TMD physics from Lattice QCD

Michael Engelhardt

New Mexico State University

In collaboration with:
B. Musch, P. Hägler, J. Negele, A. Schäfer
J. R. Green, N. Hasan, J. Peyton, C. Kallidonis, S. Krieg, S. Meinel, A. Pochinsky, G. Silvi, S. Syritsyn
T. Bhattacharya, R. Gupta, B. Yoon
M. Schlemmer, A. Vladimirov, C. Zimmermann
S. Liuti, A. Rajan
T. Izubuchi

Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv \frac{1}{2} \langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \ldots, b]\ q(b)\ |P, S\rangle$$

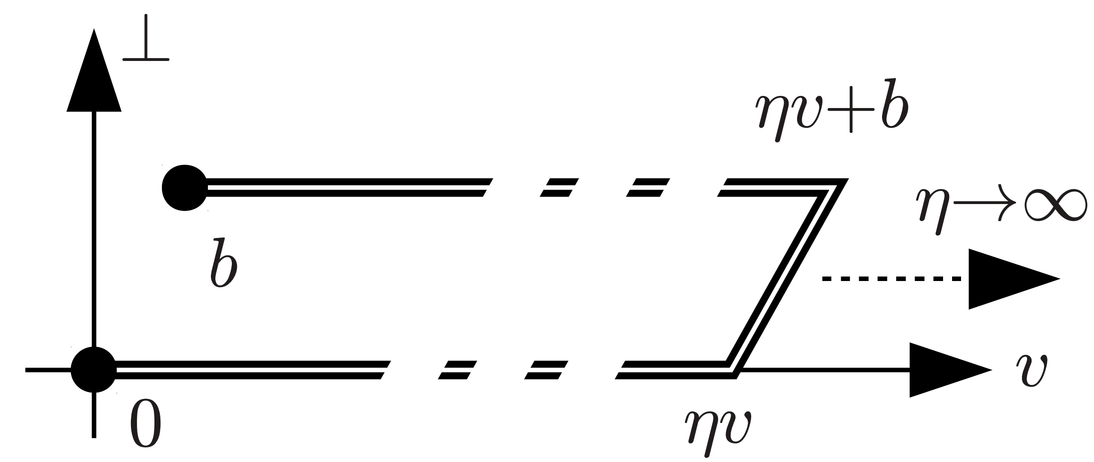
$$\Phi^{[\Gamma]}(x, k_T, P, S, \ldots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi) P^+} \exp\left(ix(b \cdot P) - ib_T \cdot k_T\right) \frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots)}{\widetilde{\mathcal{S}}(b^2, \ldots)} \Bigg|_{b^+ = 0}$$

- "Soft factor" $\widetilde{\mathcal{S}}$ required to subtract divergences of gauge link $\mathcal{U}$
- Almost all of this lecture will consider only ratios in which soft factors cancel
- Will discuss perspective for obtaining soft factors at the end

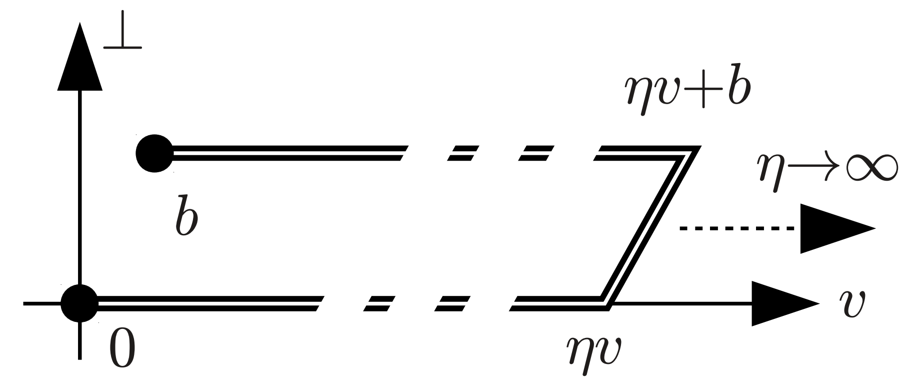
Gauge link structure motivated by factorization of physical process

SIDIS and DY: Staple-shaped gauge link $\mathcal{U}[0, \eta v, \eta v + b, b]$

- Accounts for initial/final state interactions
- Further regularization required!



Gauge link structure motivated by factorization of physical process



Beyond tree level: Rapidity divergences suggest taking staple direction slightly off the light cone. Approach of Aybat, Collins, Qiu, Rogers makes v space-like. Parametrize in terms of Collins-Soper type parameter

$$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

Light-like staple for $\hat{\zeta} \to \infty$. Perturbative evolution equations for large $\hat{\zeta}$.

"Modified universality", $f^{\text{T-odd, SIDIS}} = -f^{\text{T-odd, DY}}$

Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$$

$$\Phi^{[\Gamma]}(x, k_T, P, S, \ldots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi) P^+} \exp\left(ix(b \cdot P) - ib_T \cdot k_T\right) \frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots)}{\widetilde{\mathcal{S}}(b^2, \ldots)}\Bigg|_{b^+=0}$$

- "Soft factor" $\widetilde{\mathcal{S}}$ required to subtract divergences of gauge link $\mathcal{U}$
- Almost all of this lecture will consider only ratios in which soft factors cancel
- Will discuss perspective for obtaining soft factors at the end

Decomposition of Φ into TMDs

All leading twist structures:

$$\Phi^{[\gamma^+]} = f_1 - \left[\frac{\epsilon_{ij}k_i S_j}{m_H} f_{1T}^{\perp}\right] \mathrm{odd}$$

$$\Phi^{[\gamma^+\gamma^5]} = \Lambda g_1 + \frac{k_T \cdot S_T}{m_H} g_{1T}$$

$$\Phi^{[i\sigma^{i+}\gamma^5]} = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij})S_j}{2m_H^2} h_{1T}^{\perp} + \frac{\Lambda k_i}{m_H} h_{1L}^{\perp} + \left[\frac{\epsilon_{ij}k_j}{m_H} h_1^{\perp}\right] \mathrm{odd}$$

TMD Classification

All leading twist structures:

H ↓ \ q →	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

← Boer-Mulders (T-odd)

↑ Sivers (T-odd)

Decomposition of $\widetilde{\Phi}$ into amplitudes

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\begin{aligned}
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+]}_{\text{unsubtr.}} &= \widetilde{A}_{2B} + i m_H \epsilon_{ij} b_i S_j \widetilde{A}_{12B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+\gamma^5]}_{\text{unsubtr.}} &= -\Lambda \widetilde{A}_{6B} + i[(b\cdot P)\Lambda - m_H(b_T\cdot S_T)]\widetilde{A}_{7B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[i\sigma^{i+}\gamma^5]}_{\text{unsubtr.}} &= i m_H \epsilon_{ij} b_j \widetilde{A}_{4B} - S_i \widetilde{A}_{9B} \\
&\quad - i m_H \Lambda b_i \widetilde{A}_{10B} + m_H[(b\cdot P)\Lambda - m_H(b_T\cdot S_T)] b_i \widetilde{A}_{11B}
\end{aligned}$$

(Decompositions analogous to work by Metz et al. in momentum space)

Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \ldots) \equiv \int d^2k_T \, \exp(ib_T \cdot k_T) f(x, k_T^2, \ldots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \ldots) \equiv n! \left(-\frac{2}{m_H^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \ldots)$$

Formally, in limit $|b_T| \to 0$, recover k_T-moments:

$$\tilde{f}^{(n)}(x, 0, \ldots) \equiv \int d^2k_T \left(\frac{k_T^2}{2m_H^2} \right)^n f(x, k_T^2, \ldots) \equiv f^{(n)}(x)$$

CAREFUL: Ill-defined for large k_T, so, for now, will not attempt to extrapolate to $b_T = 0$, but give results at finite $|b_T|$.

$\longrightarrow$ Bessel-weighted asymmetries (Boer, Gamberg, Musch, Prokudin, JHEP 1110 (2011) 021)

Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \ldots) \equiv \int d^2 k_T \, \exp(i b_T \cdot k_T) f(x, k_T^2, \ldots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \ldots) \equiv n! \left(-\frac{2}{m_H^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \ldots)$$

Also, for now, only consider first x-moments (accessible at $b \cdot P = 0$), rather than scanning range of $b \cdot P$:

$$f^{[1]}(k_T^2, \ldots) \equiv \int_{-1}^{1} dx \, f(x, k_T^2, \ldots)$$

AGAIN, CAREFUL: Matching factors between unsubtracted/renormalized TMDs may depend on x

Relation between Fourier-transformed TMDs and invariant amplitudes $\widetilde{A}_i$

Invariant amplitudes directly give selected x-integrated TMDs in Fourier (b_T) space (showing just the ones relevant for Sivers, Boer-Mulders shifts), up to soft factors:

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = -2\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{h}_1^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

Generalized shifts

Form ratios in which soft factors, (Γ-independent) multiplicative renormalization factors cancel

Boer-Mulders shift:

$$\langle k_y \rangle_{UT} \equiv m_H \frac{\tilde{h}_1^{\perp [1](1)}}{\tilde{f}_1^{[1](0)}} = \left. \frac{\int dx \int d^2 k_T \, k_y \Phi^{[\gamma^+ + s^j i\sigma^{j+}\gamma^5]}(x, k_T, P, \ldots)}{\int dx \int d^2 k_T \, \Phi^{[\gamma^+ + s^j i\sigma^{j+}\gamma^5]}(x, k_T, P, \ldots)} \right|_{s_T=(1,0)}$$

Average transverse momentum of quarks polarized in the orthogonal transverse ("T") direction in an unpolarized ("U") hadron; normalized to the number of valence quarks. "Dipole moment" in $b_T^2 = 0$ limit, "shift".

Issue: k_T-moments in this ratio singular; generalize to ratio of Fourier-transformed TMDs at *nonzero* b_T^2,

$$\langle k_y \rangle_{UT}(b_T^2, \ldots) \equiv m_H \frac{\tilde{h}_1^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)}$$

(remember singular $b_T \to 0$ limit corresponds to taking k_T-moment). "Generalized shift".

Generalized shifts from amplitudes

Now, can also express this in terms of invariant amplitudes:

$$\langle k_y \rangle_{UT}(b_T^2, \ldots) \equiv m_H \frac{\tilde{h}_1^{\perp[1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = m_H \frac{\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Analogously, Sivers shift (in a polarized hadron):

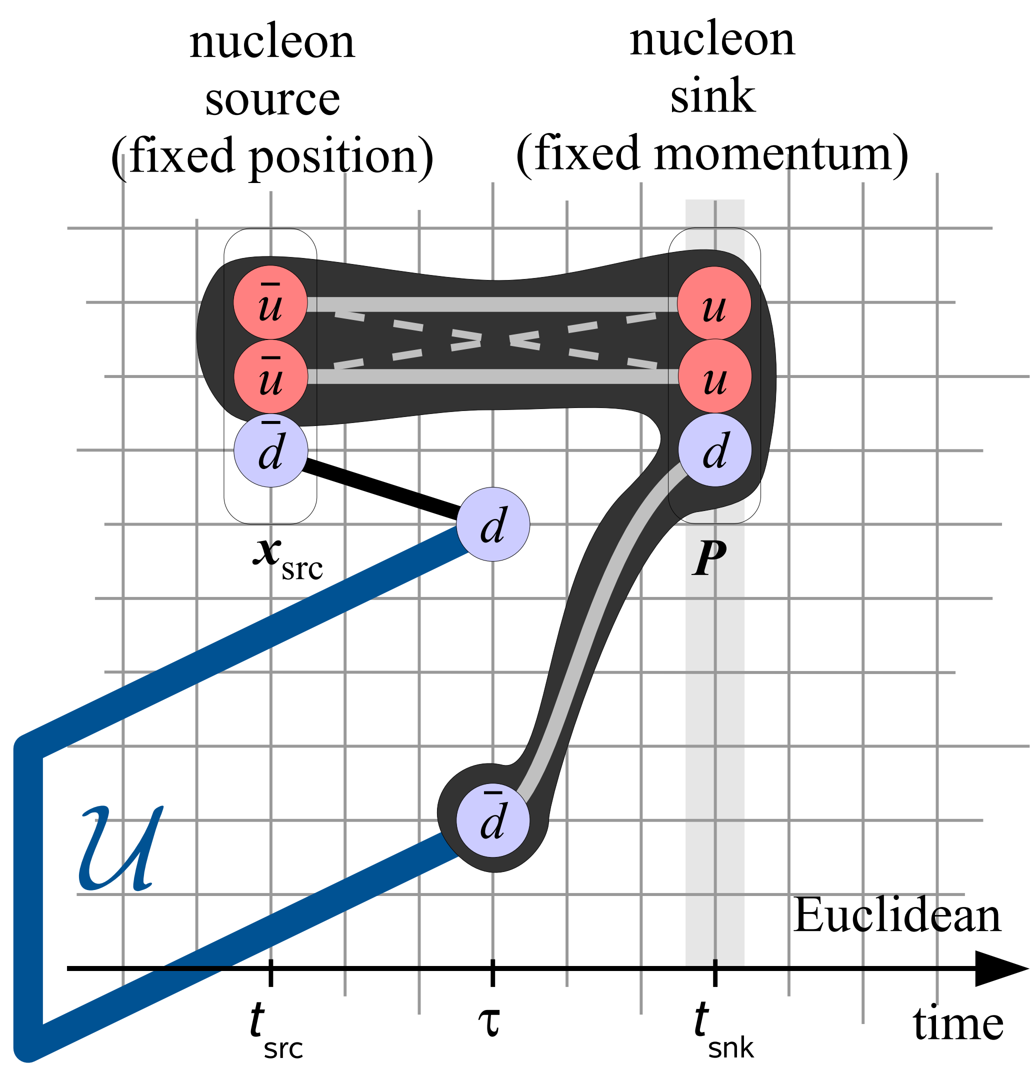
$$\langle k_y \rangle_{TU}(b_T^2, \ldots) = -m_H \frac{\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Worm-gear (g_{1T}) shift:

$$\langle k_x \rangle_{TL}(b_T^2, \ldots) = -m_N \frac{\widetilde{A}_{7B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Generalized tensor charge (no k-weighting) :

$$\frac{\tilde{h}_1^{[1](0)}}{\tilde{f}_1^{[1](0)}} = -\frac{\widetilde{A}_{9B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P) - (m_N^2 b^2/2)\widetilde{A}_{11B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$



Lattice setup

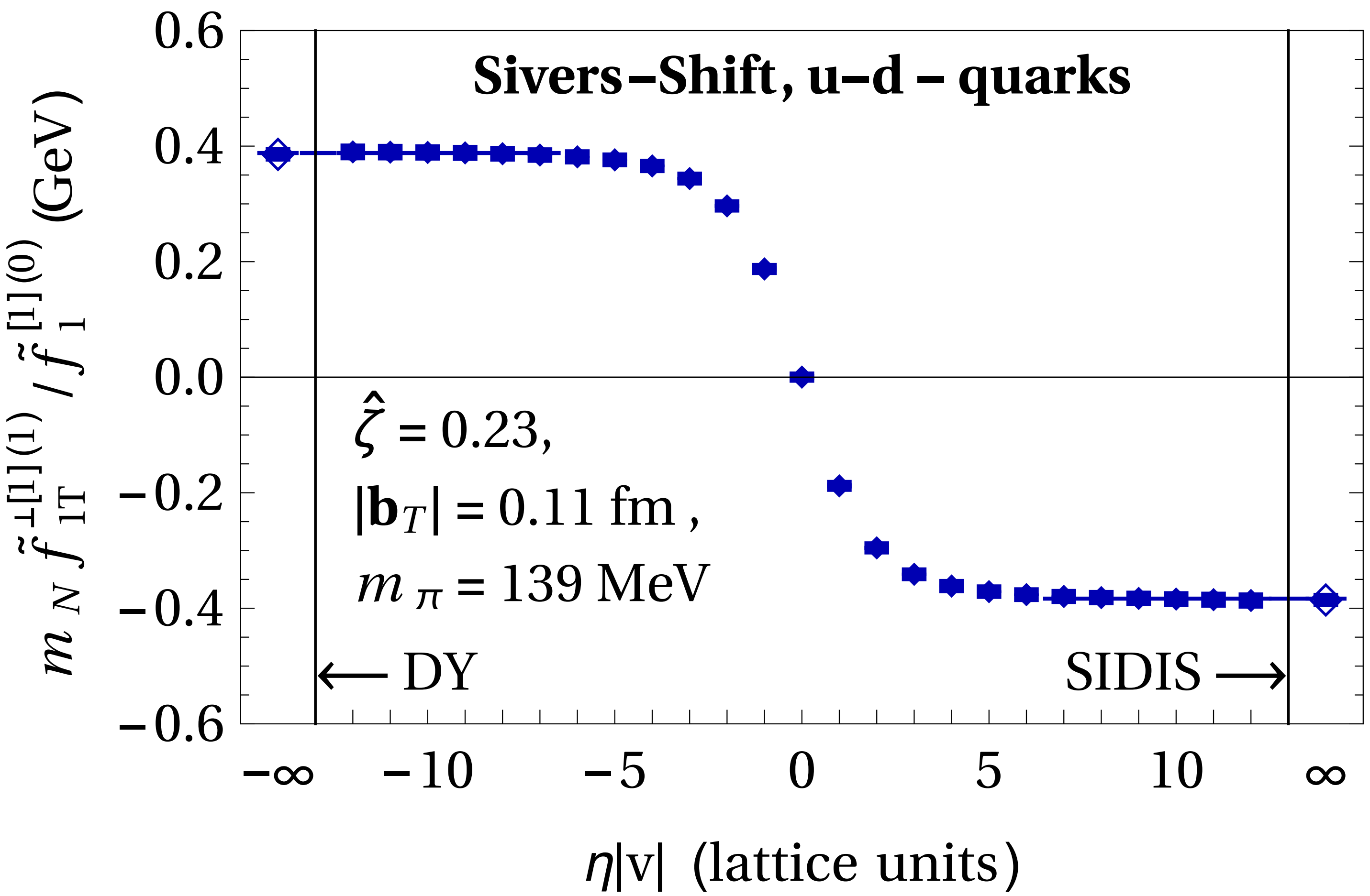
- Evaluate directly $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu)$

  $\equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$

- Euclidean time: Place entire operator at one time slice, i.e., b, ηv purely spatial
- Since generic b, v space-like, no obstacle to boosting system to such a frame!
- Parametrization of correlator in terms of $\widetilde{A}_i$ invariants permits direct translation of results back to original frame; form desired $\widetilde{A}_i$ ratios.
- Use variety of P, b, ηv; here $b \perp P$, $b \perp v$ (lowest x-moment, kinematical choices/constraints)
- Extrapolate $\eta \to \infty$, $\hat{\zeta} \to \infty$ numerically.

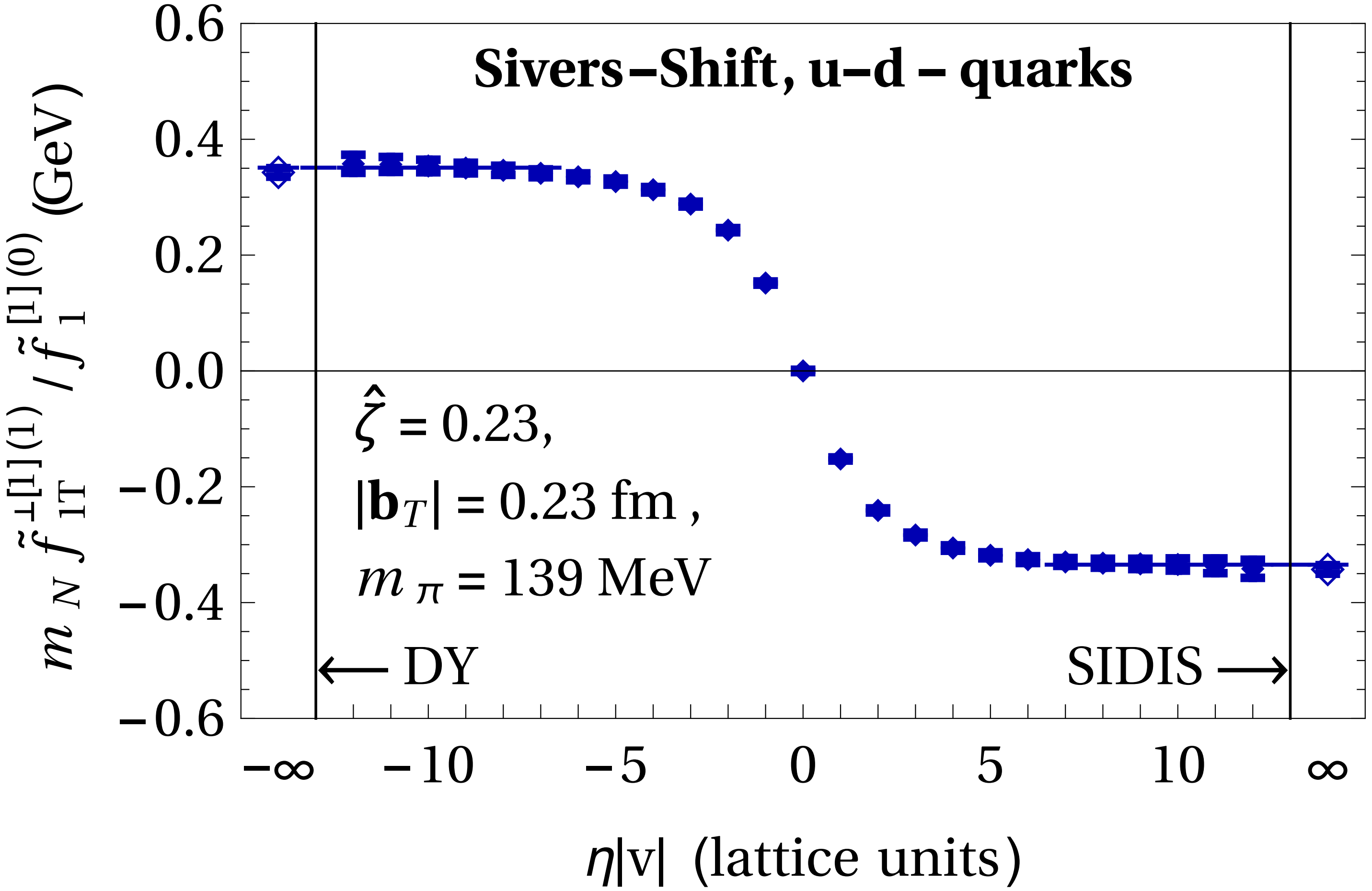
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



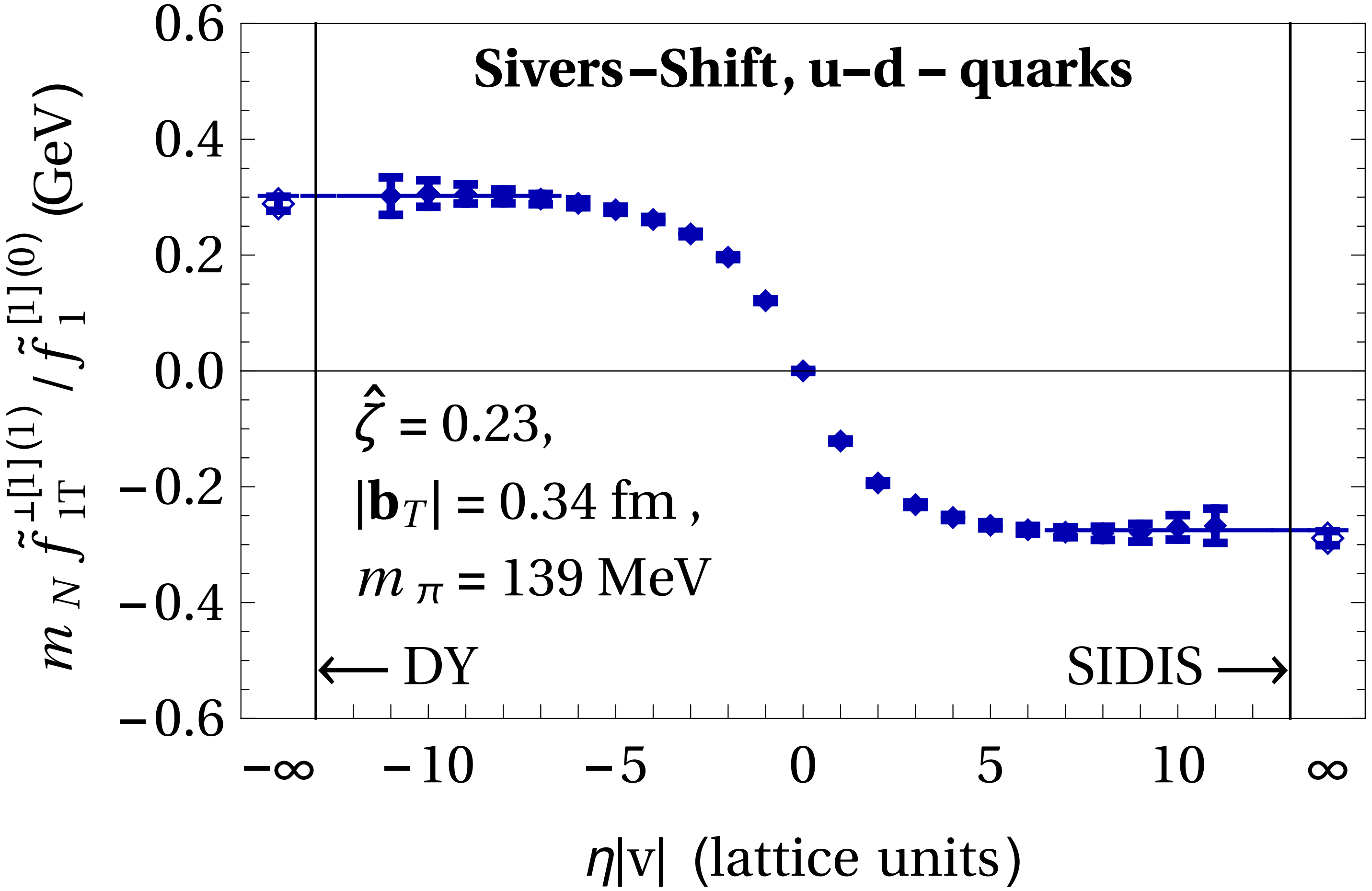
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



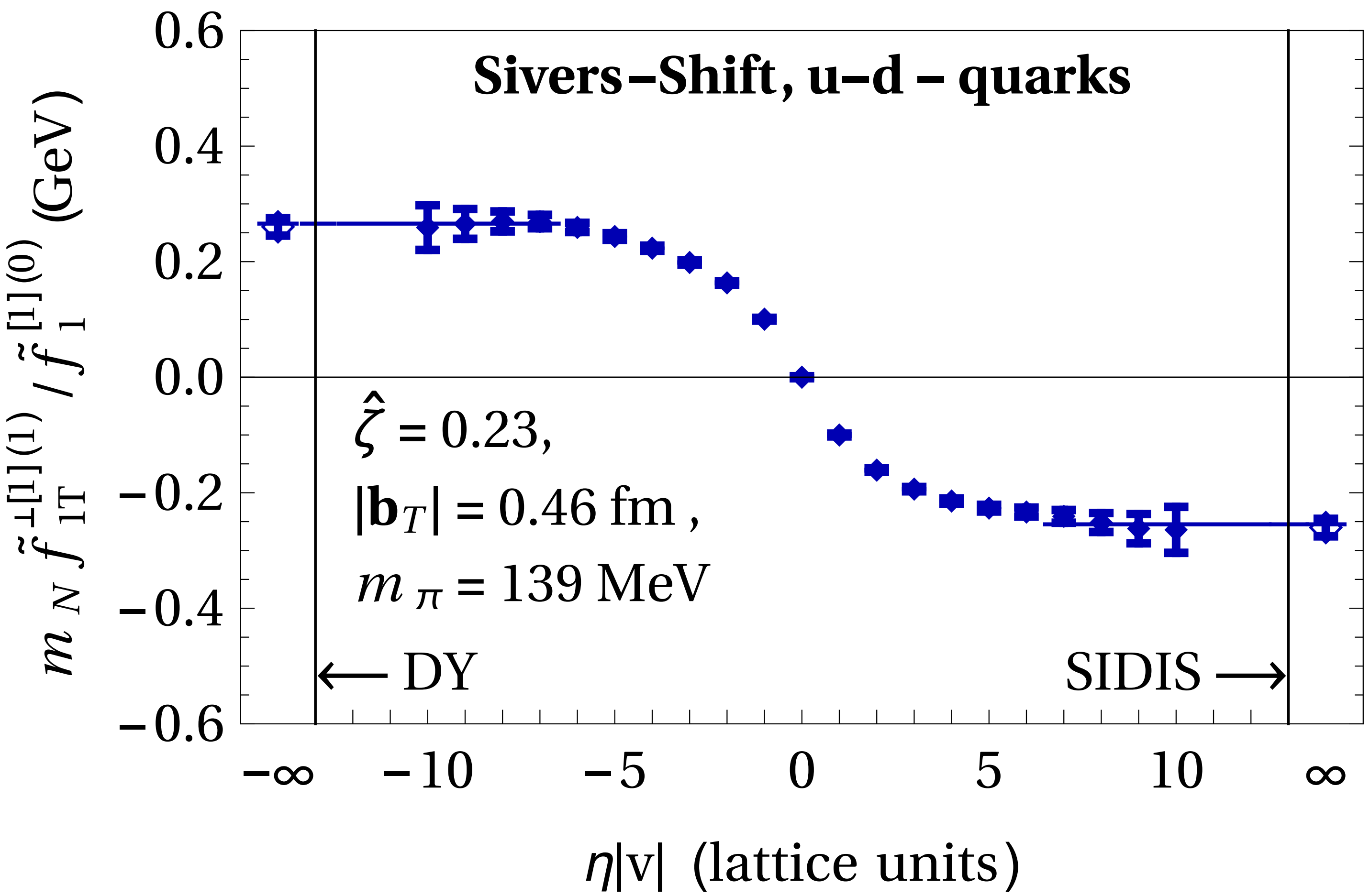
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



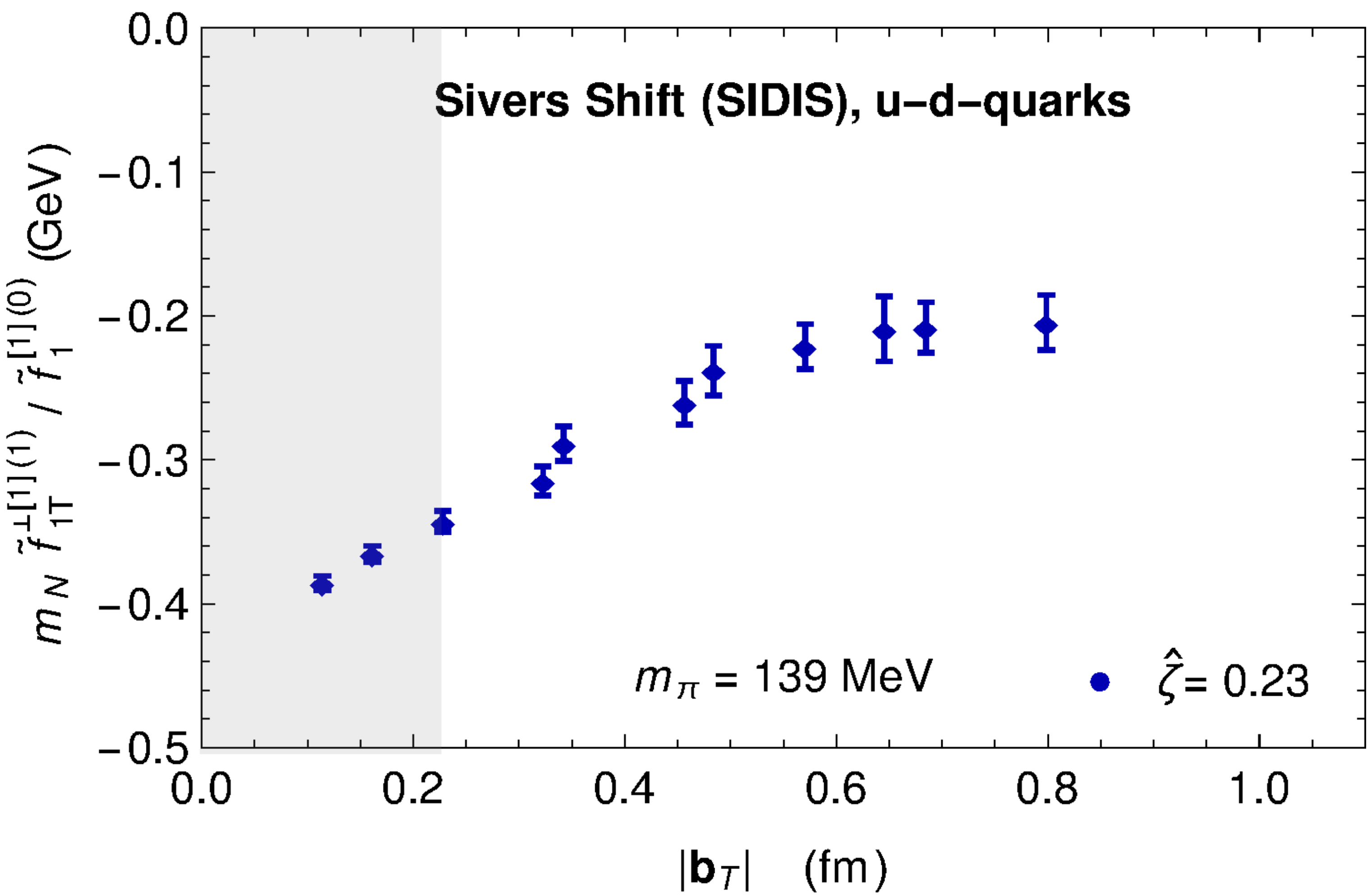
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



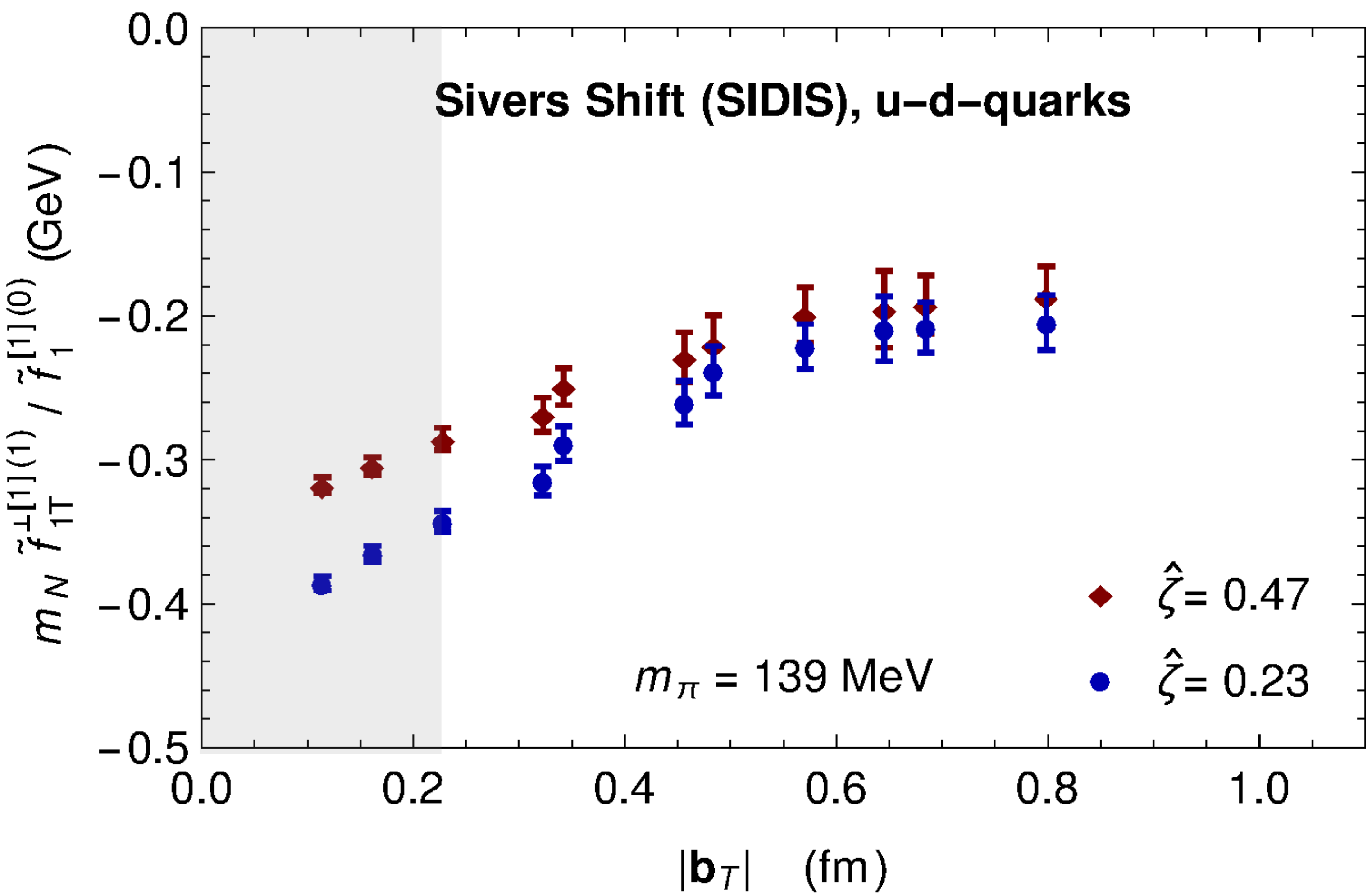
Results: Sivers shift

Dependence of SIDIS limit on $|b_T|$



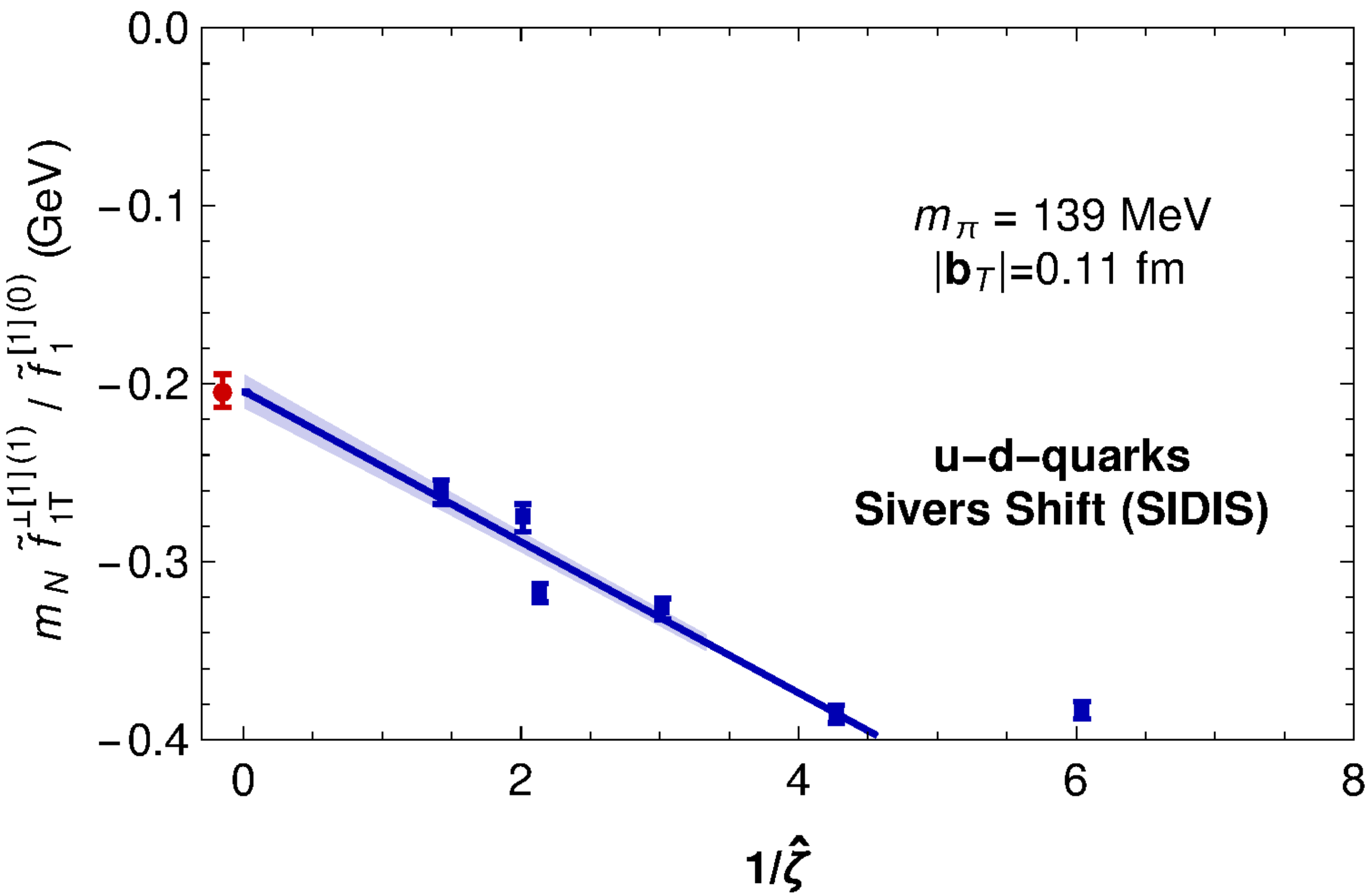
Results: Sivers shift

Dependence of SIDIS limit on $|b_T|$



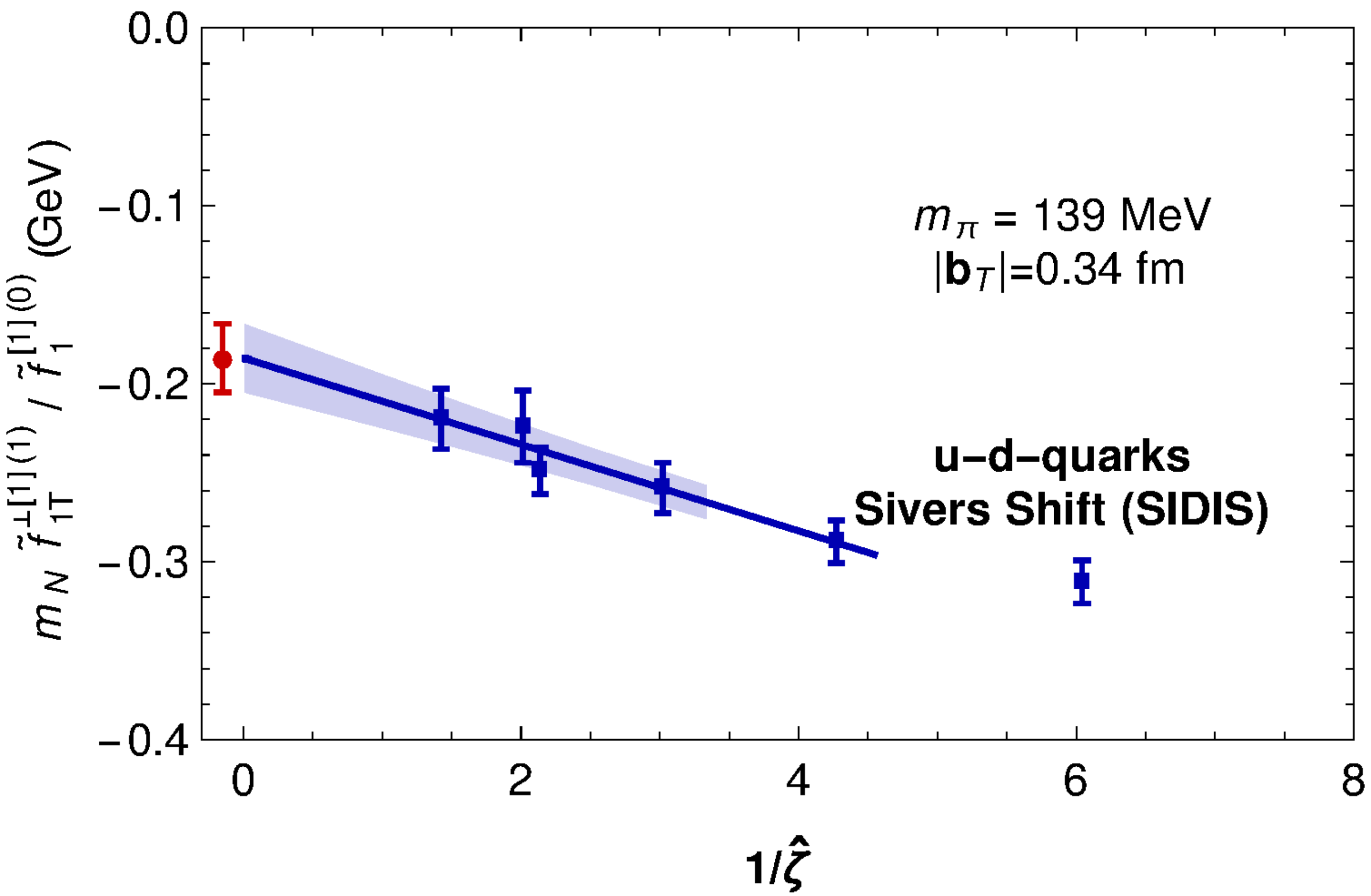
Results: Sivers shift

Extrapolation in $\hat{\zeta}$ for given $|b_T|$ – SIDIS limit



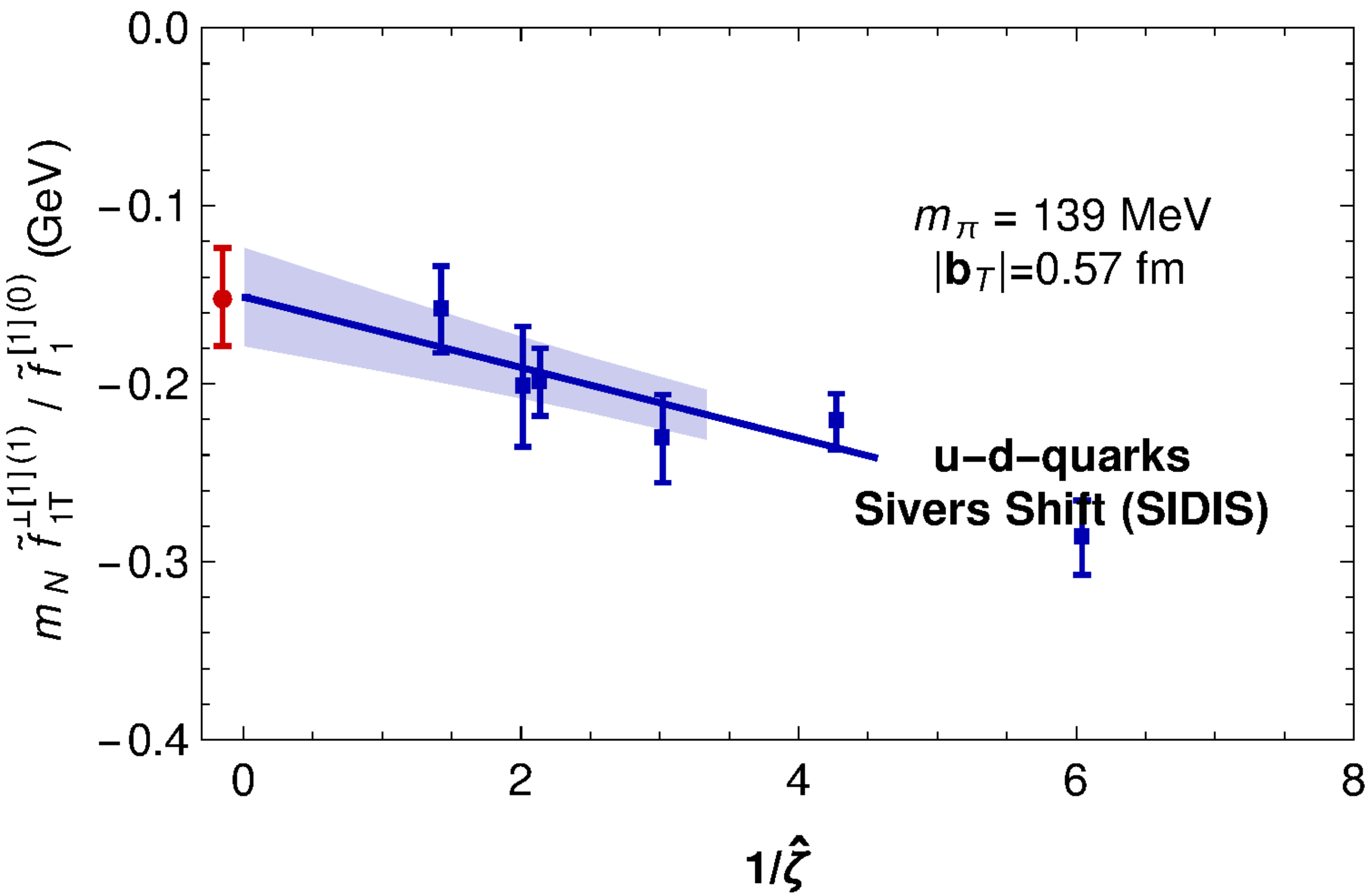
Results: Sivers shift

Extrapolation in $\hat{\zeta}$ for given $|b_T|$ – SIDIS limit



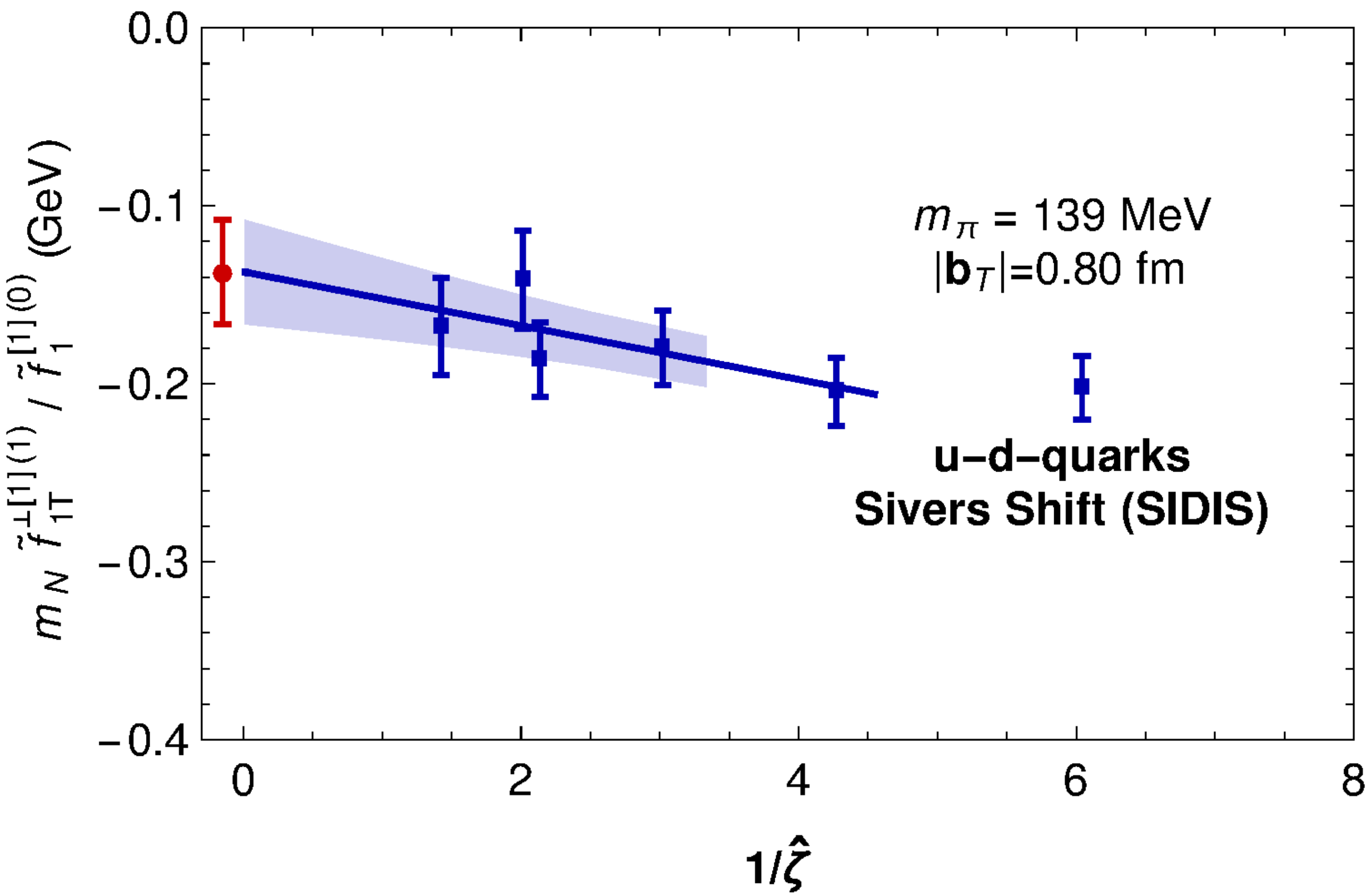
Results: Sivers shift

Extrapolation in $\hat{\zeta}$ for given $|b_T|$ – SIDIS limit



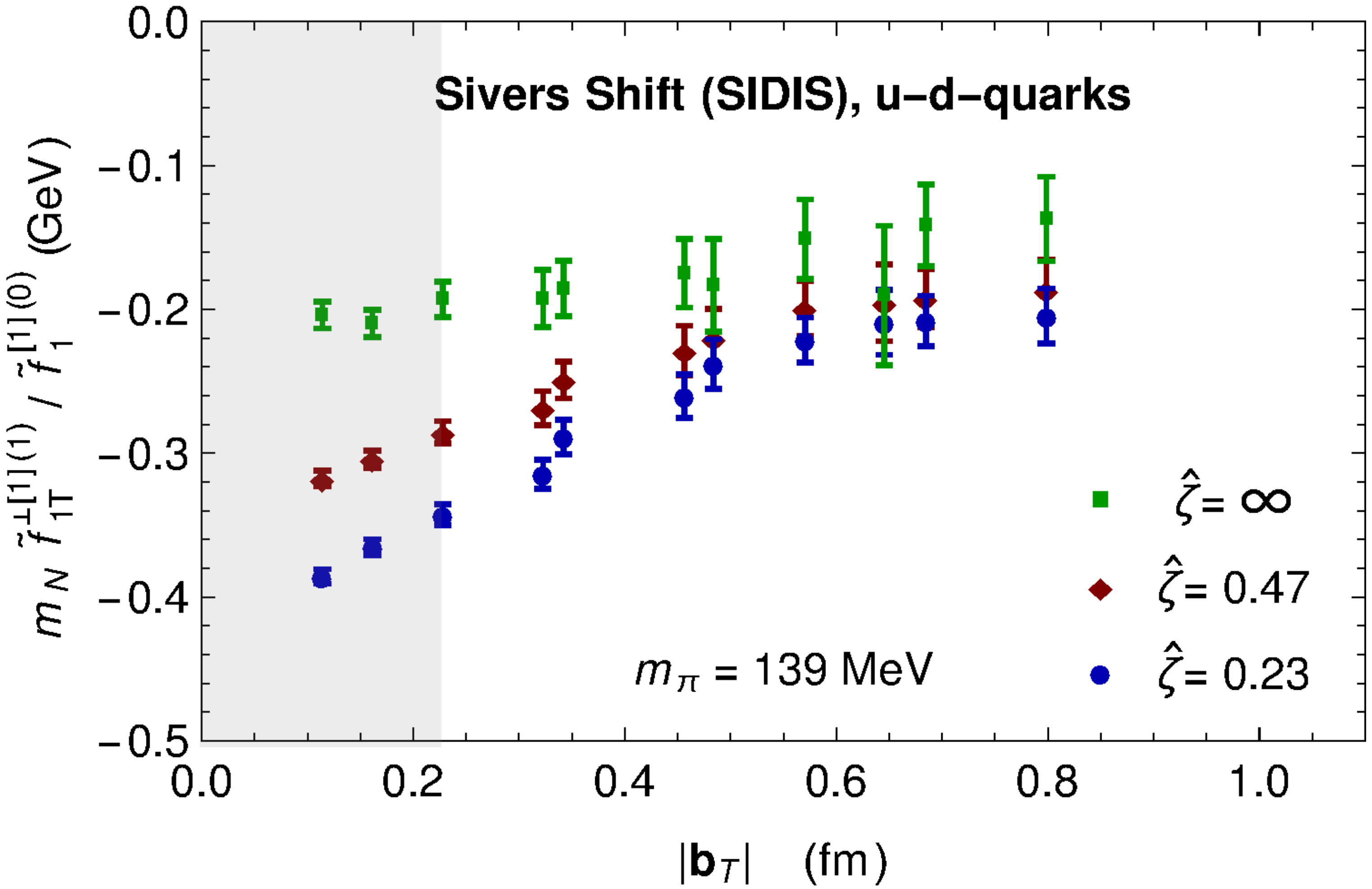
Results: Sivers shift

Extrapolation in $\hat{\zeta}$ for given $|b_T|$ – SIDIS limit



Results: Sivers shift

Dependence of SIDIS limit on $|b_T|$

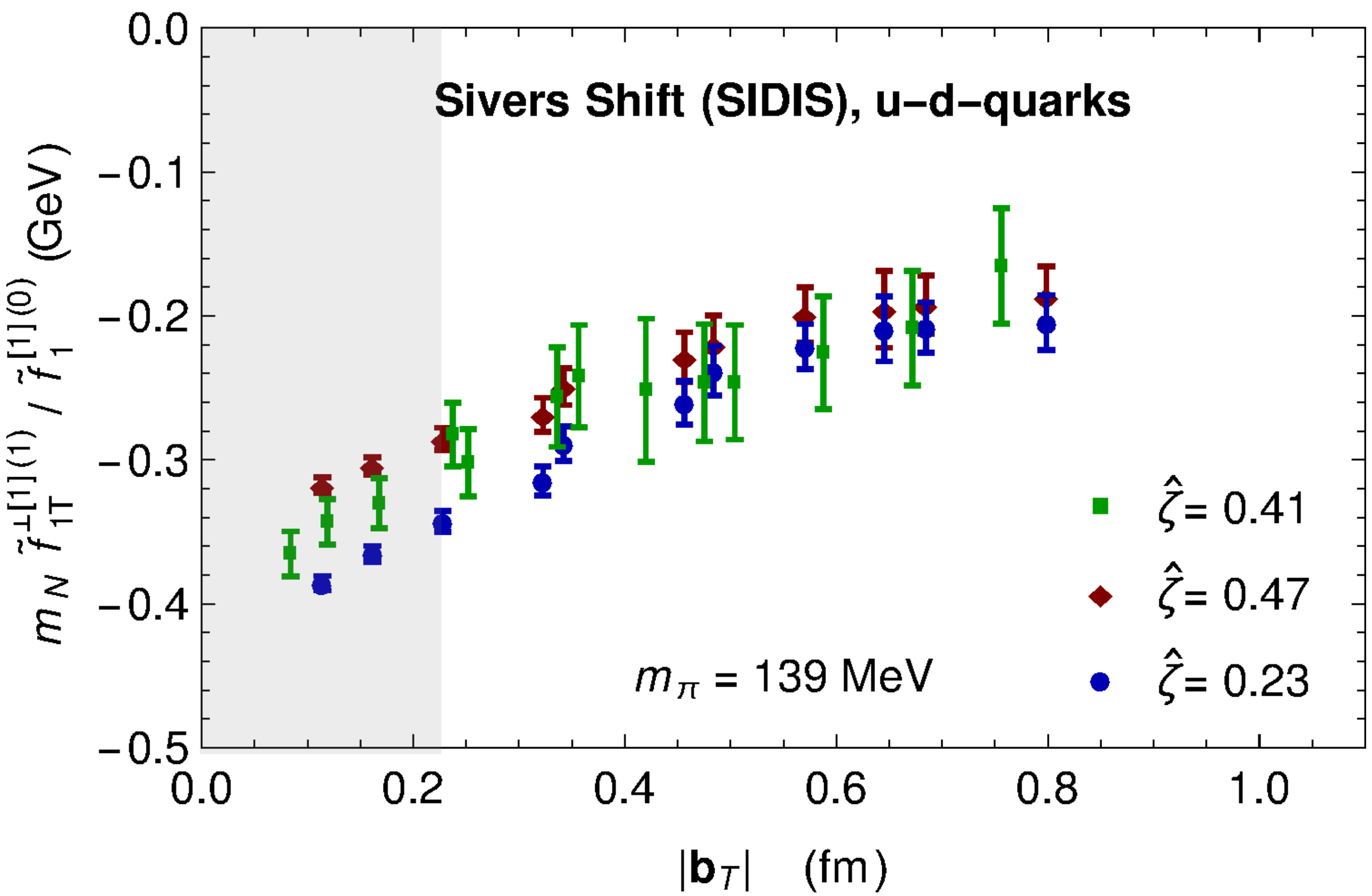


Experimental value from global fit to HERMES, COMPASS and JLab data, M. Echevarria, A. Idilbi, Z.-B. Kang and I. Vitev, Phys. Rev. D 89 (2014) 074013:

$$\langle k_y \rangle_{TU} = -0.146(49)$$

Results: Sivers shift

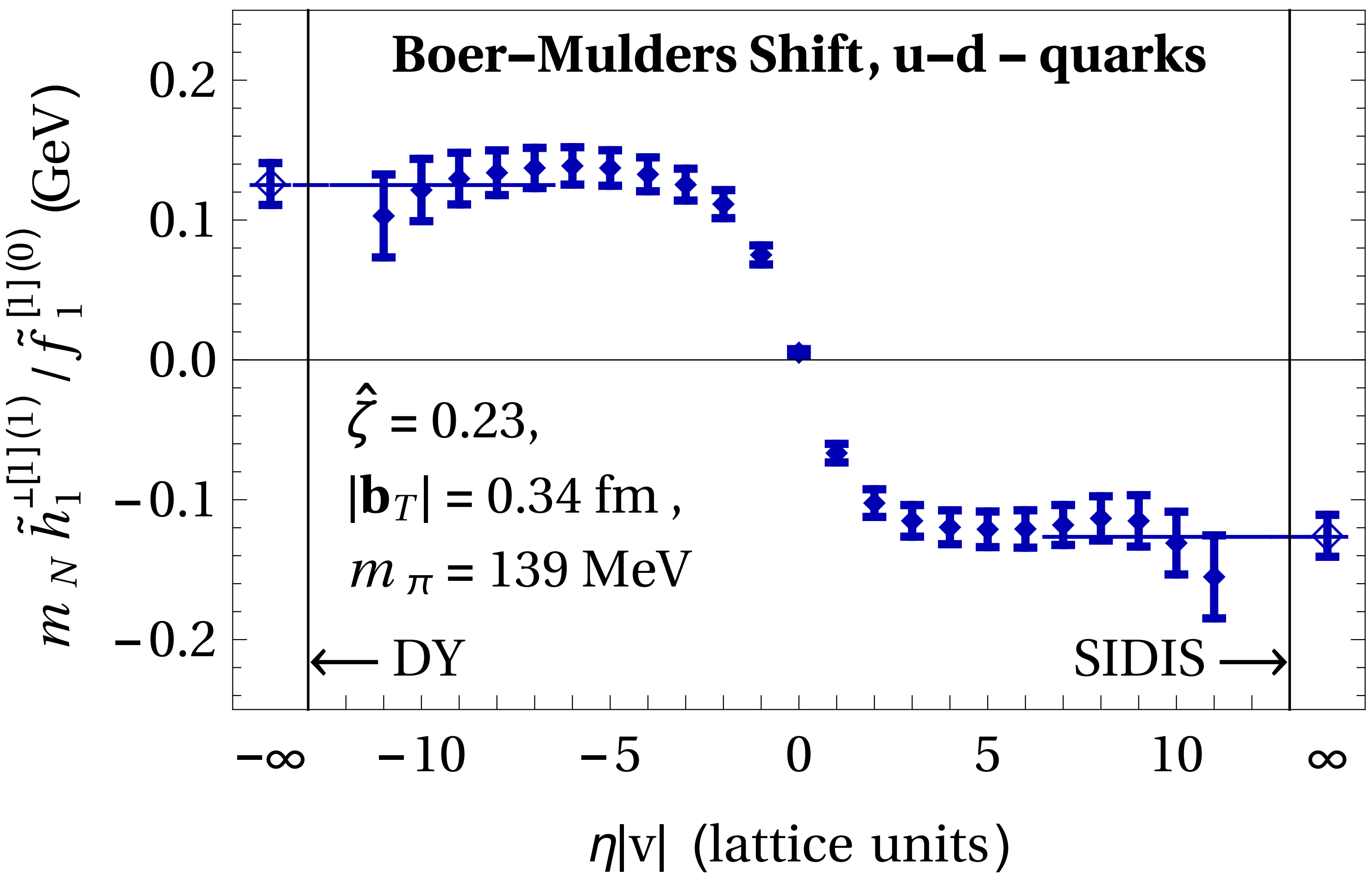
Digression: Comparison with result at heavier quark mass – Dependence of SIDIS limit on $|b_T|$



(Green data points are calculated at $m_\pi = 300\,\text{MeV}$)

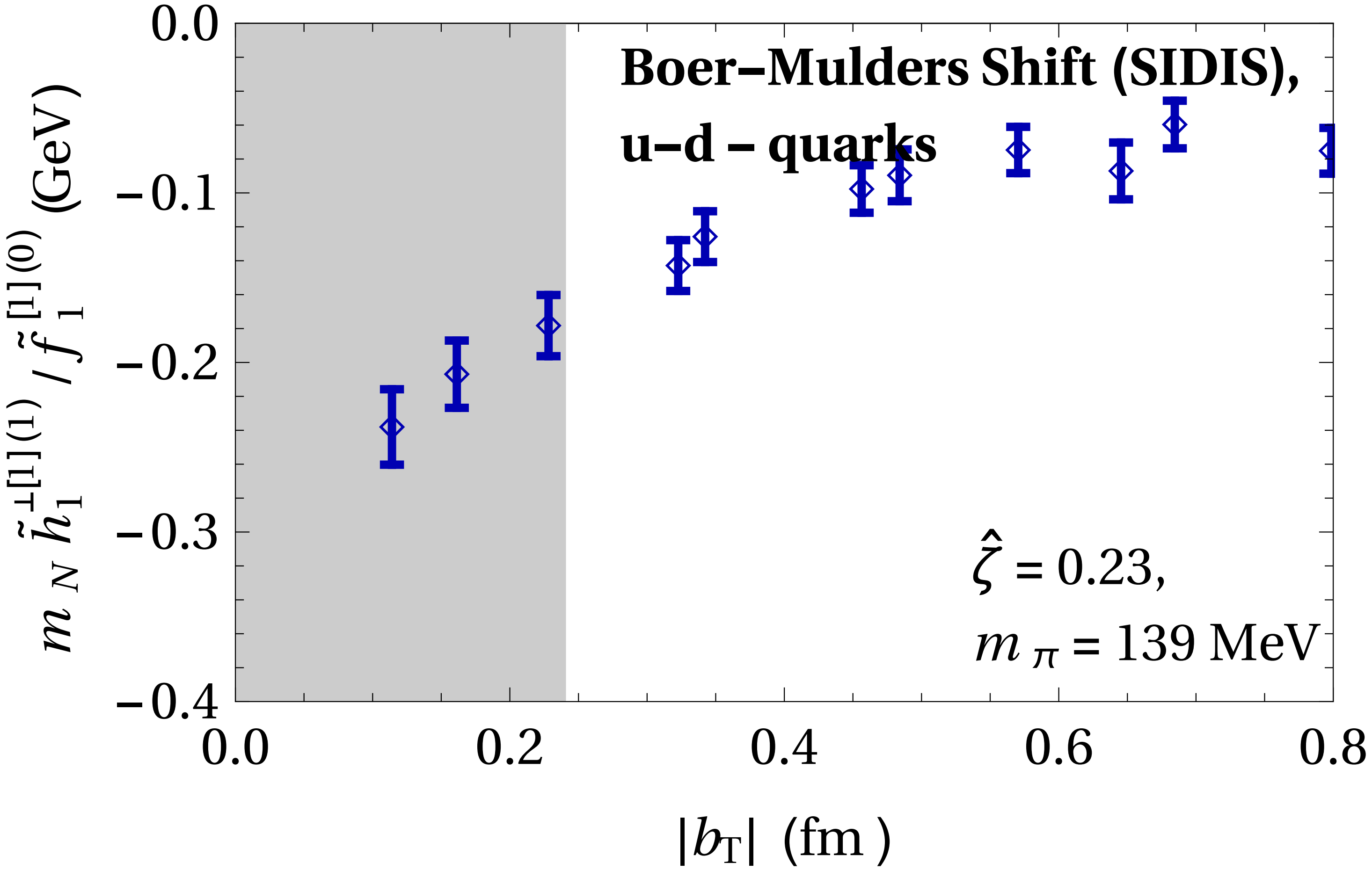
Results: Boer-Mulders shift

Dependence on staple extent



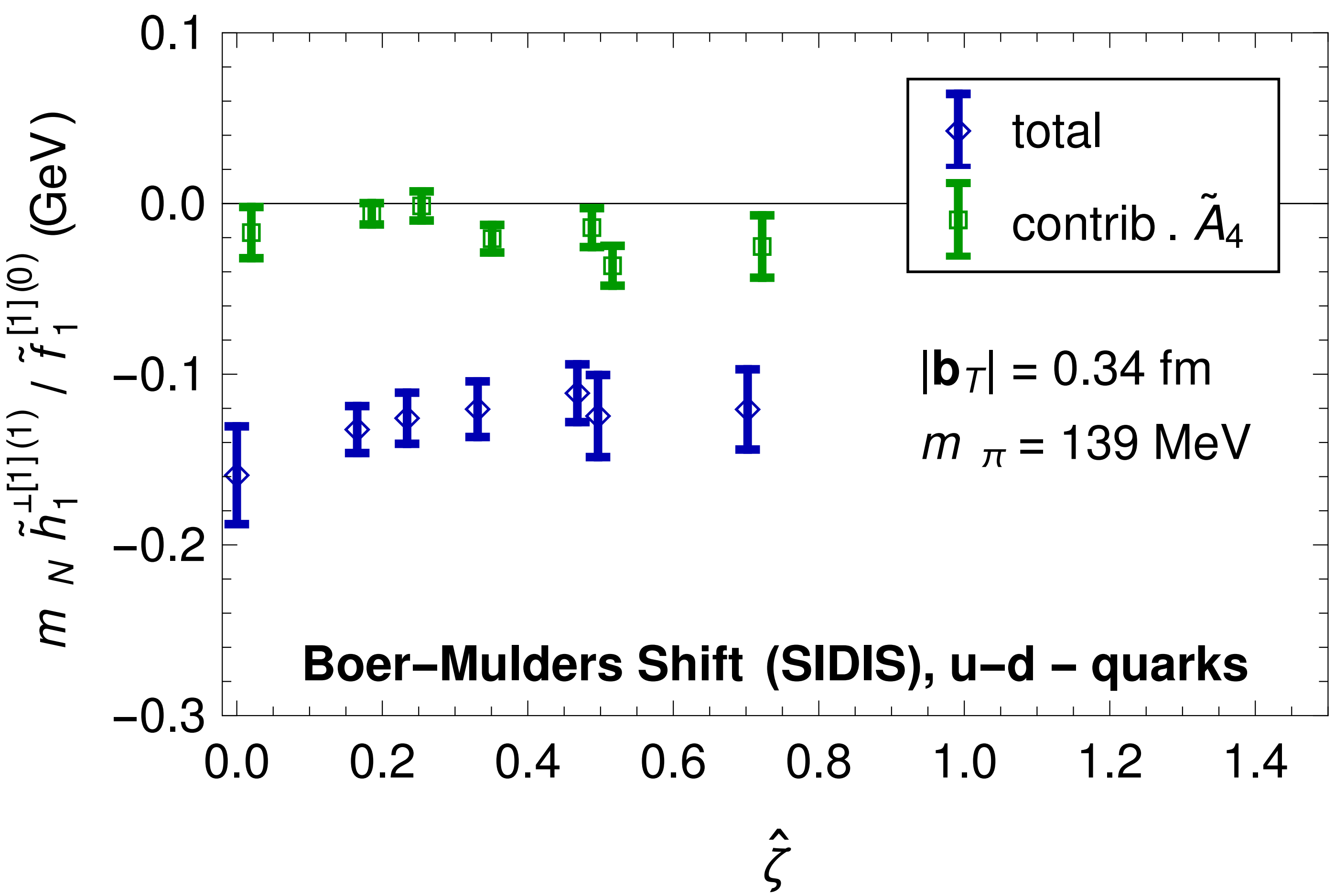
Results: Boer-Mulders shift

Dependence of SIDIS limit on $|b_T|$



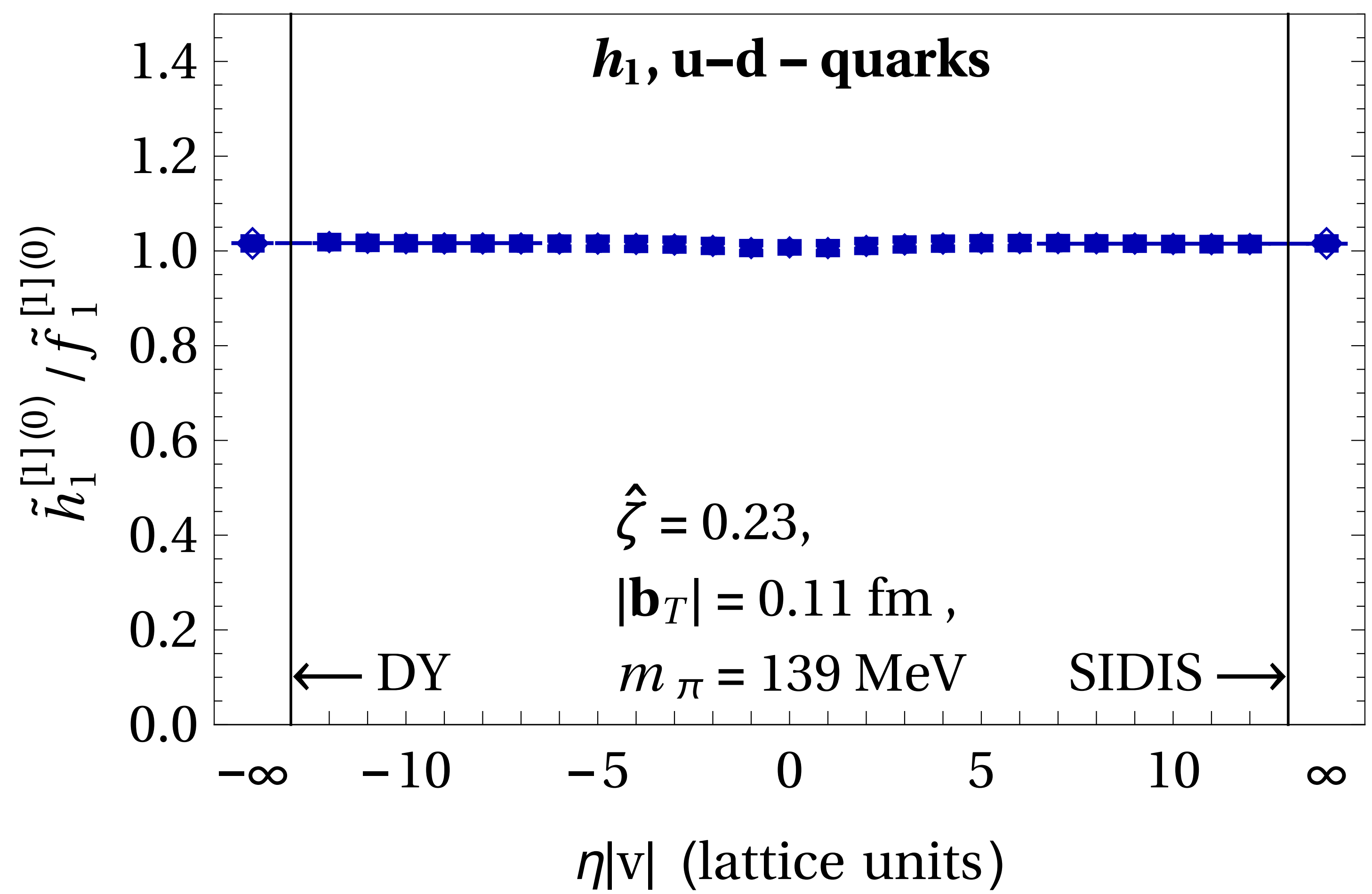
Results: Boer-Mulders shift

Dependence of SIDIS limit on $\hat{\zeta}$



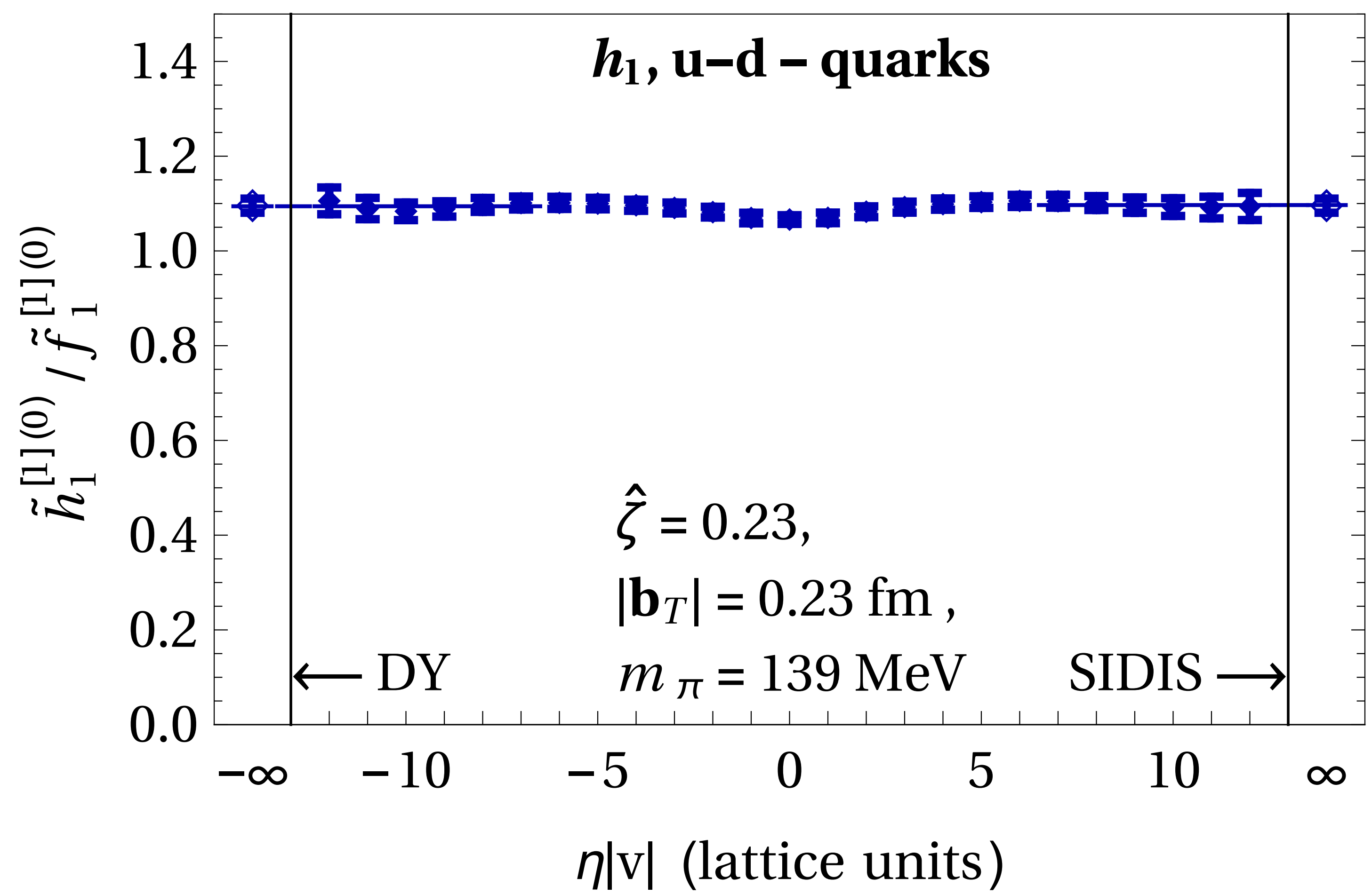
Results: Transversity

Dependence on staple extent; sequence of panels at different $|b_T|$



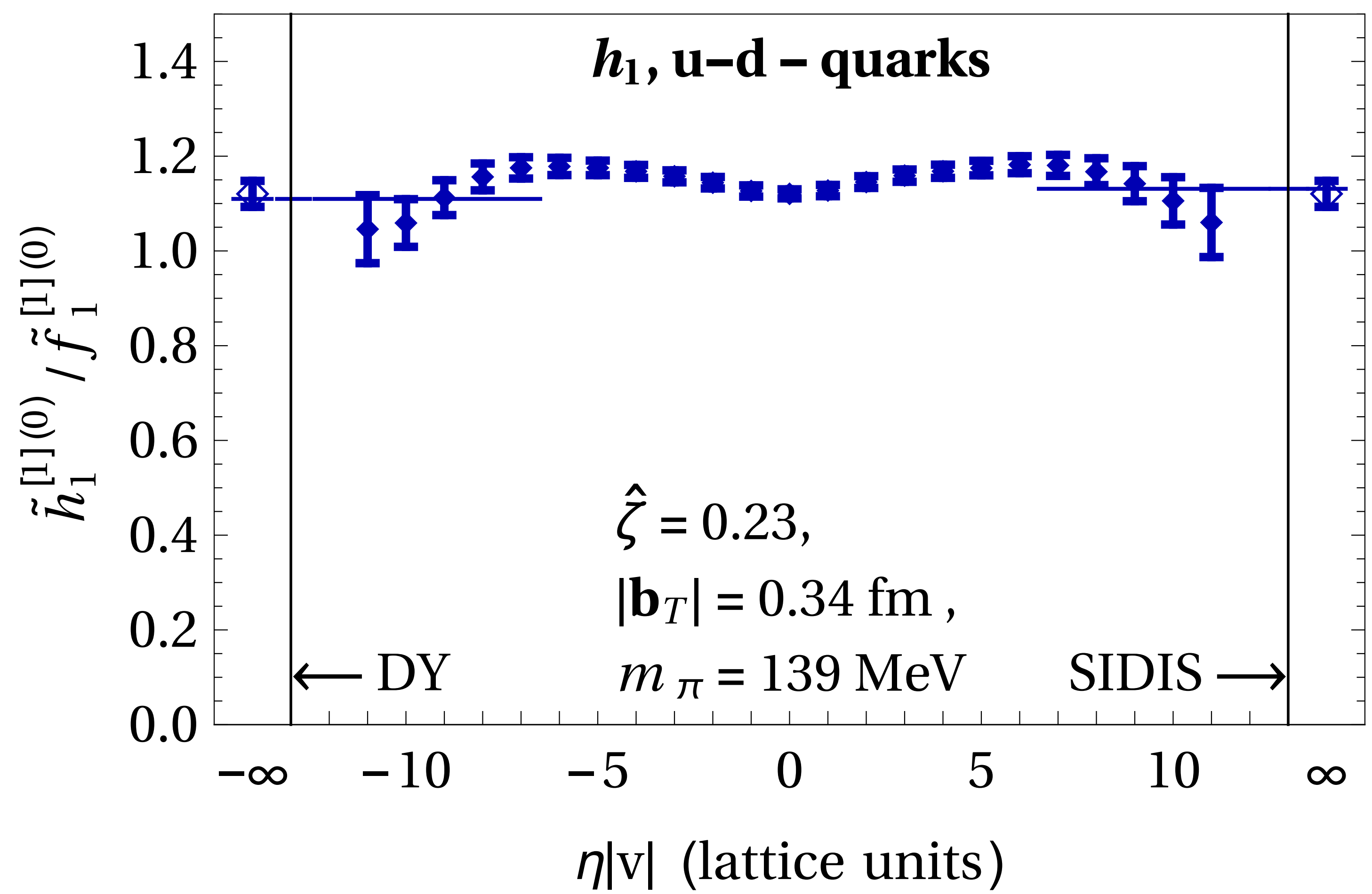
Results: Transversity

Dependence on staple extent; sequence of panels at different $|b_T|$



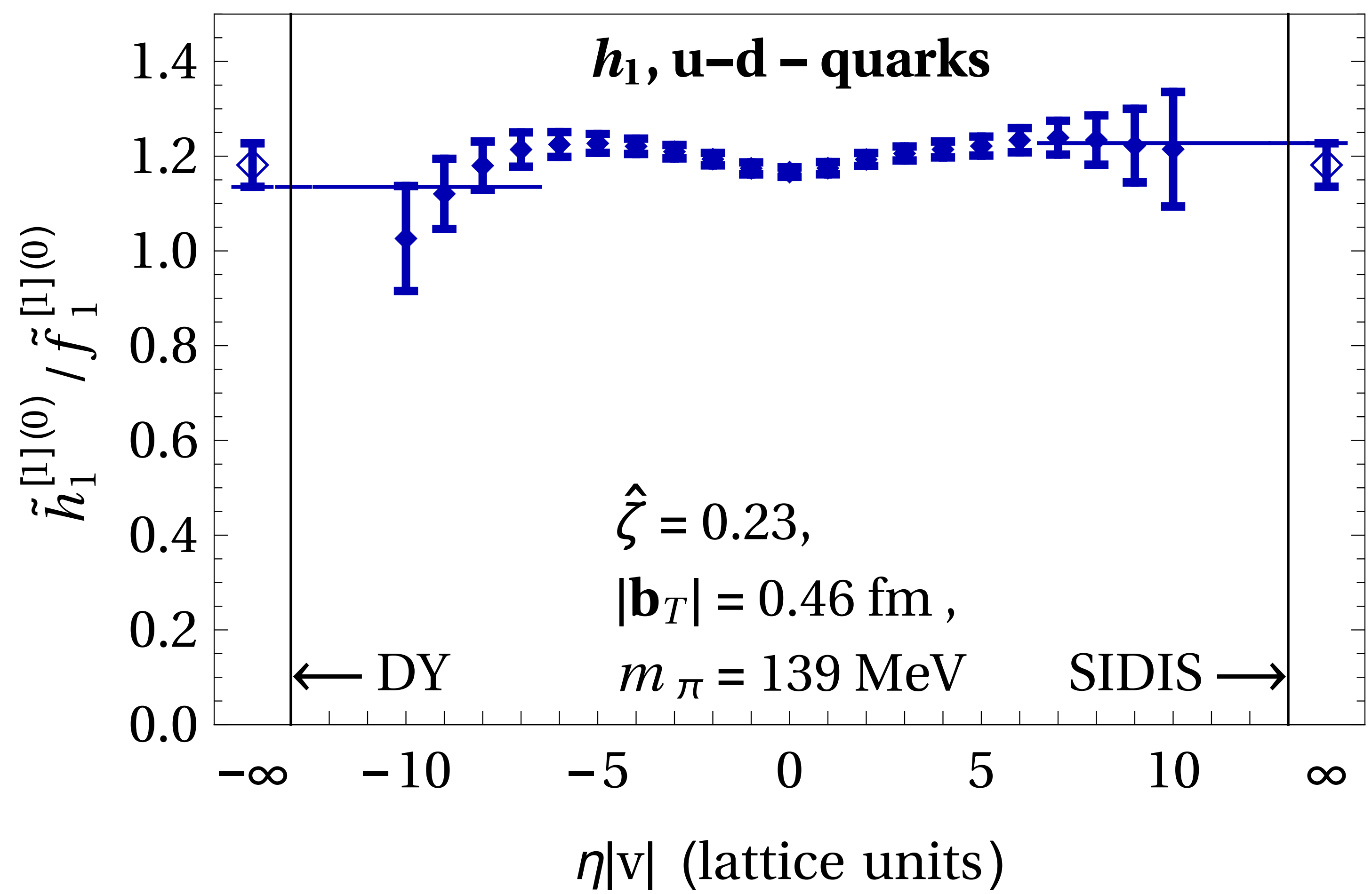
Results: Transversity

Dependence on staple extent; sequence of panels at different $|b_T|$



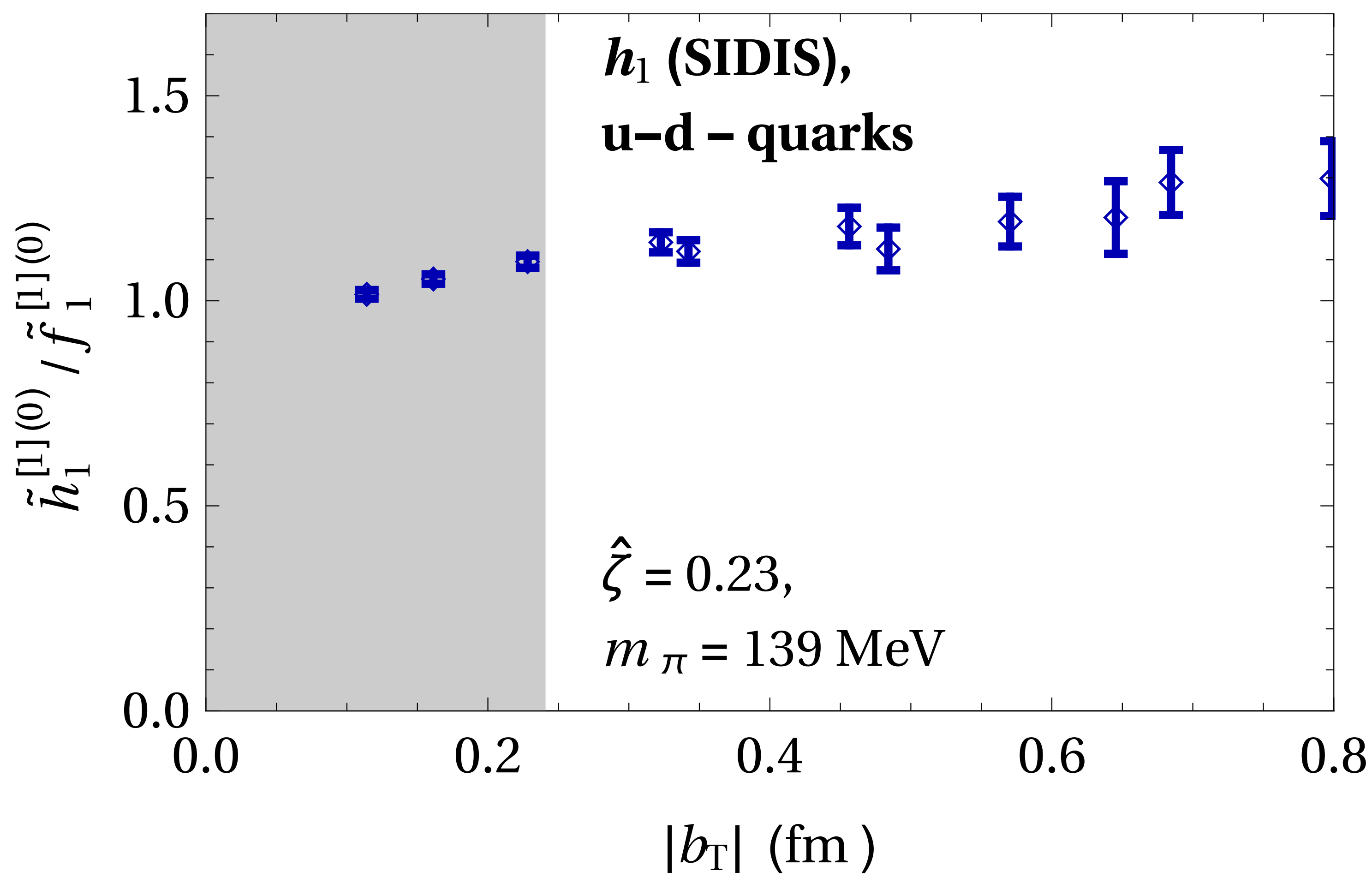
Results: Transversity

Dependence on staple extent; sequence of panels at different $|b_T|$



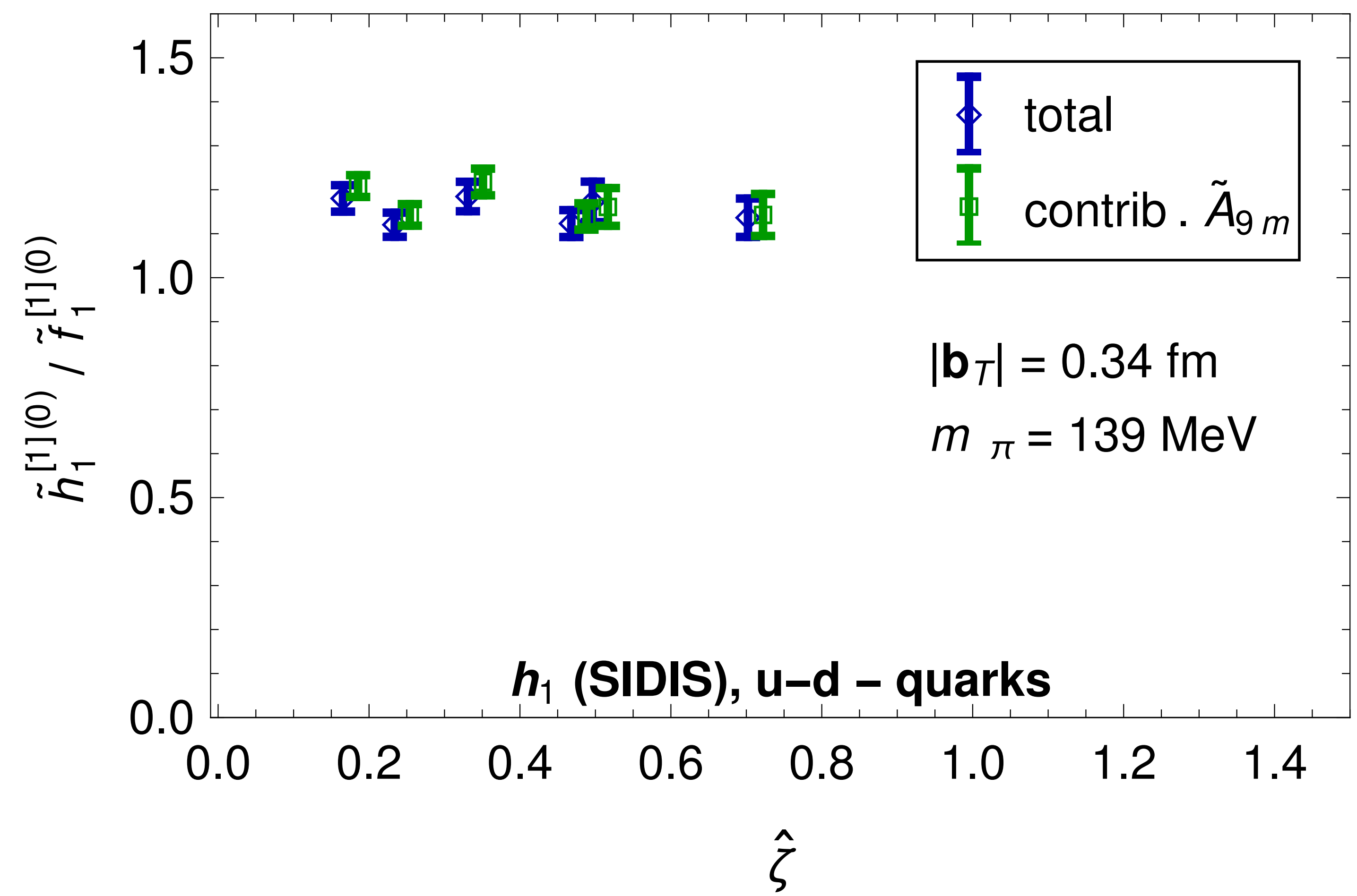
Results: Transversity

Dependence of SIDIS/DY limit on $|b_T|$



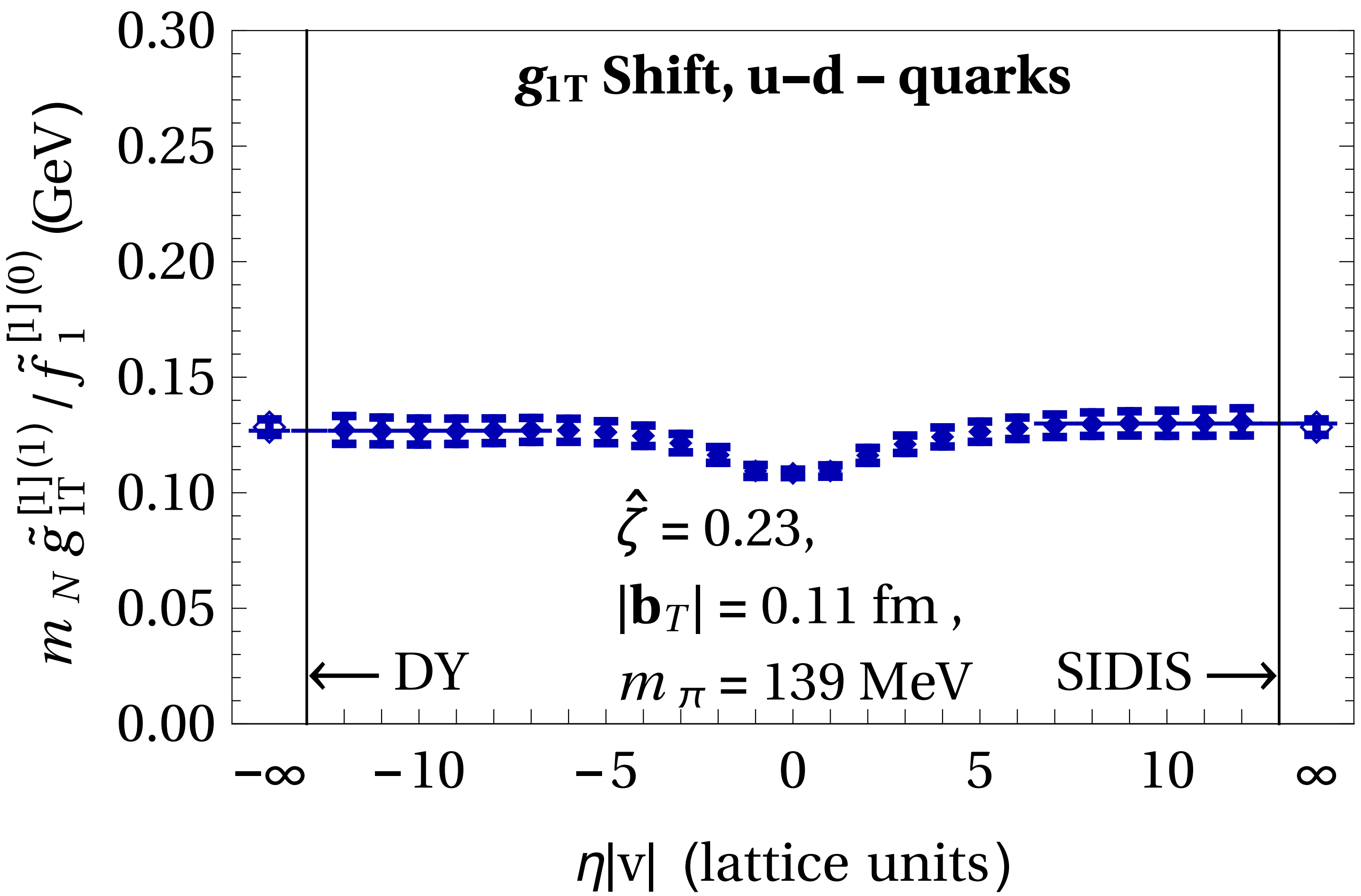
Results: Transversity

Dependence of SIDIS/DY limit on $\hat{\zeta}$



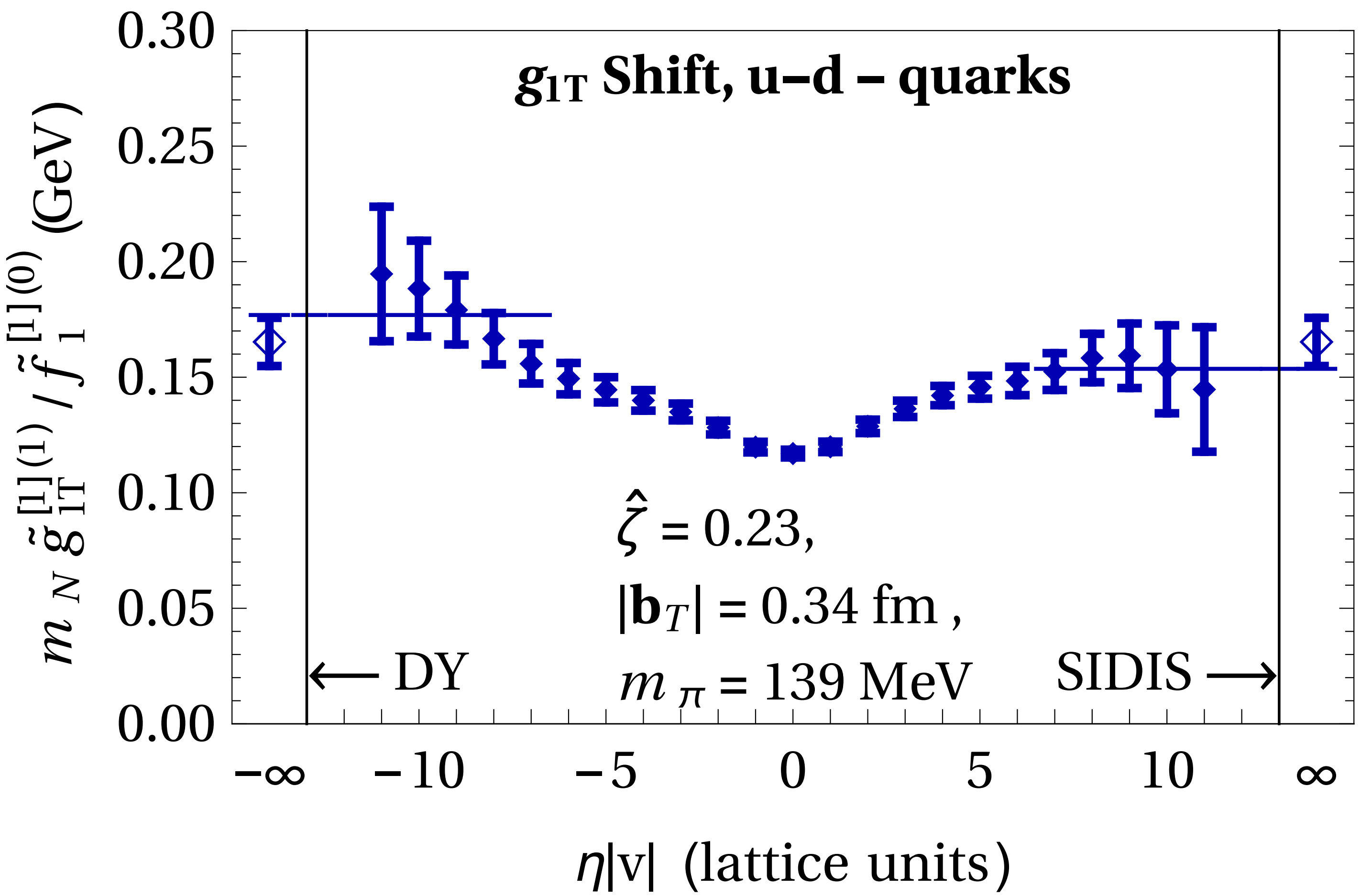
Results: g_{1T} worm gear shift

Dependence on staple extent; sequence of panels at different $|b_T|$



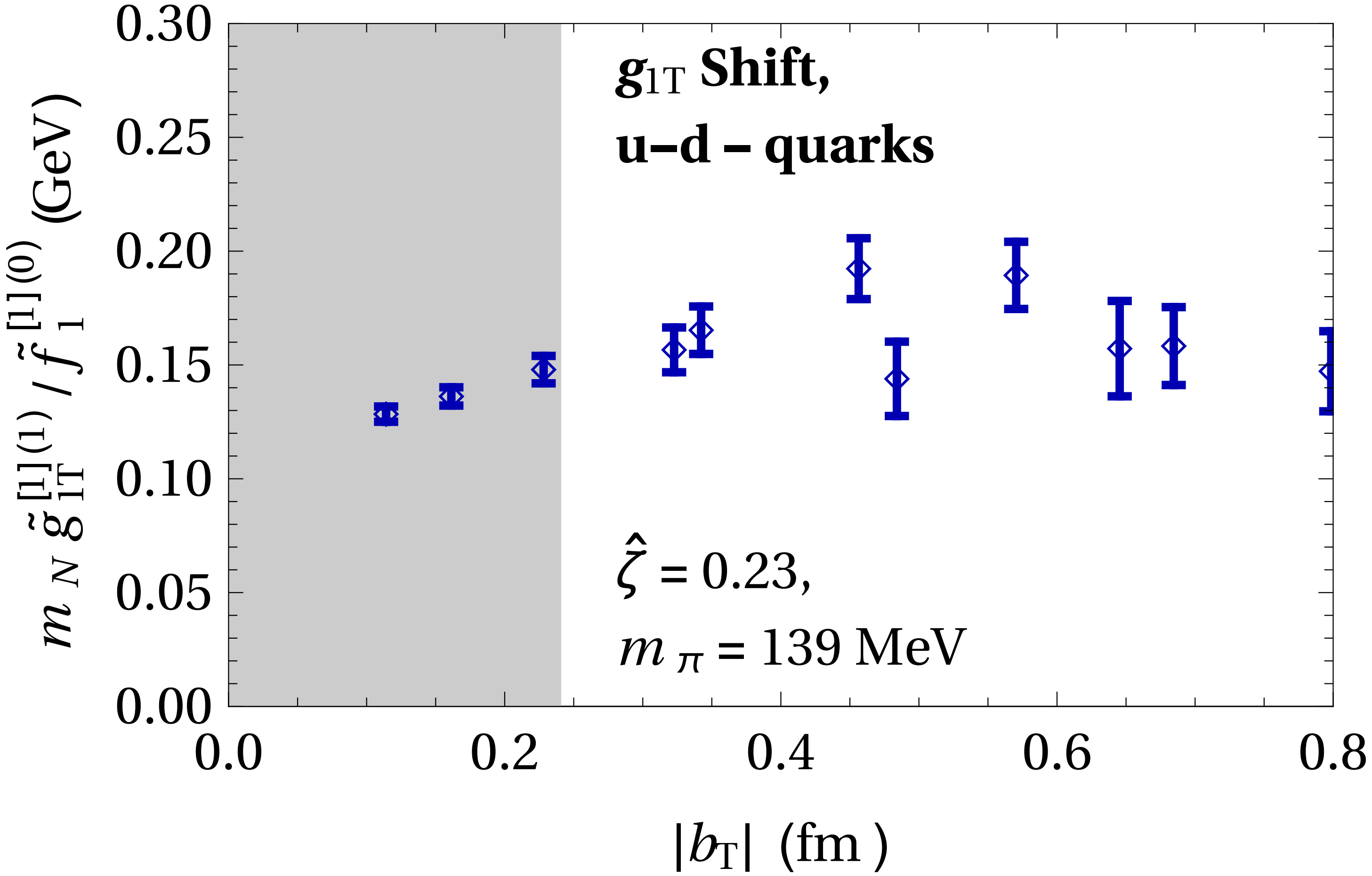
Results: g_{1T} worm gear shift

Dependence on staple extent; sequence of panels at different $|b_T|$



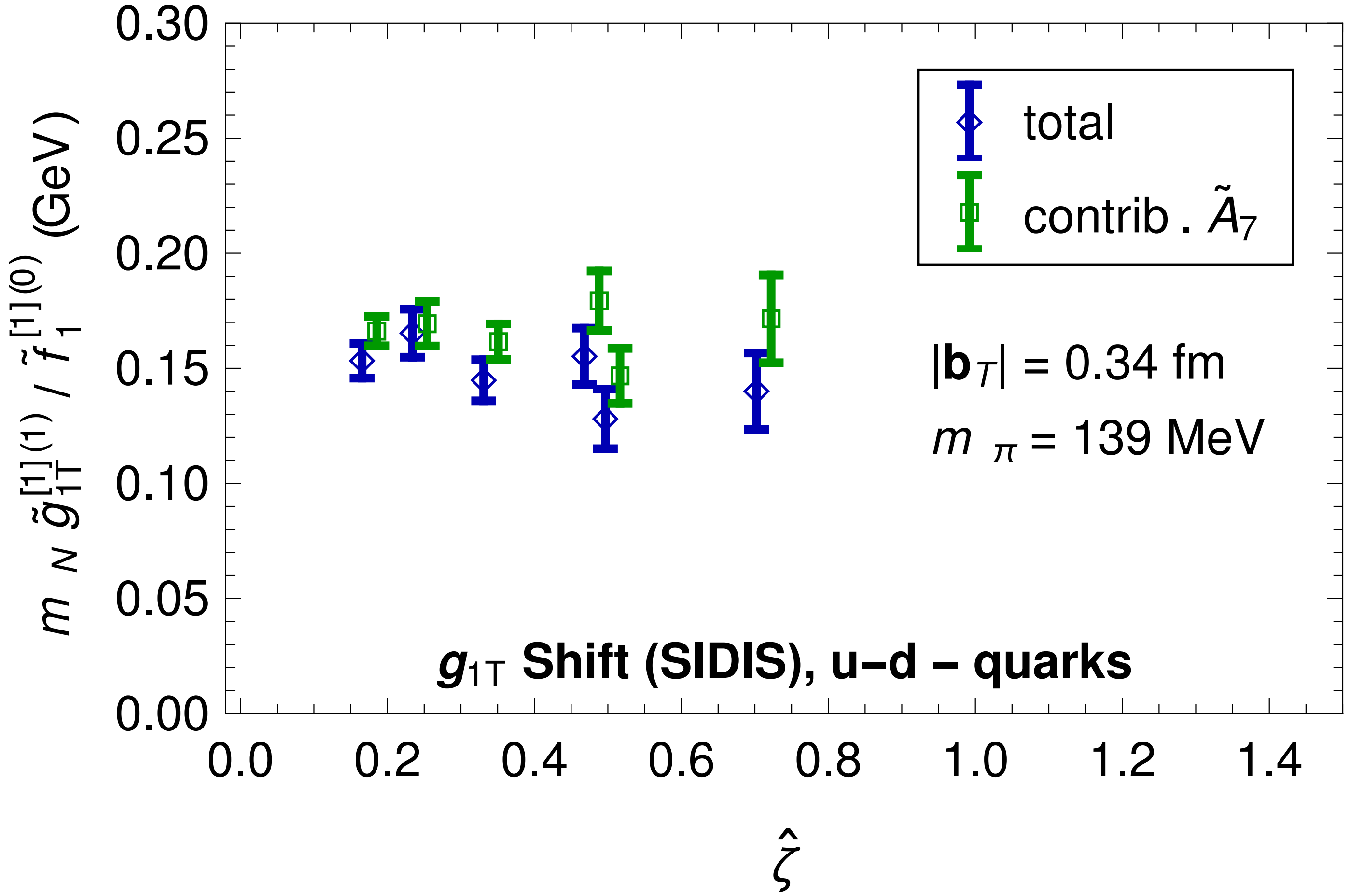
Results: g_{1T} worm gear shift

Dependence of SIDIS/DY limit on $|b_T|$



Results: g_{1T} worm gear shift

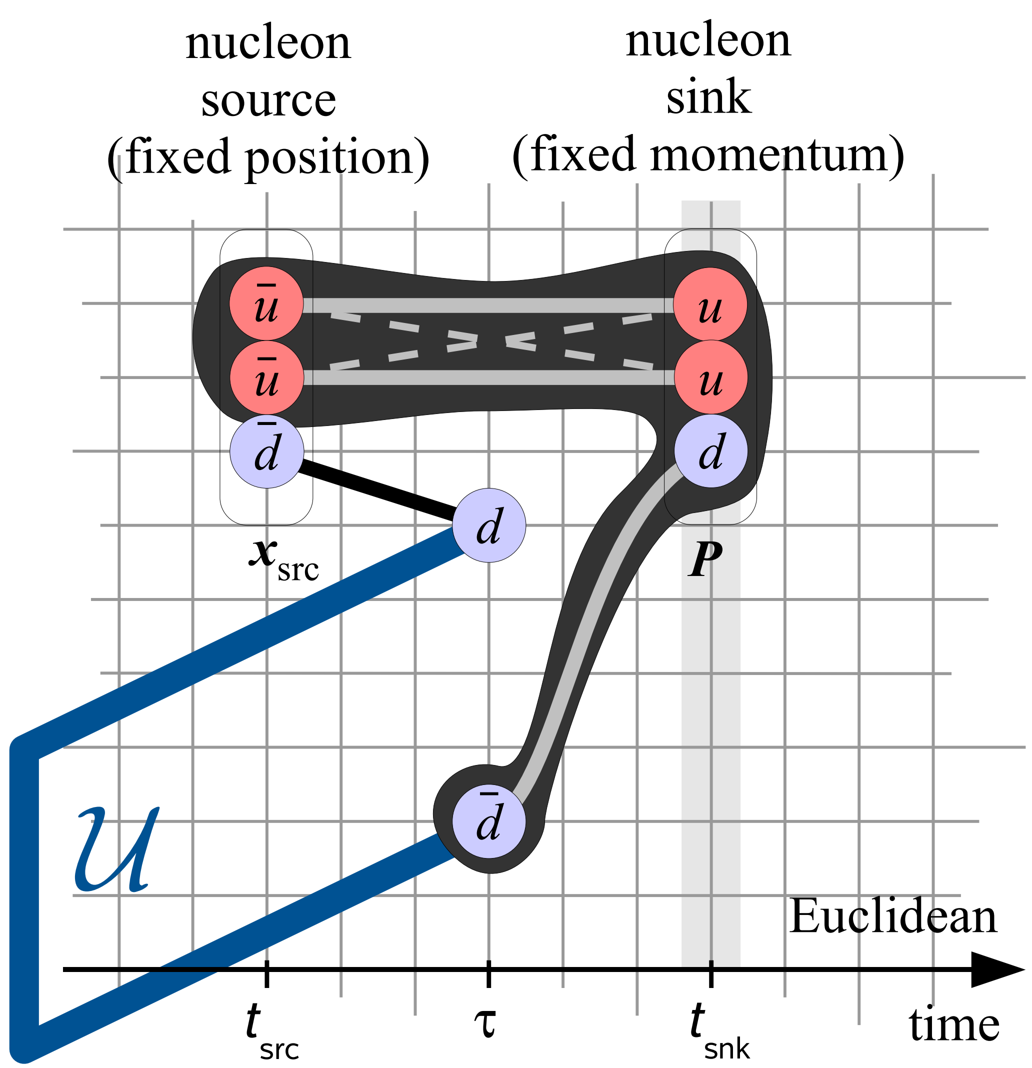
Dependence of SIDIS/DY limit on $\hat{\zeta}$



Further systematics

- Excited state contaminations
- Discretization effects, soft factor cancellation on the lattice in TMD ratios

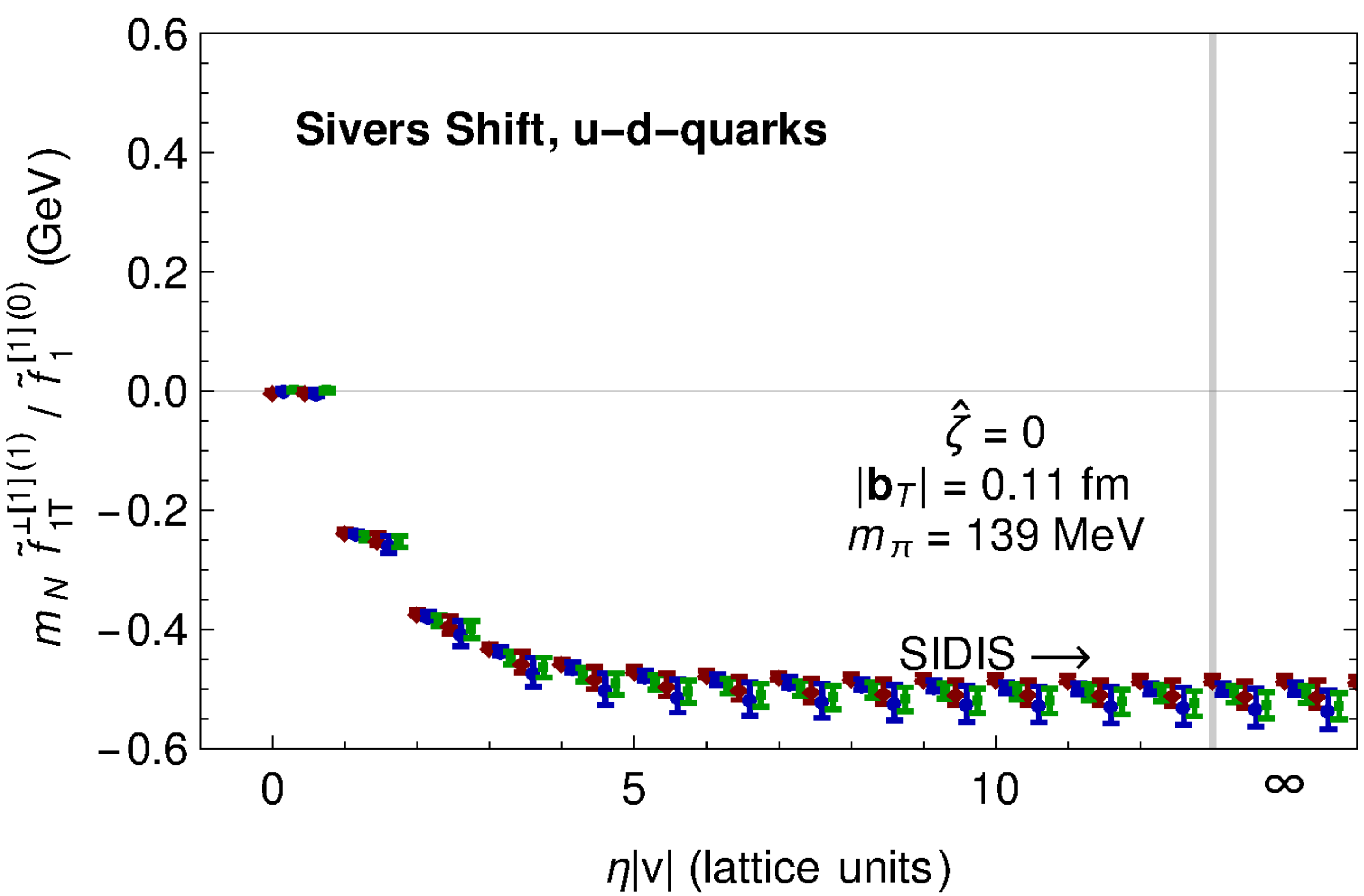
Excited state contaminations



- For finite $\tau - t_{src}$ or $t_{snk} - \tau$, the state at time τ still contains excited admixtures
- Control by performing calculations for a range of $t_{snk} - t_{src}$; extrapolate

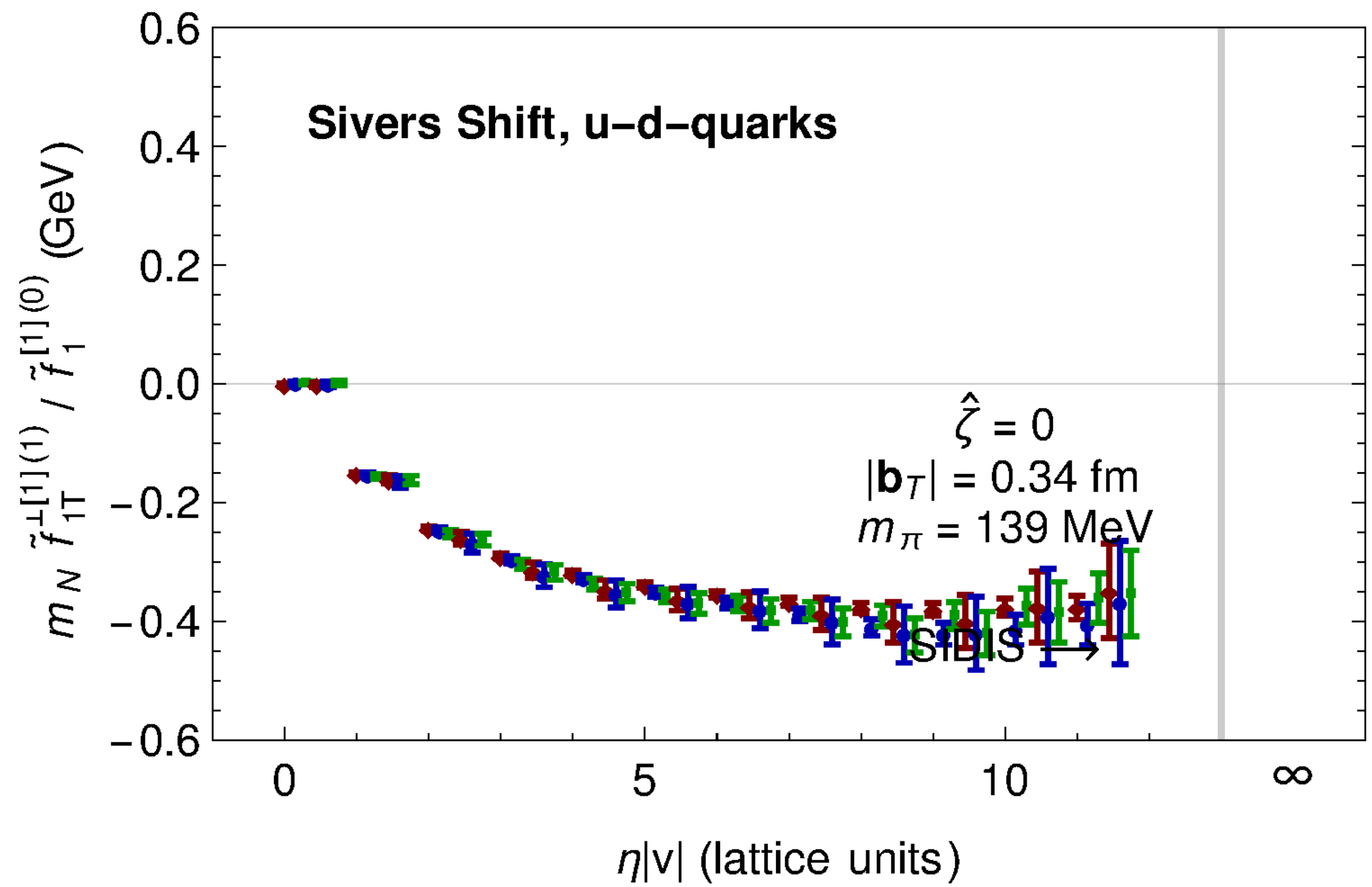
Excited state contaminations

Data obtained for $t_{snk} - t_{src} = 8, 9, 10, 11, 12$, and two-state fit



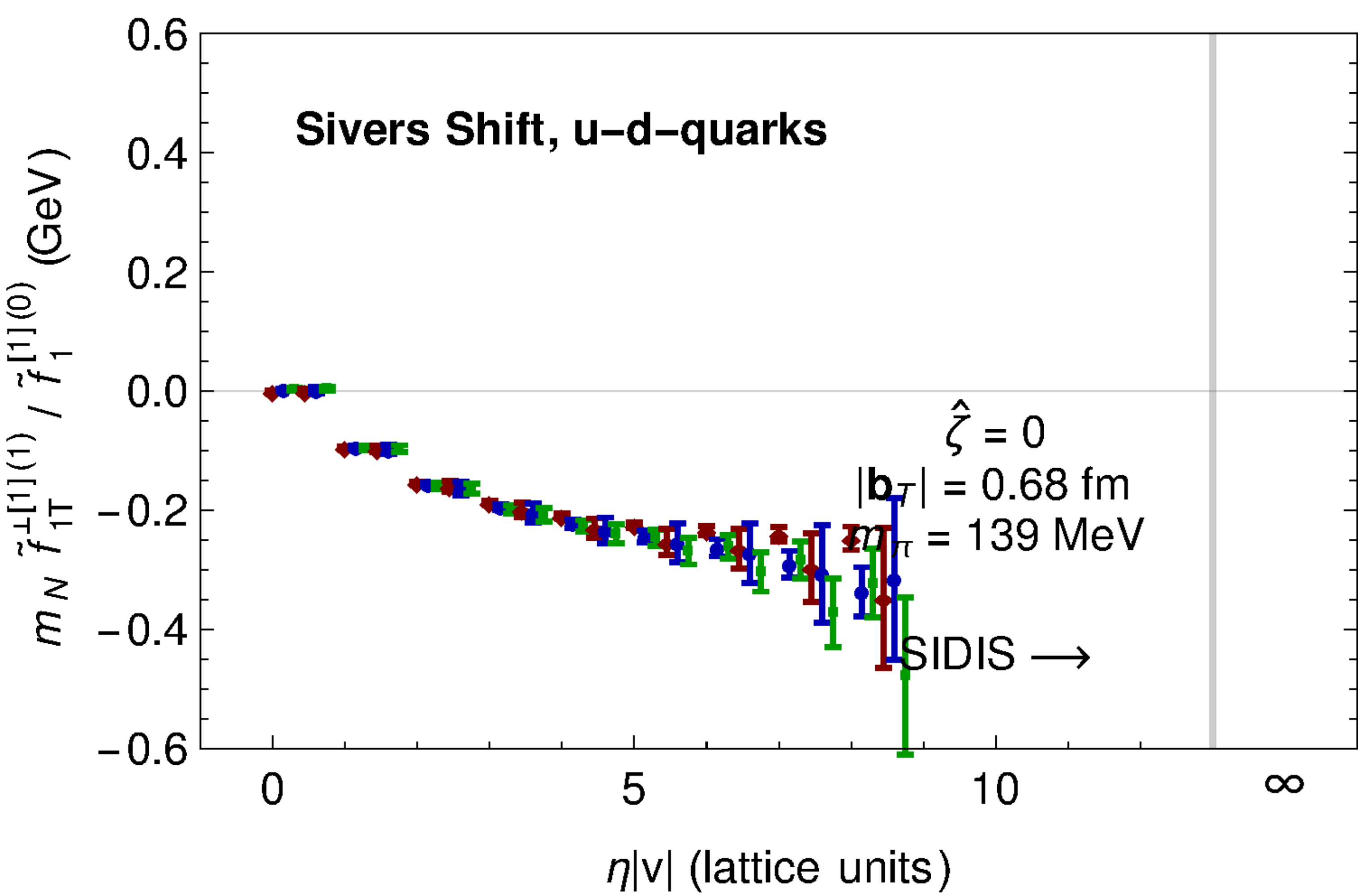
Excited state contaminations

Data obtained for $t_{snk} - t_{src} = 8, 9, 10, 11, 12$, and two-state fit



Excited state contaminations

Data obtained for $t_{snk} - t_{src} = 8, 9, 10, 11, 12$, and two-state fit



Discretization effects:

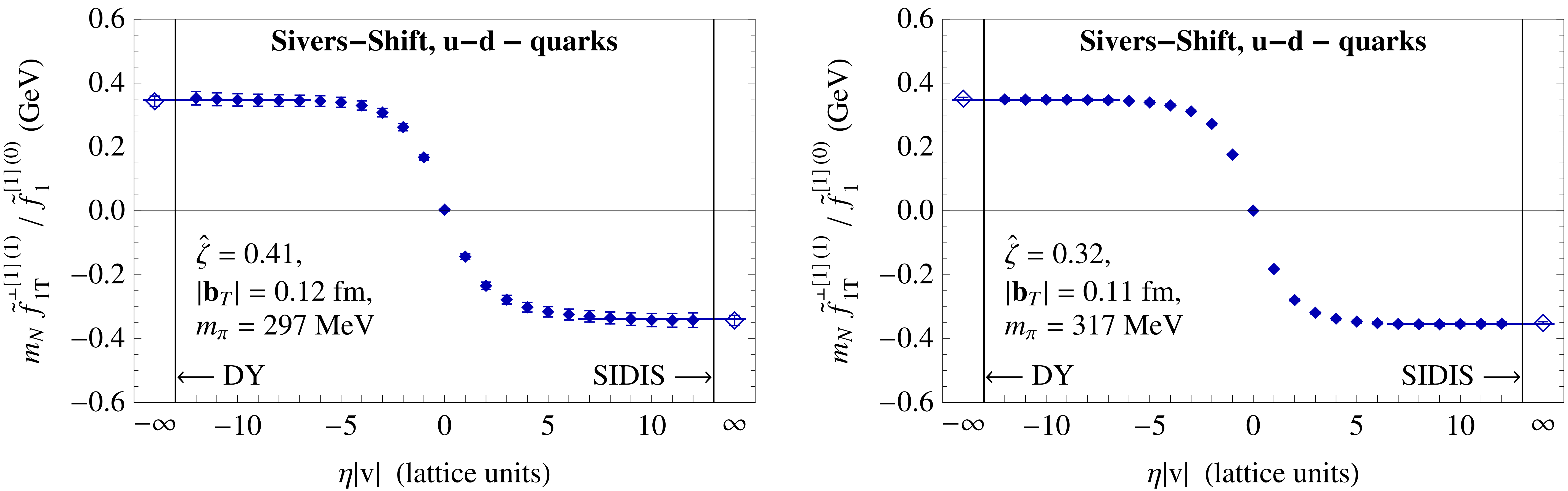
Comparison of

RBC/UKQCD DWF ensemble ($m_\pi = 297\,\mathbf{MeV}$, $a = 0.084\,\mathbf{fm}$)

with clover ensemble ($m_\pi = 317\,\mathbf{MeV}$, $a = 0.114\,\mathbf{fm}$)
produced by K. Orginos and JLab collaborators

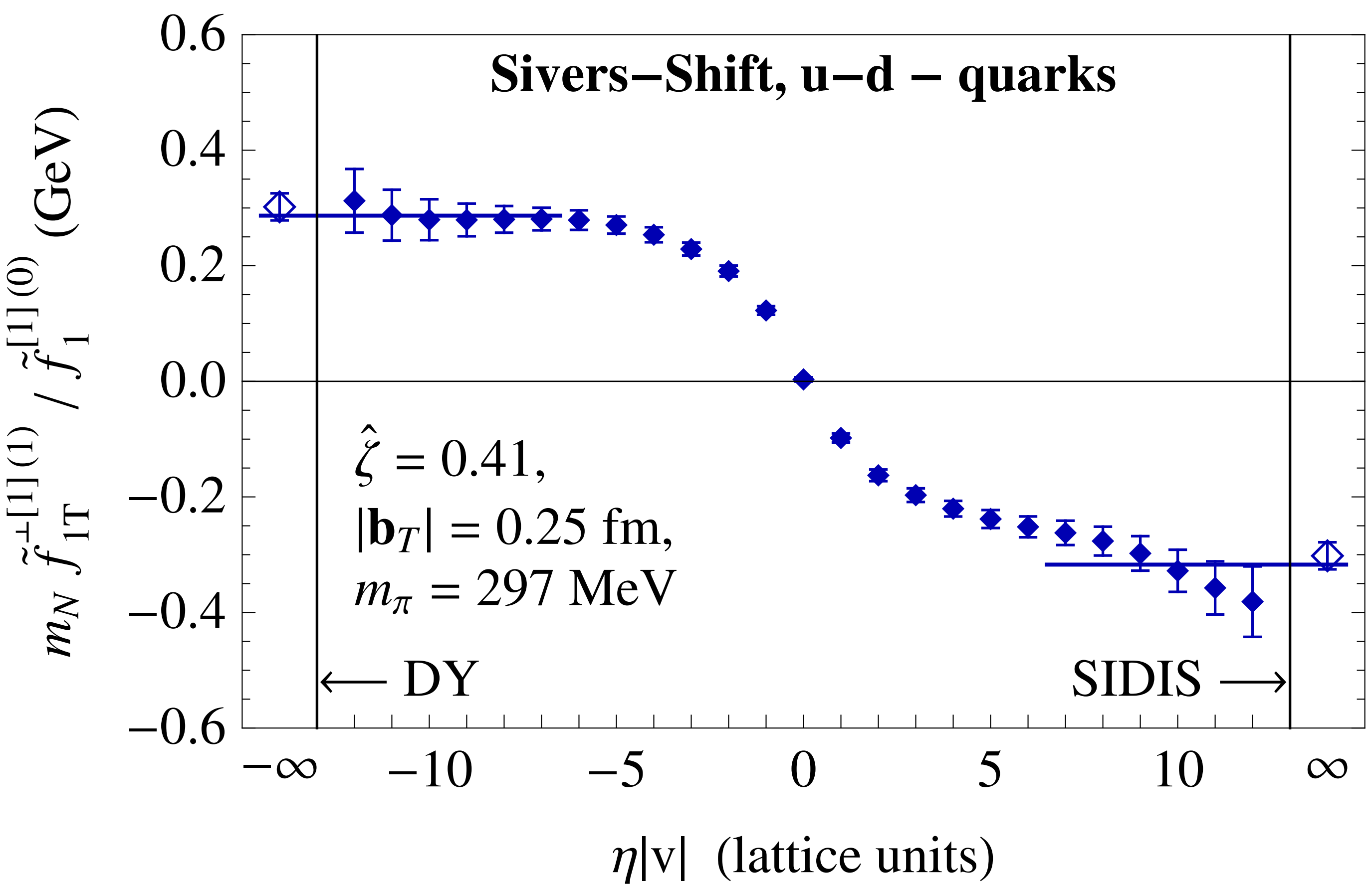
Results: Sivers shift

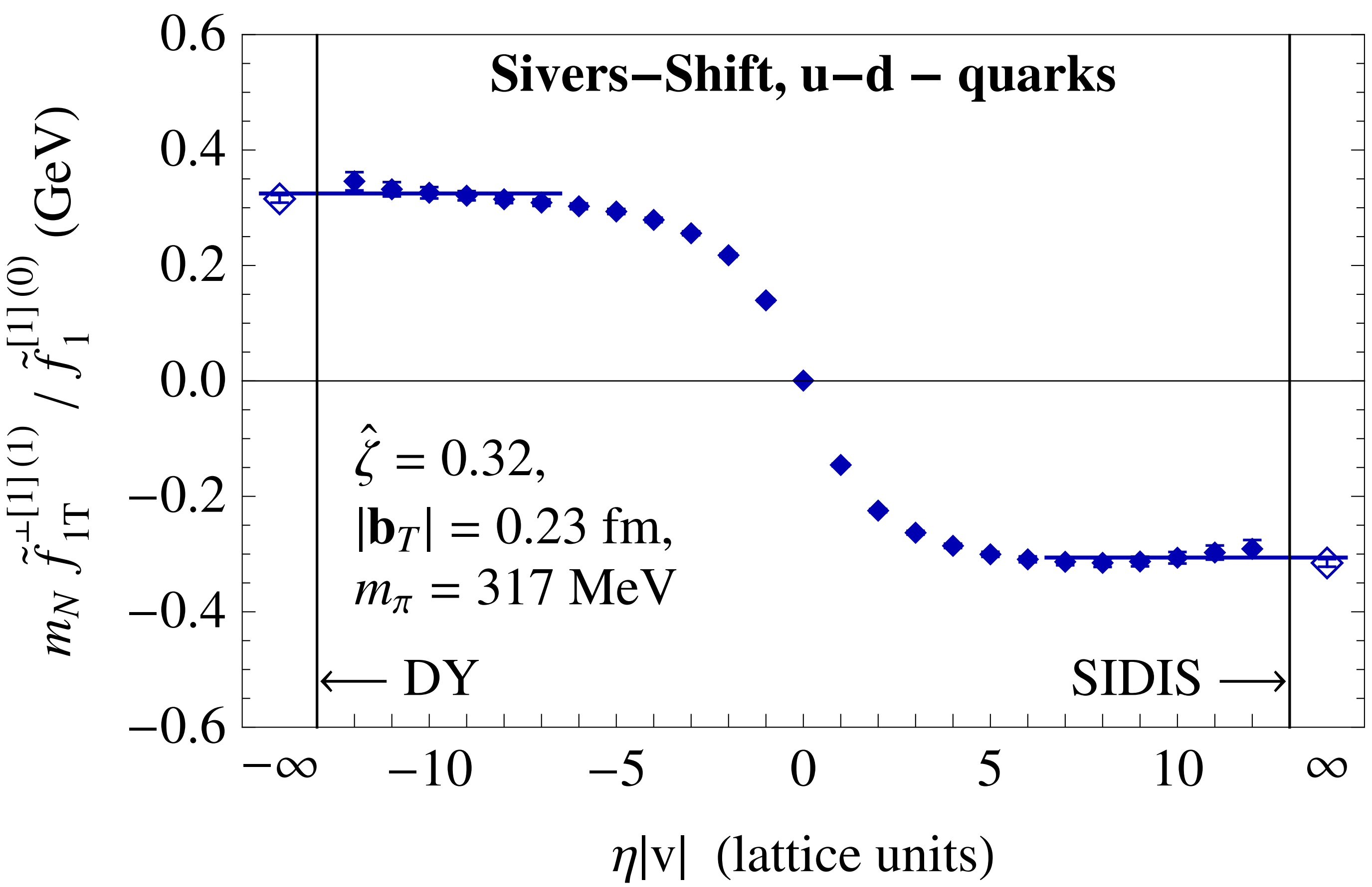
Dependence on staple extent; sequence of panels at different $|b_T|$



Results: Sivers shift

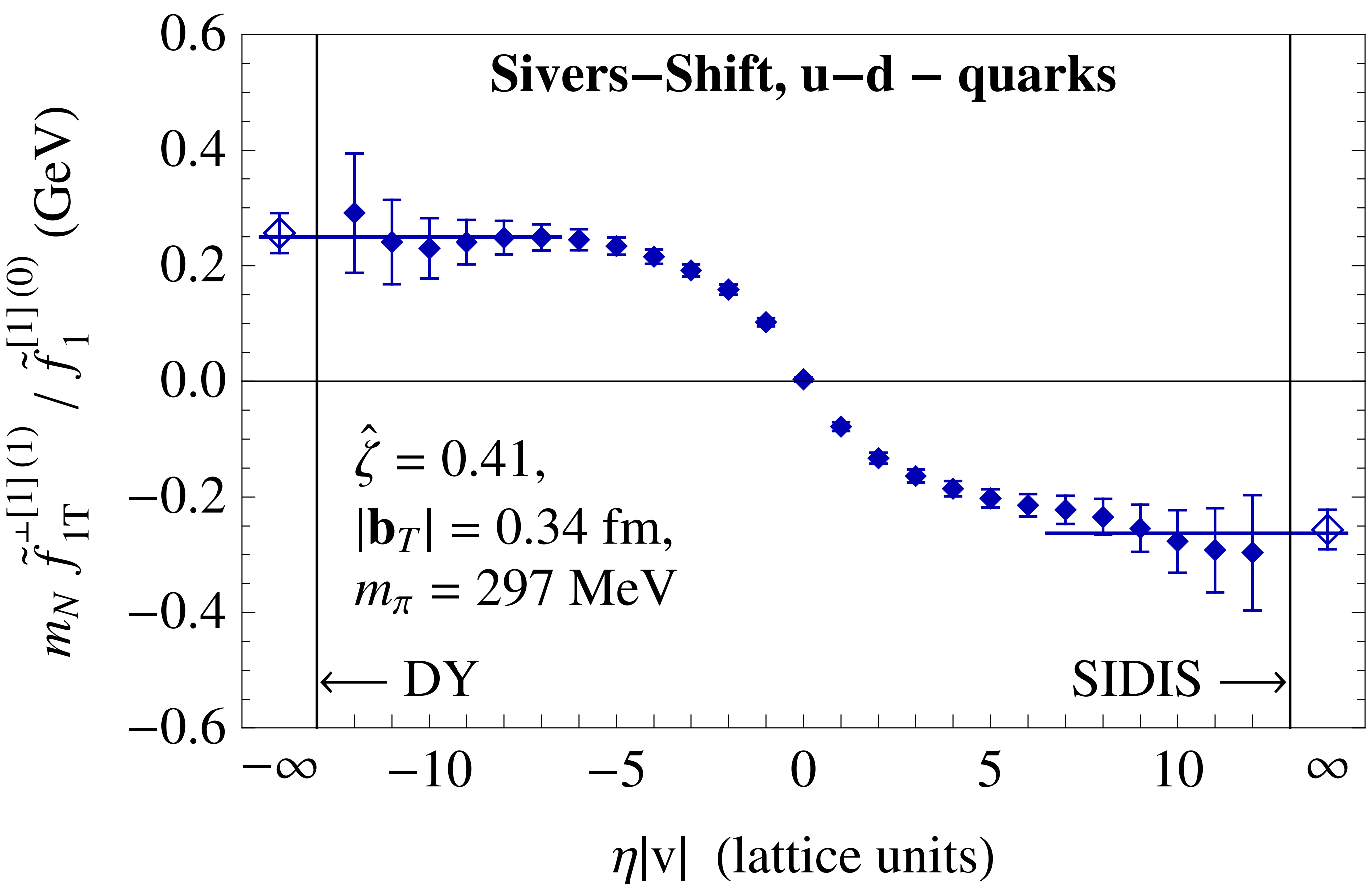
Dependence on staple extent; sequence of panels at different $|b_T|$

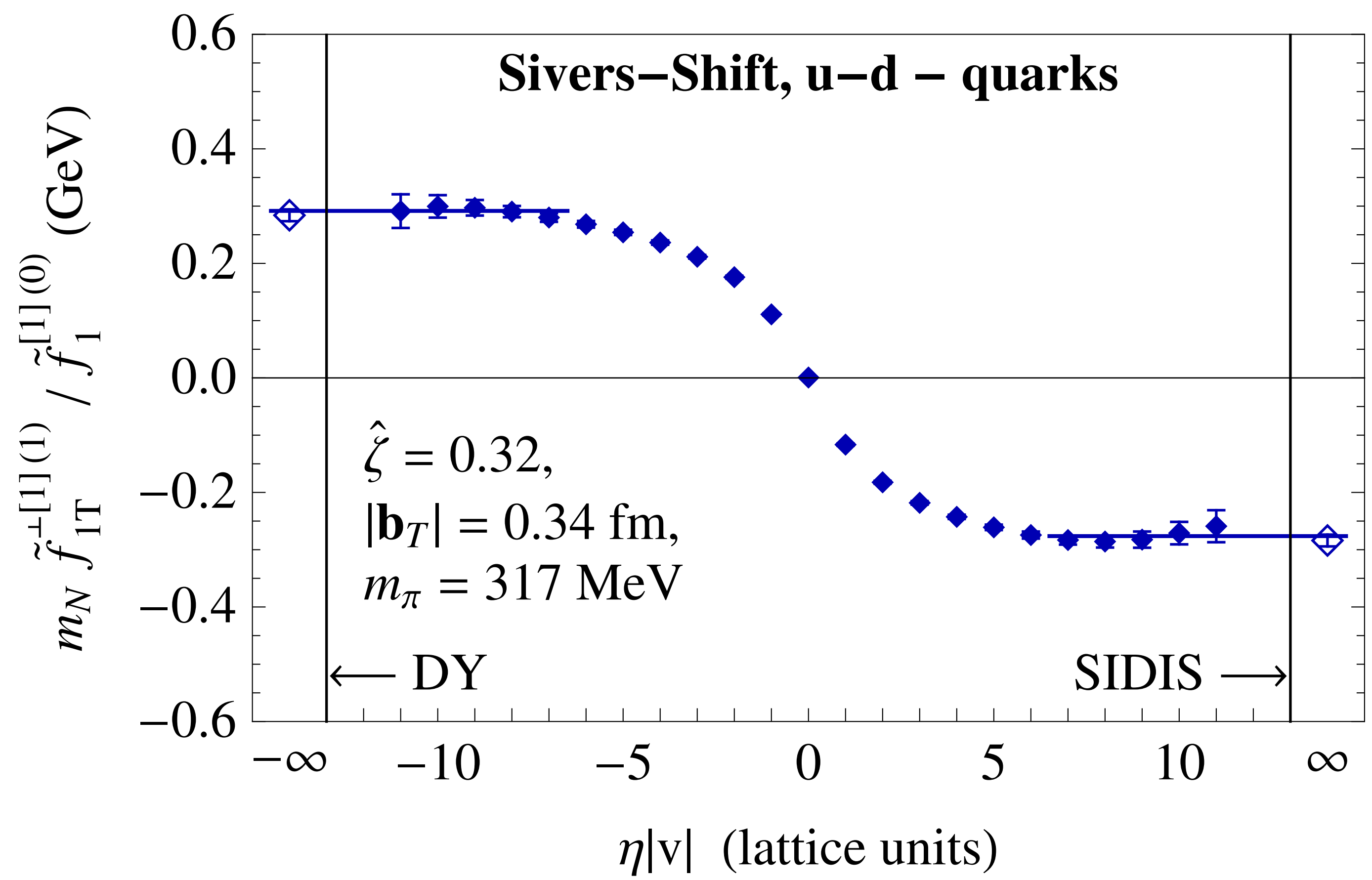




Results: Sivers shift

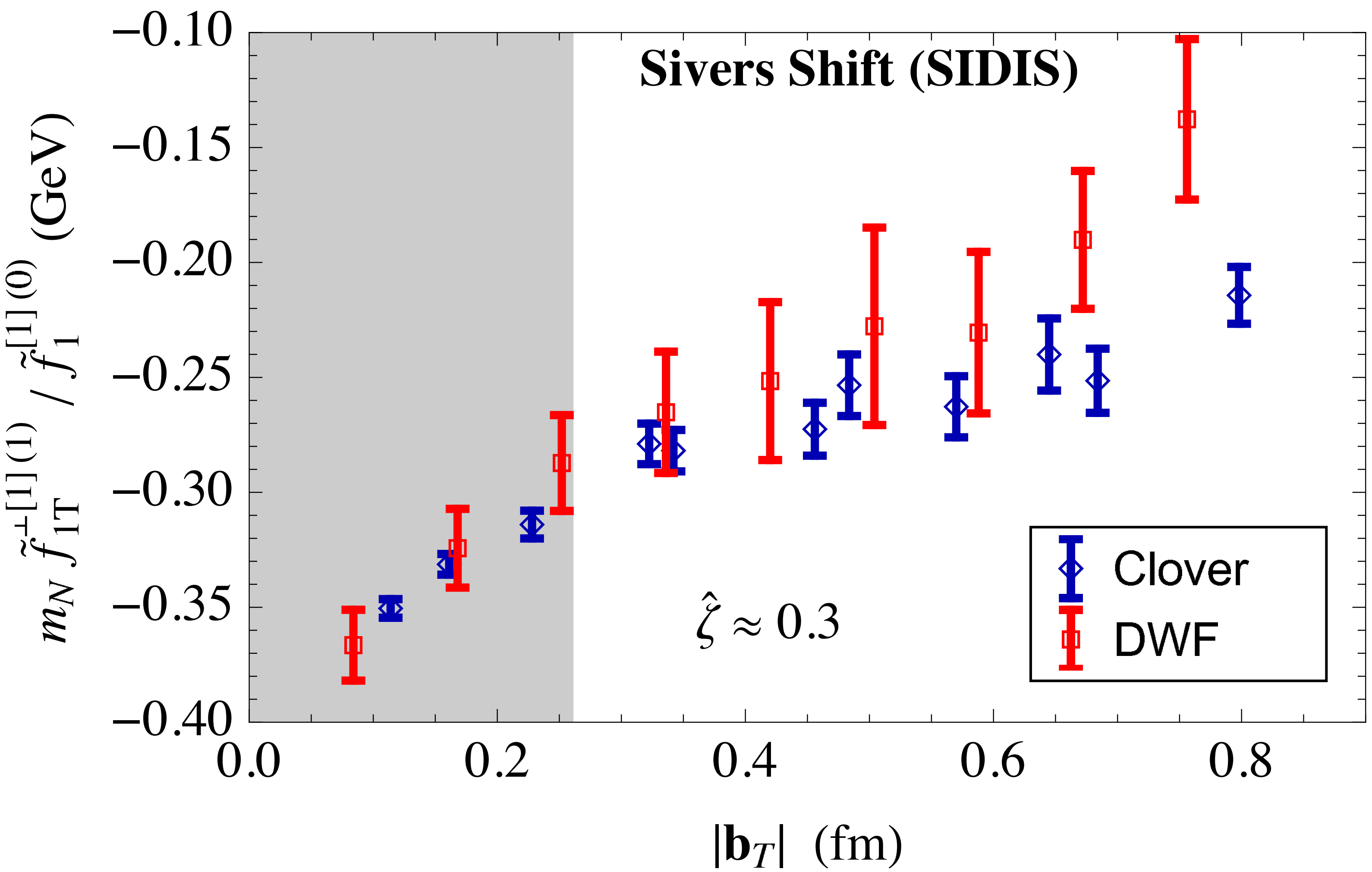
Dependence on staple extent; sequence of panels at different $|b_T|$





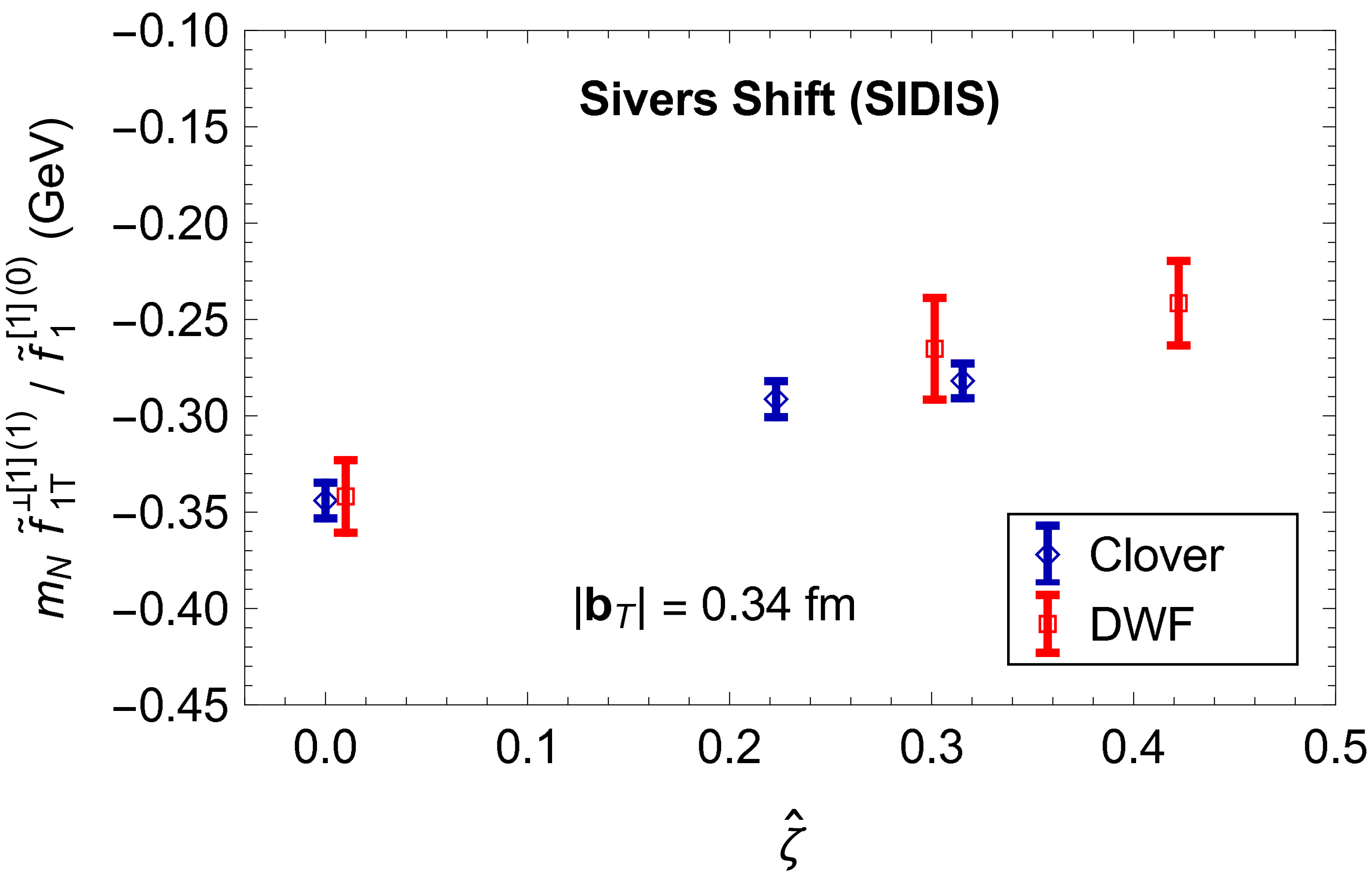
Results: Sivers shift

Dependence of SIDIS limit on $|b_T|$



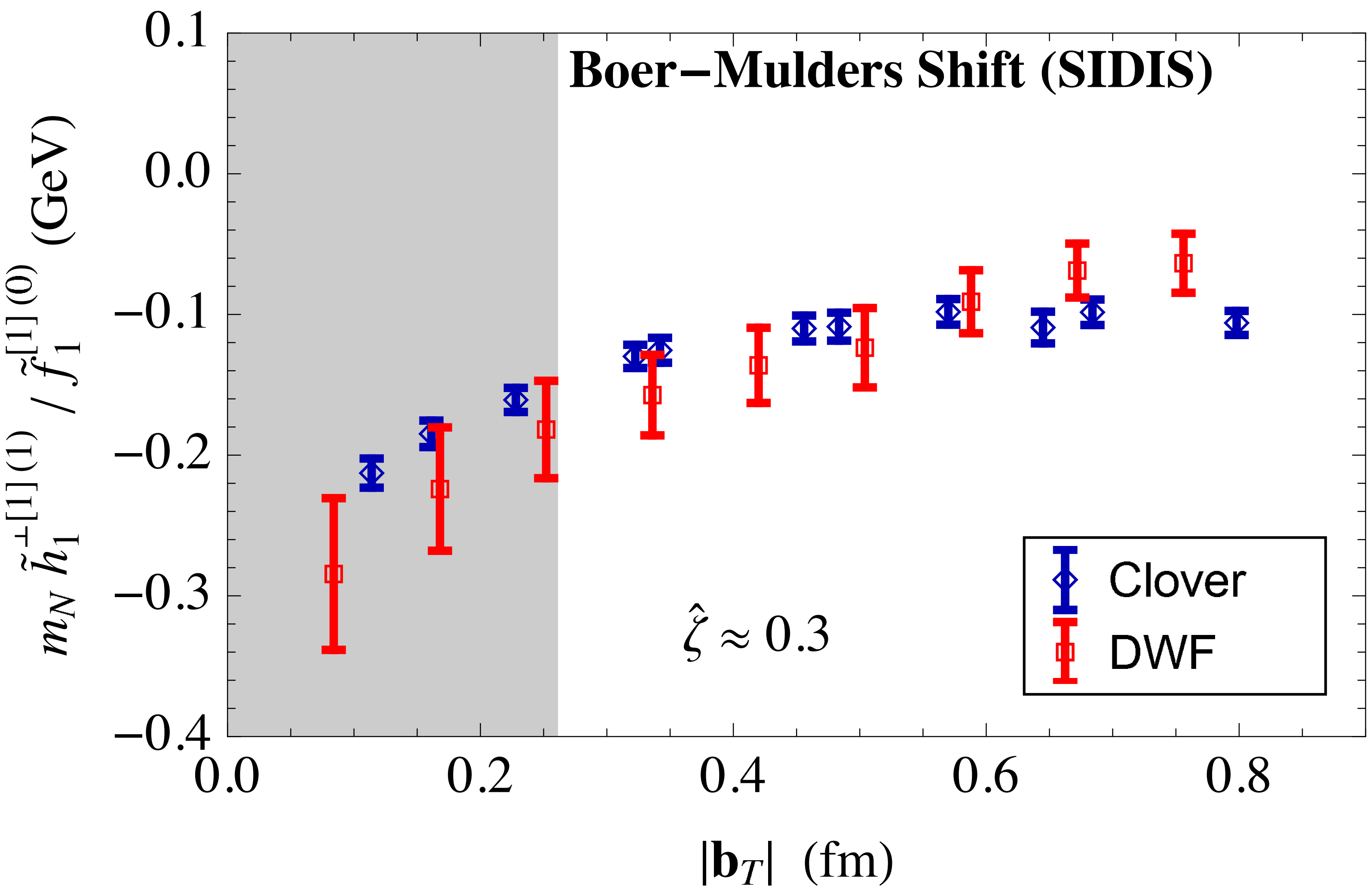
Results: Sivers shift

Dependence of SIDIS limit on $\hat{\zeta}$



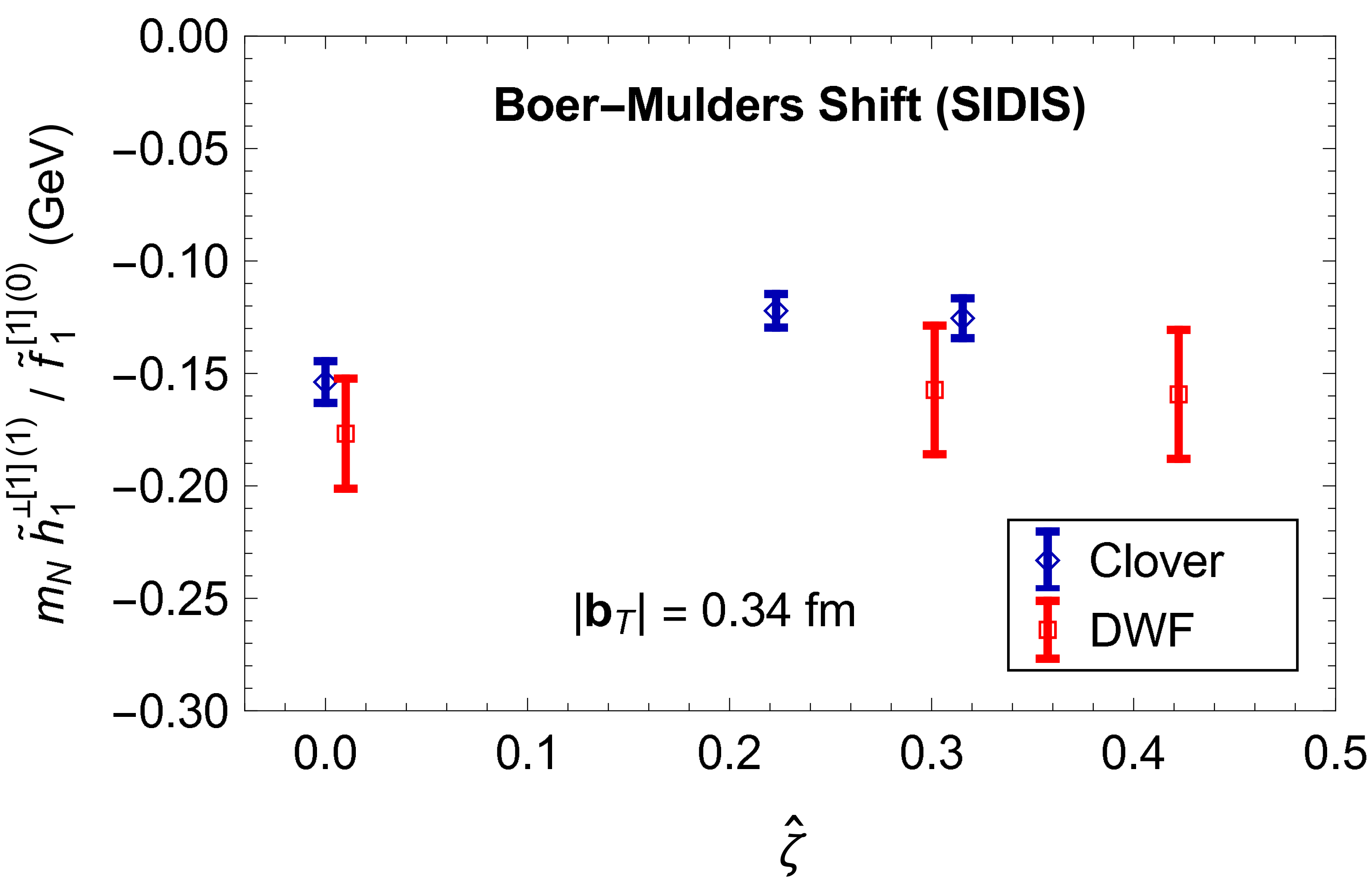
Results: Boer-Mulders shift

Dependence of SIDIS limit on $|b_T|$



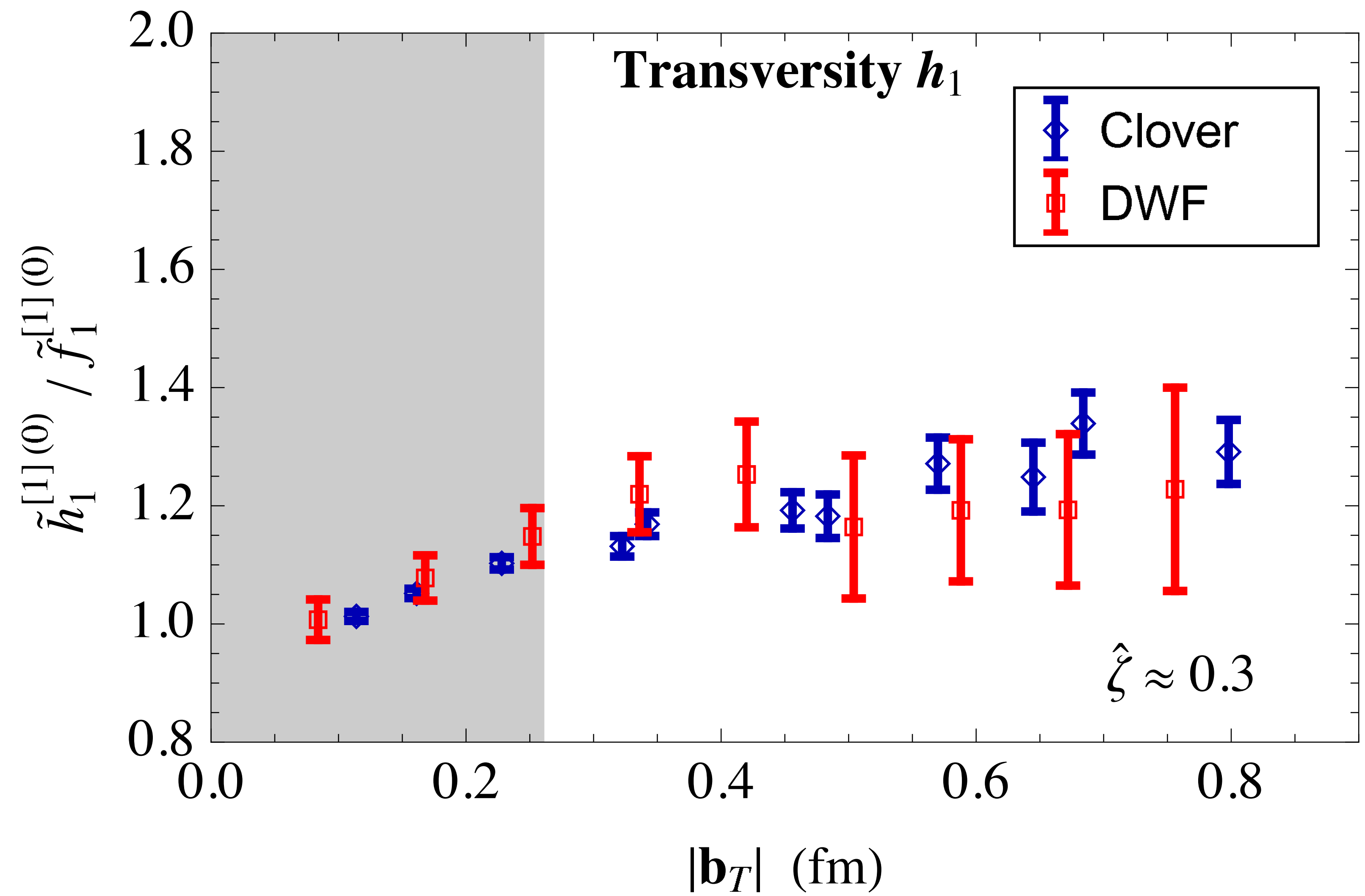
Results: Boer-Mulders shift

Dependence of SIDIS limit on $\hat{\zeta}$



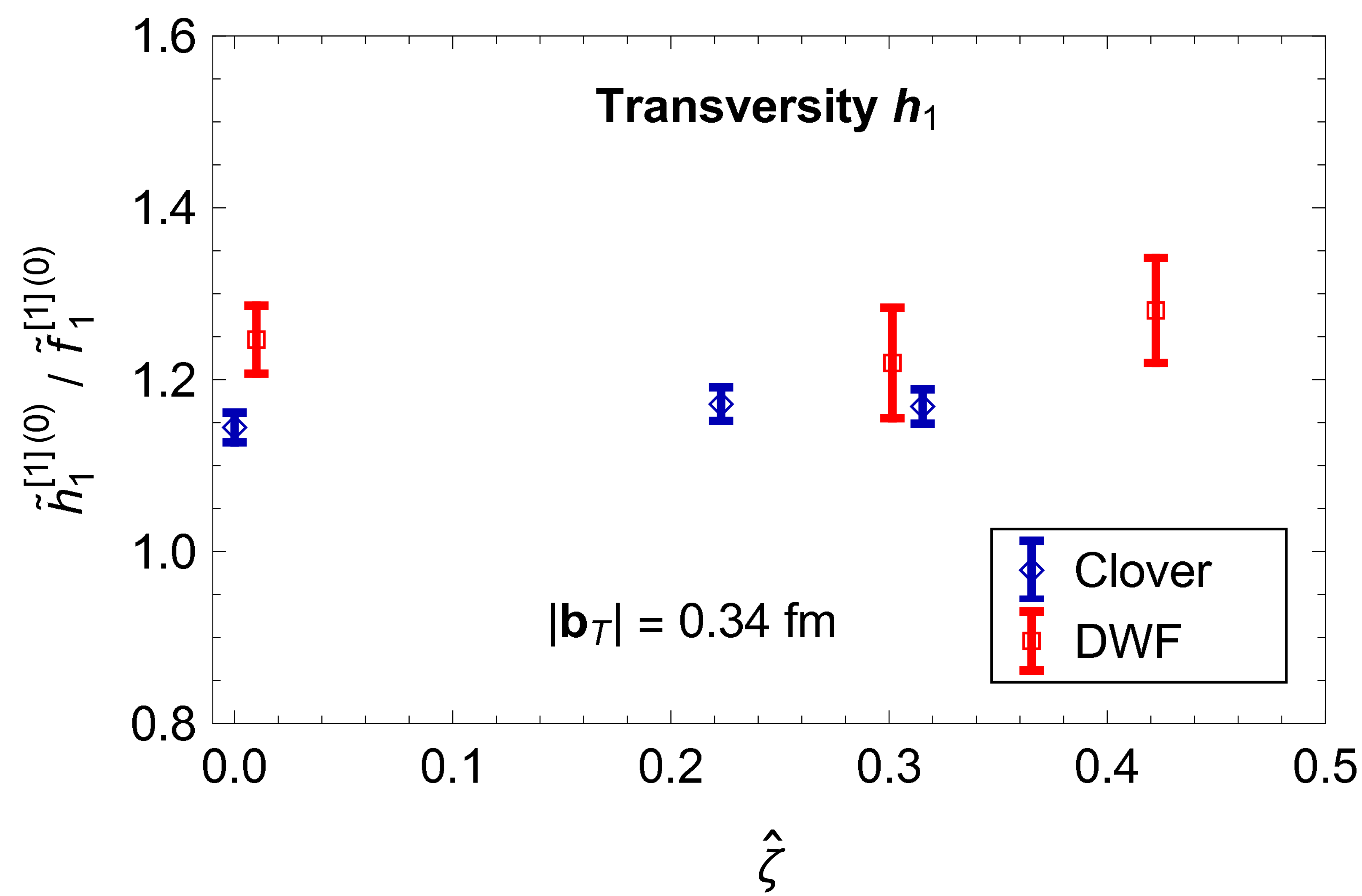
Results: Generalized Transversity

Dependence of SIDIS limit on $|b_T|$



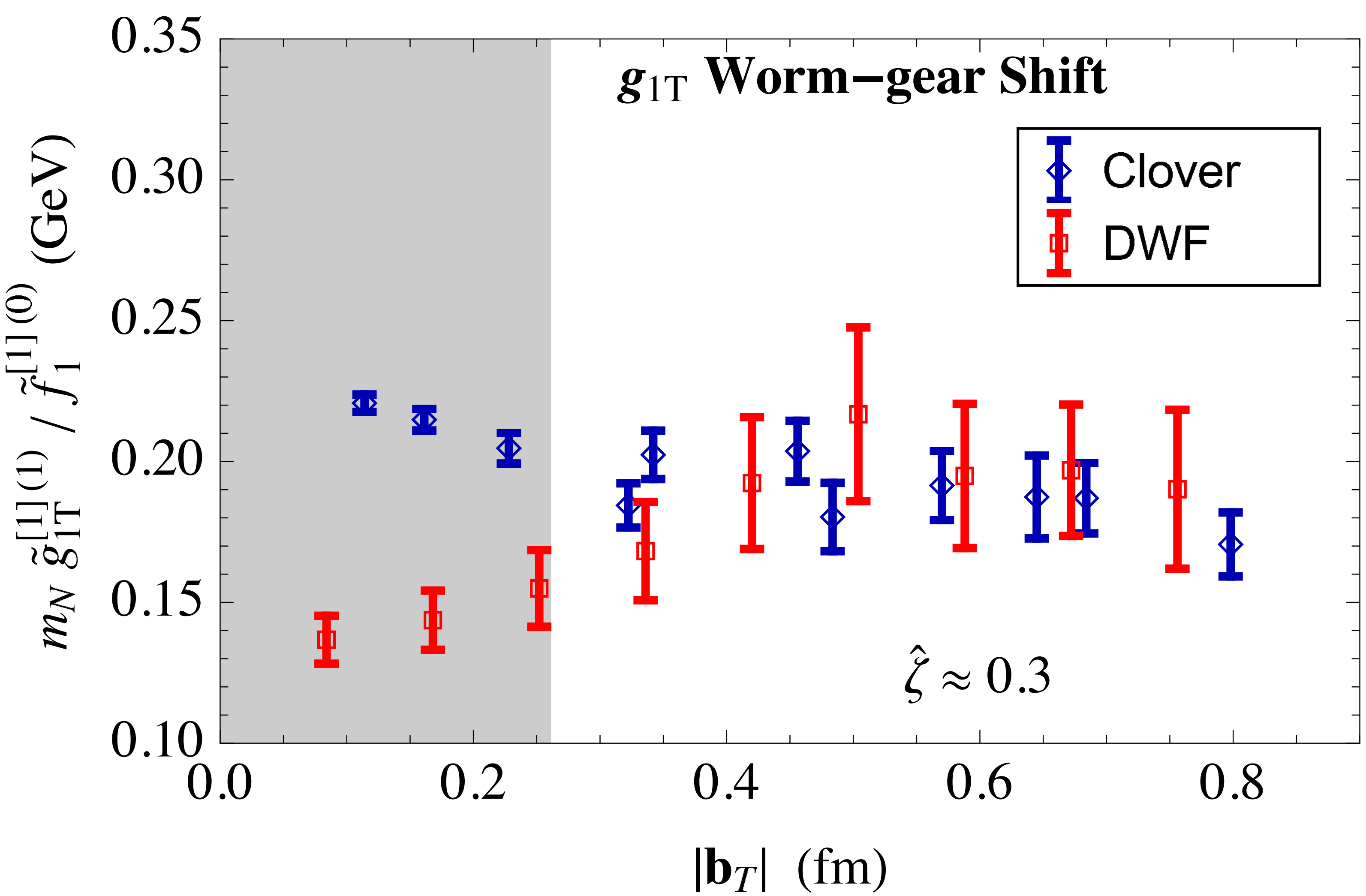
Results: Generalized Transversity

Dependence of SIDIS limit on $\hat{\zeta}$



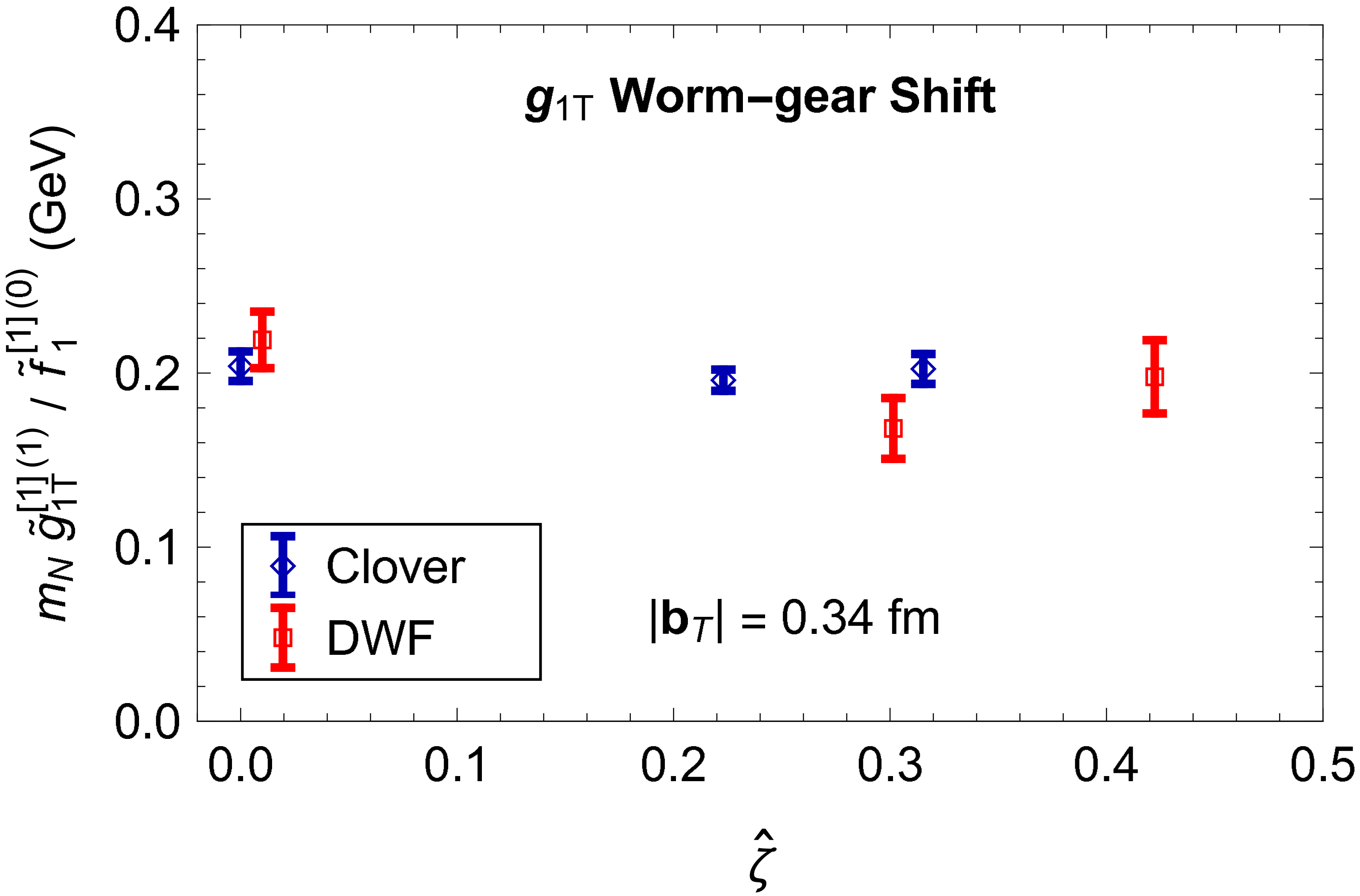
Results: g_{1T} worm gear shift

Dependence of SIDIS limit on $|b_T|$



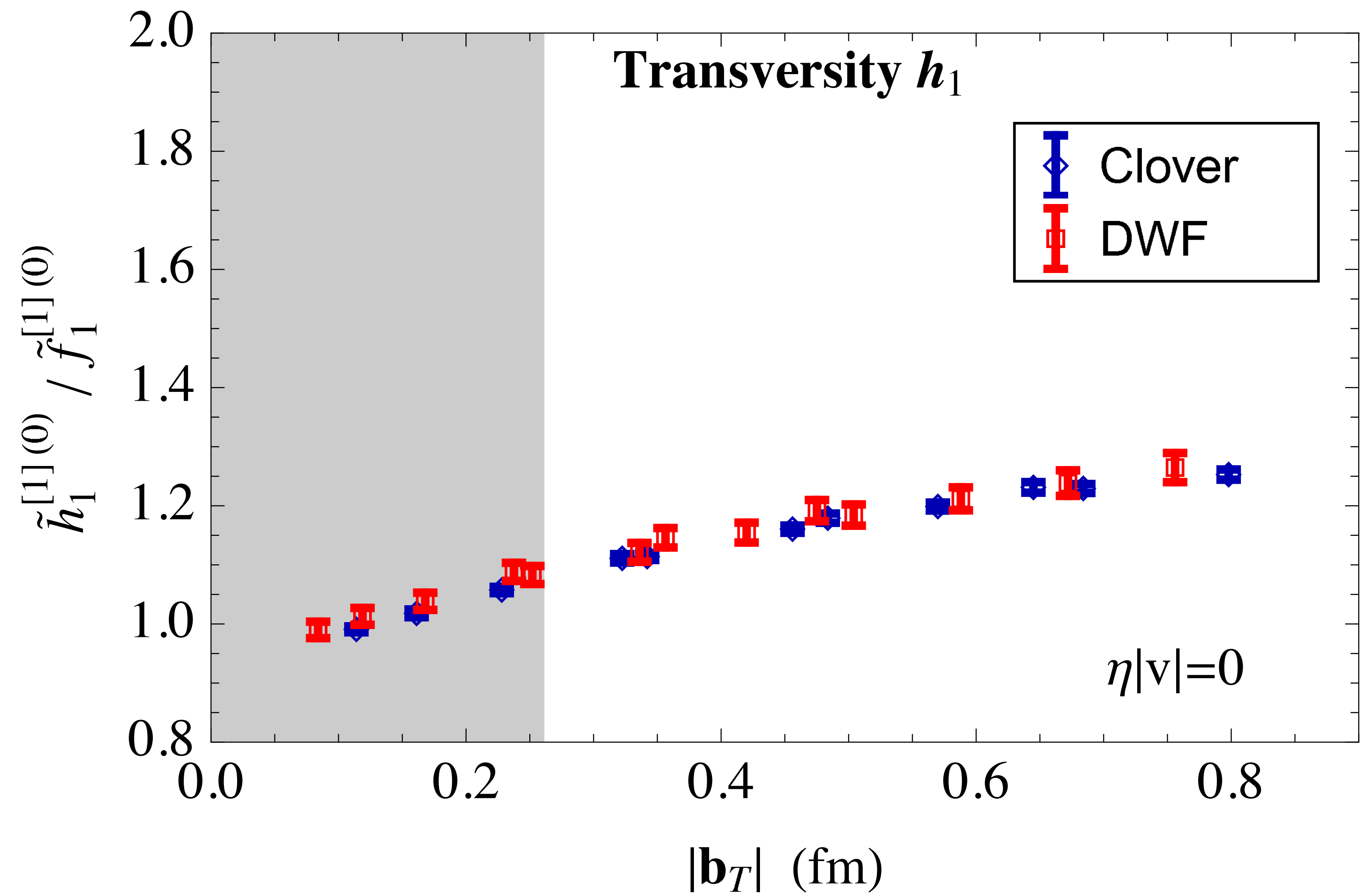
Results: g_{1T} worm gear shift

Dependence of SIDIS limit on $\hat{\zeta}$



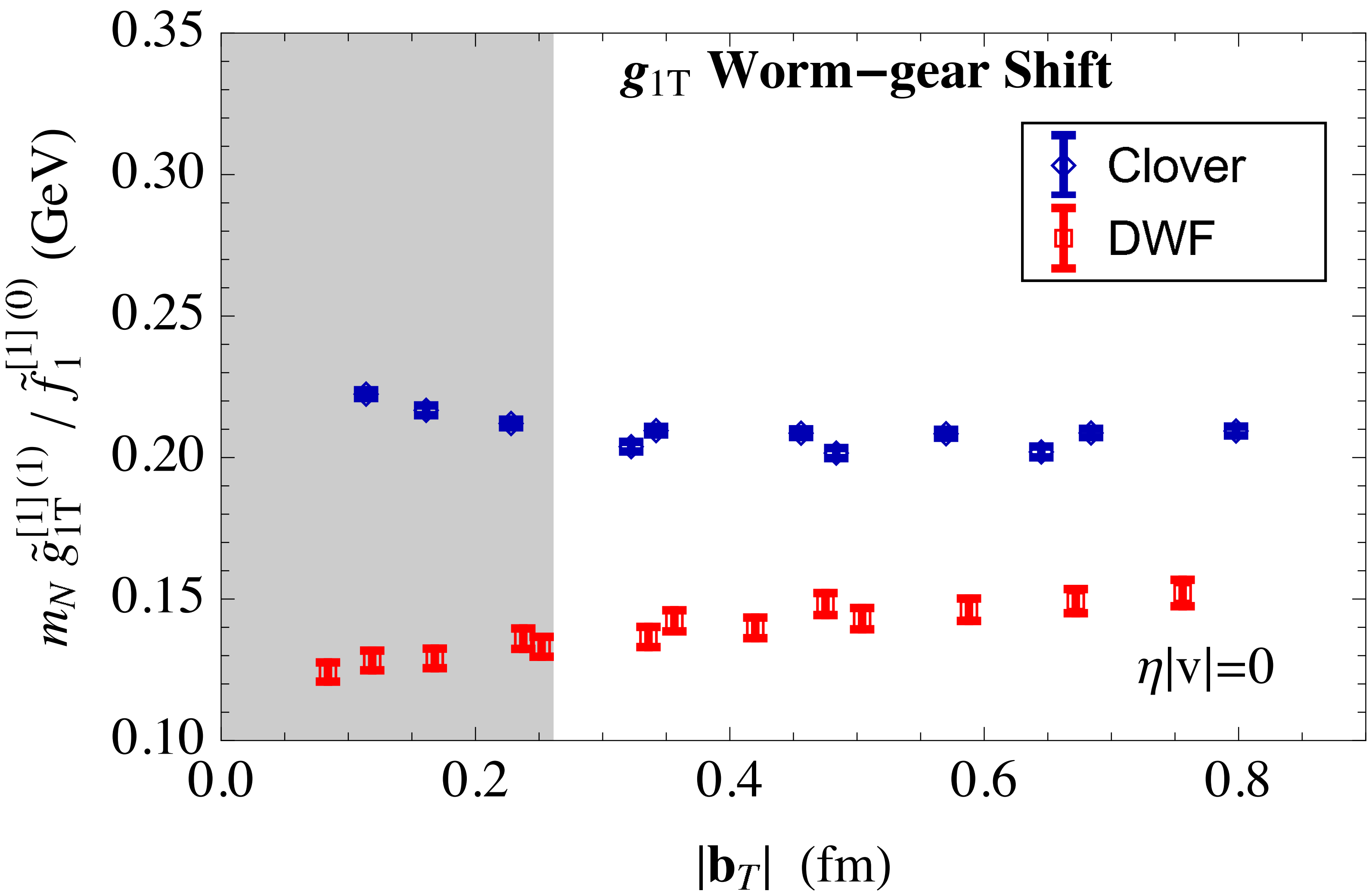
Results: Generalized Transversity, straight link

Dependence on $|b_T|$



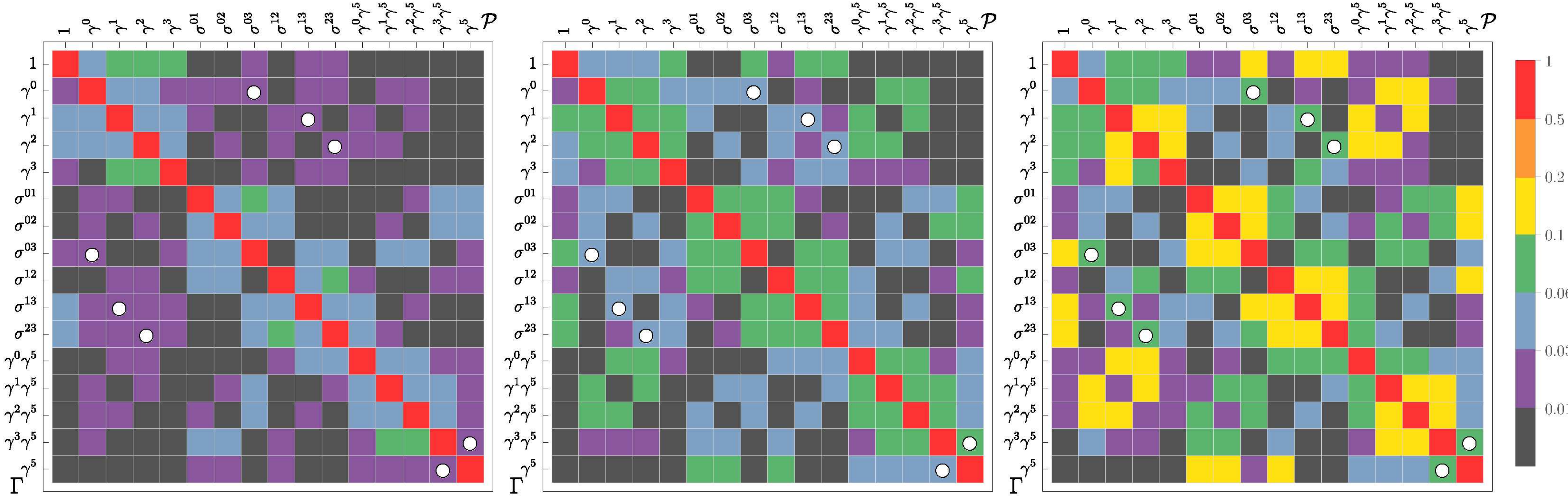
Results: g_{1T} worm gear shift, straight link

Dependence on $|b_T|$



Evidence of operator mixing? $\longrightarrow$ Lattice perturbation theory M. Constantinou et al.

Operator mixing pattern for clover fermions



$b_T/a = 3, 7, 11$ from left to right; $\eta/a = 14$.

P. Shanahan, M. Wagman and Y. Zhao, Phys. Rev. D 101 (2020) 074505.

Proton spin puzzle

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q L_q + J_g \qquad \text{(Ji)}$$

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q \mathcal{L}_q + \Delta g + \mathcal{L}_g \qquad \text{(Jaffe-Manohar)}$$

. . . and many more (in fact, we will see a continuous interpolation between the two . . .)

There isn't one unique way of separating quark and gluon orbital angular momentum – the different decompositions have different, legitimate meanings.

Quark orbital angular momentum

Interpreting terms in the energy-momentum tensor:

$$L_q \quad \sim \quad -i\psi^\dagger(\vec{r} \times \vec{D})_z\psi$$

Can be obtained from $L_q = J_q - S_q$, where S_q and J_q can be related to GPDs (Ji sum rule) – this has been used in Lattice QCD.

$$\mathcal{L}_q \quad \sim \quad -i\psi^\dagger(\vec{r} \times \vec{\partial})_z\psi \qquad\qquad \text{in light cone gauge}$$

Not accessible in Lattice QCD using traditional methods.

Quark Orbital Angular Momentum

$$L_3^{\mathcal{U}} = \int dx \int d^2k_T \int d^2r_T \, (r_T \times k_T)_3 \, \mathcal{W}^{\mathcal{U}}(x, k_T, r_T)$$

Wigner distribution

$$= - \int dx \int d^2k_T \, \frac{k_T^2}{m^2} \, F_{14}(x, k_T^2, k_T \cdot \Delta_T, \Delta_T^2) \Big|_{\Delta_T = 0}$$

Generalized transverse momentum-dependent parton distribution (GTMD)

$$= \epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2) \gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) \mid p, S \rangle \Big|_{z^+ = z^- = 0\,,\ \Delta_T = 0\,,\ z_T \to 0}$$

Y. Hatta, X. Ji, M. Burkardt:
Staple-shaped $\mathcal{U}[-z/2, z/2] \longrightarrow$ Jaffe-Manohar OAM
Straight $\mathcal{U}[-z/2, z/2] \longrightarrow$ Ji OAM

Connection to GTMDs – A. Metz, M. Schlegel, C. Lorcé, B. Pasquini . . .

Direct evaluation of quark orbital angular momentum

$$L_3^{\mathcal{U}} = \int dx \int d^2k_T \int d^2r_T \, (r_T \times k_T)_3 \, \mathcal{W}^{\mathcal{U}}(x, k_T, r_T) \qquad \text{Wigner distribution}$$

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\rightarrow 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\rightarrow 0}}$$

n : Number of valence quarks

$p' = P + \Delta_T/2$, $p = P - \Delta_T/2$, P, S in 3-direction, $P \rightarrow \infty$

This is the same type of operator as used in TMD studies – generalization to off-forward matrix element adds transverse position information

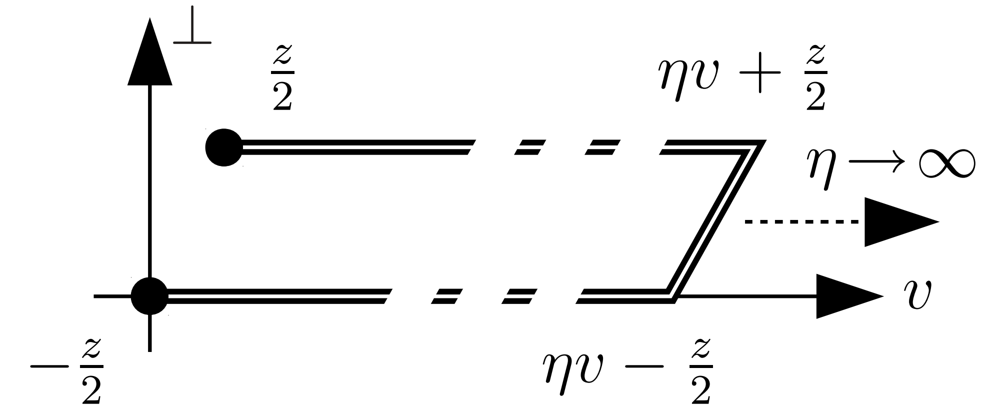
Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}}\,\langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Role of the gauge link $\mathcal{U}$:

Y. Hatta, M. Burkardt:

- Straight $\mathcal{U}[-z/2, z/2] \longrightarrow$ Ji OAM
- Staple-shaped $\mathcal{U}[-z/2, z/2] \longrightarrow$ Jaffe-Manohar OAM
- Difference is torque accumulated due to final state interaction



Direct evaluation of quark orbital angular momentum

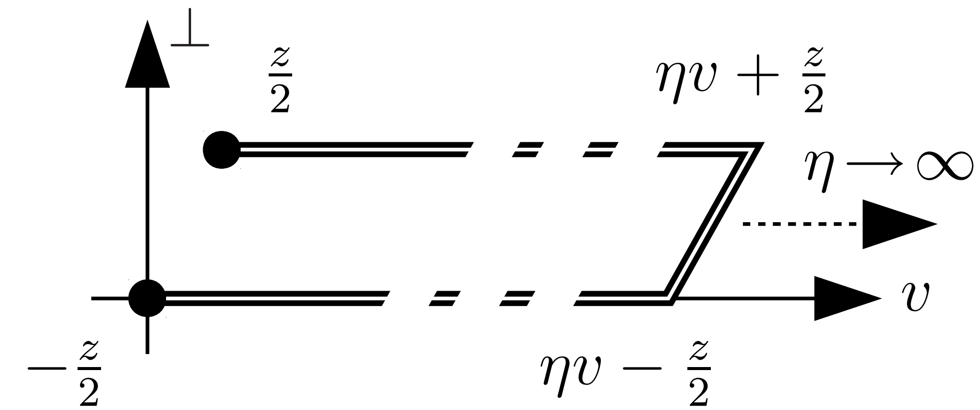
$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}}\, \langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Role of the gauge link $\mathcal{U}$:

Direction of staple taken off light cone (rapidity divergences)

Characterized by Collins-Soper parameter $\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$

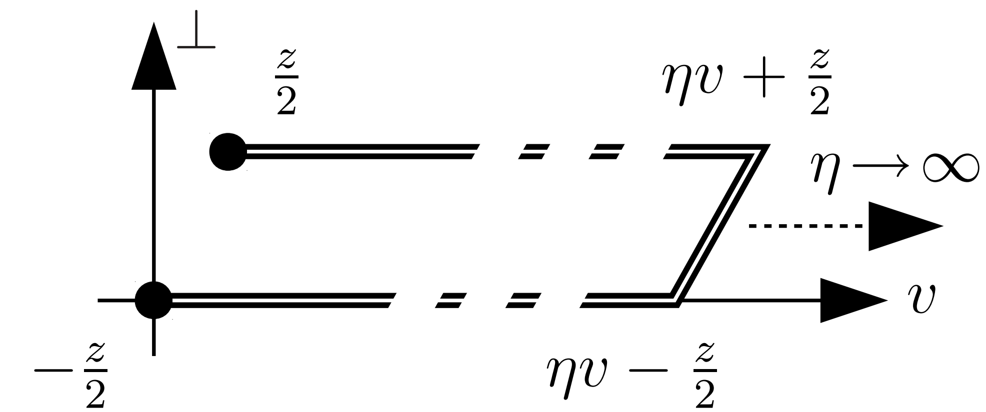
Are interested in $\hat{\zeta} \longrightarrow \infty$; synonymous with $P \longrightarrow \infty$ in the frame of the lattice calculation $(v = e_3)$



Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Parameters to consider: Δ, $\hat{\zeta}$, z, η



Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle \big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle \big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}$$

Perform Δ_T-derivative using direct derivative method

$$\left.\frac{\partial f}{\partial z_{T,i}}\right|_{z_{T,i}=0} = \frac{1}{2a}\left(f(ae_i) - f(-ae_i)\right)$$

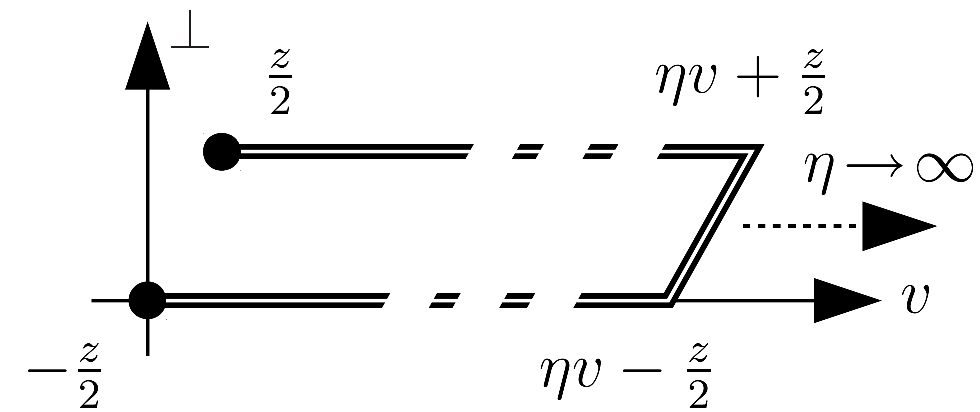
$\longrightarrow$ Corresponds to cutting off momentum integrations at the resolution scale of the calculation

$\longrightarrow$ This is not identical to $\overline{MS}$ – matching factor needed to convert

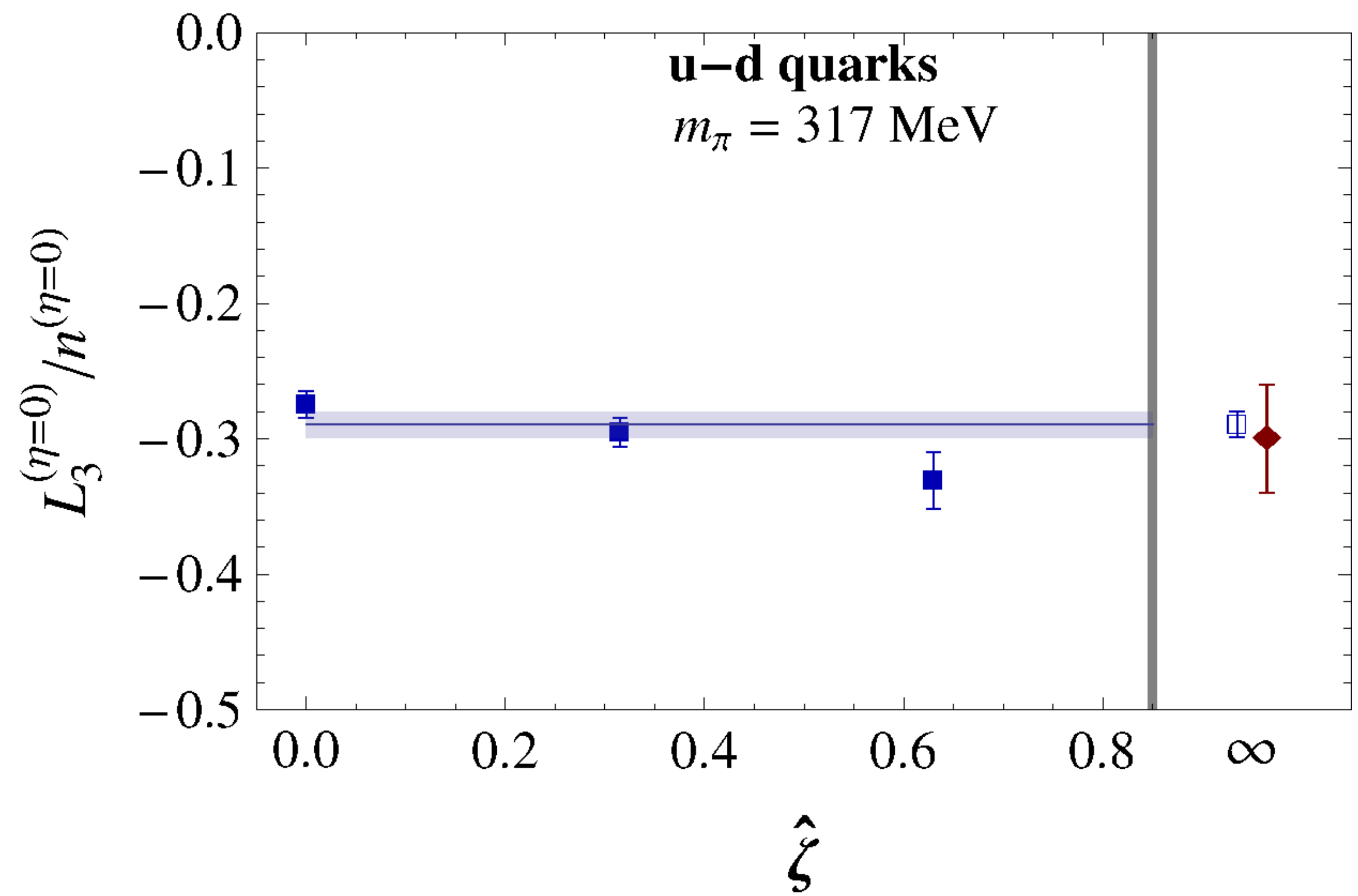
Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \left. \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle \right|_{z^+=z^-=0\, ,\ \Delta_T=0\, ,\ z_T\rightarrow 0}}{\left. \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle \right|_{z^+=z^-=0\, ,\ \Delta_T=0\, ,\ z_T\rightarrow 0}}$$

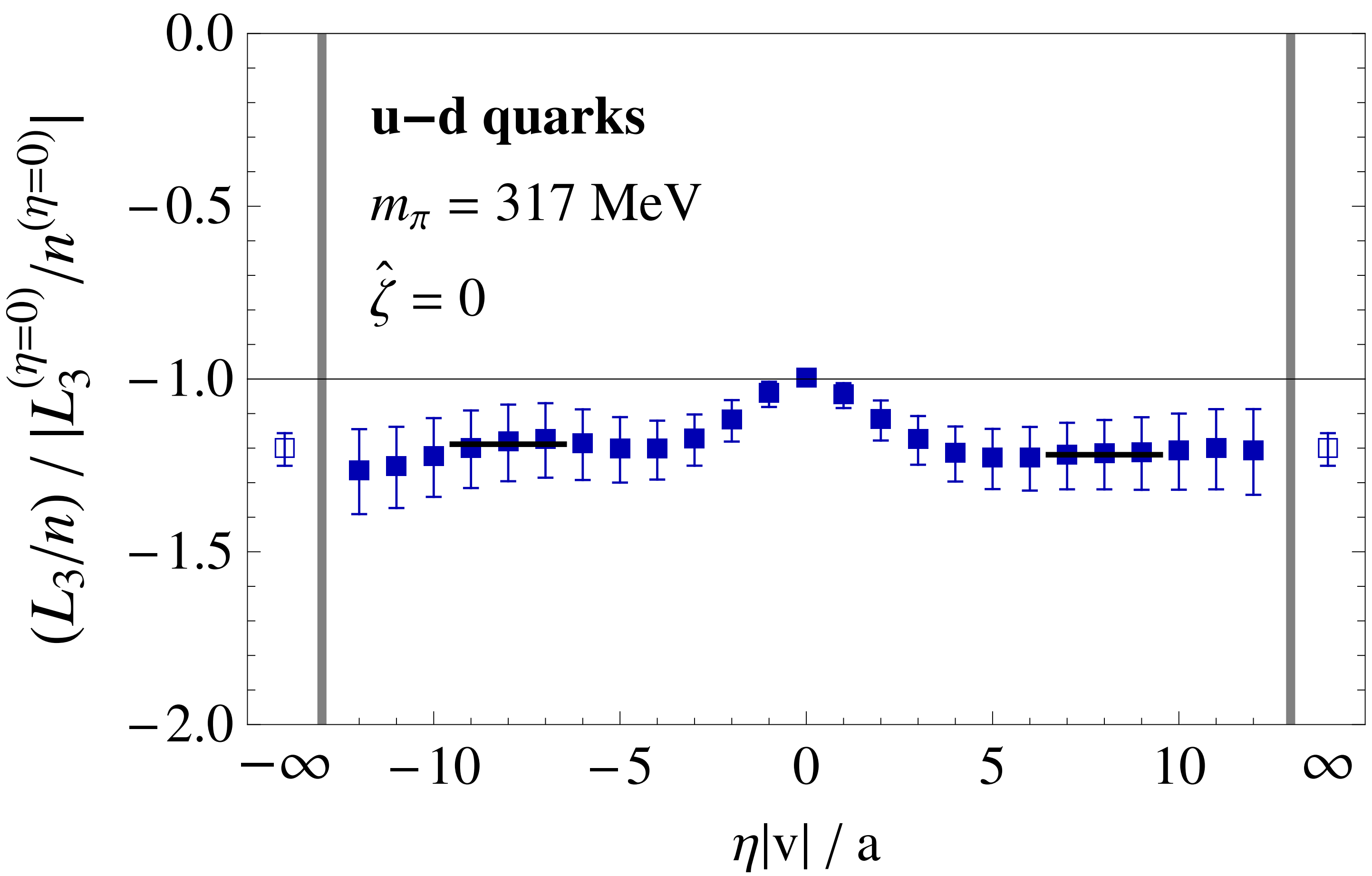
Remaining parameters to consider: $\hat{\zeta}$, η



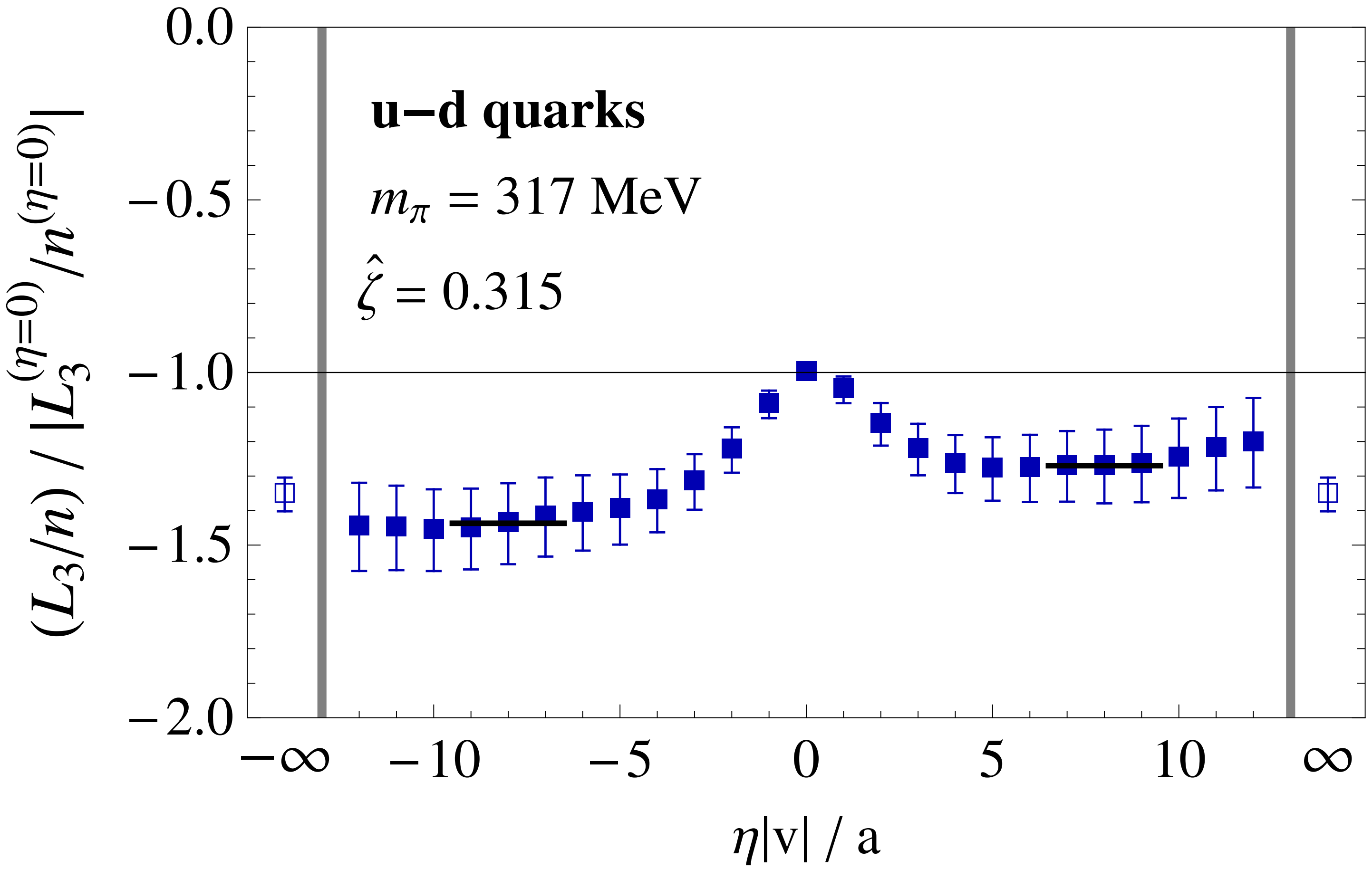
Ji quark orbital angular momentum: $\eta = 0$



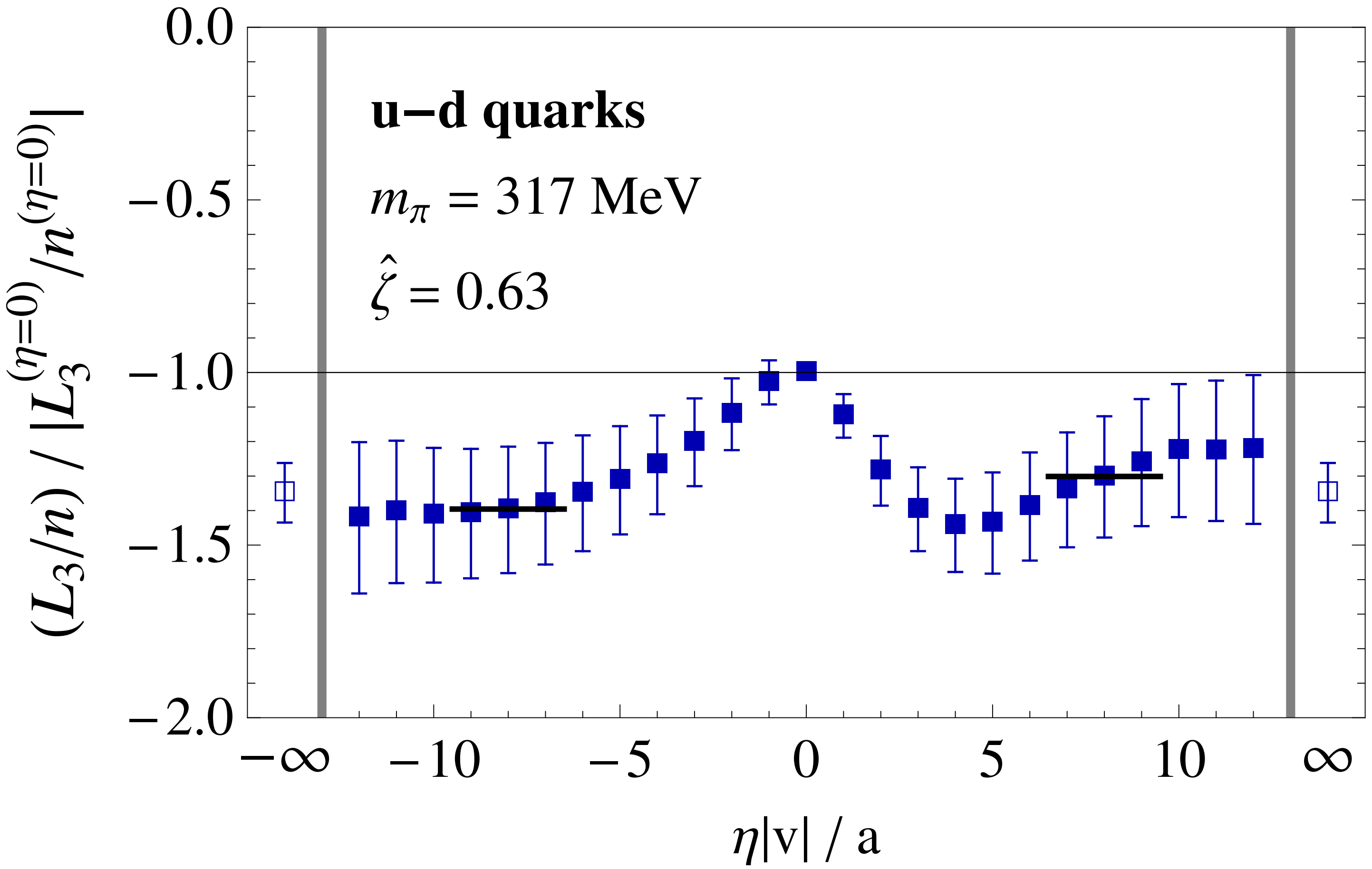
From Ji to Jaffe-Manohar quark orbital angular momentum



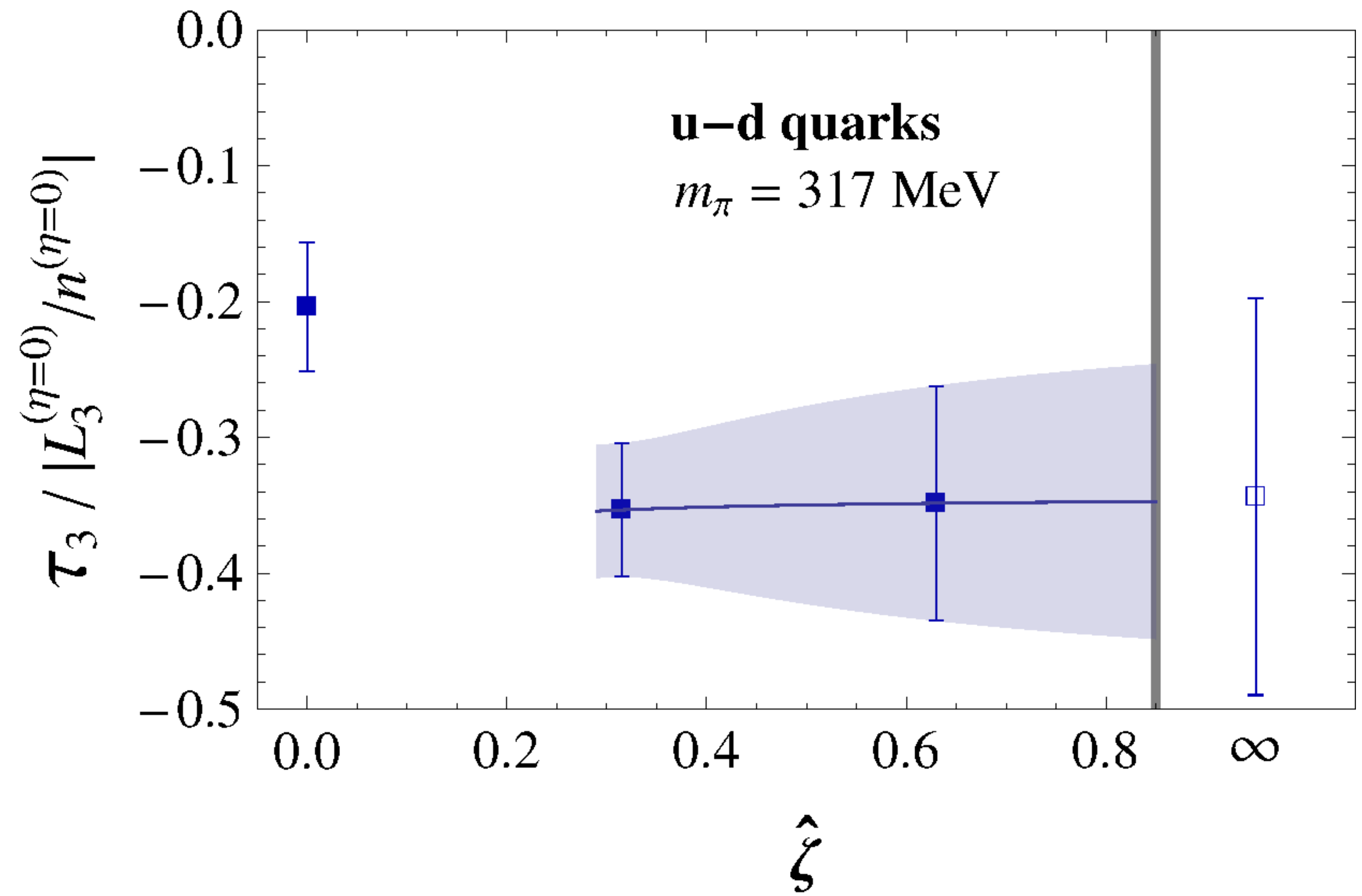
From Ji to Jaffe-Manohar quark orbital angular momentum



From Ji to Jaffe-Manohar quark orbital angular momentum



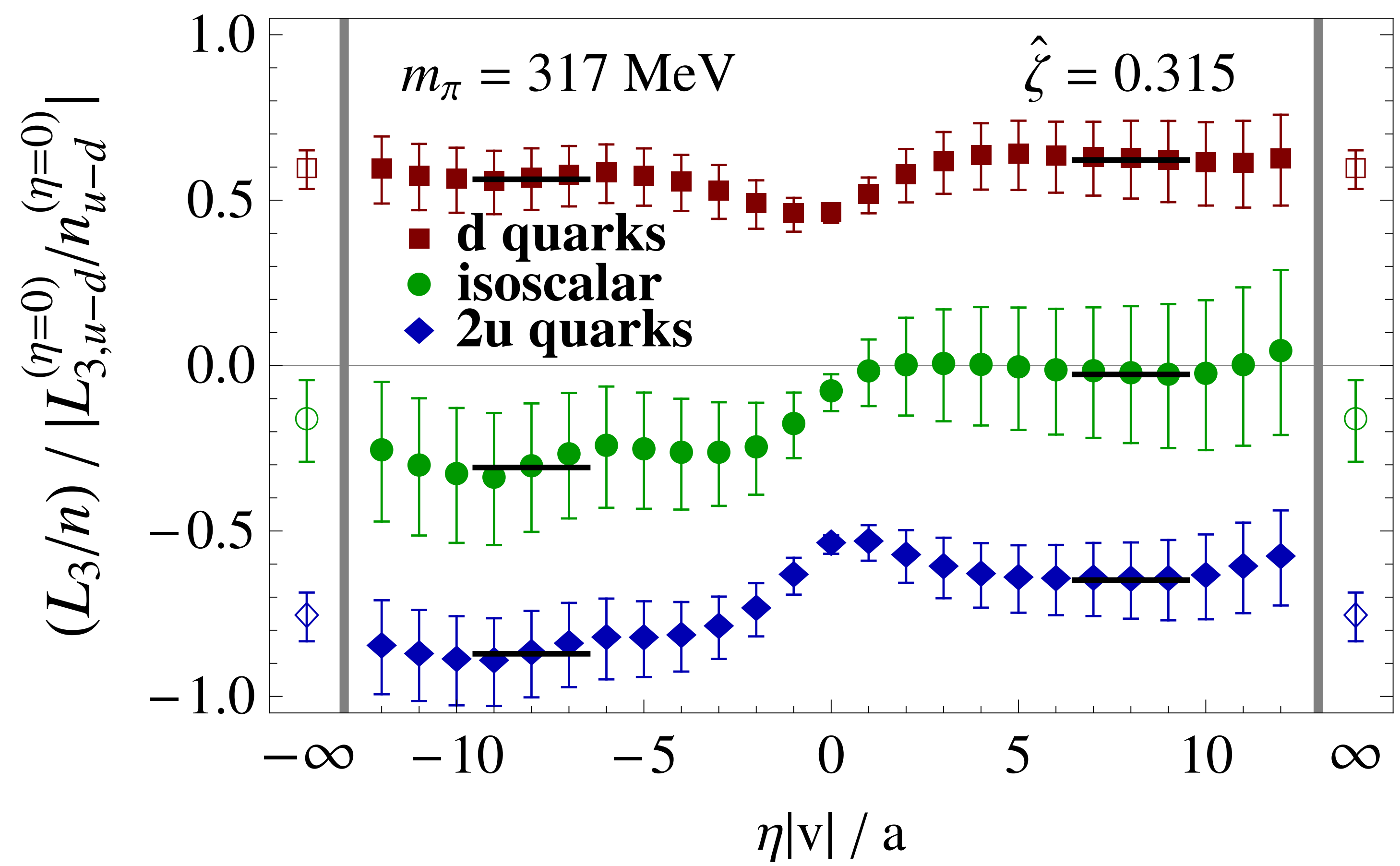
Burkardt's torque – extrapolation in $\hat{\zeta}$



$$\tau_3 = (L_3^{(\eta=\infty)}/n^{(\eta=\infty)}) - (L_3^{(\eta=0)}/n^{(\eta=0)})$$

Integrated torque accumulated by struck quark leaving proton

Flavor separation – from Ji to Jaffe-Manohar quark orbital angular momentum



Quark spin-orbit correlations

$$2L_3 S_3 \;=\; \int dx \int d^2k_T \int d^2r_T\, (r_T \times k_T)_3\, \Sigma\; \mathcal{W}^{\mathcal{U}}_{\Sigma}(x, k_T, r_T) \qquad \text{Wigner distribution}$$

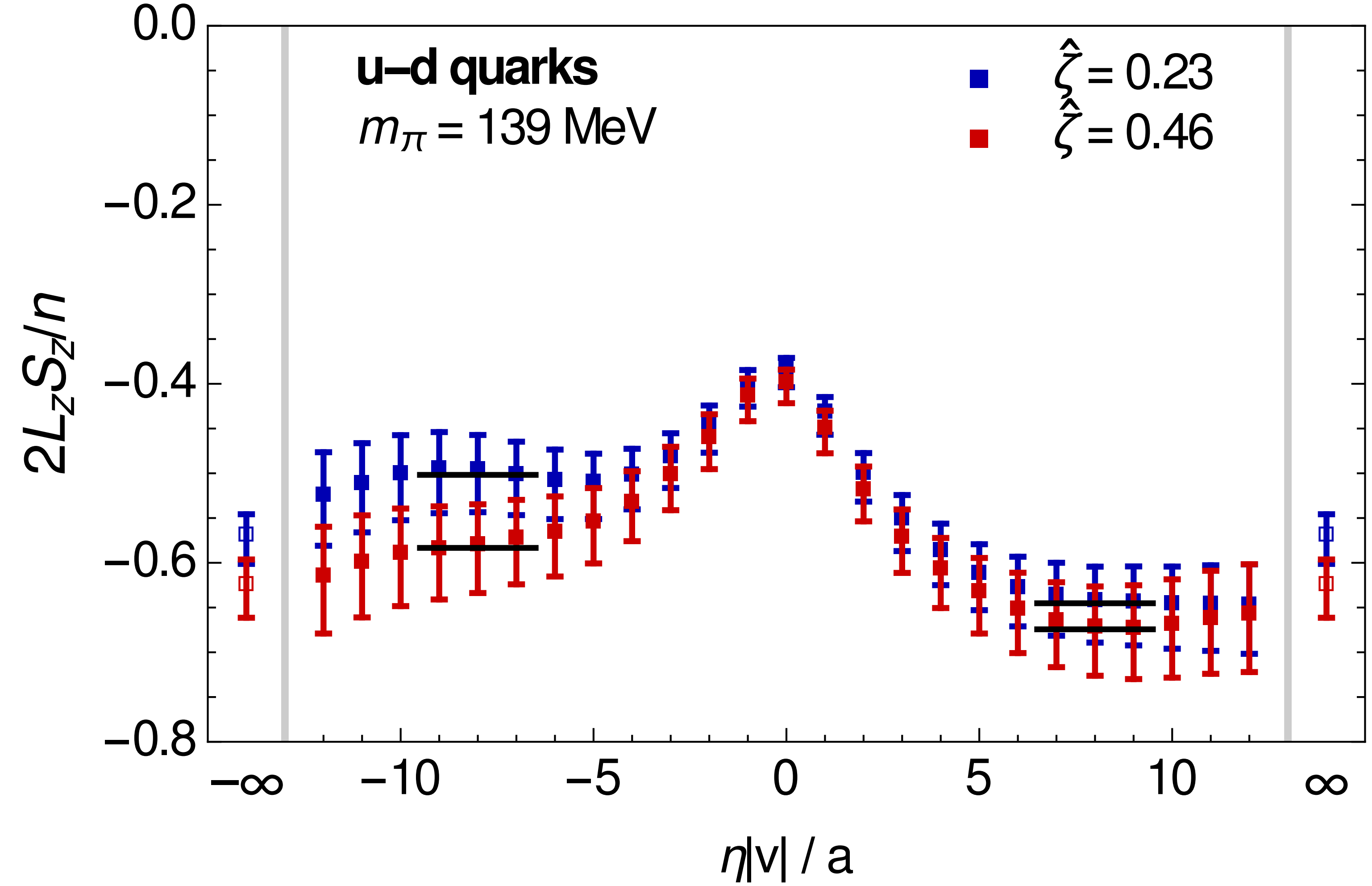
$$\frac{2L_3 S_3}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}}\, \langle P+\Delta_T/2 \mid \overline{\psi}(-z/2)\gamma^+\gamma^5\, \mathcal{U}[-z/2, z/2]\psi(z/2) \mid P-\Delta_T/2\, \rangle\big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}{\langle P+\Delta_T/2 \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid P-\Delta_T/2\, \rangle\big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}$$

P in 3-direction, $P \to \infty$

Renormalization: Form ratio with number of valence quarks n – note: are using Domain Wall Fermions!

Connection to GTMDs: $\quad 2L_3 S_3 = \int dx \int d^2k_T\, \frac{k_T^2}{M^2}\, G_{11}\Big|_{\Delta_T=0}$

Quark spin-orbit correlations



For comparison, in a polarized proton (from C. Alexandrou et al., PRD 101 (2020) 094513; 2003.08486):

$$\langle L^u \rangle = -0.22(3)\ ,\ \langle 2S^u \rangle = 0.86(2) \Rightarrow \langle L^u \rangle \langle 2S^u \rangle = -0.2$$

$$\langle L^d \rangle = 0.26(2)\ ,\ \langle 2S^d \rangle = -0.42(2) \Rightarrow \langle L^d \rangle \langle 2S^d \rangle = -0.1$$

$$\Rightarrow \langle L^u \rangle \langle 2S^u \rangle - \langle L^d \rangle \langle 2S^d \rangle = -0.1$$

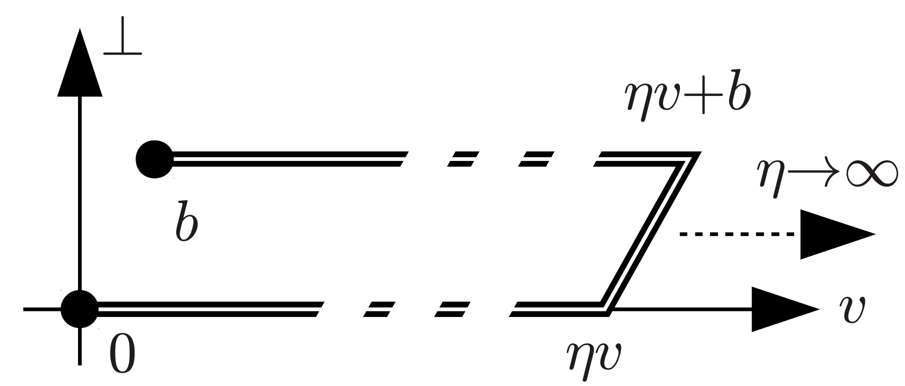
Preliminary sketch: x-dependence of Sivers shift

Sivers shift: Average transverse momentum of unpolarized quarks in a nucleon polarized in the other transverse direction

$$\frac{1}{2}\langle P,S|\ \bar{q}(0)\ \gamma^+\ \mathcal{U}[0,\ldots,b]\ q(b)\ |P,S\rangle = 2P^+\left(\widetilde{A}_{2B} + i m_N \epsilon_{ij} b_i S_j \widetilde{A}_{12B}\right)$$

$$\langle k_T\rangle_{TU}(b_T^2, x, \ldots) = m_N \frac{\widetilde{f}_{1T}^{\perp(1)}(b_T^2, x, \ldots)}{\widetilde{f}_1^{(0)}(b_T^2, x, \ldots)} = -m_N \frac{\int d(b\cdot P)\,\exp(ixb\cdot P)\widetilde{A}_{12B}(-b_T^2, b\cdot P, \hat{\zeta}, \eta v\cdot P)}{\int d(b\cdot P)\,\exp(ixb\cdot P)\widetilde{A}_{2B}(-b_T^2, b\cdot P, \hat{\zeta}, \eta v\cdot P)}$$

With a grain of salt, soft factors do not depend on $b\cdot P$ – can be factored outside the Fourier transform



Preliminary sketch: x-dependence of Sivers shift

Phenomenological frame: $P_T = v_T = 0$, $b^+ = 0$

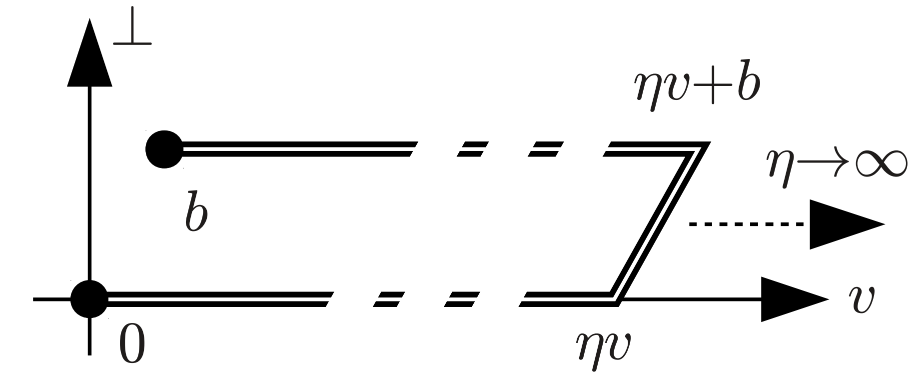
Expressed in Lorentz-invariant fashion: $\frac{v\cdot b}{v\cdot P} = \frac{b\cdot P}{m_N^2}\left(1 - \sqrt{1 + 1/\hat{\zeta}^2}\right)$

Lattice frame: b, v purely spatial

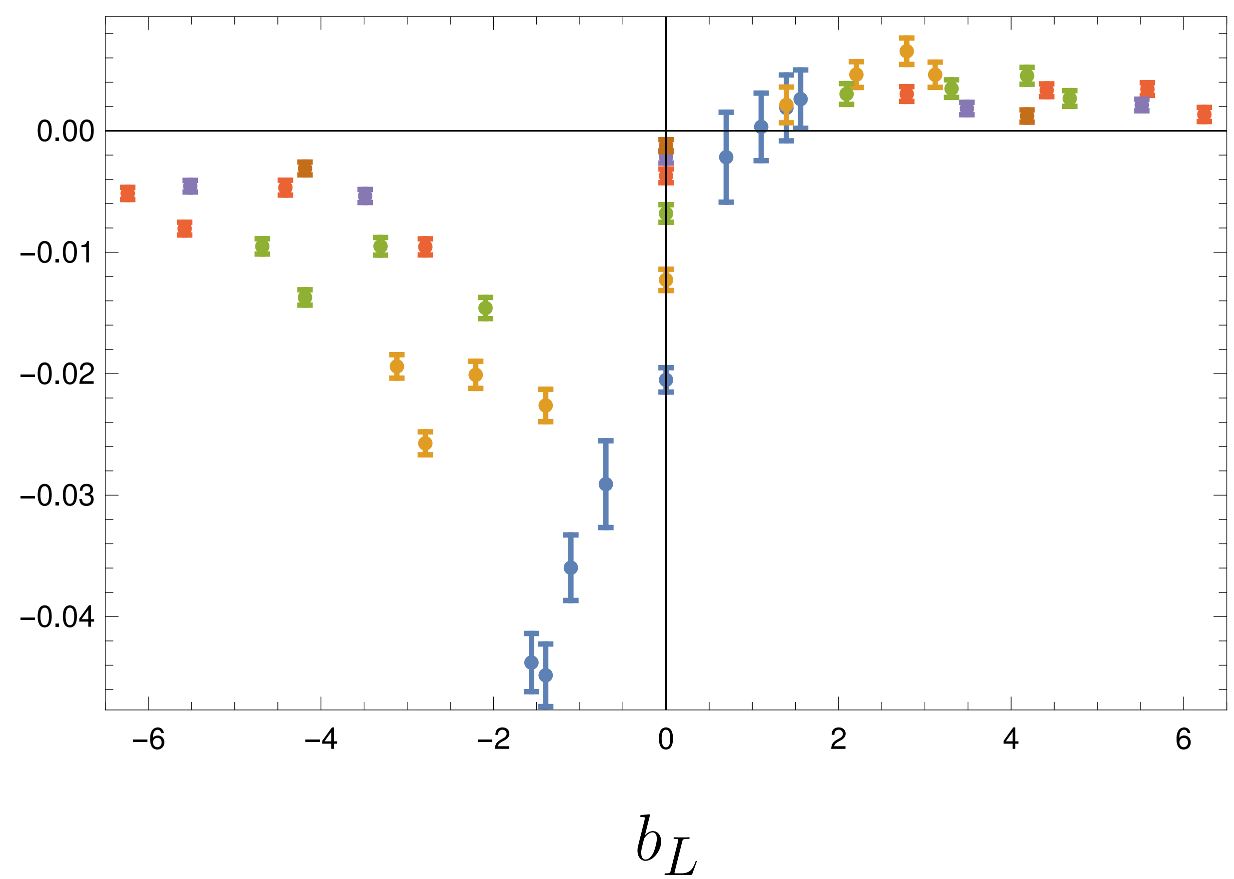
Constraint forces the use of general off-axis directions

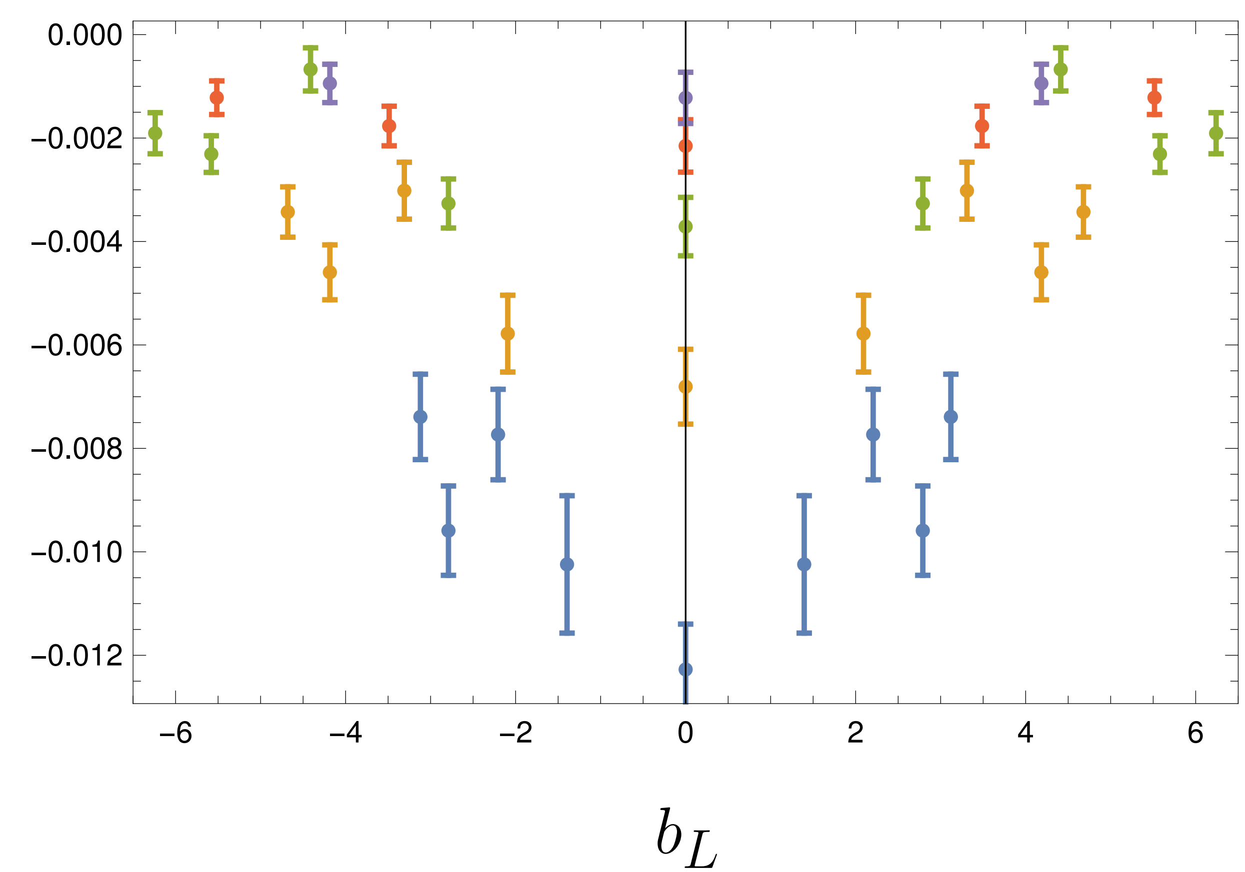
Lorentz transformation between phenomenological and lattice frames is not pure boost, contains rotation

Perform analysis at large staple length

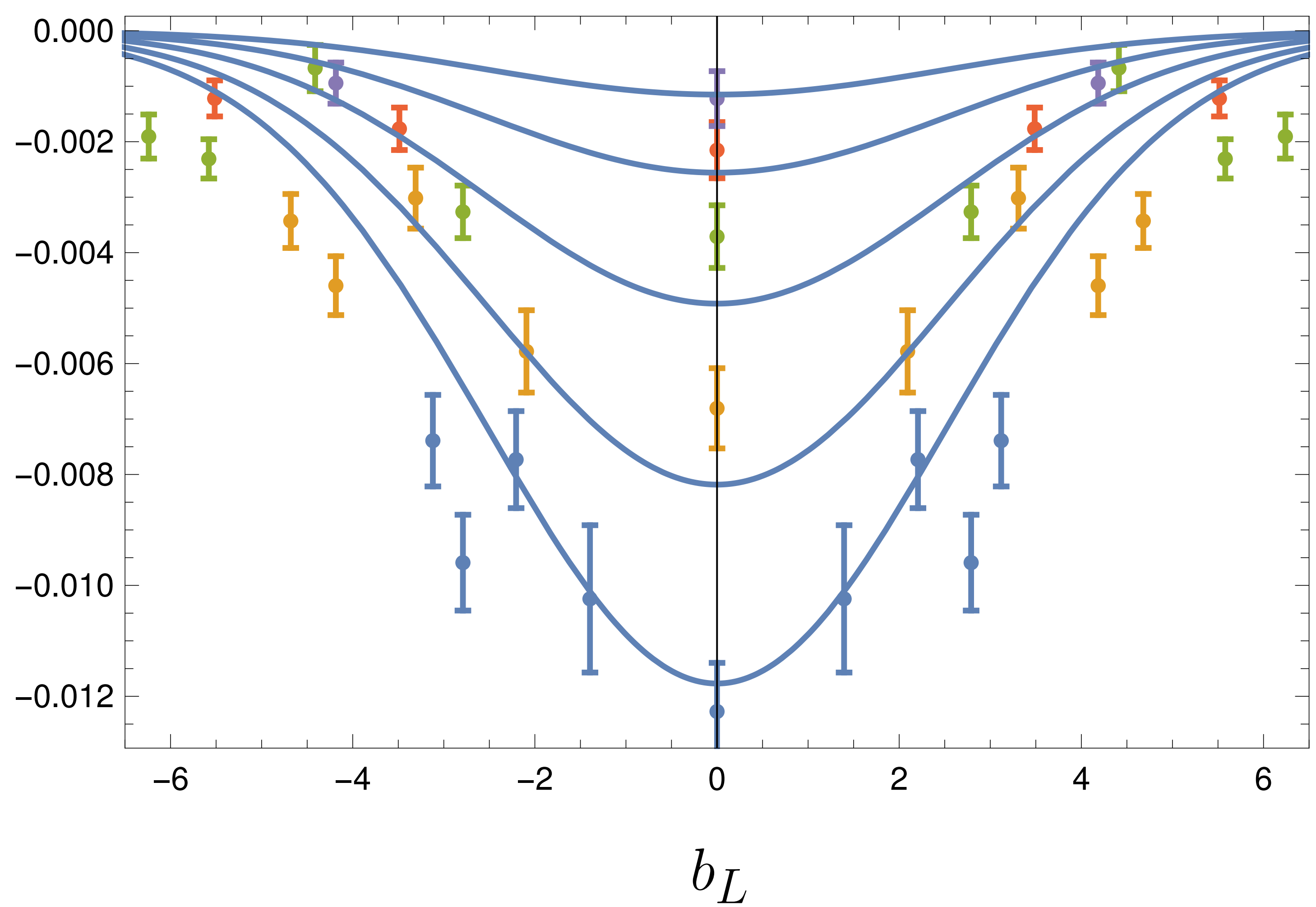


Extract b_L-even component of imaginary part of γ^+ correlator

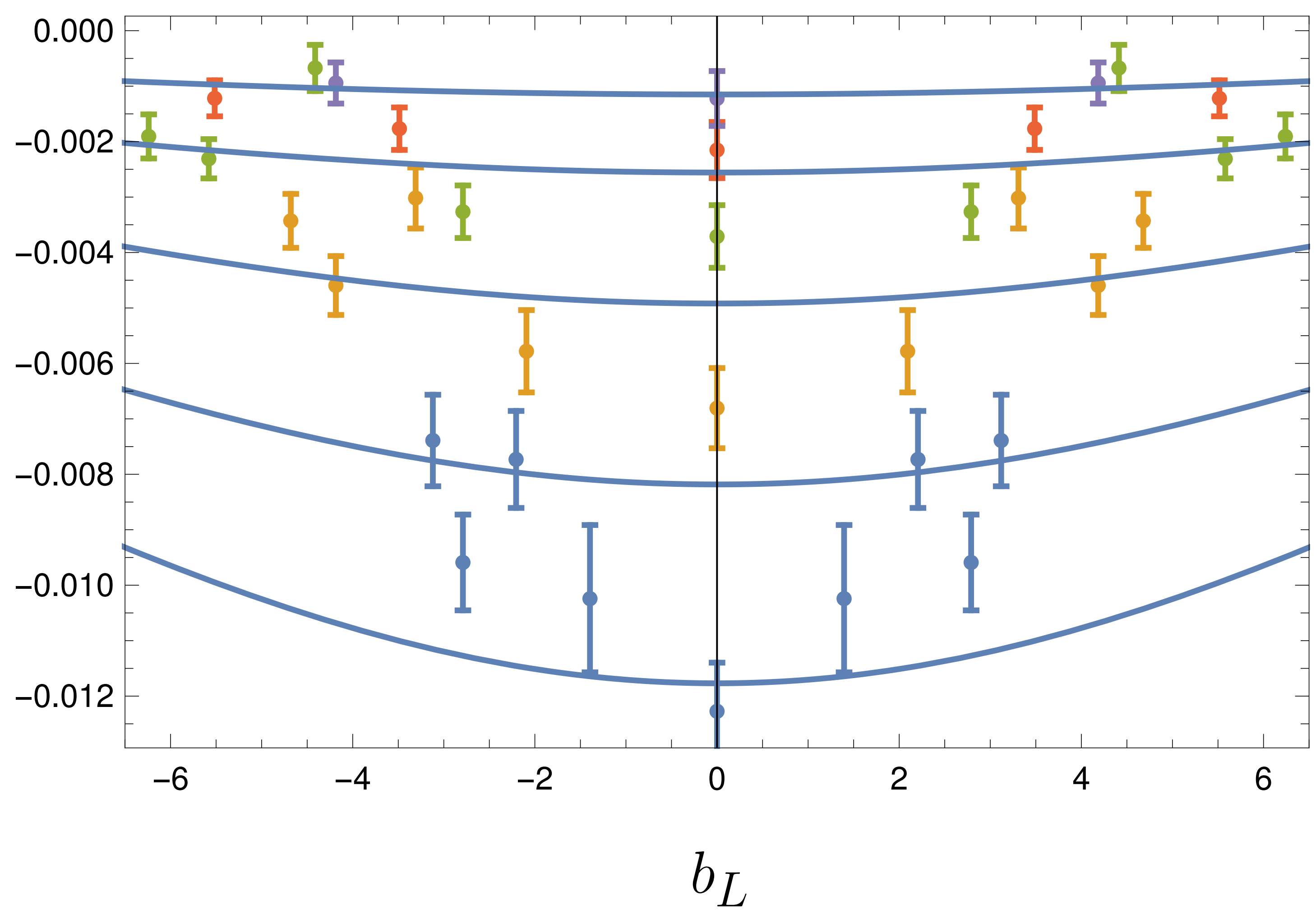




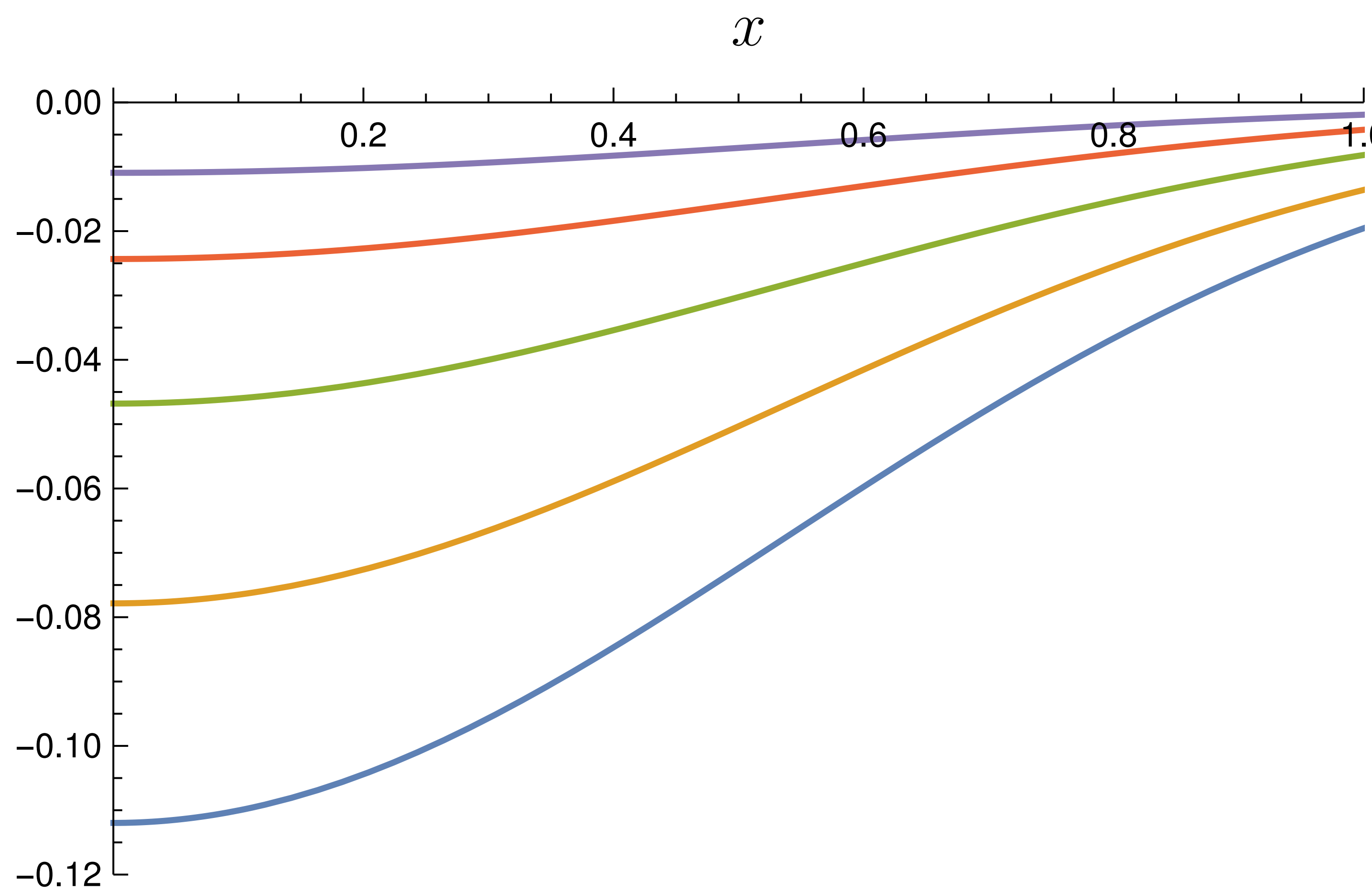
Fit dependence in b_L, $|b_T|$ space



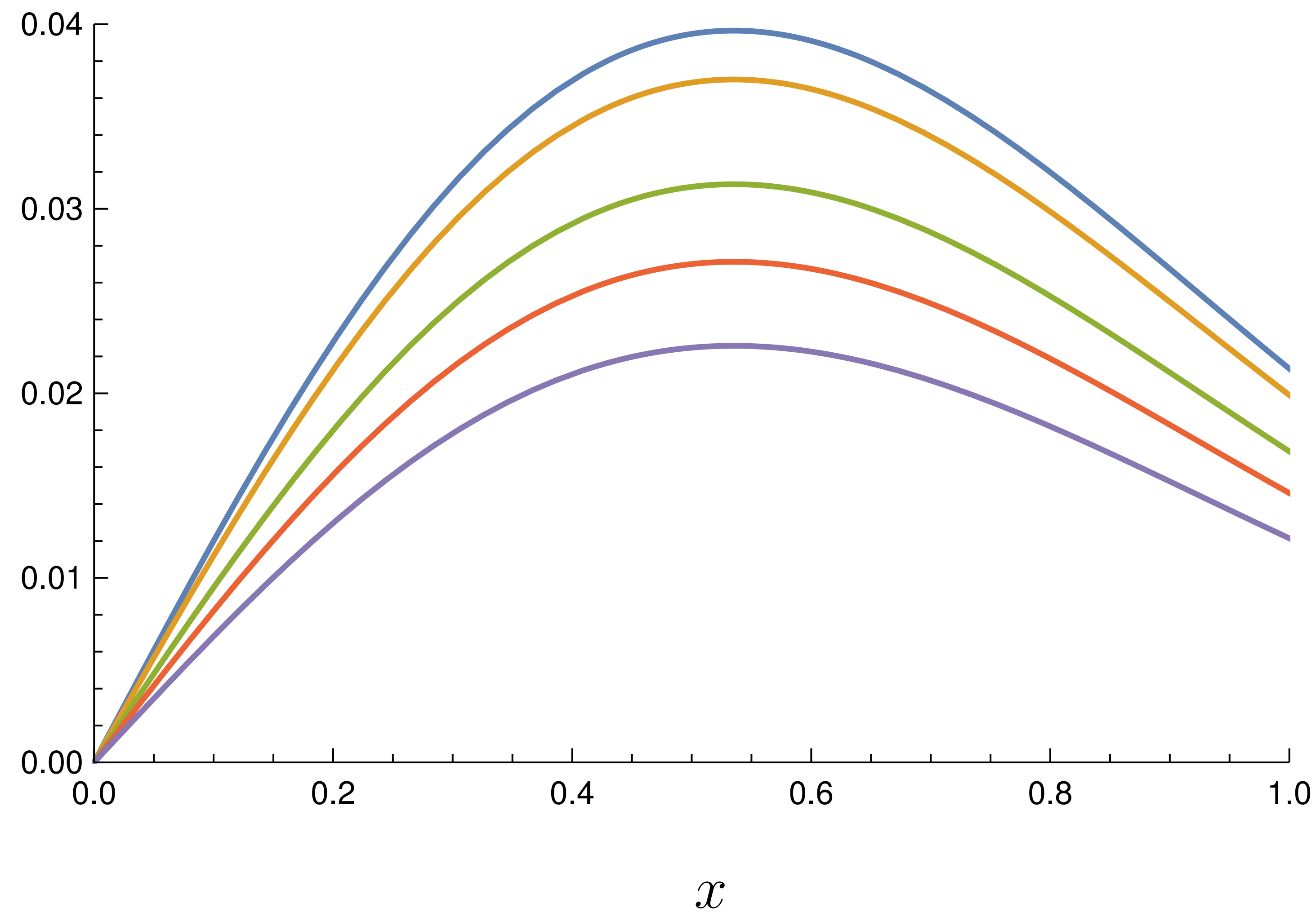
Cast in $b \cdot P$, b^2 space



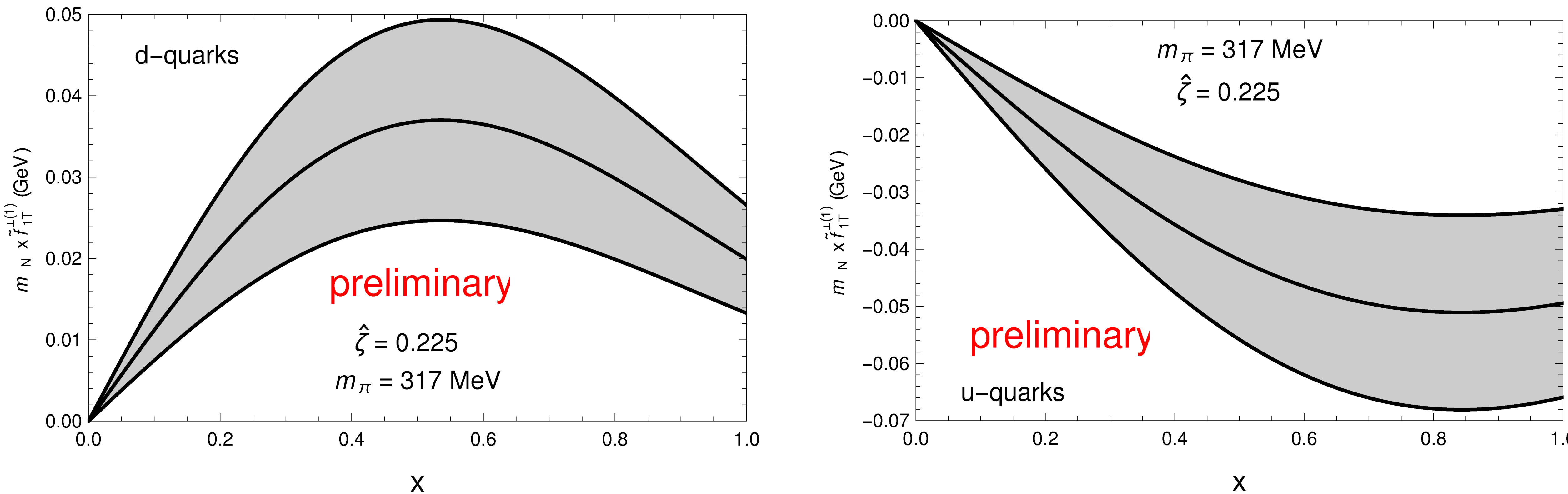
Fourier transform $b \cdot P \longrightarrow x$



Normalize to x-integrated Sivers shift, multiply by x

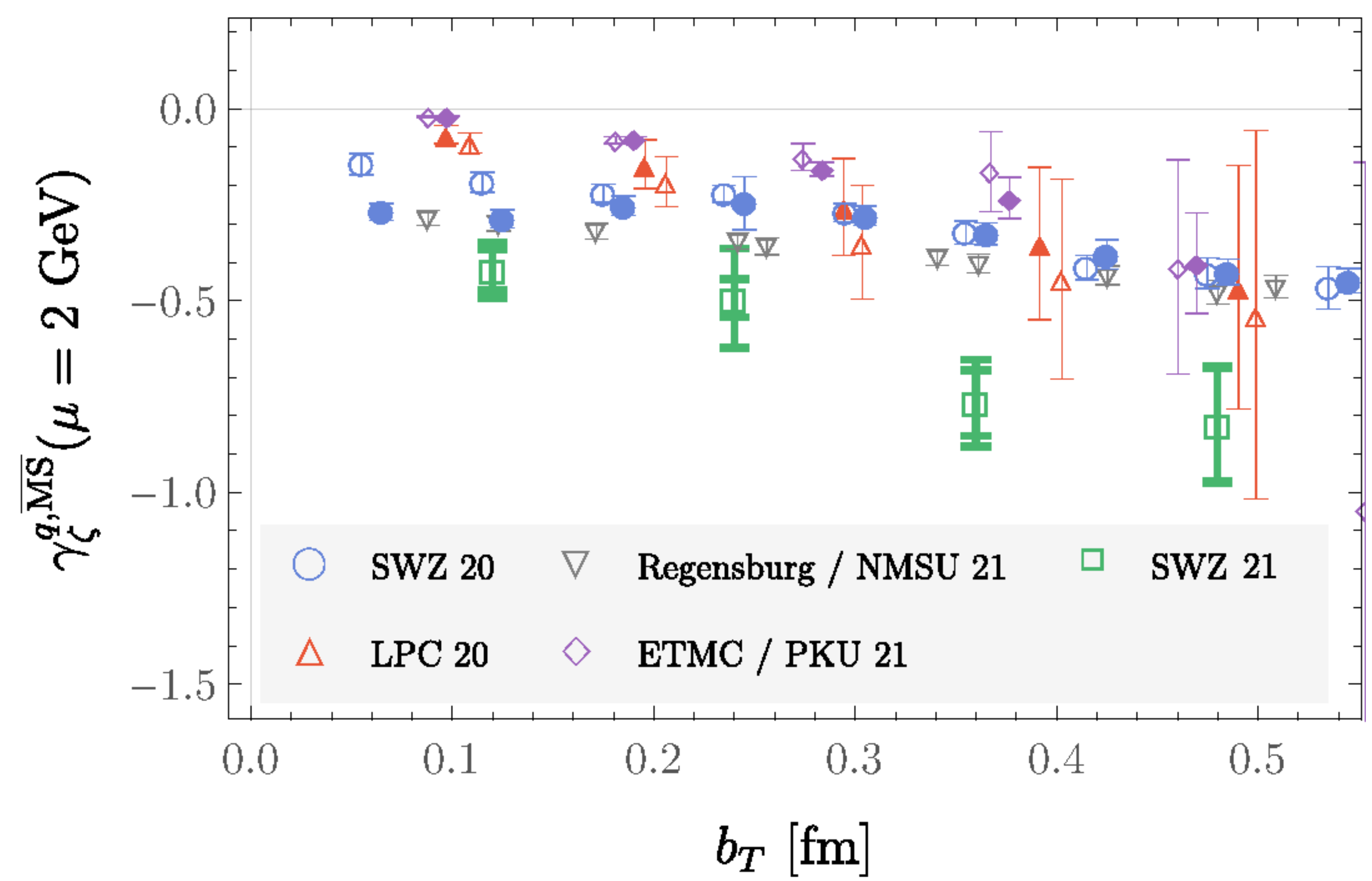


Eyeball error



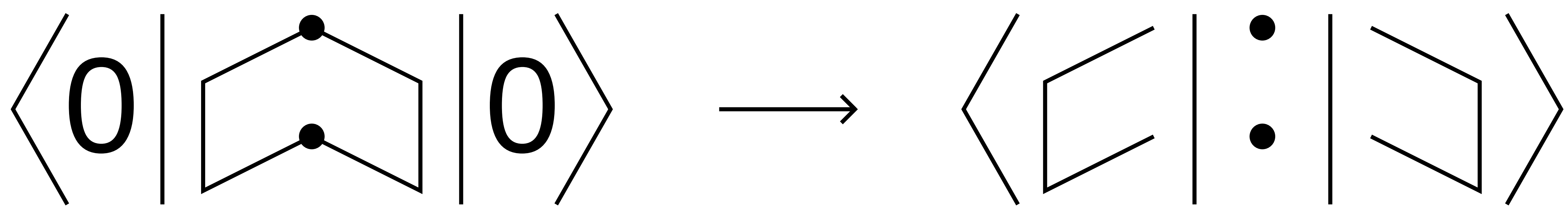
The Collins-Soper evolution kernel

- Principle: Take ratios of TMDs at different momenta (i.e., $\hat{\zeta}$), rather than different spin content
- Elaborated in a quasi-TMD framework by M. Ebert, I. Stewart and Y. Zhao, Phys. Rev. D 99 (2019) 034505, JHEP 03 (2020) 099; cf. also A. Vladimirov and A. Schäfer, Phys. Rev. D 101 (2020) 074517.



The TMD soft factor

- Problem: TMD soft factor composed of two staples with different rapidities
- Cannot boost operator to a frame in which it exists at a single time

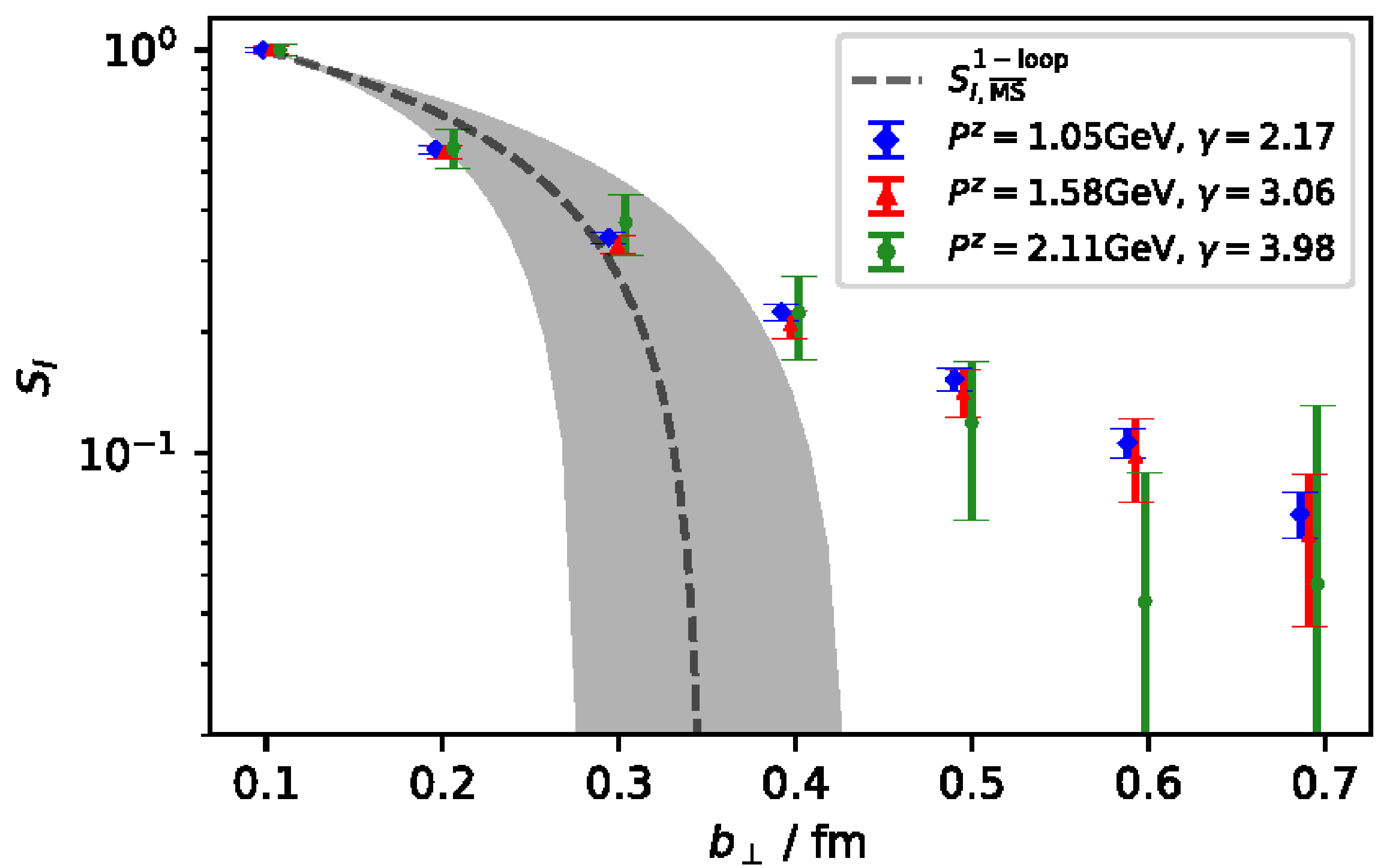


- Solution: Incorporate the Wilson lines into the external state – hard quark-antiquark states
- In-going and out-going states can be boosted to different momenta (momentum transfer at the two vertices)
- In practice, use dynamical meson states

X. Ji, Y. Liu and Y.-S. Liu, Nucl. Phys. B955 (2020) 115054

The TMD soft factor

Lattice Parton collaboration, Q.-A. Zhang et al., Phys. Rev. Lett. 125 (2020) 192001



Perspectives

Development of quasi-TMD framework:

M. Ebert, I. Stewart and Y. Zhao, JHEP 09 (2019) 037
M. Ebert, S. Schindler, I. Stewart and Y. Zhao, JHEP 09 (2020) 099
X. Ji, Y. Liu and Y.-S. Liu, Phys. Lett. B811 (2020) 135946
X. Ji, Y. Liu, A. Schäfer and F. Yuan, Phys. Rev. D 103 (2021) 074005

Complements Lorentz-invariant approach, promise to go beyond ratio observables

Questions?