SIDIS}} = +f_{1T}^{\perp\,\mathrm{DY}} \quad \textbf{gives} \quad \chi^2/N_{pt} = 1.0$$

The End