The D-term: Last Item On The Checklist

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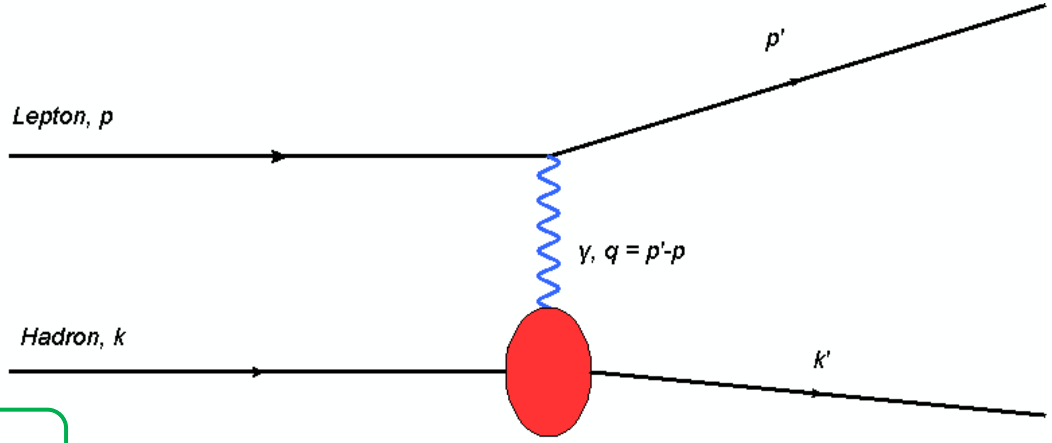
How Do Form Factors Arise in QFT?

Question: What must we include in our calculation to describe this process quantum mechanically?



We need:

- Operators to bring the lepton and hadron from their initial to their final state.
- A term characterizing how the exchange particle propagates.
- Strength of the interaction.

MATRIX ELEMENT

$$i\mathcal{M} = (-ie)^2 \left(\frac{-ig_{\mu\nu}}{q^2}\right) \langle p'|j_l^{\mu}(0)|p\rangle \langle k'|j_h^{\nu}(0)|k\rangle$$

Conserved Current

How Do Form Factors Arise in QFT?

$\frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2$, and the conserved current for the lepton is given by

Conserved current: $j_l^\mu = \bar{\psi}\gamma^\mu\psi$

What about j_h^μ?

Thesis of Nuclear Physics: "Scattering gets complicated when things are made of stuff".

We can still express the had. current as a linear combination of available tensors.

What tensors are available? $\gamma^\mu, \sigma^{\mu\nu}, q^\mu$. Can only use divergenceless combinations of these.

Our ignorance on the proportionality to those tensors is absorbed into *Form Factors (FF's):*

$$\langle p'|j_h^\mu|p\rangle = \bar{u}(p')_h\left[F_1(t)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m}q_\nu F_2(t)\right]u(p)_h$$

*Some restrictions apply.

Where Do Form Factors Come In?

$\langle k' | j_h^\nu(0) | k \rangle$ → Parameterize → $F_i(t) \times \text{Tensor}$ → Fourier Transform* → ρ's

Theory	Conserved Current	Related Densities	How to Measure
E&M	j^μ_{em}	Charge, Magnetic Moment	Elastic Scattering, exchange particle is photon.
Gravity	$T^{\mu\nu}$ (EMT)	Mass, Angular Momentum, Pressure (?)	1) Elastic Scattering , exchange particle is graviton (don't hold breath) 2) DVCS and relate to GPD's. Exchange particle is still photon☺

Burkert, V.D., Elouadrhiri, L. & Girod, F.X. The pressure distribution inside the proton. *Nature* **557,** 396–399 (2018). https://doi.org/10.1038/s41586-018-0060-z

The Energy-Momentum Tensor (EMT)

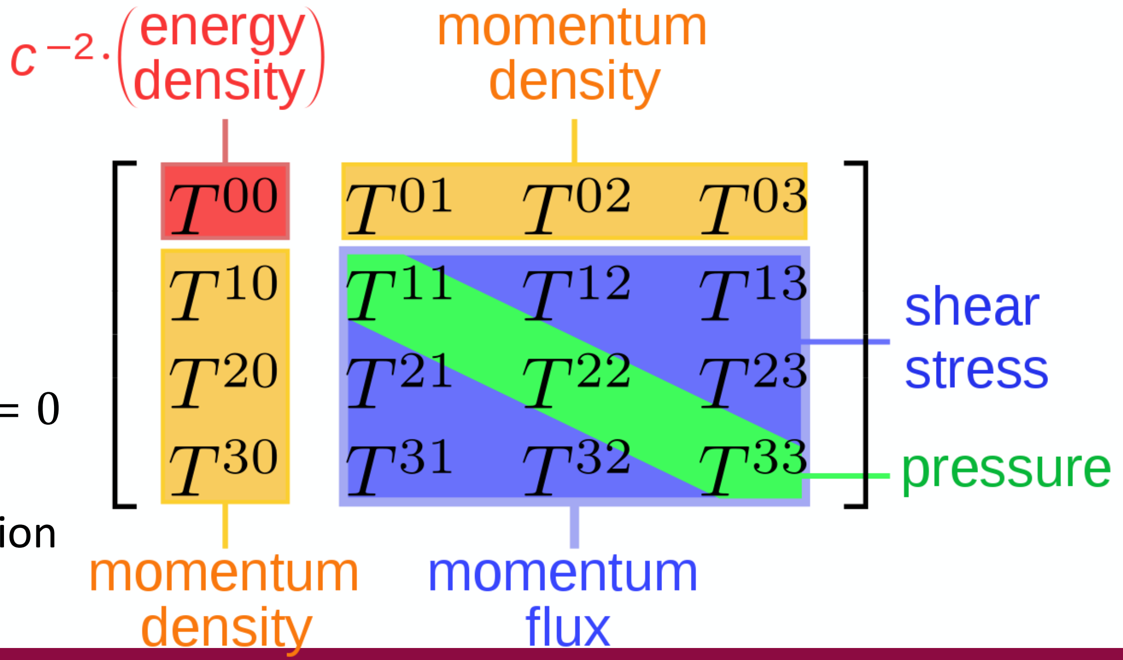
A Lagrangian that exhibits global space-time translational invariance (i.e. $x^\mu \to x^\mu + \alpha^\mu$) yields the EMT as it's conserved current:

$$T^{\mu\nu} = \sum_n \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \partial^\nu \phi_n - g^{\mu\nu}\mathcal{L}$$

Which must satisfy $\partial_\mu T^{\mu\nu} = 0$

Example: $\partial_\mu T^{\mu 0} = \frac{\partial T^{00}}{\partial t} - \nabla \cdot T^{i0} = \frac{\partial E}{\partial t} - \nabla \cdot \vec{p} = 0$

This is a statement of the conservation of energy and momentum!



The Improved Energy-Momentum Tensor

Calculations are plagued by infinities (more than usual) when using the Noether's Theorem definition of the EMT

"Infinities only exist in the minds of theorists"
- An Experimentalist, Probably

Can justify modifying the EMT 2 Equivalent Ways:

1. Renormalization of composite operators.
2. Non-minimal (conformal) coupling of field to gravity.

Dharanipragada, Sathiapalan (2021)

$$T^{\mu\nu}_{Renorm} = T^{\mu\nu} + Renorm(\phi^2, \phi^3)$$

$$S_{\text{non min}} = \int d^4x\sqrt{-g}\left[\mathcal{L}_m + hR\phi^2\right]$$

$$T^{\mu\nu}_{Renorm} = \frac{2}{\sqrt{-g}}\frac{\delta S_{non\ min.}}{\delta g_{\mu\nu}}$$

Callan, Coleman, Jackiw (1969)

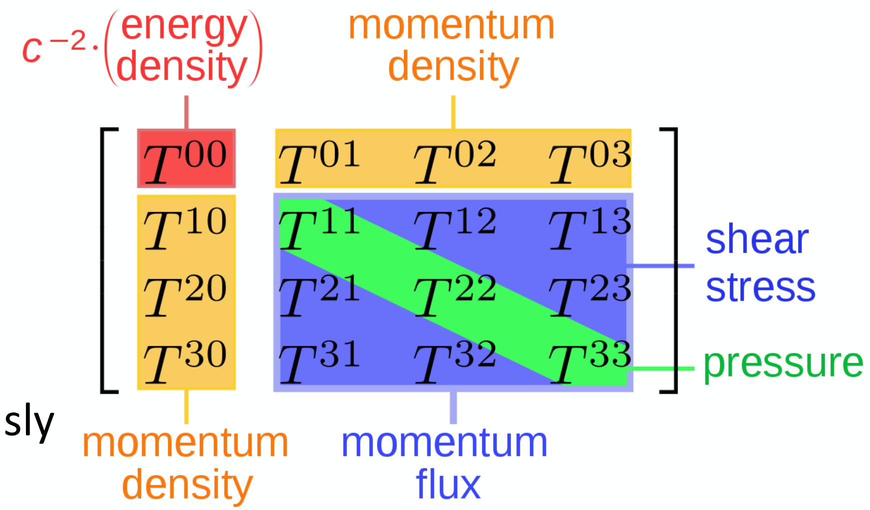
Both methods give same "Improvement term": $\Theta^{\mu\nu} = -h(\partial^\mu\partial^\nu - g^{\mu\nu}\partial_\alpha\partial^\alpha)\phi^2$ making EMT finite☺

Gravitational Form Factors of the EMT

For a real scalar field, the EMT Form Factor decomposition is (defining P = p+p', $t = \Delta^2 = (p' - p)^2$):

$$< p'|T^{\mu\nu}(0)|p > = \frac{P^\mu P^\nu}{2} A(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{2} D(t)$$

+… J(t) for non spin-0



Mass distribution: Constrained by A(0)=1

Distribution of…D-term?: Unconstrained by theory. Suspiciously negative.

For a free/non-interacting real Klein-Gordon Field: $A(t) = 1, D(t) = -1$

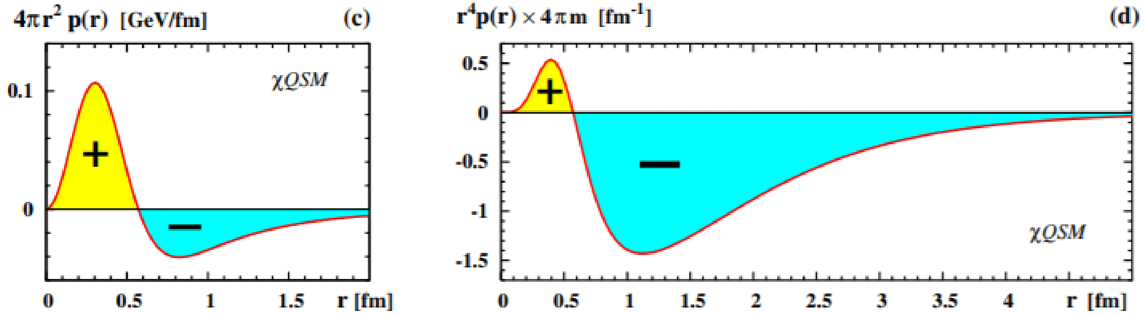
Learn more by introducing interactions?

The D-term

Goeke et al, PRD75 (2007) 094021

The ij-components of the EMT define the *Stress-Tensor:*

$$T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}p(r)$$



Where s(r) and p(r) are the shear forces and pressure distributions, respectively.

Can relate these densities to the D-term Schweitzer, Polyakov (2018)

D-Term Describes Pressure and Shear Stress inside of Nucleon!

$$s(r) = -\frac{1}{4m} r \frac{d}{dr}\frac{1}{r}\frac{d}{dr}\,\widetilde{D}(r), \qquad \mathrm{p}(r) = \frac{1}{6m}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\,\widetilde{D}(r), \quad \widetilde{D}(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-\Delta r} D(-\Delta^2)$$

EMT for ϕ^3-Theory

$$T^{\mu\nu} = (1+\delta Z_2)T_R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}\delta m^2\phi^2 - g^{\mu\nu}c\phi - h(1+\delta Z_2)(\partial^\mu\partial^\nu - g^{\mu\nu}\partial_\alpha\partial^\alpha)\phi^2$$

where

$$T_R^{\mu\nu} \equiv \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left[\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{3!}\phi^3\right]$$

Motivated by Non-minimal coupling.

Free Field contribution to the EMT is known.

For this theory, we obtain no contributions to the EMT at $\mathcal{O}(\lambda)$. That is,

$$< p' \left| T^{\mu\nu}_{\mathcal{O}(\lambda)}(x,y) \right| p > = 0$$

Must go to $\mathcal{O}(\lambda^2)$!

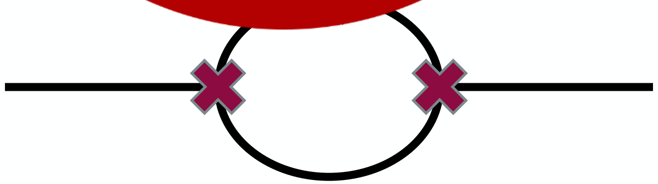
Relevant Diagrams

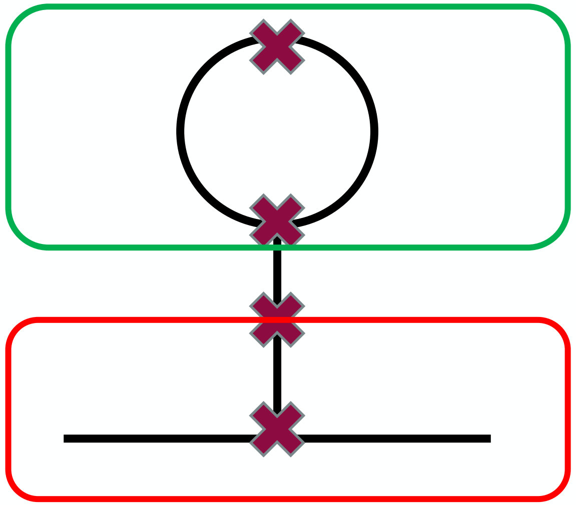
Theory are:

Can Insert EMT Into:
- **Propagators**
- **Vertices**

"Tadpoles"

$$\langle p'|T^{\mu\nu}|p\rangle = \mathcal{L}^{\mu\nu} + \mathcal{T}^{\mu\nu} + \mathcal{C}^{\mu\nu} + \Theta^{\mu\nu}$$





NOT a Tadpole
$q = \Delta$

$q = 0$
Tadpole

Calculational Insight

- All $\mathcal{O}(\lambda^2)$ contributions to $\langle p'|T^{\mu\nu}|p\rangle$ calculated ("17" total diagrams).
- Improvement Term $\Theta^{\mu\nu}$ renders $\langle p'|T^{\mu\nu}|p\rangle$ finite (Hooray!)
- "Loop" Contributions made finite through mass renormalization.
- Not all tadpoles created equal. (see backup slides)
- A-term defined from first principles, not modified by Improvement term.
- Parameter integrals make factoring out Form Factors very challenging.

Summary

The D-term is related to how pressure is distributed inside of particles. Linked to the stability of particles.

ϕ^3 Theory - surprisingly insightful! Know what subtleties to expect when going to a model like Scalar-Diquark.

Future:

- After extracting FF's, $\rightarrow$ investigate behavior in position space, perhaps in IMF for 2-D interpretation.
- Replicate calculation using GPD's / Compton Form Factor Dispersion relations.

Imagine how much physics we'd miss out on if we never investigated spin, and then how much we're currently missing by not understanding the D-term. The pressure is on☺

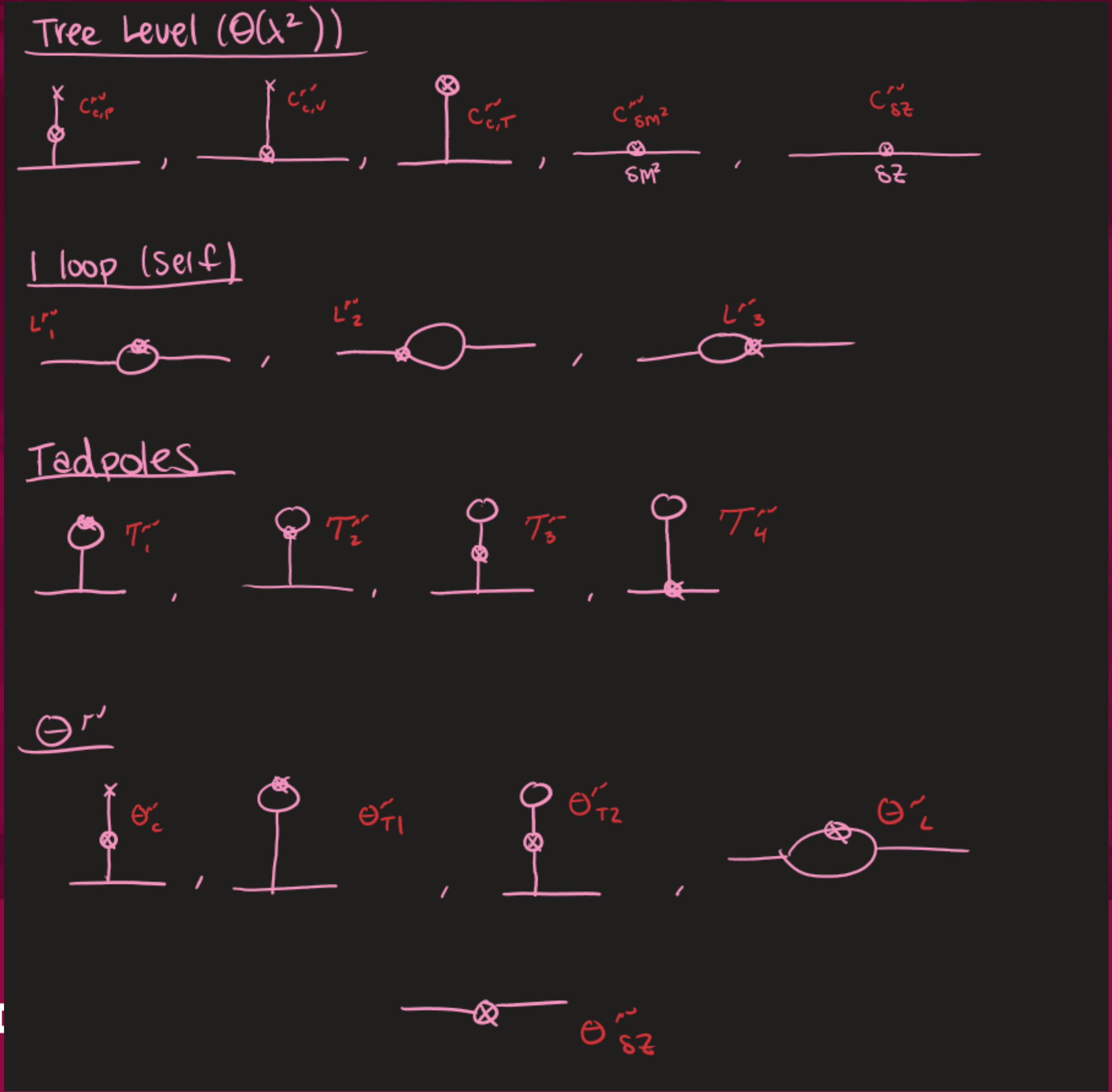
References

- Reaching for the Horizon: The 2015 Long Range Plan for Nuclear Science. United States: N. p., 2015. Web.
- Burkert, V.D., Elouadrhiri, L. & Girod, F.X. The pressure distribution inside the proton. Nature 557, 396–399 (2018). https://doi.org/10.1038/s41586-018-0060-z
- M. V. Polyakov, P. Schweitzer arXiv:1805.06596 [hep-ph] 14 Sep 2018
- L. S. Brown and J. C. Collins, Annals Phys. 130, 215 (1980).
- K. Goeke, J. Grabis, J. Ossmann, M. V. Polyakov, P. Schweitzer, A. Silva and D. Urbano, Phys. Rev. D 75, 094021 (2007)

Back Up Slides

Diagrams (jaxodraw in progress)

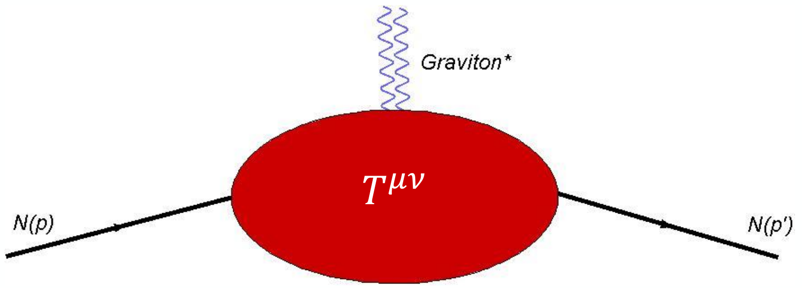
- $L_1 + L_2 + L_3 + C_{\delta m^2}$ = Finite
- $T_1 + \theta_{T1}$=Finite
- $\theta_{T2} + \theta_c$=Finite
- $T_2 + T_3$=Finite
- $T_4 + C_{c,V}$=Finite
- $C_{c,P} + C_{c,T}$=Finite
- $C_{\delta Z}, \theta_{\delta Z}, \theta_L$ are individually finite



How These Form Factors May Be Extracted Experimentally

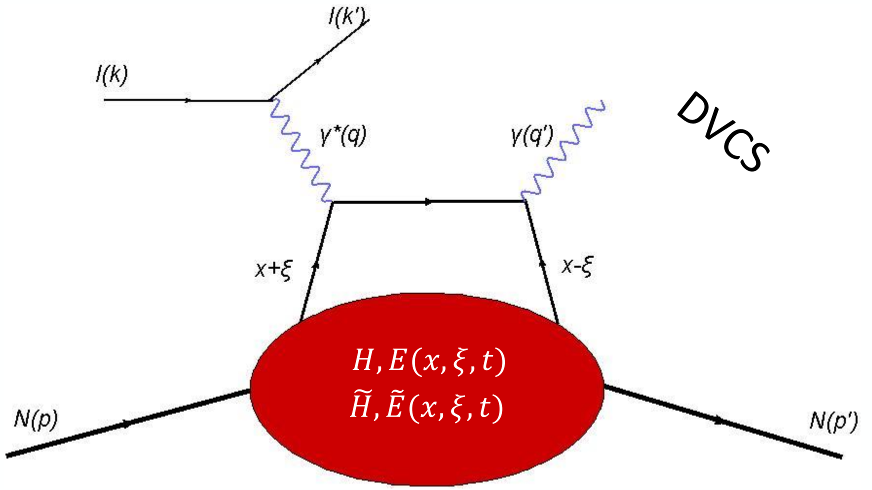
1) **Graviton scattering**

Direct access, and completely analogous to probing F_1 and F_2, but practically impossible…



2) **Generalized Parton Distributions (GPD's)**

Indirect access and non-trivial to extract, but more practical. Describes the transverse position of the partons and their longitudinal momentum.



Relating GPD's to the Form Factors

GPD's are not directly observable, but are related to many measurable quantities:

$$H_q(x,0,0) = q(x)$$
$$\widetilde{H}(x,0,0) = \Delta q(x)$$

Momentum and Angular Momentum Distributions

We also have

$$\int \mathrm{d}x \; x \, H^q(x,\xi,t) = \boldsymbol{A^q(t) + \xi^2 D^q(t)}$$
$$\int \mathrm{d}x \; x \, E^q(x,\xi,t) = \boldsymbol{B^q(t) - \xi^2 D^q(t)}$$

EMT Form Factors

Ji's Sum Rule: $\int dxx\big(H(x,\xi,t) + E(x,\xi,t)\big) = 2J(t)$

Relating GPD's to the Form Factors

1. Calculate the Compton Form Factor (CFF) from GPD H

$$\mathcal{H}(\xi,t) = \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H(x,\xi,t)$$

2. Relate real and imaginary parts of CFF through dispersion relation:

$$\mathrm{Re}\mathcal{H}(\xi,t) = \frac{1}{\pi}\mathcal{P}\int_{0}^{1} d\xi' \left[\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right] \mathrm{Im}\mathcal{H}(\xi',t) + 4D(t)$$

3. Subtraction "constant" proportional to D-term

References

- Reaching for the Horizon: The 2015 Long Range Plan for Nuclear Science. United States: N. p., 2015. Web.
- Burkert, V.D., Elouadrhiri, L. & Girod, F.X. The pressure distribution inside the proton. Nature 557, 396–399 (2018). https://doi.org/10.1038/s41586-018-0060-z
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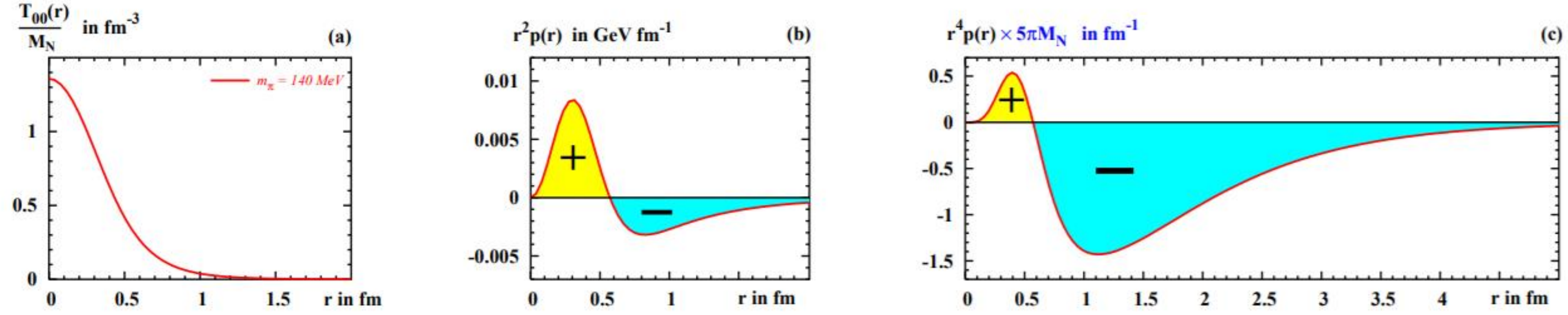
From: K. Goeke, J. Grabis, J. Ossmann, M. V. Polyakov, P. Schweitzer, A. Silva, and D. Urbano
Phys. Rev. D **75**, 094021

↪ necessary condition for stability $\int_0^\infty \mathrm{d}r\, r^2 p(r) = 0$ (von Laue, 1911)

$$D = -\frac{16\pi}{15} M_N \int_0^\infty \mathrm{d}r\, r^4 s(r) = 4\pi M_N \int_0^\infty \mathrm{d}r\, r^4 p(r)$$

↪ shows how internal forces balance

- lessons from model



energy density — pressure: $r^2 p(r)$ — $r^4 p(r) \rightsquigarrow$ sign of d_1 negative

Projection Operators

$$a^{\mu\nu} = \frac{P^\mu P^\nu}{P^2}, \quad P^2 = 4m^2 - t\,.$$

$$\left[(n-1)\,a^{\mu\nu} - g^{\mu\nu}\right]\langle \vec{p}\,'\,|\hat{T}_{\mu\nu}(0)|\vec{p}\rangle = \frac{n-2}{2}\,P^2 A(t),$$

$$\left[a^{\mu\nu} - g^{\mu\nu}\right]\langle \vec{p}\,'\,|\hat{T}_{\mu\nu}(0)|\vec{p}\rangle = \frac{n-2}{2}\,\Delta^2 D(t).$$

Back Up S

10.1 A1: Noether vs Einstein Definition of EMT

We know Noether's theorem tells us the energy-momentum tensor for a $\lambda\phi^n$ theory is given by

$$T^{\mu\nu} = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)}\partial^\nu \phi - g^{\mu\nu}\mathcal{L} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\mathcal{L} \tag{10.1}$$

I'll now show that through minimal coupling, the same EMT can be calculated using

$$T^{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta S_m}{\delta g_{\mu\nu}} \tag{10.2}$$

We need the following identities:

$$\frac{\delta\sqrt{-g}}{\delta g_{\mu\nu}} = -\frac{1}{2}\sqrt{-g}g^{\mu\nu} \tag{10.3a}$$

$$\frac{\delta g_{\alpha\beta}}{\delta g_{\mu\nu}} = \frac{1}{2}(\delta^\mu_\alpha\delta^\nu_\beta + \delta^\mu_\beta\delta^\nu_\alpha) \tag{10.3b}$$

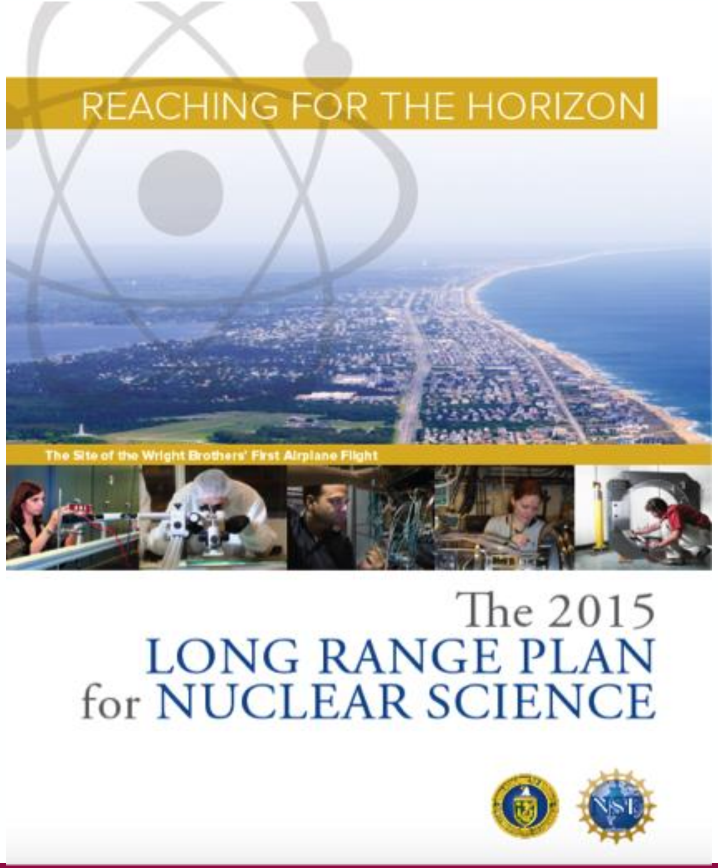
Define the matter lagrangian to be

$$\mathcal{L}_m = \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}m^2\phi^2 - V(\phi) \tag{10.4}$$

If we vary the matter-action minimally coupled to gravity, dropping the integral as we're interested in recovering the energy-momentum tensor density, we get

$$\begin{aligned}
\frac{\delta S_m}{\delta g_{\mu\nu}} &= \frac{\delta}{\delta g_{\mu\nu}}\sqrt{-g}\mathcal{L}_m \\
&= \left[\mathcal{L}_m\frac{\delta\sqrt{-g}}{\delta g_{\mu\nu}} + \sqrt{-g}\frac{\delta\mathcal{L}_m}{\delta g_{\mu\nu}}\right] \\
&= \left[\mathcal{L}_m\left(-\frac{1}{2}\sqrt{-g}g^{\mu\nu}\right) + \sqrt{-g}\frac{\delta\mathcal{L}_m}{\delta g_{\mu\nu}}\right] \\
&= \sqrt{-g}\left[\mathcal{L}_m\left(-\frac{1}{2}g^{\mu\nu}\right) + \frac{\delta}{\delta g_{\mu\nu}}\left(\frac{1}{2}g_{\alpha\beta}\partial^\alpha\phi\partial^\beta\phi - \frac{1}{2}m^2\phi^2 - V(\phi)\right)\right] \\
&= \sqrt{-g}\left[\mathcal{L}_m\left(-\frac{1}{2}g^{\mu\nu}\right) + \frac{1}{2}\frac{\delta g_{\alpha\beta}}{\delta g_{\mu\nu}}\partial^\alpha\phi\partial^\beta\phi\right] \\
&= \sqrt{-g}\left[\mathcal{L}_m\left(-\frac{1}{2}g^{\mu\nu}\right) + \frac{1}{2}\partial^\mu\phi\partial^\nu\phi\right] \\
&= \frac{1}{2}\sqrt{-g}\left[\partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\mathcal{L}_m\right] \\
&= \frac{1}{2}\sqrt{-g}T^{\mu\nu} \rightarrow T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g_{\mu\nu}}
\end{aligned} \tag{10.5}$$

Priorities in Nuclear Physics



Overarching Theme: Where Do Properties Of The Nucleon Come From, And How Are They Distributed?

That information is contained in **Form Factors (FF’s) (et. al)**

Form Factors: Parameterize our ignorance on how the conserved current relates to other available tensors. This parameterization **depends on the momentum transfer t.**