SCET & $_{\text{maybe}}^{\wedge}$ TMD-EVO

EFT
- D.O.F.
- Symmetries
- Power Counting

Example: Jelly

Flavor

- Amphorous

D.O.F.

$$\Phi^I(\vec{x},t)$$

Comoving Jelly element

$I = 1, \ldots, d-1$

d spacetime

$$\vec{\Phi}(\vec{x},0) = \vec{x}$$

initial conditions

Symmetries

- translation

$$\vec{\Phi} \to \vec{\Phi} + \vec{a}$$

- Rotations

$$\vec{\Phi} \to R\vec{\Phi}$$

$R \in SO(d-1)$

- Lorentz symmetry

Invariants

physically measured

& Dynamics in terms of invariants.

$$B^{IJ} = \partial_\mu \Phi^I \partial^\mu \Phi^J$$

$$X_1 = tr[B],$$

$$X_k = \frac{tr[B^k]}{(tr[B])^{k-1}}$$

$SO(d-1)$

$\hookrightarrow k = 1, \ldots, d-1$ complete!

$$S = \int d^d x \, F(X_1, \ldots, X_{d-1})$$

F is smooth & sufficiently bounded for physical $\langle \Phi \rangle$

BIG DATA

∞ experiments

Find F.

POWER COUNTING

$$\vec{\Phi}(\vec{x},t) = \vec{x}_{\parallel} + \vec{\pi}(\vec{x},t)$$

$$|\partial\vec{\pi}| \ll 1 \qquad \vec{\nabla} \text{ spatial derivatives}$$

$$X_k = \text{const} + a_k(\vec{\nabla}\cdot\vec{\pi}) + b_k(\partial_t\vec{\pi}\cdot\partial_t\vec{\pi}) + c_k\sum_{IJ}\nabla^I\pi^J\nabla^I\pi^J + d_k(\nabla\cdot\pi)^2 + \ldots \quad \mathcal{O}(\partial\pi^3)$$

$$S = \int d^d x \Big\{ \partial_t \vec{\pi} \cdot \partial_t \vec{\pi}$$

$$\partial_t^2 - \nabla^2 \qquad - c_T^2 \sum_{I,J} \nabla^I \pi^J \nabla^I \pi^J$$

$$\partial^2$$

$$- \left(c_L^2 - c_T^2\right)\left(\nabla \cdot \vec{\pi}\right)^2 \Big\}$$

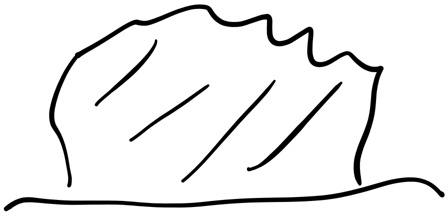
$$+ \mathcal{O}(\partial \pi^3)$$

$$\vec{\pi} = \vec{\pi}_L + \vec{\pi}_T$$

$$\nabla \cdot \pi = \nabla \cdot \pi_L \quad \& \quad \nabla \cdot \pi_T = 0$$

$$\partial_t^2 \vec{\pi}_T = c_T^2 \nabla^2 \vec{\pi}_T$$

$$\partial_t^2 (\nabla \cdot \pi_L) = c_L^2 \nabla^2 (\nabla \cdot \pi_L)$$



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QCD $\rightsquigarrow$ SCET

- D.O.F
- Symmetries

QCD itself
NRQCD
HQET
SCET
χPT

- Power counting

"Ground state" + Allowed departures

D.O.F.

- Quarks $\bar{\psi}_f(x)$, $\psi_f(x)$

$f = u\ d\ s\ c\ b \ldots$

- Gluons

↳ $iD^\mu = i\partial^\mu + gA^\mu(x)$

Symmetries Gauge

$$\psi_f(x) \to U(x)\,\psi_f(x)$$

$$F^{\mu\nu} = [D^\mu, D^\nu]$$

$U \in SU(N_c)$ & Fund. rep.

$$F^{\mu\nu} \to \mathcal{U}(x)\,F^{\mu\nu}(x)$$

$\mathcal{U} = U T^A U^\dagger$ ← Fundamental Generators

$\mathcal{U} \in SU(N_c)$ ↪ Adjoint Rep

$$\mathrm{tr}[F^{\mu\nu} F_{\mu\nu}] \overset{X}{\longleftrightarrow} \bar{\psi}_f(x)\,\psi_f(x)$$

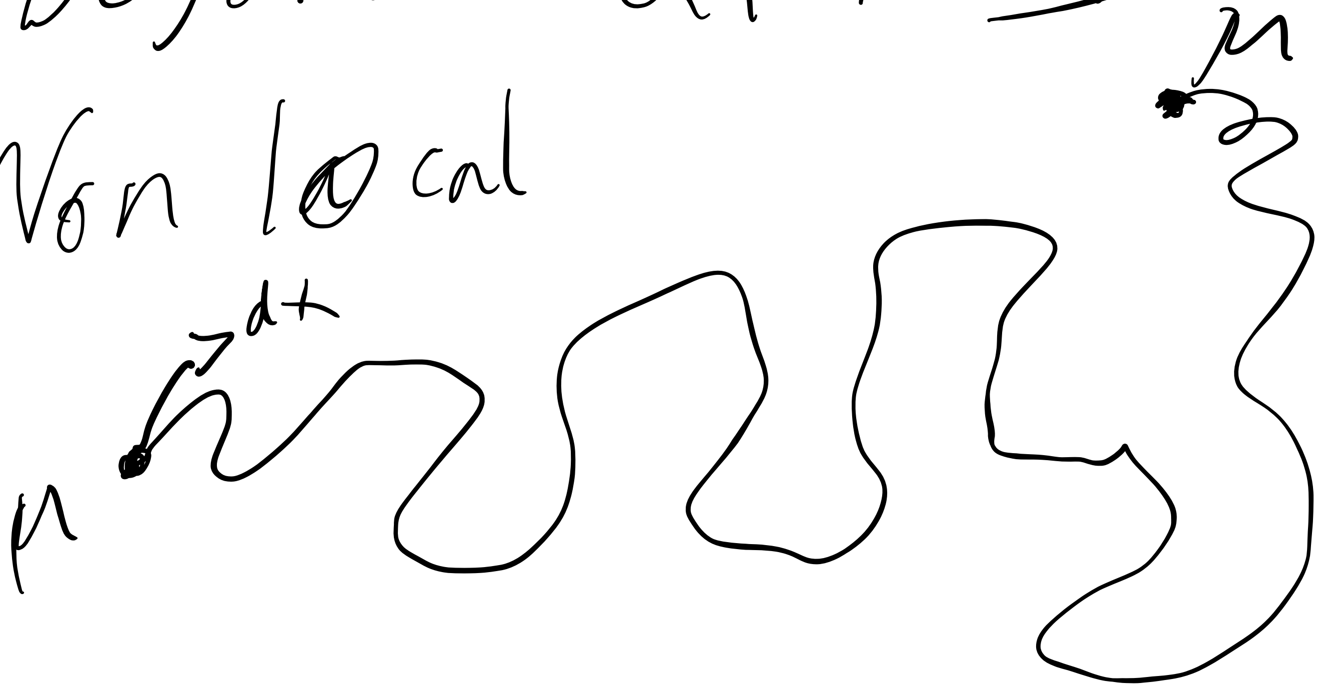
$$\bar{\psi}_f \to \bar{\psi}_f(x)\,U^\dagger(x)$$

In general

Local invariants ⍤

Beyond QFT II

Non local



Derivatives tell me how to move!

$$\phi(x) \longrightarrow U(x)\,\phi(x)$$

Assume

$$\boxed{i\,dx\cdot D\,\phi(x)=0}$$

$$\phi(x+dx)=\phi(x)+dx\cdot\partial\,\phi(x)+\dots$$

$$iD = i\partial + gA(x)$$

$$= \phi(x) - ig\, dx\cdot A(x)\,\phi(x)$$

$$= e^{-ig\, dx\cdot A(x)}\,\phi(x) + \dots$$

$$\phi(x+dx)$$

$$\hookrightarrow U(x+dx)\,\phi(x+dx)$$

$$= U(x+dx)\, e^{-ig\, dx\cdot A(x)}\,\phi(x)$$

$$\phi(x) \to U(x)\,\phi(x)$$

$$e^{-ig\, dx\cdot A(x)} \to U(x+dx)\, e^{-ig\, dx\cdot A(x)}\, U^\dagger(x)$$

$$S_\Gamma(x_f, x_i) = \lim_{k\to\infty} e^{-ig\, dx_k\cdot A(x_i+\Delta_k)}$$

$$dx\ A(x \dots)$$

$$e^{-igdx_{k-1}\cdot A(x_i+\Delta_{k-1})}$$

$$\Delta_k = \sum_{l=1}^{k-1} dx_l$$

$$e^{-igdx_1\cdot A(x_i)}$$

$$S_\Gamma(x_f, x_i) \to U(x_f) S_\Gamma(x_f, x_i) U^\dagger(x_i)$$

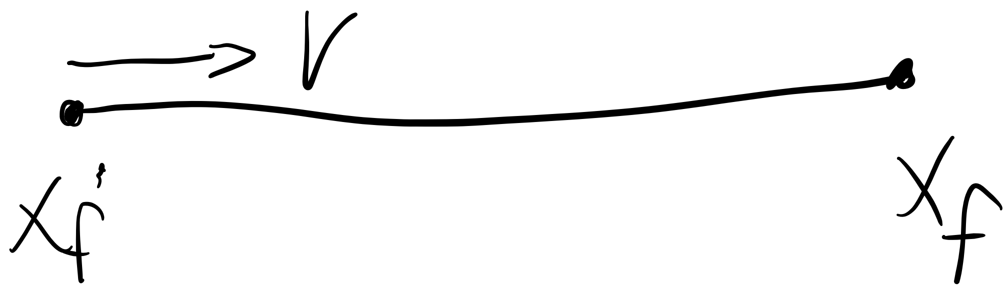
wilson line

Polyakov line (Euclidean)

<u>Cold WAR</u>

$$S_\Gamma(x_f, x_i) = P\exp\left(-ig\int_\Gamma dx\cdot A(x)\right)$$

$$\bar{\psi}_f(x_f) S_\Gamma(x_f, x_i) \psi_{f'}(x_i)$$



$v^2 = 0$

$v^2 = -1$

$v^2 = 1$

$$S_v^{(ij)}(x, t) = P\exp\left(-ig\int_0^t dt'\, v\cdot A(x+vt')\right)$$

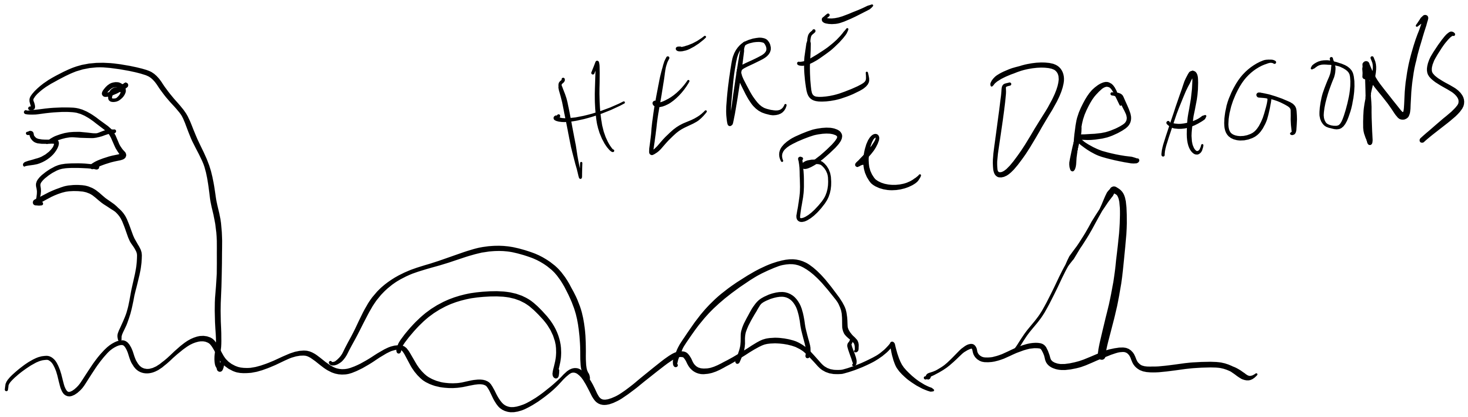
$$x_f = x + vt$$

$$= 1 - ig\int_0^t dt'\, v\cdot A(vt'+x) \longrightarrow A^A T^A_{ij}$$

$$+ (ig)^2 \int_0^t dt_1 \int_0^{t_1} dt_2\, vA(x+vt_1) \times v\cdot A(x+vt_2)$$

$$A^A T^A_{ik}\, A^B T^B_{kj}$$

+ . . . .



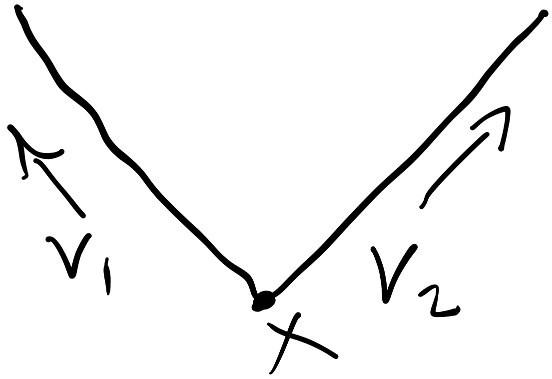
$$S_v(x,\infty)$$

No gauge transformations

At ∞ → Glauber, Transverse W-Lines

$$S_v(x,\infty)\, U^\dagger(x)$$

$$T\{S_{v_1}(x,\infty)\, S^\dagger_{v_2}(x,\infty)\}$$



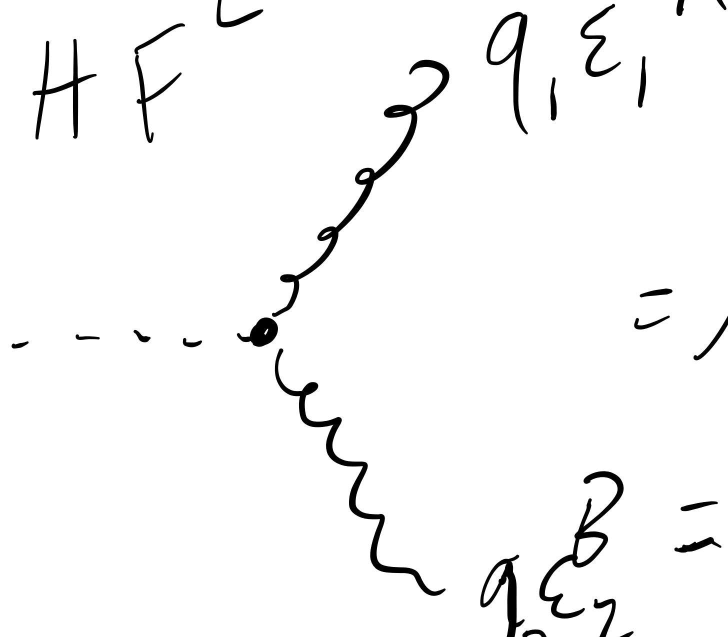
Power Counting

"Ground State"

& Allowed fluctuations

$H \to Zg$. "Ground State"

HF^2



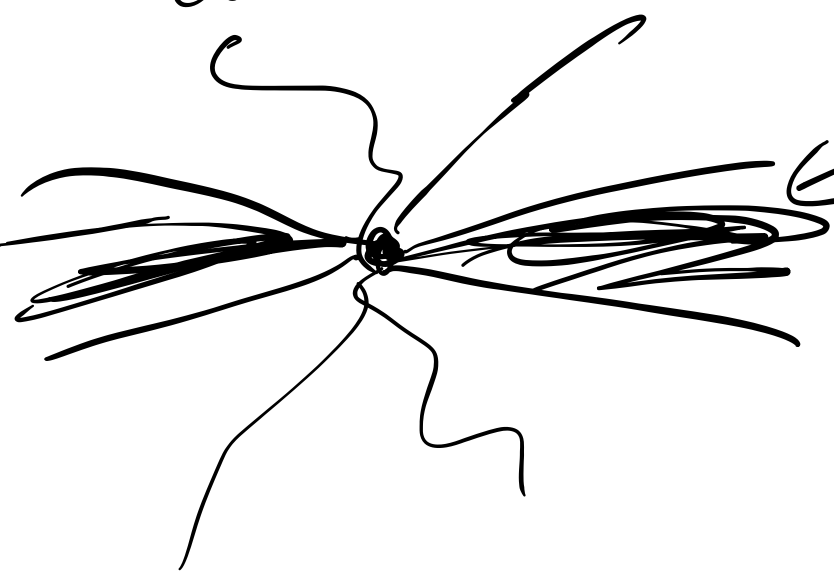
$$= A^{(4)}(1^A, 2^B)$$

$$= \delta^{AB} N\left(q_1 \cdot q_2\, \varepsilon_1 \cdot \varepsilon_2 - \varepsilon_1 \cdot q_2\, \varepsilon_2 \cdot q_1\right)$$

$$H \to 2g + \underbrace{g + g + g + g + \dots + g}_{\text{sufficiently soft}}$$

OR Collinear to q_1 & q_2

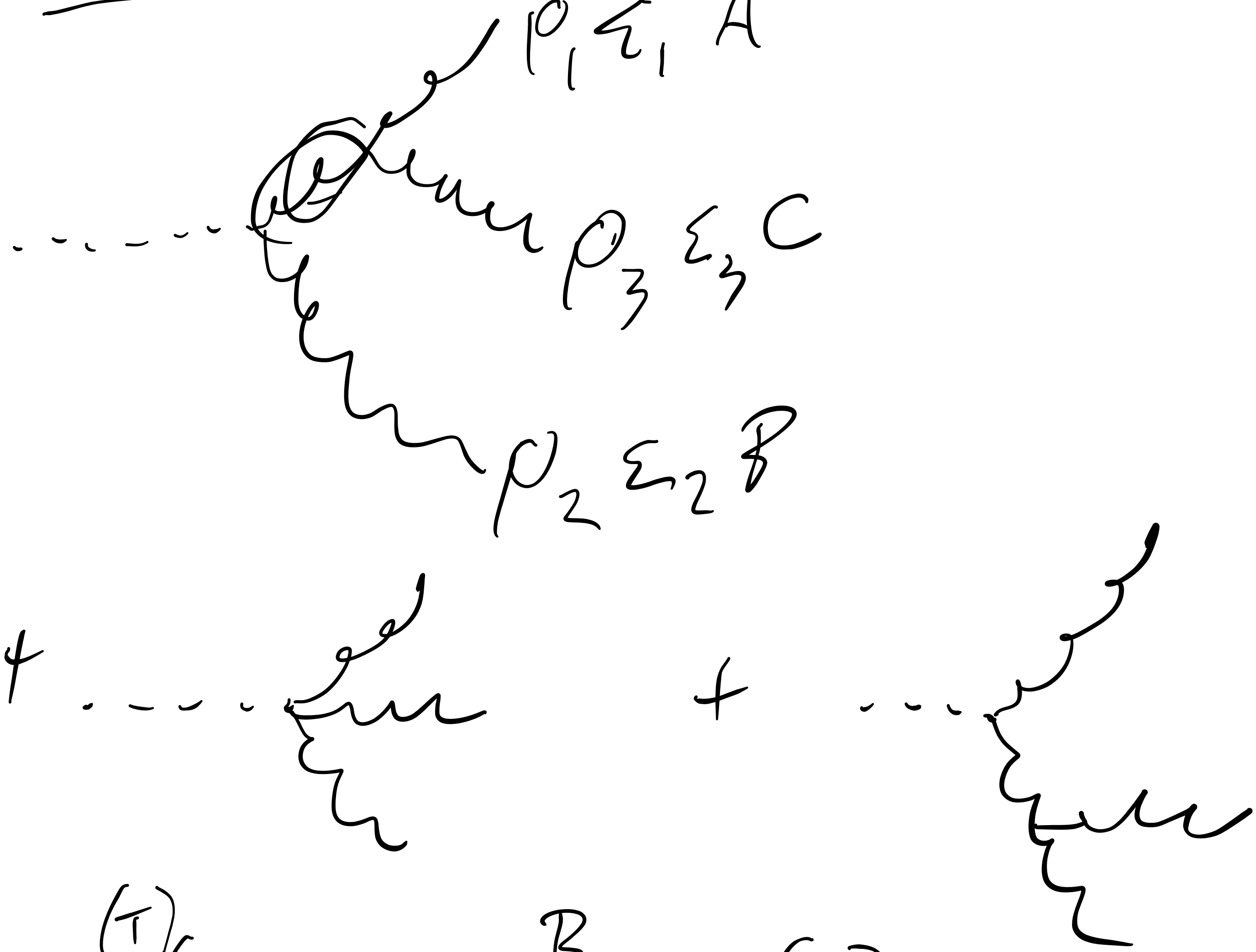
↳ same basic Momenta configuration as $H \to 2g$



$\approx q_1 = \sum_{i \in C} p_i$

Want to build which describes All "two-jet" Higgs Decays!

Amplitude Level



$= A^{(T)}(p_1 \epsilon_1^A, p_2 \epsilon_2^B, p_3 \epsilon_3^C)$ Full QCD

$p_3 \to$ soft

Soft!

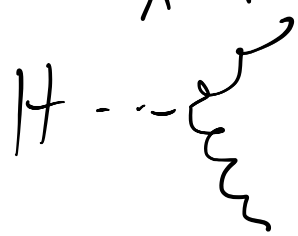
$$\boxed{\frac{p_1 \cdot p_3}{p_1 \cdot p_2} \text{ \& } \frac{p_2 \cdot p_3}{p_1 \cdot p_2} \ll 1}$$

$$p_1 \to q_1 \qquad p_2 \to q_2$$

$$p_1 + p_2 + p_3 = q_1 + q_2 + \dots$$

$$A^{(T)}\left(p_1 \varepsilon_1^{A}, p_2 \varepsilon_2^{B}, p_3 \varepsilon_3^{C}\right)$$

f structure



$$= \sum_{s_1 s_2} A^{(T)}\left(q_1 \varepsilon_1^{s_1}, q_2 \varepsilon_2^{s_2}\right)$$

eikonal

$$\times (ig)\left(f^{A s_1 C} \frac{q_1 \cdot \varepsilon_3}{q_1 \cdot p_3 + i\varepsilon} \delta^{s_2 B} + f^{B s_2 C} \delta^{s_1 A} \frac{q_2 \cdot \varepsilon_3}{q_2 \cdot p_3 + i\varepsilon} \right)$$

$$+ \mathcal{O}\left((p_3^0)^0\right)$$

OPERATOR THAT REPRODUCES this.

$$\lim_{p_3 \text{ soft}} A^{T}(1,2,3) = N \langle 0 | \hat{O}_{HC} | q_1 \varepsilon_1^{A}, q_2 \varepsilon_2^{B} \rangle \otimes \langle 0 | \hat{O}_s | p_3 \varepsilon_3^{C} \rangle$$

$+ \ldots$

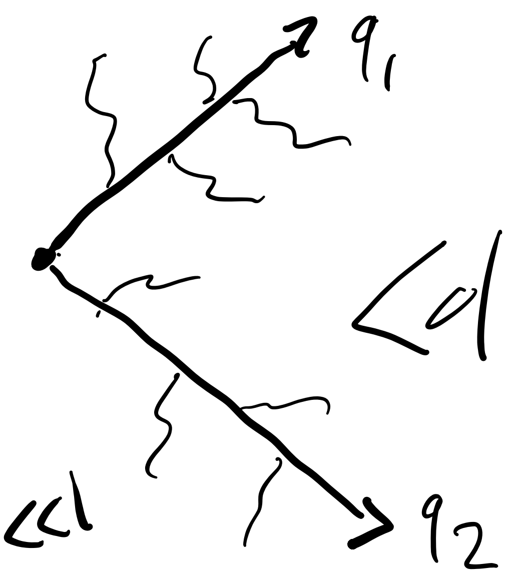
$\hat{O}_{HE} \otimes \hat{O}_S$ $\quad \otimes \begin{cases} \text{- contractions} \\ \text{color} \\ \text{- convolution?} \end{cases}$

two jet Higgs Decays

$\hat{O}_S \begin{cases} \text{Need invariants} \\ \text{Cannot be } \underline{\underline{\text{local}}}. \end{cases}$

Soft expansion not polynomial!

$$\hat{O}_S = T\left\{ S^{s_1 A}_{q_1}(0,\infty)\, S^{s_2 B}_{q_2}(0,\infty) \right\}$$



$\lambda \ll 1$

$$\langle 0| -ig \int_0^\infty dt'\, q_1 \cdot A^{s_1 A}(q_1 t' + x) \,|\, p_3 \varepsilon_3^{C} \rangle$$

$\lambda \to 0 \quad \dfrac{p_i \cdot p_3 \sim O(\lambda^\alpha)}{p_i \cdot p_2} =$ $\quad q_1 \cdot \varepsilon_3 \; f^{A s_1 C}\, \delta^{s_2 B}$

$\lambda \to 0$

$\lambda^{2} \to 0$ $\quad i=1,2$ $\quad q_1 \cdot p_3$

$\lambda^{40} \to 0$

$$\hat{O}_S = T\{ S_{q_1}(0,\infty)\, S_{q_2}(0,\infty) \}$$

Soft was Easy!

$$\langle 0 | \hat{O}_{HC} | q_1 \varepsilon_1^A, q_2 \varepsilon_2^B \rangle$$

$A^\mu(x) \to$ evil

$F^{\mu\nu} = [D^\mu, D^\nu] \to$ Less evil

why gauge transform

Collinear Power Counting

Let $\lambda \ll 1$, & Q energy scale $\quad Q \sim m_H$

$Q^2 \sim q_1 \cdot q_2$

Then momenta

$p_1, p_2, \ldots\ldots, p_k$ are

collinear to null direction n

with conjugate null direction $\bar{n}$

if

(A) $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$, $\perp$ transverse to n & $\bar{n}$ plane

(B) $$p_i^\mu = \bar{n} \cdot p_i \frac{n^\mu}{2} + n \cdot p_i \frac{\bar{n}^\mu}{2} + p_{i\perp}^\mu$$

$i = 1 \dots k$

$$\bar{n} \cdot p_i \sim \mathcal{O}(Q)$$

$$n \cdot p_i \sim \mathcal{O}(Q\lambda^2)$$

$$p_{\perp i} \sim \mathcal{O}(Q\lambda)$$

$$p_i \cdot p_j \sim \mathcal{O}(Q^2\lambda^2)$$ For any i, j

$$p_i^2 = n \cdot p_i \, \bar{n} \cdot p_i - p_{\perp i}^2$$

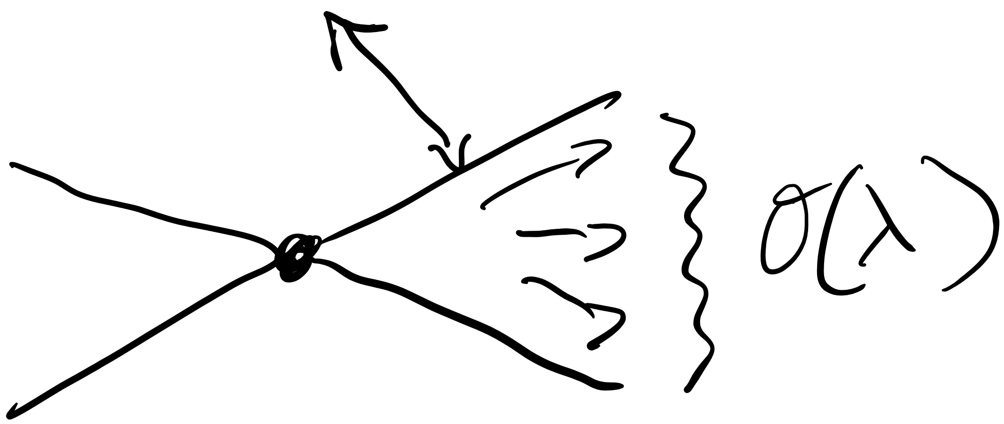
$\mathcal{O}(Q^2\lambda^2)$

On-shell

$$\frac{1}{k^2}$$

$\lambda \to 0$

$$q_1 \simeq p_1 + \dots + p_k + \mathcal{O}(\lambda)$$



To build eft to collinear sector

n & $\bar{n}$ ← start building non-local invariants!

$\hat{\mathcal{O}}$ is a collinear operator if

$$\bar{n}\cdot iD\,\hat{\mathcal{O}} \sim \mathcal{O}(Q)\times \mathcal{O}(\hat{\mathcal{O}})$$

$$n\cdot iD\,\hat{\mathcal{O}} \sim \mathcal{O}(Q\lambda^2)\times \mathcal{O}(\hat{\mathcal{O}})$$

$$iD_\perp^\mu\,\hat{\mathcal{O}} \sim \mathcal{O}(Q\lambda)\times \mathcal{O}(\hat{\mathcal{O}})$$

$$\hat{\mathcal{O}} \longrightarrow f(\bar{n}\cdot iD)\,\hat{\mathcal{O}}$$

Q

$\ddot{n}$

This will be fixed!
non-local operators

$$\bar{n}\cdot iD\, W_n = 0$$

$$W_n(x) = P\exp\left[-ig\int_0^\infty dt'\, \bar{n}\cdot A(\bar{n}t'+x)\right]$$

Now Build invariants with $W_n(x)$

Physically
css
W_n is the rest of the hard interaction

$$f(i\bar{n}\cdot D)\hat{O} = W_n(x)\left[f(i\bar{n}\cdot\partial)\, W_n^\dagger(x)\hat{O}\right]$$

$$f(i\bar{n}D) = i\bar{n}\cdot D \qquad W_n^\dagger W_n = 1$$

$$f(i\bar{n}D) = (i\bar{n}\cdot D)^k$$

Check on test operator $\hat{O}$

$$i\bar{n}\cdot D\hat{O} = (i\bar{n}\cdot\partial + g\bar{n}\cdot A)\hat{O}$$

$$= i\bar{n}\cdot\partial\hat{O} + W_n\left[(i\bar{n}\cdot\partial W_n^\dagger)\right]\hat{O}$$

$$= W_n W_n^\dagger\, i\bar{n}\cdot\partial\hat{O} + W_n\left[i\bar{n}\cdot\partial W_n^\dagger\right]\hat{O}$$

$$= W_n\left[i\bar{n}\cdot\partial\,(W_n^\dagger\hat{O})\right]$$

QED

Now build Leading Power operators.

$$\chi_n^i(x) = \mathrm{tr}\left[W_n(x)^\dagger \phi(x) \right]$$

$$B_n^{\mu\nu} = \mathrm{tr}\left[W_n^\dagger T^A F^{A\mu\nu}(x) W_n \right]$$

not homogeneous in power counting

$\bar{n}\ n$

$\bar{n}\ \perp$

$n\ \perp$

$$F^{\mu\nu} = \left[D^\mu , D^\nu \right]$$

$$g\,\underline{\bar{n}\cdot i\partial}\, B_{\perp n}^{\nu} = \bar{n}_\mu B_n^{\mu\nu_\perp} \rightarrow O(Q\lambda)$$

$\bar{n}\cdot A = 0$

$$B_{\perp n}^{\nu} = A_\perp^{\nu}$$

$$S = \int \underline{d^d x}\, \mathcal{L}$$

$O(1)$

$$\hat{O}_{HC} = \int d\omega_1 \, d\omega_2 \, C^{\mu\nu}(\omega_1 n_1, \omega_2 n_2)^{s_1,s_2}_{c_1 c_2}$$

$$\left[B^{\mu c_1}_{n_1 \perp}(0) \, \delta(\omega_1 - i\bar{n}_1 \cdot \partial) \right]$$

$$\left[B^{\nu c_2}_{n_2 \perp}(0) \, \delta(\omega_2 - i\bar{n}_2 \cdot \partial) \right]$$

$$\begin{array}{ccc} n_1 & n_2 & n_1 = \bar{n}_2 \\ \downarrow & \downarrow & \\ q_1 & q_2 & \bar{n}_2 = n_1 \end{array}$$

$$\langle 0 | B^{\mu c_1}_{n_1 \perp}(0) \, \delta(\omega_1 - i\bar{n}_1 \cdot \partial) | q_1 \, \varepsilon_1^A \rangle$$

$$= \varepsilon_1^\mu \, \delta(\omega_1 - \bar{n} \cdot q_1) \, \delta^{c_1 A}$$

$s_1 \, s_2$ contracted

$$\langle 0 | \hat{O}_{HC} | q_1 \, \varepsilon_1^A, q_2 \, \varepsilon_2^B \rangle \otimes \langle 0 | \hat{O}_s | 0 \rangle$$

$$= \varepsilon_1^\mu \varepsilon_2^\nu \, C^{\mu\nu}(\bar{n}_1 \cdot q_1 n_1, \bar{n}_2 \cdot q_2 n_2, {}^{s_1 s_2})$$

$$= A^{(T)}(q_1 \varepsilon_1 A, q_2 \varepsilon_2 B) + \dots$$

$$\bar{n}_i \cdot q_i \, n_i \approx q_i \qquad i = 1, 2$$

Collinear is done!

$$\hat{O}_{HC} \otimes \hat{O}_S =$$

$$\int d\omega_1 \int d\omega_2 \, C^{\mu\nu}(n_1\omega_1, n_2\omega_2, {}^{s_1 s_2})$$

$$[B_{\perp n_1}^{\mu c_1} \delta(\omega_1 - i\bar{n}_1 \cdot \partial)]$$

$$[B_{\perp n_2}^{\nu c_2} \delta(\omega_2 - i\bar{n}_2 \cdot \partial)]$$

$$T\{S_{n_1}^{s_1 c_1}(0) \, S_{n_2}^{s_2 c_2}(0)\}$$

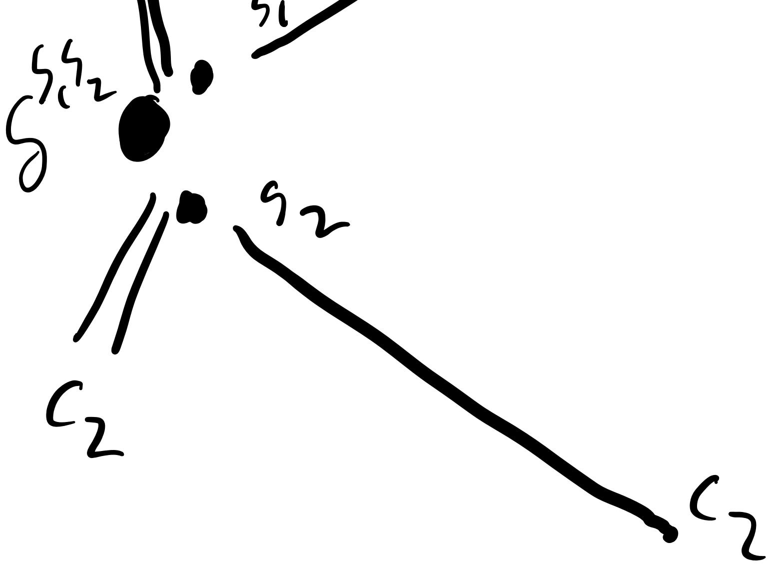
$SU(N_c)$ invariant

$\downarrow$

$\delta^{s_1 s_2}$

c_1

c_1



Subtle point

Leonard described $SCET_{II}$

NOT $SCET_{I}$

$$\begin{array}{cccc} & \bar{n} & n & \perp \\ p_s \sim Q\big(& \lambda, & \lambda, & \lambda \big) \end{array}$$

$$p_c \sim Q(1, \lambda^2, \lambda)$$

$$p_s + p_c \sim Q(1, \lambda, \lambda)$$

$$(p_s+p_c)^2 \sim n\cdot(p_s+p_c)\,\bar{n}\cdot(p_s+p_c) - p_{\perp sc}^2$$

λ $\mathcal{O}(\lambda)$ $\uparrow \mathcal{O}(\lambda^2)$

$$\frac{1}{(p_s+p_c)^2}$$

SCET I

$$p_c \sim Q\left(\overset{\bar{n}}{1}, \overset{n}{\lambda^2}, \overset{\perp}{\lambda}\right)$$

$$p_{us} \sim Q\left(\lambda^2, \lambda^2, \lambda^2\right)$$

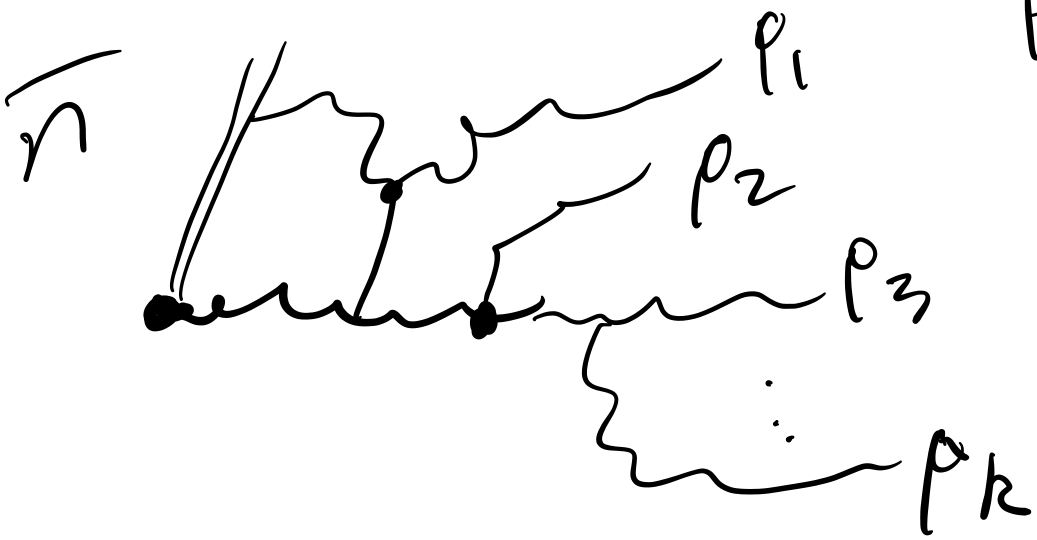
$$p_c + p_{us} \to p_c$$

$\bar{n}\cdot\partial$

$$n \cdot D_{us}$$

$$\underset{1_n}{\vee} \quad \underset{1_n}{\vee}$$

$$\langle 0|\, \mathcal{B}_{\perp n}^{\nu}(0)\, e^{iS_{SCET}}\, \delta(\omega - i\bar{n}\cdot\partial)\, |p_1, p_2, \ldots, p_k\rangle$$



Fun fact
At leading power
$S_{SCET} = S_{QCD}$

Splitting amplitude

$1 \to 2$ known 3 Loops

$1 \to 1$ 4 Loops

$1 \to 3$ known 1-loop

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1. D.O.F.
2. Symmetries
3. Power counting

③ Power counting
"State of affairs"

Found

$$B^{\mu}_{n\perp}(x) \qquad S_n(x)$$

$$\chi_n(x) = W_n^{\dagger} \frac{\bar{\not{n}}}{2}\not{\xi}(x)$$

$$iD^{\mu}_{c\perp} \sim \mathcal{O}(\lambda) \times Q$$

$$in\cdot D_c \sim \mathcal{O}(\lambda^2) \times Q$$

$$i\bar{n}\cdot D_c \sim \mathcal{O}(1) \times Q \leftarrow \quad i\bar{n}\cdot\partial \text{ via } W_n(x)$$

$$D^{\mu}_s \sim Q \times \mathcal{O}(\lambda^a)$$

$$a \begin{cases} =1 & \text{SCET}_{II} \\ =2 & \text{SCET}_{I} \end{cases}$$

$\lambda \ll 1$ is power counting parameter → controls how far from "ground state" we can go.

– What actually fixes λ?

(4) The ultimate logical conclusion of power counting:

THE MULTIPOLE EXPANSION

↳ NRQCD
GRINSTEIN
→ ROTHSTEIN

- λ is fixed by the observable

- So suppose we $pp \to \ell^+\ell^- + X$

we observe: p_n $p_{\bar{n}}$

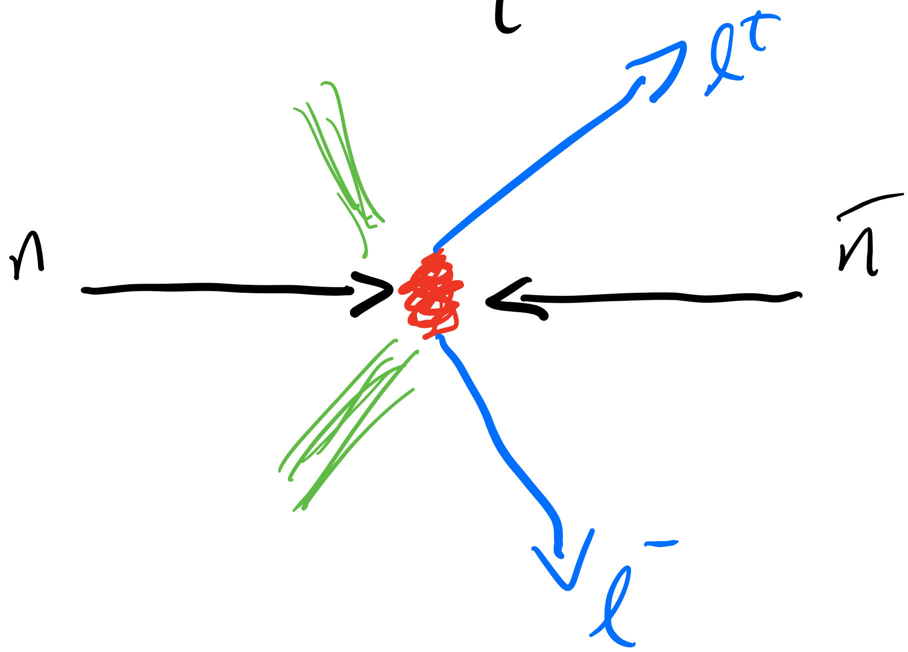
$q = p_{\ell^-} + p_{\ell^+}$

$$p_n^\mu \sim \sqrt{s}\,\frac{\bar{n}^\mu}{2}$$

$$p_{\bar{n}}^\mu \sim \sqrt{s}\,\frac{n^\mu}{2}$$

$$q = \left(e^{Y}\sqrt{q_T^2+Q^2},\ e^{-Y}\sqrt{q_T^2+Q^2},\ \vec{q}_T\right)$$

$\bar{n} \cdot q$ $\quad$ $n \cdot q$ $\quad$ $\perp$



Now assume

$$|\vec{q}_T| \ll Q$$

$$\lambda \sim \frac{|\vec{q}_T|}{Q}$$

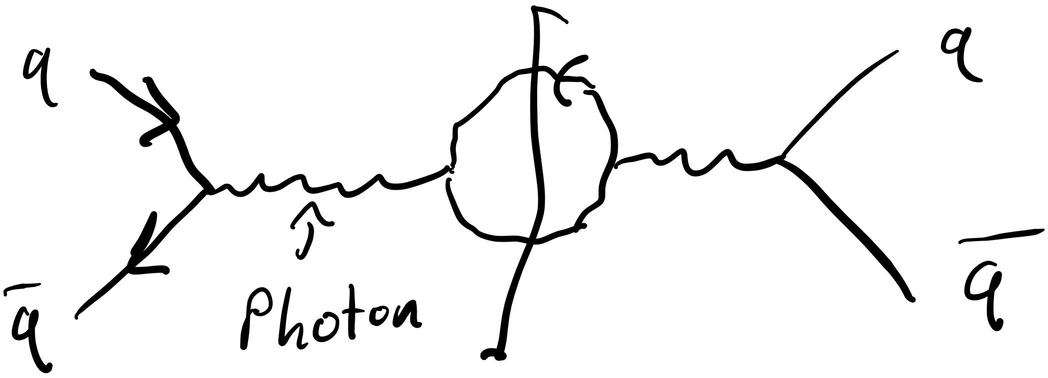
$Q\lambda$ or smaller

$$\vec{q}_T = -\sum_{i \in \text{Final state hadrons}} \vec{p}_{i\perp}$$

$Q\lambda$

Yes, But subdominate

Think about types of final state radiation in the sum that can go on-shell as $\lambda \to 0$



$\lambda \to 0$ "ground state" config.

(EFT's ARE Built ONLY FROM on-shell d.o.f.)

NRQCD & SCET → make potential "Coloumbs Glouns"

$$p^2 \sim Q^2 \lambda^{\#} \quad \lambda \to 0$$

$$\begin{array}{cccc} & \bar{n}\cdot p, & n\cdot p, & \perp \\ p_c \sim Q(& 1, & \lambda^2, & \lambda) \end{array}$$

$$p_{\bar{c}} \sim Q(\lambda^2, 1, \lambda)$$

$$p_s \sim Q(\lambda, \lambda, \lambda)$$

$$q_T \sim (p_{c\perp} + p_{\bar{c}\perp} + p_{s\perp})$$

$$p^2_{c/\bar{c}/s} \sim n\cdot p\, \bar{n}\cdot p - p_\perp^2$$

$\mathcal{O}(\lambda^2)$

$$\delta(n\cdot p\,\bar{n}\cdot p - p_\perp^2)$$

$(p_s + p_c)^2$ not homogeneous

eikonal propagator

$$(p_c + p_{\bar{c}} + p_s)^2 \sim \mathcal{O}(Q^2)$$

ASK what is x-sec in full QCD?

$\hookrightarrow$ Then match to SCET

↳ [illegible]
↳ MULTIPOLE EXPAND
↳ Factorized x-sec

$$\frac{d\sigma}{dy\, d^2\vec{q}_T\, dQ^2} = \sum_X \int d^d z\, e^{iq\cdot z} L^{\mu\nu}(q) \langle p_n p_{\bar{n}} | \hat{J}^\mu(z) | X \rangle \langle X | \hat{J}^{\dagger\nu}(0) | p_n p_{\bar{n}} \rangle$$

happening at Amplitude²

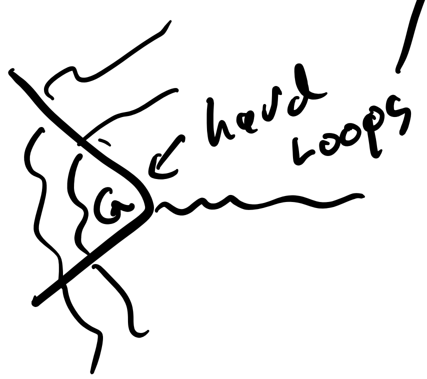
$\hat{J}$ is e.m. current

$$\hat{J}^\mu(z) = \sum_f e_f \bar{\psi}_f(z) \Gamma^\mu \psi_f(z)$$

$q_T \to 0$

match $\hat{J}^\mu(z)$ onto SCET E.M. current for two-operators

$$J^\mu(z) = \int d^d p_c \int d^d p_{\bar{c}} \int d^d p_s$$
$$\exp\left[i z \cdot (p_c + p_{\bar{c}} + p_s)\right]$$
$$C\left([p_c + p_{\bar{c}} + p_s]^2\right)$$
$$\left[\bar{\chi}_n(0)\, \delta^{(d)}(p_c - i\partial)\right]\Gamma^\mu$$
$$\left[\chi_{\bar{n}}(0)\, \delta^{(d)}(p_{\bar{c}} - i\partial)\right]$$
$$\left[T\{S_n(0) S_{\bar{n}}^\dagger(0)\, \delta^{(d)}(p_s - i\partial)\right]$$



not homogeneous

most general two jet-op constructed from leading

GET TO HOMOGENEOUS OP.

multipole expand

n, $\bar{n}$, $\perp$ $\quad O(Q)$ $\quad O(Q)$

$$p_c + p_s + p_{\bar c} \longrightarrow \frac{\bar n\cdot p_c}{2}\, n + n\cdot p_{\bar c}\,\frac{\bar n}{2} + \dots$$

$$z\cdot(p_c + p_{\bar c} + p_s)$$

$$\hookrightarrow \frac{n\cdot z}{2}\Big[\overset{1}{\bar n\cdot p_c} + \overset{\lambda^2}{\cancel{\bar n\cdot p_{\bar c}}} + \overset{\lambda}{\cancel{\bar n\cdot p_s}}\Big]$$

$$\frac{\bar n\cdot z}{2}\Big[\overset{\lambda^2}{\cancel{n\cdot p_c}} + \overset{1}{n\cdot p_{\bar c}} + \overset{\lambda}{\cancel{n\cdot p_s}}\Big]$$

$$+\ z_\perp\cdot\Big[\underset{\lambda}{p_{c\perp}} + \underset{\lambda}{p_{\bar c\perp}} + \underset{\lambda}{p_{s\perp}}\Big]$$

$$\hat J^\mu(z) = \int d\bar n\cdot p_c\, d^{d-2}p_{c\perp}\int dn\cdot p_{\bar c}\, d^{d-2}p_{\bar c\perp}$$

$$\int d^{d-2}p_{\perp s}\ C(\bar n\cdot p_c\, n\cdot p_{\bar c})$$

$$\exp\Big[i\frac{n\cdot z}{2}\bar n\cdot p_c + i\frac{\bar n\cdot z}{2}\, n\cdot p_{\bar c}$$

$$+ iz_\perp\cdot(p_{\perp c} + p_{\perp \bar c} + p_{s\perp}\Big]$$

$$\left[\bar\chi_n(0)\,\delta(\bar n\cdot p_c - \bar n\cdot i\partial)\,\delta^{(d-2)}(p_{\perp c} - i\partial_\perp)\right]\Gamma^\mu$$
$$\left[\chi_n(0)\,\delta(n\cdot p_{\bar c} - n\cdot i\partial)\,\delta^{(d-2)}(p_{\perp\bar c} - i\partial_\perp)\right]$$
$$\left[T\{S_n(0)\,S^\dagger_{\bar n}(0)\}\,\delta^{(d-2)}(p_{\perp s} - i\partial_\perp)\right] + \ldots$$

Then $L^{\mu\nu} \to g^{\mu\nu}$

$$\sum_X \langle p_n\, p_{\bar n}|\, J^\mu(x)\,|X\rangle\langle X|\, J^{\dagger\nu}(0)\,|p_n\, p_{\bar n}\rangle$$

Fierz also!

$$= \int d\bar n\cdot p_c\; dn\cdot p_{\bar c} \int dp_{\perp s}\, dp_{\perp c}\, dp_{\perp \bar c}$$

$$|C(\bar n\cdot p_c\, n\cdot p_{\bar c})|^2\, \mathrm{Exp}\left[i\frac{n\cdot z}{2}\bar n\cdot p_c + i\frac{\bar n\cdot z}{2} n\cdot p_{\bar c} + z_\perp(\sim)\right]$$

$$\sum_{X_c} \langle p_n|\left[\bar\chi_n(0)\,\delta(\bar n\cdot p_c - i\bar n\cdot\partial)\,\delta^{(d-2)}(p_{c\perp} - i\partial_\perp)\right]|X_c\rangle\langle X_c|\, \chi_n(0)\,|p_n\rangle$$

$\delta(n\cdot p_c - in\cdot\partial)$

$$\sum_{X_{\bar c}} \langle p_{\bar n}|\left[\chi_{\bar n}(0)\,\delta(n\cdot p_{\bar c} - in\cdot\partial)\,\delta^{(d-2)}(p_{\bar c\perp} - i\partial_\perp)\right]|X_{\bar c}\rangle\langle X_{\bar c}|\, \bar\chi_{\bar n}(0)\,|p_{\bar n}\rangle$$

$$\sum_{X_s} \langle 0|\,T\{S_n(0)\,S^\dagger_n(0)\}\,\delta^{(d-2)}(p_{\perp s} - i\partial_\perp)\,|X_s\rangle\langle X_s|\, \bar T\{S^\dagger_n(0)\,S_{\bar n}(0)\}\,|0\rangle$$

🙂 ! 🙁 not done

Everything above is naive.

Because multipole → gives op. meaning & "knowledge" of their scaling

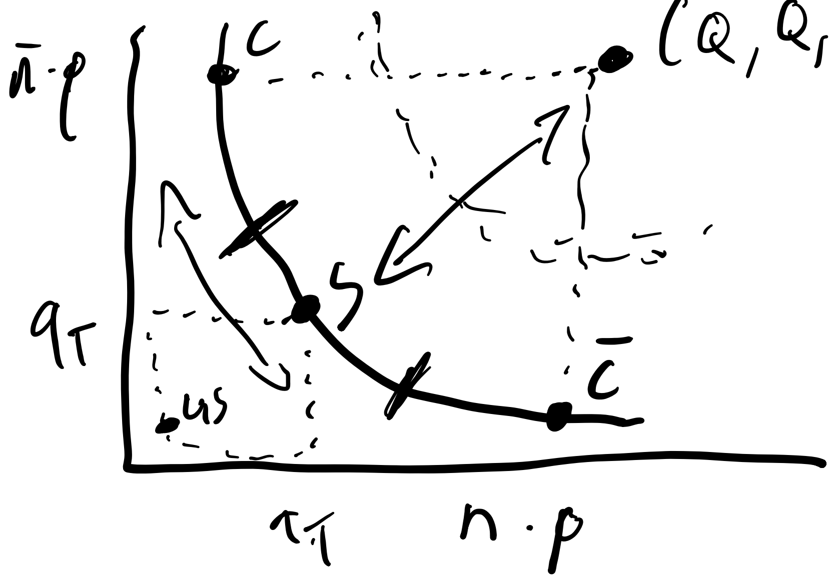
→ Induces divergences associtiate with the boundary between modes

→ Regulate

→ Renormalize

→ Resum!

$$\log\left(\frac{Q^2}{q_T^2}\right) \qquad \log\left(\frac{n\cdot p_{\bar{c}}}{q_T}\right) \qquad \log\left(\frac{\bar{n}\cdot p_c}{q_T}\right)$$



Each Boundary

Large log

$n\cdot p\,\bar{n}\cdot p \sim Q^2\lambda^2$

- Regulate $\to$ Loops

dimensional Regularization

$4 \to 4-2\varepsilon = d$

what does dim Reg Break?

Dilatations! ☺

But not Boosts ☹

- to fully Regulate I

will an additional $\int d^d z\, e^{izq}$

$$\frac{d\sigma}{dy\, dQ^2\, dq_\perp^2} = N H(n\cdot q\,\bar{n}\cdot q)$$

Hard F

$z_\perp \to b_\perp$

$$\int d^{d-2} b_\perp\, e^{i b_\perp\cdot q_T}$$

$$B(\bar{n}\cdot q, b_\perp, 0)$$

Beam Functions $\to B(n\cdot q, b_\perp, 0)$

soft $\to S(0, 0, b_\perp)$

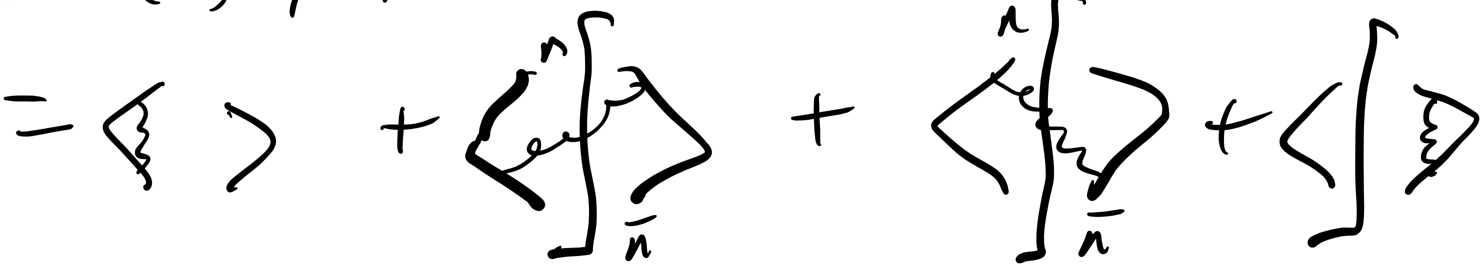
Perform m.p.e

$n\cdot b = \bar{n}\cdot b = 0$

$$S(\bar{n}\cdot b, n\cdot b, b_\perp) =$$

$$\sum_{X_s} \langle 0| T\{S_n(0)\, S_{\bar{n}}^{\dagger}(0)\}\, e^{i\cdot b\cdot(i\partial)} |X_s\rangle\langle X_s| \bar{T}\{S_{\bar{n}}(0)\, S_n^{\dagger}(0)\}|0\rangle$$

$$S^{(1)}(0,0,b_\perp)$$



$$= \mu^{2\varepsilon}\, 2g^2 C_F \int \frac{d^d p}{(2\pi)^{d-1}} \frac{n\cdot\bar{n}\ \delta(p^2)\,\theta(p^0)}{\{n\cdot p\ \bar{n}\cdot p\}} e^{ib_\perp\cdot p_\perp}$$

$$p^2 = n\cdot p\ \bar{n}\cdot p - p_\perp^2$$

$$\int^{\infty} \frac{d\bar{n}\cdot p}{\bar{n}\cdot p} \int \frac{d^{d-2} p_\perp}{\ } e^{ib_\perp\cdot p_\perp}$$

$\to n \cdot p$ $|p_\perp^2|$

- Collins 1 axial gauge
- Collins 2 spacelike wilsons
- Ji timelike
- Becker & Neubert 1 & 2 analytic
- Scimemi / Manohar $\hookrightarrow$ 1 & 2 δ-regulators "eikonal mass term"
- Duff & friends 1 $\rightsquigarrow$ analytic style $\updownarrow \neq$
- Duff & friends #2 $\rightsquigarrow$ The Best!!

Calculate highest Loops & Space-time!

$\hookrightarrow$ Highlights that

TMD anamalous dimensions ARE the same as threshold

$$S(n\cdot b,\ \bar{n}\cdot b,\ b_\perp)$$

$n\cdot b = \bar{n}\cdot b = 0$

TMD $\Rightarrow$ Resummation

$b_\perp = 0$

threshold soft

none are zero

Joint

$\to$ These are your Regulator

expand function as

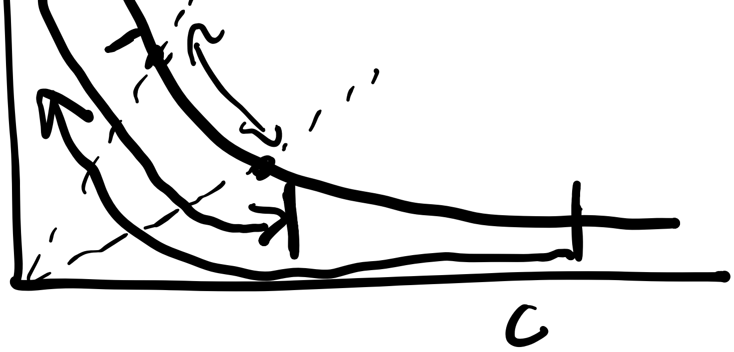
$\bar{n}\cdot b$ & $n\cdot b \to 0$

Regulators are not enough to separate modes

must have subtractions

Space time regulator

Drew wrong



$\bar{n}\cdot b = n\cdot b = i b_0 t$

μ

$t \to 0$

$$\lim_{t\to 0} \frac{B(\bar{n}\cdot p,\ b_\perp,\ i b_0 t)}{S(i t b_0,\ i t b_0,\ b_\perp)}$$ is true Beam function!

regulator & renorm. parameter!

$$\int \frac{d^d p}{(2\pi)^{d-1}} \delta(p^2)\ e$$