Independent amplitudes in the quark and antiquark correlators and TMDs in the covariant parton model

Fatma Aslan (speaker), Saman Bastami, Peter Schweitzer

TMD Collaboration Meeting

June 15 2022

Parton model description of quark and antiquark correlators and TMDs

F. Aslan[1], S. Bastami[1], P. Schweitzer[1]
[1] *Department of Physics, University of Connecticut, Storrs, CT 06269, U.S.A.*

Dedicated to Anatoly Vasilievich Efremov

properties of transverse momentum dependent parton distribution functions (TMDs). The parton model is often a helpful fra... ...ns of TMD properties and the description of deep-inela... ...ed. Based on a systematic exploration of the parton ... ...terature that there are 2 independent structures in t... ...he claim that there are 3, and explain the underlying ... ...ns. We also systematically explore the antiquark correla... ...first time derive the model expressions for all T-even lea... ...monstrate the consistency of the framework which can ... ...isticated TMD modelling.

Feynman's intuitive parton model ... ...n establishing QCD as the theory of strong interactions. In many situations ... ...zeroth order approximation" to QCD [3, 4]. As such it constitutes a valuabl... ...s also the case for TMDs. Based on a rigorous TMD factorization and evo... ...has witnessed impressive progress in modern phenomenology of deep-inelast... ...t the way to this progress was paved by, among others, important phenomenological work based on the "generalized parton model" of Refs. [20–24].

1933–2021

A systematic exploration of the parton model ... the purpose of describing DIS processes but the nonperturbative properties of TMDs *per se* was undertaken in Refs. [25–39], interestingly with conflicting results.

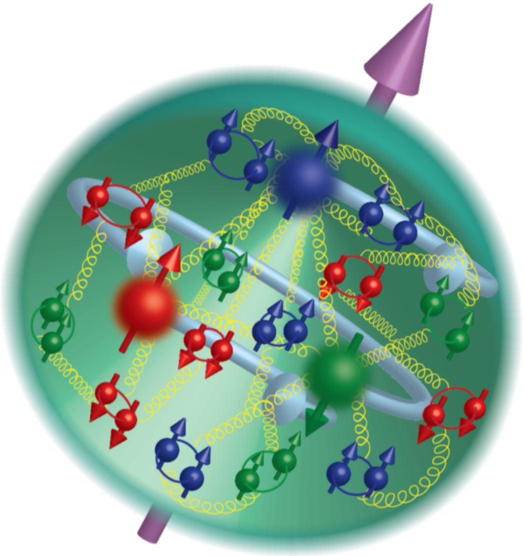
Outline

- What is a covariant parton model (CPM)?
- The assumptions
- Derivations
- Limitations
- Consistency
- Summary

Parton Models

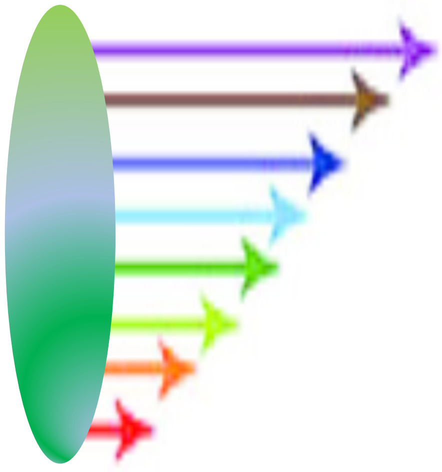
Neglect the binding effects and treat the partons as being free during the short interaction time.

The nucleon has a complex structure



Valence quarks, sea quarks, sea antiquarks and gluons all of which are spinning and also orbiting each other, bounded in the nucleon

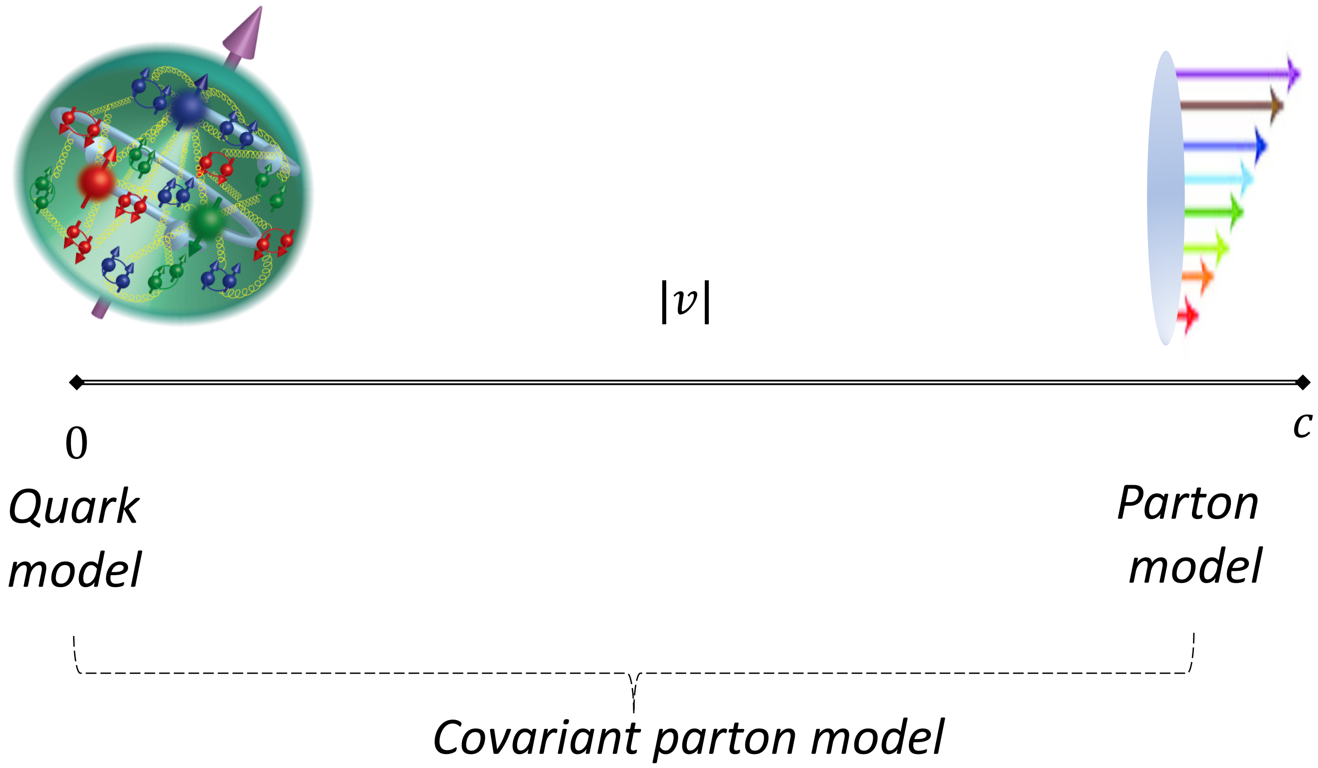
The complex structure is simplified when boosted to the IMF → Parton model



Stream of free partons, each carrying a fraction of the longitudinal momentum

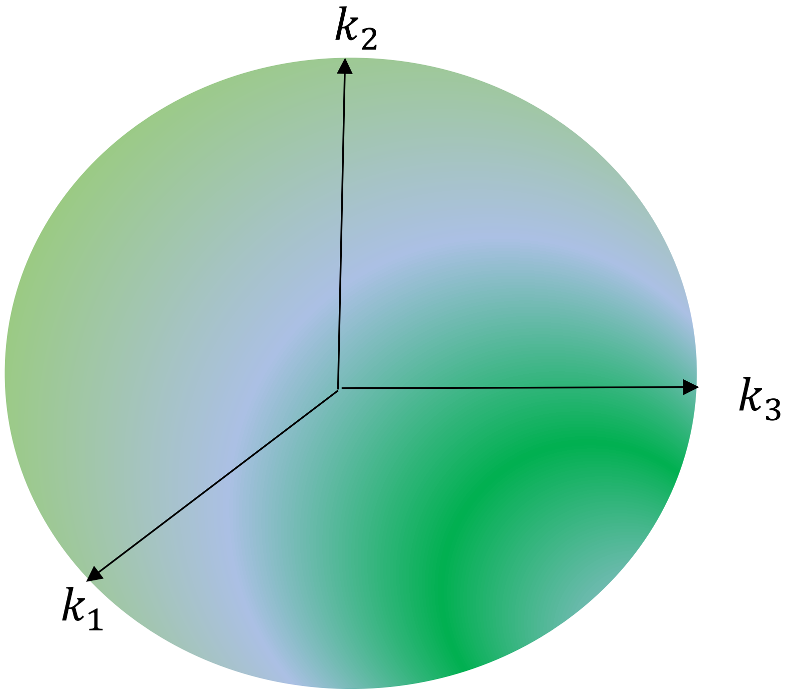
Covariant Parton Model

Formulate a covariant theory that does not prefer any special reference system like IMF and produces the quark model for slow hadrons, the parton model for fast hadrons…



Covariant Parton Model

Nucleon Rest Frame



$$k \equiv (k^0, k^1, k^2, k^3)$$
$$P \equiv (M, 0, 0, 0)$$

Petr Zavada-The structure functions and parton momenta distribution in the hadron rest system,1996

The assumptions of CPM

1- Spherical phase space is assumed:

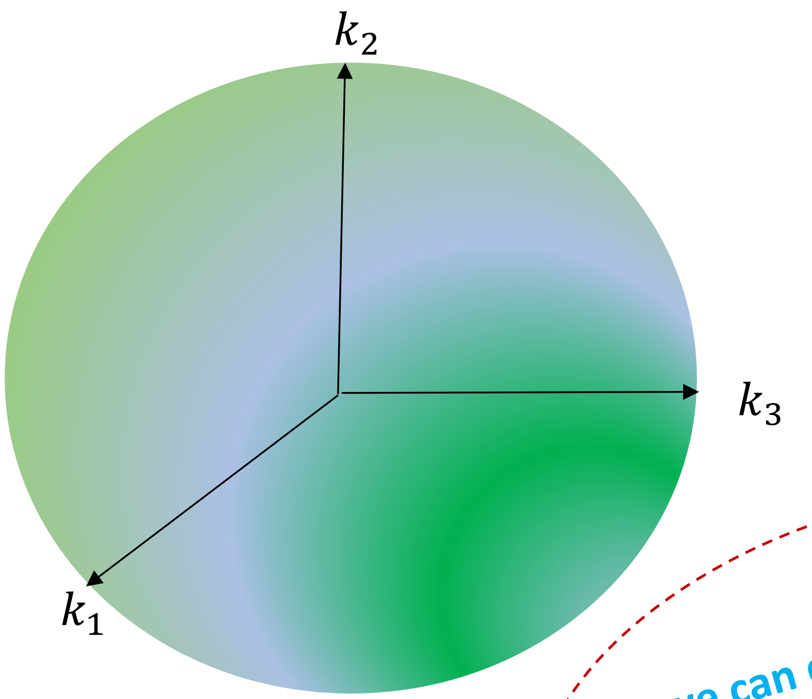
$$\sqrt{k_1^2 + k_2^2 + k_3^2} \leq k_m$$

2- Quasi free partons are on mass shell:

$$k^2 = m^2$$

Covariant Parton Model

Nucleon Rest Frame



$$k \equiv (k^0, k^1, k^2, k^3)$$
$$P \equiv (M, 0, 0, 0)$$

Petr Zavada-The structure functions and parton momenta distribution in the hadron rest system,1996

The assumptions of CPM

1- Spherical phase space is assumed:

$$\sqrt{k_1^2 + k_2^2 + k_3^2} \leq k_m$$

2- Quasi free partons are on mass shell:

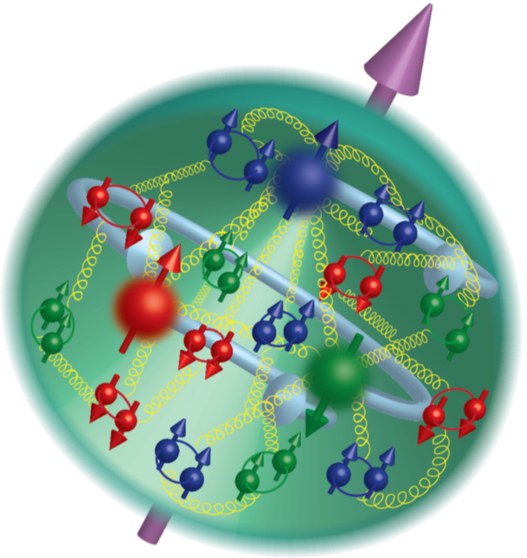
$$k^2 = m^2$$

Now we can define a polarization vector for partons:

$$\omega_\mu = AP_\mu + BS_\mu + Ck_\mu$$

Covariant Parton Model

Nucleon Rest Frame

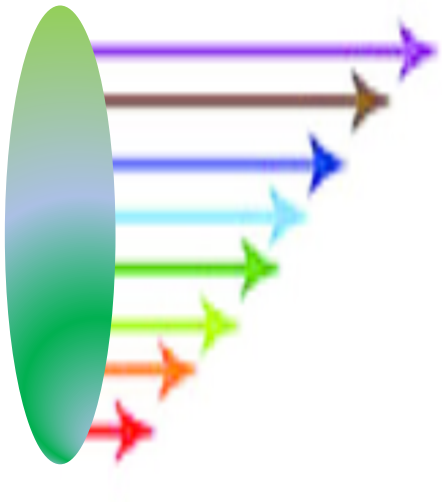


$$k \equiv (k^0, k^1, k^2, k^3)$$

$$P \equiv (M, 0, 0, 0)$$

$$x = \frac{k^0 + k^3}{M}$$

Infinite Momentum Frame



$$k' \equiv (k'^0, k^1, k^2, k'^3)$$

$$P' \equiv (P'^0, P^1, P^2, P'^3)$$

$$x = \frac{k^0 + k^3}{P^0 + P^3}$$

Covariant Parton Model

Assumptions | **Limits**

- Spherical phase space

$$\frac{m^2}{M^2} \leq x \leq 1$$

- On-shell partons

$$\frac{m^2 - M^2}{2M} \leq k_3 \leq \frac{M^2 - m^2}{2M}$$

- Consistency with IMF

$$0 \leq k_T^2 \leq M^2\left(x - \frac{m^2}{M^2}\right)(1 - x)$$

Covariant Parton Model - History

- Description of the hadronic tensor

P. Zavada, Phys. Rev. D 55, 4290 (1997) [hep-ph/9609372]
P. Zavada, Phys. Rev. D 65, 054040 (2002) [hep-ph/0106215]
P. Zavada, Phys. Rev. D 67, 014019 (2003) [hep-ph/0210141]

$\Big\} \; f_1(x),\; g_1(x),\; g_T(x)$

- Auxiliary polarized process due to the interference of vector and scalar currents

V. Efremov, O. V. Teryaev and P. Zavada, Phys. Rev. D 70, 054018 (2004) [hep-ph/0405225].

$\Big\} \; \ldots + \; h_1(x)$

- Unintegrated structure functions," to describe twist-2 T-even TMDs.

A. V. Efremov, P. Schweitzer, O. V. Teryaev and P. Zavada,
Phys. Rev. D 80, 014021 (2009) [arXiv:0903.3490 [hep-ph]].

$\Big\} \; \ldots + f_1(x,k_T),\, g_1(x,k_T),\; h_1\,(x,k_T),\, g_{1T}^{\perp}(x,k_T)$

$,\, h_{1L}^{\perp}(x,k_T),\, h_{1T}^{\perp}(x,k_T)$

Still no access to the twist-3 TMDs !

Quark correlator in covariant parton model

Structure of the nucleon at leading and subleading twist in the covariant parton model - Bastami, Efremov, Schweitzer, Teryaev, Zavada-2020

$$\Phi(k,P,S) = (\not{k}+m)\Big(\mathcal{G}(k.P) + \mathcal{H}(k.P)\gamma_5\not{S}\Big)M\delta(k^2-m^2)\Theta_{\rm kin}(k.P)$$

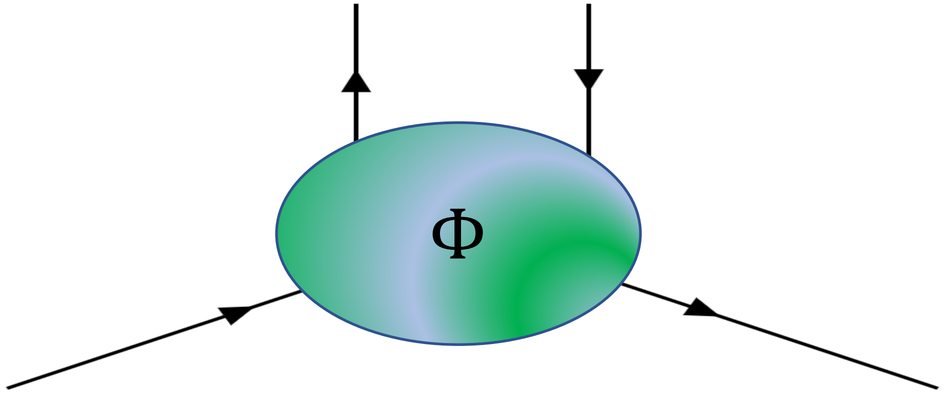
$\mathcal{G}(k.P)$: The momentum distribution of unpolarized quarks in the nucleon rest frame

$\mathcal{H}(k.P)$: The momentum distribution of polarized quarks in the nucleon rest frame along $S^\mu = (0, \mathbf{S})$

Quark correlator

$$\Phi(P,k,S,\eta) = MA_1 + \not{P}A_2 + \not{k}A_3 + \frac{1}{2M}[\not{P},\not{k}]A_4 + i(k\cdot S)\gamma_5 A_5 + M\not{S}\gamma_5 A_6 + \frac{k\cdot S}{M}\not{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\not{k}\gamma_5 A_8$$

$$+\frac{[\not{P},\not{S}]}{2}\gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5 A_{10} + \frac{k\cdot S}{2M^2}[\not{P},\not{k}]\gamma_5 A_{11} + \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma A_{12} + \mathcal{O}(B_i)$$



P, S: Hadron momentum, spin
k: Parton momentum
M: Hadron mass

$$Amplitudes: \quad A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12}, B_i$$

Quark correlator - No gauge field

In models without gauge field degrees of freedom
<u>T-odd</u> amplitudes, A_4 , A_5 , A_{12} and all the B_i amplitudes
are absent.

$$Amplitudes: \quad A_1, A_2, A_3, A_6, A_7, A_8, A_9, A_{10}, A_{11}$$

$$\begin{aligned}\Phi(k,P,S) \quad &= \quad M A_1 + \not{P} A_2 + \not{k} A_3 + M \not{S} \gamma_5 A_6 + \frac{k.S}{M} \not{P} \gamma_5 A_7 + \frac{k.S}{M} \not{k} \gamma_5 A_8 \\ &+ \frac{[\not{P},\not{S}]}{2} \gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2} \gamma_5 A_{10} + \frac{k.S}{2M^2} [\not{P},\not{k}] \gamma_5 A_{11}\end{aligned}$$

Quark correlator - No gauge field + On mass shell

Assuming the partons are on-shell, $Tr[\Phi\Gamma(\gamma.k+m)]=0$, leads to

$$A_2 = A_9 = 0$$

$$A_1 = \frac{m}{M}A_3 \quad A_6 = \frac{m}{M}A_{10} \quad A_7 = -\frac{m}{M}A_{11}$$

$$A_{10} = \frac{k.P}{M^2}A_{11} - \frac{m}{M}A_8$$

$$Amplitudes: \quad A_3, A_8, A_{11}$$

$$\Phi(k,P,S) = (\not{k}+m)\left\{A_3 + \underbrace{\frac{[(k.P)A_{11} - mMA_8]}{M^2}}_{A_{polarized}} \underbrace{\left[\not{S} - \frac{k.S}{(k.P)A_{11} - mMA_8}\not{P}A_{11} + \frac{M}{m}\frac{k.S}{(k.P)A_{11} - mMA_8}\not{k}A_8\right]}_{\not{a}}\gamma_5\right\}$$

Quark correlator – No gauge field + On mass shell + Pure spin states

ω^2	
Mixed spin state	$-1 \leq \omega^2 \leq 0$
Pure spin state	$\omega^2 = -1$

Assuming pure spin states, $\omega^2 = -1$, leads to
$A_8 = \mp A_{11}$

$$Amplitudes: \quad A_3, A_{11}$$

Choosing $A_8 = -A_{11}$

$$\Phi(k,P,S) = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)+mM\Big]A_{11}\not{\omega}\gamma_5$$

$$\omega^\mu(k,P,S) = \left\{S^\mu - \frac{(k.S)}{\Big[(P\cdot k)+mM\Big]}P^\mu - \frac{M}{m}\frac{(k\cdot S)}{\Big[(P\cdot k)+mM\Big]}k^\mu\right\}$$

Covariant Parton Model - The amplitudes for quarks

- Model

$$\Phi(k,P,S) = (\not{k}+m)\Big(\mathcal{G}(k.P)+\mathcal{H}(k.P)\gamma_5\not{S}\Big)M\delta(k^2-m^2)\Theta_{\rm kin}(k.P)$$

- Quark correlator with no gauge field + on mass shell + pure spin states

$$\Phi(k,P,S) = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)+mM\Big]A_{11}\not{S}\gamma_5$$

- Amplitudes obtained in terms of the covariant distribution functions

$$A_3(k.P) = M\delta(k^2-m^2)\Theta_{\rm kin}(k.P)\mathcal{G}(k.P)$$

$$A_{11}(k.P) = M\delta(k^2-m^2)\Theta_{\rm kin}\mathcal{H}(k.P)\Big(-\frac{M^2}{k.P+mM}\Big)$$

The covariant functions $\mathcal{G}(k.P)$ and $\mathcal{H}(k.P)$

$$\mathcal{G}^q(P\cdot k) = -\frac{1}{\pi M^3}\left[\frac{d}{dx}\,\frac{f_1^a(x)}{x}\right]$$

$$\mathcal{H}^q(P\cdot k) = \frac{1}{\pi x^2 M^3}\left[3g_1^a(x) + 2\int_x^1 \frac{dy}{y}\,g_1^a(y) - x\,\frac{dg_1^a(x)}{dx}\right]$$

P. Zavada, Phys. Rev. D **83**, 014022 (2011) [arXiv:0908.2316 [hep-ph]].
A. V. Efremov, P. Schweitzer, O. V. Teryaev and P. Zavada, [arXiv:0912.3380 [hep-ph]].
A. V. Efremov, P. Schweitzer, O. V. Teryaev and P. Zavada, Phys. Rev. D **83**, 054025 (2011) [arXiv:1012.5296 [hep-ph]].

Antiquark correlator in covariant parton model

Parton model description of quark and antiquark correlators and TMDs.
Aslan, Bastami, Schweitzer-2022

$$\bar{\Phi}(k,P,S) = (\not{k} - m)\Big(\bar{\mathcal{G}}(k.P) + \bar{\mathcal{H}}(k.P)\gamma_5 \bar{\not{\psi}}\Big) M\delta(k^2 - m^2)\Theta_{\rm kin}(k.P)$$

$\bar{\mathcal{G}}(k.P)$: The momentum distribution of unpolarized antiquarks in the nucleon rest frame

$\bar{\mathcal{H}}(k.P)$: The momentum distribution of polarized antiquarks in the nucleon rest frame along $S^\mu = (0, \mathbf{S})$

Antiquark correlator

$$\bar{\Phi}(k,P,S,\eta) = M\bar{A}_1 + \not{P}\bar{A}_2 + \not{k}\bar{A}_3 + \frac{[\not{P},\not{k}]}{2M}\bar{A}_4 + i(k.S)\gamma_5\bar{A}_5 + M\not{S}\gamma_5\bar{A}_6 + \frac{k.S}{M}\not{P}\gamma_5\bar{A}_7 + \frac{k.S}{M}\not{k}\gamma_5\bar{A}_8 + \frac{[\not{P},\not{S}]}{2}\gamma_5\bar{A}_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5\bar{A}_{10} + \frac{k.S}{2M^2}[\not{P},\not{k}]\gamma_5\bar{A}_{11} + \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma \bar{A}_{12} + \mathcal{O}(\bar{B}_i)$$

I. In models without gauge field degrees of freedom T-odd amplitudes, $\bar{A}_4$, $\bar{A}_5$, $\bar{A}_{12}$ and all the $\bar{B}_i$ amplitudes are absent

II. Assuming the partons are on-shell, $Tr[\Phi\Gamma(\gamma.k - m)] = 0$, leads to the relations between some amplitudes

III. Assuming pure spin states, $\omega^2 = -1$, leads to $\bar{A}_8 = \mp\bar{A}_{11}$

$$\bar{\Phi}(k,P,S) = (\not{k} - m)\bar{A}_3 + \frac{(\not{k} - m)}{M^2}\Big[(P\cdot k) - mM\Big]\bar{A}_{11}\bar{\not{\omega}}\gamma_5$$

$$\bar{\omega}^\mu(k,P,S) = \left\{ S^\mu - \frac{(k.S)}{\Big[(P\cdot k) - mM\Big]}P^\mu + \frac{M}{m}\frac{(k\cdot S)}{\Big[(P\cdot k) - mM\Big]}k^\mu \right\}$$

Covariant Parton Model - The amplitudes for antiquarks

- Model

$$\bar{\Phi}(k,P,S) = (\not{k} - m)\Big(\bar{\mathcal{G}}(k.P) + \bar{\mathcal{H}}(k.P)\gamma_5\bar{\not{\psi}}\Big)M\delta(k^2 - m^2)\Theta_{\rm kin}(k.P)$$

- Anti Quark correlator with no gauge field + on mass shell + pure spin states

$$\bar{\Phi}(k,P,S) = (\not{k} - m)\bar{A}_3 + \frac{(\not{k} - m)}{M^2}\Big[(P\cdot k) - mM\Big]\bar{A}_{11}\bar{\not{\psi}}\gamma_5$$

- Amplitudes obtained in terms of the covariant distribution functions

$$\bar{A}_3(k.P) = M\delta(k^2 - m^2)\Theta_{\rm kin}(k.P)\bar{\mathcal{G}}(k.P)$$

$$\bar{A}_{11}(k.P) = M\delta(k^2 - m^2)\Theta_{\rm kin}(k.P)\bar{\mathcal{H}}(k.P)\Big(\frac{M^2}{k.P - mM}\Big)$$

Covariant Parton Model - Twist-2 TMDs

		Quark polarization		
Nucleon polarization		U	L	T
	U	f_1		$h_1^\perp$
	L		g_1	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	$g_{1T}^\perp$	h_1 $h_{1T}^\perp$

T-even TMDs (in blue color) can be computed in models based on quark degrees of freedom only. T-odd TMDs (in red color) require explicit gauge field degrees of freedom, and cannot be modeled in the approach used in this model.

$h_1^\perp$ *and* $f_{1T}^\perp$ *cannot be calculated in CPM because they are T-odd*

Covariant Parton Model - Twist-3 TMDs

$$\phi^{[1]} = \frac{M}{P^+}\left[e - \frac{\varepsilon^{jk} k_T^j S_T^k}{M} e_T^\perp\right],$$

$$\phi^{[i\gamma^5]} = \frac{M}{P^+}\left[S_L e_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} e_T\right],$$

$$\phi^{[\gamma^j]} = \frac{M}{P^+}\left[\frac{k_T^j}{M} f^\perp + \varepsilon^{jk} S_T^k f_T + S_L \frac{\varepsilon^{jk} k_T^k}{M} f_L^\perp - \frac{\kappa^{jk}\varepsilon^{kl} S_T^l}{M^2} f_T^\perp\right],$$

$$\phi^{[\gamma^j\gamma^5]} = \frac{M}{P^+}\left[S_T^j g_T + S_L \frac{k_T^j}{M} g_L^\perp + \frac{\kappa^{jk} S_T^k}{M^2} g_T^\perp + \frac{\varepsilon^{jk} k_T^k}{M} g^\perp\right],$$

$$\phi^{[i\sigma^{jk}\gamma^5]} = \frac{M}{P^+}\left[\frac{S_T^j k_T^k - S_T^k k_t^j}{M} h_T^\perp - \varepsilon^{jk} h\right],$$

$$\phi^{[\gamma^j]} = \frac{M}{P^+}\left[S_L h_L + \frac{\mathbf{k_T}.\mathbf{S_T}}{M} h_T\right].$$

$e_T^\perp, e_L, e_T, f_T, f_L^\perp, f_T^\perp, g_T,$ and h cannot be calculated in CPM because they are T-odd

Consistency of the covariant parton model – Lorentz invariance relations

Lorentz invariance relations (LIRs) **connect** the twist-2 and twist-3 parton distribution functions (PDFs) and weighted moments of transverse momentum dependent (TMD) correlation functions

LIRs are satisfied in the covariant parton model. ✓

$$
\begin{aligned}
g_T^q(x) &\overset{\text{LIR}}{=} g_1^q(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{\perp(1)q}(x)\,, \\
h_L^q(x) &\overset{\text{LIR}}{=} h_1^q(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)q}(x)\,, \\
h_T^q(x) &\overset{\text{LIR}}{=} -\frac{\mathrm{d}}{\mathrm{d}x} h_{1T}^{\perp(1)q}(x)\,, \\
g_L^{\perp q}(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_T^{\perp(1)q}(x) &\overset{\text{LIR}}{=} 0\,, \\
h_T^q(x, p_T) - h_T^{\perp q}(x, p_T) &\overset{\text{LIR}}{=} h_{1L}^{\perp q}(x, p_T)\,,
\end{aligned}
$$

$$g_{1T}^{(1)}(x) = \int d^2\mathbf{k}_T \frac{\mathbf{k}_T^2}{2M^2} g_{1T}(x, \mathbf{k}_T^2), \quad \text{etc.}$$

P.J. Mulders and R.D. Tangerman, Nucl. Phys. B 461, 197-237 (1996).
D. Boer and P.J. Mulders, Phys. Rev. D 57, 5780 (1998).

Consistency of the covariant parton model – Equation of motion relations

Twist-3 TMDs = Contribution from (Genuine twist 3 TMDs + Twist-2 TMDs + Mass terms)

Equation of motion relations are satisfied in the covariant parton model when genuine twist-3 terms are set to zero. ✓

$$xe = x\tilde{e} + \frac{m}{M} f_1$$

$$xf^{\perp} = x\tilde{f}^{\perp} + f_1$$

$$xg_L^{\perp} = x\tilde{g}_L^{\perp} + g_1 + \frac{m}{M} h_{1L}^{\perp}$$

$$xg_T = \tilde{g}_T + g_{1T}^{\perp(1)} + \frac{m}{M} h_1$$

$$xg_T^{\perp} = x\tilde{g}_T^{\perp} + g_{1T}^{\perp} + \frac{m}{M} h_{1T}^{\perp}$$

$$xh_L = x\tilde{h}_L - 2h_{1L}^{\perp(1)} + \frac{m}{M} g_1$$

$$xh_T = x\tilde{h}_T - h_1 - h_{1T}^{\perp(1)} + \frac{m}{M} g_{1T}^{\perp}$$

$$xh_T^{\perp} = x\tilde{h}_T^{\perp} + h_1 - h_{1T}^{\perp(1)}$$

Consistency of the covariant parton model – WW relations

$$g_T^q(x) \overset{\text{WW}}{=} \int_x^1 \frac{\mathrm{d}y}{y}\, g_1^q(y) + \frac{m}{M}\left[-\frac{h_1^q(x)}{x} + \int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1^q(y)\right],$$

$$h_L^q(x) \overset{\text{WW}}{=} 2x\int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1^q(y) + \frac{m}{M}\left[\frac{g_1^q(x)}{x} - 2x\int_x^1 \frac{\mathrm{d}y}{y^3}\, g_1^q(y)\right].$$

✓

Quark model relations in Covariant Parton Model

$$\begin{aligned}
g_{1T}^{\perp q}(x,p_T) &= -h_{1L}^{\perp q}(x,p_T),\\
g_{T}^{\perp q}(x,p_T) &= -h_{1T}^{\perp q}(x,p_T),\\
g_{L}^{\perp q}(x,p_T) &= -h_{T}^{q}(x,p_T),\\
g_1^q(x,p_T) - h_1^q(x,p_T) &= h_{1T}^{\perp(1)q}(x,p_T),\\
g_T^q(x,p_T) - h_L^q(x,p_T) &= h_{1T}^{\perp(1)q}(x,p_T),\\
h_T^q(x,p_T) - h_T^{\perp q}(x,p_T) &= h_{1L}^{\perp q}(x,p_T).
\end{aligned}$$

These relations are valid in a large class of quark models, including spectator models, bag model, light-front constituent quark model ✓

Summary

Covariant parton model

- Spherical phase space in the rest frame
- On-shell partons in pure spin states
- Consistency with IMF
- Covariant functions G(k.P) and H(k.P)

Quarks	Antiquarks
$\Phi = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)+mM\Big]A_{11}\not{\omega}\gamma_5$	$\bar{\Phi} = (\not{k}-m)\bar{A}_3 + \frac{(\not{k}-m)}{M^2}\Big[(P\cdot k)-mM\Big]\bar{A}_{11}\bar{\not{\omega}}\gamma_5$
$\omega^\mu = \Big\{S^\mu - \frac{(k.S)}{\Big[(P\cdot k)+mM\Big]}P^\mu - \frac{M}{m}\frac{(k\cdot S)}{\Big[(P\cdot k)+mM\Big]}k^\mu\Big\}$	$\bar{\omega}^\mu = \Big\{S^\mu - \frac{(k.S)}{\Big[(P\cdot k)-mM\Big]}P^\mu + \frac{M}{m}\frac{(k\cdot S)}{\Big[(P\cdot k)-mM\Big]}k^\mu\Big\}$
$A_3(k.P) = P^0\delta_+(k^2-m^2)\mathcal{G}(k.P)$	$\bar{A}_3(k.P) = P^0\delta_-(k^2-m^2)\bar{\mathcal{G}}(k.P)$
$A_{11}(k.P) = P^0\delta_+(k^2-m^2)\mathcal{H}(k.P)\Big(-\frac{M^2}{k.P+mM}\Big)$	$\bar{A}_{11}(k.P) = P^0\delta_-(k^2-m^2)\bar{\mathcal{H}}(k.P)\Big(\frac{M^2}{k.P-mM}\Big)$

Table 1: The quark and antiquark correlators, polarization vectors and amplitudes for $A_8 = -A_{11}$ and $\bar{A}_8 = -\bar{A}_{11}$

- All polarized and unpolarized T-even TMDs are systematically obtained for quarks and antiquarks,
- TMD relations expected in QCD or supported by other quark models are satisfied

Summary

Covariant parton model

- Spherical phase space in the rest frame
- On-shell partons in pure spin states
- Consistency with IMF
- Covariant functions G(k.P) and H(k.P)

Quarks	Antiquarks
$\Phi = (\not{k}+m)A_3 + \frac{(\not{k}+m)}{M^2}\Big[(P\cdot k)+mM\Big]A_{11}\not{\omega}\gamma_5$	$\bar{\Phi} = (\not{k}-m)\bar{A}_3 + \frac{(\not{k}-m)}{M^2}\Big[(P\cdot k)-mM\Big]\bar{A}_{11}\bar{\not{\omega}}\gamma_5$
$\omega^\mu = \left\{S^\mu - \frac{(k.S)}{\Big[(P\cdot k)+mM\Big]}P^\mu - \frac{M}{m}\frac{(k\cdot S)}{\Big[(P\cdot k)+mM\Big]}k^\mu\right\}$	$\bar{\omega}^\mu = \left\{S^\mu - \frac{(k.S)}{\Big[(P\cdot k)-mM\Big]}P^\mu + \frac{M}{m}\frac{(k\cdot S)}{\Big[(P\cdot k)-mM\Big]}k^\mu\right\}$
$A_3(k.P) = P^0\delta_+(k^2-m^2)\mathcal{G}(k.P)$	$\bar{A}_3(k.P) = P^0\delta_-(k^2-m^2)\bar{\mathcal{G}}(k.P)$
$A_{11}(k.P) = P^0\delta_+(k^2-m^2)\mathcal{H}(k.P)\Big(-\frac{M^2}{k.P+mM}\Big)$	$\bar{A}_{11}(k.P) = P^0\delta_-(k^2-m^2)\bar{\mathcal{H}}(k.P)\Big(\frac{M^2}{k.P-mM}\Big)$

Table 1: The quark and antiquark correlators, polarization vectors and amplitudes for $A_8 = -A_{11}$ and $\bar{A}_8 = -\bar{A}_{11}$

Outlook

- Generalization to include off-shell-ness effects
- Wish to access T-odd TMDs
- Calculating the GPDs

THANK YOU FOR YOUR ATTENTION