

Factorization for TMDs in SIDIS at Subleading Power

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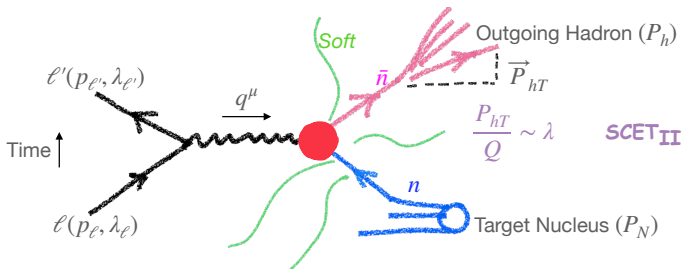
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Basics



- Only consider one photon exchange

- Lorentz invariants $Q = \sqrt{-q^2}$, $x = \frac{Q^2}{2P_N \cdot q}$, $y = \frac{P_N \cdot q}{P_N \cdot p_\ell}$, $z = \frac{P_N \cdot P_h}{P_N \cdot q}$

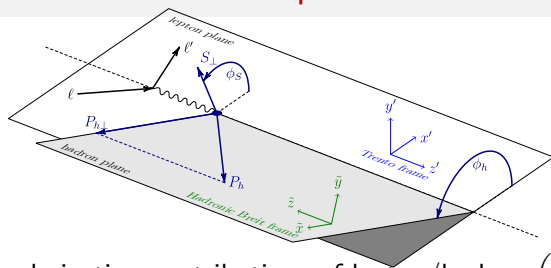
$$\frac{d\sigma}{dx dy dz d^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{2Q^4} \frac{y}{z} L_{\mu\nu}(p_\ell, p_{\ell'}) W^{\mu\nu}(q, P_N, P_h)$$

$$W^{\mu\nu}(q, P_N, P_h) = \sum_X \int \frac{d^4b}{(2\pi)^4} e^{ib \cdot q} \langle N | J^\dagger{}^\mu(b) | h, X \rangle \langle h, X | J^\nu(0) | N \rangle$$

$$L^{\mu\nu}(p_\ell, p_{\ell'}) = \langle \ell | J_e^\dagger{}^\mu | \ell' \rangle \langle \ell' | J_e^\nu | \ell \rangle$$

$$= 2\delta_{\lambda_\ell \lambda_{\ell'}} [(p_\ell^\mu p_{\ell'}^\nu + p_\ell^\nu p_{\ell'}^\mu - p_\ell \cdot p_{\ell'} g^{\mu\nu}) + i\lambda_\ell \epsilon^{\mu\nu\rho\sigma} p_{\ell\rho} p_{\ell'\sigma}]$$

Kinematics and Tensor Decomposition



- Different polarization contributions of lepton/hadron $\left(\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}\right)$

$$\frac{d\sigma}{dx dy dz d^2\vec{P}_{hT}} = \frac{\pi\alpha^2 y}{Q^2} \frac{\kappa_\gamma}{z} \frac{1}{1-\epsilon} \left[(L \cdot W)_{UU} + \lambda_\ell (L \cdot W)_{LU} \right. \\ \left. + S_L (L \cdot W)_{UL} + \lambda_\ell S_L (L \cdot W)_{LL} + S_T (L \cdot W)_{UT} + \lambda_\ell S_T (L \cdot W)_{LT} \right]$$
- Standard structure function decomposition [Bacchetta et al '06]

$$(L \cdot W)_{UU} = W_{UU,T} + \epsilon W_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi_h) W_{UU}^{\cos(\phi_h)} + \epsilon \cos(2\phi_h) W_{UU}^{\cos(2\phi_h)},$$

$$(L \cdot W)_{LU} = \sqrt{2\epsilon(1-\epsilon)} \sin(\phi_h) W_{LU}^{\sin(\phi_h)},$$

$$(L \cdot W)_{LT} = \sqrt{1-\epsilon^2} \cos(\phi_h - \phi_S) W_{LT}^{\cos(\phi_h - \phi_S)} \\ + \sqrt{2\epsilon(1-\epsilon)} \left[\cos(\phi_S) W_{LT}^{\cos(\phi_S)} + \cos(2\phi_h - \phi_S) W_{LT}^{\cos(2\phi_h - \phi_S)} \right],$$

.....

Tensor Decomposition and Motivation

- Standard structure function decomposition [Bacchetta et al '06]

$$\frac{d\sigma}{dx dy dz d^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{Q^2} \frac{y}{z} \frac{\delta_{\lambda_\ell\lambda_{\ell'}}}{1-\epsilon} \left[(W_{-1} + \epsilon W_0) + \epsilon \cos(2\phi_h) W_3 \right. \\ \left. + \sqrt{2\epsilon(1+\epsilon)} \cos\phi_h W_1 + \lambda \sqrt{2\epsilon(1-\epsilon)} \sin\phi_h W_2 \right] + \dots$$

- $\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}$
- $W_i = P_i^{\mu\nu} W_{\mu\nu}$ with projectors $P_i^{\mu\nu}$ (defined in the hadronic Breit frame)
- $P_{-1}^{\mu\nu} = (\tilde{x}^\mu \tilde{x}^\nu + \tilde{y}^\mu \tilde{y}^\nu)$, $P_3^{\mu\nu} = \tilde{x}^\mu \tilde{x}^\nu - \tilde{y}^\mu \tilde{y}^\nu$,
 $P_1^{\mu\nu} = -(\tilde{t}^\mu \tilde{x}^\nu + \tilde{x}^\mu \tilde{t}^\nu)$, $P_2^{\mu\nu} = i(\tilde{t}^\mu \tilde{x}^\nu - \tilde{x}^\mu \tilde{t}^\nu)$, $P_0^{\mu\nu} = \tilde{t}^\mu \tilde{t}^\nu$,
- $W_{-1}, W_3 \sim \mathcal{O}((\frac{P_{hT}}{Q})^0)$, standard factorization theorems (CSS, SCET)
- $W_1, W_2 \sim \mathcal{O}(\frac{P_{hT}}{Q})$
 - ▷ First treated in parton model (tree level matching) [Mulders, Tangerman '95]
 - ▷ Mismatch with perturbative results at tree level [Bacchetta et al '08]
 - ▷ Conjecture: Resolved by adding a LP soft function [Bacchetta et al '19]
- $W_0 \sim \mathcal{O}((\frac{P_{hT}}{Q})^2)$, not considered in this talk

⇒ Use SCET to derive all-order factorization at subleading power

Factorization: the General Procedure

$$W^{\mu\nu}(q, P_N, P_h) = \sum_X \int \frac{d^4b}{(2\pi)^4} e^{ib \cdot q} \langle N | J^{\dagger \mu}(b) | h, X \rangle \langle h, X | J^\nu(0) | N \rangle$$

Goal: factorize $W_i = P_i^{\mu\nu} W_{\mu\nu}$ at next-to-leading power

- Match SCET currents (operators) with QCD: $J^\mu = J^{(0)\mu} + \sum_k J_k^{(1)\mu} + \dots$
(in the **factorization frame**: $P_N^\mu = P_N^- \frac{n^\mu}{2}$, $P_h^\mu = P_h^+ \frac{\bar{n}^\mu}{2}$, $g_{F\perp}^{\mu\nu} = g_{B\perp}^{\mu\nu} + \mathcal{O}(P_{hT}/Q)$)
- $W^{(0)\mu\nu} \sim J^{(0)\dagger\mu} J^{(0)\nu}$, $W^{(1)\mu\nu} \sim \sum_k J^{(0)\dagger\mu} J_k^{(1)\nu} + J_k^{(1)\dagger\mu} J^{(0)\nu}$
- Expand **projectors** in the **factorization frame**: $P_i^{\mu\nu} = P_i^{(0)\mu\nu} + P_i^{(1)\mu\nu} + \dots$

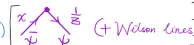
Categories of power corrections

- 1) Subleading current contributions, $P_i^{(0)} \cdot W^{(1)}$
- 2) Kinematic correction, $P_i^{(1)} \cdot W^{(0)}$
- 3) Subleading soft contributions including SCET_{II} subleading Lagrangians
 $\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1/2)} + \mathcal{L}^{(1)} + \dots$ see e.g. [Moult et al '17]

- Assumption: Glauber Lagrangian $\mathcal{L}_G^{(0)}$ doesn't spoil factorization

Factorization at Leading Power

LP current $J^{(0)\mu} \sim \sum_f (\gamma_\perp^\mu)^{\alpha\beta} C_f^{(0)}(Q) \bar{\chi}_{\bar{n},\omega_b}^\alpha [S_n^\dagger S_n] \chi_{n,\omega_a}^\beta \sim C_f^{(0)}(Q) \left[\text{diagram} \right] (+ \text{Wilson lines})$



- Plug it into $W^{(0)\mu\nu} \sim \langle N | J^{(0)\dagger\mu} | h, X \rangle \langle h, X | J^{(0)\nu} | N \rangle$
- Collinear fields yield quark correlators

$$\hat{B}_f^{\beta'\beta}(x, \vec{b}_T) = \langle N | \bar{\chi}_n^\beta(b_\perp) \delta(\omega_a - \bar{P}_n) \chi_n^{\beta'}(0) | N \rangle$$

$$\hat{G}_f^{\alpha\alpha'}(z, \vec{b}_T) = \frac{1}{2z} \not{x} \not{z} \langle 0 | \delta(\omega_b - \bar{P}_{\bar{n}}) \chi_{\bar{n}}^\alpha(b_\perp) | h, X \rangle \langle h, X | \bar{\chi}_{\bar{n}}^{\alpha'}(0) | 0 \rangle$$

- Soft Wilson lines yield the TMD soft function

$$\mathcal{S}(b_T) = \frac{1}{N_c} \text{tr} \langle 0 | [S_n^\dagger(b_\perp) S_{\bar{n}}(b_\perp)] [S_n^\dagger(0) S_n(0)] | 0 \rangle.$$

- Combine into the quark correctors

$$B_f^{\beta'\beta}(x, \vec{b}_T) = \hat{B}_f^{\beta'\beta}(x, \vec{b}_T) \sqrt{\mathcal{S}(b_T)}, \quad \mathcal{G}_f^{\alpha\alpha'}(z, \vec{b}_T) = \hat{G}_f^{\alpha\alpha'}(z, \vec{b}_T) \sqrt{\mathcal{S}(b_T)}$$

⇒ Factorized leading power hadronic tensor

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int d^2 b_T e^{i\vec{q}_T \cdot \vec{b}_T} \mathcal{H}_f^{(0)}(Q) \text{Tr} [B_f(x, \vec{b}_T) \gamma_\perp^\mu \mathcal{G}_f(z, \vec{b}_T) \gamma_\perp^\nu].$$

- Hard function: $\mathcal{H}_f^{(0)}(Q) = |C_f^{(0)}(Q)|^2$

Structure Functions at Leading Power

- In the momentum space, decompose into different Dirac structures

[Goeke, Metz, Schlegel '05]

$$B_f^{\beta'\beta}(x, \vec{p}_T) = \frac{1}{4} \left\{ f_1 \not{\epsilon} + i h_1^\perp \frac{[\not{p}_\perp, \not{\epsilon}]}{2M_N} \right\}^{\beta'\beta} + \dots,$$

$$\mathcal{G}_f^{\alpha'\alpha}(z, \vec{k}_T) = \frac{1}{4} \left\{ D_1 \not{\epsilon} + i H_1^\perp \frac{[\not{k}_\perp, \not{\epsilon}]}{2M_h} \right\}^{\alpha'\alpha} + \dots$$

- h_1^\perp Boer-Mulders function, H_1^\perp Collins function
- Contract $W^{(0)\mu\nu}$ with $P_{-1}^{(0)\mu\nu} = x^\mu x^\nu + y^\mu y^\nu$, $P_3^{(0)\mu\nu} = x^\mu x^\nu - y^\mu y^\nu$,

$$W_{-1}^{(0)} = \mathcal{F} \left[\mathcal{H}^{(0)} f_1 D_1 \right],$$

$$W_3^{(0)} = \mathcal{F} \left[-\frac{2p_{Tx} k_{Tx} - \vec{p}_T \cdot \vec{k}_T}{M_N M_h} \mathcal{H}^{(0)} h_1^\perp H_1^\perp \right],$$

[Bacchetta et al '06]

$$\mathcal{F}[\omega \mathcal{H} g D] = 2z \sum_f \int d^2 p_T d^2 k_T \delta^2(\vec{q}_T + \vec{p}_T - \vec{k}_T) \omega(\vec{p}_T, \vec{k}_T) \mathcal{H}_f(Q) g_f(x, p_T) D_f(z, k_T)$$

- Contract $W^{(0)\mu\nu}$ with $P_{1,2}^{(1)\mu\nu}$, we get the kinematic corrections for $W_{1,2}$

Subleading Current: \mathcal{P}_\perp Acting on the Collinear Fields

Unique hard operator to all orders [Feige et al '17]

$$J_{\mathcal{P}}^{(1)\mu} \sim \frac{C_f^{(0)}}{2\omega_a} \bar{\chi}_{\bar{n}, \omega_b} [S_n^\dagger S_n] \gamma^\mu \mathcal{P}_\perp \not{n} \chi_{n, \omega_a} + \text{h.c.} \sim C_f^{(0)}(\alpha) \left[\text{diagram 1} + \text{diagram 2} \right] \quad (+ \text{Wilson lines})$$

- Reparameterization of SCET relates it with the LP one

⇒ The Wilson coefficient is identical to the leading power one, $C_f^{(0)}(Q)$

- Plug these currents into $J_{\mathcal{P}}^{(1)\dagger\mu} J^{(0)\nu} + J^{(0)\dagger\mu} J_{\mathcal{P}}^{(1)\nu}$

$$\hat{W}_{\mathcal{P}}^{(1)\mu\nu} = \frac{2z}{N_c} \sum_f \int d^2 \vec{b}_T \mathcal{H}_f^{(0)}(Q) \mathcal{S}(b_T) \times \left\{ \text{Tr} \left[\hat{B}_{\mathcal{P}f}(x, \vec{b}_T) \gamma^\mu \hat{G}_f(z, \vec{b}_T) \gamma^\nu \right] + \text{Tr} \left[\hat{B}_f(x, \vec{b}_T) \gamma^\mu \hat{G}_{\mathcal{P}f}(z, \vec{b}_T) \gamma^\nu \right] \right\}.$$

$$\begin{aligned} & \hat{B}_{\mathcal{P}f}^{\beta'\beta}(x, \vec{b}_T) \\ & \equiv \frac{1}{2Q} \theta(\omega_a) \left\{ \langle N | \bar{\chi}_n^\beta(b_\perp^\mu) [\mathcal{P}_\perp \not{n} \chi_{n, \omega_a}(0)]^{\beta'} | N \rangle + \langle N | [\bar{\chi}_n(b_\perp^\mu) \not{n} \mathcal{P}_\perp^\dagger]^\beta \chi_{n, \omega_a}^{\beta'}(0) | N \rangle \right\} \\ & = i \frac{1}{2Q} \frac{\partial}{\partial b_\perp^\rho} \left[\gamma_\perp^\rho \not{n}, \hat{B}_f(x, \vec{b}_T) \right]^{\beta'\beta}, \end{aligned}$$

Subleading Current: \mathcal{P}_\perp Acting on the Collinear Fields

- Define $B_{\mathcal{P}f}$, $\mathcal{G}_{\mathcal{P}f}$ and $W_{\mathcal{P}}^{(1)\mu\nu}$

$$B_{\mathcal{P}f}^{\beta'\beta}(x, \vec{b}_T) \equiv i \frac{1}{2Q} \frac{\partial}{\partial b_\perp^\rho} \left[\gamma_\perp^\rho \not{\not{b}}_\perp, B_f(x, \vec{b}_T) \right]^{\beta'\beta}, \text{ where } B_f^{\beta'\beta}(x, \vec{b}_T) = \hat{B}_f^{\beta'\beta}(x, \vec{b}_T) \sqrt{S(b_T)}$$

$$W_{\mathcal{P}}^{(1)\mu\nu} \equiv \frac{2z}{N_c} \sum_f \int d^2 \vec{b}_T \mathcal{H}_f^{(0)}(Q) \times \left\{ \text{Tr} \left[B_{\mathcal{P}f}(x, \vec{b}_T) \gamma^\mu \mathcal{G}_f(z, \vec{b}_T) \gamma^\nu \right] + \text{Tr} \left[B_f(x, \vec{b}_T) \gamma^\mu \mathcal{G}_{\mathcal{P}f}(z, \vec{b}_T) \gamma^\nu \right] \right\}.$$

- Equivalent to $\hat{W}_{\mathcal{P}}^{(1)\mu\nu}$ (noticing that $(n_\mu - \bar{n}_\mu) P_i^{\mu\nu} = \mathcal{O}(P_{hT}/Q)$)

$$W_{\mathcal{P}}^{(1)\mu\nu} - \hat{W}_{\mathcal{P}}^{(1)\mu\nu} = \frac{2z}{N_c} \sum_f \int d^2 \vec{b}_T \mathcal{H}_f^{(0)}(Q) \frac{i}{Q} \left(\frac{\partial}{\partial b_\perp^\rho} \sqrt{S(b_T)} \right) \sqrt{S(b_T)} \times \left\{ (\bar{n}^\nu - n^\nu) \text{Tr} \left[\gamma_\perp^\rho \hat{B}_f(x, \vec{b}_T) \gamma^\mu \hat{\mathcal{G}}_f(z, \vec{b}_T) \right] + (n^\mu - \bar{n}^\mu) \text{Tr} \left[\hat{B}_f(x, \vec{b}_T) \gamma_\perp^\rho \hat{\mathcal{G}}_f(z, \vec{b}_T) \gamma^\nu \right] \right\}$$

- Same leading power functions appear, in momentum space

$$B_{\mathcal{P}f}(x, \vec{p}_T) = \frac{1}{2Q} \left[\not{p}_\perp \not{\not{b}}_\perp, B_f \right] = \frac{1}{2Q} \left\{ f_1 \not{p}_\perp - i h_1^\perp \frac{p_T^2 [\not{b}_\perp, \not{\not{b}}_\perp]}{2M_N} \right\} + \dots$$

Subleading Operators: with \mathcal{B}_\perp Insertion

- Fields and currents of definite helicity [Moult et al '15]

$$\mathcal{B}_{n\pm}^a = -\varepsilon_{\mp\mu}(n, \bar{n}) \mathcal{B}_{n\perp, \omega_c}^{a\mu}, \quad \chi_{n\pm}^\alpha = \frac{1 \pm \gamma_5}{2} \chi_{n, \omega_a}^\alpha, \quad J_{\bar{n}n\pm}^{\bar{\alpha}\beta} = \mp \sqrt{\frac{2}{\omega_a \omega_b}} \frac{\varepsilon_{\mp}^\mu(\bar{n}, n)}{\langle n \mp |\bar{n}\pm \rangle} \bar{\chi}_{\bar{n}\pm}^{\bar{\alpha}} \gamma_\mu \chi_{n\pm}^\beta$$

- The complete set of operators in the helicity basis [Feige et al '17]

$$\begin{aligned} O_{1+-}^{(1)a \bar{\alpha}\beta} &= \mathcal{B}_{n+}^a J_{\bar{n}n-}^{\bar{\alpha}\beta}, & O_{1-+}^{(1)a \bar{\alpha}\beta} &= \mathcal{B}_{n-}^a J_{\bar{n}n+}^{\bar{\alpha}\beta}, \\ O_{2--}^{(1)a \bar{\alpha}\beta} &= \mathcal{B}_{\bar{n}-}^a J_{\bar{n}n-}^{\bar{\alpha}\beta}, & O_{2++}^{(1)a \bar{\alpha}\beta} &= \mathcal{B}_{\bar{n}+}^a J_{\bar{n}n+}^{\bar{\alpha}\beta}. \end{aligned}$$

- Parity and charge conjugation invariance $\Rightarrow C_{\lambda_3 \lambda_{12}}^{(1)} = C_{-\lambda_3 -\lambda_{12}}^{(1)}$

\Rightarrow Combination of helicity operators appear as

$$\begin{aligned} \mathcal{B}_{n+} J_{\bar{n}n-} + \mathcal{B}_{n-} J_{\bar{n}n+} &= \frac{1}{\sqrt{\omega_a \omega_b}} \bar{\chi}_{\bar{n}, \omega_b} [S_{\bar{n}}^\dagger S_n] \not{\mathcal{B}}_{\perp n, -\omega_c} \chi_{n, \omega_a} \\ \mathcal{B}_{\bar{n}-} J_{\bar{n}n-} + \mathcal{B}_{\bar{n}+} J_{\bar{n}n+} &= \frac{1}{\sqrt{\omega_a \omega_b}} \bar{\chi}_{\bar{n}, \omega_b} \not{\mathcal{B}}_{\perp \bar{n}, \omega_c} [S_{\bar{n}}^\dagger S_n] \chi_{n, \omega_a} \end{aligned}$$

- Same soft Wilson lines as leading power

Subleading Current: with \mathcal{B}_\perp Insertion

$$\frac{1}{\sqrt{\omega_a \omega_b}} \bar{\chi}_{\bar{n}, \omega_b} [S_{\bar{n}}^\dagger S_n] \mathcal{B}_{\perp n, -\omega_c} \chi_{n, \omega_a}, \quad \frac{1}{\sqrt{\omega_a \omega_b}} \bar{\chi}_{\bar{n}, \omega_b} \mathcal{B}_{\perp \bar{n}, \omega_c} [S_{\bar{n}}^\dagger S_n] \chi_{n, \omega_a}$$

- Hermiticity + $n \leftrightarrow \bar{n}$ symmetry: only one Wilson coefficient $C_f^{(1)}$
- Summing over helicities gives

$$\sum_{\lambda_e, \lambda_{12}, \lambda_3} C_f^{(1)} \mathcal{B}_{\lambda_3} J_{\bar{n}n\lambda_{12}} J_{\lambda_e} \sim J_B^{(1)\mu} J_{e\mu}$$

where $J_{e\mu} = \bar{e} \gamma_\mu e$ and (denoting $\xi = \omega_c/Q$),

$$\begin{aligned} J_B^{(1)\mu} &\sim (n^\mu + \bar{n}^\mu) \int d\omega_a d\omega_b d\omega_c C_f^{(1)}(Q, \xi) \\ &\times \left[\delta(\omega_a + \omega_c - Q) \delta(\omega_b - Q) \bar{\chi}_{\bar{n}, \omega_b} [S_{\bar{n}}^\dagger S_n] \mathcal{B}_{\perp n, -\omega_c} \chi_{n, \omega_a} \right. \\ &\quad \left. + \delta(\omega_a - Q) \delta(\omega_b + \omega_c - Q) \bar{\chi}_{\bar{n}, \omega_b} \mathcal{B}_{\perp \bar{n}, \omega_c} [S_{\bar{n}}^\dagger S_n] \chi_{n, \omega_a} \right] \\ &\sim C_f^{(1)}(Q, \xi) \left[\text{diagram 1} \right]^{\frac{1}{2}} + \alpha \left[\text{diagram 2} \right]^{\frac{1}{2}} \quad (+ \text{Wilson lines}) \end{aligned}$$

Subleading Current: with \mathcal{B}_\perp Insertion

Denoting $\xi = \omega_c/Q$, define the q-g-q correlators as

$$\hat{B}_{\mathcal{B}_f}^{\rho\beta'\beta}(x, \xi, \vec{b}_T) \equiv Q \langle N | [\bar{\chi}_{n, \omega_a}^\beta \mathcal{B}_{\perp n, -\omega_c}^\rho](b_\perp^\mu) \chi_n^{\beta'}(0) | N \rangle,$$

$$\hat{G}_{\mathcal{B}_f}^{\rho\beta'\beta}(z, \xi, \vec{b}_T) \equiv \frac{Q}{2z} \sum_X \langle 0 | [\bar{\chi}_{\bar{n}, \omega_b}^\beta \mathcal{B}_{\perp \bar{n}, \omega_c}^\rho](b_\perp^\mu) | h, X \rangle \langle h, X | \chi_{\bar{n}}^{\beta'}(0) | 0 \rangle$$

$$\tilde{B}_{\mathcal{B}_f}^{\rho\beta'\beta}(x, \xi, \vec{b}_T) = \hat{B}_{\mathcal{B}_f}^{\rho\beta'\beta}(x, \xi, \vec{b}_T) \sqrt{\mathcal{S}(b_T)},$$

$$\tilde{G}_{\mathcal{B}_f}^{\rho\beta'\beta}(z, \xi, \vec{b}_T) = \hat{G}_{\mathcal{B}_f}^{\rho\beta'\beta}(z, \xi, \vec{b}_T) \sqrt{\mathcal{S}(b_T)}$$

$$W_{\mathcal{B}}^{(1)\mu\nu} = \frac{2z}{Q} \sum_f \int d^2 b_T e^{i\vec{q}_T \cdot \vec{b}_T} \int_0^1 d\xi \mathcal{H}^{(1)}(Q, \xi) (n^\mu + \bar{n}^\mu)$$

$$\times \text{Tr} \left[\tilde{B}_{\mathcal{B}_f}^\rho(x, \xi, \vec{b}_T) \gamma_\rho \mathcal{G}_f(z, \vec{b}_T) \gamma_\perp^\nu + B_f(x, \vec{b}_T) \gamma_\perp^\nu \tilde{G}_{\mathcal{B}_f}^\rho(z, \xi, \vec{b}_T) \gamma_\rho \right] + \text{h.c.}$$

$$\mathcal{H}^{(1)}(Q, \xi) = C_f^{(1)}(Q, \xi) C_f^{(0)}(Q)$$

In momentum space, $\tilde{B}_{\mathcal{B}_f}^{\rho\beta'\beta}$ can be decomposed as [Boer, Mulders, Pijlman '03]

[Bacchetta, Mulders, Pijlman '04]

$$\tilde{B}_{\mathcal{B}_f}^{\rho\beta'\beta}(x, \xi, \vec{p}_T) = \frac{xM_N}{2} \left\{ \left[(\tilde{f}^\perp - i\tilde{g}^\perp) \frac{p_{\perp\sigma}}{M_N} (g_{\perp}^{\rho\sigma} - i\epsilon_{\perp}^{\rho\sigma} \gamma_5) + i(\tilde{h} + i\tilde{e}) \gamma_\perp^\rho \right] \frac{\not{p}}{2} \right\}^{\beta'\beta} + \dots$$

Vanishing Soft Contributions

- Subleading soft contributions exist in general, and are important in other processes
- In subleading SIDIS, we showed that they all vanish:

(1) Operators involving $\mathcal{B}_{s\perp}^{(n_i)\mu}$ yield

$$\hat{S}_1^\rho(b_\perp) \sim \text{tr} \left\langle 0 \left| [S_n^\dagger(b_\perp) S_{\bar{n}}(b_\perp)] [S_{\bar{n}}^\dagger(0) S_n(0) g \mathcal{B}_{s\perp}^{(n)\rho}(0)] \right| 0 \right\rangle$$

Vanish due to charge conjugation & parity invariance of the vacuum

(2) Operators involving a $n \cdot \partial_s$, $n \cdot \mathcal{B}_s^{(\bar{n})}$, ... give $\left. \frac{\partial}{\partial b_s^\mp} \mathcal{S}(b_T, b_s^+ b_s^-) \right|_{b_s^\pm \rightarrow 0}$,

which scales linear in \bar{n} or n under RPI-III ($n \rightarrow e^\alpha n$, $\bar{n} \rightarrow e^{-\alpha} \bar{n}$) of SCET, thus vanishes

Vanishing Soft Contributions

(3) SCET_{II} Subleading Lagrangian insertions

$$W_{\mathcal{L}}^{(1)\mu\nu} \sim \langle N | J^{(0)\dagger\mu}(b) | h, X \rangle \langle h, X | \int d^4x d^4y T [J^{(0)\nu}(0) \mathcal{L}^{(1/2)}(x) \mathcal{L}^{(1/2)}(y)] | N \rangle \\ + \langle N | J^{(0)\dagger\mu}(b) | h, X \rangle \langle h, X | \int d^4x T [J^{(0)\nu}(0) \mathcal{L}^{(1)}(x)] | N \rangle + \dots$$

Since μ, ν are transverse ($J^{(0)\mu} \sim (\gamma_{\perp}^{\mu})^{\alpha\beta} C_f^{(0)}(Q) \bar{\chi}_{\bar{n}, \omega_b}^{\alpha} [S_{\bar{n}}^{\dagger} S_n] \chi_{n, \omega_a}^{\beta}$), when contracting with $P_1^{\mu\nu} = -(\tilde{t}^{\mu} \tilde{x}^{\nu} + \tilde{x}^{\mu} \tilde{t}^{\nu})$, $P_2^{\mu\nu} = i(\tilde{t}^{\mu} \tilde{x}^{\nu} - \tilde{x}^{\mu} \tilde{t}^{\nu})$, ... such contributions vanish

(4) $T[J_I^{(0)\mu} \mathcal{L}_I^{(1)}]$, $T[J_I^{(0)\mu} \mathcal{L}_I^{(2)}]$, $T[J_I^{(0)\mu} \mathcal{L}_I^{(1)} \mathcal{L}_I^{(1)}]$ in SCET_I

→ hard scattering operators in SCET_{II}

Vanish since μ, ν in $J^{(0)}$ are (again) transverse

Results

$$\begin{aligned}
 W_1 = \mathcal{F} & \left\{ -\frac{P_{hT}}{zQ} \mathcal{H}^{(0)} \left[f_1 D_1 - \frac{2 p_{Tx} k_{Tx} - \vec{p}_T \cdot \vec{k}_T}{M_N M_h} h_1^\perp H_1^\perp \right] \right. && \text{(Kinematic corrections)} \\
 & - \mathcal{H}^{(0)} \left[\frac{p_{Tx} + k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2 k_{Tx} + k_T^2 p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp \right] && \text{(From the } \mathcal{P}_\perp \text{ operators)} \\
 & \left. + \mathcal{H}^{(1)} \left[\frac{2x}{Q} \left(p_{Tx} \tilde{f}^\perp D_1 + \frac{M_N}{M_h} k_{Tx} \tilde{h} H_1^\perp \right) + \frac{2}{zQ} \left(k_{Tx} f_1 \tilde{D}^\perp + \frac{M_h}{M_N} p_{Tx} h_1^\perp \tilde{H} \right) \right] \right\} && \text{(From the } \mathcal{B}_\perp \text{ operators)} \\
 W_2 = \mathcal{F} & \left\{ \mathcal{H}^{(1)} \left[\frac{2x}{Q} \left(p_{Tx} \tilde{g}^\perp D_1 + \frac{M_N}{M_h} k_{Tx} \tilde{e} H_1^\perp \right) + \frac{2}{zQ} \left(k_{Tx} f_1 \tilde{G}^\perp + \frac{M_h}{M_N} p_{Tx} h_1^\perp \tilde{E} \right) \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}[\omega \mathcal{H} g D] = 2z \sum_f \int d^2 p_T d^2 k_T \delta^2(\vec{q}_T + \vec{p}_T - \vec{k}_T) \omega(\vec{p}_T, \vec{k}_T) \\
 \times \int_0^1 d\xi \mathcal{H}_f(Q, (\xi)) g_f(x, (\xi), p_T) D_f(z, (\xi), k_T)
 \end{aligned}$$

New in our results

- Vanishing of the subleading soft contributions
- Soft function, same as leading power (as conjectured in [Bacchetta et al '19])
- Appearance of two hard functions, $\mathcal{H}^{(0)}(Q)$ and $\mathcal{H}^{(1)}(Q, \xi)$
- Dependence on ξ in $\mathcal{H}^{(1)}(Q, \xi)$ and the functions $\tilde{f}^\perp, \tilde{D}^\perp, \dots$

Structure Functions with Full Spin Dependence

$$\frac{d\sigma}{dx dy dz d\psi d^2\vec{P}_{hT}} = \frac{\alpha^2}{2Q^2} \frac{y}{z} \frac{\kappa_\gamma}{1-\epsilon} \left[(L \cdot W)_{UU} + S_L(L \cdot W)_{UL} \right. \\ \left. + \lambda_\ell(L \cdot W)_{LU} + \lambda_\ell S_L(L \cdot W)_{LL} + S_T(L \cdot W)_{UT} + \lambda_\ell S_T(L \cdot W)_{LT} \right],$$

$$(L \cdot W)_{UU} = W_{UU,T} + \epsilon W_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi_h) W_{UU}^{\cos(\phi_h)} + \epsilon \cos(2\phi_h) W_{UU}^{\cos(2\phi_h)},$$

$$(L \cdot W)_{UL} = \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_h) W_{UL}^{\sin(\phi_h)} + \epsilon \sin(2\phi_h) W_{UL}^{\sin(2\phi_h)},$$

$$(L \cdot W)_{LU} = \sqrt{2\epsilon(1-\epsilon)} \sin(\phi_h) W_{LU}^{\sin(\phi_h)},$$

$$(L \cdot W)_{LL} = \sqrt{1-\epsilon^2} W_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_h) W_{LL}^{\cos(\phi_h)},$$

$$(L \cdot W)_{UT} = \sin(\phi_h - \phi_S) \left[W_{UT,T}^{\sin(\phi_h - \phi_S)} + \epsilon W_{UT,L}^{\sin(\phi_h - \phi_S)} \right] \\ + \epsilon \left[\sin(\phi_h + \phi_S) W_{UT}^{\sin(\phi_h + \phi_S)} + \sin(3\phi_h - \phi_S) W_{UT}^{\sin(3\phi_h - \phi_S)} \right] \\ + \sqrt{2\epsilon(1+\epsilon)} \left[\sin(\phi_S) W_{UT}^{\sin(\phi_S)} + \sin(2\phi_h - \phi_S) W_{UT}^{\sin(2\phi_h - \phi_S)} \right],$$

$$(L \cdot W)_{LT} = \sqrt{1-\epsilon^2} \cos(\phi_h - \phi_S) W_{LT}^{\cos(\phi_h - \phi_S)} \\ + \sqrt{2\epsilon(1-\epsilon)} \left[\cos(\phi_S) W_{LT}^{\cos(\phi_S)} + \cos(2\phi_h - \phi_S) W_{LT}^{\cos(2\phi_h - \phi_S)} \right].$$

- We also have results for spin-dependent $\mathcal{O}(P_{hT}/Q)$ structure functions

Anomalous dimensions

- Rapidity anomalous dimension is the same as at leading power

$$\tilde{B}_{\mathcal{B}f}^{\rho\beta'\beta}(x, \xi, \vec{b}_T, \mu, \zeta) = \hat{\tilde{B}}_{\mathcal{B}f}^{\rho\beta'\beta}(x, \xi, \vec{b}_T, \mu, \nu^2/\zeta) \sqrt{\mathcal{S}(b_T, \mu, \nu)},$$

$$\Rightarrow \frac{d \log \tilde{B}_{\mathcal{B}f}^{\rho\beta'\beta}}{d \log \zeta} = \frac{1}{4} \frac{d \log \mathcal{S}}{d \log \nu} = \frac{1}{4} \gamma_\nu(\mu, b_T)$$

- Anomalous dimension of $C_f^{(1)}$ have been calculated to one loop, with single log dependence on ξ [Beneke et al, '17 '18]

$$\mu \frac{d}{d\mu} C_f^{(1)}(Q, \xi, \mu) = \int \frac{d\xi'}{\xi'} \gamma_{ff'}^{(1)}(\xi, \xi', Q, \mu) C_{f'}^{(1)}(\xi', Q, \mu)$$

Discussion

- At leading order, $C_f^{(1)}$ is independent on ξ from tree level matching, ξ can be integrated in q-g-q correlators. W_1, W_2, \dots then fully agree with [Bacchetta et al '06] at leading order (after inclusion of the soft function, as conjectured in [Bacchetta et al '19])
 - Anomalous dimension results for $C_f^{(1)}(Q, \xi, \mu)$ confirms the nontrivial ξ dependence
- ⇒ Disproves the simpler factorization theorem in [Bacchetta et al '19]
- Our results includes radiative corrections: fixed order + resummed logs
LO $\xrightarrow{\text{Fact.}}$ NLO + LL + NLL + ...!
 - Recent overlapping work on operator basis, perturbative corrections and anomalous dimensions [Vladimirov, Moos, Scimemi '21]

Summary & Outlook

- Derived factorization of $W^{\mu\nu}$ at subleading power, including contribution from subleading operators with insertion of \mathcal{P}_\perp and \mathcal{B}_\perp
- Showed the factorization formulae of subleading structure functions W_1, W_2 , including contributions from the kinematic correction and the subleading operators
- Future Directions: phenomenology including perturbative and resummation effects for W_1, W_2, \dots

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Thanks for your attention!