Ivan Vitev

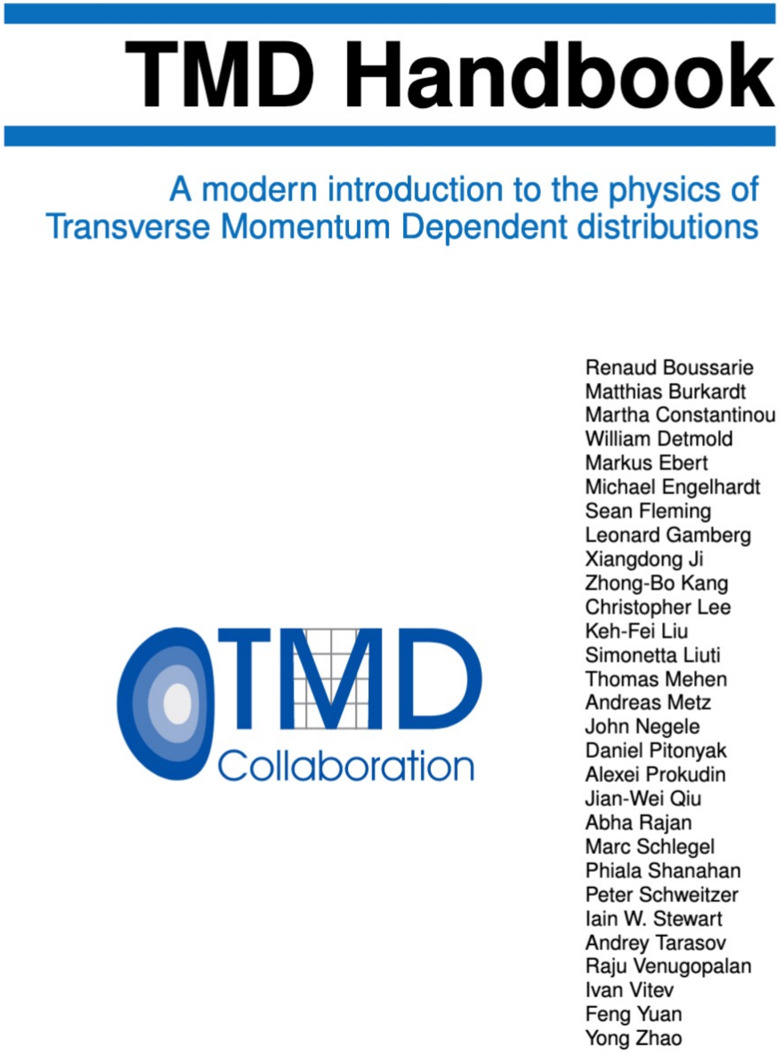
TMD Physics in Dense Matter

TMD Winter School
Santa Fe, NM, **December 20-26, 2022**

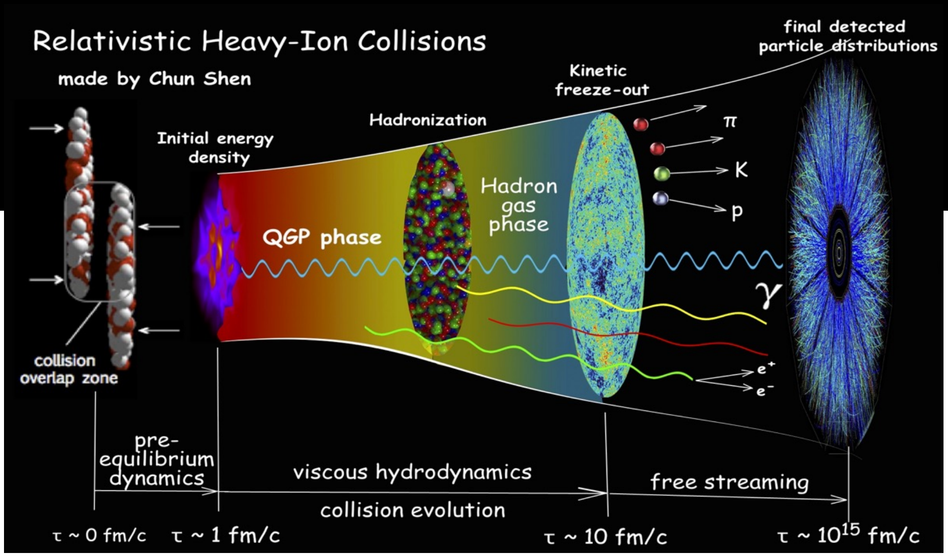
Los Alamos
NATIONAL LABORATORY

Outline of the lecture

- Process dependence of TMD physics in nuclear matter
- Scattering in dense matter
- Radiation in dense matter
- Observables in dense matter
- Conclusions



Dense matter



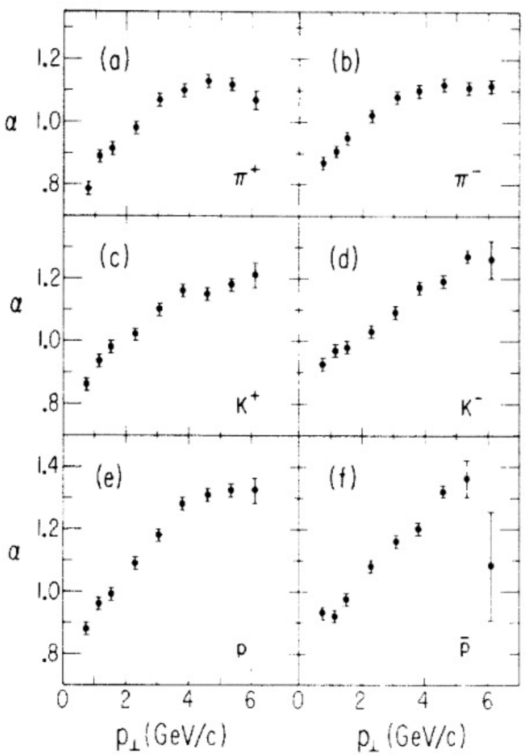
- We have different forms of dense matter – cold nuclear matter, quark-gluon plasma, hadron gas, neutron stars, …

- A+A collisions
- p+A collisions (fixed target, RHIC, LHC)
- e+A collisions (HERMES, EIC)

Discovery of transverse momentum broadening

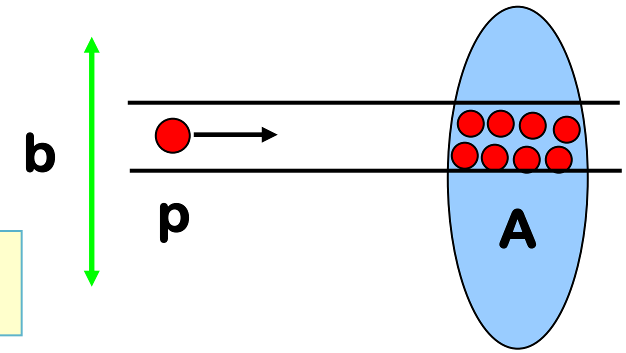
- Original measurement Jim Cronin – enhancement of particle production at intermediate p_T in p+A vs p+p collisions

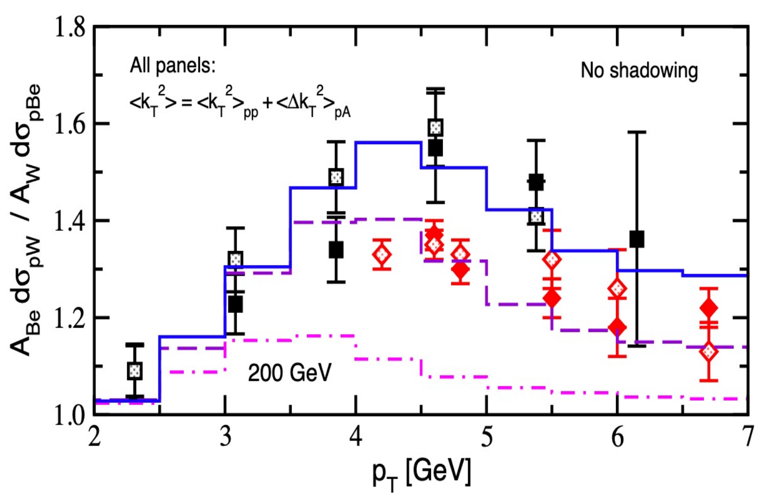
$$d\sigma^{p+A} \simeq d\sigma^{p+p} \left(N_{coll}\right)^{\alpha}, \quad \alpha = \alpha(p_T)$$



J. Cronin et al. (1975)

$$R_{pA}(p_T, b) = A^{\alpha(p_T, b) - 1}$$

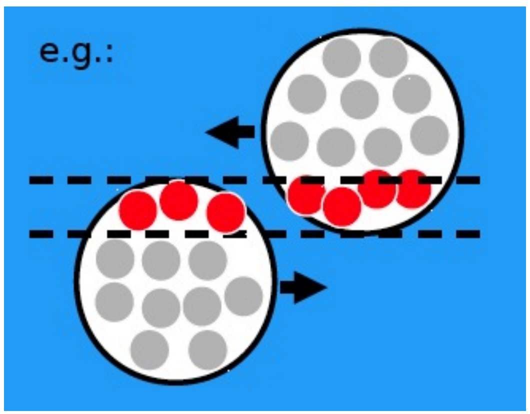




- Soft physics, with nuclear enhancement $A^{1/3}$ manifest at ~few GeV

Non-universal TMD effects in dense matter

- Reminder about the geometry in heavy ion collisions

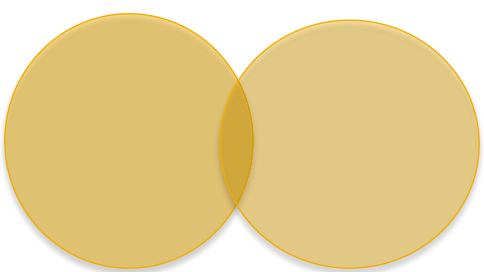


Number of binary collisions

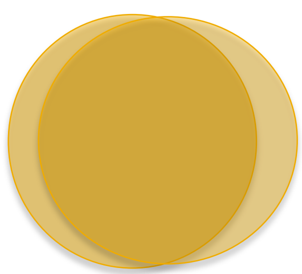
$N_1 \times N_2$

Number of participants

$N_1 + N_2$

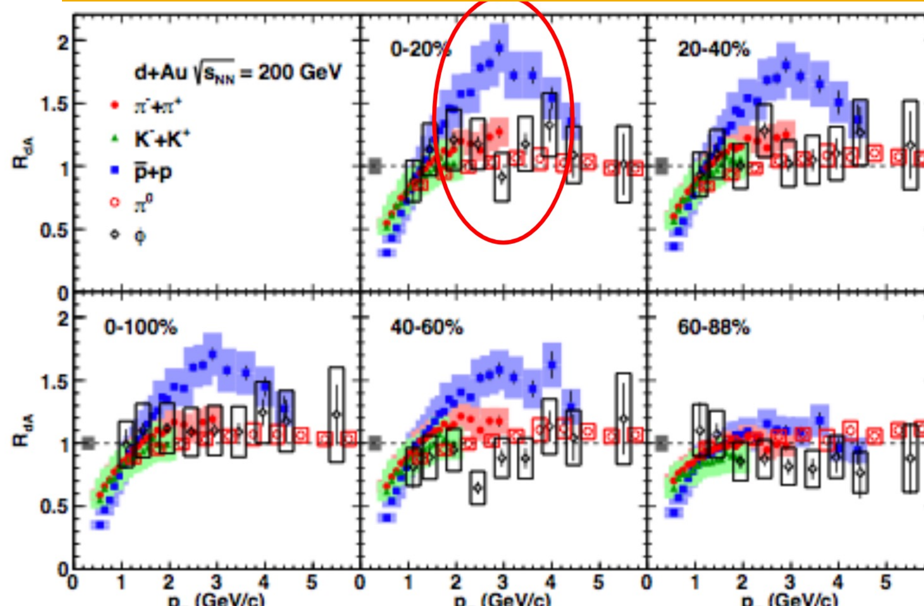


Peripheral



Central

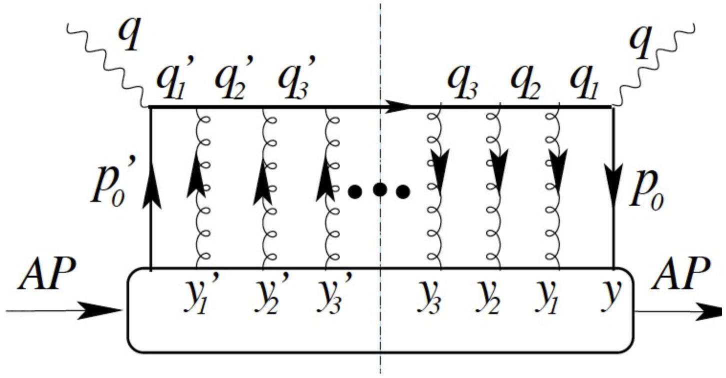
- The broadening effects are different for different nuclei
- They are different for different impact parameters – if you want to parameterize becomes a 4D problem



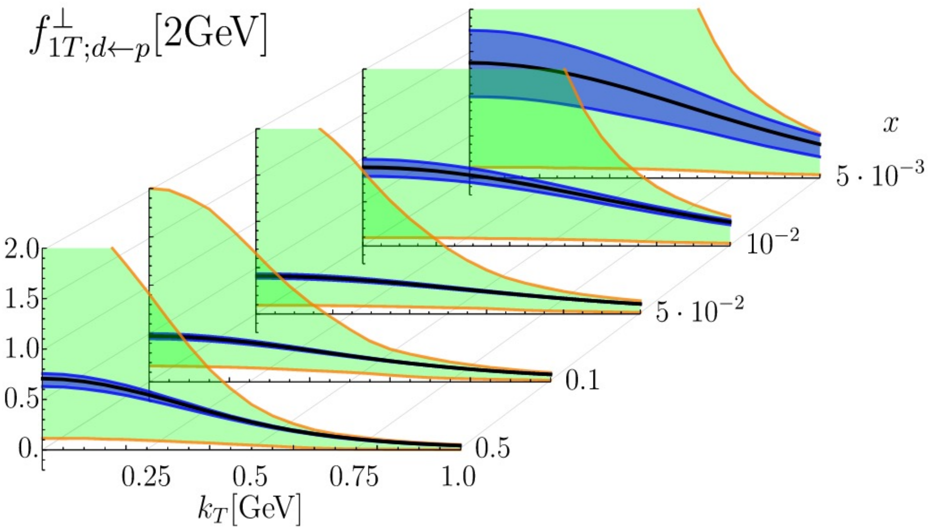
PHENIX Collab. (1975)

Reducing the complexity and broadening to scattering

- Final state parton broadening in semi-inclusive DIS.



We already have 2 dimensions



- Formulation of a transport coefficient as a Wilson line

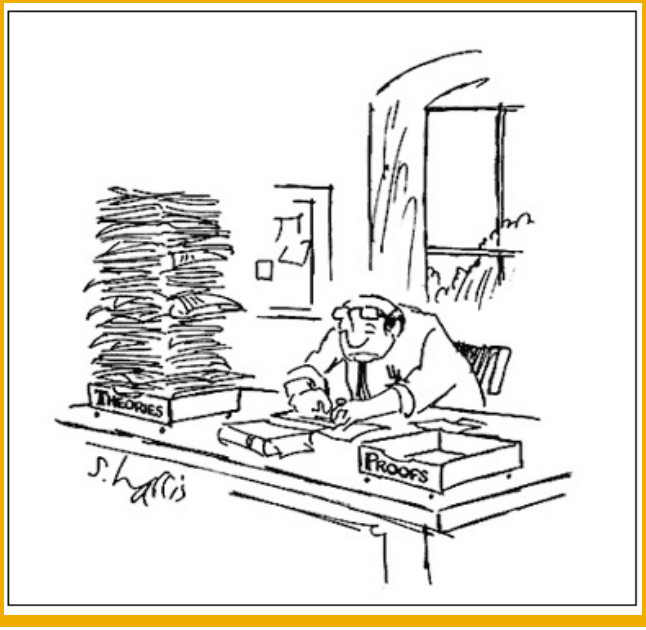
$$\hat{q} = \langle q_\perp^2 \rangle / \lambda_g$$

F. D'Eramo et al. (2010)

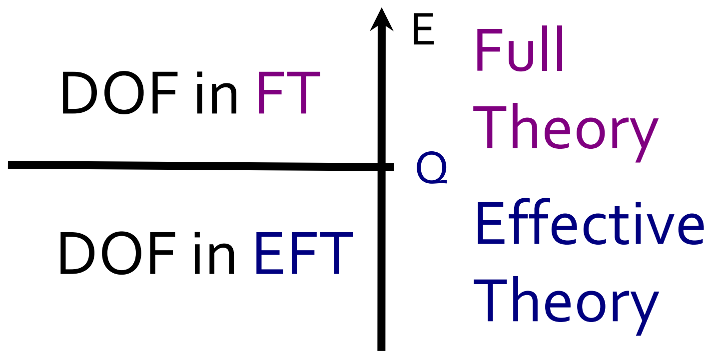
$$W_F[y^+, y_\perp] \equiv P\left\{\exp\left[ig\int_0^{L^-} dy^- A^+(y^+, y^-, y_\perp)\right]\right\}$$

$$\sum_{m=1,n=1}^{\infty} \frac{d^2 A_{nm}}{d^2 k_\perp} = \frac{\sqrt{2}}{L^3 N_c}\int dy^+ dy_\perp dy'_\perp\, e^{-ik_\perp\cdot(y_\perp - y'_\perp)} \left\langle \mathrm{Tr}\left[\left(W_F^\dagger[y^+, y'_\perp] - 1\right)\left(W_F[y^+, y_\perp] - 1\right)\right]\right\rangle$$

Transverse momentum of partons in matter



Examples of effective field theories [EFTs]

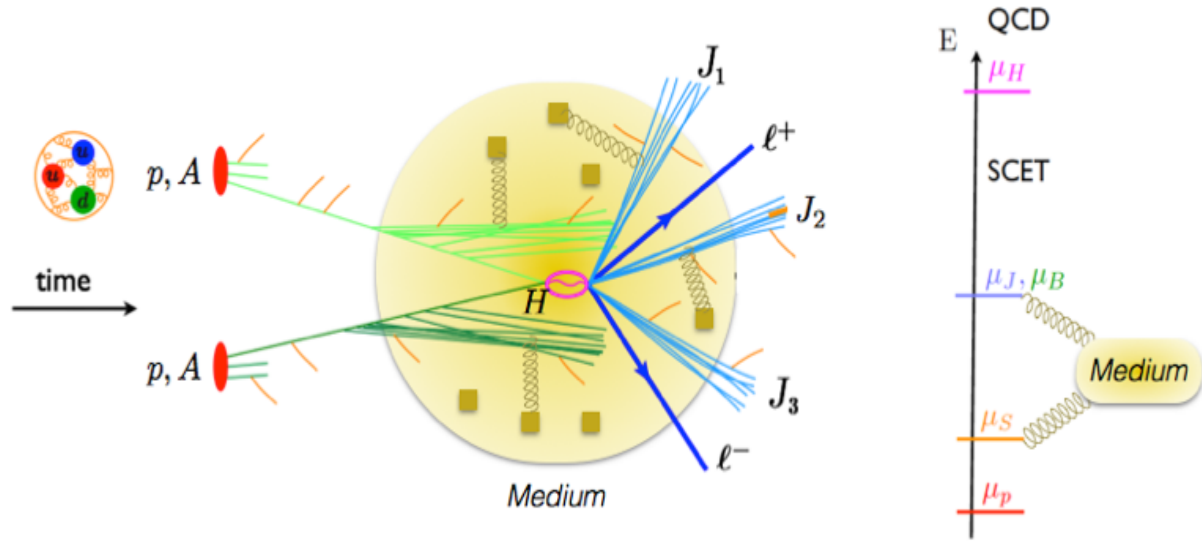


- Focus on the significant degrees of freedom [DOF]. Manifest power counting

	Q	power counting	DOF in FT	DOF in EFT
Chiral Perturbation Theory (ChPT)	Λ_{QCD}	p/Λ_{QCD}	q, g	K,π
Heavy Quark Effective Theory (HQET)	m_b	Λ_{QCD}/m_b	ψ,A	h_v, A_s
Soft Collinear Effective Theory (SCET)	Q	$p_\perp/Q$	ψ,A	ξ_n, A_n, A_s
Non-Relativistic QCD (NRQCD)	m_Q	p/m_Q	ψ,A	ψ_Q, A_s, A_{us}

Example of successful EFT in matter

RHIC (though not the first HI machine) has played a very important role in truly developing a new field – interaction of hard probes in matter



Energy loss approach

- M. Gyulassy et al. (1993)
- B. Zakharov (1995)
- R. Baier et al. (1997)
- M. Gyulassy et al. (2000)
- X. Guo et al. (2001)
- P. Arnold et al. (2003)

EFT approach

- Idilbi et al. (2008)

- Factorization, with modified J (jet), B (beam), S (soft) functions

- Ovanesyan et al. (2011)
- Z. Kang et al. (2016)

$$\sigma = \mathrm{Tr}(HS) \otimes \prod_{i=1}^{n_B} B_i \otimes \prod_{j=1}^{N} J_j + \text{power corrections}$$

- QCD in the medium remains a multi-scale problem. I will focus on x+A reactions

Origin of transverse momentum physics in dense matter

- What is missing in the YM Lagrangian is the interaction between the jet and the medium

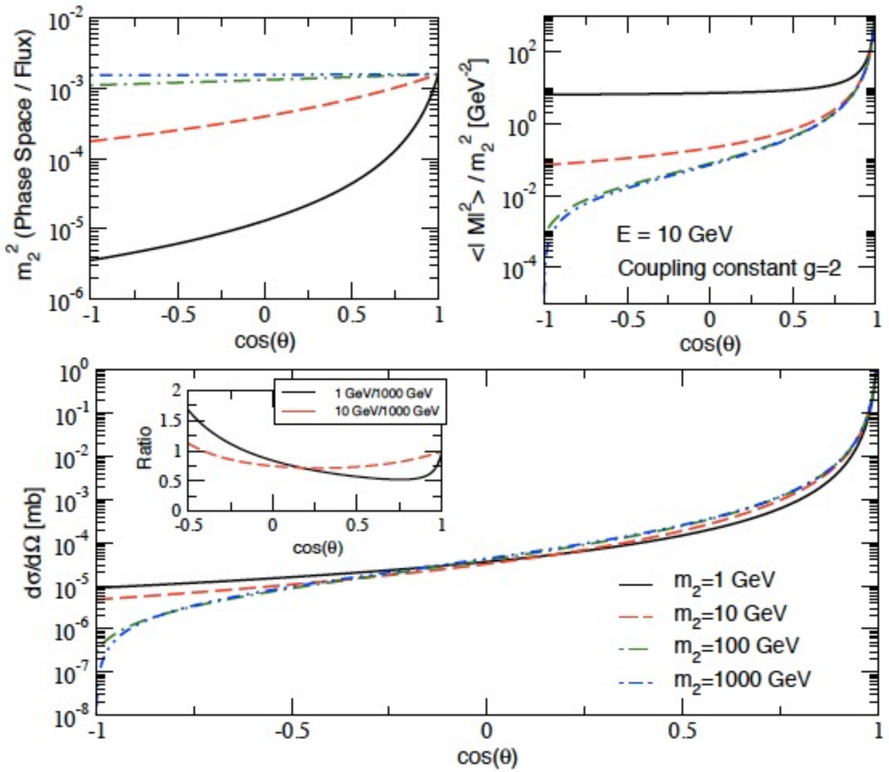
- Kinematics and channels

t – jet broadening and energy loss
s– isotropisation
u – backward hard scattering

- Fully dynamic medium recoil, cross section reduction (5% – 15%). Completely dominated by forward scattering

$$\frac{d\sigma}{d\Omega} \to \frac{d\sigma}{d^2\mathbf{q}_\perp} = \frac{C_2(R)C_2(T)}{d_A}\frac{|v(\mathbf{q}_\perp;E,m_1,m_2)|^2}{(2\pi)^2}$$

G. Ovanesyan et al. (2011)



The Glauber gluon Lagrangian

- Glauber gluons (transverse)

$$q \sim [\lambda^2, \lambda^2, \boldsymbol{\lambda}]$$

A. Idilbi et al. (2008)

$$\mathcal{L}_{\rm G}\left(\xi_n, A_n, \eta\right) = \sum_{p,p',q} e^{-i(p-p'+q)x} \left(\bar{\xi}_{n,p'} \Gamma^{\mu,a}_{\rm qqA_G} \frac{\bar{n\!\!\!/}}{2} \xi_{n,p} - i \Gamma^{\mu\nu\lambda,abc}_{\rm ggA_G} \left(A^c_{n,p'}\right)_\lambda \left(A^b_{n,p}\right)_\nu \right) A^\mu_G$$

- Feynman rules for different sources and gauges

Gauge	Object	Collinear source	Static source	Soft source
	p	$[\lambda^2, 1, \boldsymbol{\lambda}]$	$[1, 1, \boldsymbol{\lambda}]$	$[\lambda, \lambda, \boldsymbol{\lambda}]$
	$a_{\boldsymbol{p}}, a^\dagger_{\boldsymbol{p}}$	λ^{-1}	$\lambda^{-3/2}$	$\lambda^{-3/2}$
	$u(p)$	1	1	$\lambda^{1/2}$
	$\bar{u}(p_2)\gamma_\nu u(p_1)$	$[\lambda^2, 1, \boldsymbol{\lambda}]$	$[1, 1, \boldsymbol{\lambda}]$	$[\lambda, \lambda, \boldsymbol{\lambda}]$
R_ξ	$A^\mu(x)$	$[\lambda^4, \lambda^2, \boldsymbol{\lambda}^3]$	$[\lambda^2, \lambda^2, \boldsymbol{\lambda}^3]$	$[\lambda, \lambda, \boldsymbol{\lambda}]$
	$\Gamma_{\rm qqA_G}$	Γ^μ_1	Γ^μ_1	Γ^μ_1
	$\Gamma_{\rm ggA_G}$	$\Sigma^{\mu\nu\lambda}_1$	$\Sigma^{\mu\nu\lambda}_1$	$\Sigma^{\mu\nu\lambda}_1$
	$\Gamma_{\rm s}$	$\Gamma^\mu_1\ (n \leftrightarrow \bar{n})$	Γ^μ_3	Γ^μ_4
$A^+ = 0$	$A^\mu(x)$	$[0, \lambda^2, \boldsymbol{\lambda}^3]$	$[0, \lambda^2, \boldsymbol{\lambda}]$	$[0, \lambda, \mathbf{1}]$
	$\Gamma_{\rm qqA_G}$	Γ^μ_1	$\Gamma^\mu_1 + \Gamma^\mu_2$	$\Gamma^\mu_1 + \Gamma^\mu_2$
	$\Gamma_{\rm ggA_G}$	$\Sigma^{\mu\nu\lambda}_2$	$\Sigma^{\mu\nu\lambda}_2$	$\Sigma^{\mu\nu\lambda}_2$
	$\Gamma_{\rm s}$	$\Gamma^\mu_2\ (n \leftrightarrow \bar{n})$	Γ^μ_3	Γ^μ_4
$A^- = 0$	$A^\mu(x)$	$[\lambda^2, 0, \boldsymbol{\lambda}]$	$[\lambda^2, 0, \boldsymbol{\lambda}]$	$[\lambda, 0, \mathbf{1}]$
	$\Gamma_{\rm qqA_G}$	Γ^μ_2	Γ^μ_2	Γ^μ_2
	$\Gamma_{\rm ggA_G}$	$\Sigma^{\mu\nu\lambda}_3$	$\Sigma^{\mu\nu\lambda}_3$	$\Sigma^{\mu\nu\lambda}_3$
	$\Gamma_{\rm s}$	$\Gamma^\mu_1\ (n \leftrightarrow \bar{n})$	Γ^μ_3	Γ^μ_4

G. Ovanesyan et al. (2011)

$$\Gamma^{\mu,a}_1 = igT^a\, n^\mu \frac{\bar{n\!\!\!/}}{2}\,,$$

$$\Gamma^{\mu,a}_2 = igT^a \frac{\gamma^\mu_\perp p\!\!\!/_\perp + p\!\!\!/'_\perp \gamma^\mu_\perp}{\bar{n}\cdot p} \frac{\bar{n\!\!\!/}}{2}\,,$$

$$\Gamma^{\mu,a}_3 = igT^a\, v^\mu\,,$$

$$\Gamma^{\mu,a}_4 = igT^a\, \gamma^\mu\,,$$

$$\Sigma^{\mu\nu\lambda,abc}_1 = gf^{abc} n^\mu \left[g^{\nu\lambda}\, \bar{n}\cdot p + \bar{n}^\nu \left(p'^\lambda_\perp - p^\lambda_\perp\right) - \bar{n}^\lambda \left(p'^\nu_\perp - p^\nu_\perp\right) - \frac{1-\frac{1}{\xi}}{2} \left(\bar{n}^\lambda p^\nu + \bar{n}^\nu p'^\lambda\right) \right],$$

$$\Sigma^{\mu\nu\lambda,abc}_2 = gf^{abc} \left[g^{\mu\lambda}_\perp \left(-\frac{n^\nu}{2} p^+ + p^\nu_\perp - 2p'^\nu_\perp \right) + g^{\mu\nu}_\perp \left(-\frac{n^\lambda}{2} p^+ + p'^\lambda_\perp - 2p^\lambda_\perp \right) + g^{\nu\lambda}_\perp \left(n^\mu\, \bar{n}\cdot p + p^\mu_\perp + p'^\mu_\perp \right) \right],$$

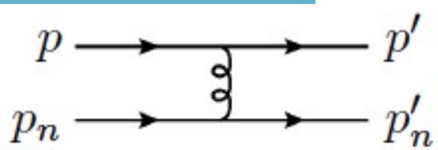
$$\Sigma^{\mu\nu\lambda,abc}_3 = gf^{abc} \left[g^{\mu\lambda}_\perp \left(\frac{\bar{n}^\nu}{2}(p^- - 2p'^-) + p^\nu_\perp - 2p'^\nu_\perp \right) + g^{\mu\nu}_\perp \left(\frac{\bar{n}^\lambda}{2}(p'^- - 2p^-) + p'^\lambda_\perp - 2p^\lambda_\perp \right) + g^{\nu\lambda}_\perp \left(p^\mu_\perp + p'^\mu_\perp \right) \right].$$

The QCD forward scattering diagram expansion

- What I described is the background field method. I did not dwell too much on the microscopic physics of the Glauber gluon field - lots of non-perturbative physics in matter

- Consider the target as active degrees of freedom

I. Rothstein et al. (2015)

$t_{coll.} =$ 

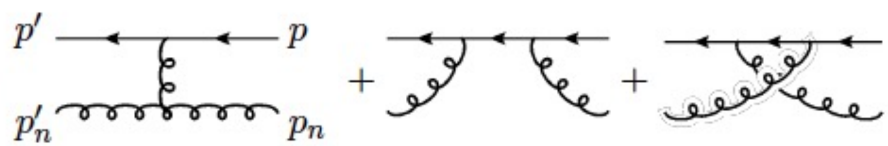
Glauber field for collinear source

$$A_G^{\mu,a} = \frac{n^\mu}{\mathbf{q}_T^2}\sum_\ell \bar{\xi}_{n,\ell-\mathbf{q}_T}\frac{\bar{\not n}}{2}(gT^a)\xi_{n,\ell}$$

Coulomb field for soft source

$$A_C^{\mu,a} \equiv \frac{1}{\mathbf{q}^2}\sum_\ell \bar{\phi}_{\ell-\mathbf{q}}\gamma^\mu(gT^A)\phi_\ell$$

Y. Makris et al. (2019)

$t_{g-coll.} =$ 

$$= t^{(0)}_{g-coll.} + t^{(1)}_{g-coll.} + \mathcal{O}(\lambda^2)\ .$$

The field is a expansion of gauge invariant operator

$$\mathbf{B}^{a,(0)}_{n\perp,\ell} \equiv \mathbf{A}^a_{n\perp,\ell} - \mathbf{p}_{n\perp}\frac{A^{-,a}_{n,\ell}}{p^-_n}$$

$$B^\mu_{n\perp} \equiv \frac{1}{g}\left[W_n^\dagger(\mathcal{P}^\mu_\perp - gA^\mu_{n\perp})W_n\right] = B^{\mu,a(0)}_{n\perp}T^a + \mathcal{O}(g)$$

Glauber field for collinear source

$$A_G^{\mu,a} = \frac{i}{2}gf^{abc}\frac{n^\mu}{\mathbf{q}_T^2}\sum_\ell\left[\bar{n}\cdot\mathcal{P}\ (\mathbf{B}^{b(0)}_{n\perp,\ell-\mathbf{q}_T}\cdot\mathbf{B}^{c(0)}_{n\perp,\ell})\right]$$

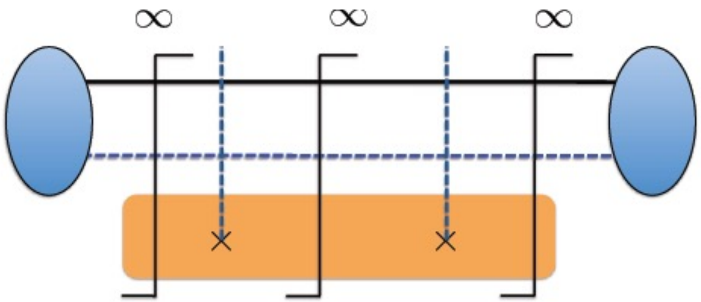
Coulomb field for soft source

$$A_C^{\mu,a} = f^{abc}\frac{ig}{2\,\mathbf{q}^2}\sum_\ell\left\{\left[\mathcal{P}^\mu\ (\mathbf{B}^{b(0)}_{s,\ell-\mathbf{q}}\cdot\mathbf{B}^{c(0)}_{s,\ell})\right] - 2(\mathbf{B}^{c(0)}_{s,\ell}\cdot\left[\boldsymbol{\mathcal{P}})B^{\mu,b(0)}_{s,\ell-\mathbf{q}}\right] - 2(\mathbf{B}^{b(0)}_{s,\ell-\mathbf{q}}\cdot\left[\boldsymbol{\mathcal{P}})B^{\mu,c(0)}_{s,\ell}\right]\right\}$$

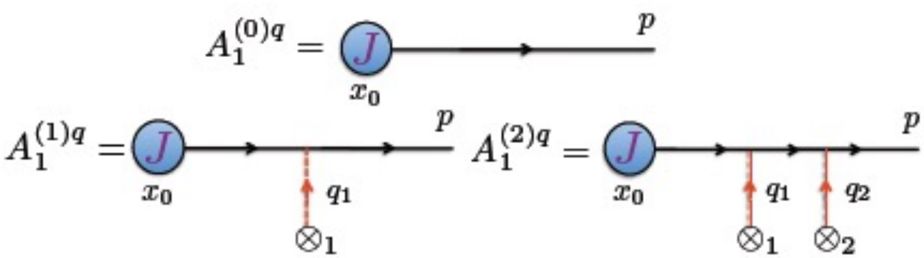
Jet broadening – unit interaction

- Need two Glauber gluon exchanges to build 1 power of the scattering cross section in matter

M. Gyulassy et al. (2001)



Classes of diagrams (single Born, double Born). Reaction Operator



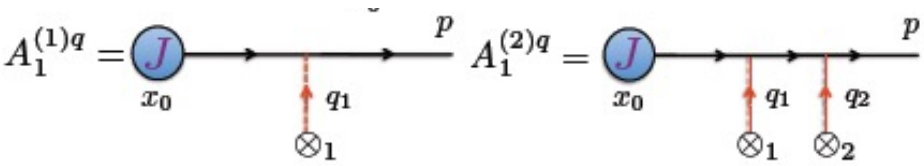
Any momentum dependence we put in J(p). E.g. I can choose unit strength and the quark not having transverse momentum initially

$$A_1^{(0)q} = \bar{\chi}_{n,p}\, iJ(p)\, e^{ipx_0}$$

$$d\sigma \propto \frac{1}{d_R d_T} \sum_{\text{spin,color}} |A_1^{(0)q}|^2 = \mathrm{Tr}\left(\frac{\not{n}}{2} J(p)\bar{J}(p)\right) \bar{n}\cdot p$$

There is a phase – our propagating particle is a plane wave. Here is notion of coherence and interference.

Single and double Born terms



- Need two Glauber gluon exchanges to build 1 power of the scattering cross section in matter

$$d\Phi_i = \frac{d^4 q_i}{(2\pi)^4}\, \mathrm{e}^{i q_i \delta x_i}\, v(q_i)\ ,$$

$$d\boldsymbol{\Phi}_{i\perp} = \frac{d^2 \boldsymbol{q}_{i\perp}}{(2\pi)^2}\, \mathrm{e}^{-i \boldsymbol{q}_{i\perp} \delta \boldsymbol{x}_{i\perp}}\, \tilde{v}(\boldsymbol{q}_{i\perp})\ ,$$

Have to integrate over the phase space
Important part of the propagator

$$\Delta_g(p,q) \equiv \frac{1}{p^- - q^- - \frac{(\boldsymbol{p}_\perp - \boldsymbol{q}_\perp)^2 - i\varepsilon}{p^+}}$$

$$\bar{\chi}_{n,p}\left(\frac{\bar{n\!\!\!/}}{2}\frac{n\!\!\!/}{2}\right)^k = \bar{\chi}_{n,p}\left(\frac{\bar{n\!\!\!/}}{2}\frac{n\!\!\!/}{2}\right) = \bar{\chi}_{n,p}$$

$$\omega_1 = \Omega(p,q_1) = p^- - \frac{(\boldsymbol{p}_\perp - \boldsymbol{q}_{1\perp})^2 - i\varepsilon}{p^+}$$

$$\Omega(p, q_1+q_2) = p^- - \frac{(\boldsymbol{p}_\perp - \boldsymbol{q}_{1\perp} - \boldsymbol{q}_{2\perp})^2 - i\varepsilon}{p^+}$$

$$I_1^{(1)} = \int \frac{dq_1^-}{2\pi}\, \mathrm{e}^{i q_1^- \delta z_1}\, \Delta_g(p,q_1) = \int \frac{dq_1^-}{2\pi}\, \mathrm{e}^{i q_1^- \delta z_1}\, \frac{1}{\omega_1 - q_1^-} = -i\mathrm{e}^{i\omega_1 \delta z_1}$$

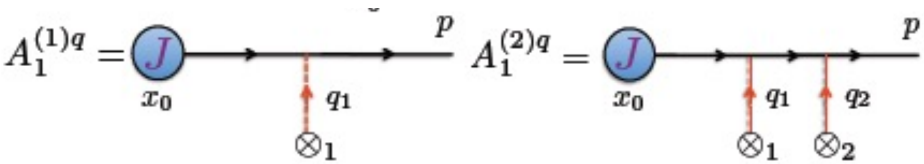
Contour integration

$$I_1^{(2c)} = \int \frac{dq_1^-}{2\pi}\frac{dq_2^-}{2\pi}\, \mathrm{e}^{i(q_1^- + q_2^-)\delta z_1}\, \Delta_g(p,q_2)\, \Delta_g(p,q_1+q_2) = (-i)\int \frac{dq_2^-}{2\pi}\, e^{i(\omega_{12} - q_2^- + q_2^-)\delta z_1}\, \Delta_g(p,q_2)$$

$$= -i\mathrm{e}^{i\omega_{12}\delta z_1} \int \frac{dq_2^-}{2\pi}\frac{1}{\omega_2 - q_2^-} = \frac{i\mathrm{e}^{i\omega_{12}\delta z_1}}{2\pi}\left(\ln(\infty - \omega_2) - \ln(-\infty - \omega_2)\right) = -\frac{1}{2}\mathrm{e}^{i\omega_{12}\delta z_1}\ . \qquad ($$

Can take the phases to 1 (momentum highly suppressed)

Single and double Born terms



- Need to keep track of the momentum shift in the initial distribution

Average over scattering centers positions $\langle \cdots \rangle = \int \frac{d^2\mathbf{b}}{A_\perp} \cdots$

$$\langle e^{-i(\mathbf{q}-\mathbf{q}')\cdot\mathbf{b}} \rangle = \frac{(2\pi)^2}{A_\perp} \delta^2(\mathbf{q}-\mathbf{q}')$$

$$A_{\rm coll}^{(k)q} = \bar{\chi}_{n,p} \int \prod_{m=1}^{k} d\Phi_m \, B^{(k)q} \, iJ\left(p - \sum_{l=1}^{k} q_l\right) e^{ipx_0}$$

We are left with the transverse integrals

$$\frac{1}{d_R\, d_T} \mathrm{Tr}|A_1^{(1)q}|^2 = \frac{N}{A_\perp} \frac{C_2(R)C_2(T)}{d_A} \int \frac{d^2\boldsymbol{q}_\perp}{(2\pi)^2} |\tilde{v}(\boldsymbol{q}_\perp)|^2 \times e^{-\mathbf{q}_\perp\cdot\vec{\nabla}_{\mathbf{p}_\perp}}$$

$$\frac{1}{d_R\, d_T} \mathrm{Tr} \left(A_1^{(0)q}\right)^\dagger A_1^{(2c)q} = \left(-\frac{1}{2}\right) \frac{N}{A_\perp} \frac{C_2(R)C_2(T)}{d_A} \int \frac{d^2\boldsymbol{q}_\perp}{(2\pi)^2} |\tilde{v}(\boldsymbol{q}_\perp)|^2$$

The two transverse momenta become equal

The two transverse momenta add to o

$$\frac{1}{d_R\, d_T} \mathrm{Tr}\left(|A_1^{(0)q}|^2 + |A_1^{(1)q}|^2 + 2\,\mathrm{Re}\left(A_1^{(0)q}\right)^\dagger A_1^{(2c)q}\right)$$
$$= 1 + \frac{N}{A_\perp} \int d^2\boldsymbol{q}_\perp \left[\frac{d\sigma_{\rm el}(R,T)}{d^2\boldsymbol{q}_\perp} e^{-\mathbf{q}_\perp\cdot\vec{\nabla}_{\mathbf{p}_\perp}} - \sigma_{\rm el}\delta^{(2)}(\boldsymbol{q}_\perp)\right]$$

This is the effect of one scattering (averaged over the transverse plane)

Main results: jet broadening

- We can also consider many scatterings along the path of propagation

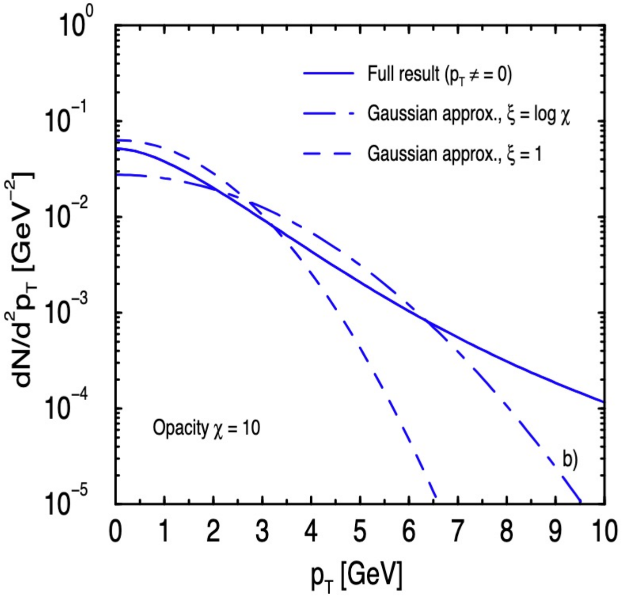
$$\frac{dN^{(n)}(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp} = \prod_{i=1}^{n} \int_{z_{i-1}}^{L} \frac{dz_i}{\lambda} \int d^2\mathbf{q}_{\perp i} \left[\frac{1}{\sigma_{el}(z_i)} \frac{d\sigma_{el}(z_i)}{d^2\mathbf{q}_{\perp i}} \left(e^{-\mathbf{q}_{\perp i} \cdot \vec{\nabla}_{\mathbf{p}_\perp}} \right) - \delta^2(\mathbf{q}_\perp) \right] \frac{dN^{(0)}(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp} \qquad \chi = \frac{L}{\lambda}$$

$$\frac{dN(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp} = \sum_{n=0}^{\infty} \frac{dN^{(n)}(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp} = \sum_{n=0}^{\infty} e^{-\chi} \frac{\chi^n}{n!} \int \prod_{i=1}^{n} d^2\mathbf{q}_i \frac{1}{\sigma_{el}} \frac{d\sigma_{el}}{d^2\mathbf{q}_{\perp i}} \, dN^{(0)}(\mathbf{p}_\perp - \mathbf{q}_{\perp 1} - \cdots - \mathbf{q}_{\perp n})$$

- In special cases such as constant density and the Gaussian approximation – carry out resummation in impact parameter space

$$\frac{d\tilde{\sigma}_{el}}{d^2\mathbf{q}_\perp}(\mathbf{b}) = \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} \, e^{-i\mathbf{q}_\perp \cdot \mathbf{b}} \frac{1}{\pi} \frac{\mu^2}{(\mathbf{q}_\perp^2 + \mu^2)^2} = \frac{\mu\, b}{4\pi^2} K_1(\mu\, b) \approx \frac{1}{4\pi^2} \left(1 - \frac{\xi\, \mu^2\, b^2}{2} + \mathcal{O}(b^3) \right)$$

$$\frac{dN(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp} = \int d^2\mathbf{b} \; e^{i\mathbf{p}_\perp \cdot \mathbf{b}} \frac{1}{(2\pi)^2} e^{-\frac{\chi\, \mu^2\, \xi\, b^2}{2}} = \frac{1}{2\pi} \frac{e^{-\frac{p_\perp^2}{2\chi\, \mu^2\, \xi}}}{\chi\, \mu^2\, \xi} \qquad \log 2/(1.08\, \mu\, b)$$



We obtained Gaussian distribution (in reality has a power law tail beyond the mean width)

M. Gylassy et al. (2002)

Phenomenology

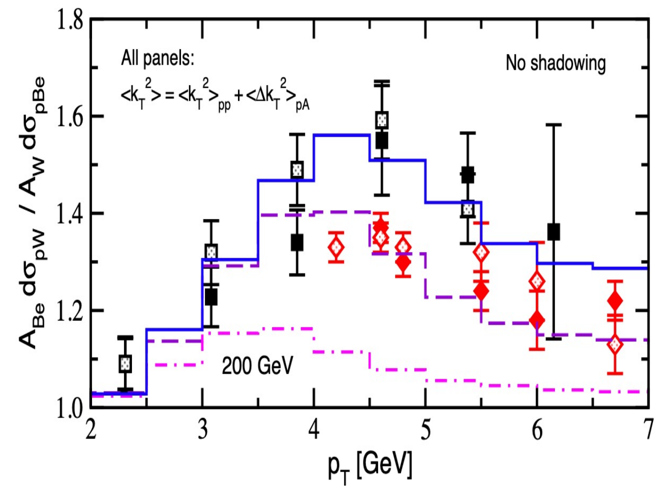
- What are the values of the transport parameter q-hat in nuclear matter

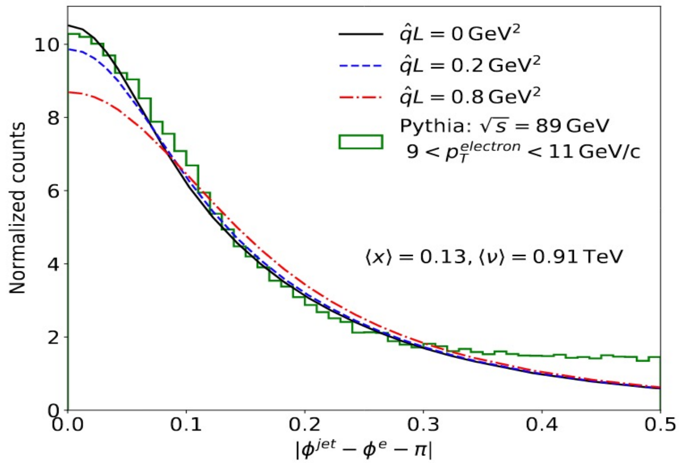
I. Vitev et al. (2002)

$$\langle p_T^2 \rangle = \frac{2\mu^2}{\lambda} L\xi \qquad \hat{q} = \frac{2\mu^2}{\lambda} = \quad 0.05 \; - 0.1 \; GeV^2/fm$$

For quarks. For gluons it is 2.25 times larger

The reason we see the Cronin effect extend to a few GeV is steeply falling spectra. Generally limited to small transverse momenta



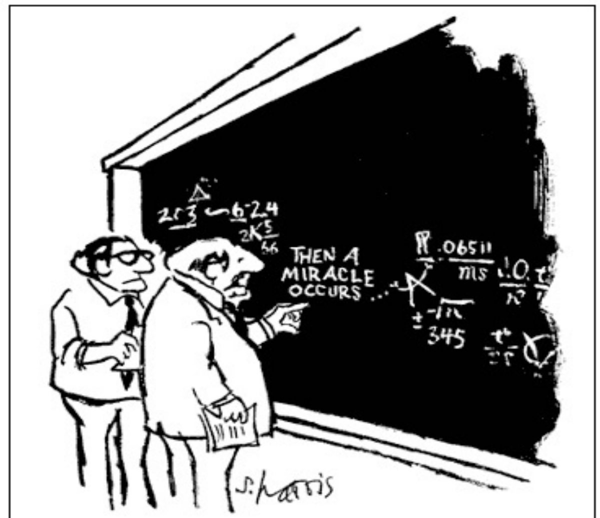


- Broadening of lepton jet correlations

Rather small effect. Perhaps twice smaller as indicated

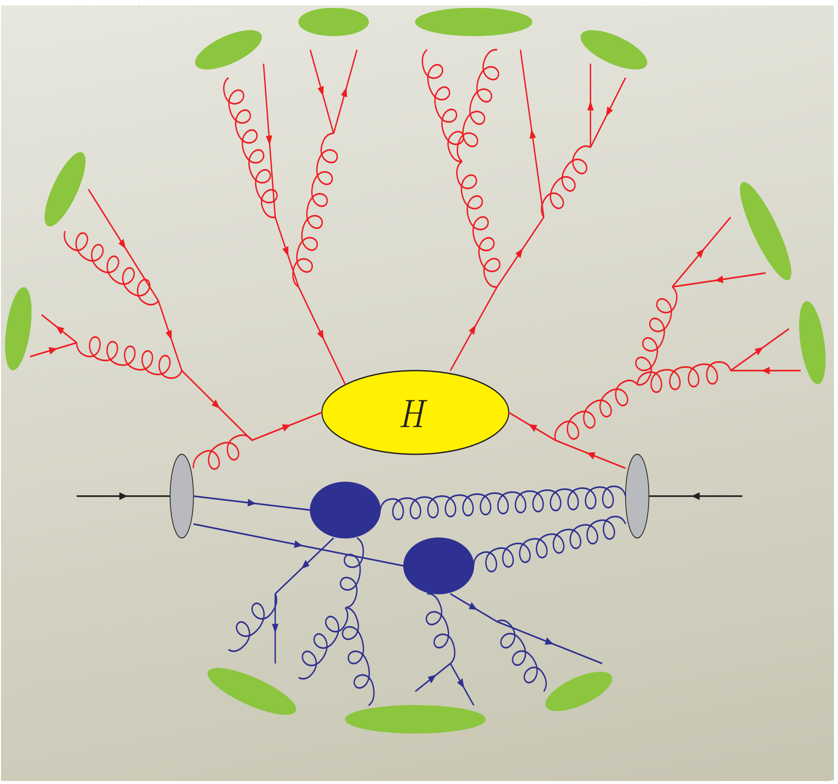
M. Arratia et al. (2019)

Medium-induced radiative corrections



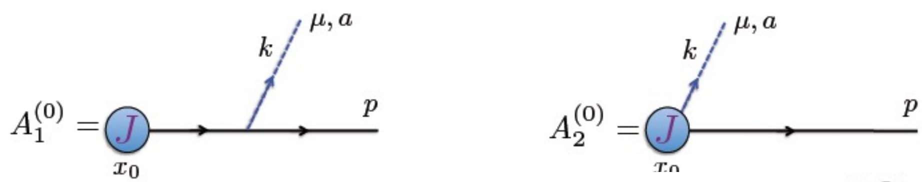
"I think you should be more explicit here in step two."

If not scattering and broadening then what?



- Splitting functions are related to beam (B) and jet (J) functions in SCET

$$
\begin{aligned}
A_{q\to qg} &= \langle J|\,T\,\bar{\chi}_n(x_0)\,e^{iS}\,|q(\boldsymbol{p})g(\boldsymbol{k})\rangle \\
A_{g\to q\bar{q}} &= \langle J|\,T\,\mathcal{B}^{\lambda c}(x_0)\,e^{iS}\,|q(\boldsymbol{p})\bar{q}(\boldsymbol{k})\rangle \\
A_{g\to gg} &= \langle J|\,T\,\mathcal{B}^{\lambda c}(x_0)\,e^{iS}\,|g(\boldsymbol{p})g(\boldsymbol{k})\rangle
\end{aligned}
$$

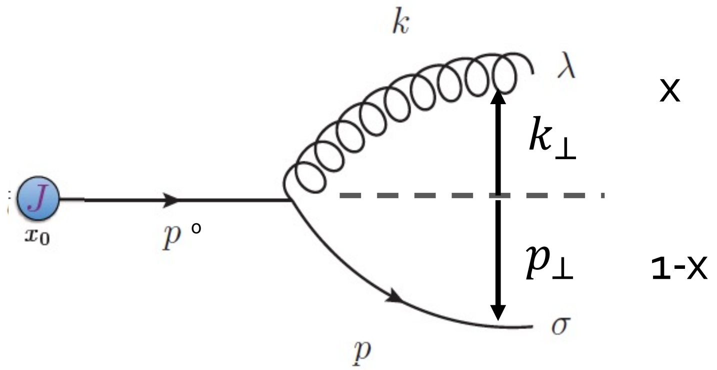


$$\Gamma_W^{\alpha,a}(k) = gT_r^a\,\frac{\bar{n}^\alpha}{k^+ + i\epsilon}$$

Gribov et al. (1972)
G. Altarelli et al. (1977)
Y. Dokshitzer (1977)

- In the vacuum we have the DGLAP splitting kernels that factorize from the hard scattering cross section and are process independent

Calculation of the quark splitting function



Chose physical polarization, lightcone gauge

$$\varepsilon_i^\mu(k) = \left(0, \frac{2\varepsilon_{i\perp}\cdot \mathbf{k}_\perp}{k^+}, \boldsymbol{\varepsilon}_{i\perp}\right), \quad i = 1,2$$

- Note: relative to standard notation x <-> 1-x
- This was done to make connection with the traditional energy loss approaches.

$$R_1^{(0)\mu,a} = i T^a \left(n^\mu + \frac{\gamma_\perp^\mu (\not{p}_\perp + \not{k}_\perp)}{\bar{n}\cdot(p+k)} + \frac{\not{p}_\perp \gamma_\perp^\mu}{\bar{n}\cdot p} - \frac{\not{p}_\perp (\not{p}+\not{k})_\perp}{\bar{n}\cdot p\ \bar{n}\cdot(p+k)} \bar{n}^\mu \right) i \frac{\bar{n}(p+k)}{(p+k)^2}$$

$$R_1^{(0)\mu,a}\varepsilon_\mu = -T^a \left[\frac{2\boldsymbol{A}_\perp^i}{\boldsymbol{A}_\perp^2} + \frac{x}{\boldsymbol{A}_\perp^2} \boldsymbol{A}_\perp^j \gamma_\perp^i \gamma_\perp^j \right] \boldsymbol{\varepsilon}_\perp^i$$

$$\boldsymbol{A}_\perp \equiv \boldsymbol{k}_\perp (1-x) - \boldsymbol{p}_\perp x ,$$

$$\frac{1}{d_R} |A_{Jq\to qg}|^2 = (1-x)\mathrm{Tr}\left(\frac{\not{n}}{2} p_0^+ J(0)\bar{J}(0) \right) \times 4g^2 C_F \left(1 - x + \frac{x^2}{2} \right) \frac{1}{\boldsymbol{k}_\perp^2} = |A_{Jq}|^2 \times |M_0^{\mathrm{rad}}|^2$$

Supplementing the 2 body phase space in the final state we can identify

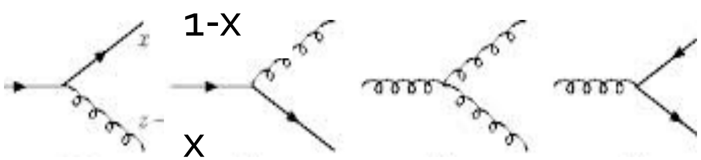
$$\frac{dN^g}{dx d^2\boldsymbol{k}_\perp} = C_F \frac{\alpha_s}{\pi^2} \frac{\left(1 - x + \frac{x^2}{2}\right)}{x} \frac{1}{\boldsymbol{k}_\perp^2}$$

- We can read off the splitting kernel (continuous part of it)

Diagonal splitting functions have singular contributions

Splitting kernel results

- Explicitly verified the gauge invariance and factorization in QCD



Reversed convention

- The singular pieces A, B can be obtained form flavor and momentum conservation sum rules

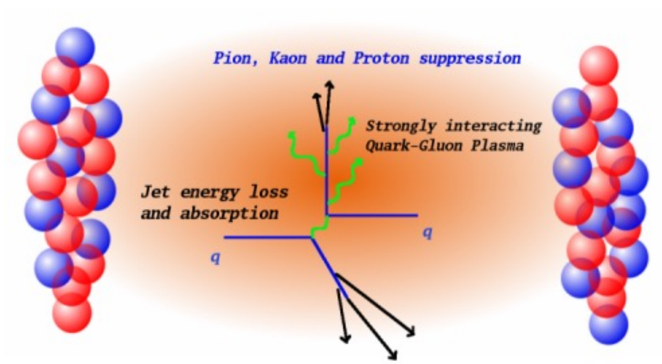
$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{q\to qg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1+(1-x)^2}{x}\frac{1}{\mathbf{k}_\perp^2}, \quad (\ldots]_+ + A\delta(x))$$

$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{g\to gg} = \frac{\alpha_s}{2\pi^2} 2C_A \Big(\frac{1-x}{x} + \frac{x}{1-x} + x(1-x)\Big)\frac{1}{\mathbf{k}_\perp^2}, \quad (\ldots]_+ + B\delta(x))$$

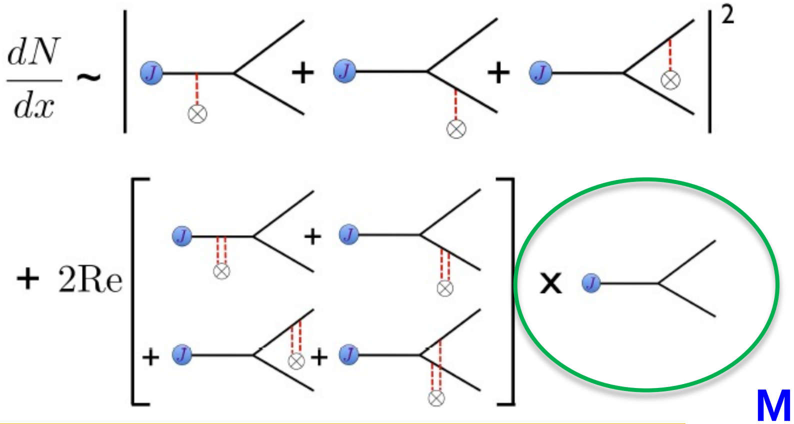
$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{g\to q\bar{q}} = \frac{\alpha_s}{2\pi^2} T_R \; \left(x^2 + (1-x)^2\right) \frac{1}{\mathbf{k}_\perp^2}$$

$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{q\to gq} = \left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{q\to qg} \quad (x \to 1-x)$$

What we want to compute is that



Medium–induced contribution to parton splitting



- First we note that the topology of all splittings is same

- The importance of formation time

- What this tells us is that processes take time - the splitting is not instantaneous. If the time for the splitting is comparable to the distance between the scattering centers we have interference – Landu –Pomeranchink-Migdal effect in QCD

M. Gyulassy et al. (1993)

Momenta in the propagators

$$\boldsymbol{A}_\perp = \boldsymbol{k}_\perp,\ \boldsymbol{B}_\perp = \boldsymbol{k}_\perp + x\boldsymbol{q}_\perp,\ \boldsymbol{C}_\perp = \boldsymbol{k}_\perp - (1-x)\boldsymbol{q}_\perp,$$

$$\boldsymbol{D}_\perp = \boldsymbol{k}_\perp - \boldsymbol{q}_\perp,$$

Interference phases or inverse formation times

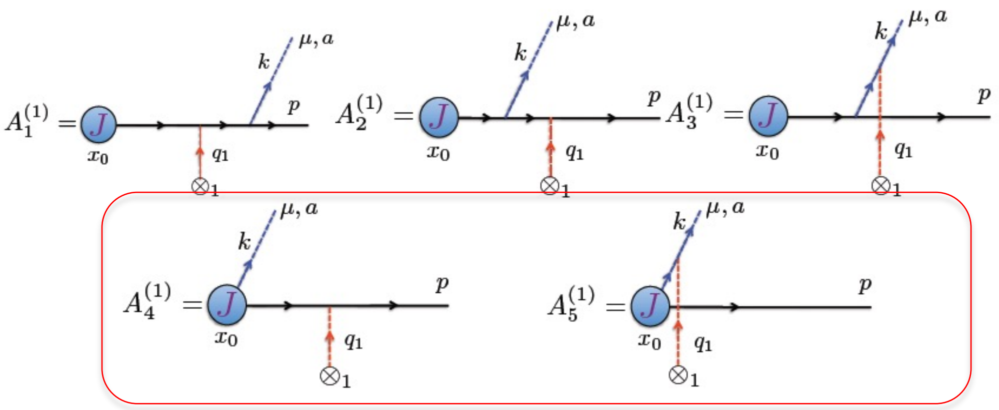
$$\Omega_1 - \Omega_2 = \frac{\boldsymbol{B}_\perp^2}{p_0^+ x(1-x)},\ \Omega_1 - \Omega_3 = \frac{\boldsymbol{C}_\perp^2}{p_0^+ x(1-x)},$$

$$\Omega_2 - \Omega_3 = \frac{\boldsymbol{C}_\perp^2 - \boldsymbol{B}_\perp^2}{p_0^+ x(1-x)},\ \Omega_4 = \frac{\boldsymbol{A}_\perp^2}{p_0^+ x(1-x)},$$

$$\Omega_5 = \frac{\boldsymbol{A}_\perp^2 - \boldsymbol{D}_\perp^2}{p_0^+ x(1-x)},$$

First order in opacity single Born Diagrams

In a moment we will discuss subtleties of the calculation



For the physical polarization vector The contribution for the last two diagrams vanishes

$$I_1^{(1)} = \int \frac{dq_1^-}{2\pi}\, e^{iq_1^-\delta z_1}\, \Delta_g(p+k, q_1) \ ,$$

$$I_2^{(1)} = \int \frac{dq_1^-}{2\pi}\, e^{iq_1^-\delta z_1}\, \Delta_g(p, q_1)\Delta_g(p+k, q_1)$$

$$I_3^{(1)} = \int \frac{dq_1^-}{2\pi}\, e^{iq_1^-\delta z_1}\, \Delta_g(k, q_1)\Delta_g(p+k, q_1)$$

$$I_1^{(1)} = -i\, e^{i\Omega_1\delta z_1} \ ,$$

$$I_2^{(1)} = \frac{i}{\Omega_1 - \Omega_2}\left(e^{i\Omega_1\delta z} - e^{i\Omega_2\delta z_1}\right)$$

$$I_3^{(1)} \equiv \frac{i}{\Omega_1 - \Omega_3}\left(e^{i\Omega_1\delta z} - e^{i\Omega_3\delta z_1}\right)$$

$$\Omega_1 = \Omega(p+k, q_1) = p^- + k^- - \frac{(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp - \boldsymbol{q}_{1\perp})^2 - i\varepsilon}{p^+ + k^+} \ ,$$

$$\Omega_2 = \Omega(p, q_1) = p^- - \frac{(\boldsymbol{p}_\perp - \boldsymbol{q}_{1\perp})^2 - i\varepsilon}{p^+} \ ,$$

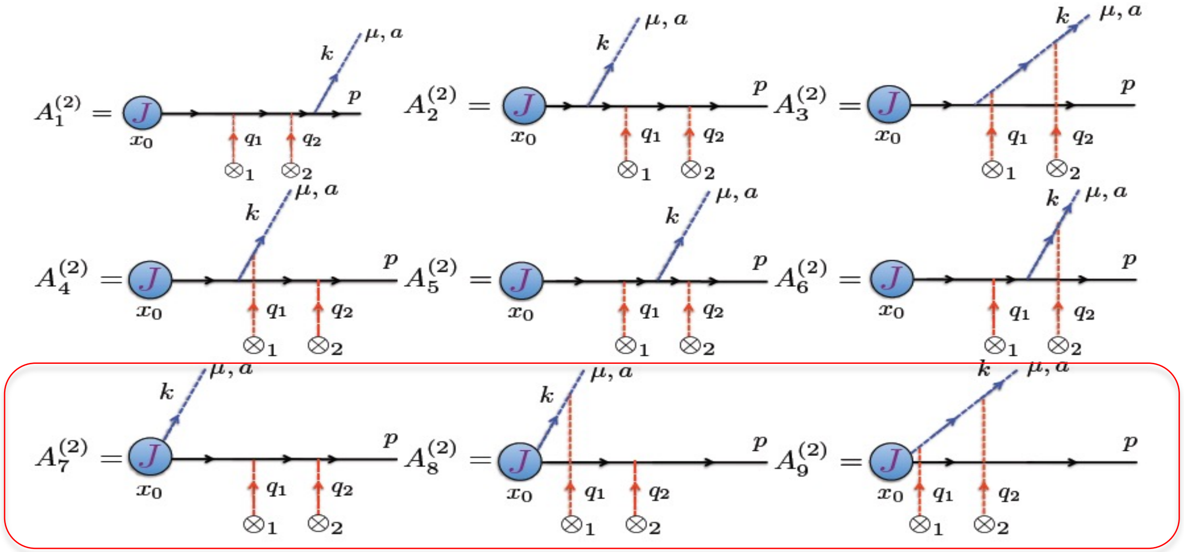
$$\Omega_3 = \Omega(k, q_1) = k^- - \frac{(\boldsymbol{k}_\perp - \boldsymbol{q}_{1\perp})^2 - i\varepsilon}{k^+} \ .$$

- Note that a collinear Wilson line appears in the R_ξ gauge

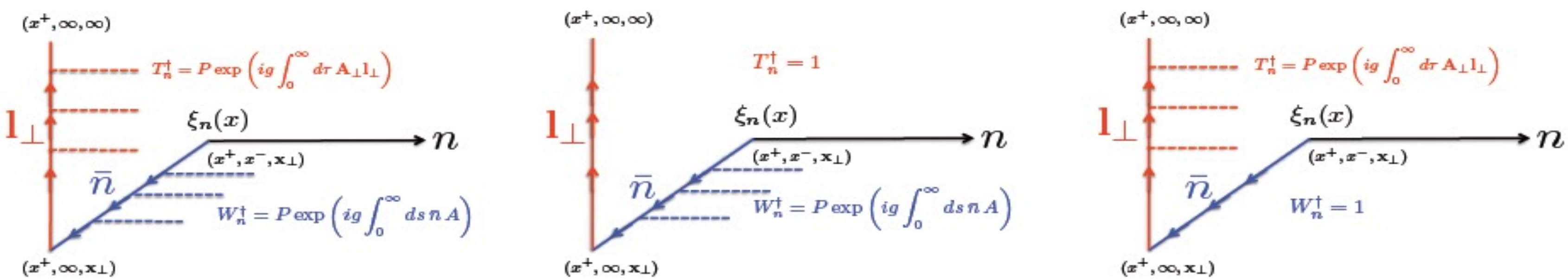
Main results: in-medium splitting

Double Born diagrams

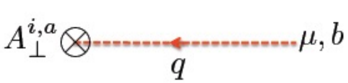
G. Ovanesyan et al. (2011)



- The lightcone gauge



- New Feynman rule



$$= \; i\,\delta^{ab}\,\frac{\bar{n}^\mu q^i}{q^2 + i\varepsilon} C_\infty^{(\mathrm{Pres})}\left(\frac{1}{q^+ + i\varepsilon} - \frac{1}{q^+ - i\varepsilon}\right)$$

A. Idilbi et al. (2010)

In-medium parton splitting and gauge independence

	R_ξ	A^+	Hyb.
W^+	✓	×	×
T_n	×	✓	×

G. Ovanesyan et al. , (2011)
G. Ovanesyan et al. , (2012)

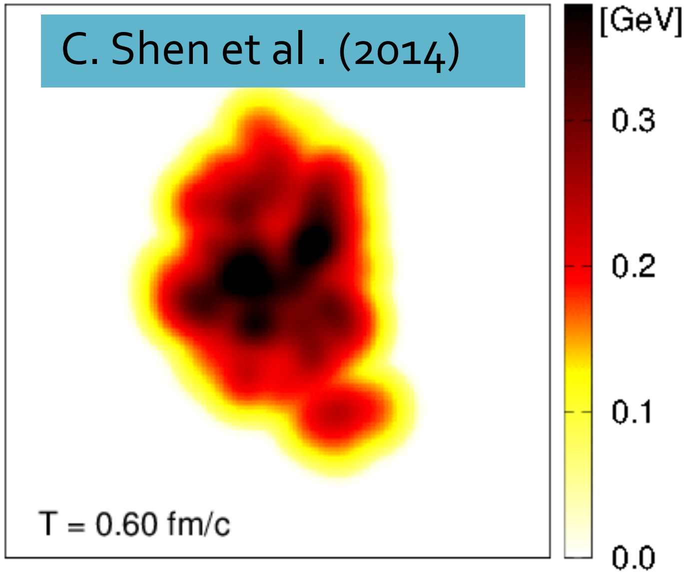
- The two sectors – the collinear and Glauber – decouple. One can simplify the calculations considerably by using the hybrid gauge

- **Proportional to the vacuum splitting**
- **Depend on the medium properties**
- **Vanish if there is no medium**
- **Explicitly have the LPM effect differentially**
- **Kinematics x, k d not decouple**

- Properties of in-medium splittings

$$\left(\frac{dN}{dxd^2\boldsymbol{k}_\perp}\right)_{q\to qg} = \frac{\alpha_s}{2\pi^2}C_F\frac{1+(1-x)^2}{x}\int\frac{d\Delta z}{\lambda_g(z)}\int d^2\mathbf{q}_\perp\frac{1}{\sigma_{el}}\frac{d\sigma_{el}^{\,\mathrm{medium}}}{d^2\mathbf{q}_\perp}\left[-\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\right)^2+\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}-\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right)\right.$$
$$\times(1-\cos[(\Omega_1-\Omega_2)\Delta z])+\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\cdot\left(2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}-\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\right)(1-\cos[(\Omega_1-\Omega_3)\Delta z])$$
$$+\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}(1-\cos[(\Omega_2-\Omega_3)\Delta z])+\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}-\frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2}\right)\cos[\Omega_4\Delta z]$$
$$\left.+\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\cdot\frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2}\cos[\Omega_5\Delta z]+\frac{1}{N_c^2}\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}-\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z])\right].$$

Improvements in physics & code




```
if (split_id==1)  //Quark--->Quark, Gluon
{
if (int_id==1)
{
  Vegas(NDIM, NCOMP, Integrand_qqgnocuts, USERDATA,
    EPSREL, EPSABS, verbose, SEED,
    MINEVAL, MAXEVAL, NSTART, NINCREASE, NBATCH,
    GRIDNO, STATEFILE,
    &neval, &fail, integral, error, prob);
}

if (int_id==2)
{
  Suave(NDIM, NCOMP, Integrand_qqgnocuts, USERDATA,
    EPSREL, EPSABS, verbose | LAST, SEED,
    MINEVAL, MAXEVAL, NNEW, FLATNESS,
    STATEFILE,
    &nregions, &neval, &fail, integral, error, prob);
}

if (int_id==3)
{
  Divonne(NDIM, NCOMP, Integrand_qqgnocuts, USERDATA,
    EPSREL, EPSABS, verbose, SEED,
    MINEVAL, MAXEVAL, KEY1, KEY2, KEY3, MAXPASS,
    BORDER, MAXCHISQ, MINDEVIATION,
    NGIVEN, LDXGIVEN, NULL, NEXTRA, NULL,
    STATEFILE,
    &nregions, &neval, &fail, integral, error, prob);
}

if (int_id==4)
{
  Cuhre(NDIM, NCOMP, Integrand_qqgnocuts, USERDATA,
    EPSREL, EPSABS, verbose | LAST,
    MINEVAL, MAXEVAL, KEY,
    STATEFILE,
    &nregions, &neval, &fail, integral, error, prob);
}
}

if (split_id==2)  //Gluon--->Gluon, Gluon
{
if (int_id==1)
{
  Vegas(NDIM, NCOMP, Integrand_gggnocuts, USERDATA,
    EPSREL, EPSABS, verbose, SEED,
    MINEVAL, MAXEVAL, NSTART, NINCREASE, NBATCH,
    GRIDNO, STATEFILE,
< 2018/01/27-14:27 col:1 ascii:32 pos:71863 lin:3292,4378 74%
```

8. boram@pn1615176.lanl.gov: ~/work/qcd/hydrodynamics/splitting_funct..

```
    if(cut_id == 1)
    {
      switch(split_id)
      {
        case 1:
          func = &Integrand_qqgnocuts; break;
        case 2:
          func = &Integrand_gggnocuts; break;
        case 3:
          func = &Integrand_gqqnocuts; break;
        case 4:
          func = &Integrand_qgqnocuts; break;
        default:
          printf("Error: Unknown split_id %d\n", split_id);
          exit(0);
      } // switch(split_id)
    } // cur_id == 1
< 2018/01/27-14:28 col:20 ascii:49 pos:44971 lin:1459,1631 91%
```

Refactoring

- Code is **restructured** (in C++) and shortened (**24K → 8K lines**). 20x speed improvement

Effective incorporation of simulated QGP medium

- Reduced overhead for calling QGP medium grid function. 2x speed improvement

Efficient on-node parallelization

- New parallelization shows much better scaling 10x speed improvement

Overall improvement:

18 days → 1 hour

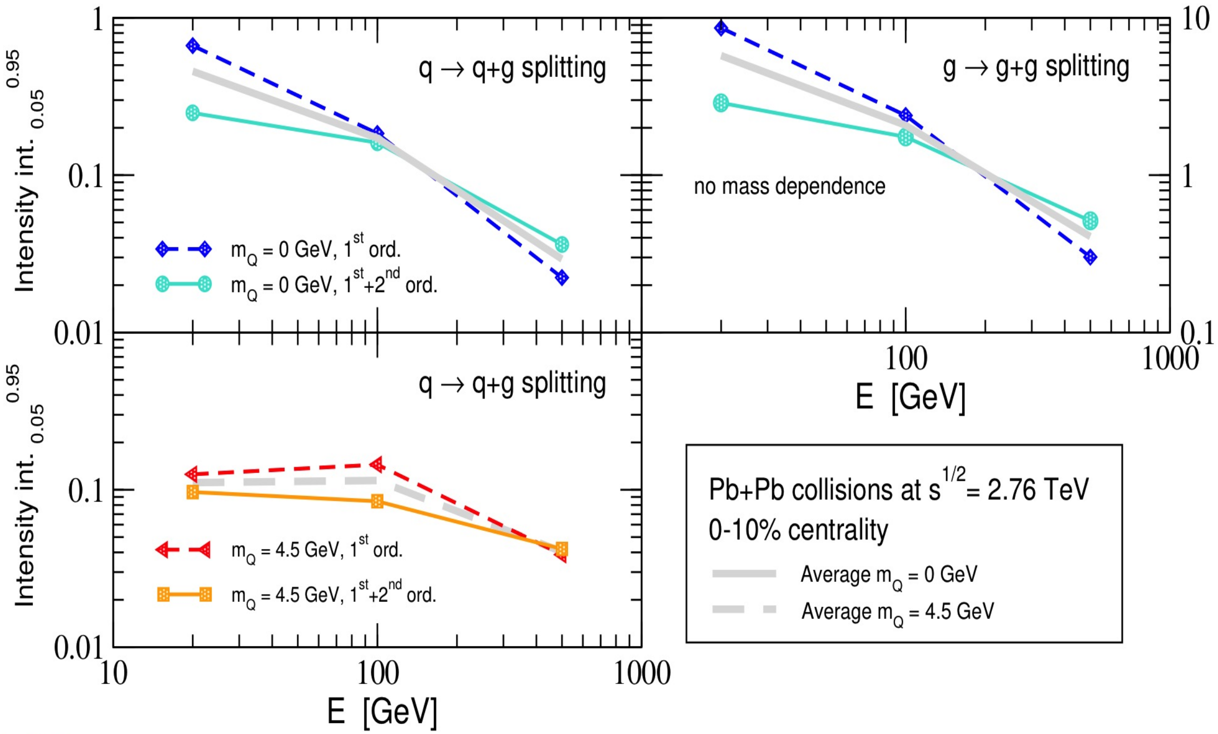
Medium-induced splitting intensity

Porting to code

- Results are directly exported from Mathematica to C++

Challenges

- Arise from larger number of evaluations



$$\mathcal{I}_{x_{\min}}^{x_{\max}} = \int_{x_{\min}}^{x_{\max}} dx \int d^2k \; x \frac{dN}{d^2k \, dx}$$

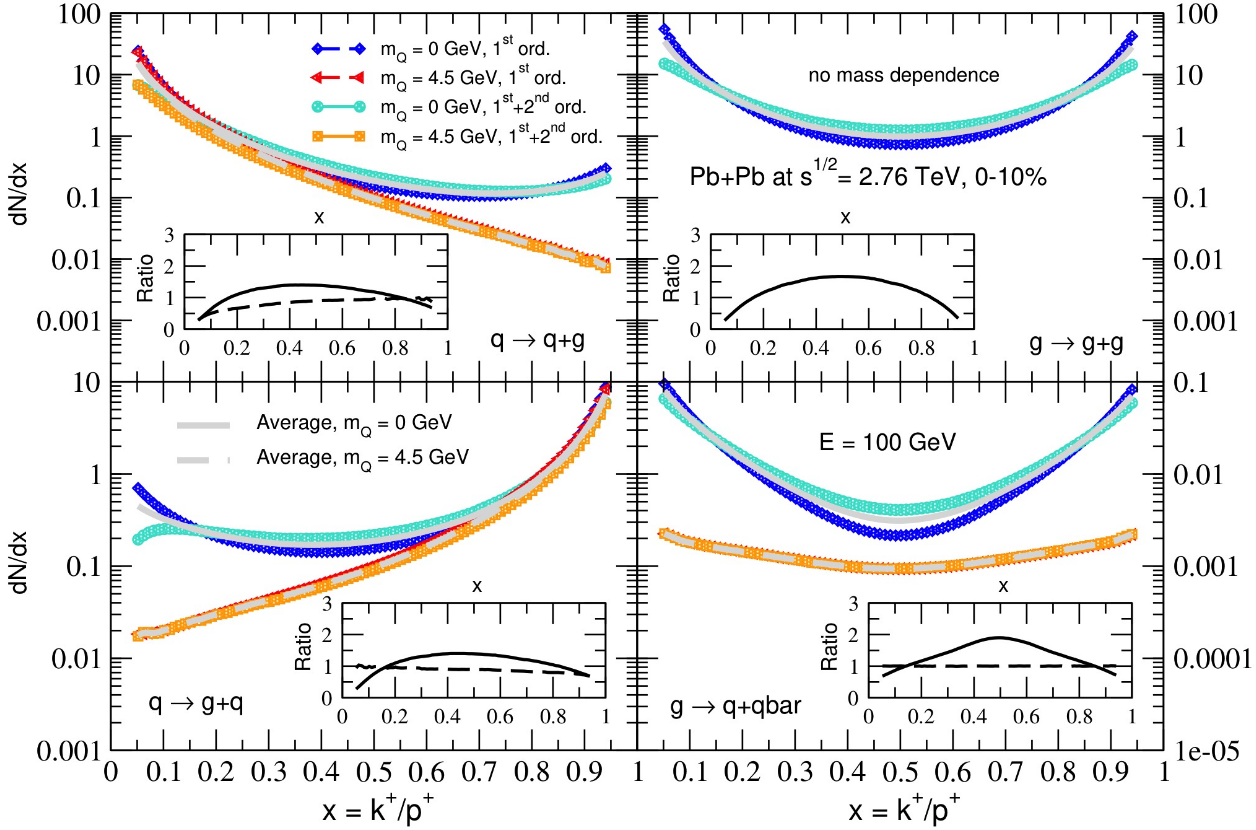
Energy loss – not a well defined concept for parton shower processes - define splitting intensity

- The main result is a change in the energy dependence of the splitting intensity – smother, or more slowly varying with E (understand jet modification with p_T)

Differential branching spectra

- Reduction of small-x and large-x probabilities (assymptotics modulated by thermal mass)
- Enhancement of democratic branching (x~0.5)

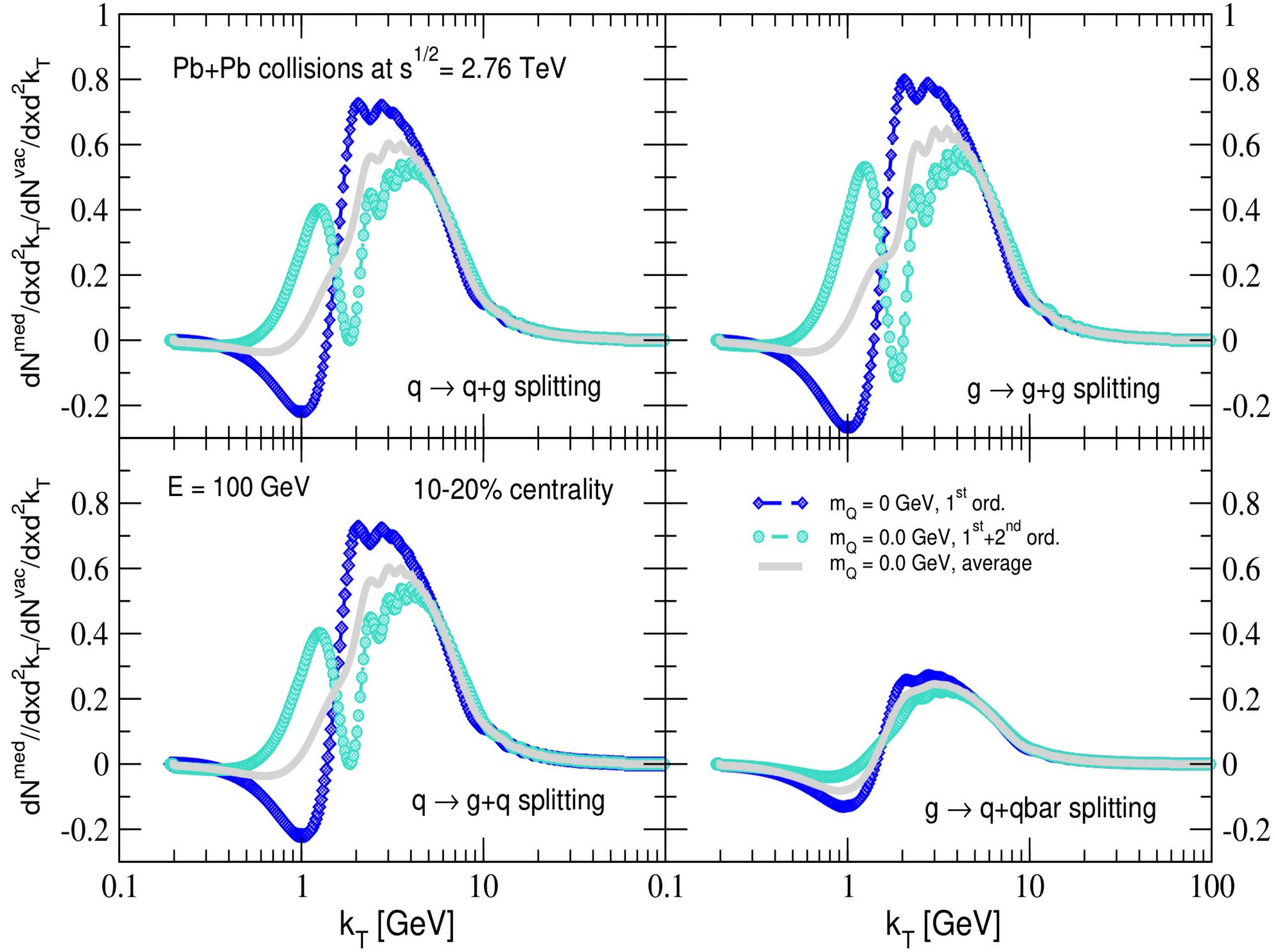
Parton showers in matter are <u>softer</u> than the ones in the vacuum



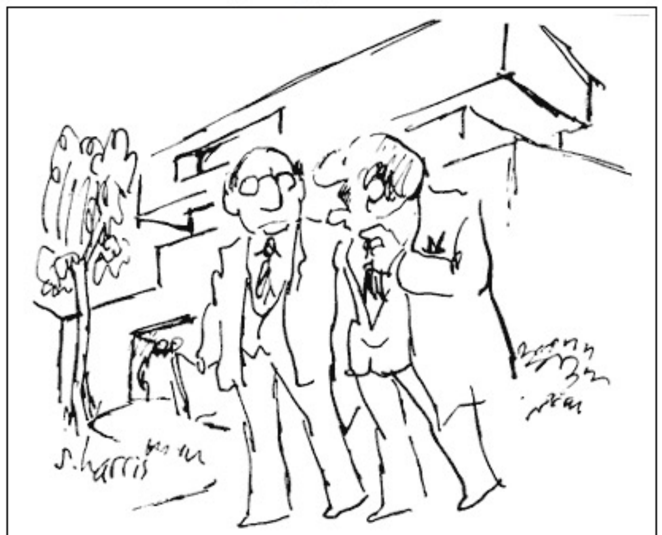
Differential branching spectra

Parton showers in matter are <u>broader</u> than the ones in the vacuum

- Broder angular enhancement region
- Oscillating series – the average of 1st and 1st+2nd order-candidate for pheno.



Applications



"I'm firmly convinced that behind every great man is a great computer."

In-medium evolution of the fragmentation functions - hadrons

Medium induced scaling violations

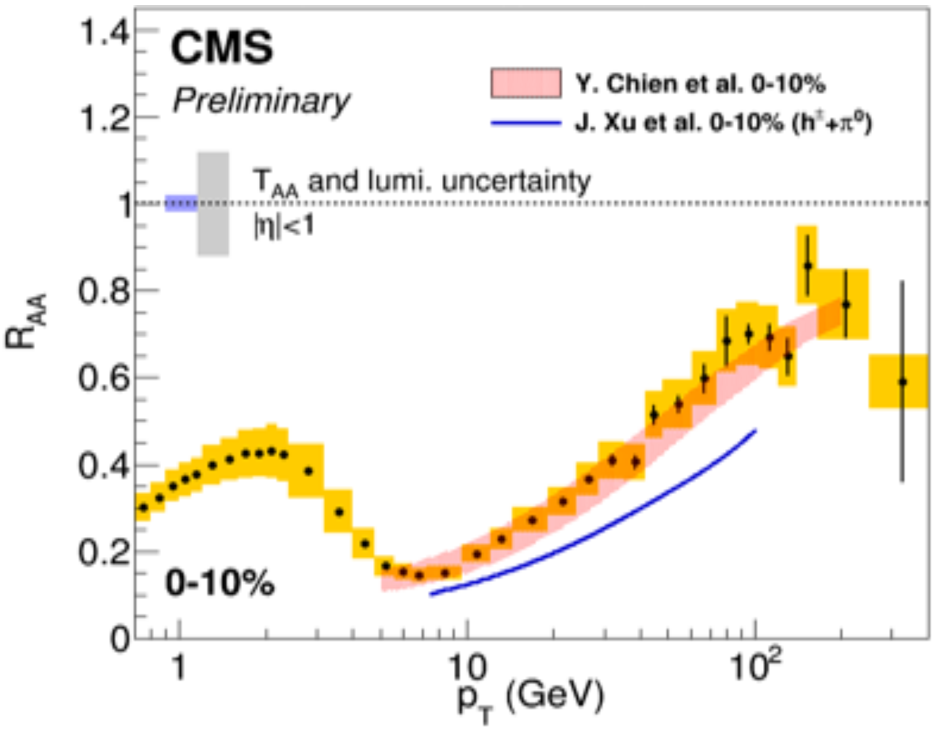
Y.T-Chien et al. (2015)

$$\frac{\mathrm{d}D_q(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{q\to qg}(z',Q)D_q\left(\frac{z}{z'},Q\right) + P_{q\to gq}(z',Q)D_g\left(\frac{z}{z'},Q\right)\right\},$$

$$\frac{\mathrm{d}D_{\bar q}(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{q\to qg}(z',Q)D_{\bar q}\left(\frac{z}{z'},Q\right) + P_{q\to gq}(z',Q)D_g\left(\frac{z}{z'},Q\right)\right\},$$

$$\frac{\mathrm{d}D_g(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{g\to gg}(z',Q)D_g\left(\frac{z}{z'},Q\right)\right.$$

$$\left. + P_{g\to q\bar q}(z',Q)\left(D_q\left(\frac{z}{z'},Q\right) + \bar q \text{ term }\right)\right\}.$$



Implement medium –induced splittings as corrections to vacuum evolution

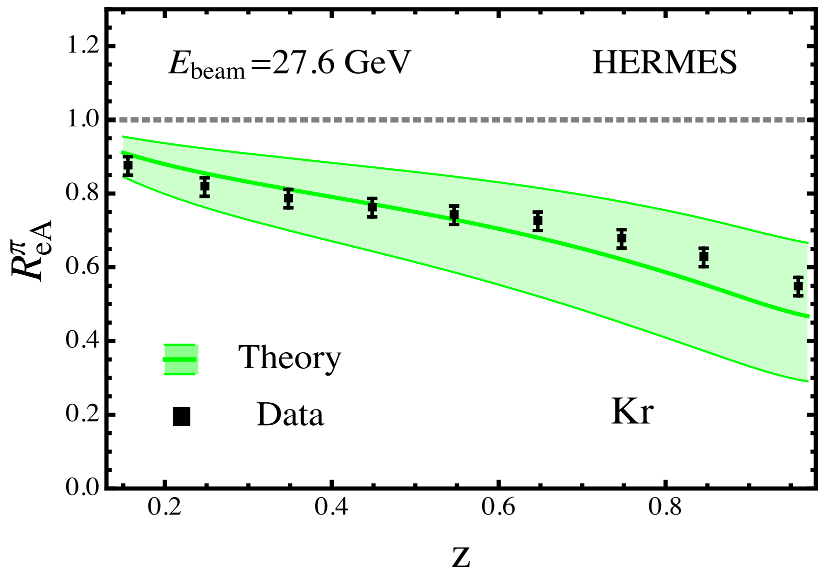
Predictions - very good description of data at 2.76 TeV

$$D^{\text{med.}}_{h/c}(z,Q) = D_{h/c}(z,Q)e^{-[n(z)-1]\left\langle\frac{\Delta E}{E}\right\rangle_z - \langle \tilde{N^g}\rangle_z}$$

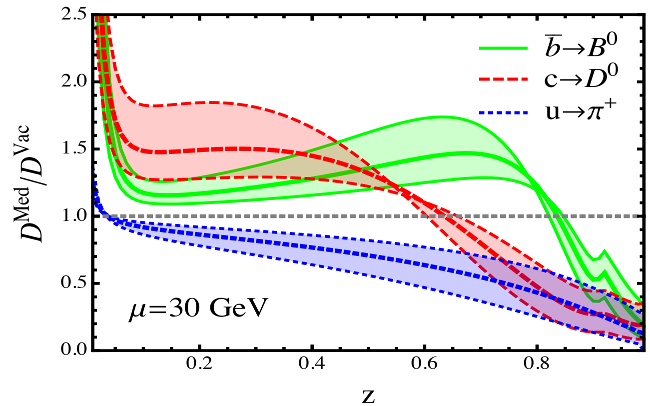
Predictions for e+A at the EIC

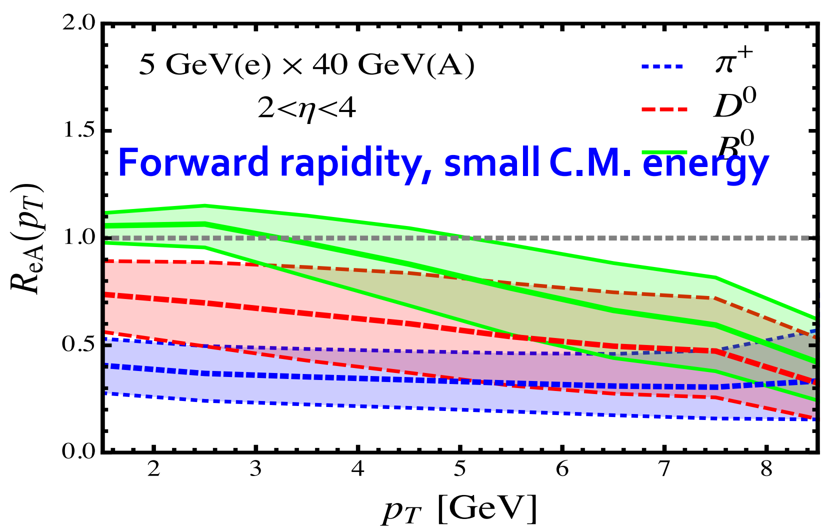
Vacuum splitting functions provide correction to vacuum showers and correspondingly modification to DGLAP evolution for FFs

$$R^{\pi}_{eA}(\nu, Q^2, z) = \frac{\left.\frac{N^{\pi}(\nu,Q^2,z)}{N^{e}(\nu,Q^2)}\right|_A}{\left.\frac{N^{\pi}(\nu,Q^2,z)}{N^{e}(\nu,Q^2)}\right|_D}$$



Z. Liu et al. (2020)





Transport properties – q-hat = 0.05 GeV²/fm
Still significant effects

Jet production

Z. Kang et al. (2016)

L. Dai et al. (2016)

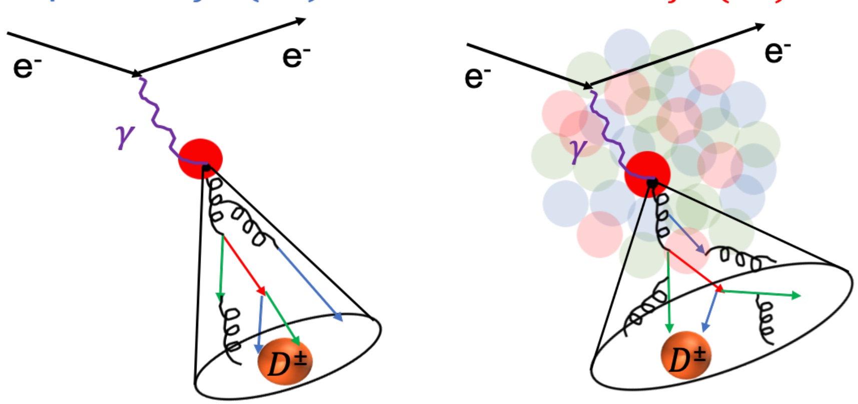
A useful modern way (though not unique) to calculate jet cross sections

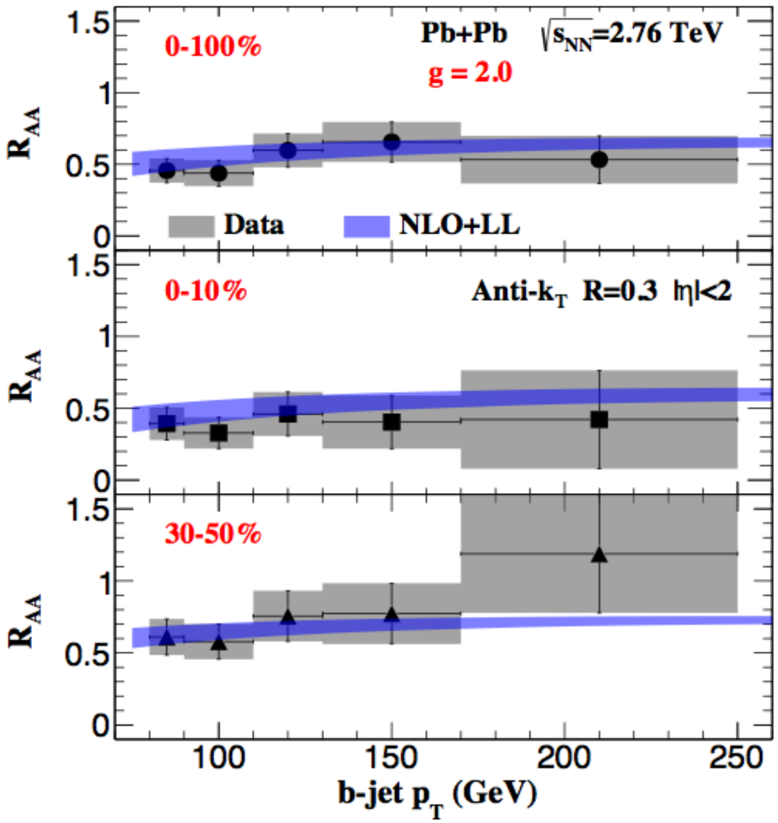
Factorization formula

$$E_J \frac{d^3\sigma^{lN\to jX}}{d^3P_J} = \frac{1}{S}\sum_{i,f}\int_0^1 \frac{dx}{x}\int_0^1 \frac{dz}{z^2} f_{i/N}(x,\mu) \times \hat{\sigma}^{i\to f}(s,t,u,\mu) J_f(z, p_T R, \mu) \,,$$

$$\mu_J = \omega_J \tan\frac{\mathcal{R}}{2} = (2p_T\cosh\eta)\tan\left(\frac{R}{2\cosh\eta}\right) \approx p_T R$$

$e^- + \mathrm{p} \to e^- + jet(D^\pm) + X$ $\quad$ $e^- + \mathrm{Au} \to e^- + jet(D^\pm) + X$



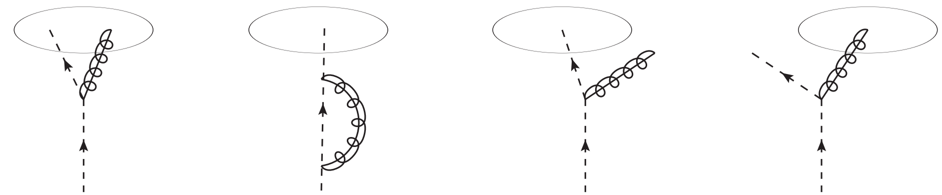


H. Li et al. (2019)

Large factor of 2 suppression for jets. Light jets and heavy jets

Evaluating the in-medium jet function

- Can we formulate the evaluation of the jet function in a way suitable for numerical implementation



(B) (C) (D) (E)

Z. Kang et al. (2017)

$$(\mathrm{B}) = \delta(1-z)\int_0^1 dx \int_0^{x(1-x)\omega\tan(R/2)} dq_\perp P_{qq}(x,q_\perp)$$

$$(\mathrm{C}) = -\delta(1-z)\int_0^1 dx \int_0^{\mu} dq_\perp P_{qq}(x,q_\perp)$$

Sum rules

$$(\mathrm{D}) = \int_{z(1-z)\omega\tan(R/2)}^{\mu} dq_\perp P_{qq}(z,q_\perp)$$

$$(\mathrm{E}) = \int_{z(1-z)\omega\tan(R/2)}^{\mu} dq_\perp P_{gq}(z,q_\perp)$$

Can be combined.

$$\frac{d\sigma^{pp\to \mathrm{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes J_c$$

NB has to be understood in the sense of convolution

$$J_q^{\mathrm{med},(1)}(z,\omega R,\mu) = \left[\int_{z(1-z)\omega\tan(R/2)}^{\mu} dq_\perp P_{qq}(z,q_\perp)\right]_+ + \int_{z(1-z)\omega\tan(R/2)}^{\mu} dq_\perp P_{gq}(z,q_\perp)\,.$$

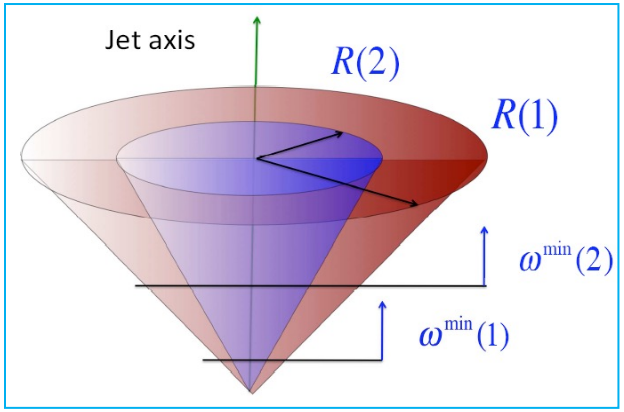
- Stable in numerical implementation
- Similarly for gluon jets

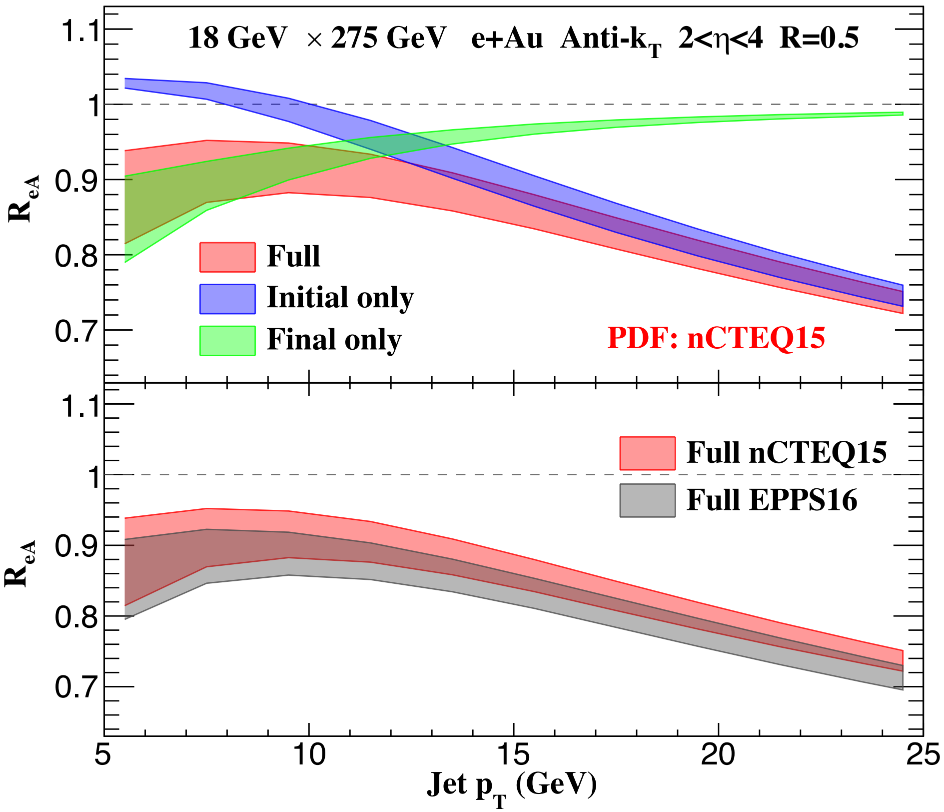
Jet results at the EIC

- The physics of reconstructed jet modification

H. Li et al. (2020)

$$R_{\mathrm{eA}}(R) = \frac{1}{A}\frac{\int_{\eta 1}^{\eta 2} d\sigma/d\eta dp_T\big|_{e+A}}{\int_{\eta 1}^{\eta 2} d\sigma/d\eta dp_T\big|_{e+p}}$$





Two types of nuclear effect play a role

- **Initial-state effects parametrized in nuclear parton distribution functions or nPDFs**
- **Final-state effects from the interaction of the jet and the nuclear medium – in-medium parton showers and jet energy loss**

- Net modification 20-30% even at the highest CM energy
- E-loss has larger role at lower p_T. The EMC effect at larger p_T

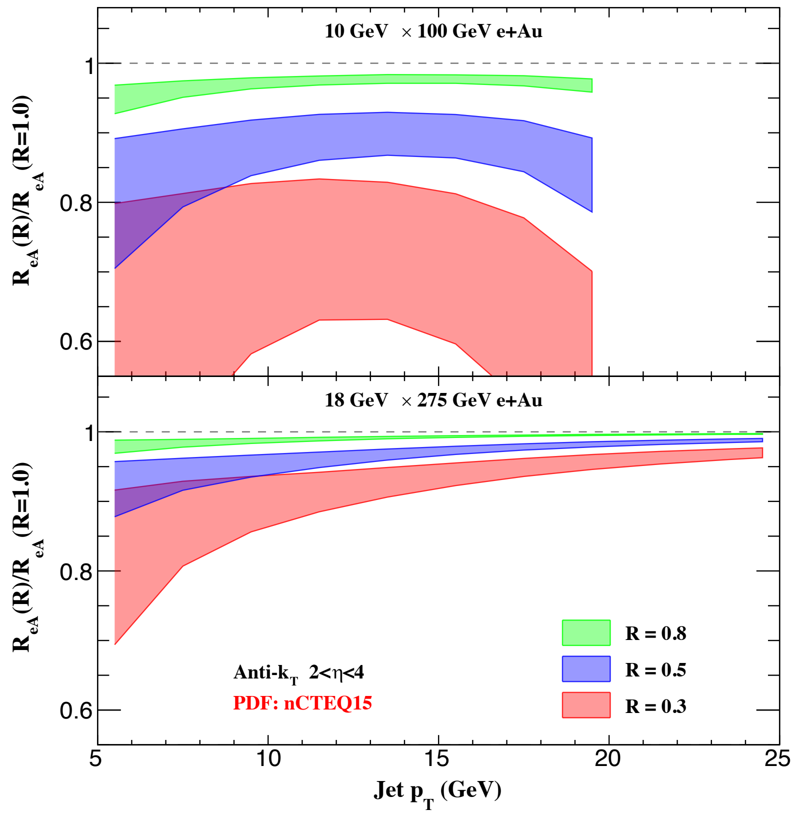
Separating initial-state from final-state effects at EIC

A key question – will benefit both nPDF extraction and understanding hadronization / nuclear matter transport properties - how to separate initial-state and final-state effects?

Define the ratio of modifications for 2 radii (it is a double ratio)

$$R_R = R_{eA}(R)/\ R_{eA}(R=1)$$

- Jet energy loss effects are larger at smaller center of mass energies (electron-nuclear beam combinations)
- Effects can be almost a factor of 2 for small radii. Remarkable as it approaches magnitudes observed in heavy ion collisions (QGP)



H. Li et al. (2020)

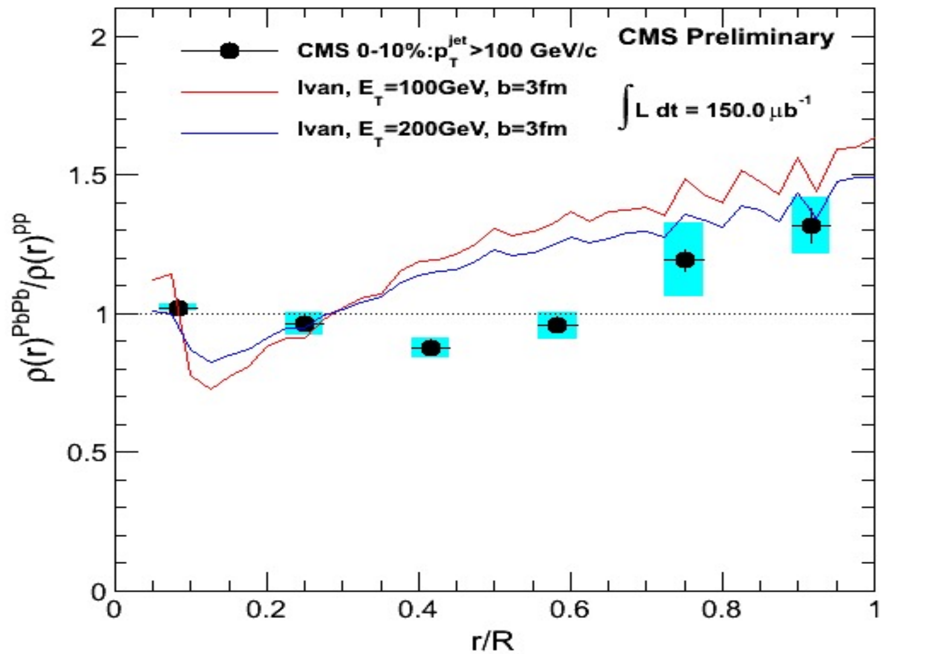
Initial-state effects are successfully eliminated

Applications of $SCET_G$ to jet shapes

- Jet shapes reflect the energy density inside the jet and the structure of the parton shower

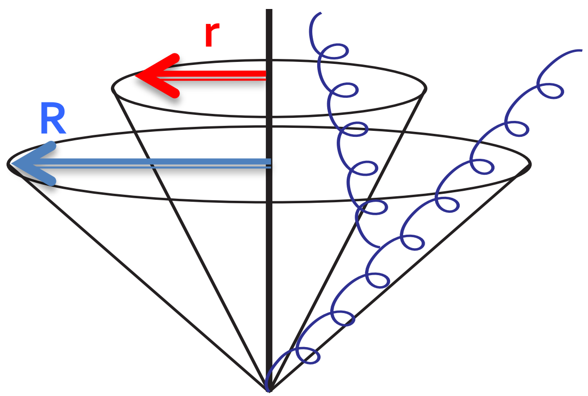
$$\Psi_{\rm int}(r;R) = \frac{\sum_i (E_T)_i \Theta(r-(R_{\rm jet})_i)}{\sum_i (E_T)_i \Theta(R-(R_{\rm jet})_i)} ,$$

$$\psi(r;R) = \frac{d\Psi_{\rm int}(r;R)}{dr} .$$



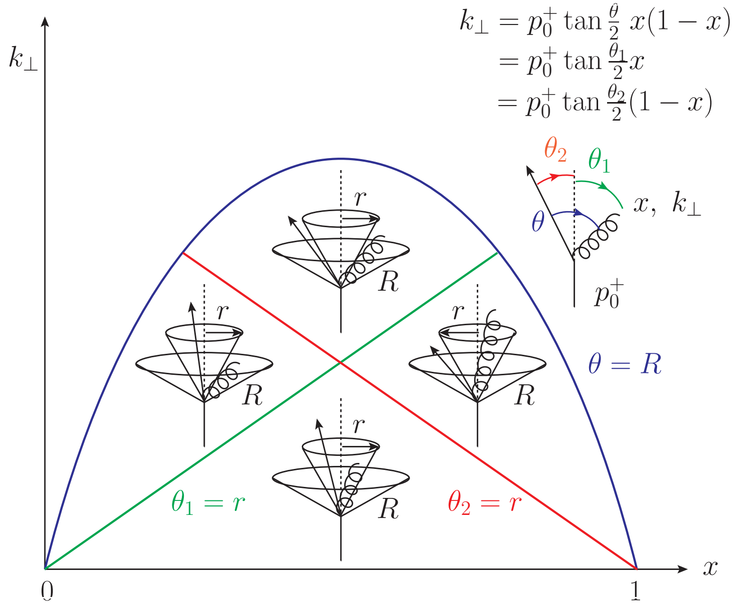
- First proposed as an observable that can test the understanding of the quenching of reconstructed jets and the

- Predicted in the energy loss approach ~5 years before measurement



I. Vitev et al. (2008)

Medium-modified jet shapes at NLL



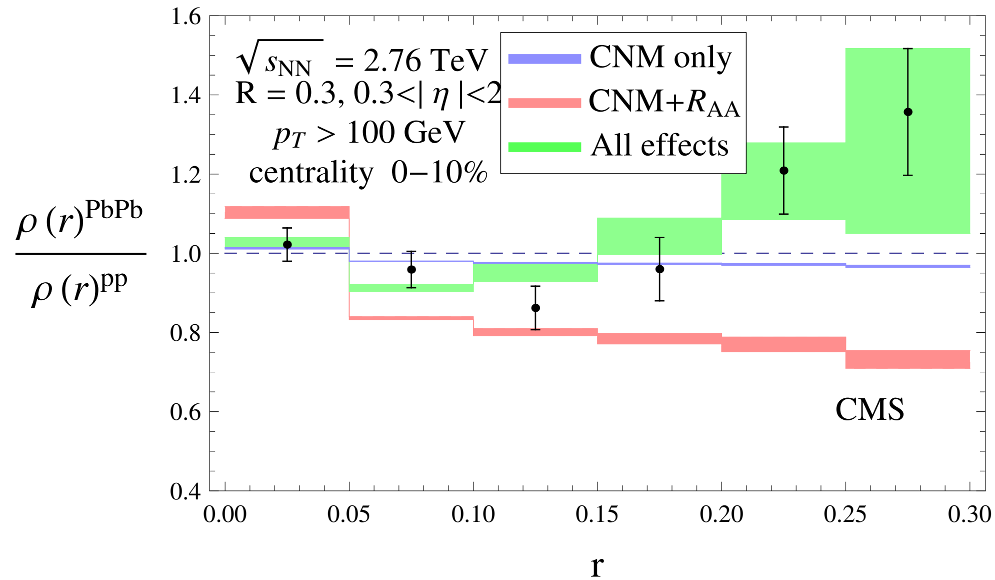
$$E_r(x, k_\perp) = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4$$

Measurement operator – tells us how the above configurations contribute energy to J (jet function)

- One can evaluate the jet energy functions from the splitting functions

$$J^i_{\omega,E_r}(\mu) = \sum_{j,k} \int_{PS} dx dk_\perp \mathcal{P}_{i\to jk}(x,k_\perp) E_r(x,k_\perp)$$

$$J_{\omega,E_r}(\mu) = J^{vac}_{\omega,E_r}(\mu) + J^{med}_{\omega,E_r}(\mu).$$

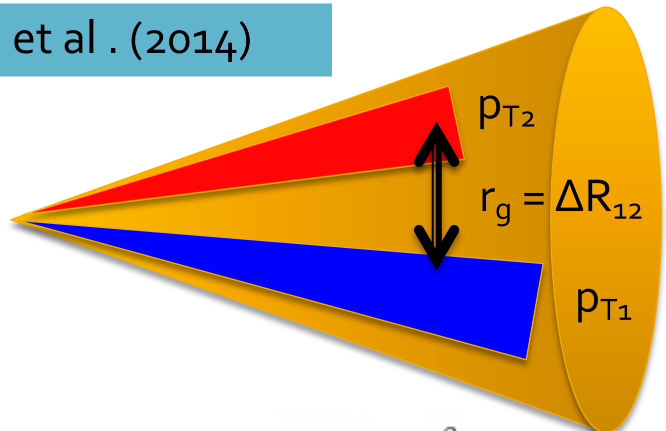


- First quantitative pQCD/SCET description of jet shapes in HI

Groomed soft dropped distributions in SCET_G

- Groomed jet distribution using "soft drop"

A. Larkoski et al . (2014)



$$z_g = \frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\rm cut} \left(\frac{\Delta R_{12}}{R_0}\right)^{\beta}$$

The great utility of these new distributions: probe the early time dynamics / splitting

$$\frac{dN_j^{\rm vac,MLL}}{dz_g d\theta_g} = \sum_i \left(\frac{dN^{\rm vac}}{dz_g d\theta_g}\right)_{j\to i\bar{i}} \underbrace{\exp\left[-\int_{\theta_g}^{1} d\theta \int_{z_{\rm cut}}^{1/2} dz \sum_i \left(\frac{dN^{\rm vac}}{dz d\theta}\right)_{j\to i\bar{i}}\right]}_{\text{Sudakov Factor}}$$

Directly proportional to the splitting functions, + resummation for small angles

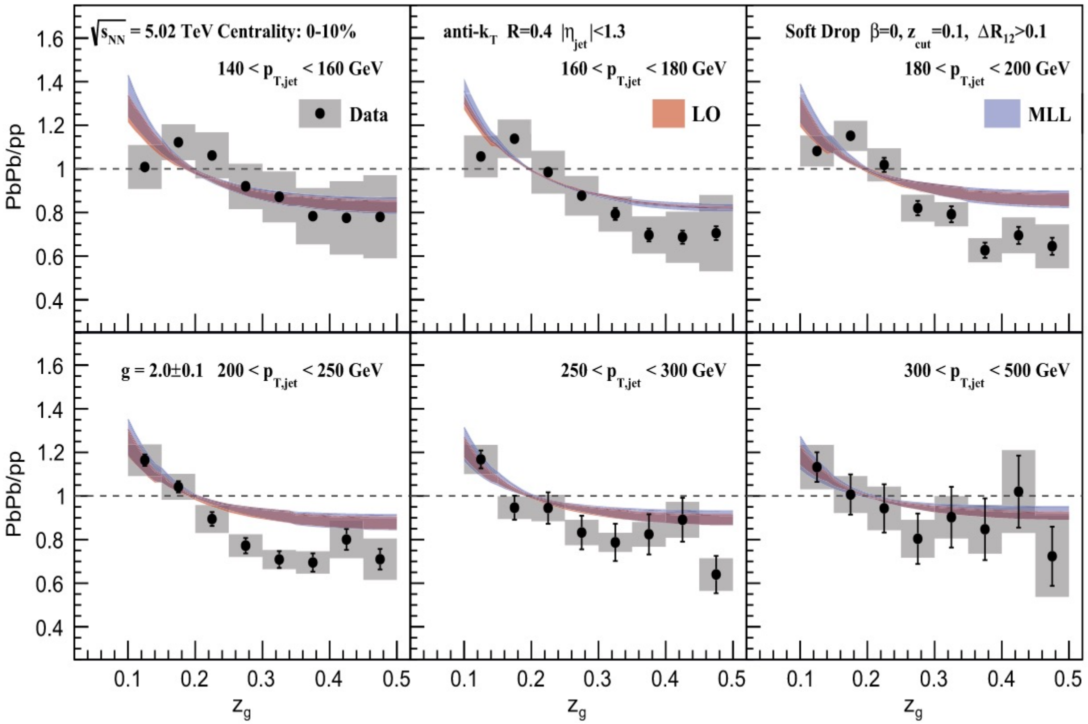
$$\tau_{\rm br}[{\rm fm}] = \frac{0.197\ {\rm GeV\ fm}}{z_g(1-z_g)\,\omega[{\rm GeV}]\,\tan^2(r_g/2)}$$

Typical situation: E=200 GeV, r_g = 0.1

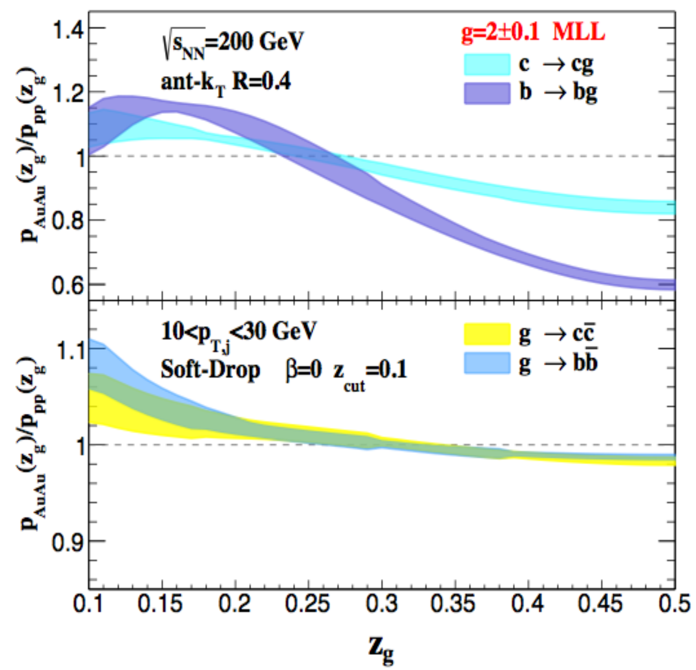
Branching time < 2 fm for z_g studied

Y. T. Chien et al . (2016)

Heavy flavor jet substructure



H. Li et al. (2018)



Analytic predictions for low jet momenta

$$\frac{p_{med}^{Q\to Qg}(z_g)}{p_{pp}^{Q\to Qg}(z_g)} \sim \frac{1}{z_g^2}, \quad \frac{p_{med}^{j\to i\bar{i}}(z_g)}{p_{pp}^{j\to i\bar{i}}(z_g)} \sim \frac{1}{z_g}, \quad \frac{p_{med}^{g\to Q\bar{Q}}(z_g)}{p_{pp}^{g\to Q\bar{Q}}(z_g)} \sim \text{const.}$$

- A unique inversion of the mass hierarchy of jet quenching effects,
- Can be used to constrain the still not well understood dead cone effect in matter

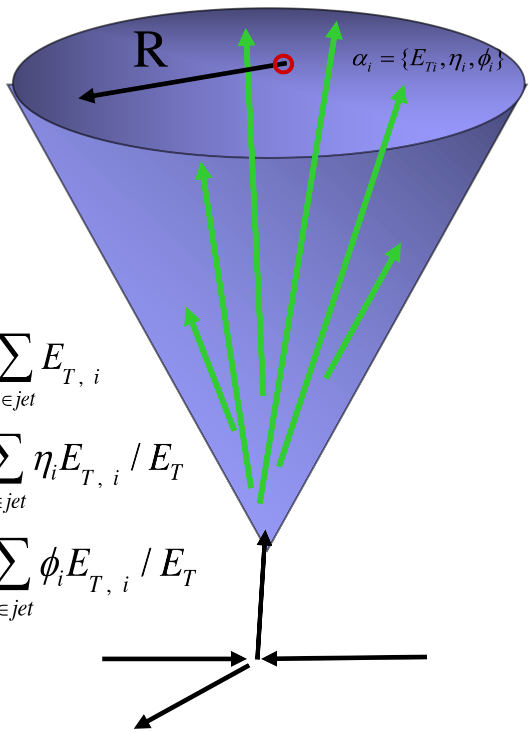
Summary

- Learned about dense matter. The need to include a new mode in SCET to describe parton/jet interactions in matter
- Learned about transverse momentum broadening in dense matter. Broader transverse momentum distributions in reactions with nuclei. Effects limited to low transverse momenta
- Learned about medium induced radiative corrections. Characteristics of parton shower – broader and softer than the ones in the vacuum
- Learned about phenomenological applications – suppression of hadron and jet cross sections. Modification of jet substructure observables – jet shapes, jet splitting functions, jet fragmentation functions, jet charge

Jet definitions and jet finding algorithms

- Jets: collimated showers of energetic particles that carry a large fraction of the energy available in the collisions

G. Sterman, S. Weinberg (1977)



$$E_T = \sum_{i \in jet} E_{T,\,i}$$

$$\eta = \sum_{i \in jet} \eta_i E_{T,\,i} / E_T$$

$$\phi = \sum_{i \in jet} \phi_i E_{T,\,i} / E_T$$

$$R = \sqrt{(\eta - \eta_{jet})^2 + (\phi - \phi_{jet})^2}$$

- Jet finding algorithms [have to satisfy collinear and infrared safety]:

1) Successive recombination algorithms

   a) k_t algorithm

   b) anti-k_t algorithm

S. Ellis et al. (1993)

2) Iterative cone algorithms:

   a) cone algorithm with "seed": CDF, D0

   b) "seedless" cone algorithm

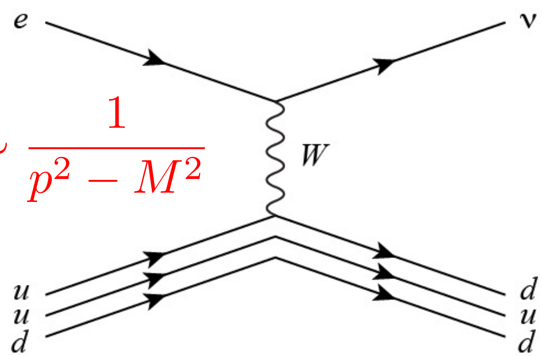
   c) midpoint cone algorithm

G. Salam et al. (2007)

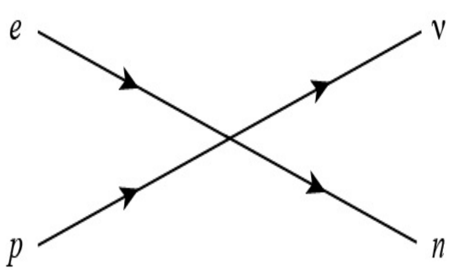
The Fermi interaction

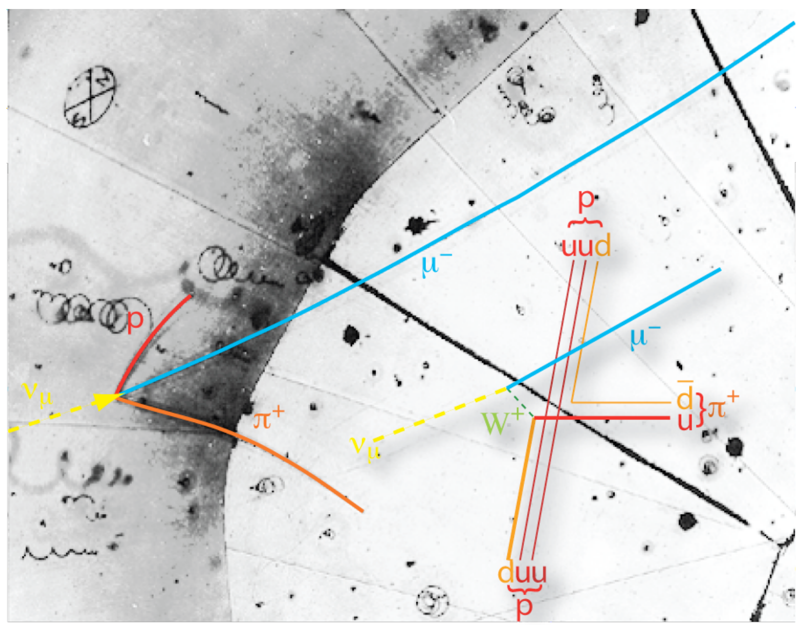
- The first, probably best known, effective theory is the Fermi interaction

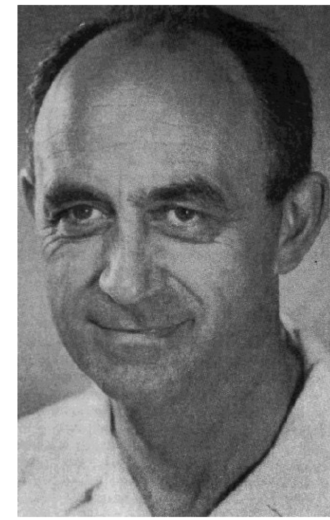
$$\sim \frac{1}{p^2 - M^2}$$



$$p \ll M$$



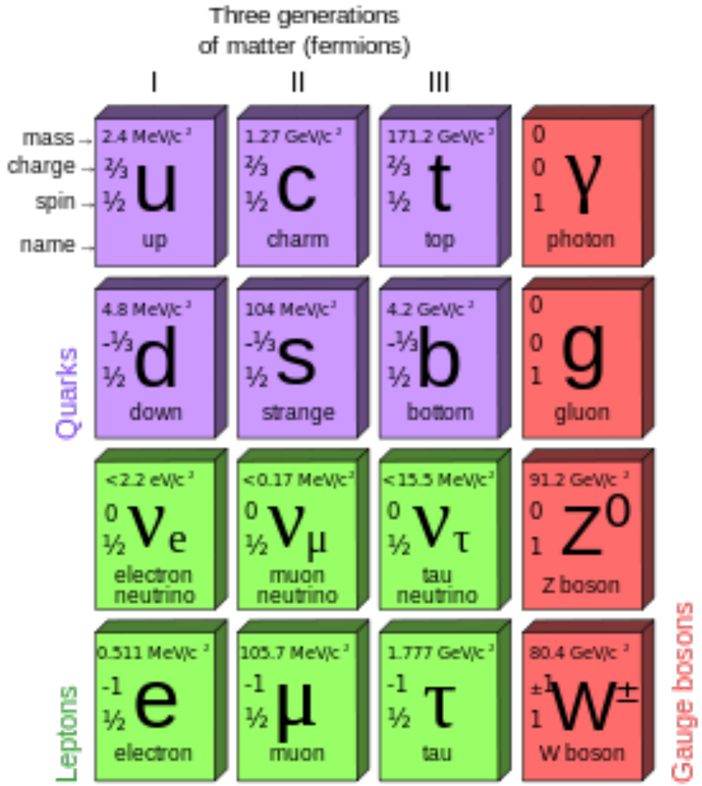




E. Fermi
(Nobel Prize)

- First direct observation of the neutrino, Nov. 1970

Effective field theories



- Powerful framework based on exploiting symmetries and controlled expansions for problems with a natural separation of energy/momentum or distance scales.

- Particularly well suited to QCD and nuclear physics

- Effective theories are ubiquitous. The Standard Model is likely a low energy EFT of a theory at a much higher scale