Disentangling Long and Short Distances in Momentum-Space TMDs

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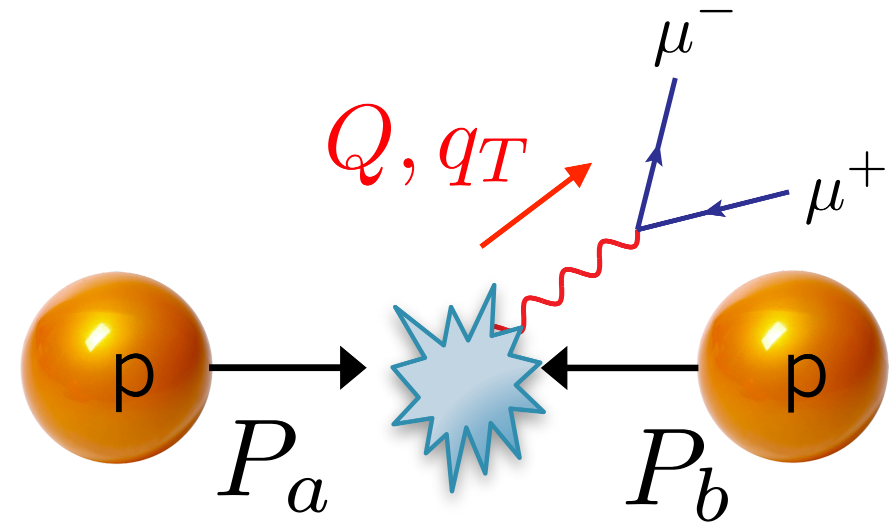
Massachusetts Institute of Technology

TMD Collaboration

TMDPDFs



- Factorization of Drell-Yan cross section:

$$\frac{d\sigma}{dQ\,dY\,d^2q_T} = H(Q,\mu) \sum_i \int d^2\vec{b}_T \; e^{i\vec{q}_T\cdot\vec{b}_T} \, f_i(x_a, b_T, \mu, \zeta_a) \; f_{\bar{i}}(x_b, b_T, \mu, \zeta_b) \times \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)\right]$$

Hard virtual corrections

Describe transverse momentum of the partons

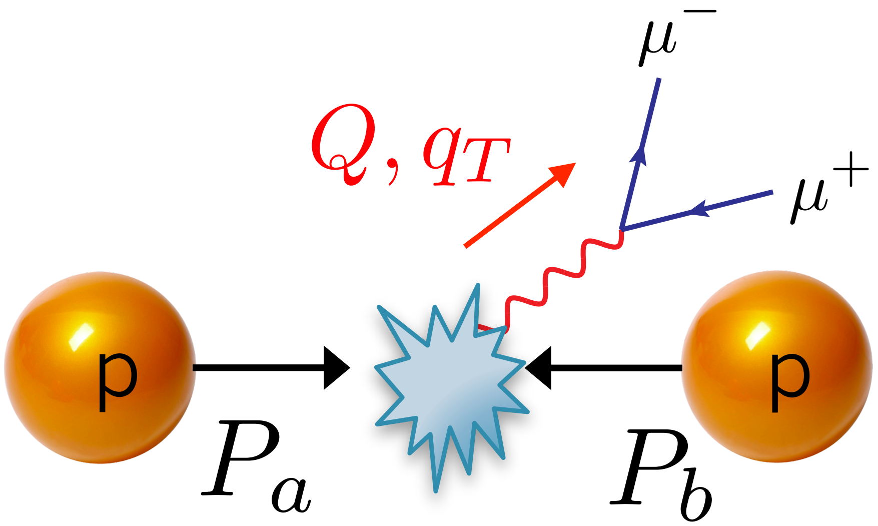
- Most easily written in position space

μ = Renormalization scale

ζ = Collins-Soper parameter

$\zeta_a \zeta_b = Q^4$

TMDPDFs

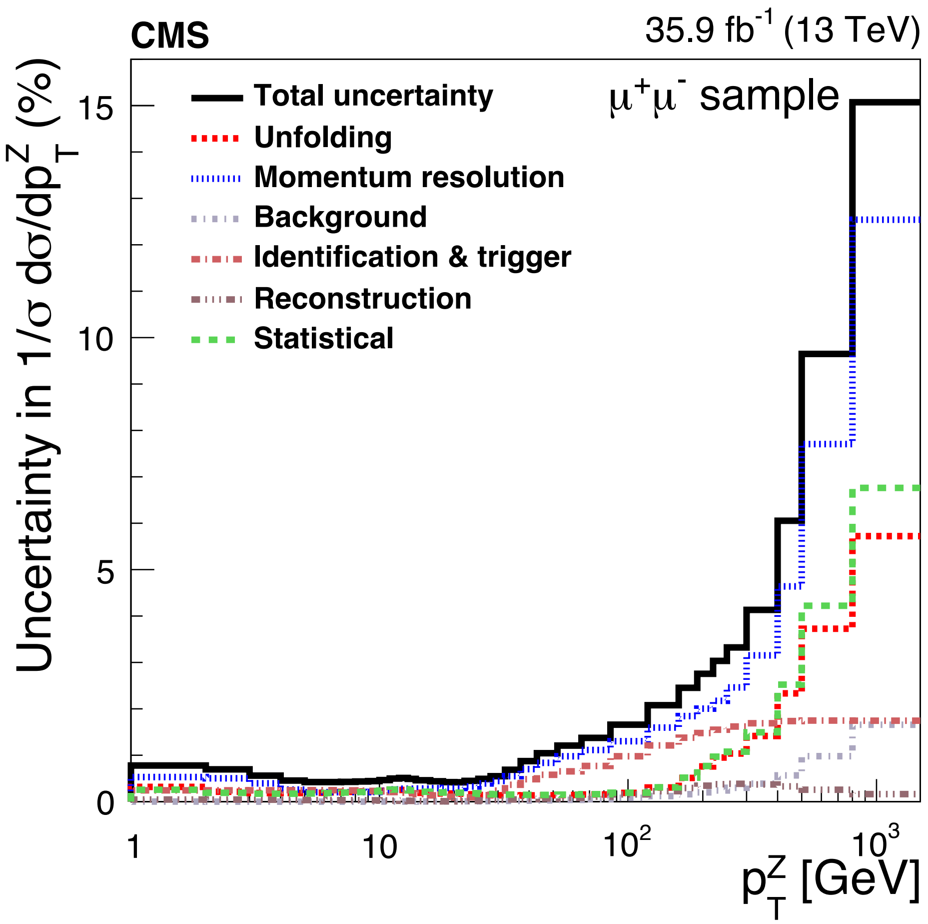


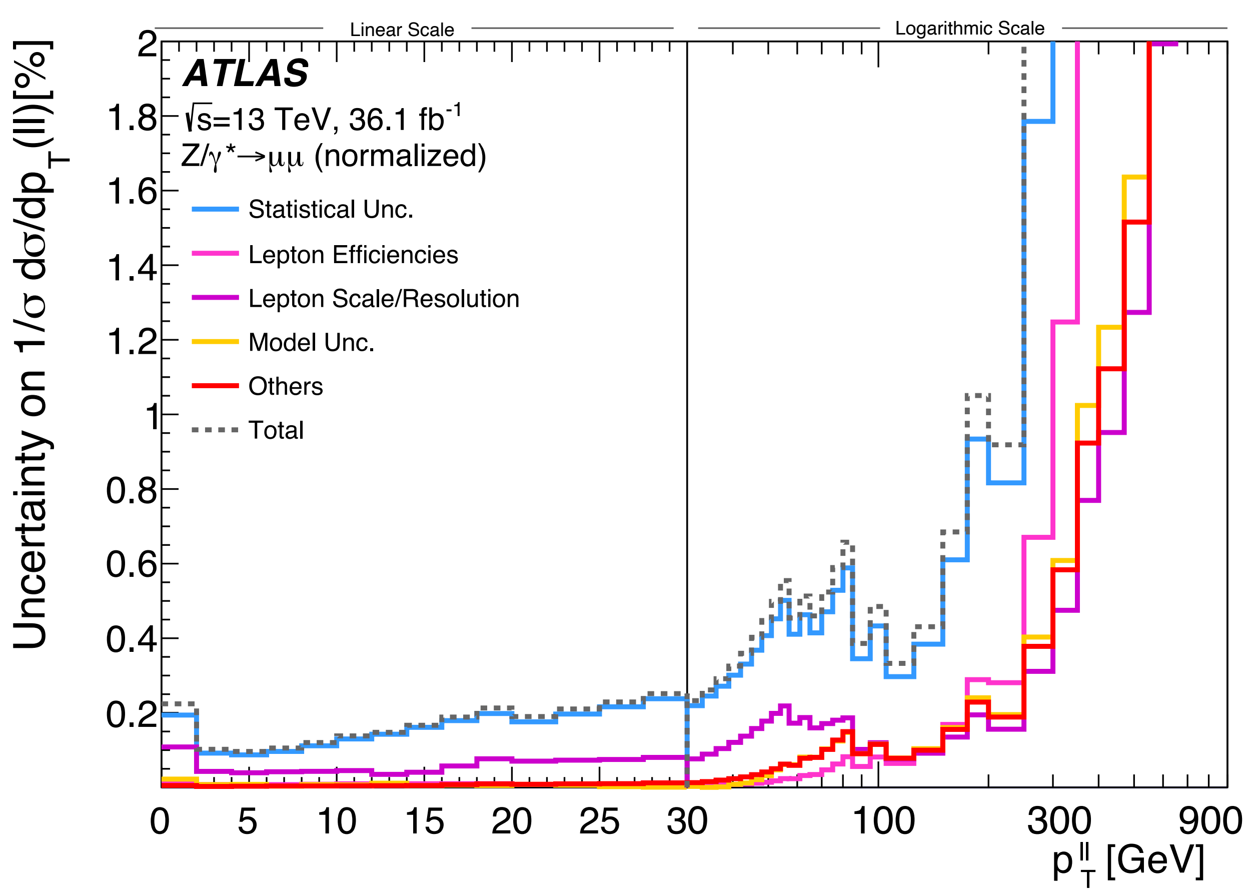
- Factorization of Drell-Yan cross section:

$$\frac{d\sigma}{dQ\,dY\,d^2q_T} = H(Q,\mu)\sum_i \int d^2\vec{b}_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\, f_i(x_a, b_T, \mu, \zeta_a)\, f_{\bar{i}}(x_b, b_T, \mu, \zeta_b) \times \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)\right]$$

- Measurements are done in momentum space!

CMS: 1909.04133
ATLAS: 1912.02844





Modeling TMDPDFs

- TMDPDFs have both perturbative and nonperturbative parts, and usually:

$$f_i(x, b_T, \mu, \zeta) = f_{\text{pert},\, i}(x, b^*(b_T), \mu, \zeta) \cdot f_{\text{NP}}(x, b_T, \zeta)$$

Calculated with expansion in $\alpha_s(1/b_T)$

- The perturbative part computed with an operator product expansion (OPE):

$$f^{\text{TMD}}_{\text{pert},\, i}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{dz}{z} C_{ij}(\frac{x}{z}, b_T, \mu, \zeta) f_j^{\text{coll}}(z, \mu)$$

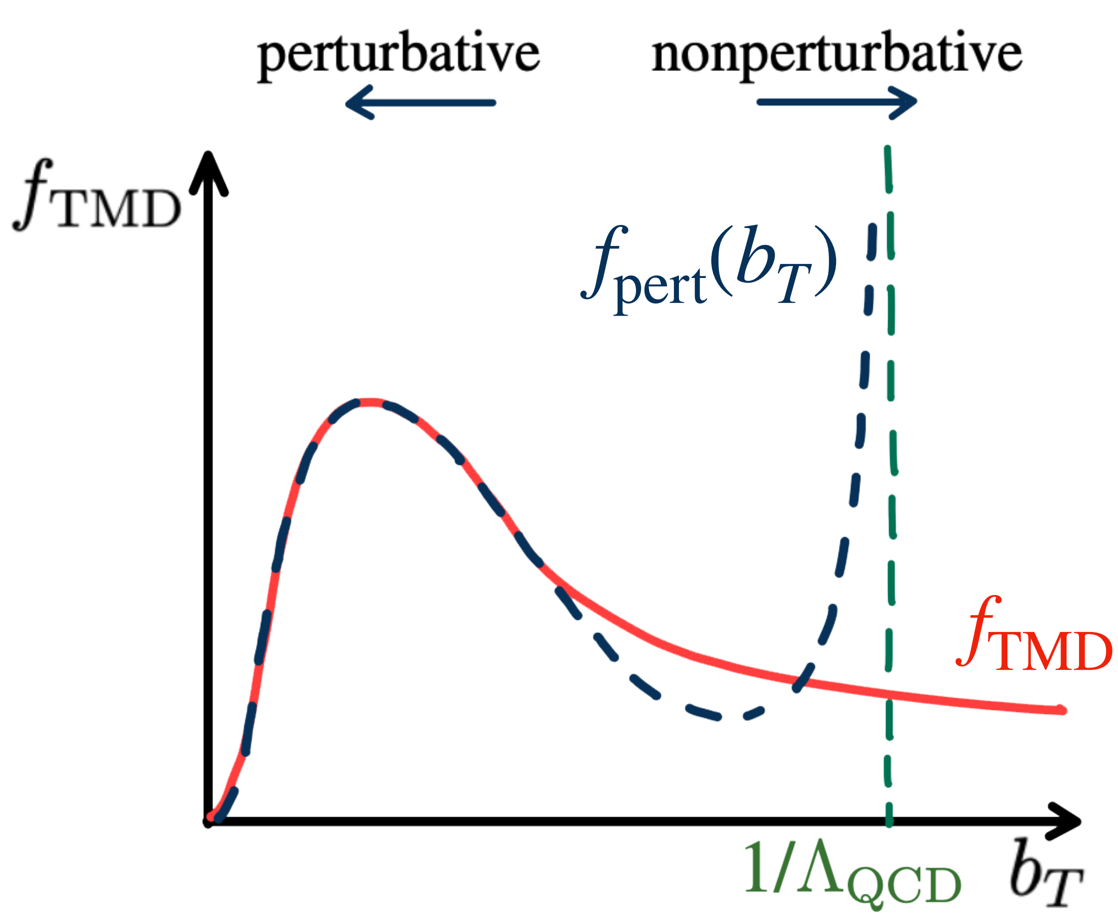
$$= f_i^{\text{coll}}(x, \mu) + \alpha_s C_{ij}^{(1)} \otimes f_j^{\text{coll}}(x, \mu) + \mathcal{O}(\alpha_s^2)$$

Modeling TMDPDFs

- TMDPDFs have both perturbative and nonperturbative parts, and usually:

$$f_i(x, b_T, \mu, \zeta) = f_{\text{pert},\, i}(x, b^*(b_T), \mu, \zeta) \cdot f_{\text{NP}}(x, b_T, \zeta)$$

Calculated with expansion in $\alpha_s(1/b_T)$ — Has to be 1+$\mathcal{O}(b_T^2)$



- $b^*(b_T)$ shields the Landau pole
- $b_T \ll 1/\Lambda_{\text{QCD}}$: $b^*(b_T) \to b_T$, $f_{\text{NP}} \to 1$

  f_{pert} dominates
- $b_T \gg 1/\Lambda_{\text{QCD}}$: $b^*(b_T) \to$ constant

  f_{NP} dominates

Modeling TMDPDFs

- Different models of $f_{\rm NP}$ are used for fitting to data
- $b*(b_T)$ shields the Landau pole and is coupled to $f_{\rm NP}$

$$\begin{aligned} f_{\rm TMD}(x, b_T, \mu, \zeta) &= f_{\rm pert}(x, b_A^*(b_T), \mu, \zeta) \cdot f_{\rm NP}^A(x, b_T, \zeta) \\ &= f_{\rm pert}(x, b_B^*(b_T), \mu, \zeta) \cdot f_{\rm NP}^B(x, b_T, \zeta) \end{aligned}$$

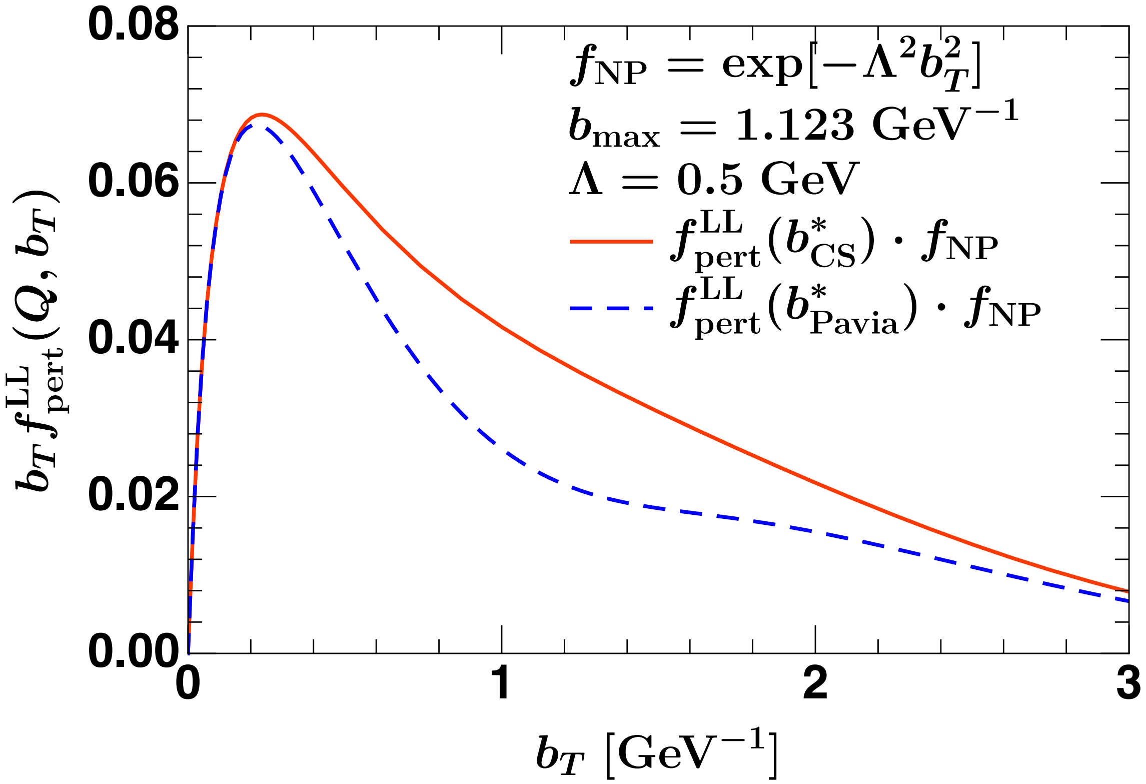
$$b_A^*(b_T) \neq b_B^*(b_T) \quad \Rightarrow \quad f_{\rm NP}^A(x, b_T) \text{ and } f_{\rm NP}^B(x, b_T) \text{ are not comparable!}$$

- The perturbative and nonperturbative effects are mixed up!

Modeling TMDPDFs

- b^* prescriptions makes different $f_{\rm NP}$ not comparable
- For example, take the same $f_{\rm NP}(b_T) = e^{-(0.5 {\rm GeV}\ b_T)^2}$,

  use either $b^*_{\rm CS}(b_T)$ or $b^*_{\rm Pavia}(b_T)$:



$$b^*_{\rm CS}(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{\rm max})^2}} = b_T\ (1 + \mathcal{O}(b_T^2))$$

Collins+Soper (1982)

$$b^*_{\rm Pavia}(b_T) = b_{\rm max}\left(1 - \exp(-\frac{b_T^4}{b_{\rm max}^4})\right)^{\frac{1}{4}} = b_T\ (1 + \mathcal{O}(b_T^4))$$

Pavia: 1703.10157

- Goal: extract nonperturbative physics without b^* contamination

Momentum Space

- Measurements are in q_T space: Fourier transform

$$\frac{d\sigma}{dq_T} = 2\pi q_T \int_0^\infty \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\sigma(b_T)$$

q_T spectrum ↗

$$= q_T \int_0^\infty db_T\, b_T \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{iq_T b_T \cos\phi}\sigma(b_T) = q_T \int_0^\infty db_T\, b_T\, J_0(q_T b_T)\sigma(b_T)$$

  - For perturbative q_T, integral still includes nonperturbative b_T !

- Intuition: perturbative q_T should be dominated by perturbative $b_T \sim 1/q_T$

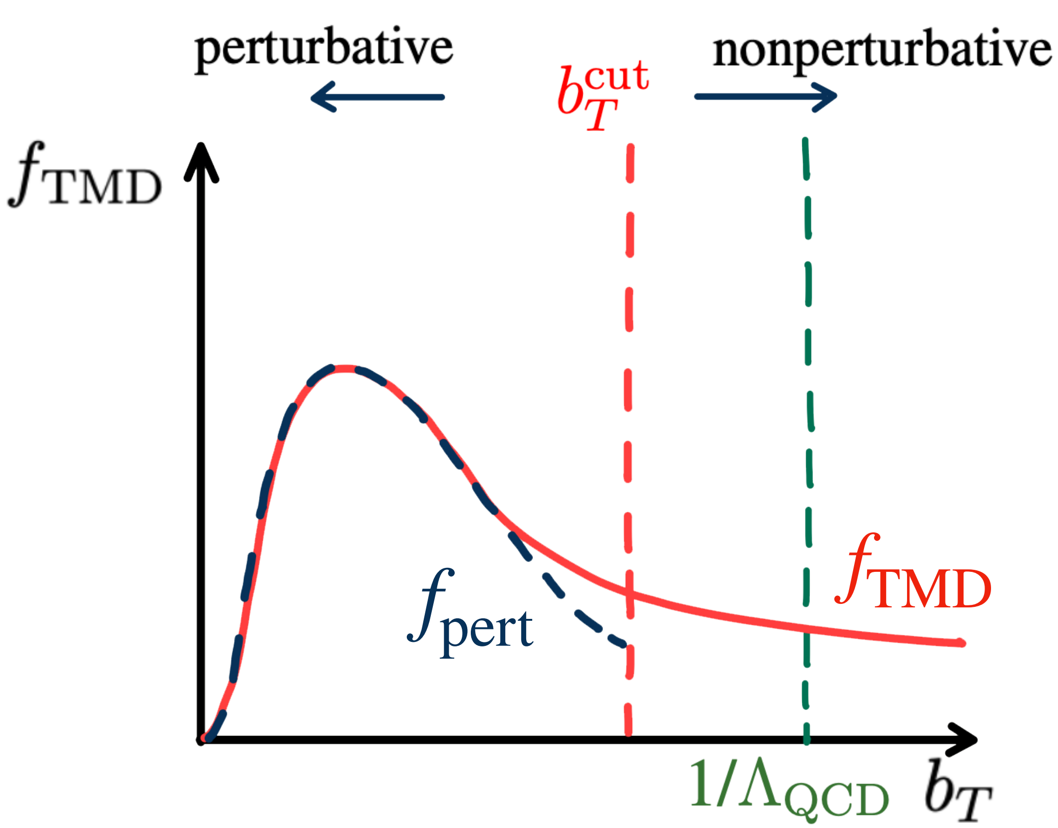
Momentum Space

- Intuition: perturbative q_T should be dominated by perturbative b_T
- Goal: make this intuition manifest
- Solution: introducing b_T^{cut}

Can use perturbative OPE — Nonperturbative physics

$$S[f](q_T) \equiv q_T \int_0^\infty db_T\, b_T J_0(q_T b_T) f(b_T) = S_<[f](q_T) + S_>[f](q_T)$$

full spectrum



$$S_<[f](q_T) \equiv q_T \int_0^{b_T^{\text{cut}}} db_T\, b_T J_0(q_T b_T) f(b_T),$$

$$S_>[f](q_T) \equiv q_T \int_{b_T^{\text{cut}}}^\infty db_T\, b_T J_0(q_T b_T) f(b_T)$$

Truncated Functionals

- Want to approximate $S[f]$ using perturbative $b_T \leq b_T^{\text{cut}}$
- Can use $S_<[f]$, but need to systematically account for $S_>[f]$

$$S_>[f](q_T, b_T^{\text{cut}}) = q_T \int_{b_T^{\text{cut}}}^{\infty} db_T\, b_T J_0(q_T b_T) f(b_T)$$

$$= -\, b_T^{\text{cut}} \underbrace{J_1(q_T b_T^{\text{cut}})} f(b_T^{\text{cut}}) - \int_{b_T^{\text{cut}}}^{\infty} db_T\, b_T J_1(q_T b_T) \underbrace{f'(b_T)}_{< b_T^{-\rho-1}}$$

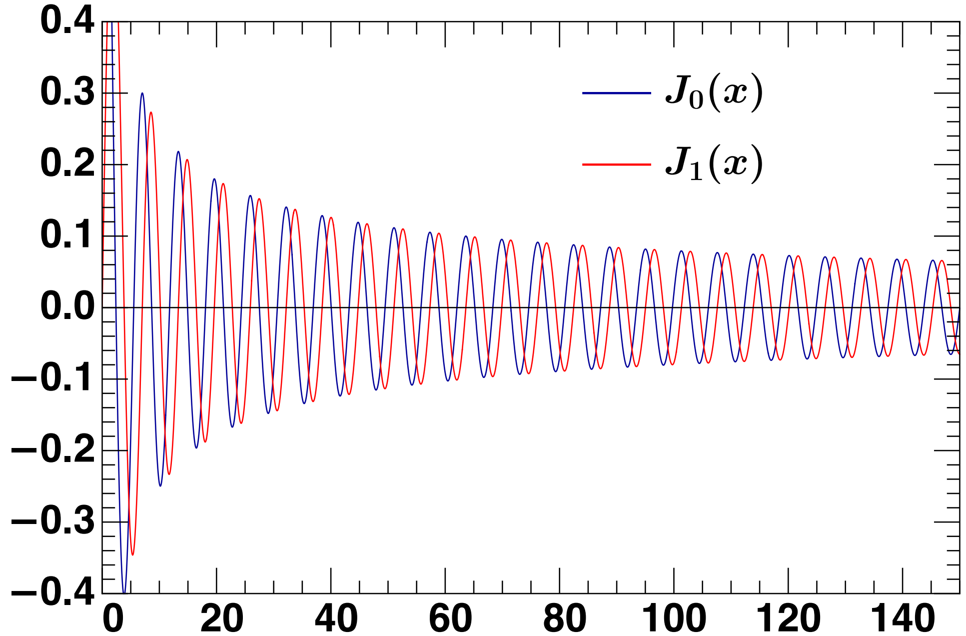
asymptotic form $\quad q_T \gg 1/b_T^{\text{cut}} > \Lambda_{\text{QCD}}$

$$= \sqrt{\frac{2 b_T^{\text{cut}}}{\pi q_T}} \cos\left(q_T b_T^{\text{cut}} + \frac{\pi}{4}\right) f(b_T^{\text{cut}}) + \mathcal{O}[(b_T^{\text{cut}} q_T)^{-\frac{3}{2}}]$$

Assumption:

a) $f(b_T \to \infty) < b_T^{-\rho}$, $\rho > \dfrac{1}{2}$

b) $f(b_T)$ differentiable at b_T^{cut}



$$J_0(x \to \infty) = \sqrt{\frac{2}{\pi x}} \cos(x - \frac{\pi}{4}) + \mathcal{O}(x^{-\frac{3}{2}})$$

$$J_1(x \to \infty) = -\sqrt{\frac{2}{\pi x}} \cos(x + \frac{\pi}{4}) + \mathcal{O}(x^{-\frac{3}{2}})$$

Truncated Functionals

Perturbative region

- Define a systematic series to approximate $S[f]$ using $b_T \leq b_T^{\text{cut}}$

$$S^{(0)}[f](q_T) \equiv S_<[f](q_T) = q_T \int_0^{b_T^{\text{cut}}} db_T\, b_T J_0(q_T b_T) f(b_T)$$

- Define $S^{(1)}[f]$ to include leading boundary contribution from $S_>[f]$

$$S^{(1)}[f](q_T) \equiv S^{(0)}[f] + \sqrt{\frac{2b_T^{\text{cut}}}{\pi q_T}} \cos\left(q_T b_T^{\text{cut}} + \frac{\pi}{4}\right) f(b_T^{\text{cut}})$$

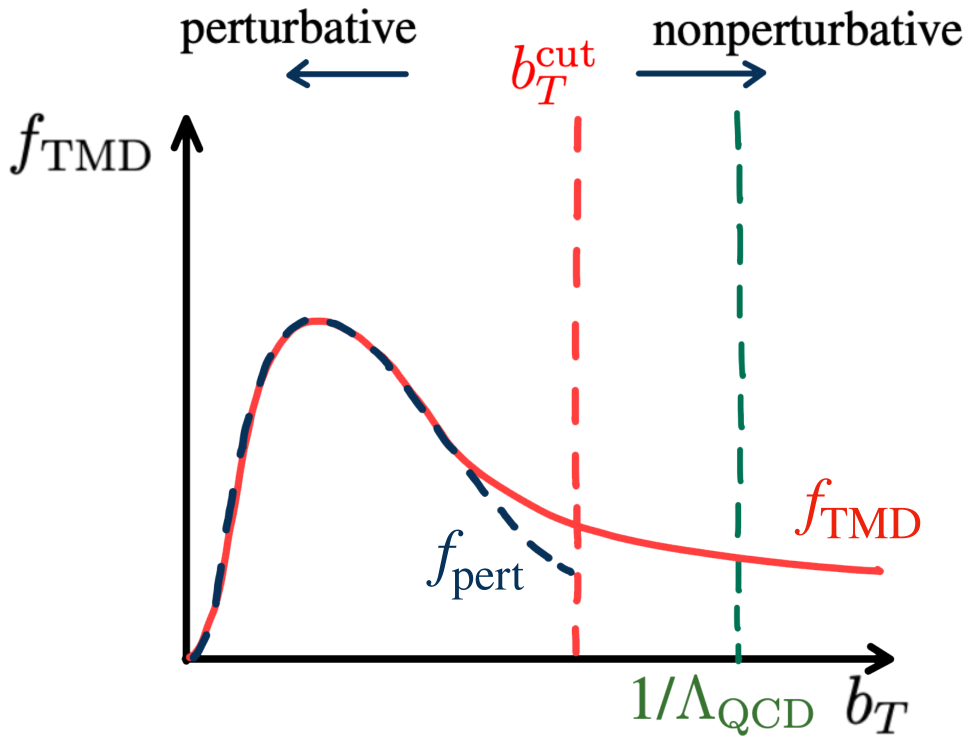
← First correction!

$$\boxed{S[f](q_T) = S^{(1)}[f](q_T, b_T^{\text{cut}}) + \frac{1}{q_T}\mathcal{O}[(b_T^{\text{cut}} q_T)^{-\frac{1}{2}}]}$$

Truncated Functionals

- Systematically add on power corrections

  so $S^{(n)}[f] \to S[f]$



$$S^{(0)}[f](q_T, b_T^{\text{cut}}) = \int_0^{b_T^{\text{cut}}} db_T\, b_T J_0(q_T b_T) f(b_T),$$

$$S^{(1)}[f](q_T, b_T^{\text{cut}}) = S^{(0)}[f] + \sqrt{\frac{2b_T^{\text{cut}}}{\pi q_T}} f(b_T^{\text{cut}}) \cdot \cos(b_T^{\text{cut}} q_T + \frac{\pi}{4})$$

$$S^{(2)}[f](q_T, b_T^{\text{cut}}) = S^{(1)}[f] - \sqrt{\frac{2b_T^{\text{cut}}}{\pi q_T}} \left(\frac{3\, f(b_T^{\text{cut}})}{8\, b_T^{\text{cut}} q_T} + \frac{f'(b_T^{\text{cut}})}{q_T} \right) \cdot \cos(b_T^{\text{cut}} q_T - \frac{\pi}{4})$$

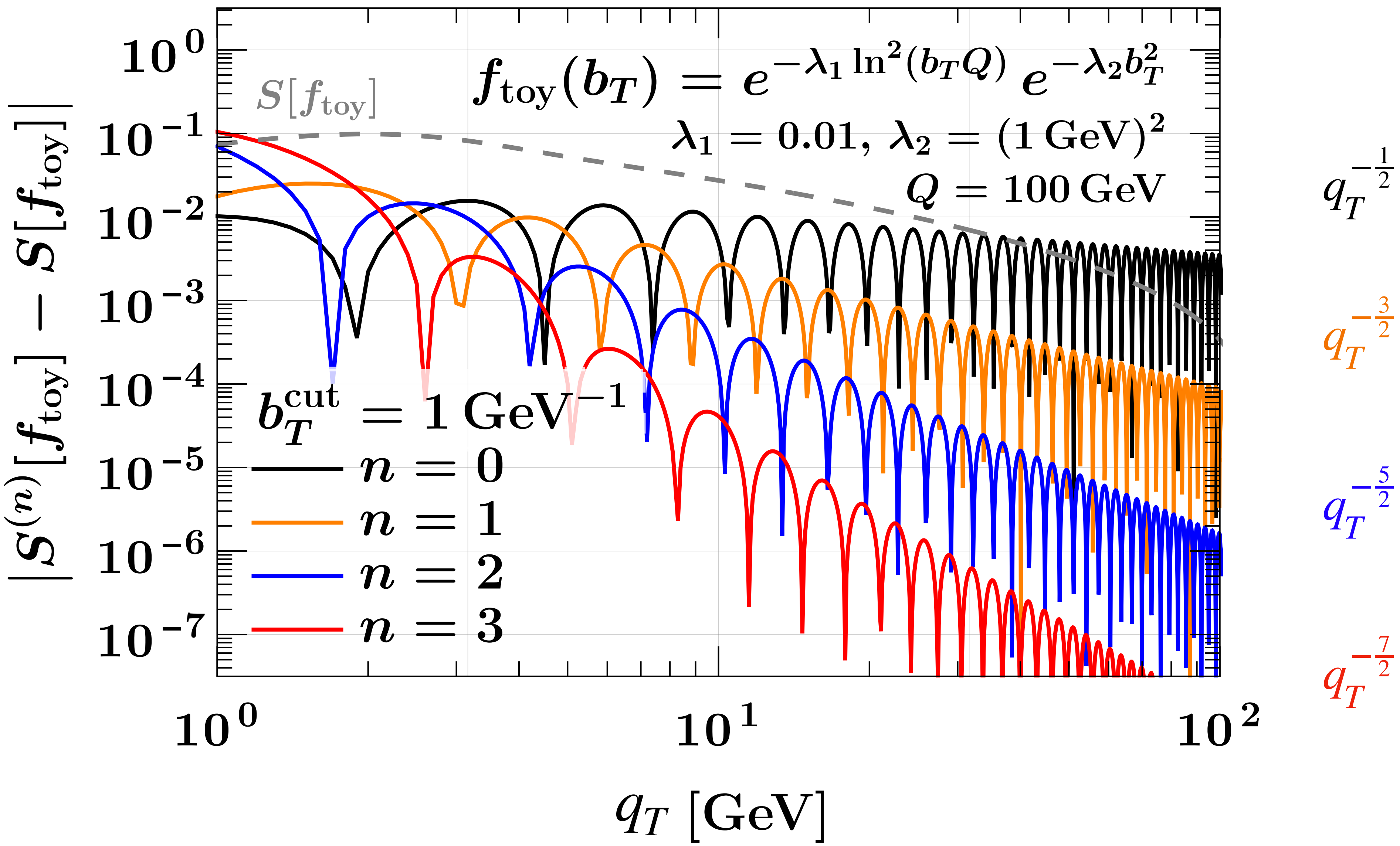
$$S^{(3)}[f](q_T, b_T^{\text{cut}}) = S^{(2)}[f] + \sqrt{\frac{2b_T^{\text{cut}}}{\pi q_T}} \left(\frac{15\, f(b_T^{\text{cut}})}{128\, b_T^{\text{cut}\,2} q_T^2} - \frac{7\, f'(b_T^{\text{cut}})}{8\, b_T^{\text{cut}} q_T^2} - \frac{f''(b_T^{\text{cut}})}{q_T^2} \right) \cdot \cos(b_T^{\text{cut}} q_T + \frac{\pi}{4})$$

$$\boxed{S[f](q_T) = S^{(n)}[f] + \frac{1}{q_T} \mathcal{O}[(b_T^{\text{cut}} q_T)^{-n+\frac{1}{2}}]}$$

Power Correction to Functionals

- Toy function $f = \exp[-\lambda_1 \ln^2(b_T Q)] \exp[-\lambda_2 b_T^2]$
- Errors of truncated functionals follow expected power law

$$S[f](q_T) = S^{(n)}[f] + \frac{1}{q_T}\mathcal{O}[(b_T^{\rm cut} q_T)^{-n+\frac{1}{2}}]$$

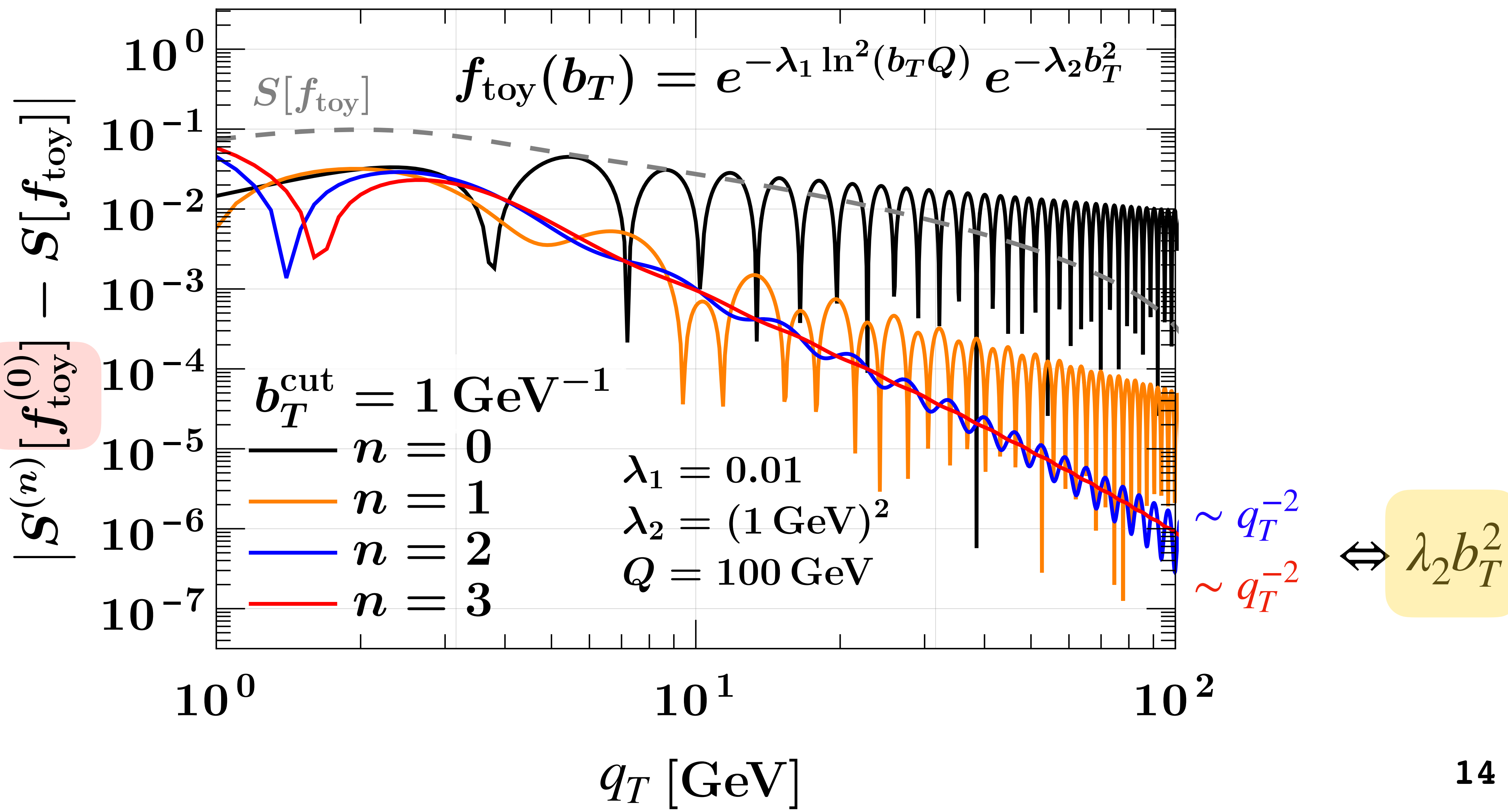


Perturbative Input

- Power expand toy function and use only "perturbative" input $f^{(0)}$

$$f = \underbrace{\exp[-\lambda_1 \ln^2(b_T Q)]}_{f^{(0)}}(1 - \lambda_2\, b_T^2 + \mathcal{O}(b_T^4))$$

- "Errors" of truncated functionals identify missing quadratic term



Cumulative Functionals

- We are often interested in the cumulative distribution:

$$\int_{|k_T|\le k_T^{\rm cut}} d^2\vec{k}_T\, f(k_T) = \int_{|k_T|\le k_T^{\rm cut}} d^2\vec{k}_T \int \frac{d^2b_T}{(2\pi)^2}\, e^{+i\vec{k}_T\cdot\vec{b}_T} f(b_T)$$

$$= \int^{k_T^{\rm cut}} dk_T\, k_T \int_0^\infty db_T\, b_T J_0(b_T k_T) f(b_T)$$

$$= \underbrace{k_T^{\rm cut} \int_0^\infty db_T\, J_1(b_T k_T^{\rm cut})\, f(b_T)}_{K[f](k_T^{\rm cut})}$$

- Approximate using perturbative region:

$$K^{(0)}[f](k_T^{\rm cut}, b_T^{\rm cut}) = k_T^{\rm cut} \int_0^{b_T^{\rm cut}} db_T\, J_1(b_T k_T^{\rm cut})\, f(b_T)$$

Cumulative Functionals

- Systematically add on power corrections so $K^{(n)}[f] \to K[f]$

$$K^{(0)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) = k_T^{\text{cut}} \int_0^{b_T^{\text{cut}}} db_T\, J_1(b_T k_T^{\text{cut}})\, f(b_T)$$

$$K^{(1)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) = K^{(0)}[f] + f(b_T^{\text{cut}}) \cdot \frac{\cos(b_T^{\text{cut}} k_T^{\text{cut}} - \frac{\pi}{4})}{\sqrt{\frac{\pi}{2}}(b_T^{\text{cut}} k_T^{\text{cut}})^{1/2}}$$

$$K^{(2)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) = K^{(1)}[f] - \left(\frac{f(b_T^{\text{cut}})}{8\, b_T^{\text{cut}} k_T^{\text{cut}}} - \frac{f'(b_T^{\text{cut}})}{k_T^{\text{cut}}}\right) \cdot \frac{\cos(b_T^{\text{cut}} k_T^{\text{cut}} + \frac{\pi}{4})}{\sqrt{\frac{\pi}{2}}(b_T^{\text{cut}} k_T^{\text{cut}})^{1/2}}$$

$$K^{(3)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) = K^{(2)}[f] - \left(\frac{9 f(b_T^{\text{cut}})}{128\, b_T^{\text{cut}\,2} k_T^{\text{cut}\,2}} - \frac{5 f'(b_T^{\text{cut}})}{8\, b_T^{\text{cut}} k_T^{\text{cut}\,2}} + \frac{f''(b_T^{\text{cut}})}{k_T^{\text{cut}\,2}}\right) \cdot \frac{\cos(b_T^{\text{cut}} k_T^{\text{cut}} - \frac{\pi}{4})}{\sqrt{\frac{\pi}{2}}(b_T^{\text{cut}} k_T^{\text{cut}})^{1/2}}$$

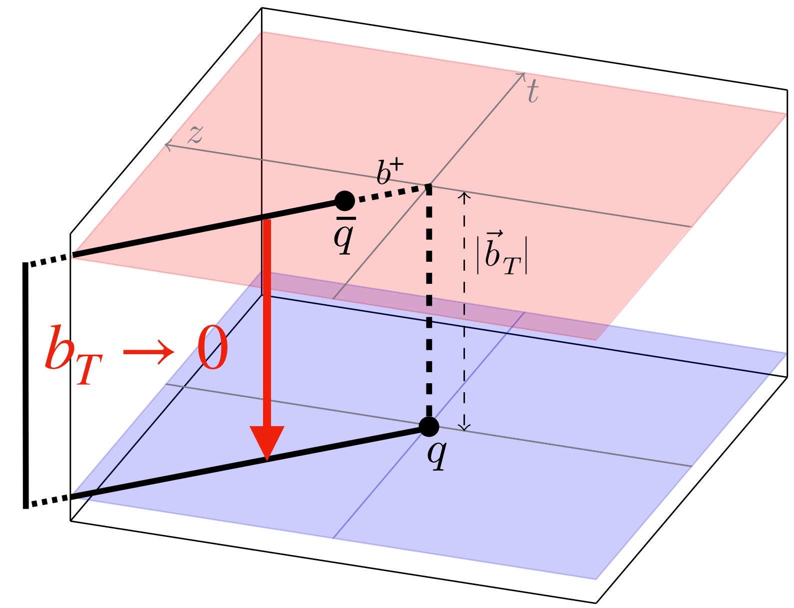
$$\boxed{K[f](k_T^{\text{cut}}) = K^{(n)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) + \mathcal{O}[(b_T^{\text{cut}} k_T^{\text{cut}})^{-n-\frac{1}{2}}]}$$

Apply to TMDPDFs

- What's the normalization of the TMDPDFs?

$$\int d^2\vec{k}_T \, f^{\rm TMD}(x, k_T, \mu, \zeta) \overset{?}{=} f^{\rm coll}(x, \mu)$$

↑ naively yes :)



- Renormalization breaks the naive expectation

$$\mu \frac{d}{d\mu} \int d^2\vec{k}_T \, f^{\rm TMD}(x, k_T, \mu, \zeta) \neq \mu \frac{d}{d\mu} f^{\rm coll}(x, \mu)$$

↑ renormalization says no :(

Expanding the TMDPDF

$$f_{\rm TMD}(x,\, b_T,\, \mu,\, \zeta) = f^{(0)}_{\rm TMD}(x,\, b_T,\, \mu,\, \zeta)\,\big(1 + \mathcal{O}(b_T^2)\big)$$

Recall: we have perturbative knowledge of $f_{\rm TMD}$ from the OPE

$$f^{(0)}_{{\rm TMD},\,i} = \sum_j \int \frac{dz}{z} C_{ij}\Big(\frac{x}{z}, b_T, \mu, \zeta\Big) f_j^{\rm coll}(z,\mu)$$

$$f_{\rm TMD}(x,\, b_T,\, \mu,\, \zeta) = f^{(0)}_{\rm TMD}(x,\, b_T,\, \mu,\, \zeta)\left[1 + b_T^2\left(\Lambda^{(2)} + \frac{1}{2}L_\zeta \gamma_\zeta^{(2)}\right)\right] + \mathcal{O}(b_T^4)$$

Intrinsic

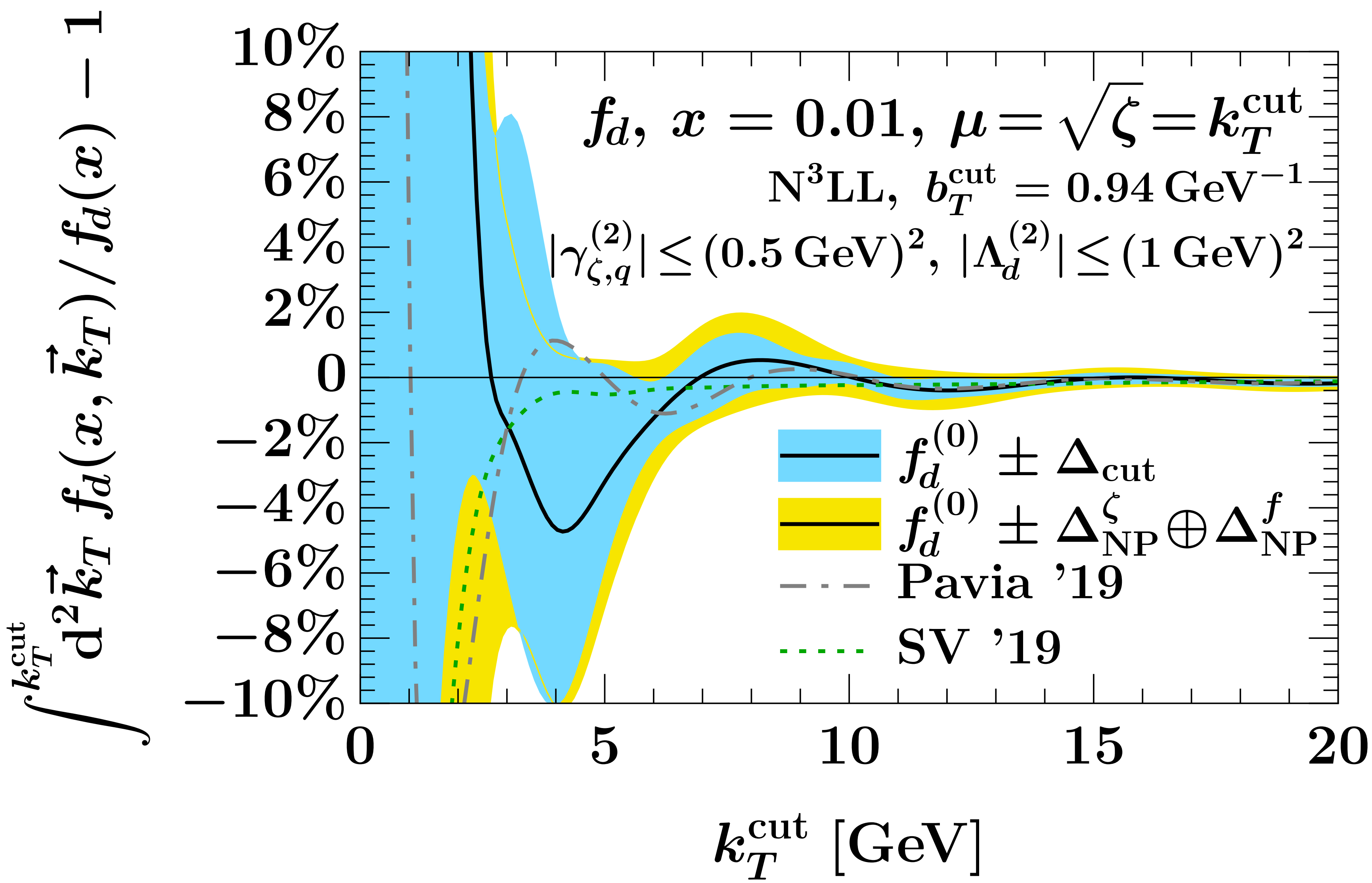
Evolution

$$K[f_{\rm TMD}] \simeq K^{(3)}[f^{(0)}_{\rm TMD}] + \left(\Lambda^{(2)} + \frac{1}{2}L_\zeta \gamma_\zeta^{(2)}\right) K^{(3)}[b_T^2]$$

Model-independent, fully perturbative

Normalization of TMDPDFs

- Approximate the cumulant using $K^{(3)}[f^{(0)}_{\rm TMD}]$ and normalize to $f^{\rm coll}$
- Deviation is small! $\int^{k_T^{\rm cut}} d^2\vec{k}_T\, f^{\rm TMD}(x, k_T, \mu = k_T^{\rm cut}, \zeta) = f^{\rm coll}(x, \mu = k_T^{\rm cut})$ YES!

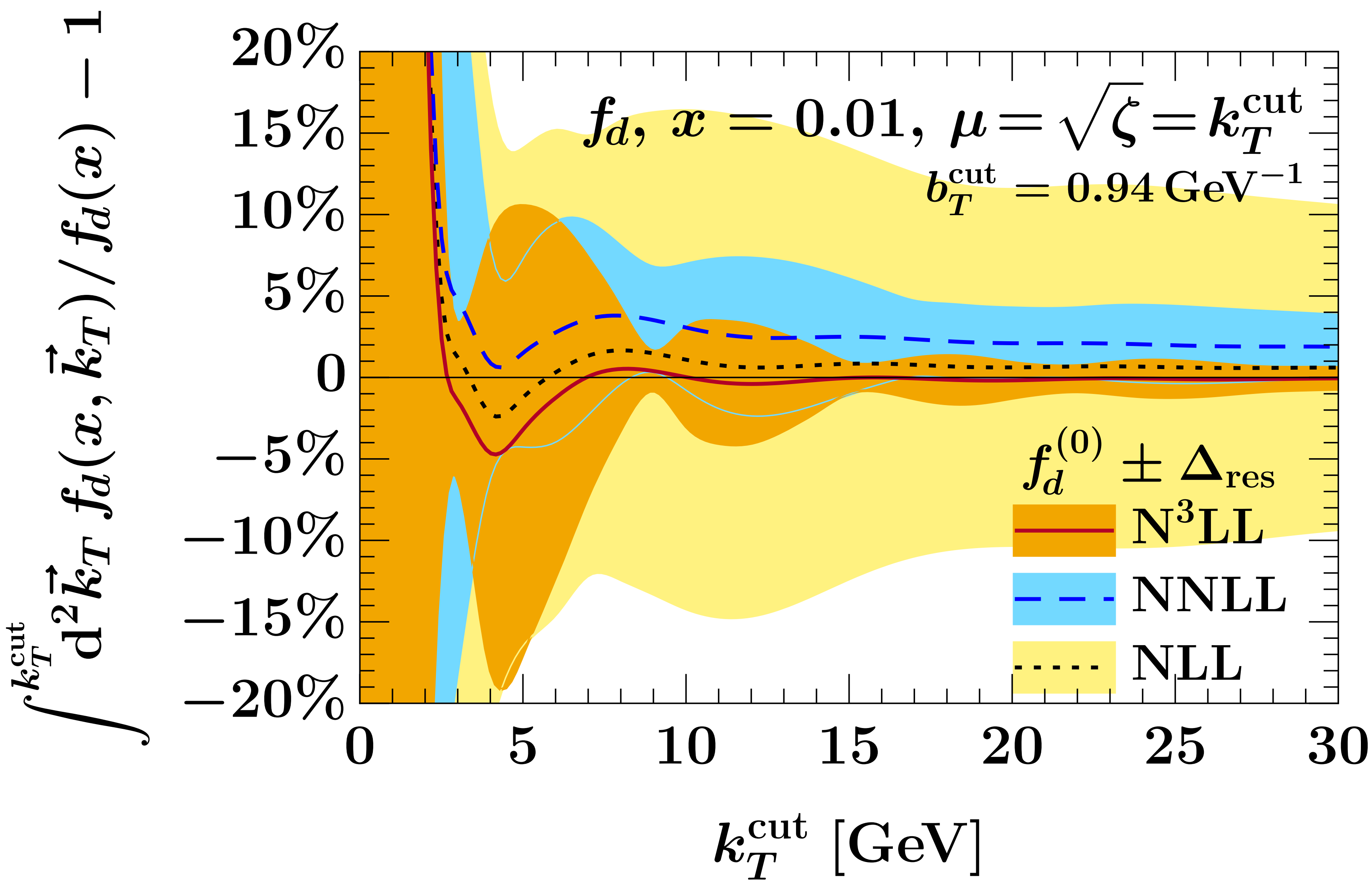


- Less than 1% corrections to intuitive expectation!
- $\Delta_{\rm cut}$ from varying $b_T^{\rm cut}$
- $\Delta_{\rm NP}$ from varying $L_\zeta \gamma_\zeta^{(2)}$ and $\Lambda^{(2)}$
- Small deviation supported by SV and Pavia global fits

SV: 1912.06532
Pavia: 1912.07550

Normalization of TMDPDFs

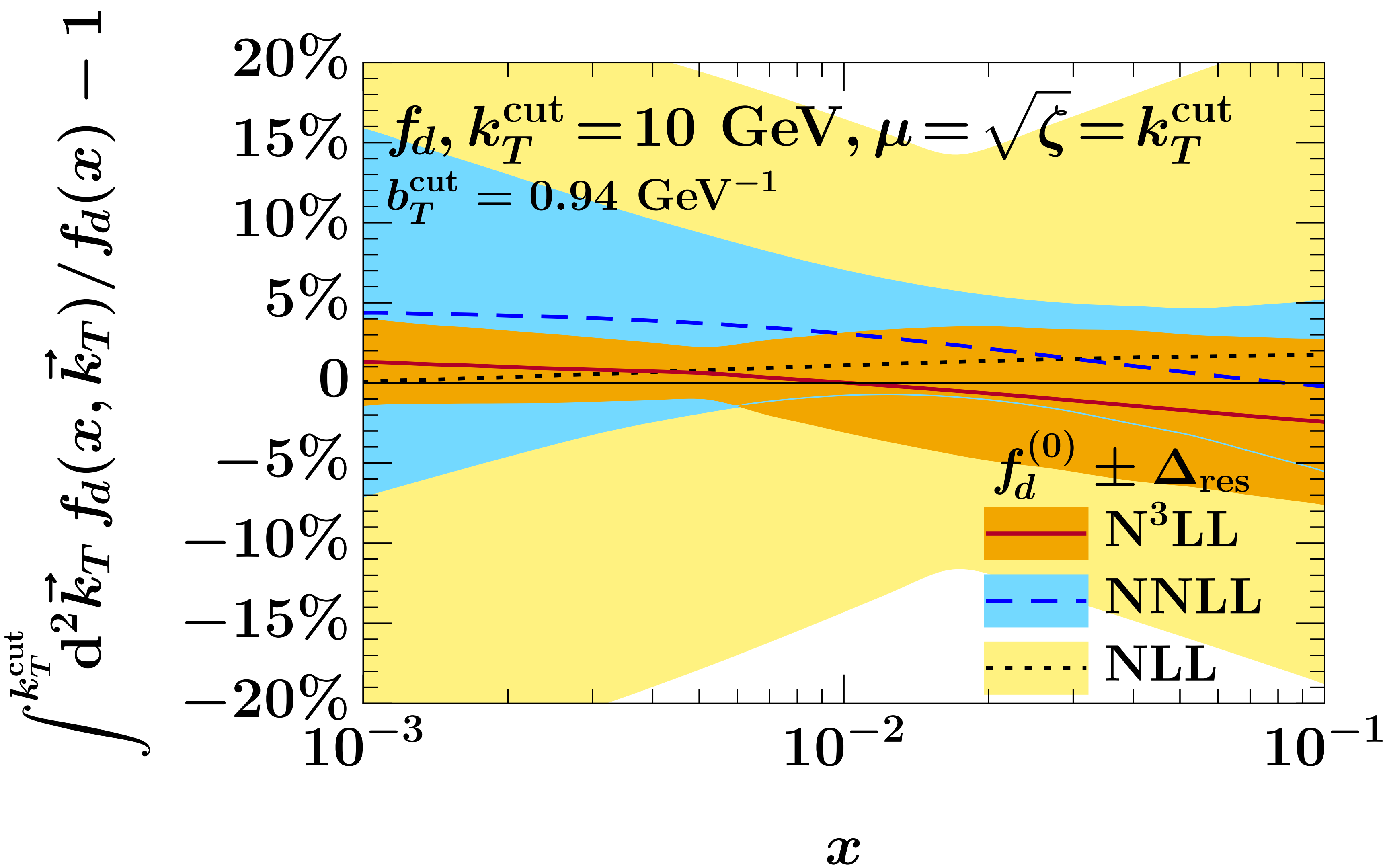
- Deviation is small! $\int^{k_T^{\rm cut}} d^2\vec{k}_T\, f^{\rm TMD}(x, k_T, \mu = k_T^{\rm cut}, \zeta) = f^{\rm coll}(x, \mu = k_T^{\rm cut})$ YES!
- Resummation orders: convergence and perturbative uncertainty



- $\Delta_{\rm res}$ is perturbative uncertainties from resummation orders
- Always consistent with intuitive result
- Important to reduce perturbative uncertainty

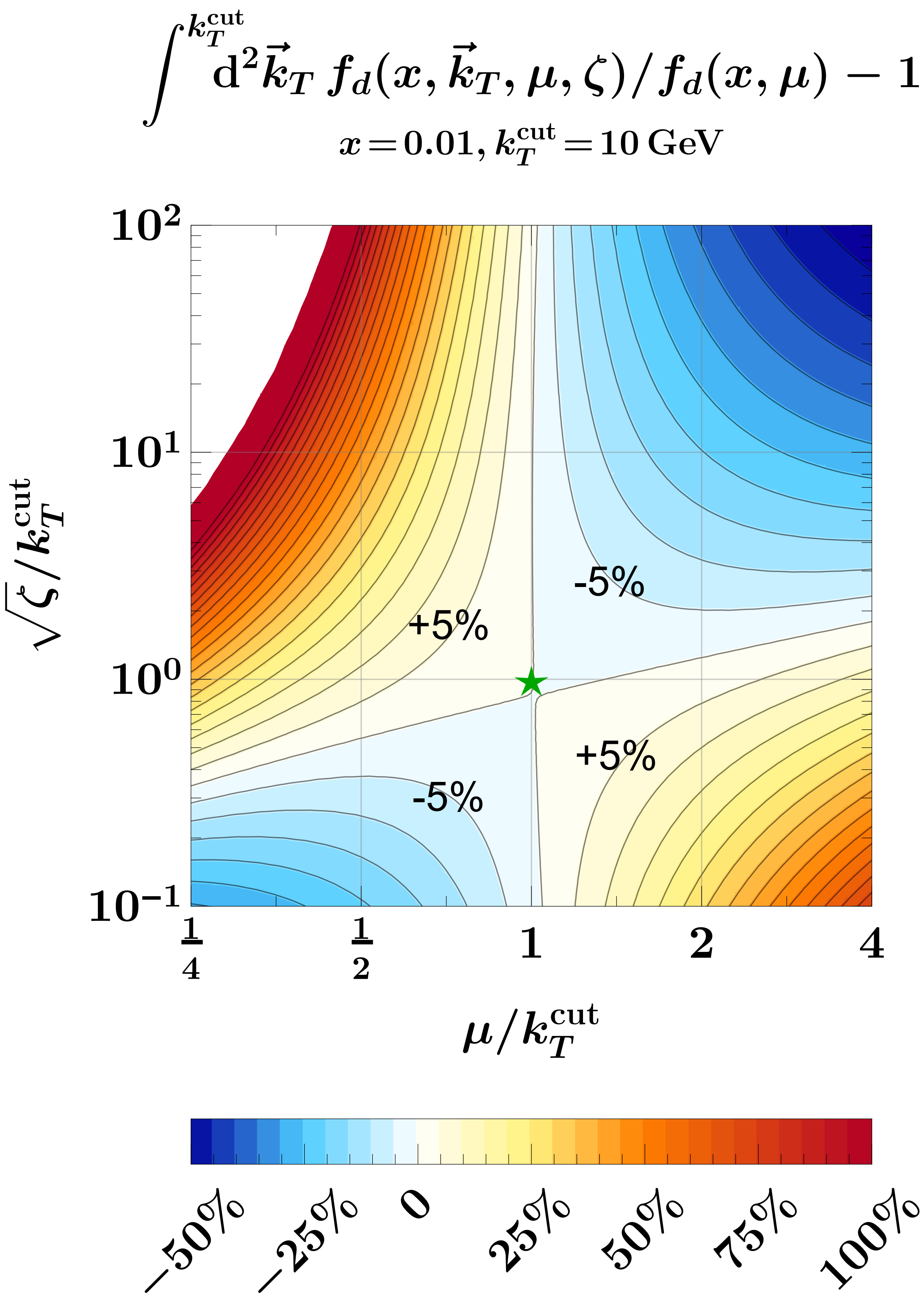
Normalization of TMDPDFs

- Deviation is small! $\int^{k_T^{\text{cut}}} d^2\vec{k}_T\, f^{\text{TMD}}(x, k_T, \mu = k_T^{\text{cut}}, \zeta) = f^{\text{coll}}(x, \mu = k_T^{\text{cut}})$ YES!
- Resummation orders: convergence and perturbative uncertainty



- Test the observation as a function of x and keep k_T^{cut} fixed
- Same conclusions about convergence
- Central value can differ from zero (± 2%)

Impact of Evolution Effects



- Intuitive expectation is robust in the vicinity of $\mu = \sqrt{\zeta} = k_T^{\rm cut}$
- For $\mu = k_T^{\rm cut}$, the ζ evolution is negligible
- Sizable corrections from evolution away from these regions, due to the cusp anomalous dimension
- Evolution effect matters, but at the natural scale $\mu = k_T^{\rm cut}$ the intuition is valid

$$\int^{k_T^{\rm cut}} d^2\vec{k}_T\, f^{\rm TMD}(x, k_T, \mu = k_T^{\rm cut}, \zeta) = f^{\rm coll}(x, \mu = k_T^{\rm cut})$$

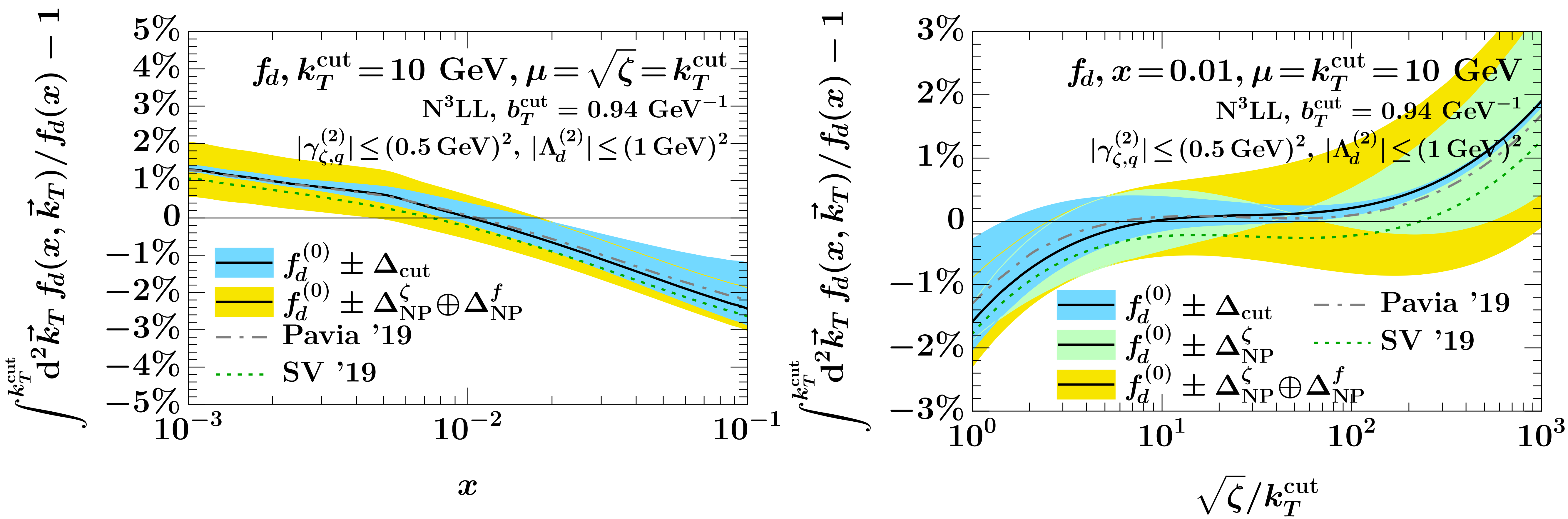
Conclusions

- Perturbative and nonperturbative physics in TMDPDFs are usually hard to disentangle because of b^* prescriptions
- Truncated functionals provide a model-independent and systematically improvable method to exploit perturbative results without use of b^*
- We construct a fully perturbative baseline of the cumulative TMDPDF as well as model-independent constraints on the quadratic coefficients
- Demonstrated that integrating the unpolarized TMDPDF over $[0,\ k_T^{\rm cut}]$ gives the collinear PDF to the percent level (when renormalization scale $\mu = k_T^{\rm cut}$)!!

Thank you!!!

Back-up Slides

More on NP effects



Evolution effects + perturbative