TMDs and Jets

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What is a Jet?

Jet Factorization Theorems (SCET)

Measuring TMDs with Jets

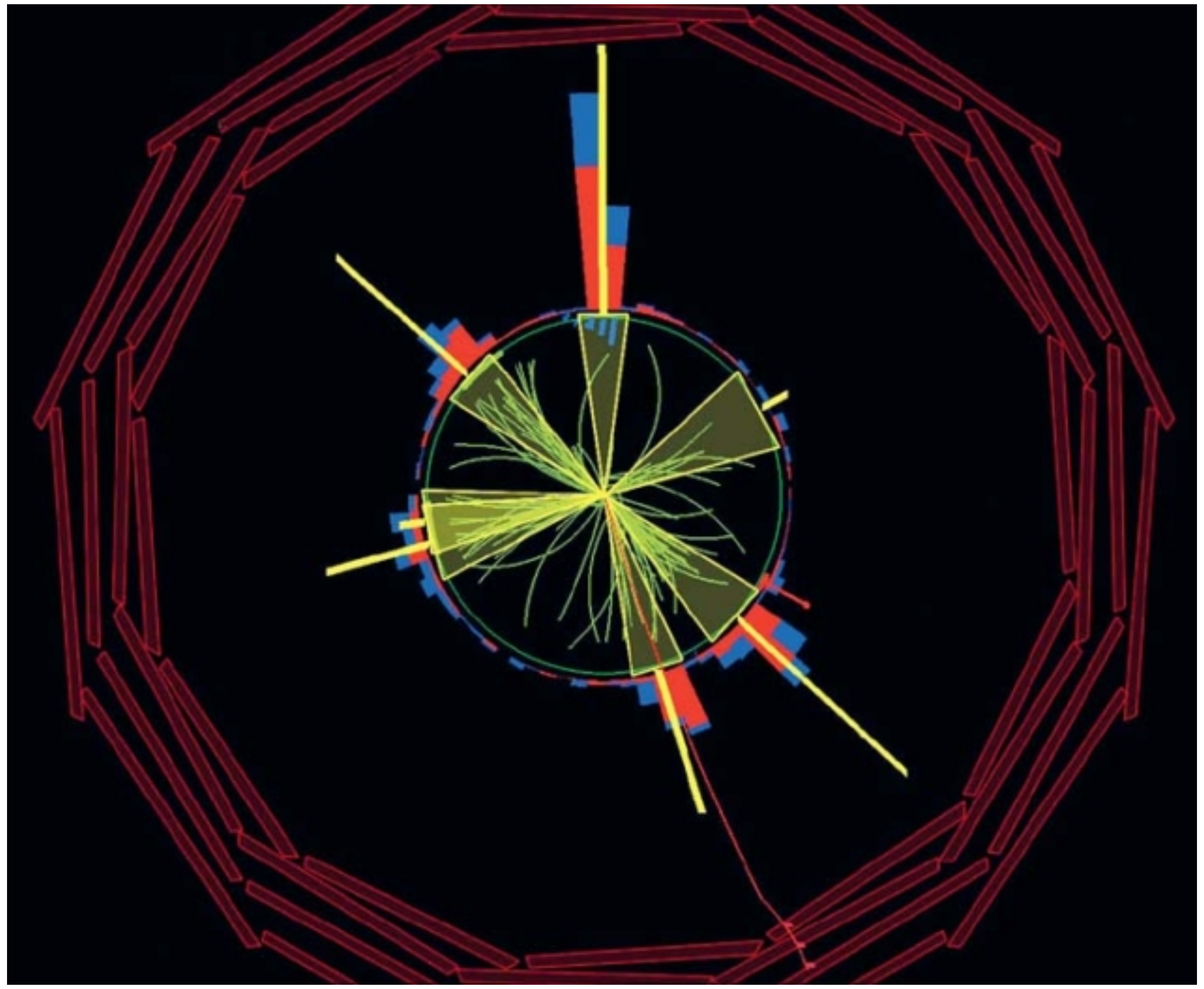
Jet Fragmentation Functions

Jets with Identified Hadrons

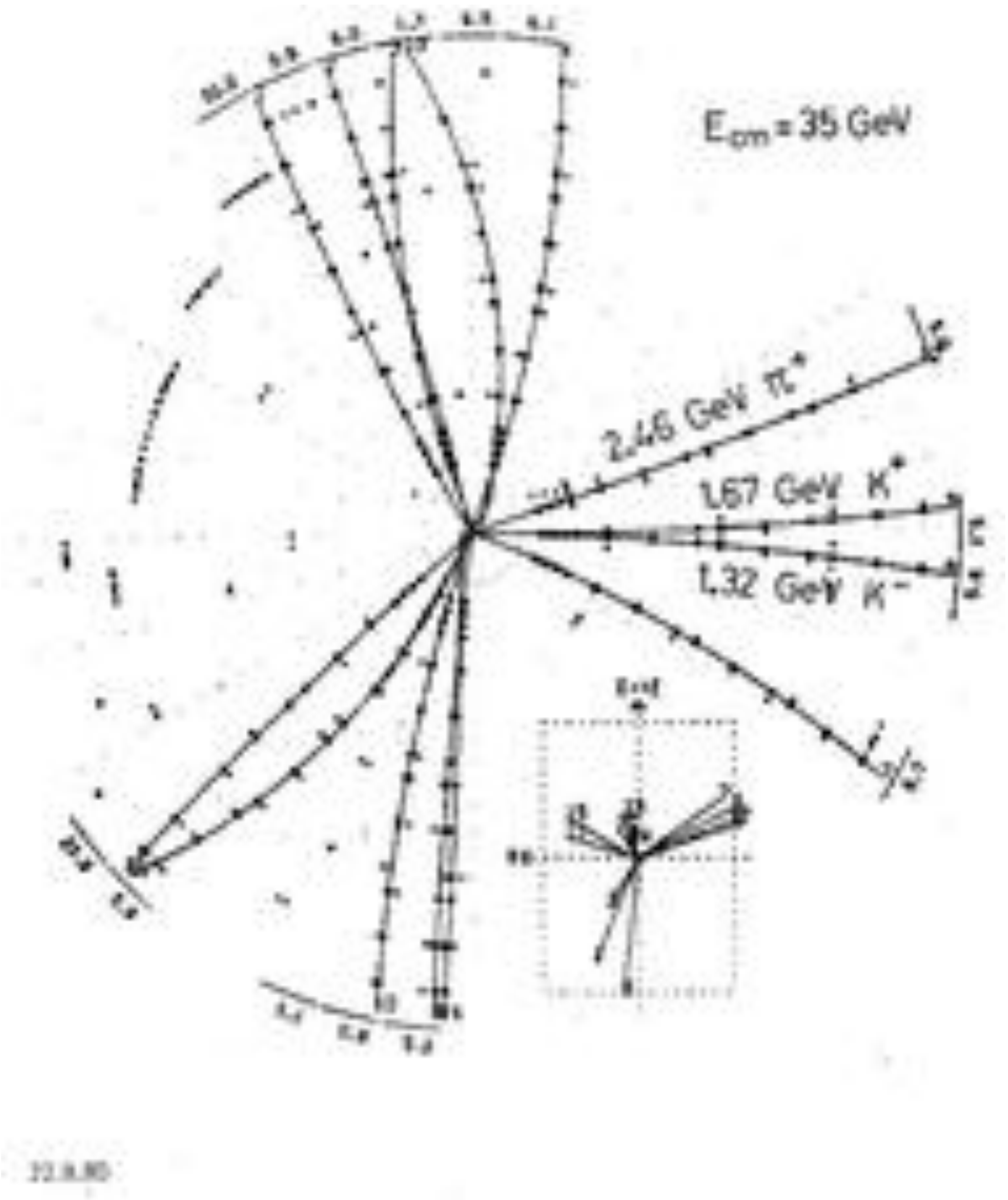
Jets with Quarkonia

What is a Jet?

Collimated shower of energetic final state particles



Jets at CMS



3 jet event at DESY
evidence for gluon

Jet Algorithms

sequential clustering algorithms

boost invariant distance measures (hadron collider)

$$d_{ij} = \min(p_{Ti}^{2p}, p_{Tj}^{2p}) \frac{\Delta R_{ij}^2}{R^2},$$
$$d_{iB} = p_{Ti}^{2p},$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

p = 1: k_T algorithm

p = 0: Cambridge-Aachen

p = -1: anti-k_T

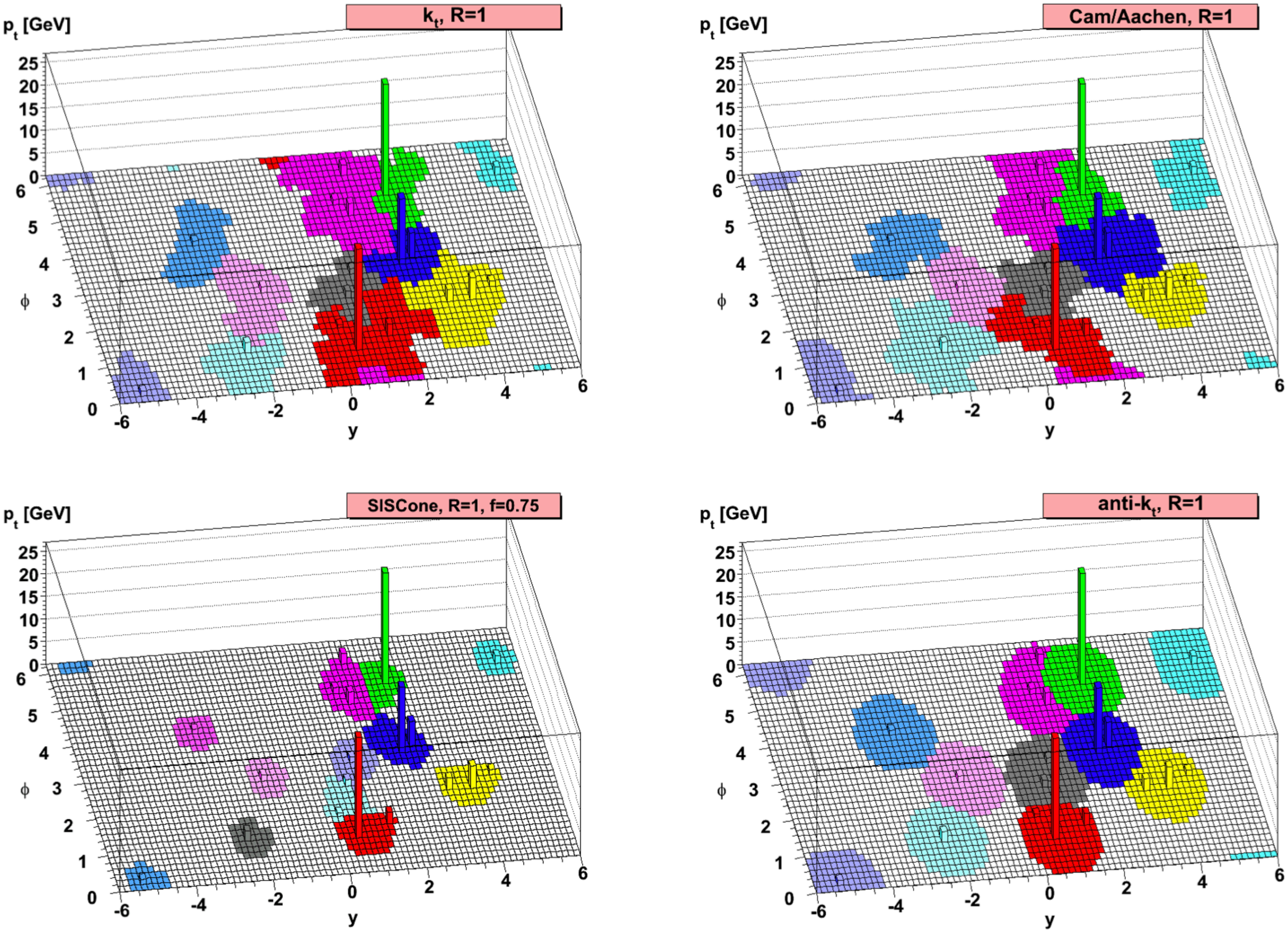
1) calculate d_{ij}, d_{iB} for all i,j

2) if d_{ij} smallest combine i and j, if d_{iB} smallest object is a jet and removed

3) repeat until every particle has been assigned to a jet

cone algorithms

more commonly used at $e^+ e^-$ colliders



M. Cacciari, G.P. Salam, G. Soyez, JHEP 0804:063,2008, arXiv:0802.1189

All jets characterized by: ω_J - light cone momentum R - jet radius

or P_T- transverse momentum

can also study jet substructure - functions of jet constituents

angularities $$\tau_a = \frac{1}{\omega}\sum_i (p_i^+)^{1-a/2}(p_i^-)^{a/2} \qquad \omega = \sum_i p_i^-$$

identified hadrons within jet

Jet Factorization Theorems

inclusive jet production, e.g. $pp \to jet + X$

$$\frac{d\sigma^{pp\to\text{jet}X}}{dp_T d\eta} = \sum_{a,b,c} \int_{\xi_a^{\min}}^{1} \frac{d\xi_a}{\xi_a} f_a(\xi_a,\mu) \int_{\xi_b^{\min}}^{1} \frac{d\xi_b}{\xi_b} f_b(\xi_b,\mu) \times \int_{z_c^{\min}}^{1} \frac{dz_c}{z_c^2} H_{ab\to c}(\hat{s},\hat{p}_T,\hat{\eta},\mu) J_c(z_c, p_T R, \mu)$$

similar to $pp \to H + X$ jet function replaces fragmentation function

jet function obeys DGLAP evolution

L. Dai, C. Kim, A. K. Leibovich, PRD 94 (2016)114023,arXiv:1606.07411

exclusive jet production, e.g, $e^+e^- \to N jets$

veto extra jets, radiation outside jets must be soft

$$\frac{d\sigma(\mathbf{P}_1,\ldots,\mathbf{P}_N)}{d\tau_1\cdots d\tau_M} = \sigma^{(0)}(\mathbf{P}_1,\ldots,\mathbf{P}_N)H(n_1,\omega_1;\cdots n_N,\omega_N;\mu)$$
$$\times\left[J_{n_1,\omega_1}(\tau_1;\mu)\cdots J_{n_M,\omega_M}(\tau_M;\mu)\right]\otimes S_{n_1\cdots n_N}(\tau_1,\ldots,\tau_M;R,\Lambda;\mu)$$
$$\times J_{n_{M+1},\omega_{M+1}}(R;\mu)\cdots J_{n_N,\omega_N}(R;\mu)\,,$$

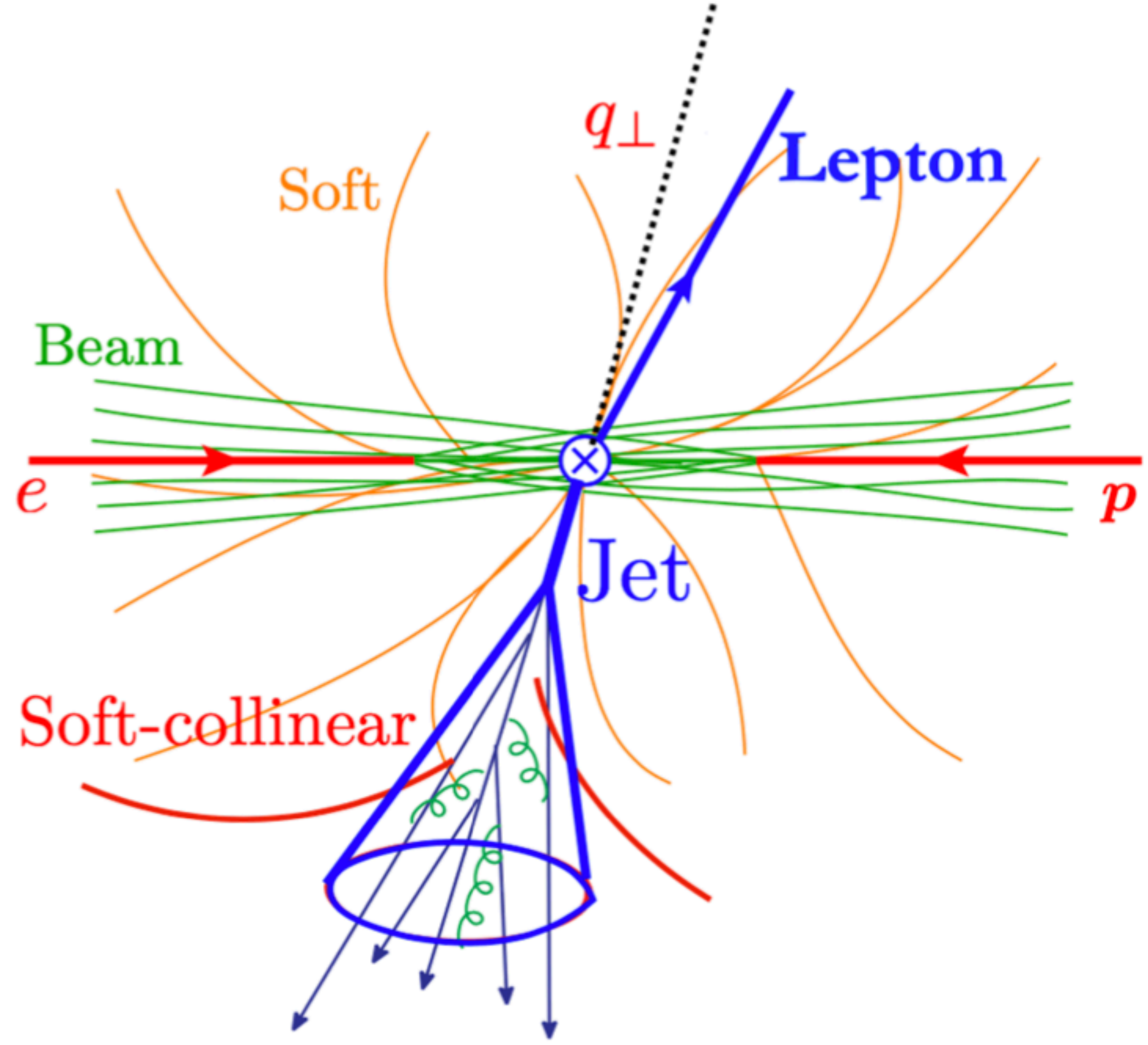
S.D. Ellis, et. al., JHEP 11 (2010) 101, arXiv:1001.0014

N-M "unmeasured" jets: ω_J, R only

M "measured" jets: also substructure

soft function convolved with measures jet functions because substructure is sensitive to soft radiation within jets

Measuring TMDs with Jets



$$p(P) + e(\ell) \to J(y_j, P_{JT}) + e(\ell') + X$$

$$P_T = (P_{jT} - \ell'_T)/2 \qquad\qquad q_T = P_{jT} + \ell'_T$$

SCET-like Factorization

$$\frac{d\sigma}{dy_J dP_T^2 d^2\boldsymbol{q}_T} = \hat{\sigma}_0 H(Q,\mu) \sum_q e_q^2 J_q(P_T R,\mu) \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}\, x\, \tilde{B}_{q/p}(x,\boldsymbol{b}_T,\mu,\zeta/\nu^2) \times \tilde{S}_q^{\text{global}}(\boldsymbol{b}_T,\mu,\nu)\, \tilde{S}_q^{\text{cs}}(\boldsymbol{b}_T,R,\mu)\,,$$

$$\tilde{S}_q^{\text{global}}(\boldsymbol{b}_T,\mu,\nu) = \tilde{S}_q^{\text{global}}(\boldsymbol{b}_T,\mu)\sqrt{\tilde{S}_{n_a n_b}(b_T,\mu,\nu)}\,.$$

$$\tilde{f}_{i/p}(x,\mathbf{b}_T,\mu,\zeta) = \tilde{B}_{i/p}(x,\mathbf{b}_T,\mu,\zeta/\nu^2)\sqrt{\tilde{S}_{n_a n_b}(b_T,\mu,\nu)}\,.$$

TMD PDF convolved with soft function instead of TMD FF

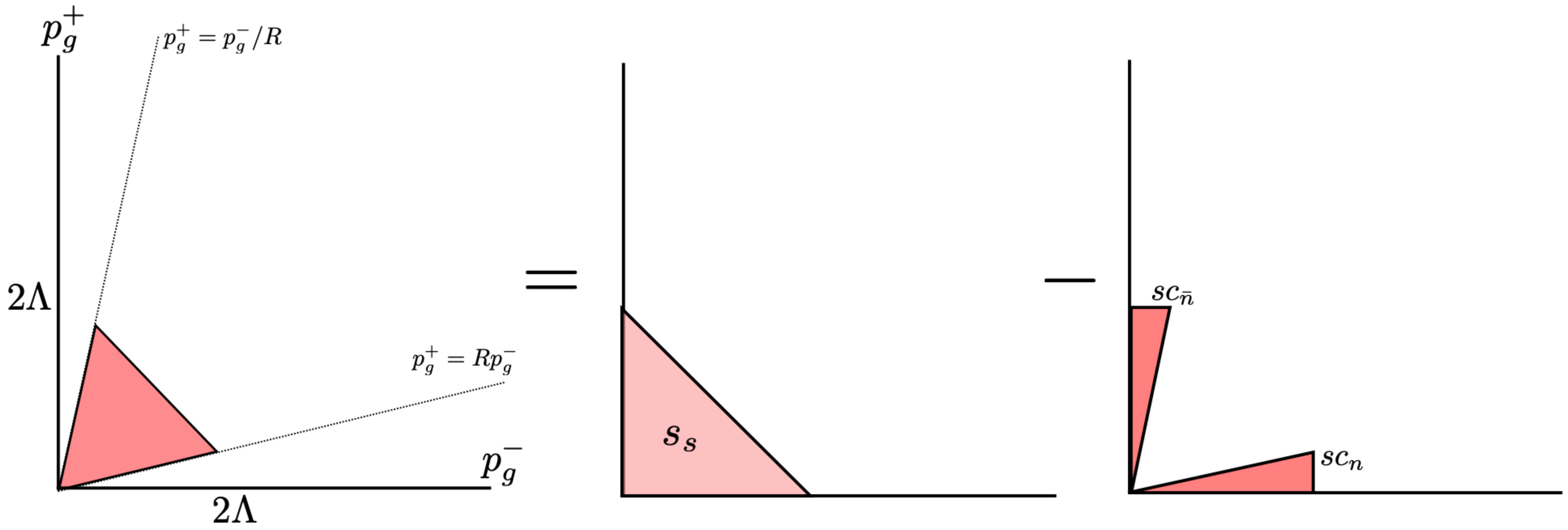
$$\frac{d\sigma}{dy_J dP_T^2 d^2\boldsymbol{q}_T} = \hat{\sigma}_0 H(Q,\mu) \sum_q e_q^2 J_q(P_T R,\mu) \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}\, x\, \tilde{f}_{q/p}(x,\boldsymbol{b}_T,\mu,\zeta) \times \tilde{S}_q^{\text{global}}(\boldsymbol{b}_T,\mu)\, \tilde{S}_q^{\text{cs}}(\boldsymbol{b}_T,R,\mu)\,.$$

Z.B. Kang, et. al., JHEP 11 (2021) 005. arXiv:2106.15624

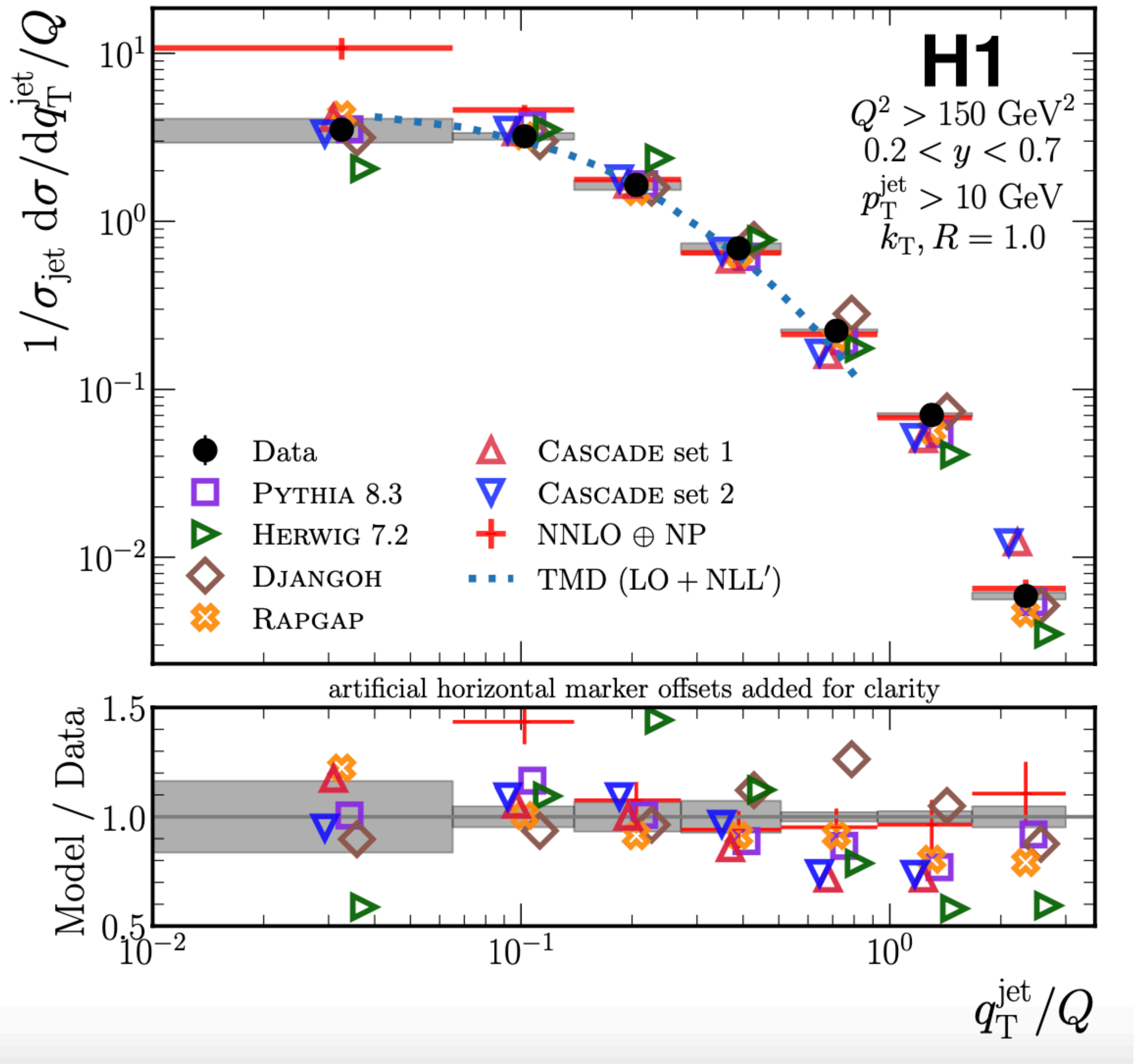
$J_q(P_T R, \mu)$ describes energetic particles collinear to jet

$\tilde{S}_q^{\mathrm{global}}(b_T, \mu)$ describes soft radiation over all phase space

$\tilde{S}_q^{\mathrm{cs}}(b_T, R, \mu)$ describes soft radiation collinear to jet

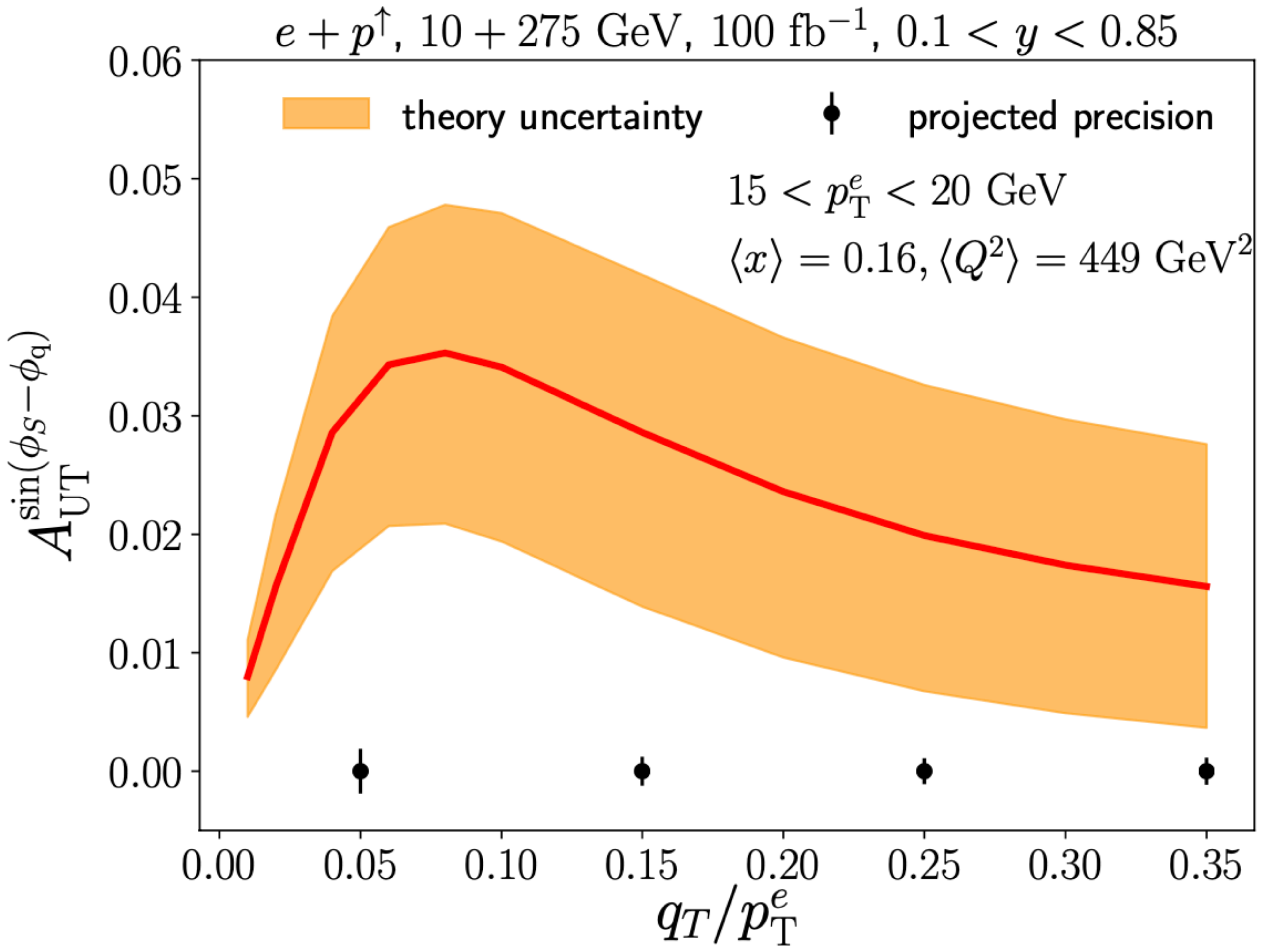


Y.-T. Chien, A. Hornig, C. Lee, PRD 93 (2016) 1, 014033, arXiv:1509.04287



H1 collaboration, V. Andreev, et. al., arXiv:2108.12376

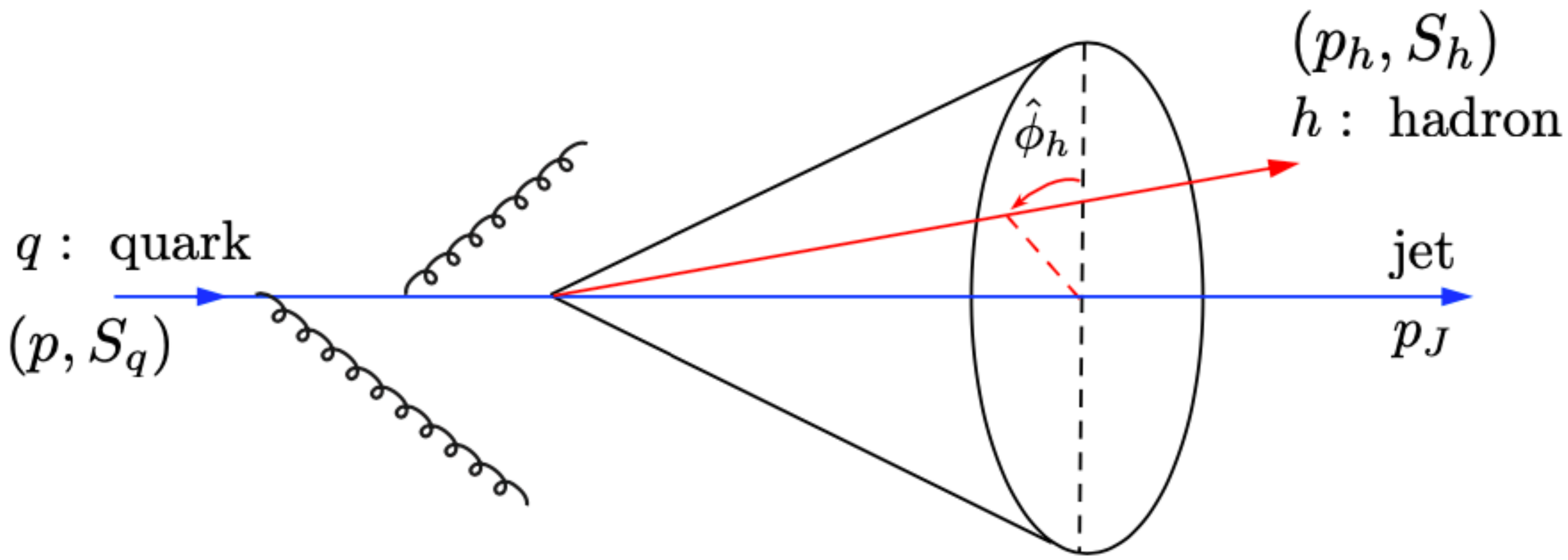
Sivers Asymmetry



considerable improvement in theory required for EIC

M. Arratia, et. al., PRD 102, 074015 (2020), arXiv:2007.07281

Jets with Identified Hadrons



$\uparrow j_T$

inclusive jet production: $z = \dfrac{\omega_J}{\omega}$ $z_h = \dfrac{\omega_h}{\omega_j}$ j_T

exclusive jet production: $z_h = \dfrac{\omega_h}{\omega_j}$ j_T

Fragmenting Jet Functions

$$\frac{d\sigma^{pp\to(\text{jet}\,h)X}}{dp_T d\eta dz_h d^2\boldsymbol{j}_\perp} = \sum_{a,b,c}\int_{\xi_a^{\min}}^1 \frac{d\xi_a}{\xi_a} f_a(\xi_a,\mu)\int_{\xi_b^{\min}}^1 \frac{d\xi_b}{\xi_b} f_b(\xi_b,\mu)$$

$$\times\int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} H_{ab\to c}(\hat{s},\hat{p}_T,\hat{\eta},\mu)\,\mathcal{G}_c^h(z_c,p_T R,z_h,\boldsymbol{j}_\perp,\mu,\zeta_J)$$

inclusive jet production, longitudinal only: $\mathcal{G}_i^h(z,z_h,p_T R,\mu)$,

$$\mu\frac{d}{d\mu}\mathcal{G}_i^h(z,z_h,p_T R,\mu) = \frac{\alpha_s(\mu)}{\pi}\sum_j\int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}\right)\mathcal{G}_j^h(z',z_h,p_T R,\mu)$$

$$\mathcal{G}_i^h(z,z_h,p_T R,\mu) = \sum_j\int_{z_h}^1 \frac{dz'_h}{z'_h}\mathcal{J}_{ij}^{\text{incl}}\left(z,z'_h,p_T R,\mu\right)D_j^h\left(\frac{z_h}{z'_h},\mu\right)\left[1+\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{p_T^2 R^2}\right)\right]$$

exclusive jet production, longitudinal only:

$$\mu\frac{d}{d\mu}\mathcal{G}_i^h(z_h, p_T R, \mu) = \gamma_J^i(\mu)\,\mathcal{G}_i^h(z_h, p_T R, \mu),$$

$$\gamma_J^i(\mu) = \Gamma_{\text{cusp}}^i\left[\alpha_s(\mu)\right]\ln\left(\frac{\mu^2}{p_T^2 R^2}\right) + \gamma^i\left[\alpha_s(\mu)\right],$$

$$\mathcal{G}_i^h(z_h, p_T R, \mu) = \sum_j \int_{z_h}^1 \frac{dz_h'}{z_h'}\mathcal{J}_{ij}\left(z_h', p_T R, \mu\right) D_j^h\left(\frac{z_h}{z_h'}, \mu\right)\left[1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{p_T^2 R^2}\right)\right]$$

if both z_h, j_T measured:

TMD FF

$$\mathcal{G}_i^h(z, p_T R, z_h, \boldsymbol{j}_\perp, \mu, \zeta_J) = \mathcal{C}_{i\to j}(z, p_T R, \mu)\, D_{h/j}(z_h, \boldsymbol{j}_\perp, \mu, \zeta_J)$$

inclusive - DGLAP evolution

$$\mu\frac{d}{d\mu}\mathcal{G}_i^h(z, p_T R, z_h, \boldsymbol{j}_\perp, \mu, \zeta_J) = \frac{\alpha_s(\mu)}{\pi}\sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}\right)\mathcal{G}_j^h(z', p_T R, z_h, \boldsymbol{j}_\perp, \mu, \zeta_J)$$

exclusive - jet-like evolution

$$\mu\frac{d}{d\mu}\mathcal{G}_i^h(p_T R, z_h, \boldsymbol{j}_\perp, R, \mu) = \gamma_J^i(\mu)\,\mathcal{G}_i^h(p_T R, z_h, \boldsymbol{j}_\perp, \mu)$$

Jets with Quarkonia

for light hadrons, singly heavy hadrons collinear FFs and TMD FFs are nonperturbative quantities

heavy quarkonia: $Q\overline{Q}$ bound states w/ $m_Q \gg \Lambda_{\rm QCD}$

collinear FFs calculable using NRQCD factorization: **more predictive power**

Quarkonia production not completely understood:
jets provide new tests of NRQCD

NRQCD and Quarkonium Production

Heavy Quarkonium FJFs

Recent Data on Quarkonia in Jets (LHCb)

Ongoing Work

Color-Singlet Model (pre-1995)

$$\sigma(pp \to J/\psi + X) = f_{g/p} \otimes f_{g/p} \otimes \sigma[gg \to c\bar{c}(^3S_1^{(1)}) + X]\,|\psi_{c\bar{c}}(0)|^2$$

$\bar{c}c$ pair produced with same quantum numbers as J/ψ

Predictive Formalism

$\sigma[gg \to c\bar{c}(^3S_1^{(1)}) + X]$ calculable in QCD perturbation theory

$|\psi_{c\bar{c}}(0)|^2$ fixed by $\Gamma[J/\psi \to \ell^+\ell^-]$

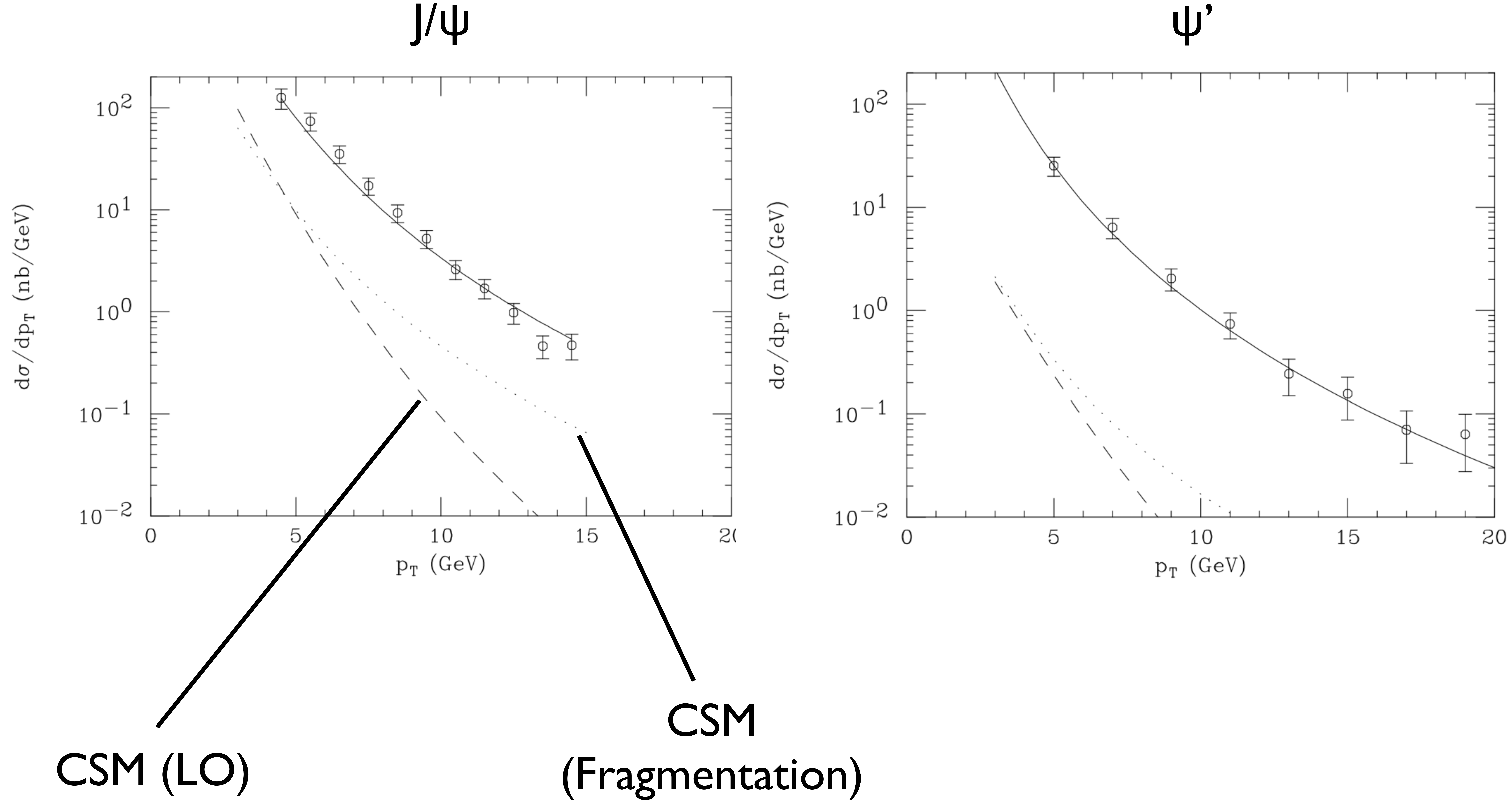
Suffers from theoretical inconsistencies when applied to χ_{cJ}

$$\Gamma[\chi_{cJ} \to \text{hadrons}] = |\psi'_{c\bar{c}}(0)|^2 \sigma(c\bar{c}(^3P_J^{(1)}) \to gg)$$

⟵ **Not IR Safe**

J/ψ production at Tevatron (1996)

CSM badly underpredicts J/ψ and ψ' production at large p_T



Non-Relativistic QCD (NRQCD) Factorization Formalism

Bodwin, Braaten, Lepage, PRD 51 (1995) 1125

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X)\langle \mathcal{O}^{J/\psi}(n)\rangle$$

$$n - {}^{2S+1}L_J^{(1,8)}$$

double expansion in α_s, v

NRQCD long-distance matrix element (LDME)

$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle \sim v^3$

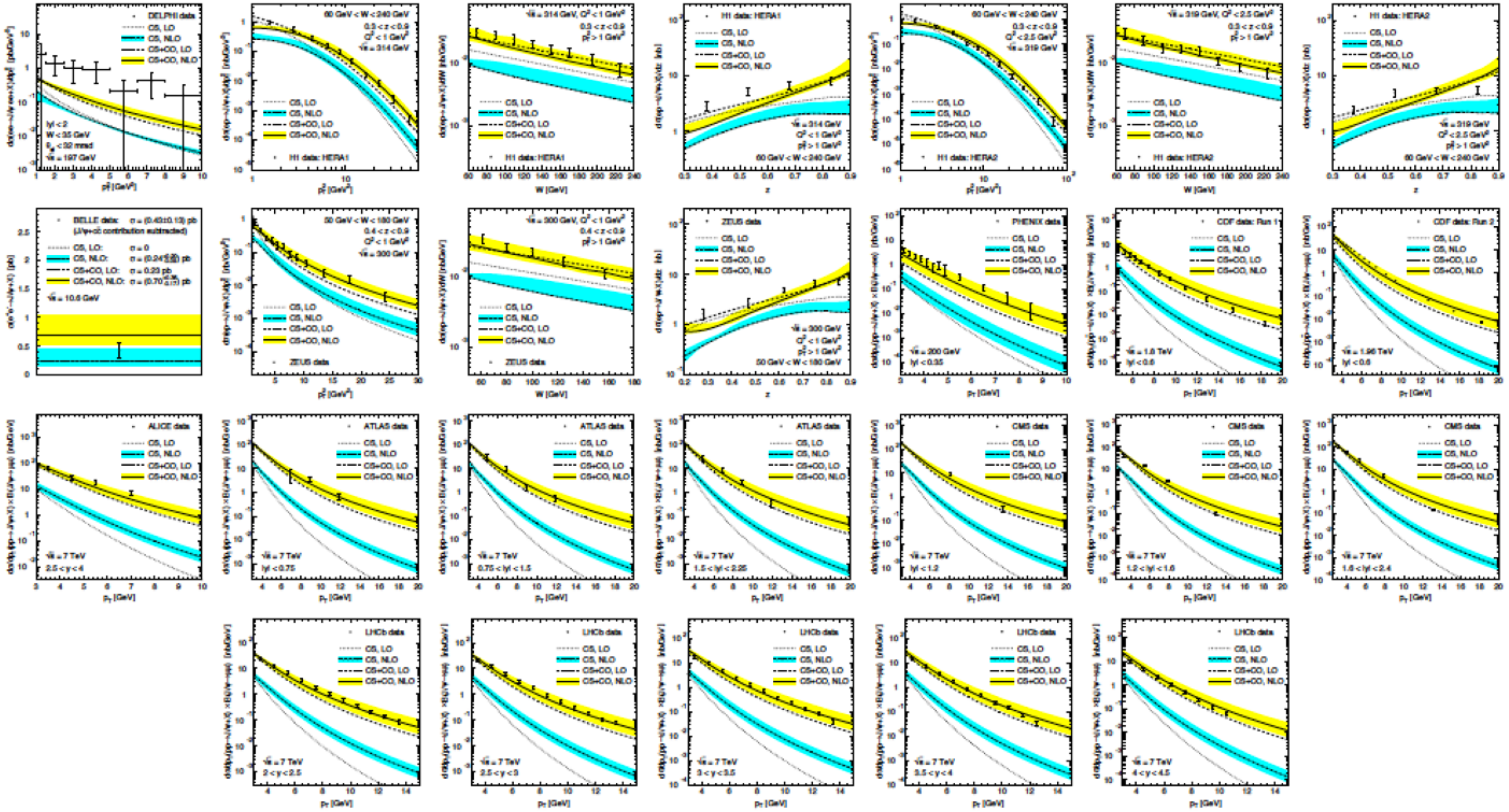
CSM - lowest order in v

$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]})\rangle \sim v^7$

color-octet mechanisms

Global Fits with NLO CSM + COM

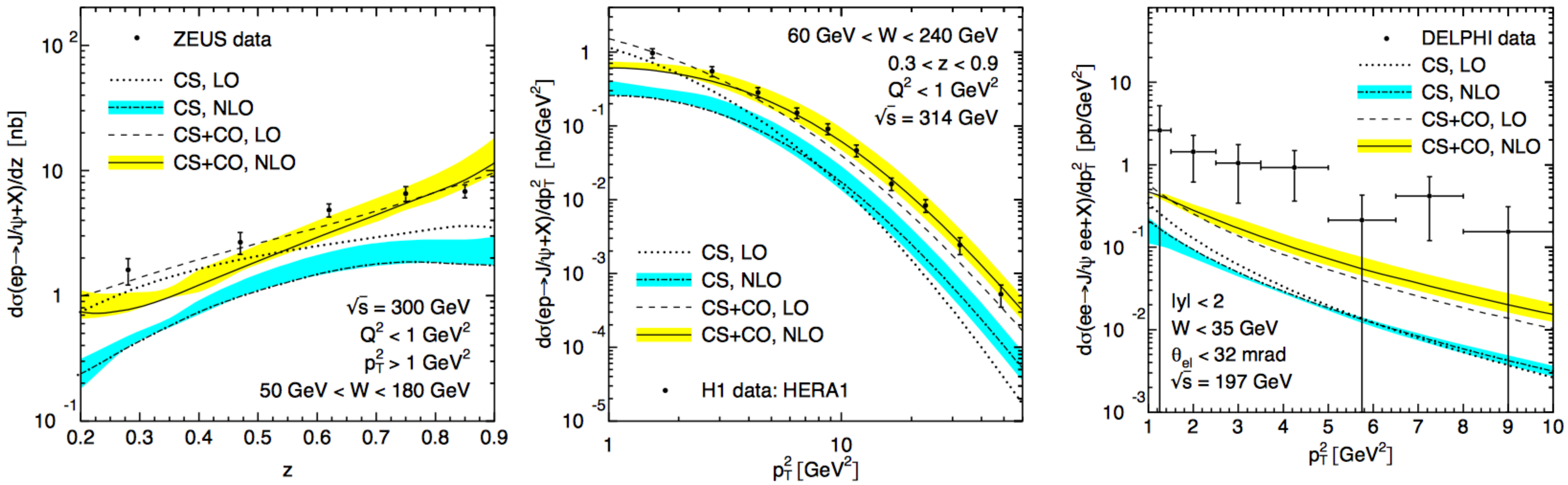
Butenschoen and Kniehl, PRD 84 (2011) 051501



$$e^+e^-, \gamma\gamma, \gamma p, p\bar{p}, pp \to J/\psi + X$$

fit to 194 data points, 26 data sets

NLO: CSM + COM Required to Fit Data



$ep \to J/\psi + X$

$\gamma^* \gamma^* \to J/\psi + X$

Status of NRQCD approach to J/ψ Production

NLO: COM + CSM required for most processes

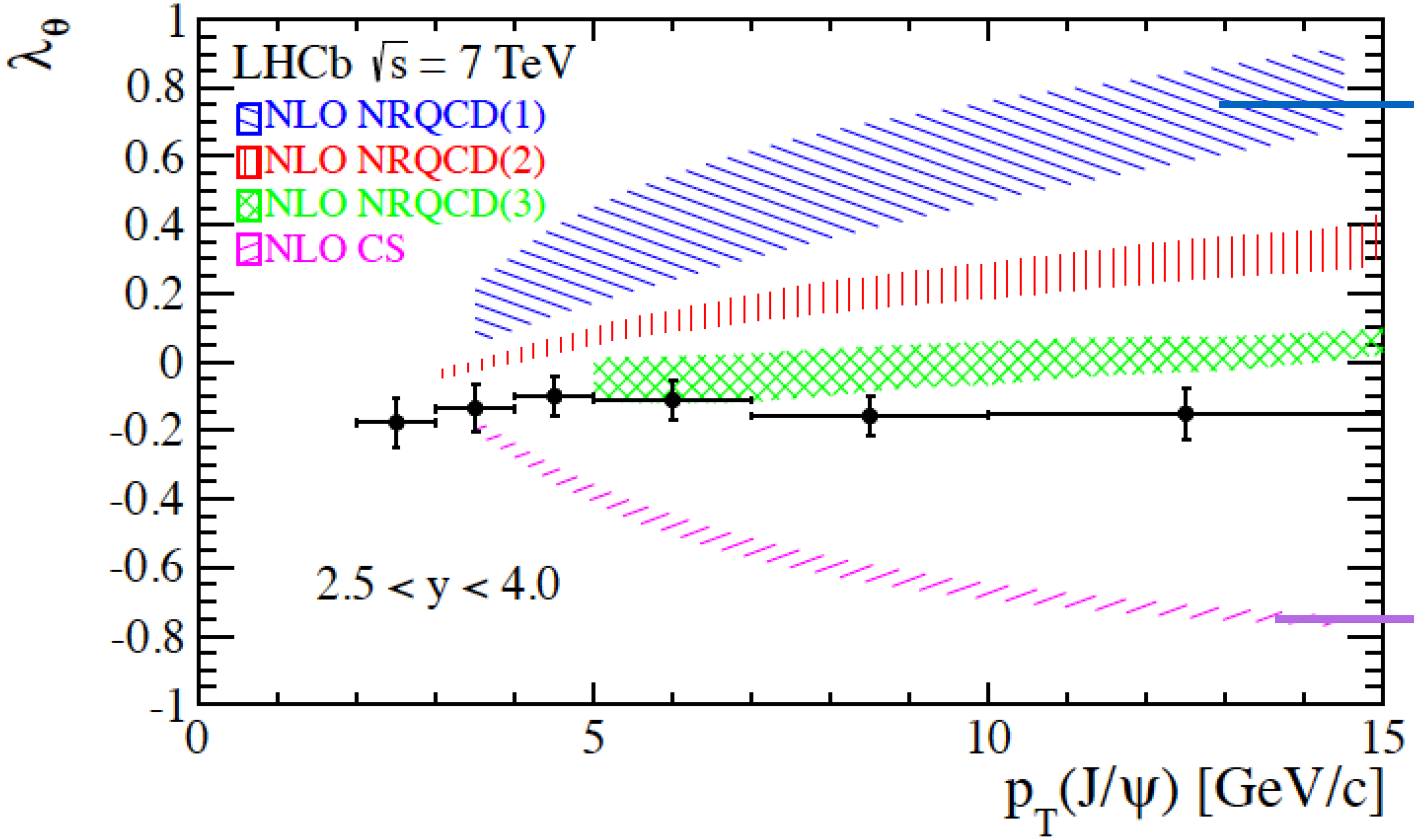
extracted LDME satisfy NRQCD v-scaling

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle = 1.32 \text{ GeV}^3$$

$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2}$ GeV^3
$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3}$ GeV^3
$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2}$ GeV^5

$$\chi^2_{\text{d.o.f.}} = 857/194 = 4.42$$

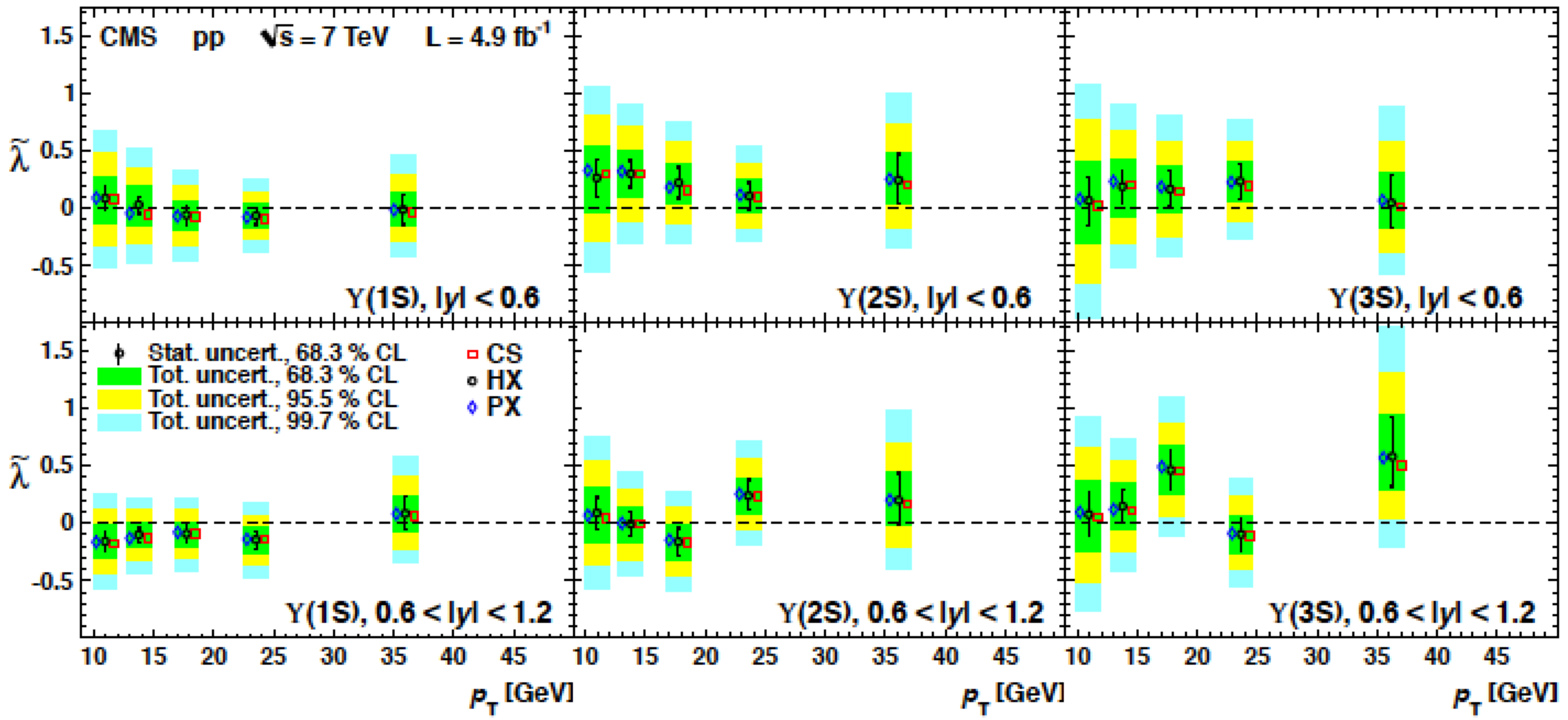
Polarization of J/ψ at LHCb



LDME from Global fits

CSM

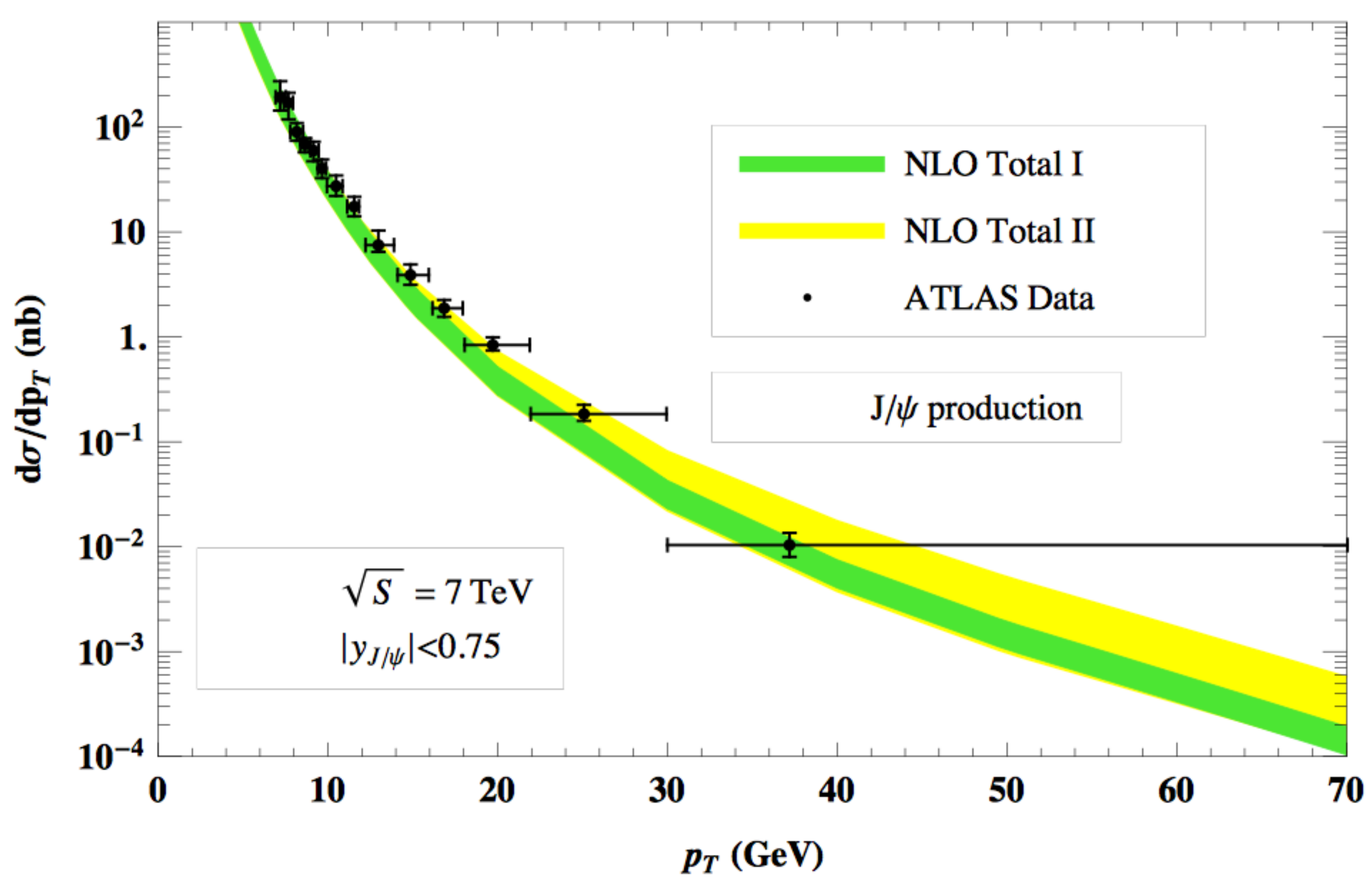
Polarization of Υ(nS) at CMS

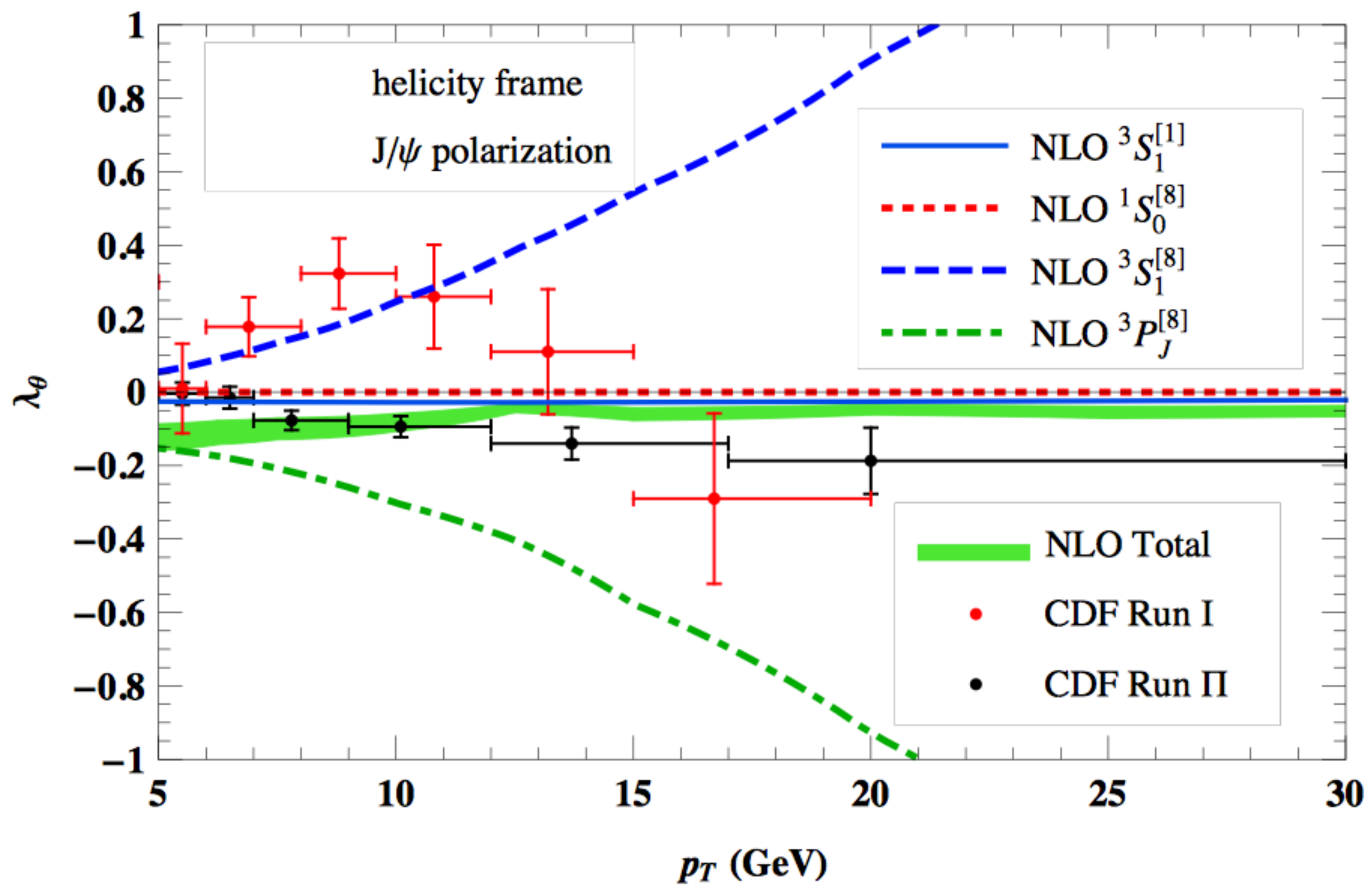


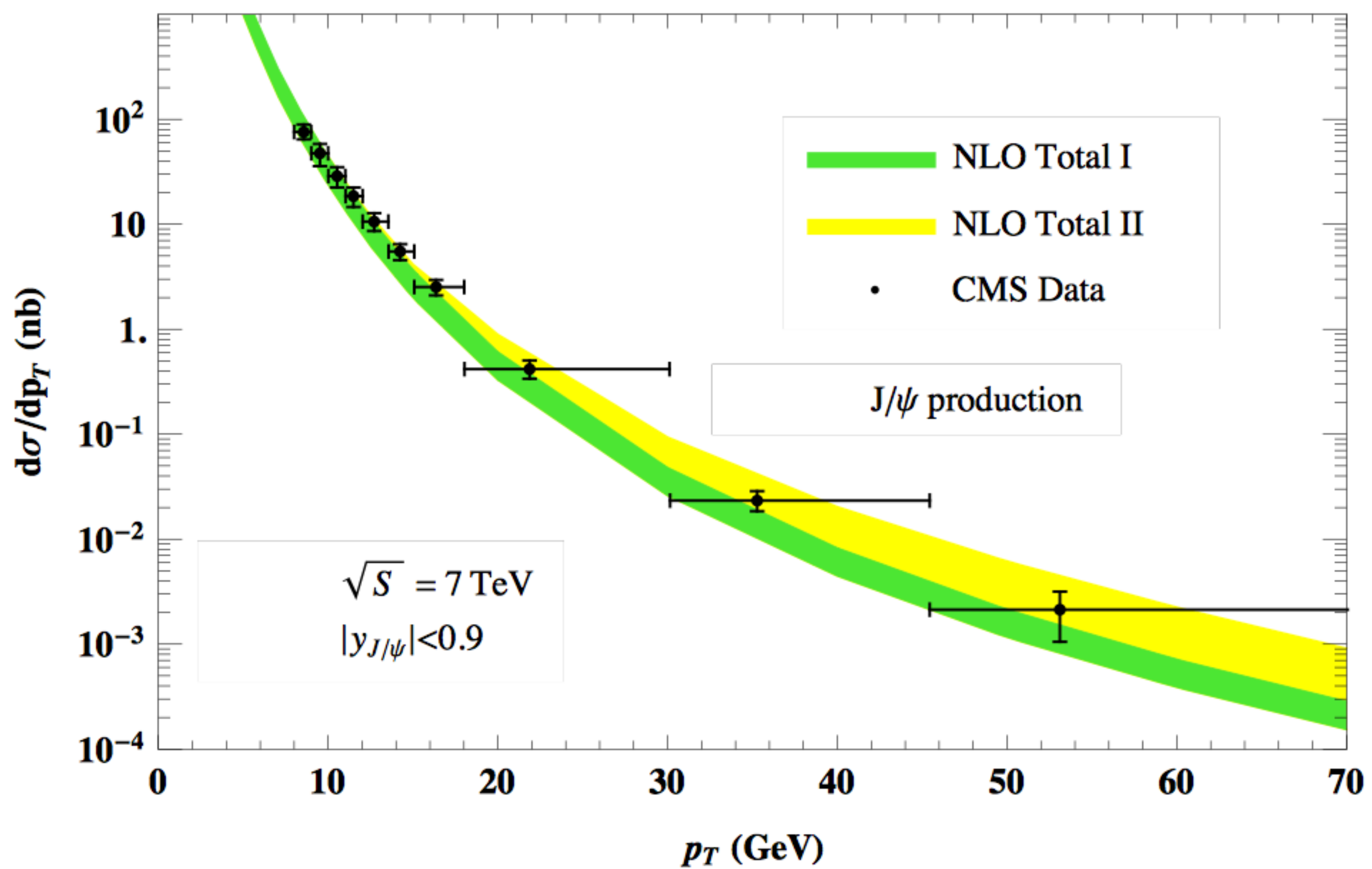
Recent Attempts to Resolve J/ψ Polarization Puzzle

simultaneous NLO fit to CMS, ATLAS high p_t production, polarization

Chao, et. al. PRL 108, 242004 (2012)







$\langle \mathcal{O}(^3S_1^{[1]}) \rangle$ GeV^3	$\langle \mathcal{O}(^1S_0^{[8]}) \rangle$ $10^{-2}\mathrm{GeV}^3$	$\langle \mathcal{O}(^3S_1^{[8]}) \rangle$ $10^{-2}\mathrm{GeV}^3$	$\langle \mathcal{O}(^3P_0^{[8]}) \rangle / m_c^2$ $10^{-2}\mathrm{GeV}^3$
1.16	8.9 ± 0.98	0.30 ± 0.12	0.56 ± 0.21
1.16	0	1.4	2.4
1.16	11	0	0

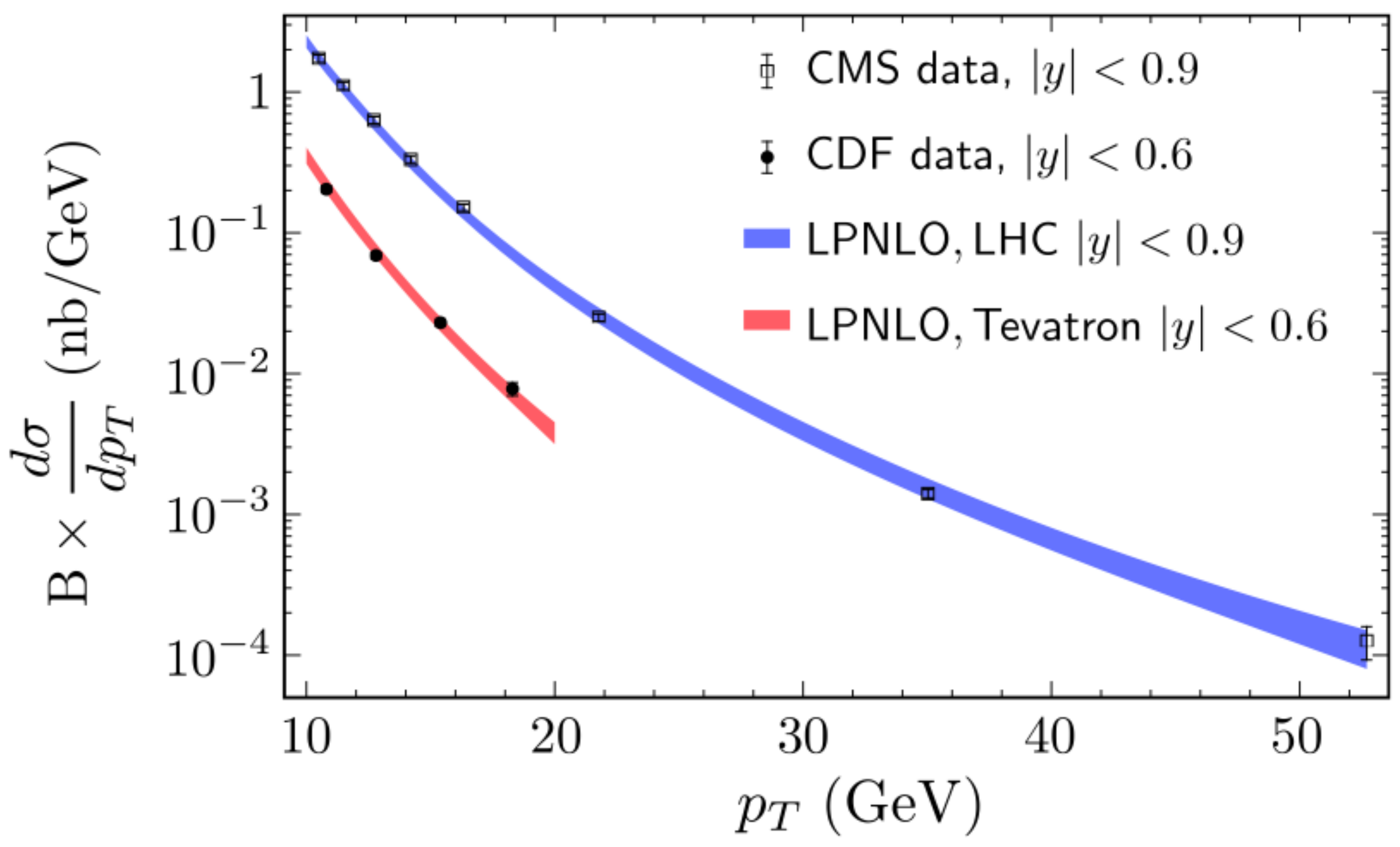
Recent Attempts to Resolve J/ψ Polarization Puzzle

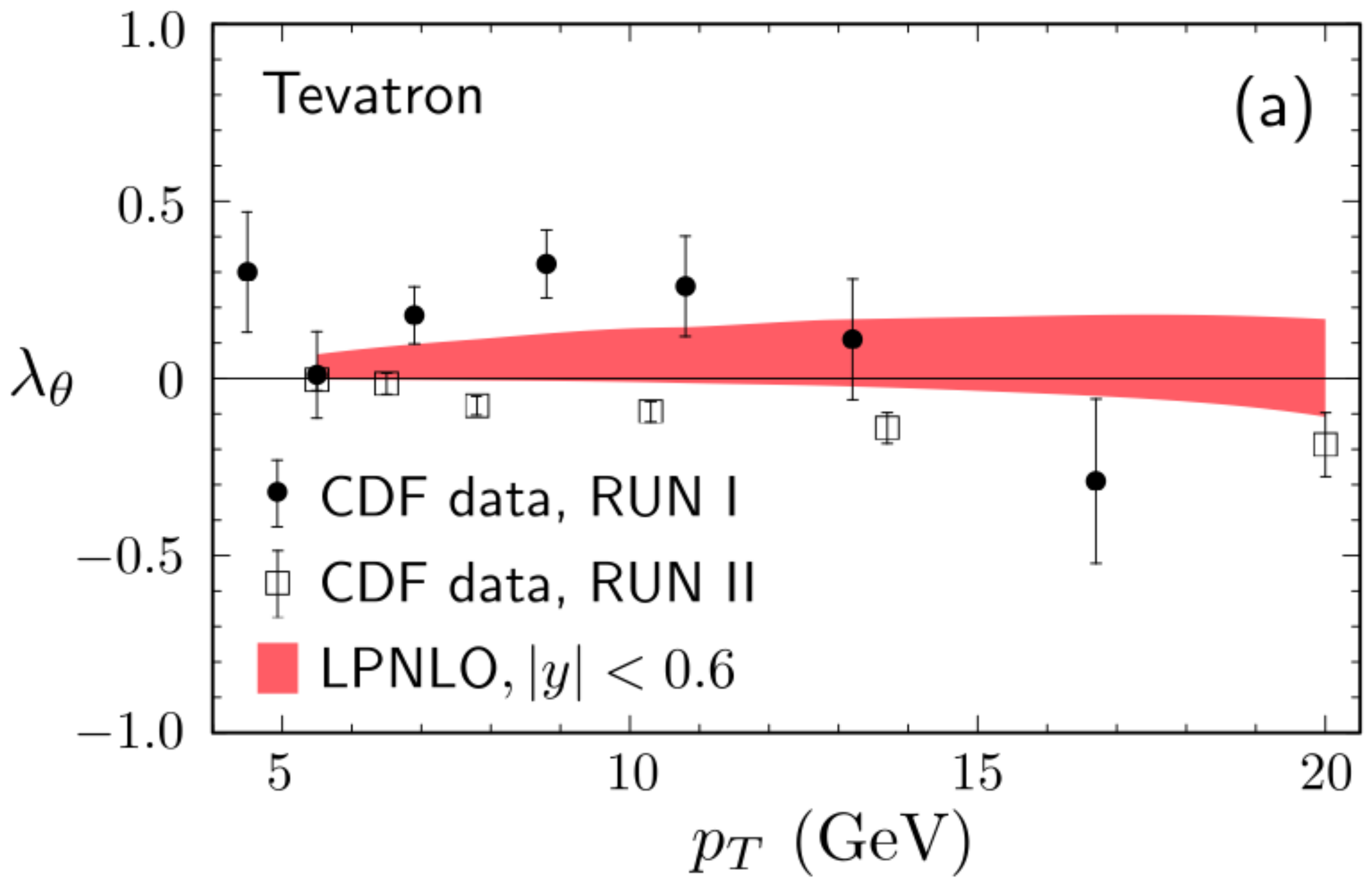
i) large p_t production at CDF

Bodwin, et. al., PRL 113, 022001(2014)

ii) resum logs of p_t/m_c using DGLAP evolution

iii) fit COME to p_t spectrum, predict basically no polarization





Extracted COME **inconsistent** with global fits

$$\langle \mathcal{O}^{J/\psi}(^1S_0^{(8)}) \rangle = 0.099 \pm 0.022\,\mathrm{GeV}^3$$

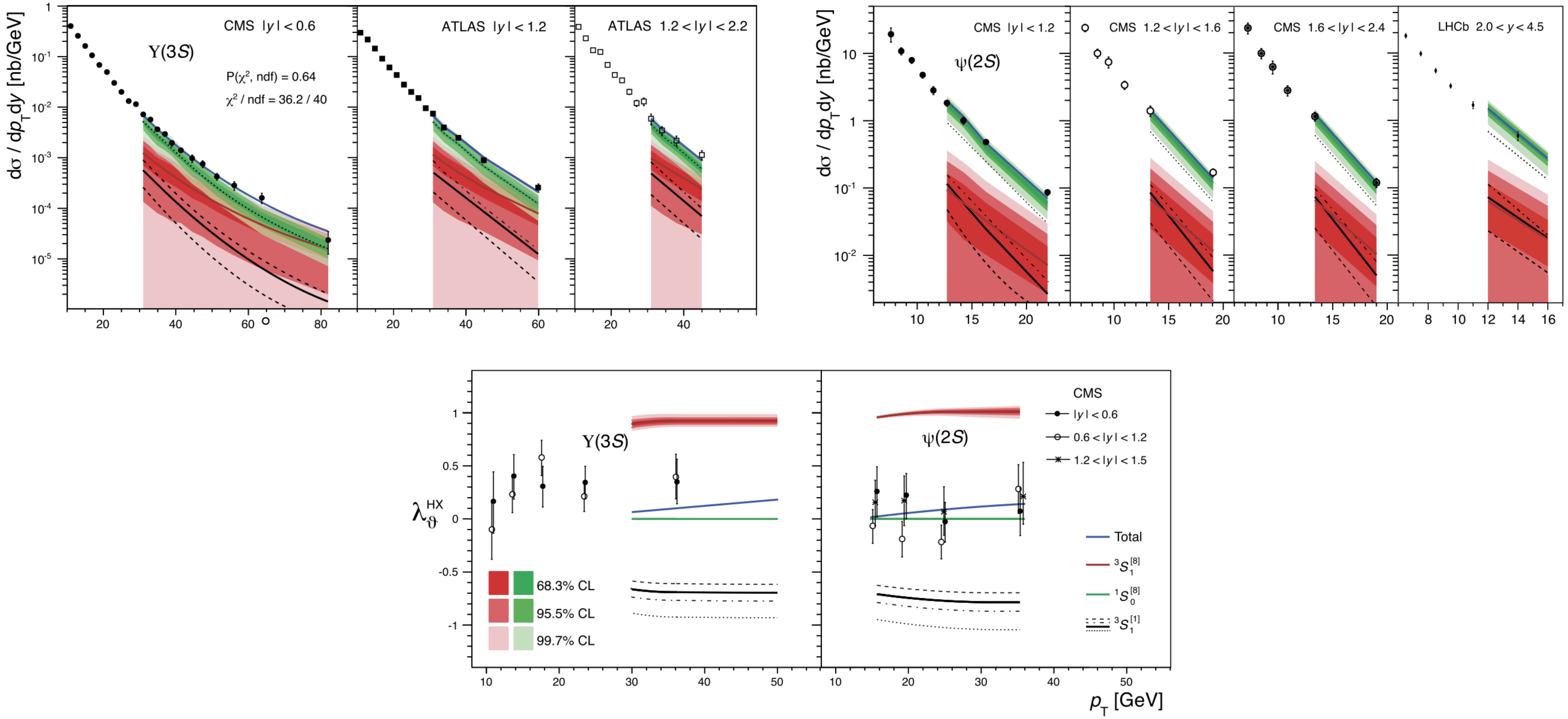
$$\langle \mathcal{O}^{J/\psi}(^3S_1^{(8)}) \rangle = 0.011 \pm 0.010\,\mathrm{GeV}^3$$

$$\langle \mathcal{O}^{J/\psi}(^3P_0^{(8)}) \rangle = 0.011 \pm 0.010\,\mathrm{GeV}^5$$

Recent Attempts to Resolve J/ψ Polarization Puzzle

Faccioli, et. al. PLB736 (2014) 98

Lourenco, et. al., NPA, in press



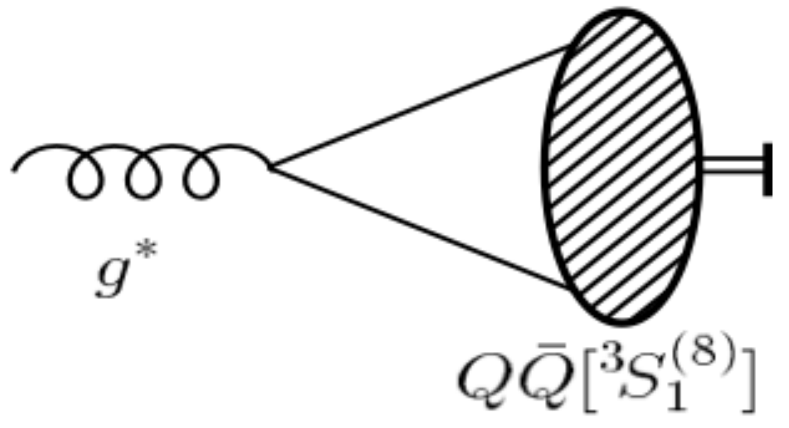
argue for $^1S_0^{(8)}$ dominance in both ψ(2S) & Υ(3S) production

NRQCD fragmentation functions

Braaten, Yuan, PRD 48 (1993) 4230
Braaten, Chen, PRD 54 (1996) 3216
Braaten, Fleming, PRL 74 (1995) 3327

Perturbatively calculable **at the scale 2m$_c$**

$$D_g^{\psi(8)}(z,2m_c) = \frac{\pi\alpha_s(2m_c)}{3M_\psi^3}\langle O^\psi(^3S_1^{(8)})\rangle\delta(1-z)$$



$$D_g^{\psi(1)}(z,2m_c) = \frac{5\alpha_s^3(2m_c)}{648\pi^2}\frac{\langle O^\psi(^3S_1^{(1)})\rangle}{M_\psi^3}\int_0^z dr\int_{(r+z^2)/2z}^{(1+r)/2} dy \frac{1}{(1-y)^2(y-r)^2(y^2-r)^2}$$
$$\sum_{i=0}^{2} z^i\left(f_i(r,y)+g_i(r,y)\frac{1+r-2y}{2(y-r)\sqrt{y^2-r}}\ln\frac{y-r+\sqrt{y^2-r}}{y-r-\sqrt{y^2-r}}\right),$$

DGLAP evolution: 2m$_c$ to 2E tan(R/2)

FJF in terms of fragmentation function

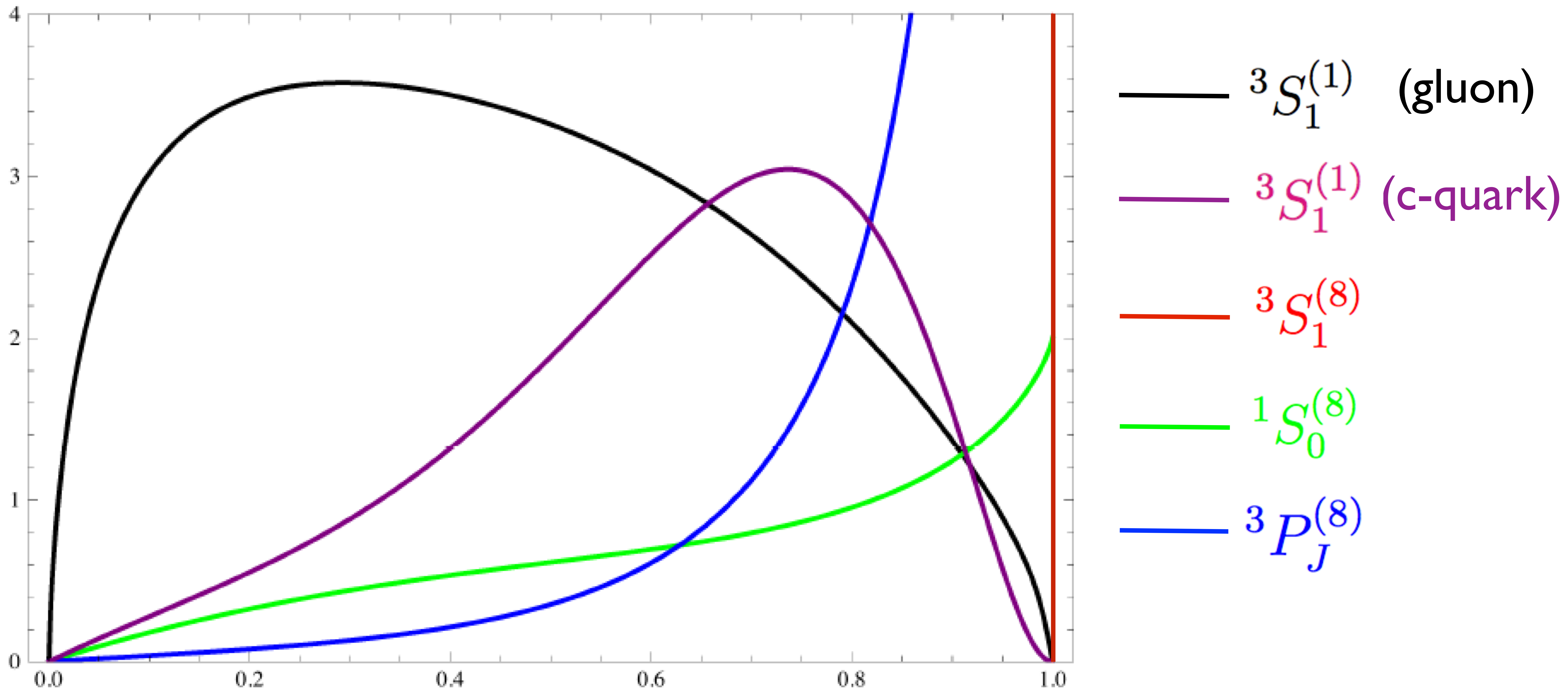
$$
\begin{aligned}
\mathcal{G}_g^{\psi}(E,R,z,\mu) \;=\;& D_{g\to\psi}(z,\mu)\left(1+\frac{C_A\alpha_s}{\pi}\left(L_{1-z}^2-\frac{\pi^2}{24}\right)\right)\\
&+\frac{C_A\alpha_s}{\pi}\left[\int_z^1\frac{dy}{y}\tilde{P}_{gg}(y)L_{1-y}D_{g\to\psi}\left(\frac{z}{y},\mu\right)\right.\\
&+2\int_z^1 dy\frac{D_{g\to\psi}(z/y,\mu)-D_{g\to\psi}(z,\mu)}{1-y}L_{1-y}\\
&\left.+\theta\left(\frac{1}{2}-z\right)\int_z^{1/2}\frac{dy}{y}\hat{P}_{gg}(y)\ln\left(\frac{y}{1-y}\right)D_{g\to\psi}\left(\frac{z}{y},\mu\right)\right]
\end{aligned}
$$

$$
L_{1-z}=\ln\left(\frac{2E\tan(R/2)(1-z)}{\mu}\right)
$$

For large E, FJF ~ NRQCD frag. function (at scale 2E tan(R/2))

$$
\mathcal{G}_g^{h}(E,R,\mu=2E\tan(R/2),z)\to D_g^{\psi}(z,2E\tan(R/2))+O(\alpha_s)
$$

NRQCD FF's (at scale $2m_c$)

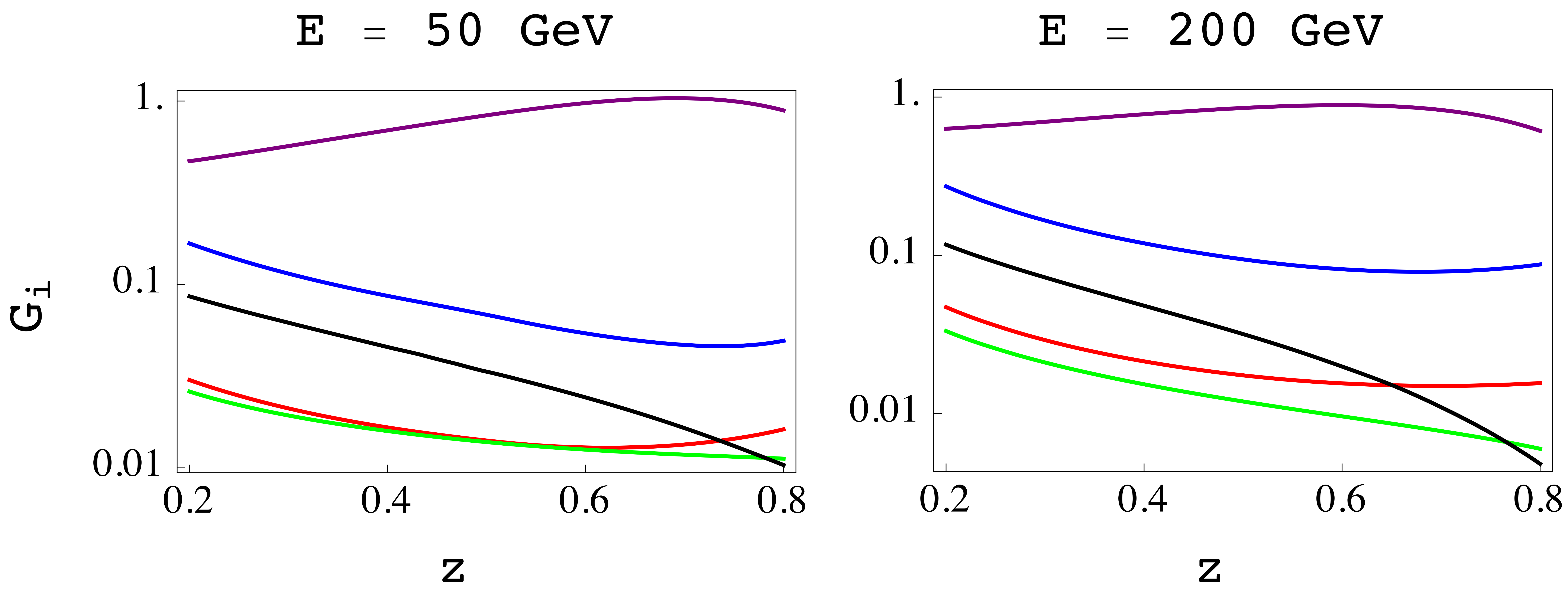


(normalization arbitrary)

Evolution to 2E tan(R/2) will soften discrepancies

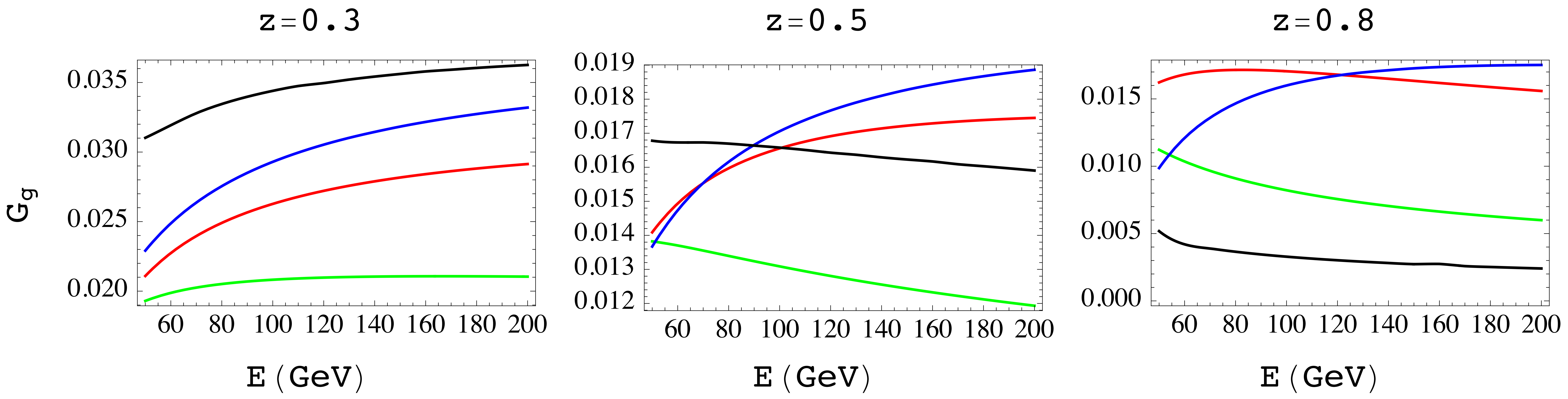
FJF's at Fixed Energy vs. z

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003



FJF's at Fixed z vs. Energy

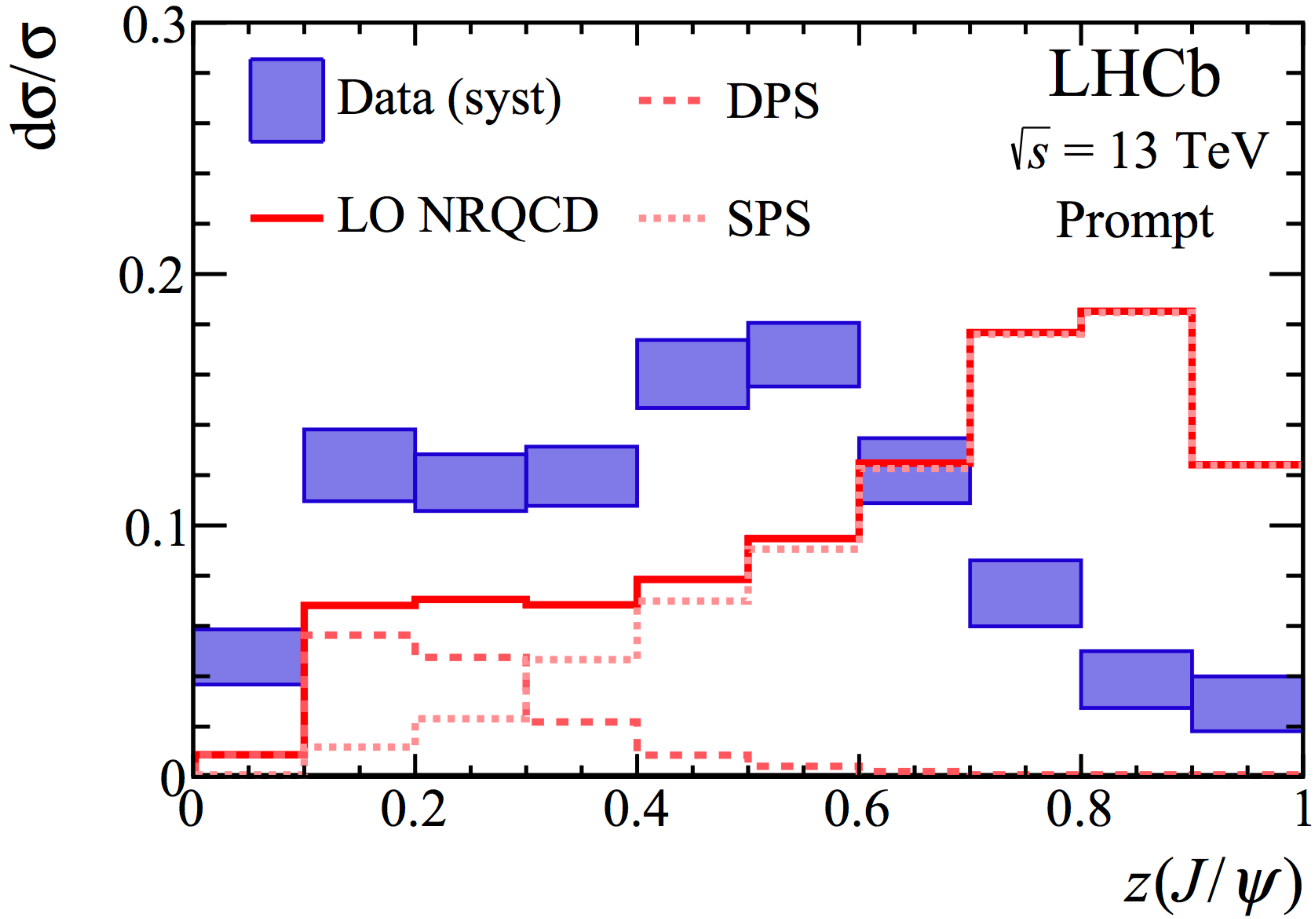
M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003



$^1S_0^{(8)}$ dominance predicts negative slope for z vs. E if z > 0.5

Recent Observations of Quarkonia within Jets

LHCb collaboration, Phys. Rev. Lett. 118 (2017) no.19, 192001



cuts: $2.5 < \eta_{\text{jet}} < 4.0 \quad p_{T,jet} > 20\,\text{GeV} \quad p(\mu) > 5\,\text{GeV}$

This result was anticipated in:

Jets w/ Heavy Mesons: NLL' vs. Monte Carlo

(w/ R. Bain, L. Dai, A. Hornig, A.Leibovich, Y. Makris)

JHEP 1606 (2016) 121 (arXiv:1601.05815)

$$e^+e^- \to b\bar{b}$$

$\hookrightarrow$ B jet

$$e^+e^- \to q\bar{q}g$$

$\hookrightarrow$ J/ψ jet

$e^+e^- \longrightarrow$ Jets in SCET

S.D. Ellis, et.al., JHEP1011(2010) 101

$$d\sigma = H \times J_q \otimes J_{\bar{q}} \otimes J_g \otimes S$$

$$\longrightarrow \quad d\sigma = H \times J_q \otimes J_{\bar{q}} \otimes \mathcal{G}_g^{J/\psi} \otimes S$$

unmeasured jets:

E, R

measured jets:

angularity: $\tau_a = \frac{1}{\omega} \sum_i (p_i^+)^{1-a/2} (p_i^-)^{a/2}$

$$\omega = \sum_i p_i^- \qquad s = \omega^2 \tau_0$$

$e^+e^- \longrightarrow$ Jets Formula (NLL')

$$
\begin{aligned}
\frac{1}{\sigma^{(0)}}\frac{d\sigma^{(i)}}{dz\,d\tau_a} &= \sum_j \int_z^1 \frac{dx}{x} D_j(x;\mu_J) H_2(\mu_H) \left(\frac{\mu_H}{\omega}\right)^{\omega_H(\mu,\mu_H)} S^{\mathrm{unmeas}}(\mu_\Lambda) J_\omega(\mu_R) \left(\frac{\mu_R}{\omega \tan\frac{R}{2}}\right)^{\omega_R(\mu,\mu_R)} \\
&\times \left\{ \left[\delta_{ij}\delta(1-z/x)(1+f_S(\tau_a,\mu_S)) + f_{\mathcal{J}}^{ij}(\tau_z, z/x;\mu_J)\right] \left(\frac{\mu_S \tan^{1-a}\frac{R}{2}}{\omega_1}\right)^{\omega_S(\mu,\mu_S)} \right. \\
&\times \left. \left(\frac{\mu_J}{\omega}\right)^{(2-a)\omega_J(\mu,\mu_J)} \frac{1}{\Gamma[-\omega_J(\mu,\mu_J)-\omega_S(\mu,\mu_S)]} \frac{1}{\tau_a^{1+\omega_J(\mu,\mu_J)+\omega_S(\mu,\mu_S)}} \right\}_+ \\
&\times \exp\left[\mathcal{K}(\mu;\mu_H,\mu_R,\mu_J,\mu_S,\mu_\Lambda) + \gamma_E \Omega(\mu;\mu_J,\mu_S)\right].
\end{aligned}
$$

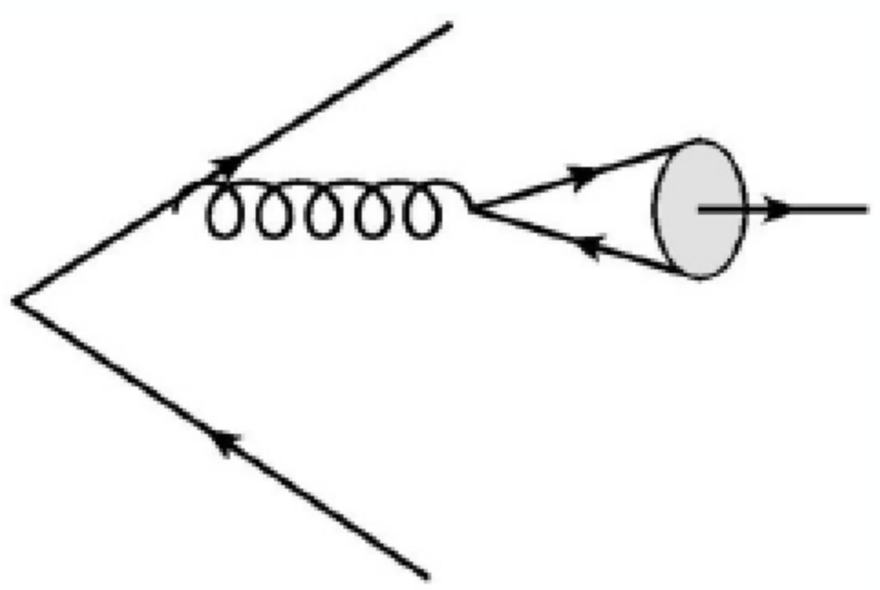
$e^+e^- \longrightarrow$ Jets Formula (NLL')

$$
\begin{aligned}
\frac{1}{\sigma^{(0)}}\frac{d\sigma^{(i)}}{dz\,d\tau_a} = {} & \sum_j \int_z^1 \frac{dx}{x} D_j(x;\mu_J) H_2(\mu_H) \left(\frac{\mu_H}{\omega}\right)^{\omega_H(\mu,\mu_H)} S^{\mathrm{unmeas}}(\mu_\Lambda) J_\omega(\mu_R) \left(\frac{\mu_R}{\omega \tan\frac{R}{2}}\right)^{\omega_R(\mu,\mu_R)} \\
& \times \Bigg\{ \Big[\delta_{ij}\delta(1-z/x)(1+f_S(\tau_a,\mu_S)) + f_J^{ij}(\tau_z, z/x;\mu_J)\Big] \left(\frac{\mu_S \tan^{1-a}\frac{R}{2}}{\omega_1}\right)^{\omega_S(\mu,\mu_S)} \\
& \times \left(\frac{\mu_J}{\omega}\right)^{(2-a)\omega_J(\mu,\mu_J)} \frac{1}{\Gamma[-\omega_J(\mu,\mu_J)-\omega_S(\mu,\mu_S)]} \frac{1}{\tau_a^{1+\omega_J(\mu,\mu_J)+\omega_S(\mu,\mu_S)}} \Bigg\}_+ \\
& \times \exp\left[\mathcal{K}(\mu;\mu_H,\mu_R,\mu_J,\mu_S,\mu_\Lambda) + \gamma_E \Omega(\mu;\mu_J,\mu_S)\right].
\end{aligned}
$$

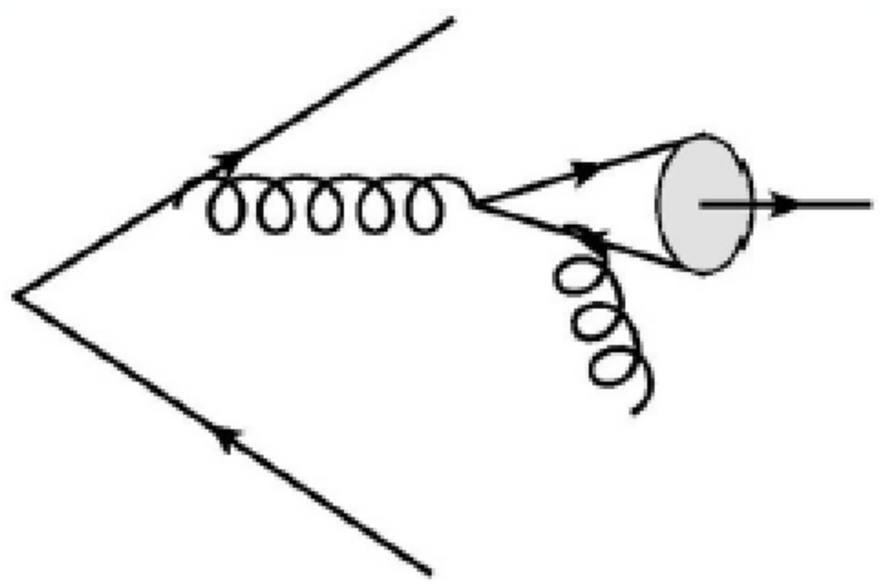
RGE evolution

Madgraph + PYTHIA

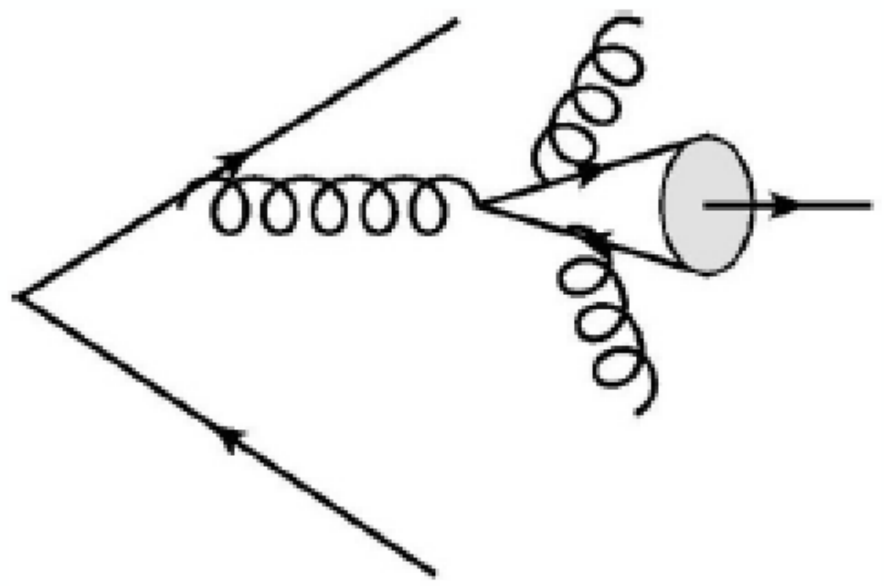
$e^+e^- \to b\,\bar{b}\,c\,\bar{c}\left[{}^3S_1^{(8)}\right]$



$e^+e^- \to b\,\bar{b}\,g\,c\,\bar{c}\left[{}^1S_0^{(8)}\right]$



$e^+e^- \to b\,\bar{b}\,g\,g\,c\,\bar{c}\left[{}^3S_1^{(1)}\right]$

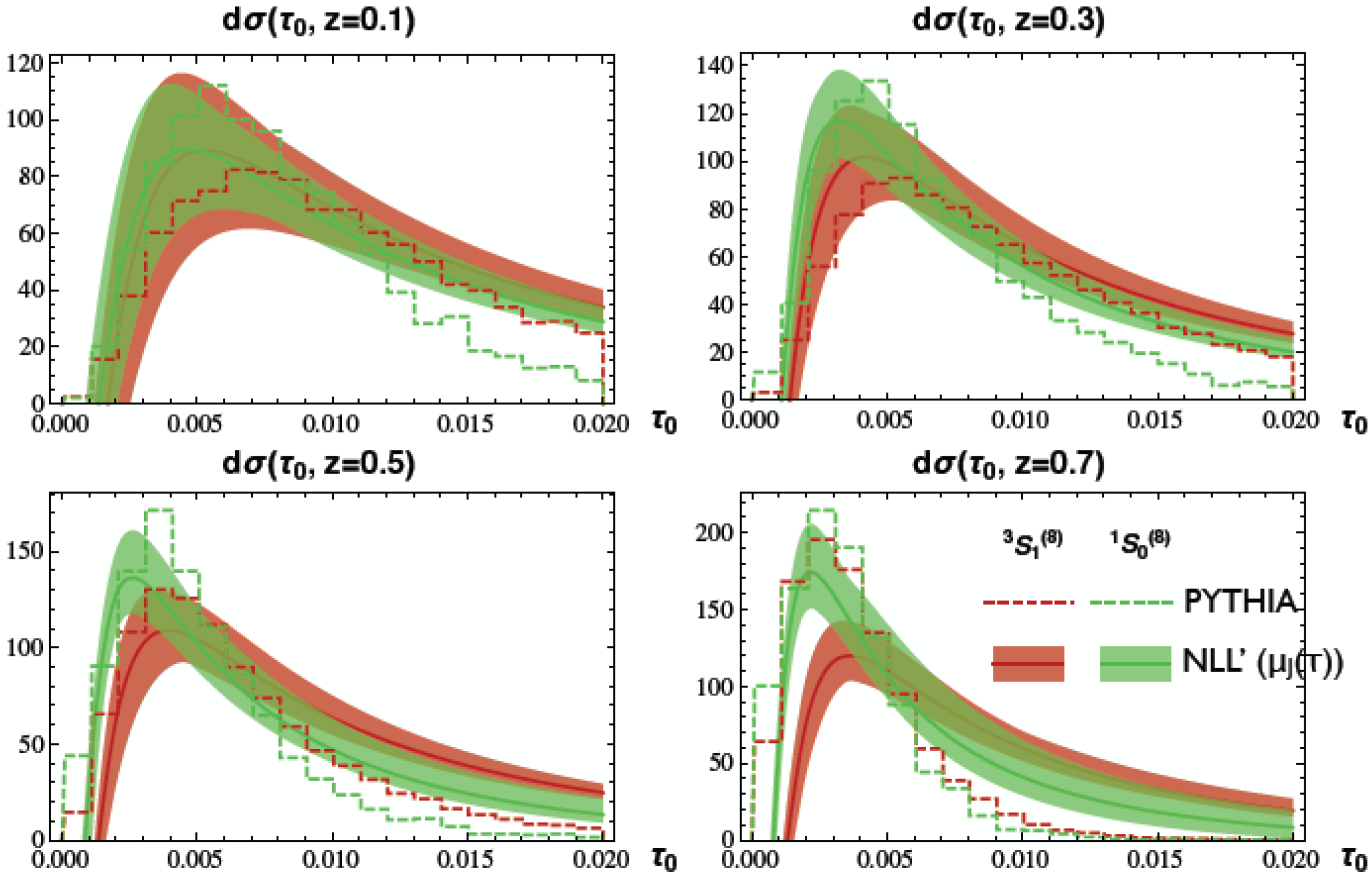


Force Madgraph to create J/ψ from gluon initiated jet

PYTHIA: parton shower, hadronization

NLL' vs. Monte Carlo

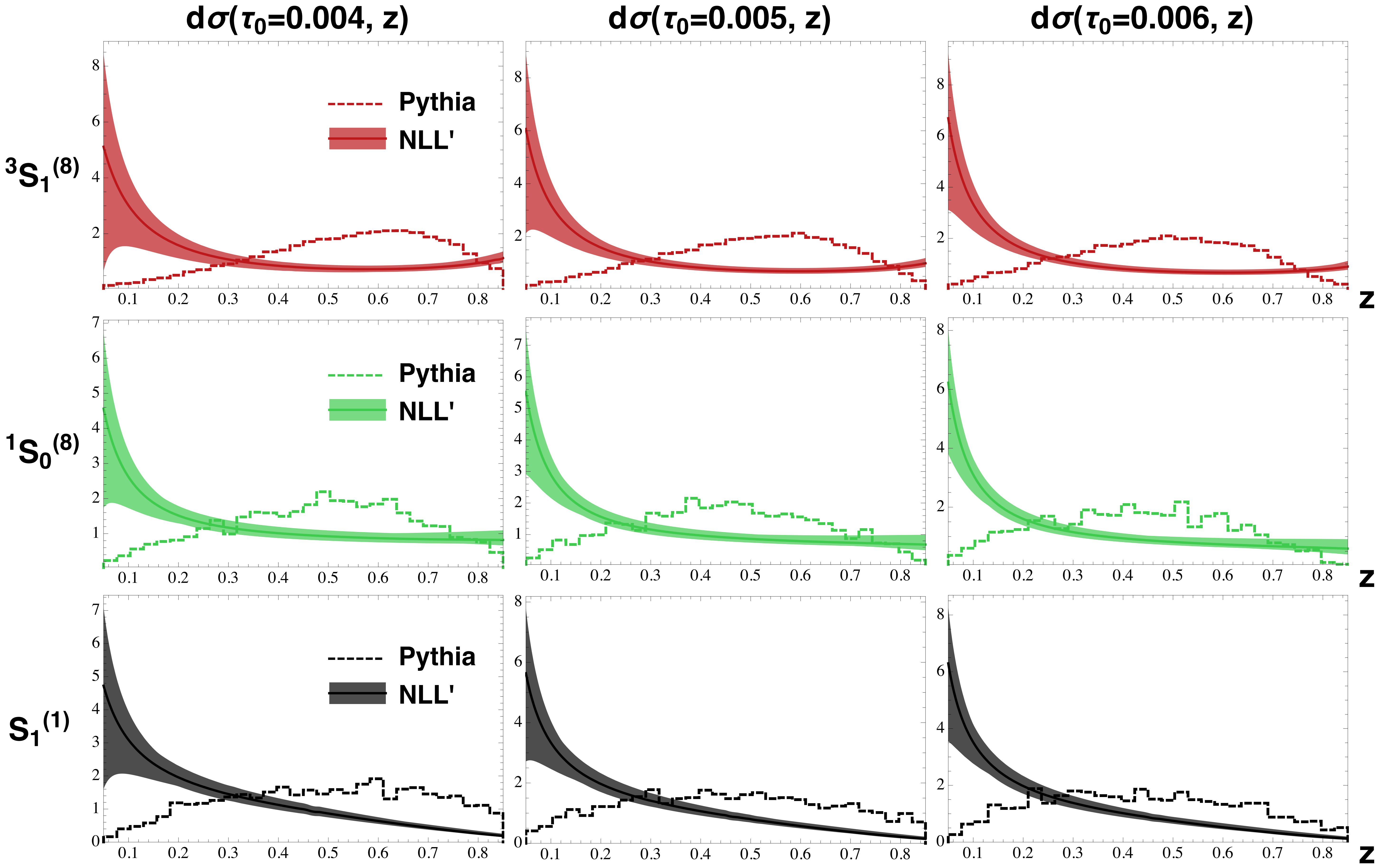
fixed z, variable τ_0



good agreement, some discrimination for large z

NLL' FJF vs. Pythia

R. Bain, L. Dai, A. Hornig, A. K. Leibovich, Y. Makris, T. Mehen JHEP 1606 (2016) 121

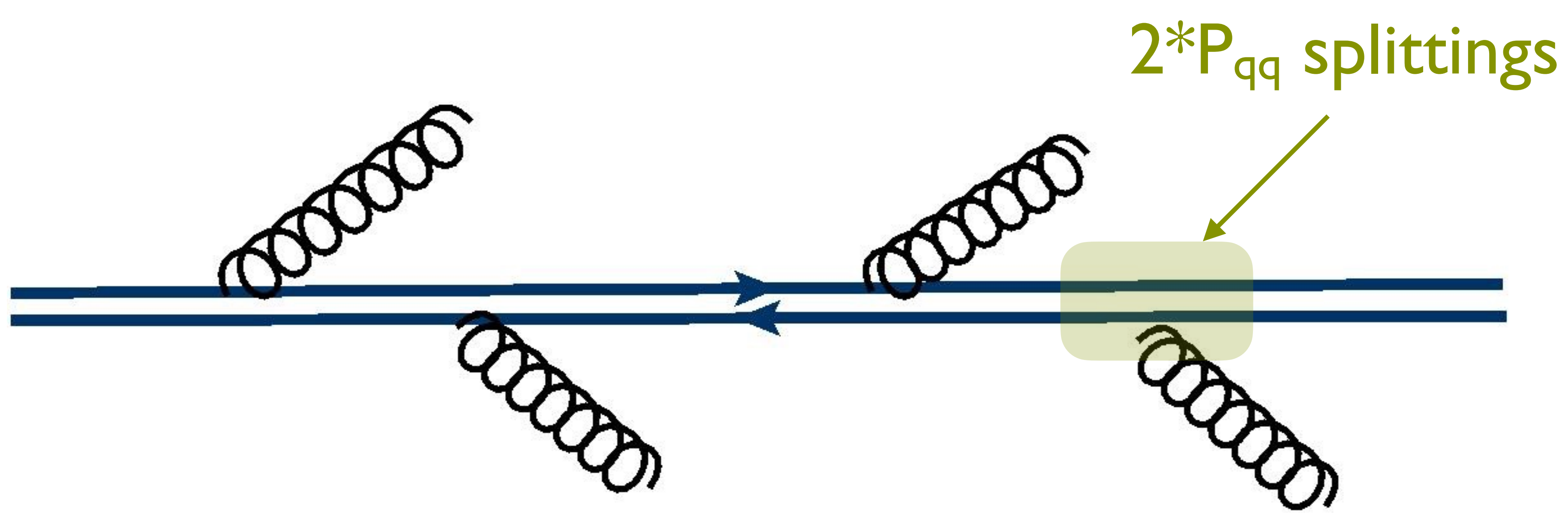


$e^+e^- \to \bar{q}qg$ $\quad E_{CM} = 250\,\text{GeV} \quad \tau_0 = s/\omega^2$

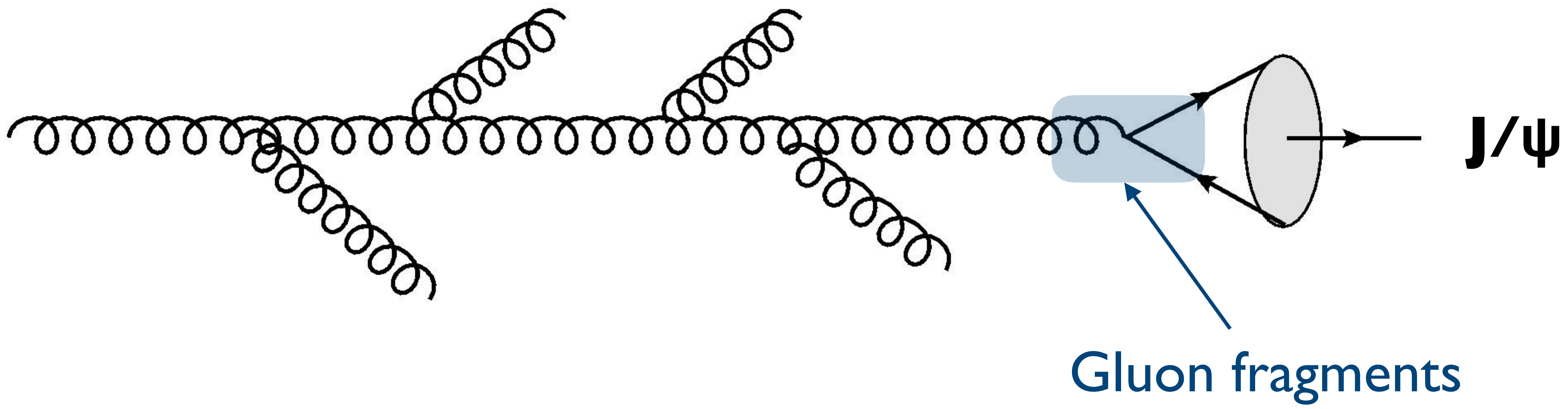
└→ jet w/ J/ψ

Explaining difference between NLL' vs Pythia

PYTHIA's model for showering color-octet $c\bar{c}$ pairs:



Physical picture of analytical calculation

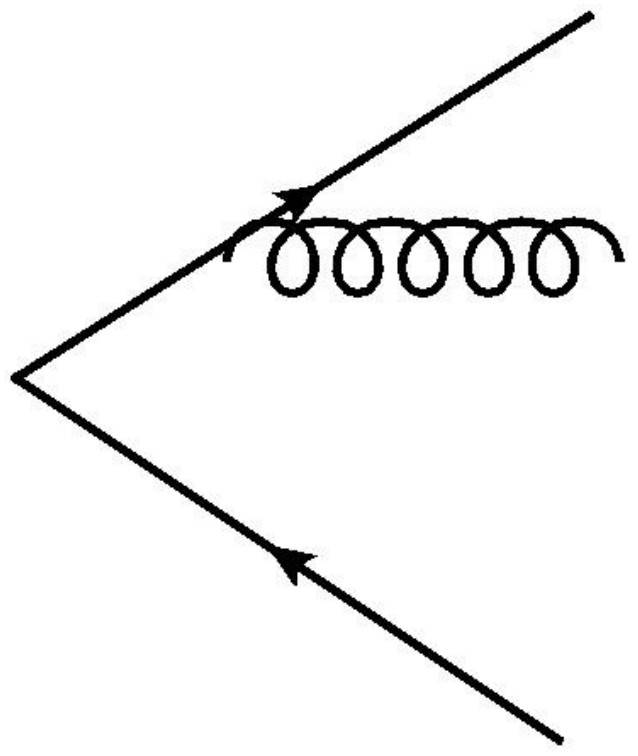


Pythia z distributions much harder than NLL' calculations

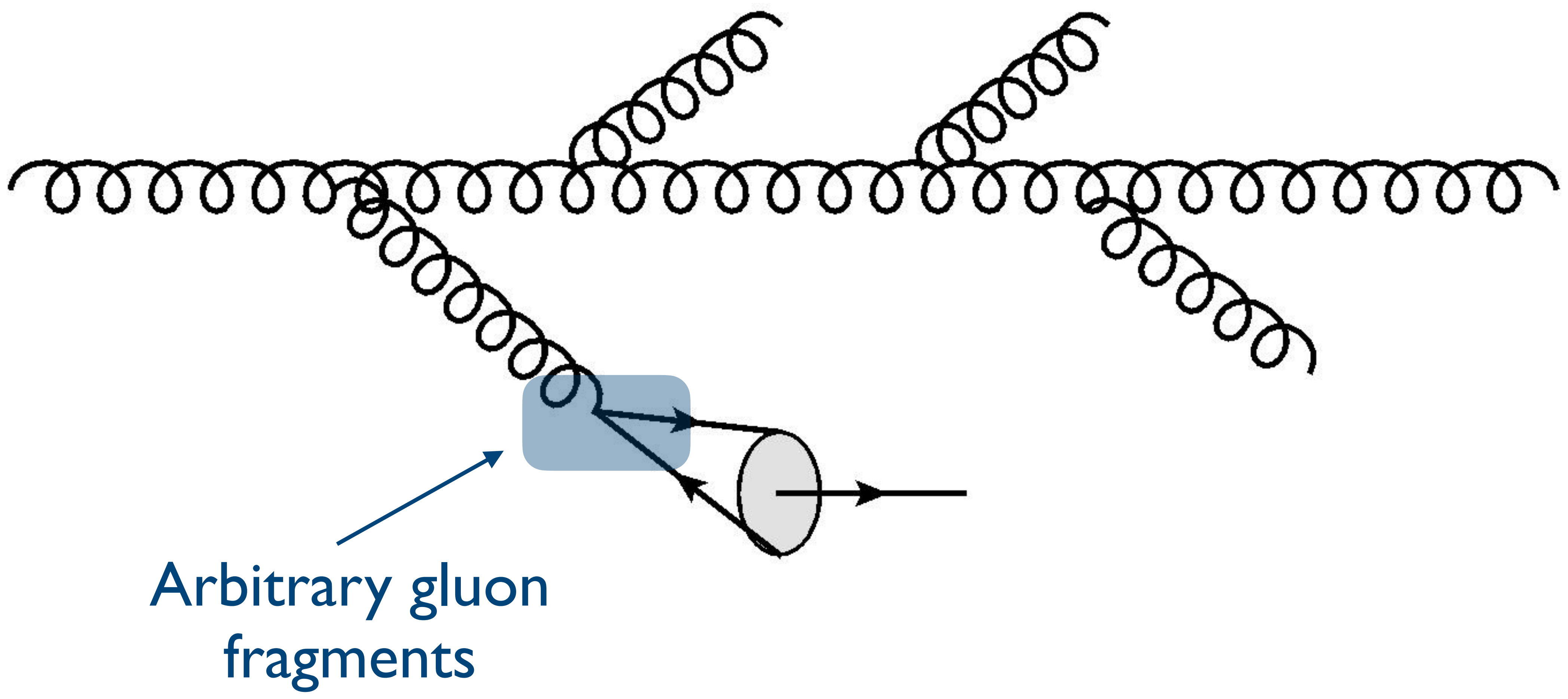
Gluon Fragmentation Improved PYTHIA (GFIP)

Madgraph 5

$$e^+e^- \to b\,\bar{b}\,g$$



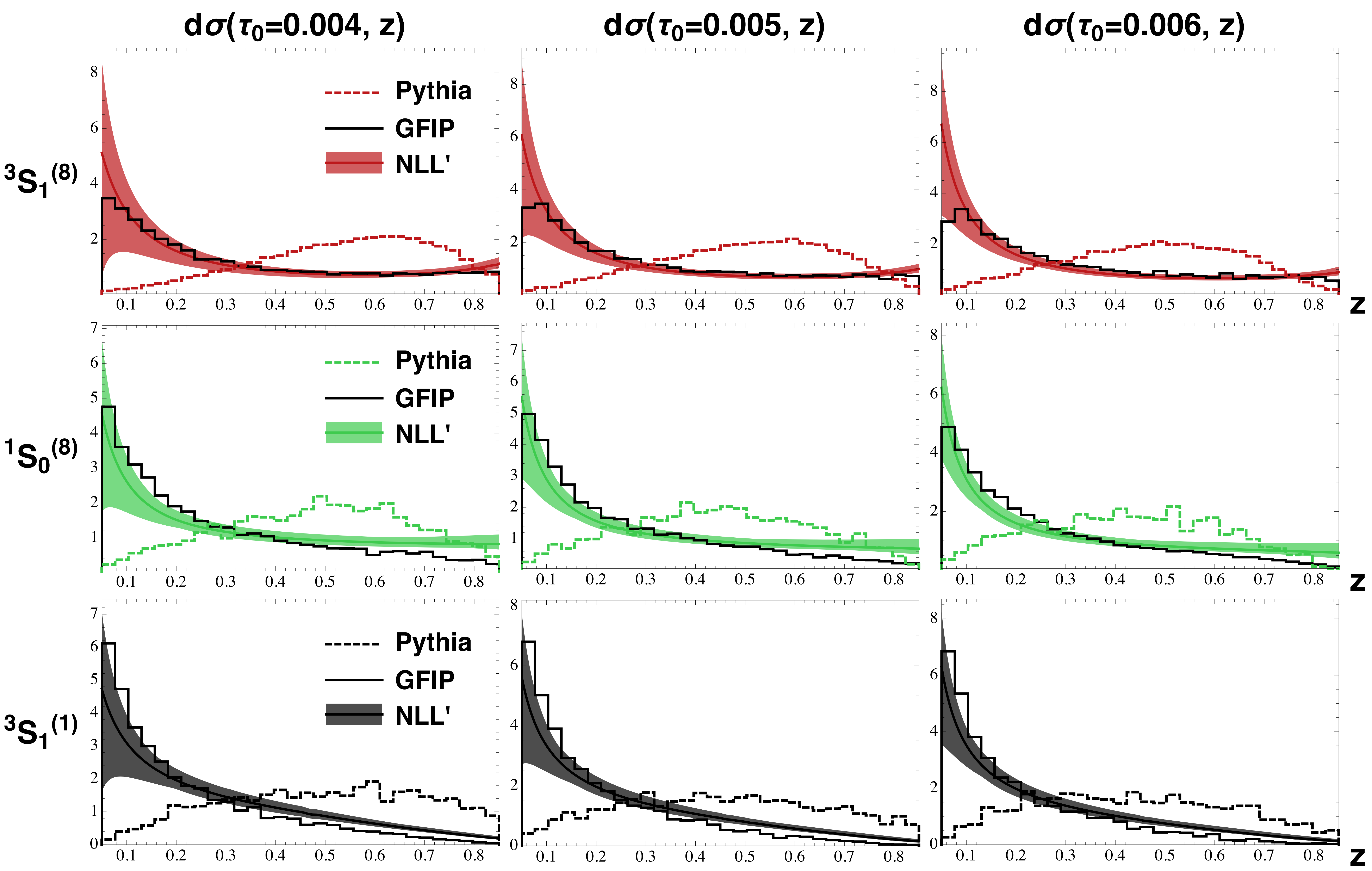
PYTHIA + Convolution



shower gluon with PYTHIA down to scale ~$2m_c$, no hadronization

convolve final state gluon distribution w/ NRQCD FFs

NLL', PYTHIA, and GFIP



GFIP and Recent LHCb Observations

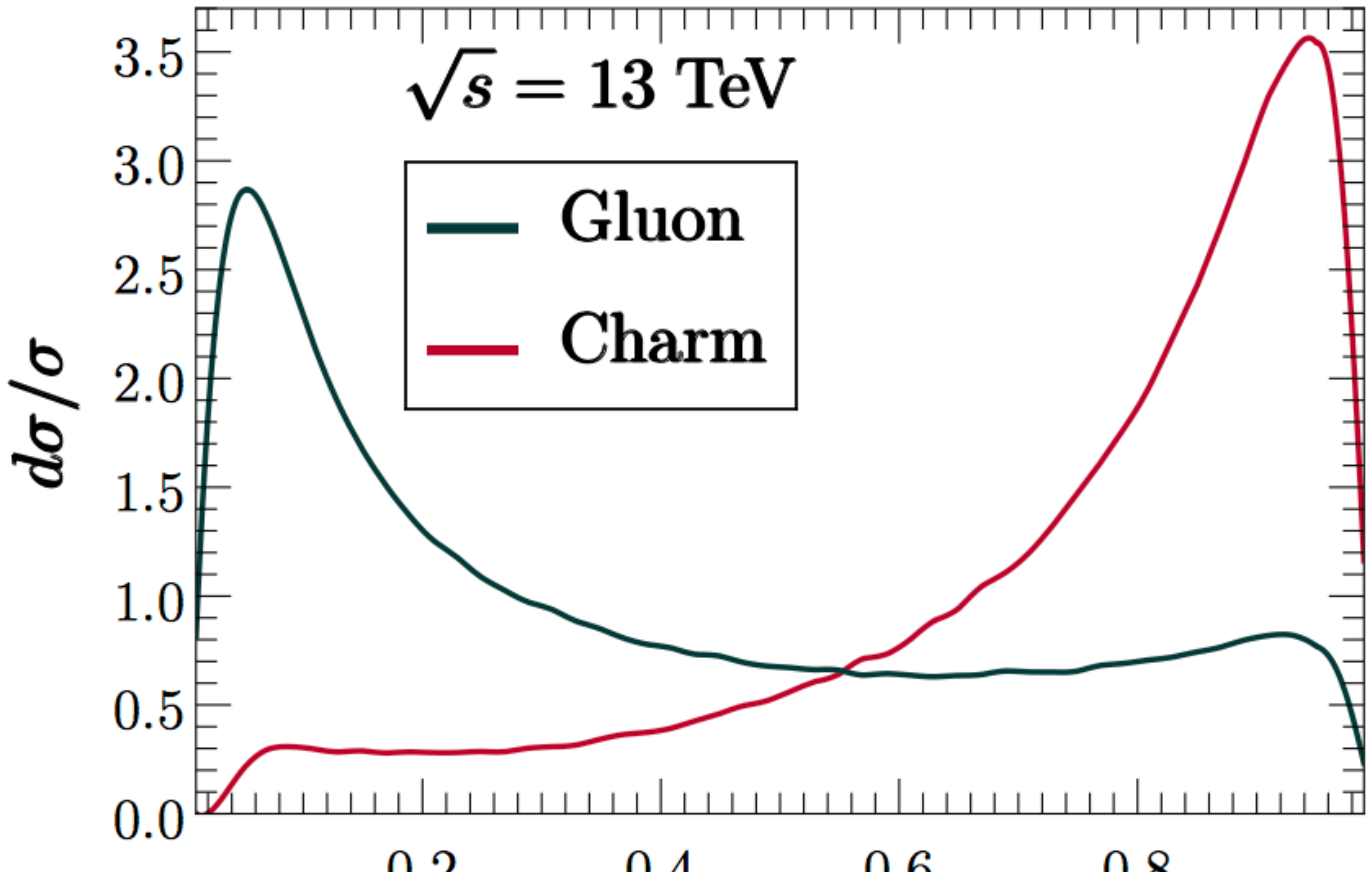
R. Bain, L. Dai, A. K. Leibovich, Y. Makris, T. Mehen, PRL 119 (2017) 3, 032002, arXiv:1702.5525

generate events with hard c-quark , gluons

LHCb: pp collisions $\sqrt{s} = 13\,\mathrm{TeV}$ cuts:

2<η<4.5
R=0.5
$p_{T,JET}$ < 20 GeV
p_{μ} < 5 GeV

evolve shower to scale ~2m_c



convolve w/ NRQCD FF for c quarks, gluons ~2m_c

LHCb data is normalized so $\sum_i \Delta z \left(\frac{d\sigma}{\sigma}\right)_i = \Delta z$

compare 0.1 < z < 0.9

Use following three sets of LDMEs

	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle$ $\times$ GeV^3	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ $\times 10^{-2}$ GeV^3	$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ $\times 10^{-2}$GeV^3	$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle/m_c^2$ $\times 10^{-2}$GeV^3
B & K [5, 6]	1.32 ± 0.20	0.224 ± 0.59	4.97 ± 0.44	-0.72 ± 0.88
Chao, et al. [12]	1.16 ± 0.20	0.30 ± 0.12	8.9 ± 0.98	0.56 ± 0.21
Bodwin et al. [13]	1.32 ± 0.20	1.1 ± 1.0	9.9 ± 2.2	0.49 ± 0.44

Butenschoen and Kniehl, PRD 84 (2011) 051501 — **global fits to world's data**

Chao, et. al. PRL 108, 242004 (2012)
Bodwin, et. al., PRL 113, 022001(2014) — **fits to high p_T hadron collider data**

FJF and Recent LHCb Observations

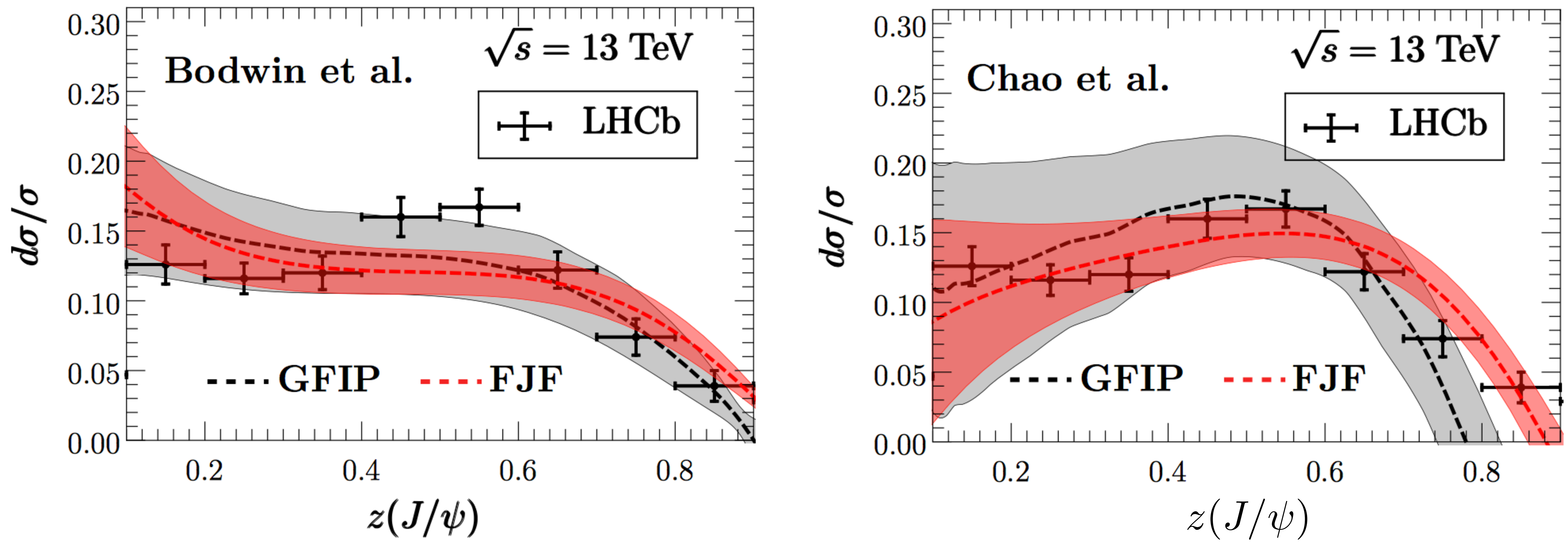
combine FJFs with hard events generated by Madgraph

NRQCD FFs evolved from $2m_c$ to jet energy scale using DGLAP

factorization theorem with tree level hard function,
trivial soft function, no NLL' resummation

FJF is only term in factorization dependent on $z(J/\psi)$

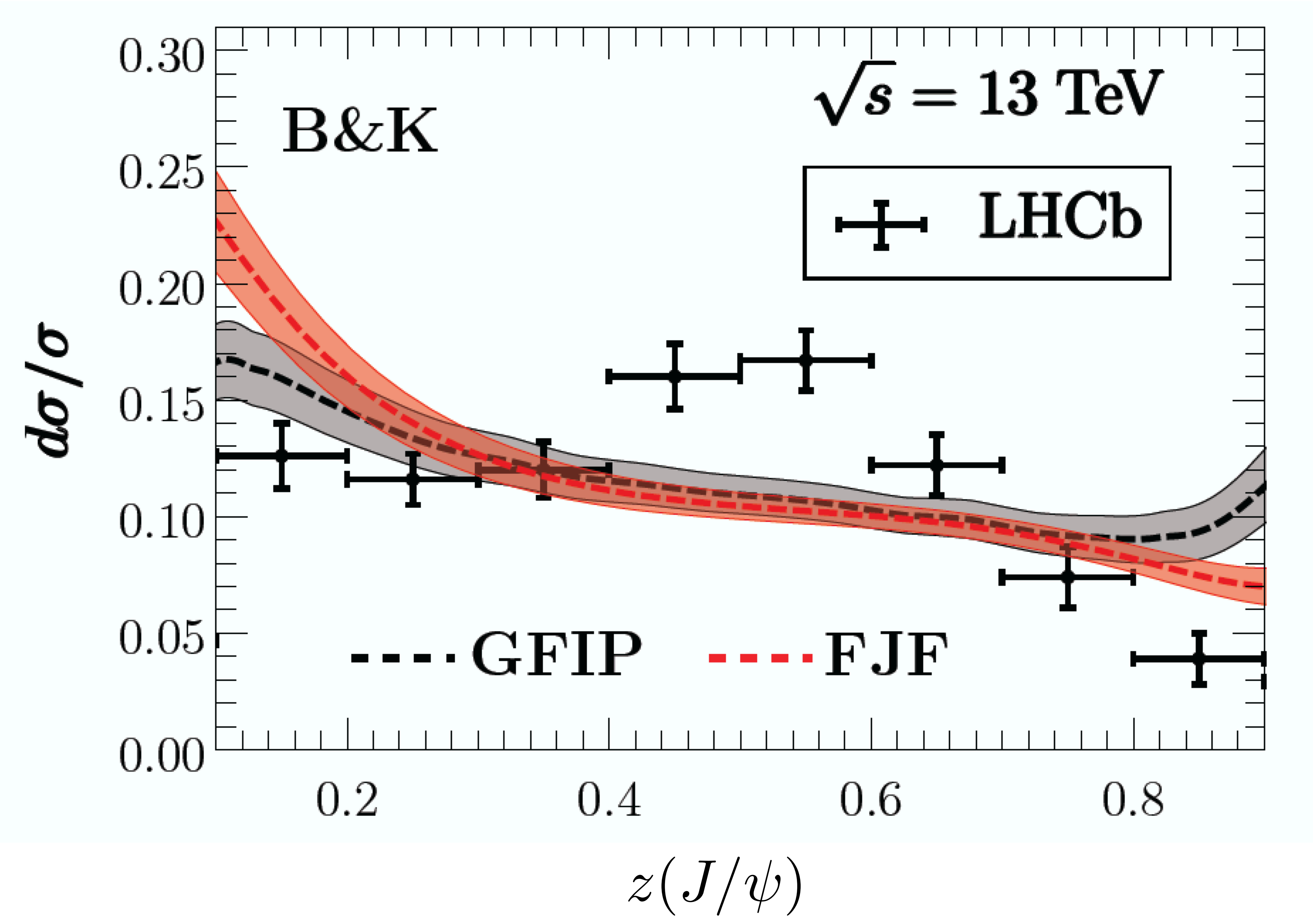
Results



FJFs, GFIP consistent

LDME from fits high p_T agree with LHCb

Results



LDME from global fits:

poorer agreement with LHCb, better than PYTHIA

Future Measurements

polarization of J/ψ in jets

Z.-B. Kang, .J.-W. Qiu, F. Ringer, H. Xing, H. Zhang, PRL 119 (2017) 032001

absolute cross sections

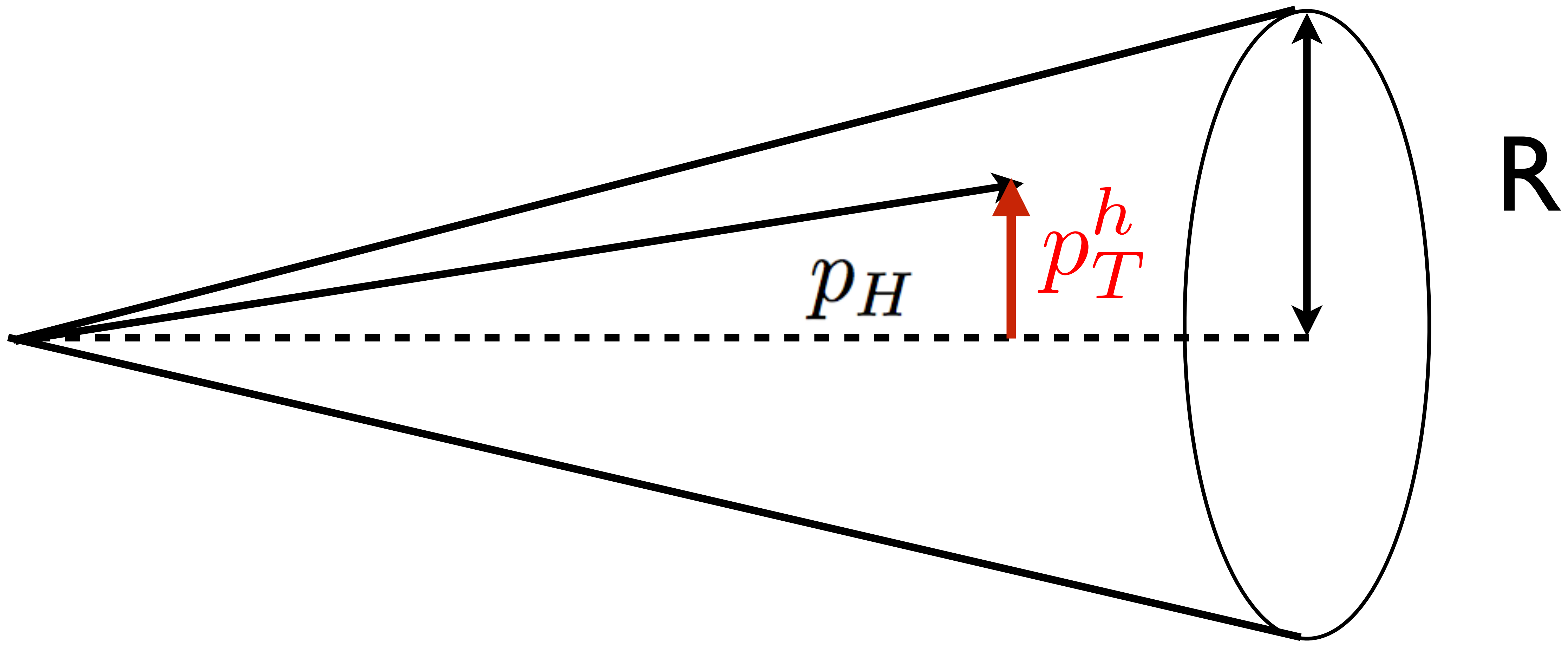
alternative jet definitions, e.g., soft drop

p_T dependent FJFs

Transverse Momentum Dependent FJFs

R. Bain, Y. Makris, TM, JHEP 1611 (2016) 144

jets with identified hadron: hadron z, p_T are both measured



Jet Energy: E

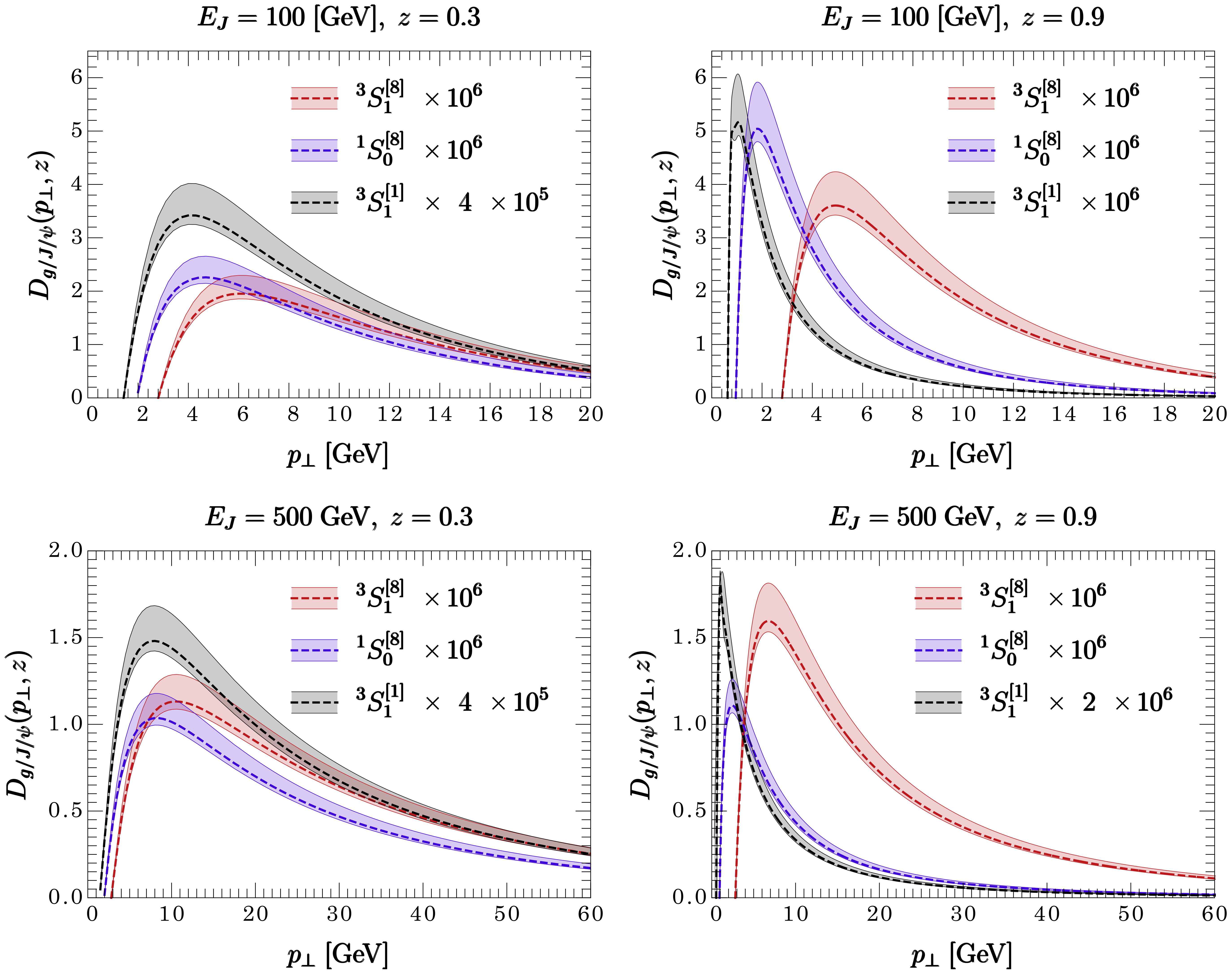
$$p_H^+ = z p_{\rm jet}^+$$

transverse momentum measured w/ rspt. to jet axis

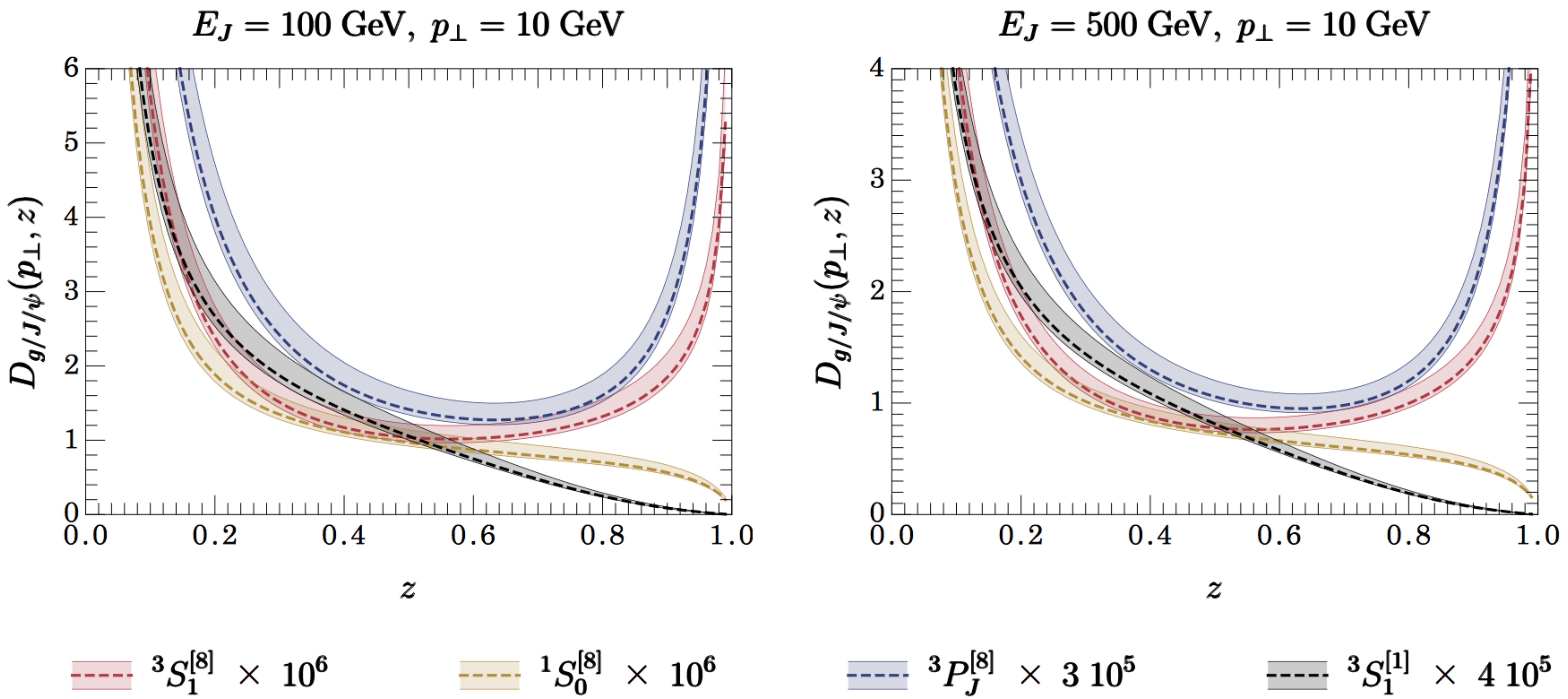
jet axis ~ parton initiating jet if out of jet radiation is ultrasoft

$$\omega \gg p_T^h \gg \Lambda \gg \Lambda_{\rm QCD}$$

Application to Quarkonium Production

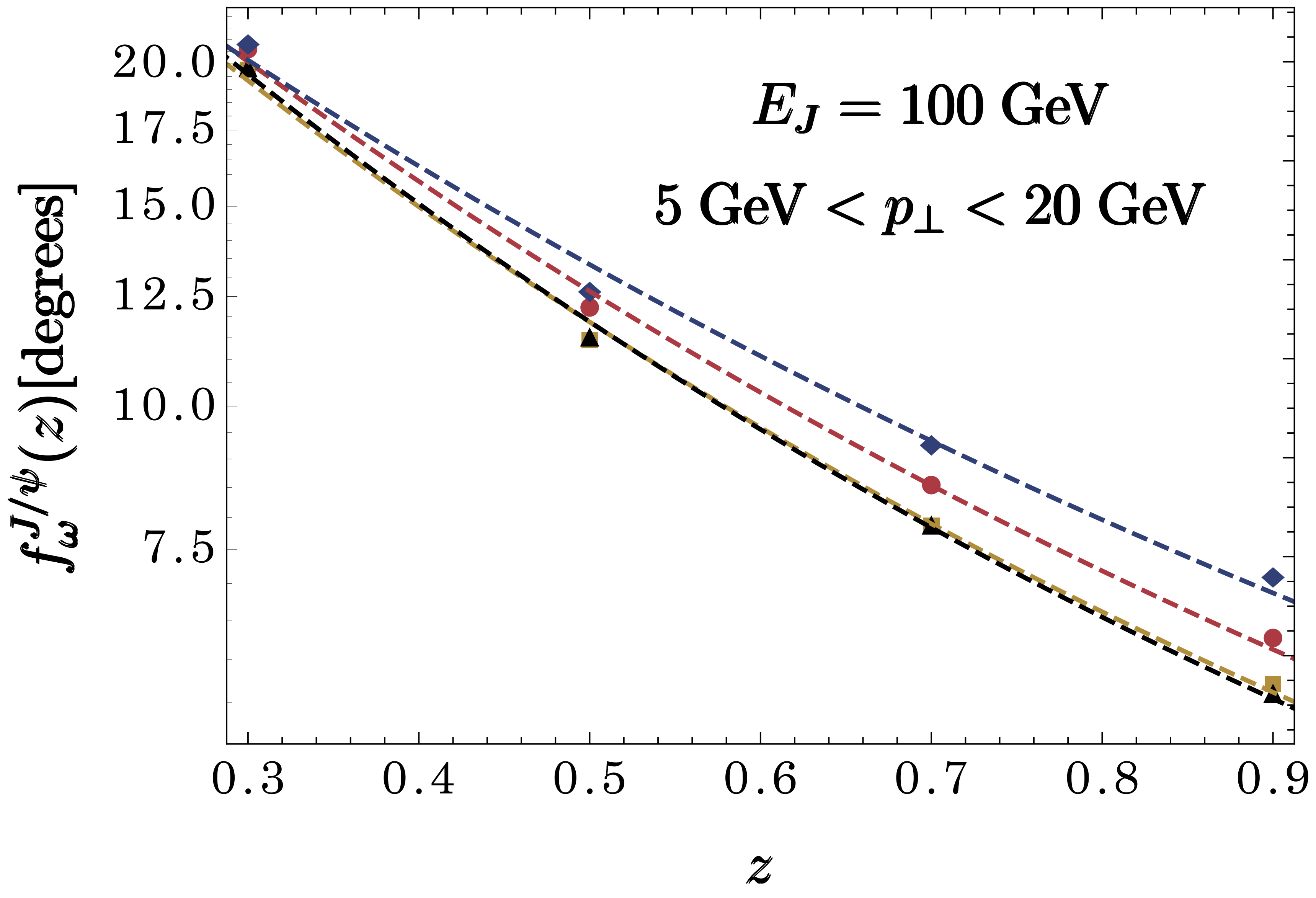


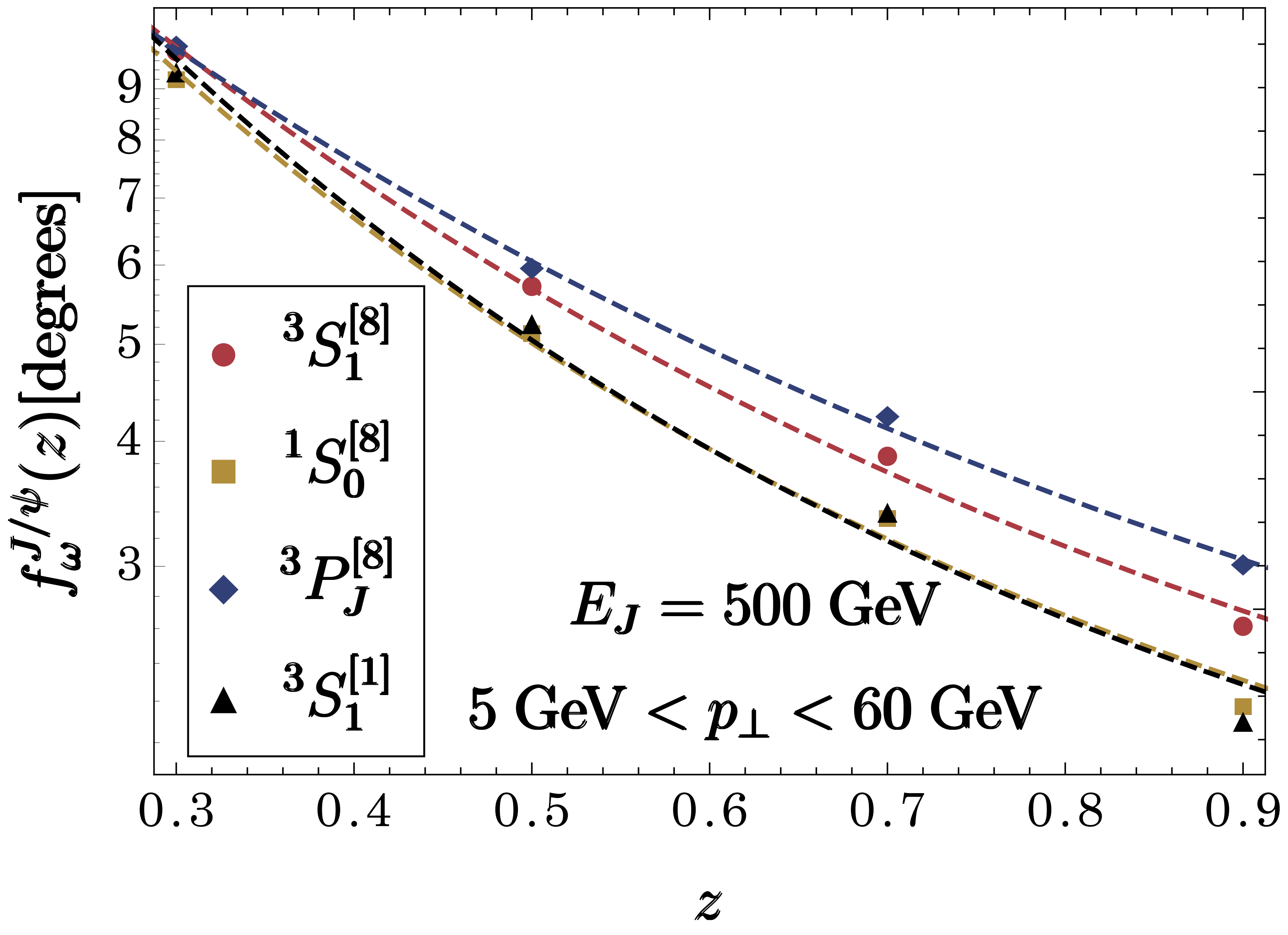
Application to Quarkonium Production



Application to Quarkonium Production

$$\langle\theta\rangle(z) \sim \frac{2\int dp_\perp\, p_\perp D_{g/h}(p_\perp, z, \mu)}{z\omega \int dp_\perp\, D_{g/h}(p_\perp, z, \mu)} \equiv f^h_\omega(z)$$





$E_J = 100$ GeV		
$^{2S+1}L_J^{[1,8]}$	C_0	C_1
$^3S_1^{[1]}$	3.92	0.92
$^3S_1^{[8]}$	3.86	0.84
$^1S_0^{[8]}$	3.88	0.90
$^3P_J^{[8]}$	3.75	0.74

$E_J = 500$ GeV		
$^{2S+1}L_J^{[1,8]}$	C_0	C_1
$^3S_1^{[1]}$	3.75	1.68
$^3S_1^{[8]}$	3.48	1.39
$^1S_0^{[8]}$	3.66	1.64
$^3P_J^{[8]}$	3.28	1.20

$$\ln(f(x)) = g(x; C_0, C_1) \text{ s.t. } g(x=0) = C_0$$

$$g_2(x) = C_0 \exp(-C_1 x)$$

Conclusions

measuring quarkonia within jets and using jet observables
should provide insights into quarkonia production

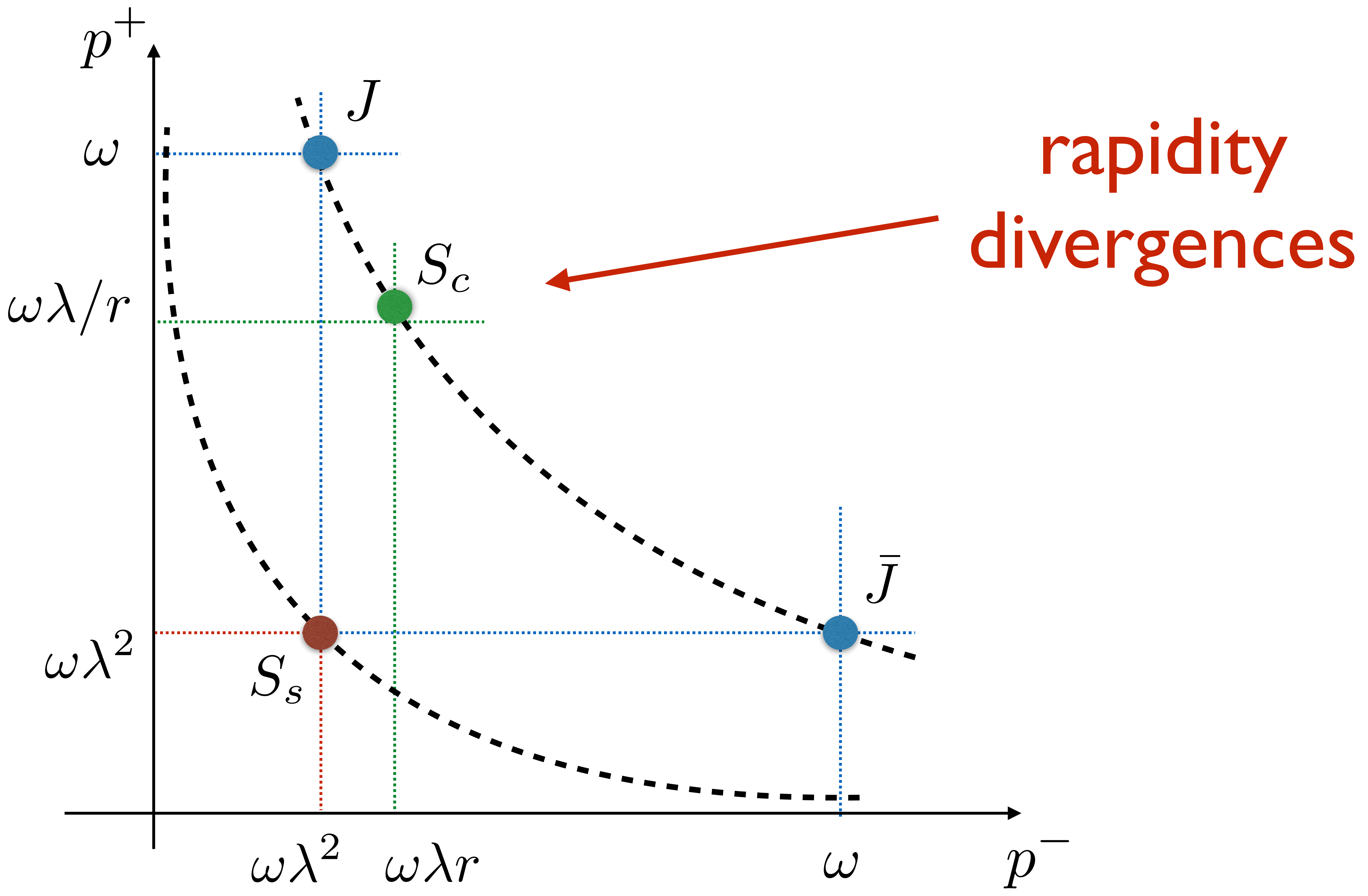
If $^1S_0^{(8)}$ mechanism dominates high p_T production
FJF should have negative slope for z(E), for z > 0.5

LHCb data on $z(J/\psi)$ well-described by FJF, GFIP
improvement over default PYTHIA, consistent w/ NLL' calculations
LDME extracted from high p_T slightly preferred

TMD FJFs: p_T^h, θ discriminate between NRQCD mechanisms

Backup

Scales in TMDFJF



$$p_c \sim \omega(\lambda^2, 1, \lambda) \qquad p_{cs} \sim p_h^\perp(r, 1/r, 1) \qquad p_{us} \sim \Lambda(1, 1, 1)$$

$$\lambda = p_h^\perp/\omega$$

Factorization Theorem

$$D_{q/h}(\mathbf{p}_\perp, z, \mu) = H_+(\mu) \times \left[\mathcal{D}_{q/h} \otimes_\perp S_C\right](\mathbf{p}_\perp, z, \mu)$$

$$H_+(\mu) = (2\pi)^2 N_c\, C_+^\dagger(\mu) C_+(\mu)$$

$$\mathcal{D}_{q/h}(\mathbf{p}_\perp^{\mathcal{D}}, z) \equiv \frac{1}{z} \sum_{X_n} \frac{1}{2N_c} \delta(p^-_{Xh;r}) \delta^{(2)}(p^\perp_{Xh;r})\, \mathrm{Tr}\left[\frac{\bar{\not{n}}}{2} \langle 0|\delta_{\omega,\overline{\mathcal{P}}} \chi_n(0) \delta^{(2)}(\mathcal{P}_\perp^{X_n} + \mathbf{p}_\perp^{\mathcal{D}})|X_n h\rangle \right.$$
$$\left. \times \langle X_n h|\bar{\chi}_n(0)|0\rangle \right]$$

$$\mathcal{D}_{i/h}(\mathbf{p}_\perp, z, \mu, \nu) = \int_z^1 \frac{dx}{x}\, \mathcal{J}_{i/j}(\mathbf{p}_\perp, x, \mu, \nu) D_{j/h}\left(\frac{z}{x}, \mu\right) \ + \ \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{|\mathbf{p}_\perp|^2}\right)$$

$$S_C(\mathbf{p}_\perp^S) \equiv \frac{1}{N_c} \sum_{X_{cs}} \mathrm{Tr}\left[\langle 0|V_n^\dagger(0) U_n(0) \delta^{(2)}(\mathcal{P}_\perp + \mathbf{p}_\perp^S)|X_{cs}\rangle\langle X_{cs}|U_n^\dagger(0) V_n(0)|0\rangle\right]$$

Anomalous Dimensions for RGE, RRGE

RGE

$$\gamma_\mu^{S_C}(\nu) = \frac{\alpha_s C_i}{\pi} \ln\left(\frac{\mu^2}{r^2\nu^2}\right)$$

$$\gamma_\mu^{\mathcal{D}}(\nu) = \frac{\alpha_s C_i}{\pi} \left(\ln\left(\frac{\nu^2}{\omega^2}\right) + \bar{\gamma}_i\right)$$

$$\gamma_\mu^{\mathcal{D}}(\nu) + \gamma_\mu^{S_C}(\nu) = \gamma_\mu^J = \frac{\alpha_s C_i}{\pi} \left(\ln\left(\frac{\mu^2}{r^2\omega^2}\right) + \bar{\gamma}_i\right)$$

Rapidity Renormalization Group

$$\gamma_\nu^{S_C}(p_\perp, \mu) = +(8\pi)\alpha_s C_i \, \mathcal{L}_0(\mathbf{p}_\perp, \mu^2)$$

$$\gamma_\nu^{\mathcal{D}}(p_\perp, \mu) = -(8\pi)\alpha_s C_i \, \mathcal{L}_0(\mathbf{p}_\perp, \mu^2)$$

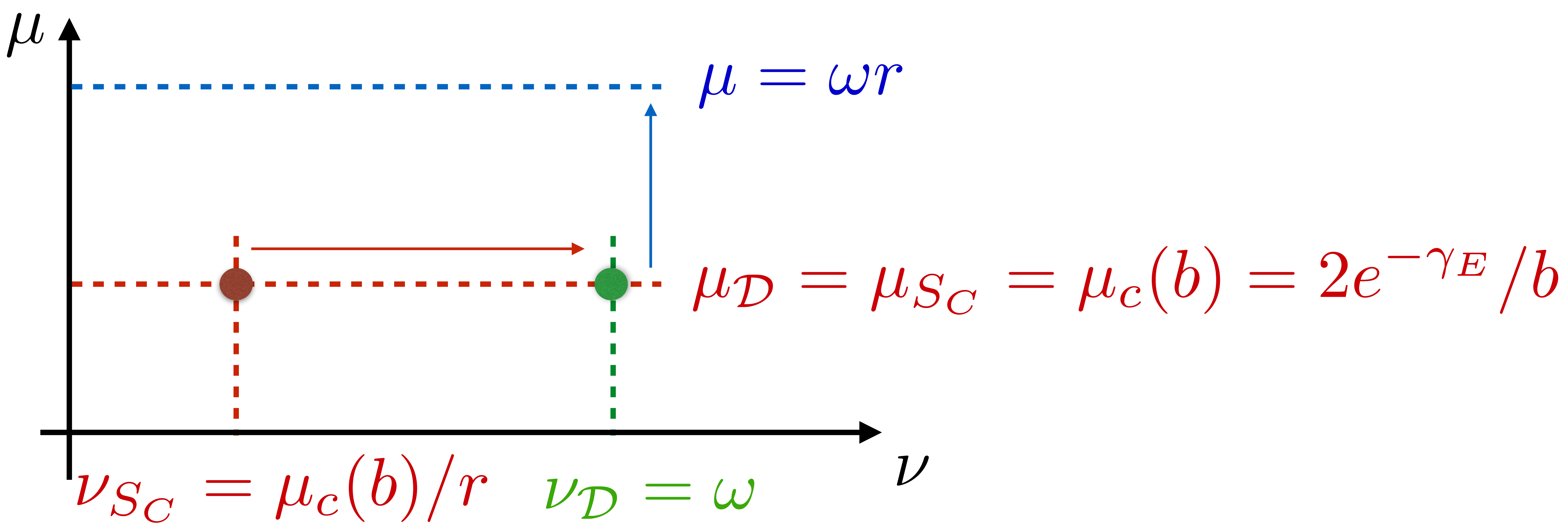
$$\gamma_\nu^{\mathcal{D}}(\mathbf{p}_\perp, \mu) + \gamma_\nu^{S}(\mathbf{p}_\perp, \mu) = 0$$

J-y. Chiu, A. Jain, D. Neill, I.Z. Rothstein, PRL108 (2012) 151601

J-y. Chiu, A. Jain, D. Neill, I.Z. Rothstein, JHEP1205 (2012) 084

Solution to Evolution Eqs. in Fourier Space

$$D_{i/h}(p_\perp, z, \mu) = (2\pi)^2\, p_\perp \int_0^\infty db\, bJ_0(bp_\perp)\mathcal{U}_{S_C}(\mu, \mu_{S_C}, m_{S_C})\mathcal{U}_{\mathcal{D}}(\mu, \mu_{\mathcal{D}}, 1)$$
$$\times\, \mathcal{V}_{S_C}(b, \mu_{S_C}, \nu_{\mathcal{D}}, \nu_{S_C})\mathcal{FT}\left[\mathcal{D}_{i/h}(\mathbf{p}_\perp, z, \mu_{\mathcal{D}}, \nu_{\mathcal{D}}) \otimes_\perp S^i_C(\mathbf{p}_\perp, \mu_{S_C}, \nu_{S_C})\right]$$



fragmentation function (QCD)

$$D_q^h(z) = z \int \frac{\mathrm{d}x^+}{4\pi}\, e^{ik^- x^+/2}\, \frac{1}{4N_c}\, \mathrm{Tr} \sum_X \langle 0| \bar{n}\!\!\!/\, \Psi(x^+,0,0_\perp)|Xh\rangle\langle Xh|\bar{\Psi}(0)|0\rangle\Big|_{p_h^\perp=0}$$

fragmentation function (SCET)

$$D_q^h\left(\frac{p_h^-}{\omega},\mu\right) = \pi\omega \int dp_h^+\, \frac{1}{4N_c}\, \mathrm{Tr} \sum_X \bar{n}\!\!\!/\, \langle 0|[\delta_{\omega,\bar{\mathcal{P}}}\, \delta_{0,\mathcal{P}_\perp}\, \chi_n(0)]|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle$$

Jet function (SCET)

$$J_u(k^+\omega) = -\frac{1}{\pi\,\omega}\, \mathrm{Im} \int d^4x\, e^{ik\cdot x}\, i\, \langle 0|\, \mathrm{T}\, \bar{\chi}_{n,\omega,0_\perp}(0)\, \frac{\bar{n}\!\!\!/}{4N_c}\, \chi_n(x)|0\rangle$$

fragmentation jet function (SCET)

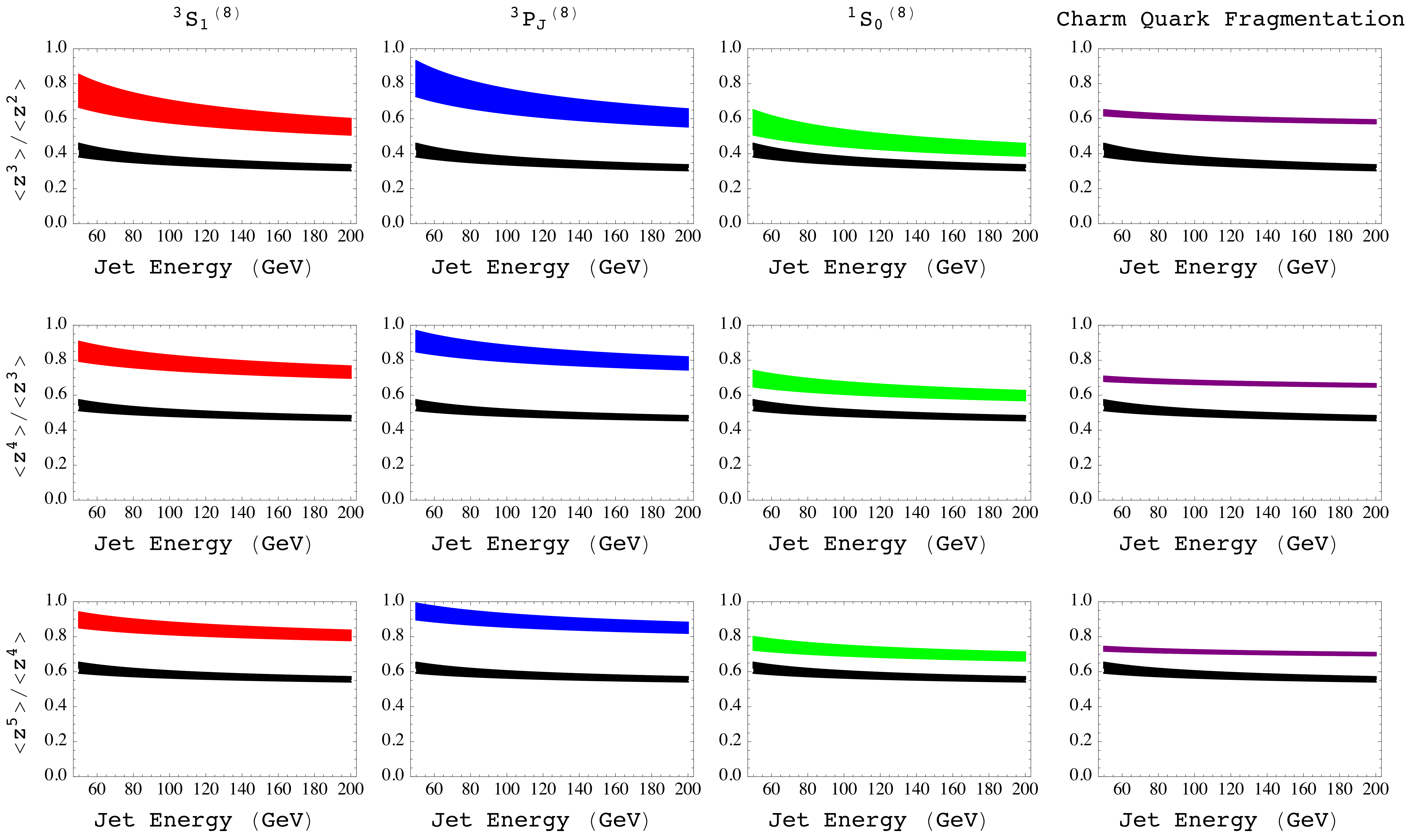
$$\mathcal{G}_{q,\mathrm{bare}}^h(s,z) = \int \mathrm{d}^4y\, e^{\mathrm{i}k^+y^-/2} \int \mathrm{d}p_h^+ \sum_X \frac{1}{4N_c}\, \mathrm{tr}\left[\frac{\bar{n}\!\!\!/}{2}\langle 0|[\delta_{\omega,\overline{\mathcal{P}}}\, \delta_{0,\mathcal{P}_\perp}\chi_n(y)]|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle\right]$$

$$\delta(p^+/z - P_H^+) \to \delta(p^+/z - P_H^+)\delta(p^- - s/p^+)$$

FF FJF

Ratios of Moments

$$E\tan(R/2) < \mu < 4E\tan(R/2)$$

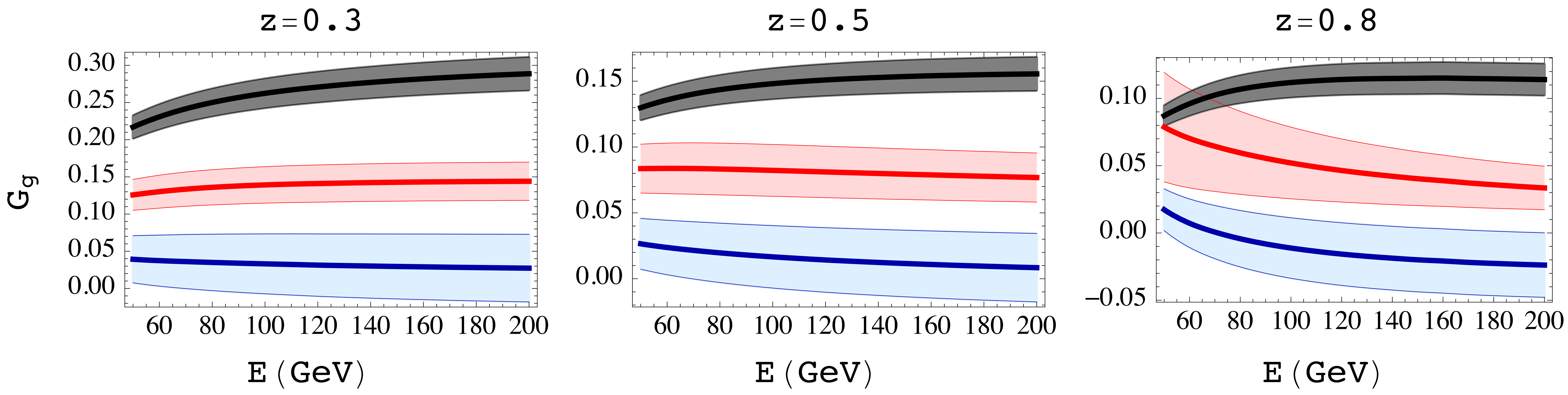


Ratios of Moments

$$\left.\frac{\langle z^{n+1}\rangle}{\langle z^n\rangle}\right|_{^3P_J^{(8)}} \approx \left.\frac{\langle z^{n+1}\rangle}{\langle z^n\rangle}\right|_{^3S_1^{(8)}} > \left.\frac{\langle z^{n+1}\rangle}{\langle z^n\rangle}\right|_{^1S_0^{(8)}} \approx \left.\frac{\langle z^{n+1}\rangle}{\langle z^n\rangle}\right|_{\mathrm{c-quark}} > \left.\frac{\langle z^{n+1}\rangle}{\langle z^n\rangle}\right|_{^3S_1^{(1)}}$$

Gluon FJF for different extractions of LDME

fix z, vary energy



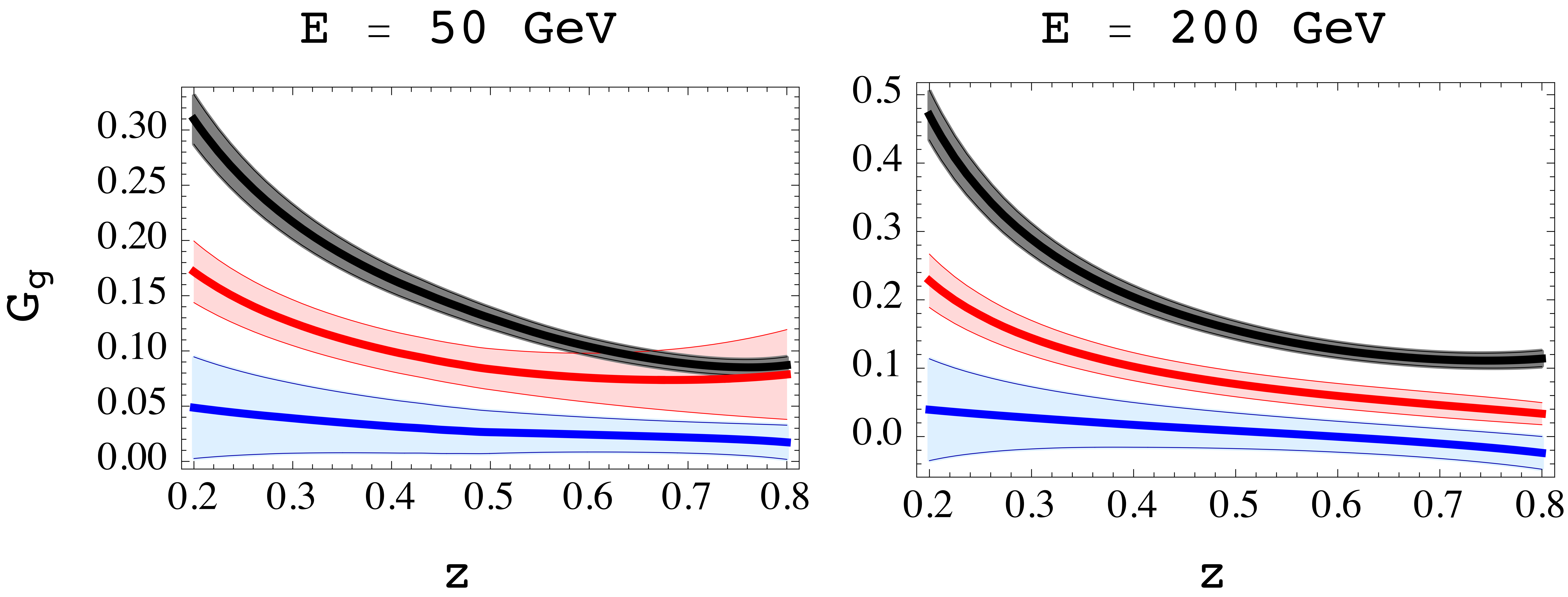
Butenschoen and Kniehl, PRD 84 (2011) 051501, arXiv:1105.0822

Bodwin, et. al. arXiv:1403.3612

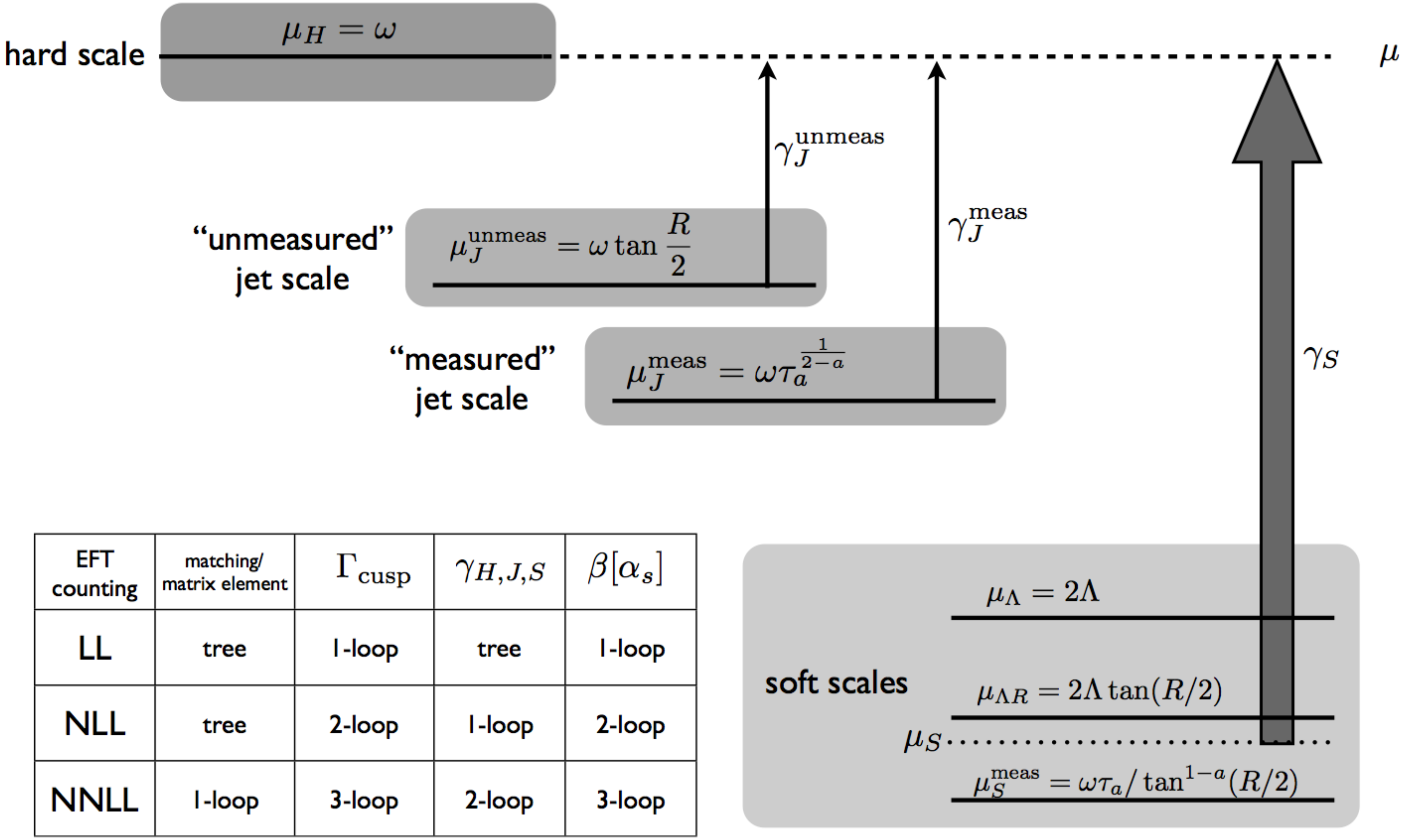
Chao, et. al. PRL 108, 242004 (2012)

Gluon FJF for different extractions of LDME

fix energy, vary z



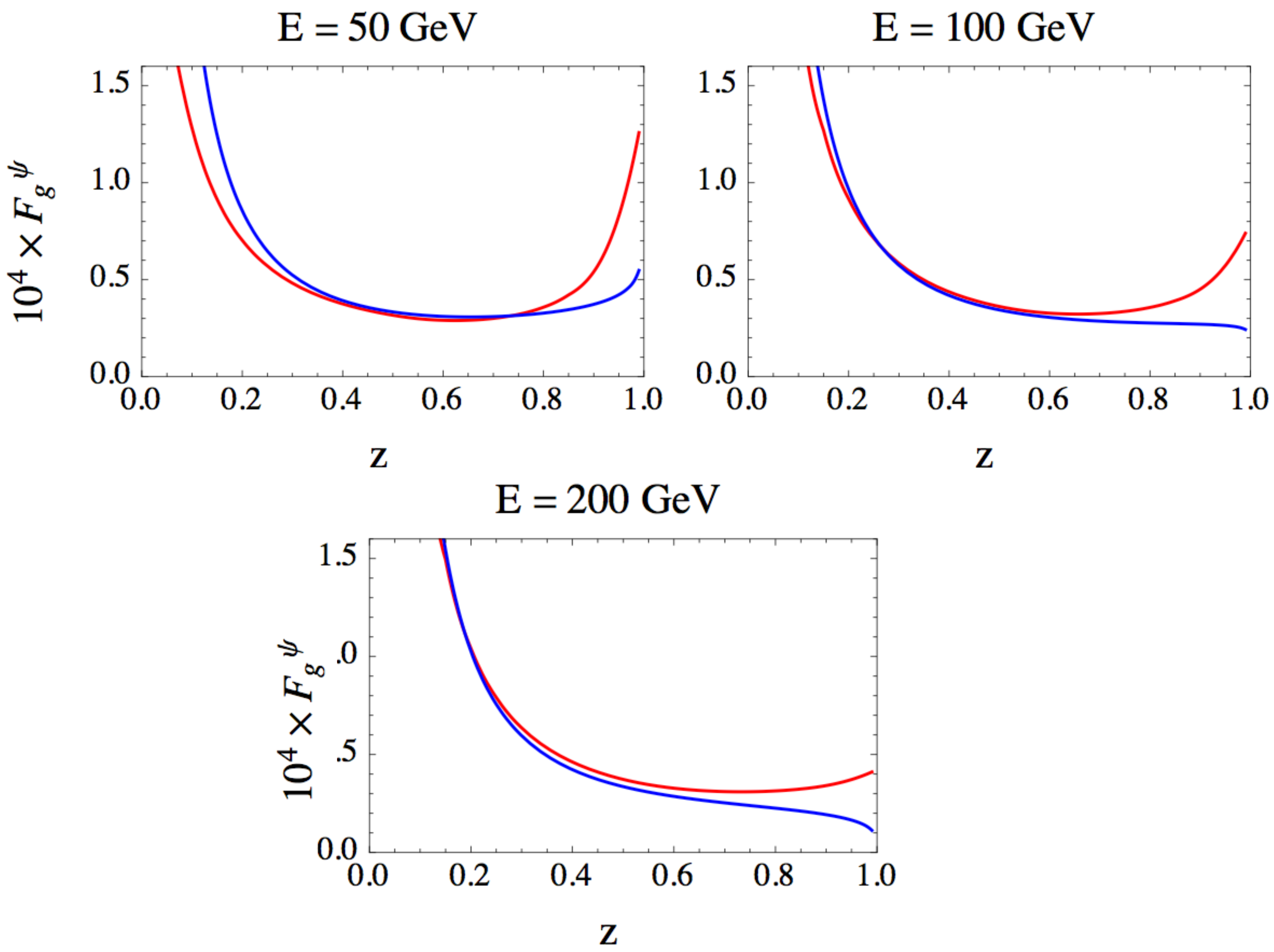
Scales in Jet Cross section



EFT counting	matching/ matrix element	Γ_{cusp}	$\gamma_{H,J,S}$	$\beta[\alpha_s]$
LL	tree	1-loop	tree	1-loop
NLL	tree	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop

Color-Octet $^3S_1$ fragmentation function, FJF

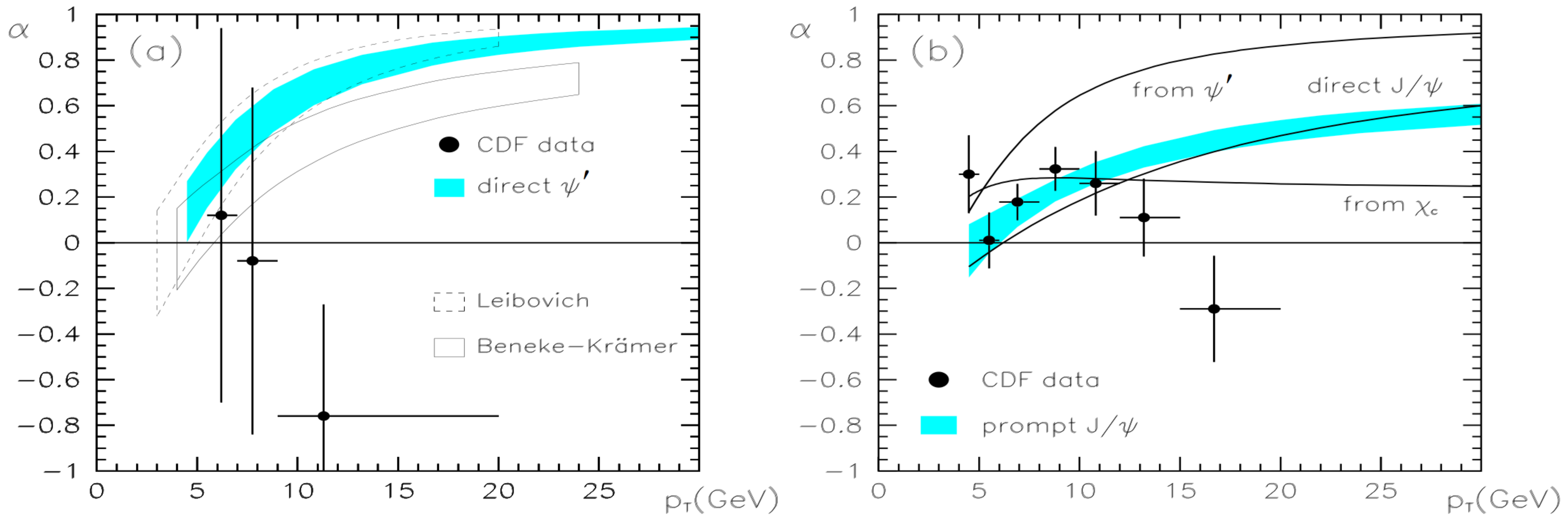
M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003



fragmentation function
fragmenting jet function

Polarization Puzzle

$^3S_1^{[8]}$ fragmentation at large p_T predicts transversely polarized J/ψ, ψ'



ψ '

J/ψ

Braaten, Kniehl, Lee, 1999

$$D_{q/h}(\mathbf{p}_\perp, z, \mu) = \frac{1}{z}\sum_X \frac{1}{2N_c}\delta(p^-_{Xh;r})\delta^{(2)}(\mathbf{p}_\perp + \mathbf{p}^X_\perp)\,\mathrm{Tr}\left[\frac{\bar{\not{n}}}{2}\langle 0|\delta_{\omega,\overline{\mathcal{P}}}\chi_n^{(0)}(0)|Xh\rangle\right.$$

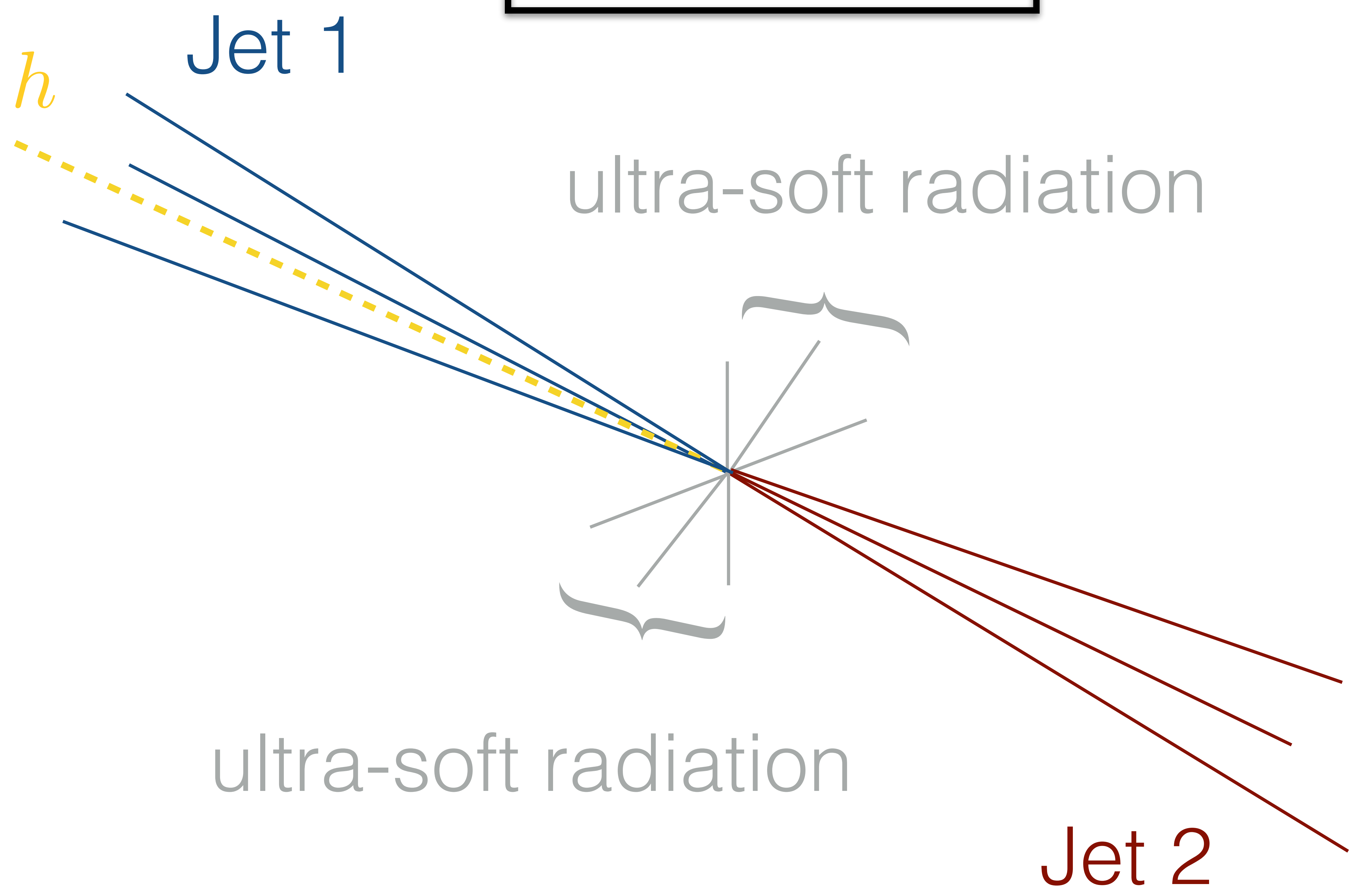
$$\left.\langle Xh|\bar{\chi}_n^{(0)}(0)|0\rangle\right]$$

$$\int d^2\mathbf{p}^h_\perp\, D_{q/h}(\mathbf{p}^h_\perp, z, \mu) = D_{q/h}(z, \mu)$$

Transverse Momentum Dependent FJFs

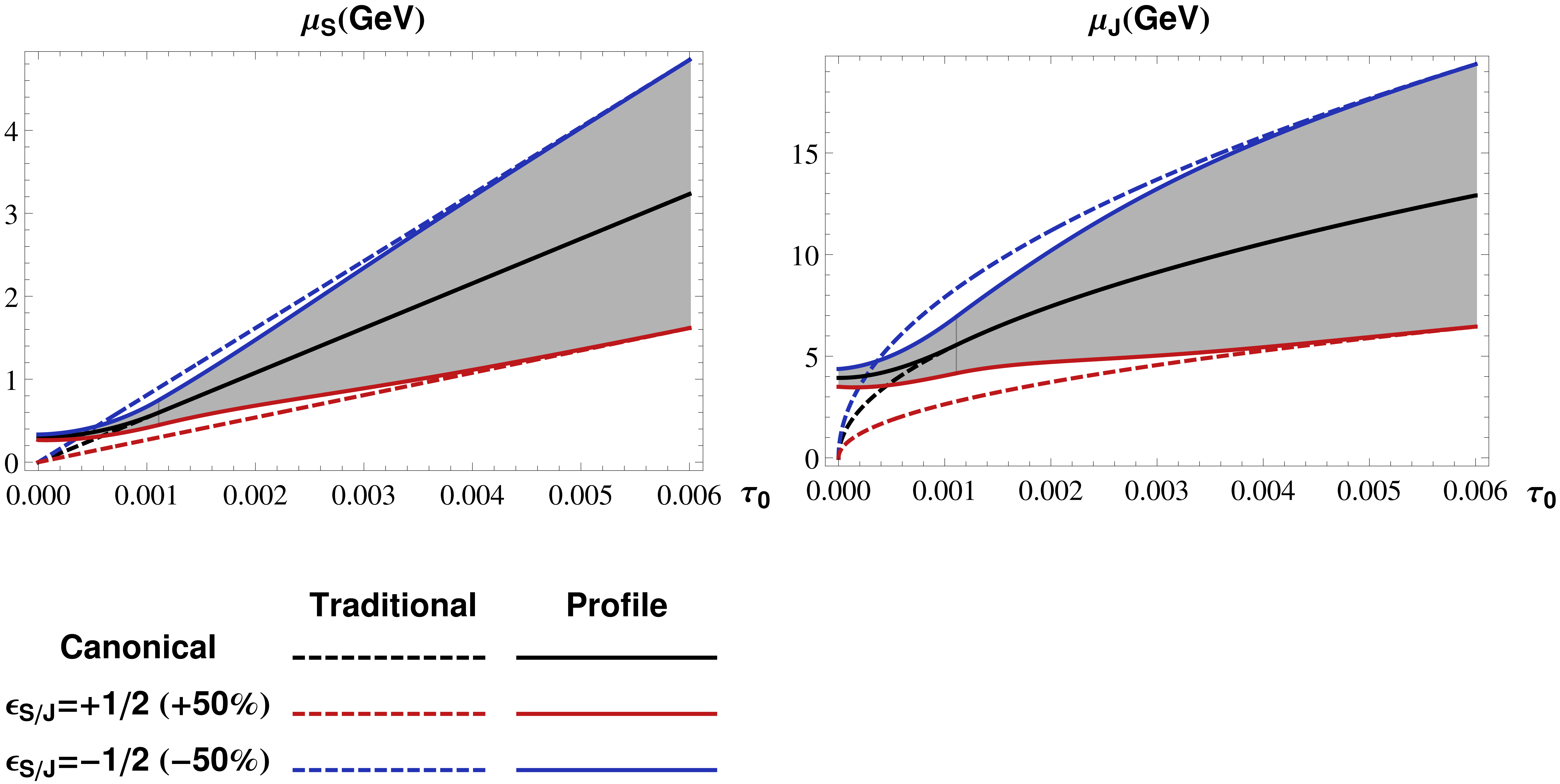
R. Bain, Y. Makris, TM, JHEP 1611 (2016) 144

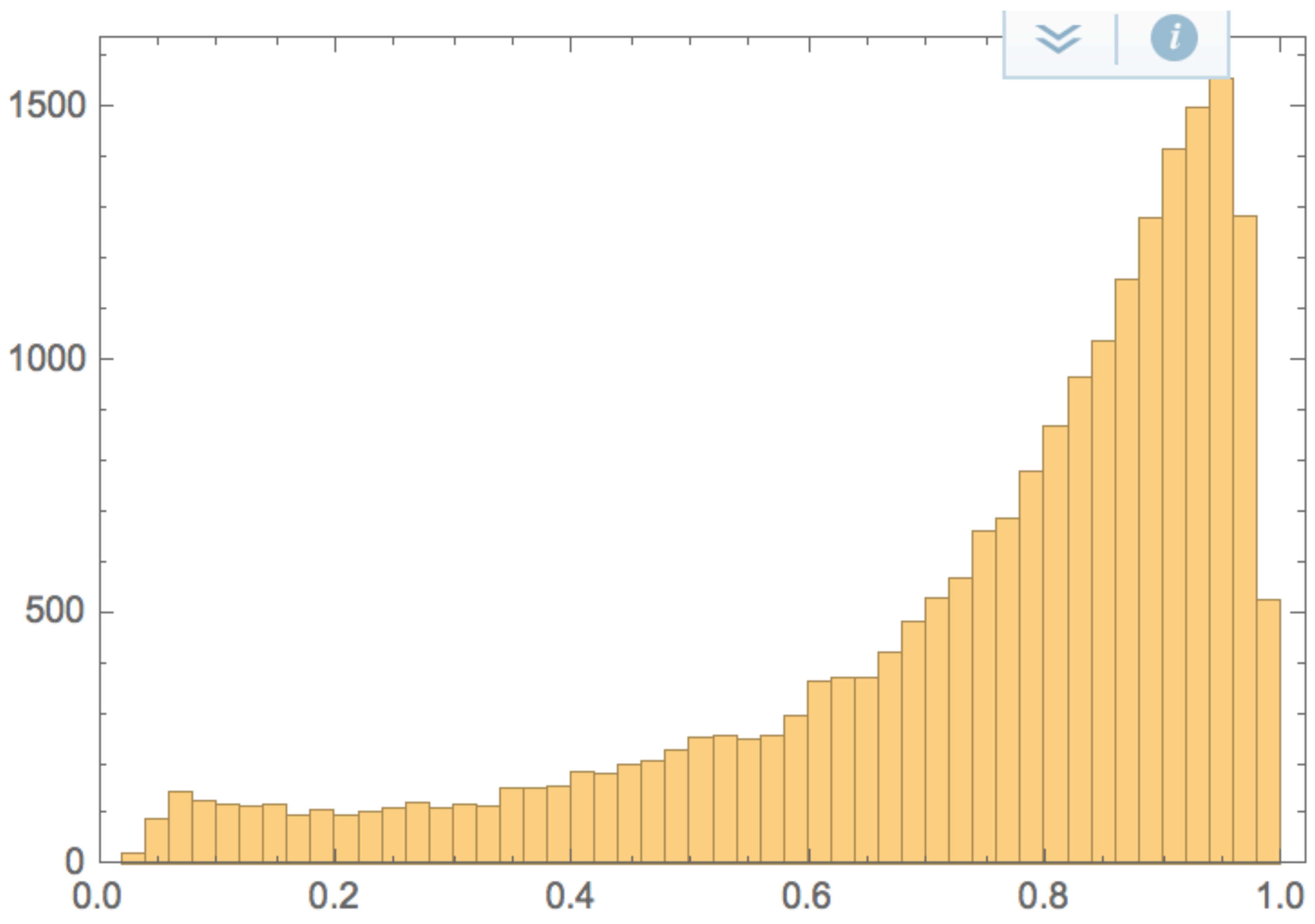
$$\omega \gg p_h^\perp \gg \Lambda$$



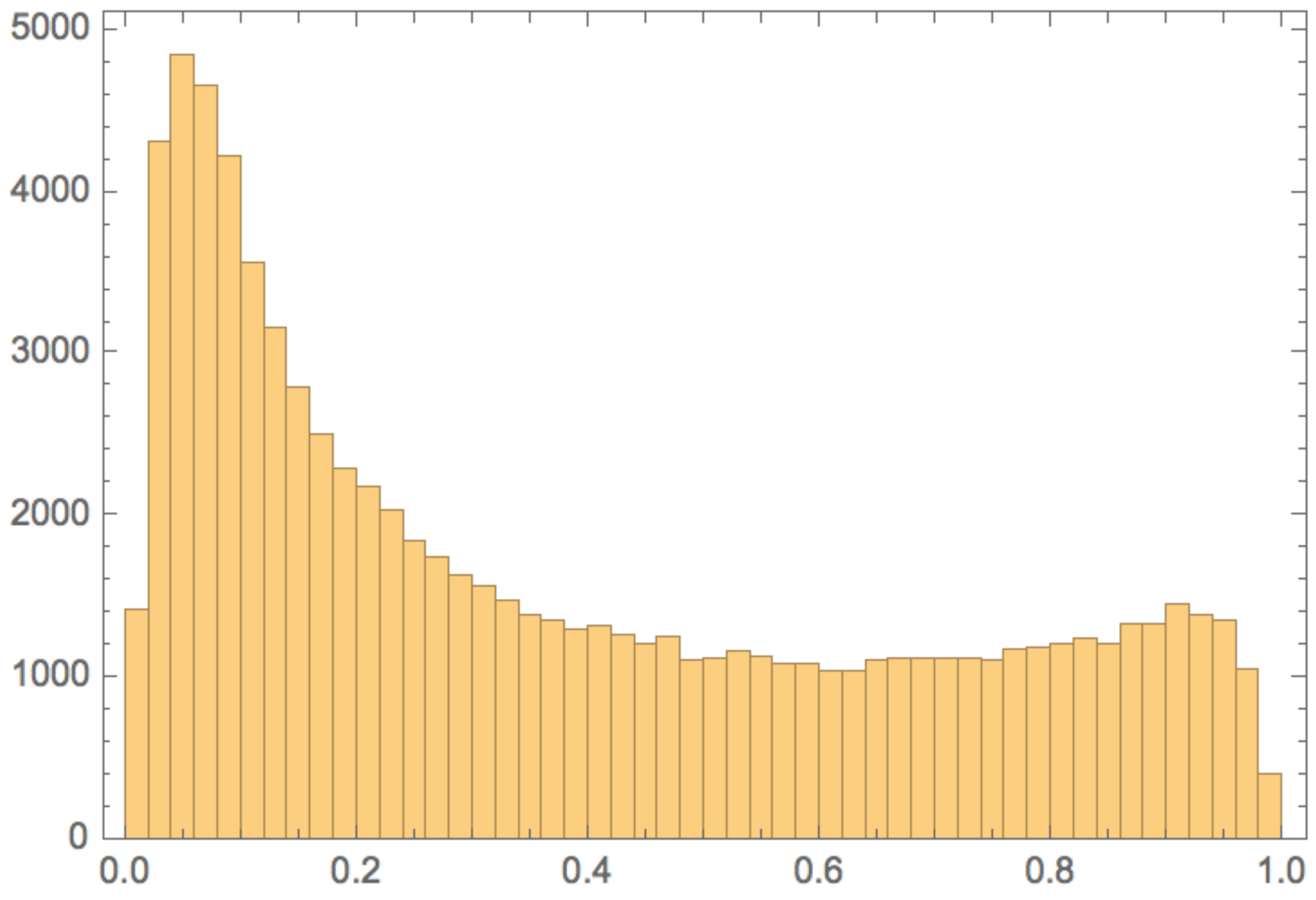
$D_{i/h}\left(z, p_h^\perp, \mu\right)$
$p_c \sim \omega(\lambda^2, 1, \lambda)$ $p_{cs} \sim p_h^\perp(r, 1/r, 1)$ $p_{us} \sim \Lambda(1, 1, 1)$
$\lambda = p_h^\perp/\omega$

Profile Functions





c distribution

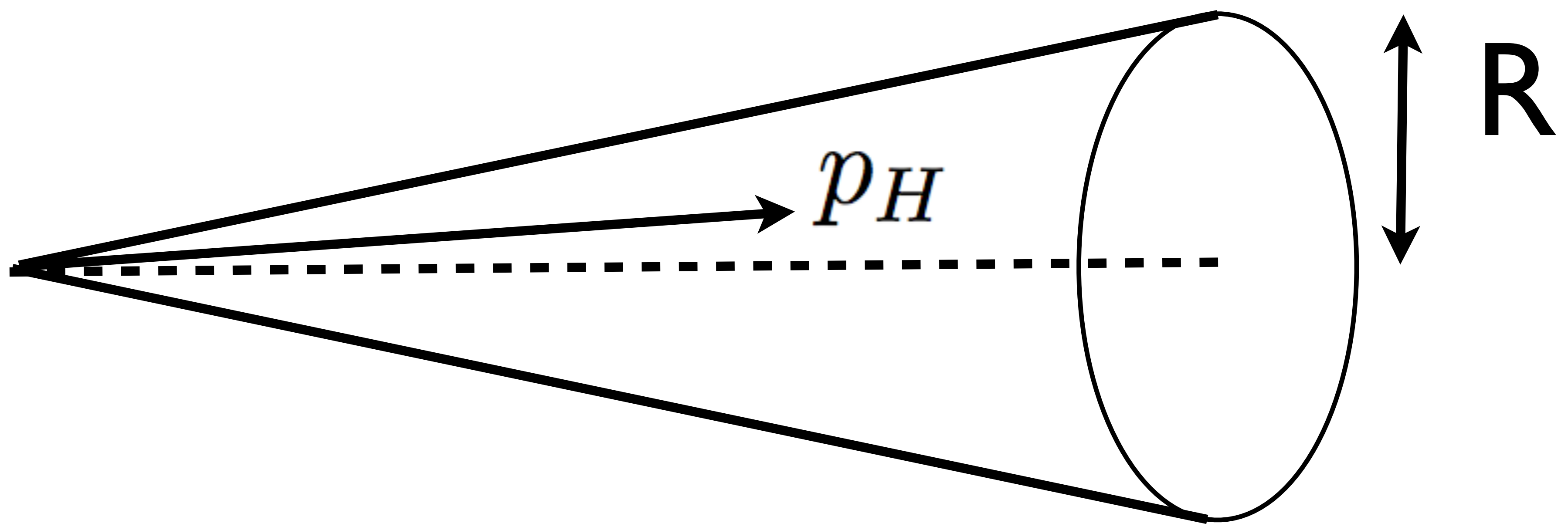


g distribution

Fragmenting Jet Functions

M. Procura, I. Stewart, PRD 81 (2010) 074009
A. Jain, M. Procura, W. Waalewijn, JHEP 1105 (2011) 035
A. Procura, W. Waalewijn, PRD 85 (2012) 114041

jets with identified hadrons



Jet Energy: E

$$p_H^+ = z p_{\rm jet}^+$$

cross sections determined by **fragmenting jet function (FJF):**

$$\mathcal{G}_g^h(E, R, \mu, z)$$

inclusive hadron production: **fragmentation functions**

$$\frac{1}{\sigma_0}\frac{d\sigma^h}{dz}(e^+e^- \to h\,X) = \sum_i \int_z^1 \frac{dx}{x}\, C_i(E_{\rm cm}, x, \mu)\, D_i^h(z/x, \mu)$$

jet cross sections: **jet functions**

$$d\sigma(E,R) = \int d\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell$$

$$\boxed{\mathcal{G}_g^h(E,R,\mu,z) \longrightarrow D_i^h(z/x,\mu)\ ,\ J_\ell}$$

relationship to jet function:

$$\sum_h \int_0^1 \mathrm{d}z\, z\, D_j^h(z,\mu) = 1$$

$$J_i(E,R,z,\mu) = \frac{1}{2}\sum_h \int \frac{dz}{(2\pi)^3}\, z\, \mathcal{G}_i^h(E,R,z,\mu)$$

cross section for jet w/ identified hadron from jet cross section

$$\frac{d\sigma}{dE} = \int d\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell J_i(E,R,\mu)$$

$$\frac{d\sigma}{dE dz} = \int d\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell\, \mathcal{G}_i^h(E,R,z,\mu)$$

relationship to fragmentation functions

$$\mathcal{G}_i^h(E,R,z,\mu) = \sum_i \int_z^1 \frac{\mathrm{d}z'}{z'} \mathcal{J}_{ij}(E,R,z',\mu) D_j^h\left(\frac{z}{z'},\mu\right) \left[1 + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{4E^2\tan^2(R/2)}\right)\right]$$

matching coefficients calculable in perturbation theory

$$\frac{\mathcal{J}_{gg}(E,R,z,\mu)}{2(2\pi)^3} = \delta(1-z) + \frac{\alpha_s(\mu)C_A}{\pi}\left[\left(L^2 - \frac{\pi^2}{24}\right)\delta(1-z) + \hat{P}_{gg}(z)L + \hat{\mathcal{J}}_{gg}(z)\right]$$

$$\hat{\mathcal{J}}_{gg}(z) = \begin{cases} \hat{P}_{gg}(z)\ln z & z \le 1/2 \\ \frac{2(1-z+z^2)^2}{z}\left(\frac{\ln(1-z)}{1-z}\right)_+ & z \ge 1/2. \end{cases}$$

$$L = \ln[2E\tan(R/2)/\mu].$$

scale for $\mathcal{J}_{ij}(E,R,z,\mu)$

sum rule for matching coefficients

$$\sum_j \int_0^1 \mathrm{d}z\, z\, \mathcal{J}_{ij}(R,z,\mu) = 2(2\pi)^3\, J_i(R,\mu)$$

Jet Shapes in Dijet Events at the LHC

A. Hornig, Y. Makris, T.M, JHEP 1604 (2016) 097

A. Hornig, D. Kang, Y. Makris, T.M, JHEP 1712 (2017) 043

boost invariant angularity

$$\tau_a^{e^+e^-} = \frac{1}{2E_J} \sum_i |p_T^{iJ}| e^{-(1-a)|y_{iJ}|}$$

$$= (2E_J)^{-(2-a)} (p_T)^{1-a} \sum_i |p_T^i| \left(\frac{\theta_{iJ}}{\sin\theta_J}\right)^{2-a} (1 + \mathcal{O}(\theta_{iJ}^2))$$

$$\tau_a^{pp} = \left(\frac{2E_J}{p_T}\right)^{2-a} \tau_a^{e^+e^-} + \mathcal{O}(\tau_a^2)$$

modified jet function

$$J_i(\tau_a) = \left(\frac{p_T}{2E_J}\right)^{2-a} J_i^{e^+e^-}\left(\left(\frac{p_T}{2E_J}\right)^{2-a} \tau_a\right)$$

angularities for inclusive jet cross sections

Z.-B. Kang, K. Lee, F. Ringer, arXiv:1801.00790

Analogous Formulae for pp collisions

Soft Function in e^+e^-

rotationally invariant cuts: $E < E_{min}$

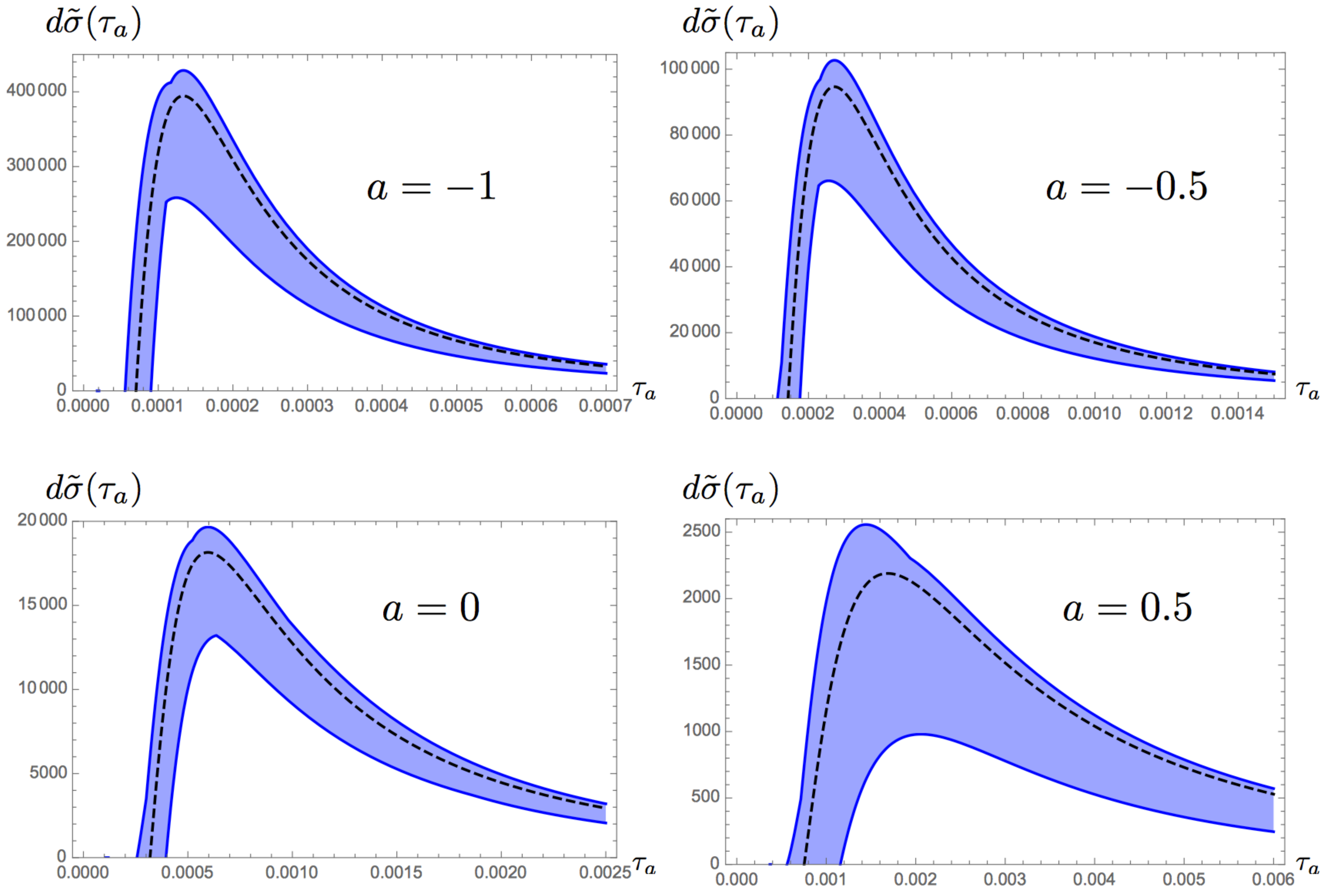
Soft Function in pp

boost invariant cuts, observables: p_T, rapidity

hard, soft functions are matrices in color space

$$d\sigma(\tau_a^1, \tau_a^2) = \frac{p_T x_1 x_2}{8\pi E_{\rm cm}^4} \frac{1}{N} B(x_1;\mu) \bar{B}(x_2;\mu) \,{\rm Tr}\{\mathbf{H}(\mu)\mathbf{S}(\tau_a^1, \tau_a^2;\mu)\} \otimes [J_1(\tau_a^1;\mu) J_2(\tau_a^2;\mu)]$$

pp→ 2 jets with boost invariant soft function



$q\bar{q}$ channel only

Study dependence on: a, R, p_T cut, scale variation