Global analysis of worm-gear function g_{1T}

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BNL

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In Collaboration with:

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Gregory Penn (Yale U./ Temple U.)
Daniel Pitonyak (Lebanon Valley College)

TMD Collaboration

2022 TMD Collaboration Meeting

June 15-17, 2022

Sante Fe, New Mexico

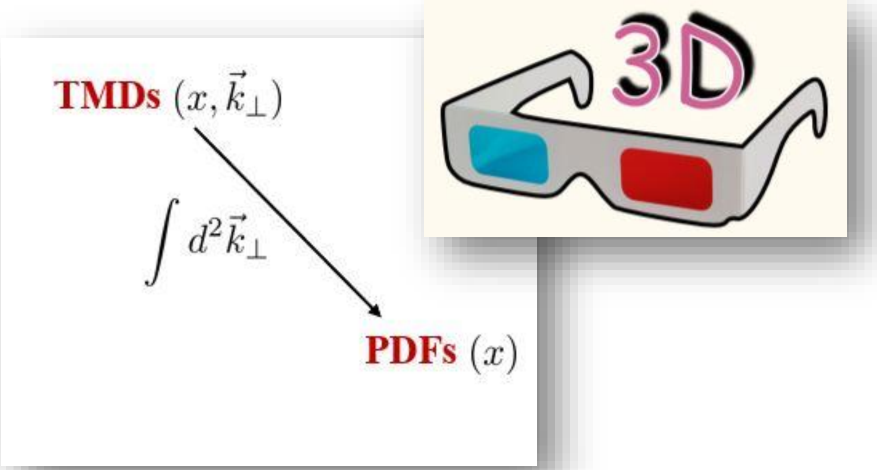
Based on Physical Review D 105 (2022) 3, 034007

Outline

- **Introduction to TMDs**
- **Previous knowledge on g_{1T} : Model calculations, Lattice Calculations, Theoretical predictions**
- **Main extraction: Fit through Monte-Carlo technique**
- **Main fit results**
- **Comparison with theoretical predictions**
- **Comparison with Lattice QCD results**
- **Summary/Outlook**

Introduction

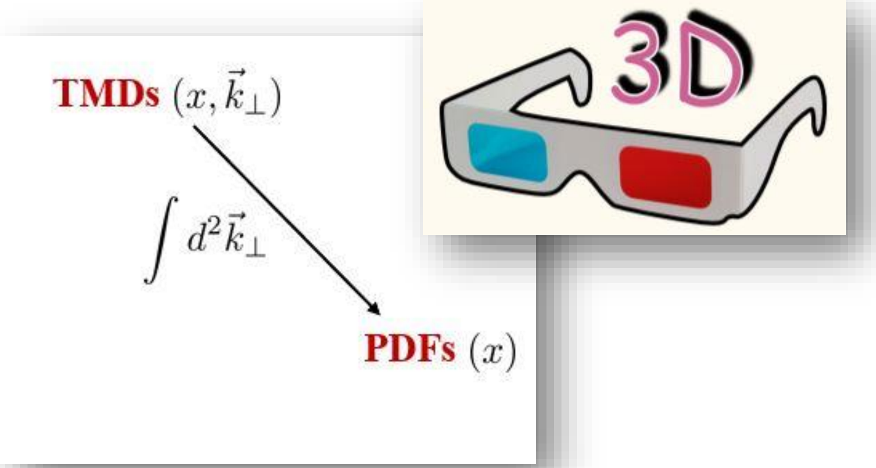
- **TMDs: Interest in 3D rather than 1D parton structure of hadrons**

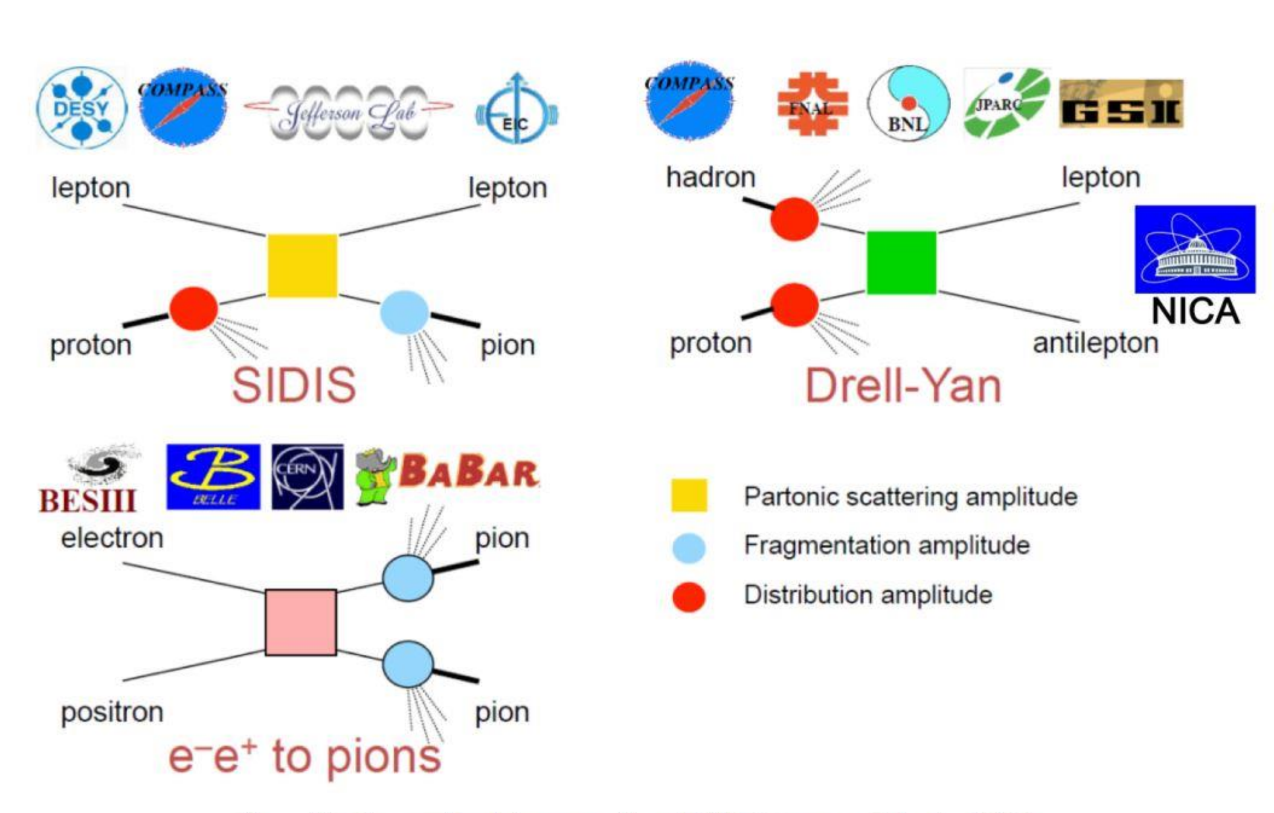


Introduction

- **TMDs: Interest in 3D rather than 1D parton structure of hadrons**

- **TMDs appear in the QCD description of many scattering processes:**





(from M. Grosse Perdekamp, talk at TMD Summer School, 2017)

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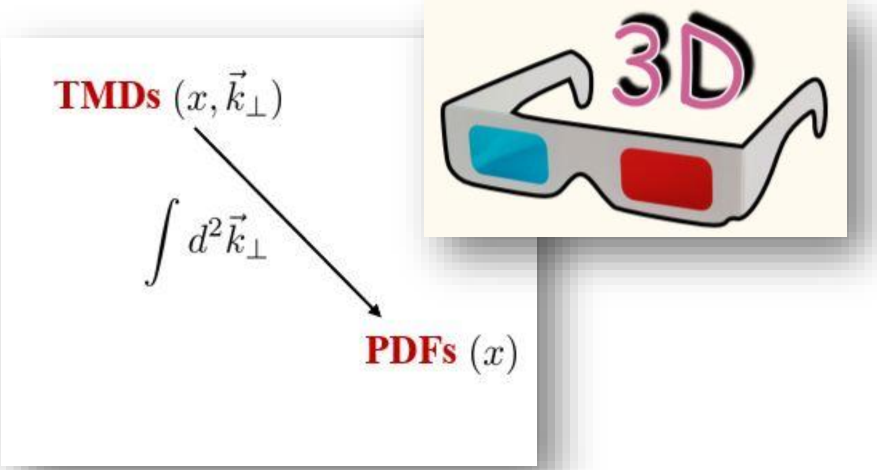

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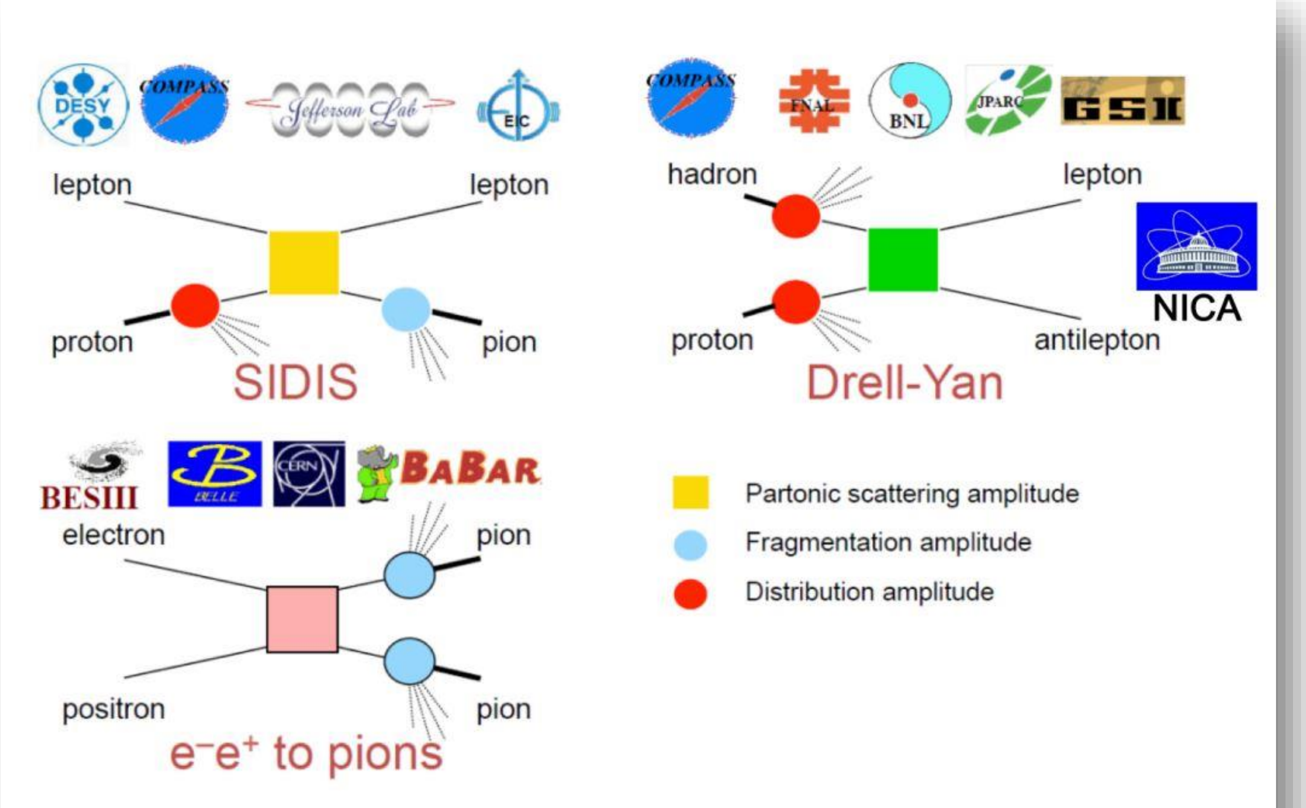

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- **TMDs allow one to study interesting non-trivial aspects like universality**

Dirac matrix

$$\mathcal{F.T.}\ \langle P,S|\bar{\psi}(0^-,0_\perp) \qquad \Gamma\,\psi(\xi^-,\xi_\perp)|P,S\rangle$$

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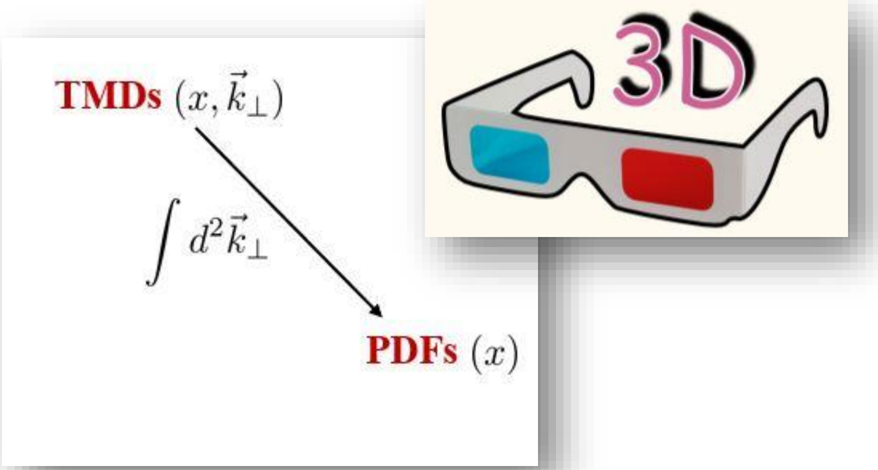

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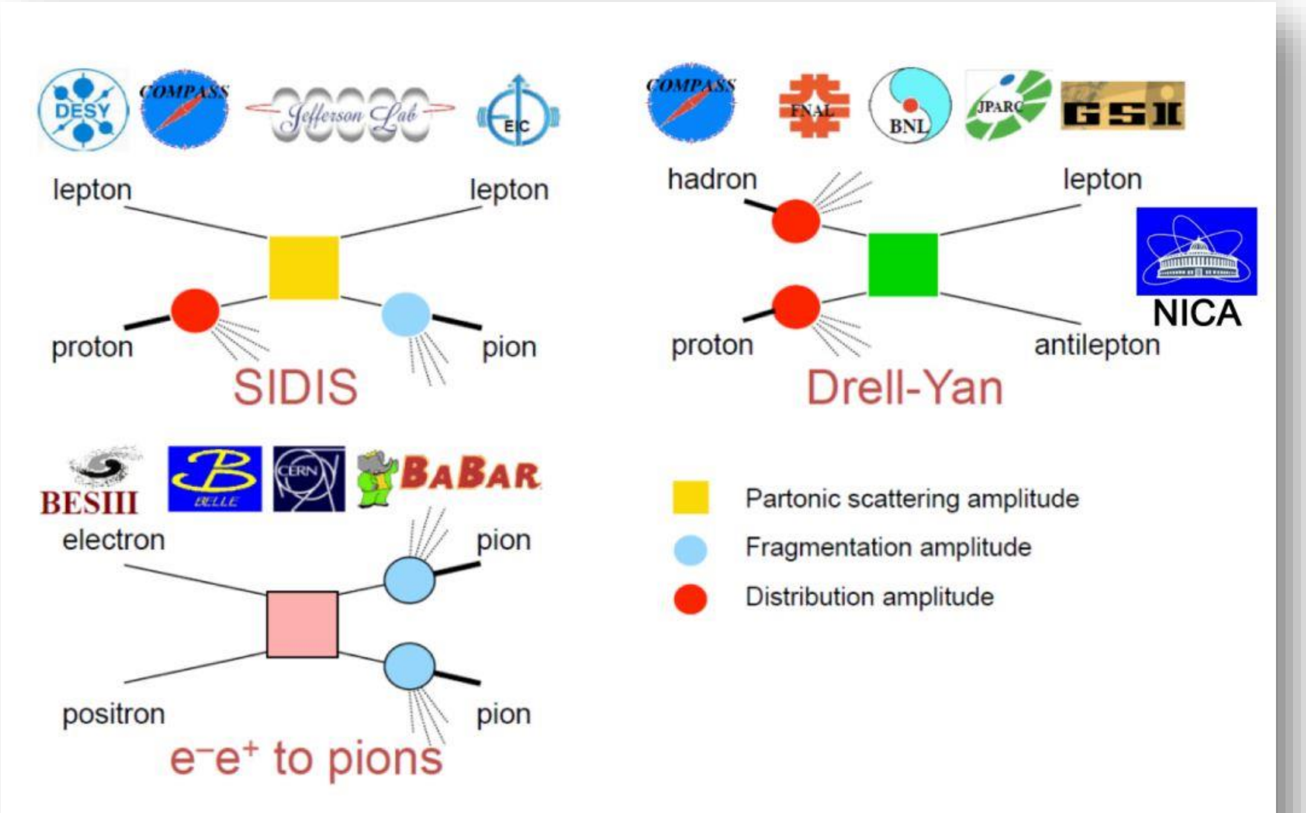

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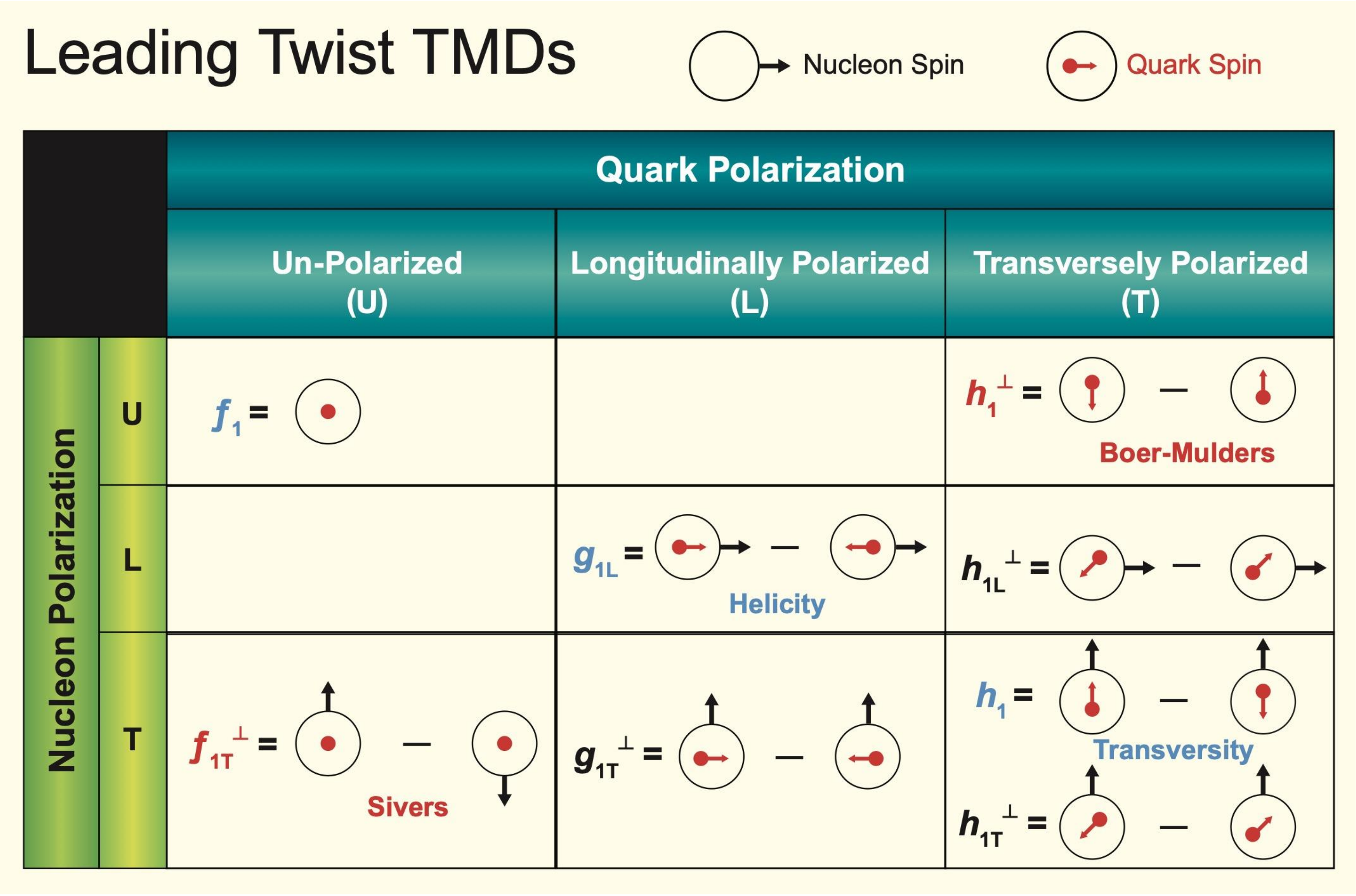
$$\mathcal{F.T.}\ \langle P, S|\bar{\psi}(0^-, 0_\perp)\mathcal{W}(0;\xi)\,\Gamma\,\psi(\xi^-, \xi_\perp)|P, S\rangle$$

Wilson line

Introduction

Fig. courtesy:
D. Pitonyak



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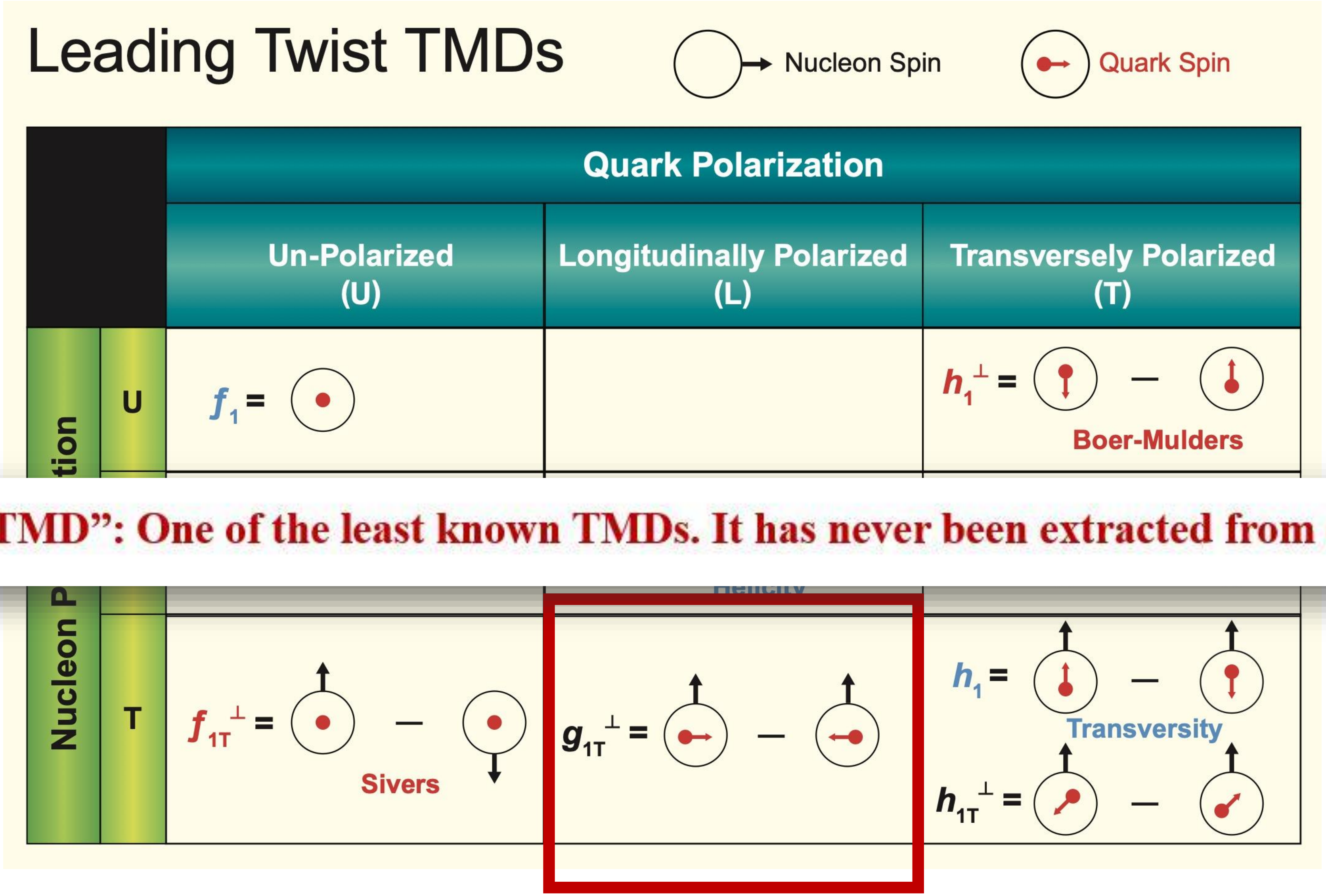
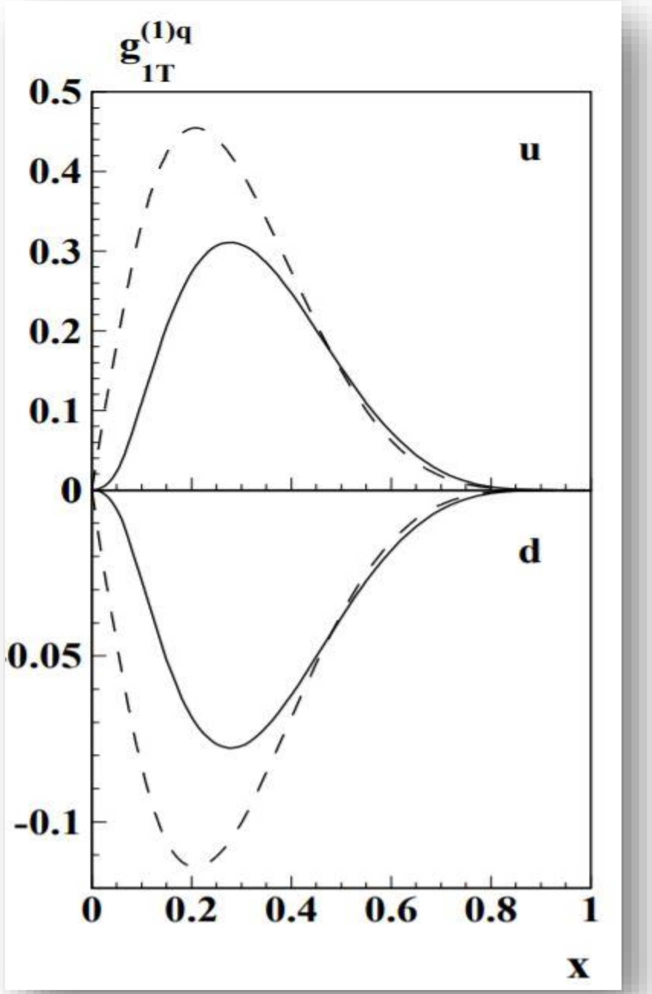


"Worm-gear TMD": One of the least known TMDs. It has never been extracted from experimental data.

Model calculations

Light-cone constituent quark model:

(Pasquini, Cazzaniga, Boffi, arXiv:0806.2298)



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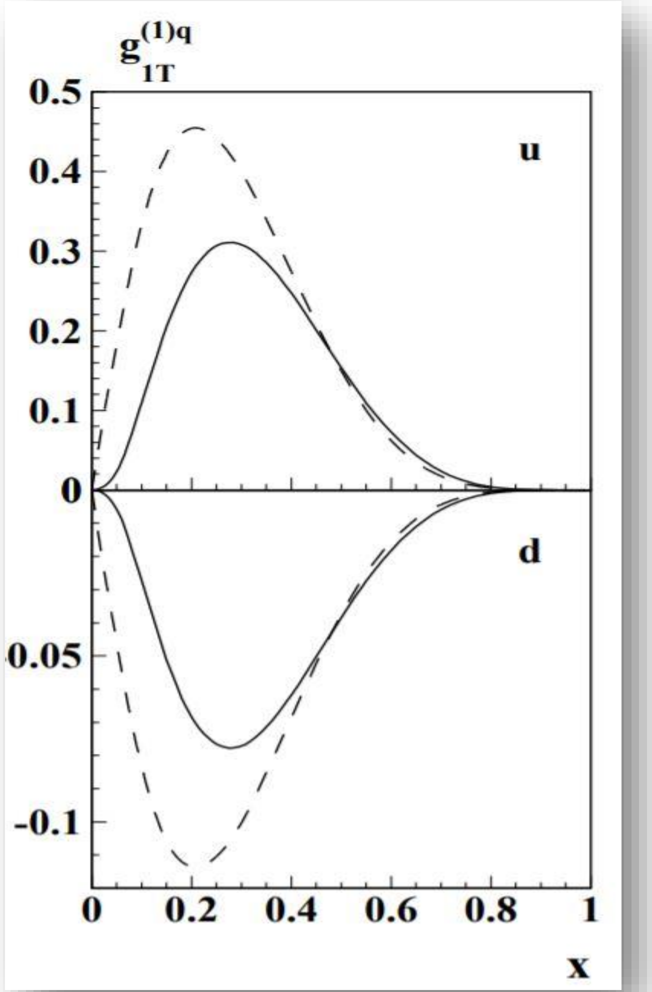
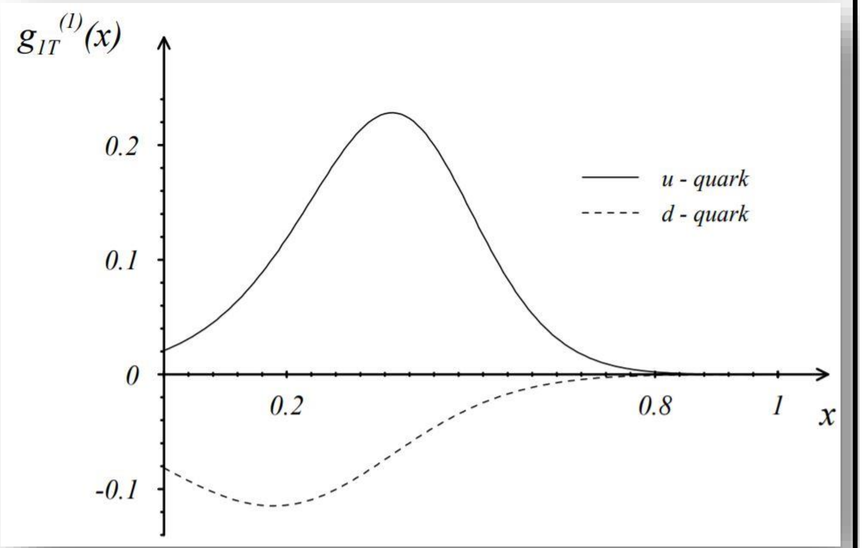


Spectator-diquark model:

(Jakob, Mulders, Rodrigues, hep-ph/9704335,
Bacchetta, Conti, Radici, arXiv:0807.0323, ...)



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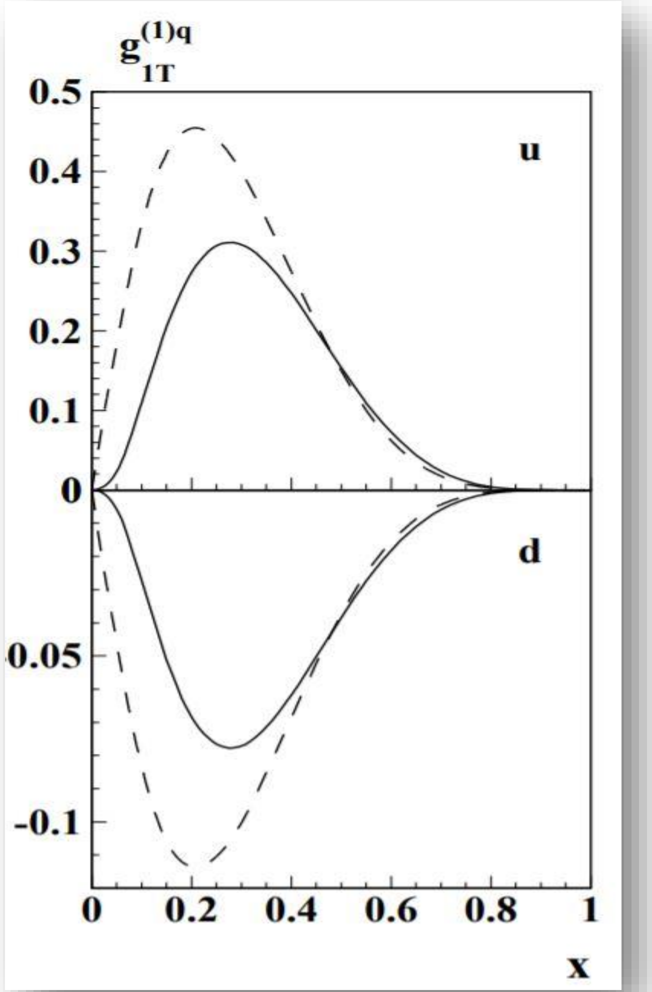

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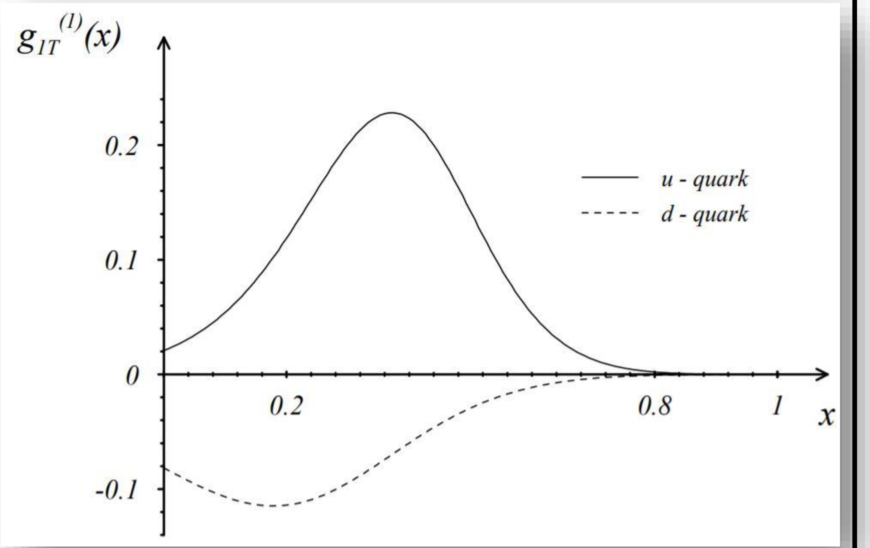
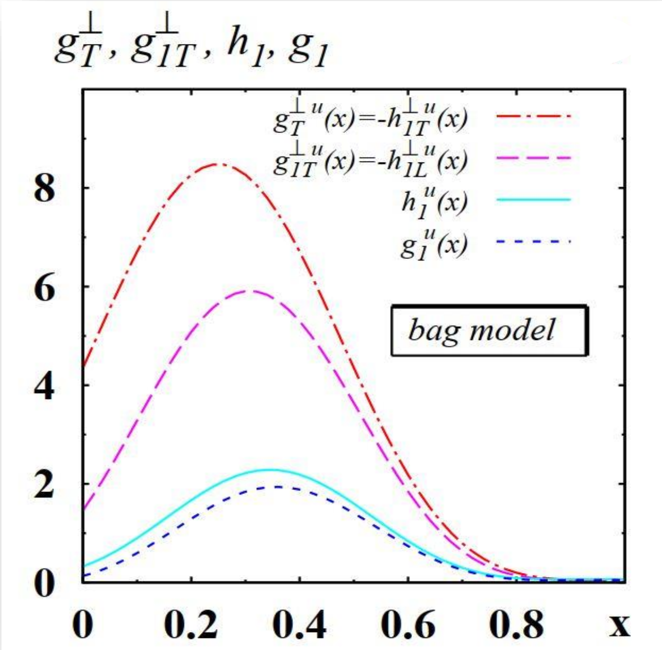


MIT bag model:

Avakian, Efremov, Schweitzer, Yuan, arXiv: 1001.5467



(The d-quark distributions have opposite signs and are smaller.)

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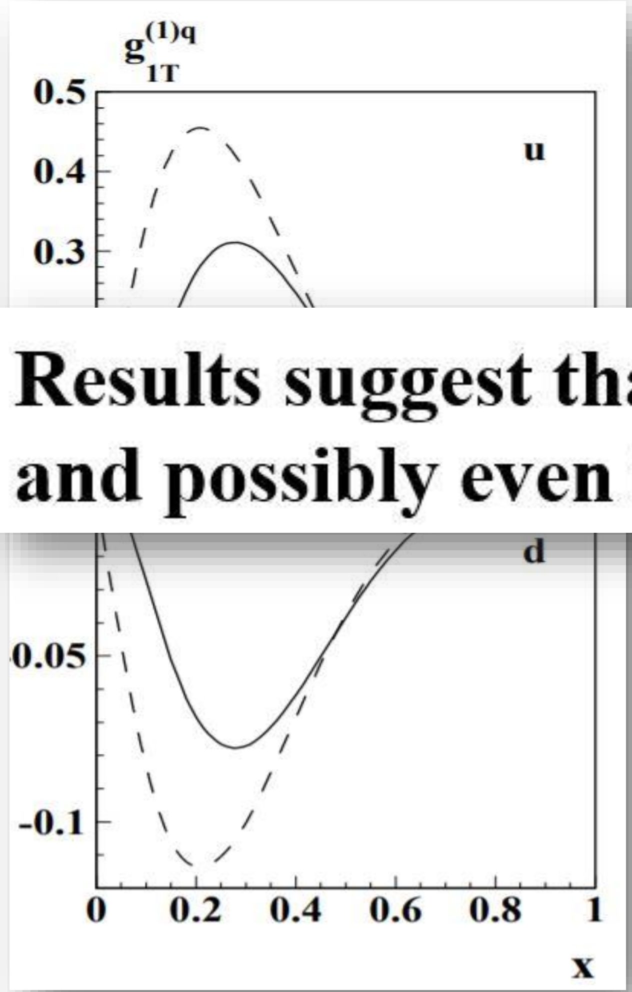

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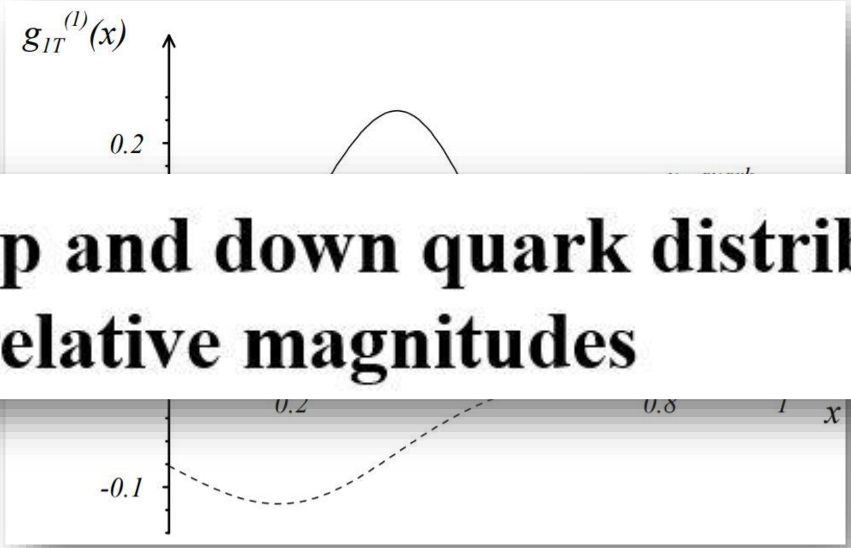

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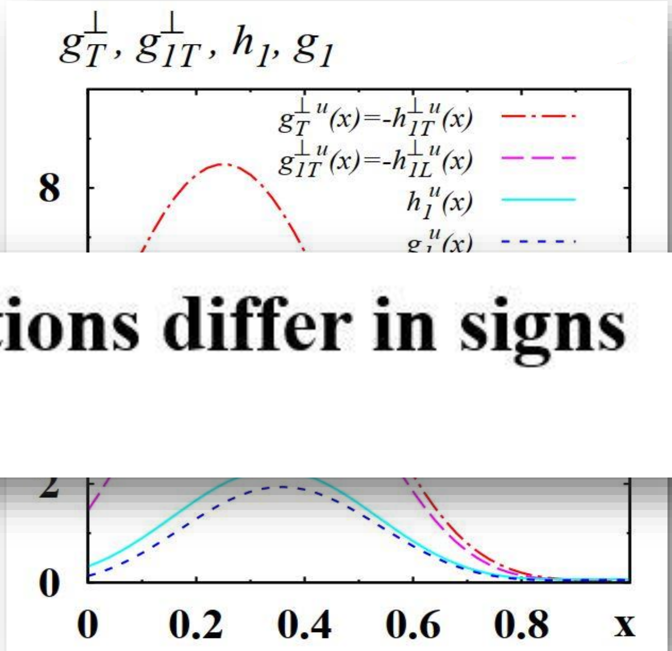

Results suggest that up and down quark distributions differ in signs and possibly even in relative magnitudes

(The d-quark distributions have opposite signs and are smaller.)

Lattice QCD calculations

Lattice QCD results (Musch et al., arXiv:0908.1283, arXiv:1011.1213)

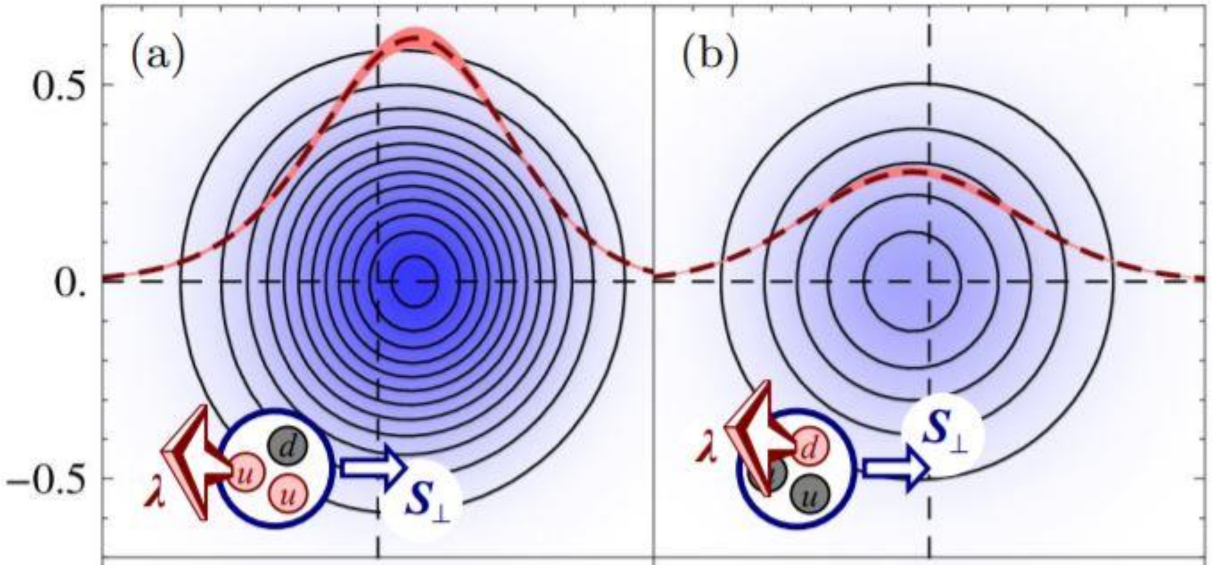

FIG. 3: Quark densities in the $\boldsymbol{k}_\perp$-plane, for $m_\pi \approx 500$ MeV. (a) ρ_L for u-quarks and $\lambda = 1$, $\boldsymbol{S}_\perp = (1, 0)$, (b) the same for d-quarks

- **Pioneering lattice QCD calculations done more than 10 years ago**
- **Lattice results support that up and down down quark distributions come in with different signs and different magnitudes**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theoretical predictions

1.

2.

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theoretical predictions

1. Large N_c analysis: (Pobylitsa, hep-ph/ 0301236)

$$g_{1T}^{u}(x, \vec{k}_{\perp}^{2}) = - g_{1T}^{d}(x, \vec{k}_{\perp}^{2}) + 1/N_c\text{-suppressed}$$

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2. Wandzura-Wilczek-type (WW-type) relation: (Avakian et. al., 0709.3253, Kanazawa et. al., 1512.07233, …)

$$g_{1T}^{(1)q}(x) \equiv \int d^2\vec{k}_\perp \left(\frac{k_\perp^2}{2M^2}\right) g_{1T}^{q}(x, \vec{k}_\perp^2) \overset{\text{EOM}}{=} x \int_x^1 \frac{dy}{y} g_1^q(y) + x\,\tilde{g}_T^q(x)$$

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Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fundamentals

Semi-inclusive Deep Inelastic Scattering: $\ell(l) + N(P,S) \to \ell'(l') + h(P_h) + X$

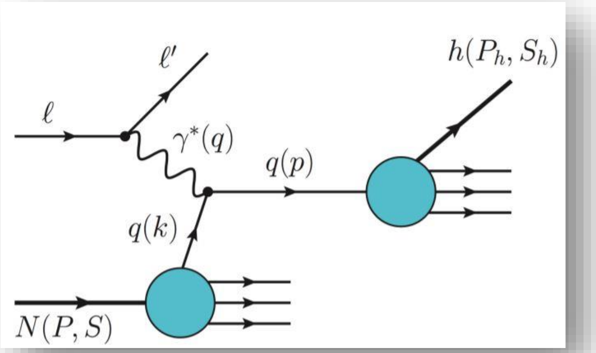

Fig. courtesy:
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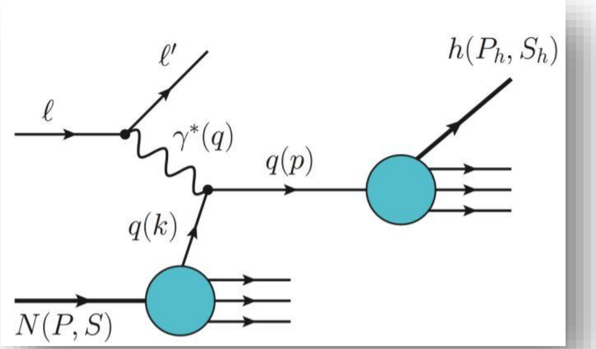

Model-independent decomposition of cross-section: **(Bacchetta et. al. 2007, …)**

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz_h\,d\phi_h\,dP_{hT}^2} = \frac{\alpha_{\rm em}^2}{x\,y\,Q^2}\Bigg\{\left(1-y+\frac{1}{2}y^2\right)\boldsymbol{F_{UU}}$$

$$+\lambda_l\,|\vec{S}_\perp|\,y\left(1-\frac{1}{2}y\right)\cos(\phi_h-\phi_S)\boldsymbol{F_{LT}^{\cos(\phi_h-\phi_S)}}+\ldots\Bigg\}$$

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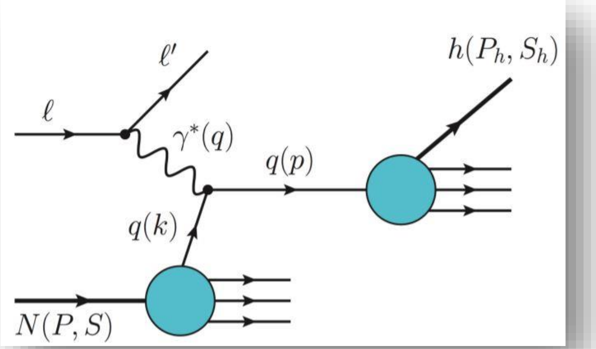
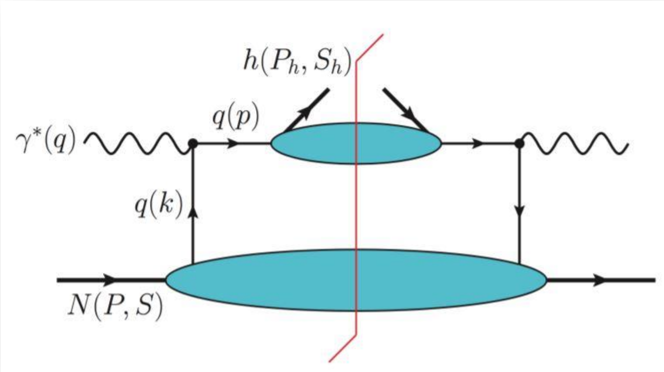


$q_T \ll Q$



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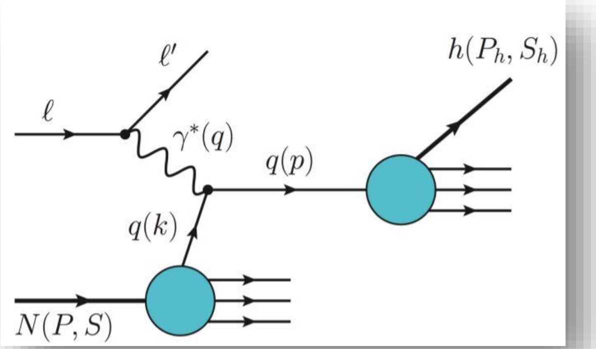

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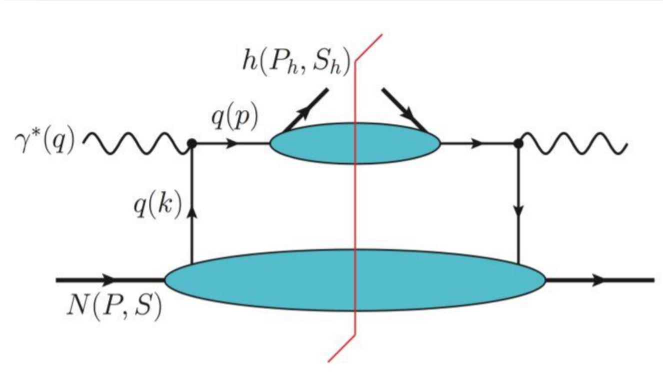

Connection between structure functions and TMDs: (Bacchetta et. al. 2007, …)

$$F_{UU} = C\left[f_1(x,\vec{k}_\perp^2)\, D_1(z,\vec{P}_\perp^2)\right] \qquad F_{LT}^{\cos(\phi_h-\phi_S)} = C\left[\frac{\vec{P}_{hT}\cdot\vec{k}_\perp}{|\vec{P}_{hT}|M}\, g_{1T}(x,\vec{k}_\perp^2)\, D_1(z,\vec{P}_\perp^2)\right]$$

$$C\left[w\, f\, D\right] = x\sum_q e_q^2 \int d^2\vec{k}_\perp \int d^2\vec{P}_\perp\, \delta^{(2)}(z\vec{k}_\perp + \vec{P}_\perp - \vec{P}_{hT})\, w(\vec{k}_\perp,\vec{P}_\perp)\, f^q(x,\vec{k}_\perp^2)\, D^q(z,\vec{P}_\perp^2)$$

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Parameterization of g_{1T}

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Gaussian ansatz:

$$g_{1T}^{q}(x,\vec{k}_{\perp}^{2},Q^{2}) = g_{1T}^{(1)q}(x,Q^{2})\,\frac{2M_{N}^{2}\,e^{-\frac{\vec{k}_{\perp}^{2}}{\pi\langle k_{\perp}^{2}\rangle}}}{\pi\big(\langle k_{\perp}^{2}\rangle\big)^{2}}$$

$q = (u,d)$

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where, $$g_{1T}^{(1)}(x, Q^2) = \frac{n}{\int_0^1 dy\, y^{\alpha+1}(1-y)^{\beta} f_1(y, Q_0^2)} x^{\alpha}(1-x)^{\beta} f_1(x, Q^2)$$

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Remark about evolution

Given the precision and range in Q of the data, a rigorous implementation of TMD evolution not needed

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Presently available data insufficient to pin down the parameters:

$$\langle k_\perp^2 \rangle, \quad \alpha^d, \quad \beta^{u/d}$$

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- **Fix TMD width:**

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- **Fix TMD width:** $$\frac{\langle k_\perp^2 \rangle\big|_{g_1}}{\langle k_\perp^2 \rangle\big|_{f_1}} \approx 0.76$$

Lattice QCD Hagler et. al., hep-lat/ 0908.1283 (See also Bastami et. al., 1807.10606 that uses this idea to get $\langle k_\perp^2 \rangle\big|_{g_{1T}}$)

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$$\approx \frac{\langle k_\perp^2 \rangle\big|_{g_{1T}}}{\langle k_\perp^2 \rangle\big|_{f_1}} \xrightarrow{\langle k_\perp^2 \rangle|_{f_1} = 0.53}$$

Cammarota et. al., arXiv 2002.08384

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$\langle k_\perp^2 \rangle\big|_{f_1} = 0.53$ → $\therefore \langle k_\perp^2 \rangle\big|_{g_{1T}} \approx 0.40$

Cammarota et. al., arXiv 2002.08384

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

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$$\langle k_\perp^2 \rangle, \quad \alpha^d, \quad \beta^{u/d}$$

Gaussian ansatz:

$$g_{1T}^q(x, \vec{k}_\perp^2, Q^2) = g_{1T}^{(1)q}(x, Q^2)\, \frac{2M_N^2\, e^{-\frac{k_\perp^2}{\pi\langle k_\perp^2\rangle}}}{\pi\big(\langle k_\perp^2\rangle\big)^2} \qquad q = (u, d)$$

where,

$$g_{1T}^{(1)}(x, Q^2) = \frac{n}{\int_0^1 dy\, y^{\alpha+1}(1-y)^\beta f_1(y, Q_0^2)}\, x^\alpha (1-x)^\beta f_1(x, Q^2)$$

- **Fix TMD width:** $\langle k_\perp^2 \rangle\big|_{g_{1T}} \approx 0.40$

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Helicity & unpolarized PDFs have similar large-x behavior

$$g_1^q(x)\big|_{x\to 1} \propto f_1^q(x)\big|_{x\to 1}$$

$$g_{1T}^{(1)q}(x) \overset{x\to 1}{\approx} (1-x)\, g_1^q(x)$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

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- **Fix beta from WW approximation:**

$$g_{1T}^{(1)q}(x) \overset{x \to 1}{\approx} (1-x) f_1^q(x) \longrightarrow \therefore \beta^u = \beta^d = 1$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Presently available data insufficient to pin down the parameters:

$$\langle k_\perp^2 \rangle, \quad \alpha^d, \quad \beta^{u/d}$$

Gaussian ansatz: $$g_{1T}^q(x, \vec{k}_\perp^2, Q^2) = g_{1T}^{(1)q}(x, Q^2) \frac{2M_N^2\, e^{-\frac{k_\perp^2}{\pi\langle k_\perp^2 \rangle}}}{\pi\left(\langle k_\perp^2 \rangle\right)^2}$$ $q = (u, d)$

We checked explicitly that using different values of TMD width and beta does not change the qualitative conclusions of our results

- **Fix TMD width:** $\langle k_\perp^2 \rangle\big|_{g_{1T}} \approx 0.40$
- **Set alphas equal:** $\alpha^d = \alpha^u$
- **Fix beta from WW approximation:**

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Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Presently available data insufficient to pin down the parameters:

$$\langle k_\perp^2 \rangle, \quad \alpha^d, \quad \beta^{u/d}$$

Gaussian ansatz: $g_{1T}^q(x, \vec{k}_\perp^2, Q^2)$ … $q = (u, d)$

where, $g_{1T}^{(1)}(x, Q^2)$ … $(1-x)^\beta f_1(x, Q^2)$

3 free parameters:

$$n^u, \quad n^d, \quad \alpha$$

- **Fix TMD width:** $\langle k_\perp^2 \rangle \big|_{g_{1T}} \approx 0.40$
- **Set alphas equal:** $\alpha^d = \alpha^u$

- **Fix beta from WW approximation:**

$$g_{1T}^{(1)q}(x) \overset{x\to 1}{\approx} (1-x) f_1^q(x) \longrightarrow \therefore \beta^u = \beta^d = 1$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Experimental data

Dataset	Target	Identified hadron	No. of points
HERMES	p	π^+	26
Airapetian et. al., arXiv: 2007.07755		π^-	26
		π^0	8
COMPASS	p	$h^+ \approx (\pi^+, \mathrm{K}^+)$	33
Parsamyan, PoS: QCDEV2017		$h^- \approx (\pi^-, \mathrm{K}^-)$	31
JLab	n	π^+	2
Huang, arXiv: 1108.0489		π^-	2

Cut:

$$\frac{q_T}{Q} < 0.50$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

Theory

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

Theory

Fit to exp. data

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

Theory

Fit to exp. data

Minimize:

$$\chi^2 = \sum_{\mathrm{H+C+J}} \frac{(\mathrm{exp.\,data} - \mathrm{theory})^2}{(\mathrm{exp.\,error})^2}$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

Theory

Fit to exp. data

Minimize weighted chi-squared:

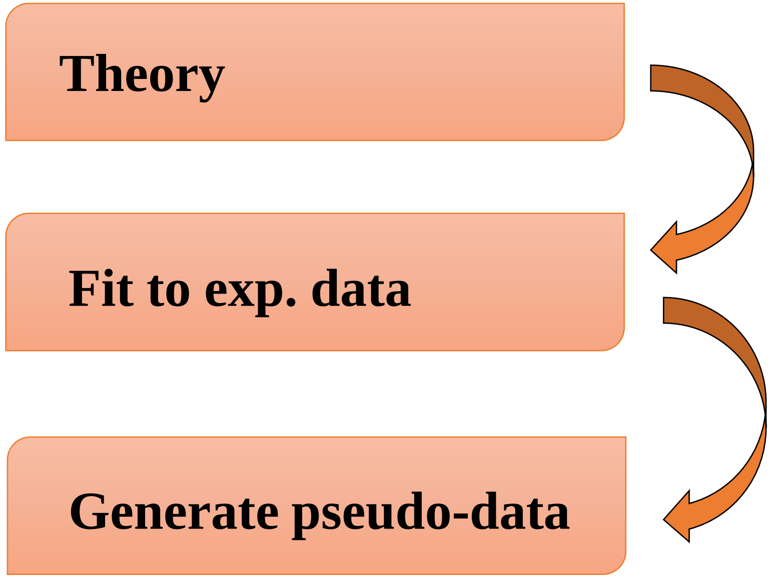
$$\chi^2 = \sum_{\mathrm{H+C}} \frac{(\mathrm{exp.\,data} - \mathrm{theory})^2}{(\mathrm{exp.\,error})^2} + w \sum_{\mathrm{J}} \frac{(\mathrm{exp.\,data} - \mathrm{theory})^2}{(\mathrm{exp.\,error})^2}$$

(Echevarria, Kang, Terry, arXiv: 2009.10710)

Give JLab data weight similar to HERMES & COMPASS data

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique



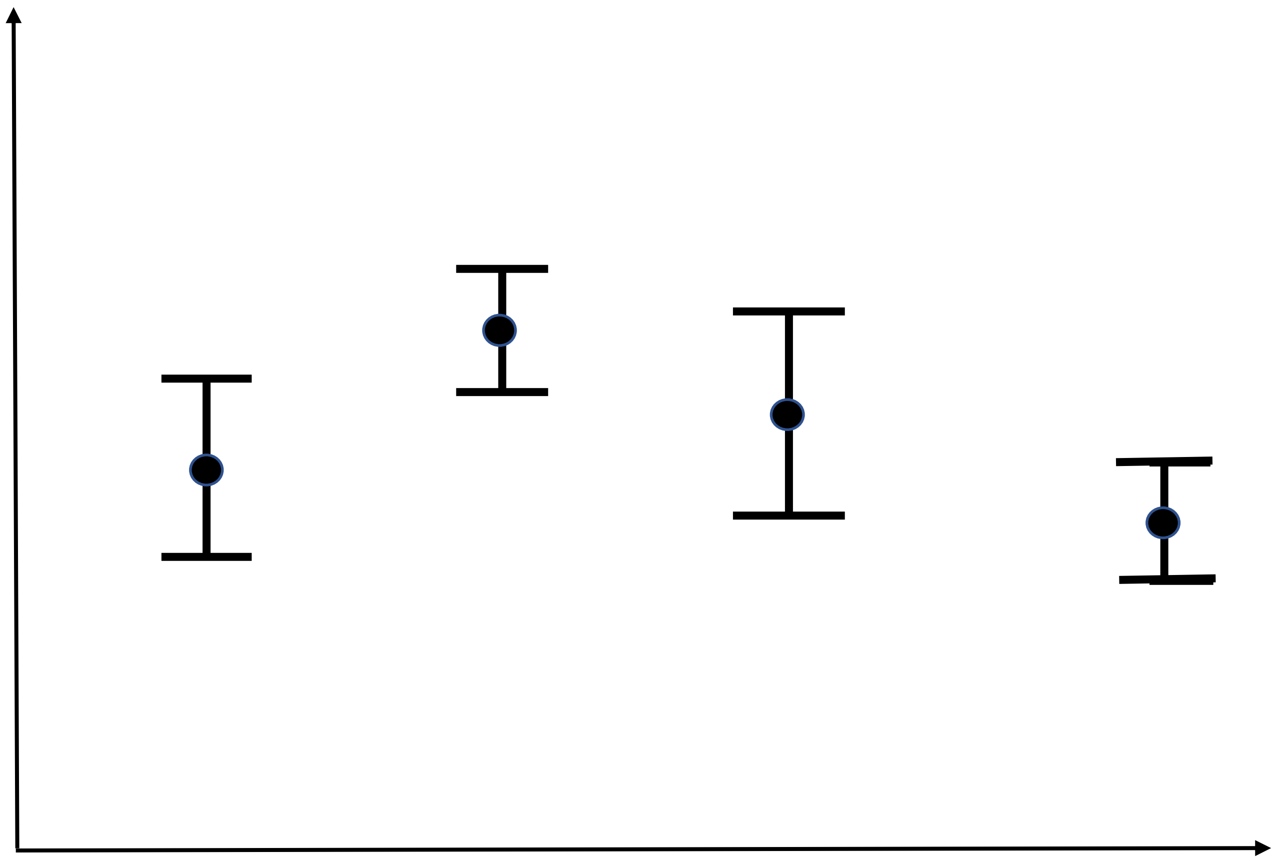
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

Theory

Fit to exp. data

Generate pseudo-data



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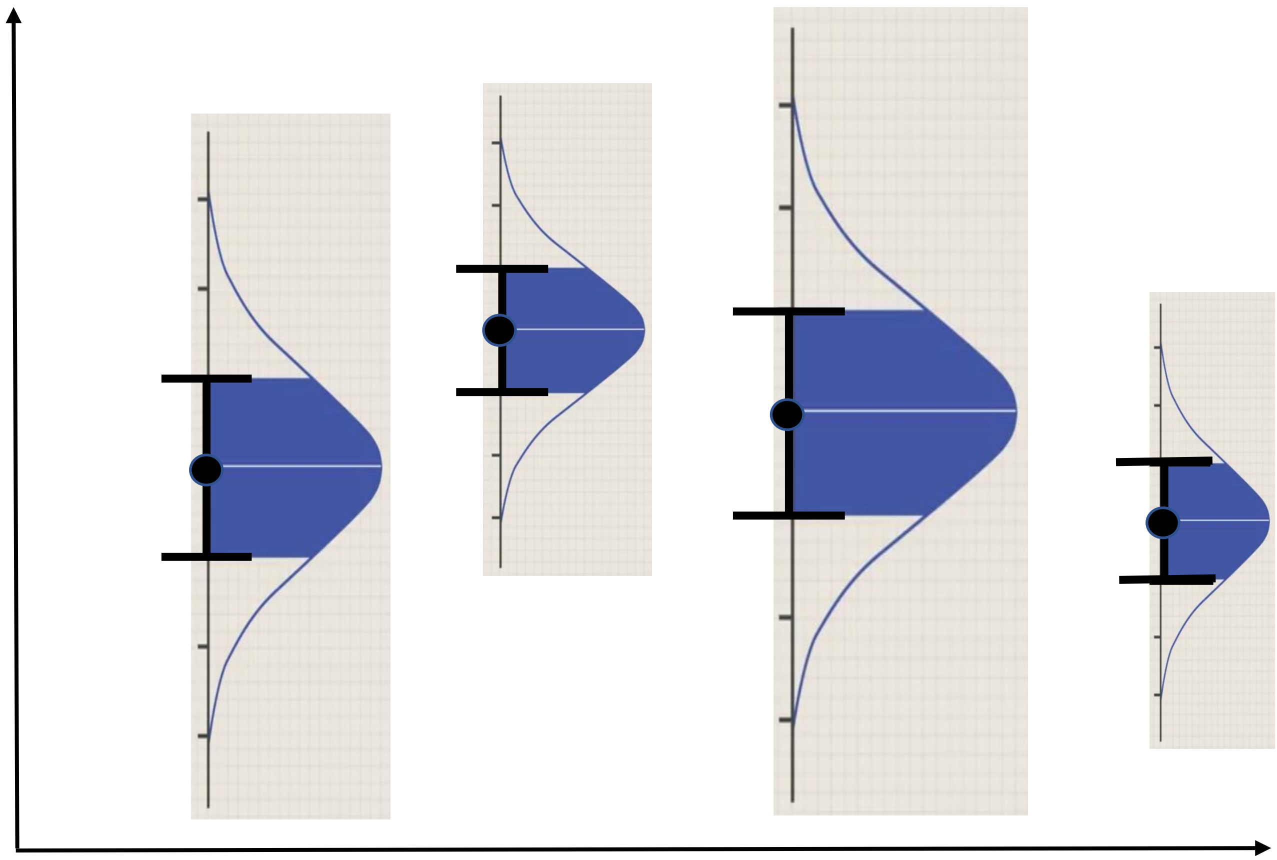
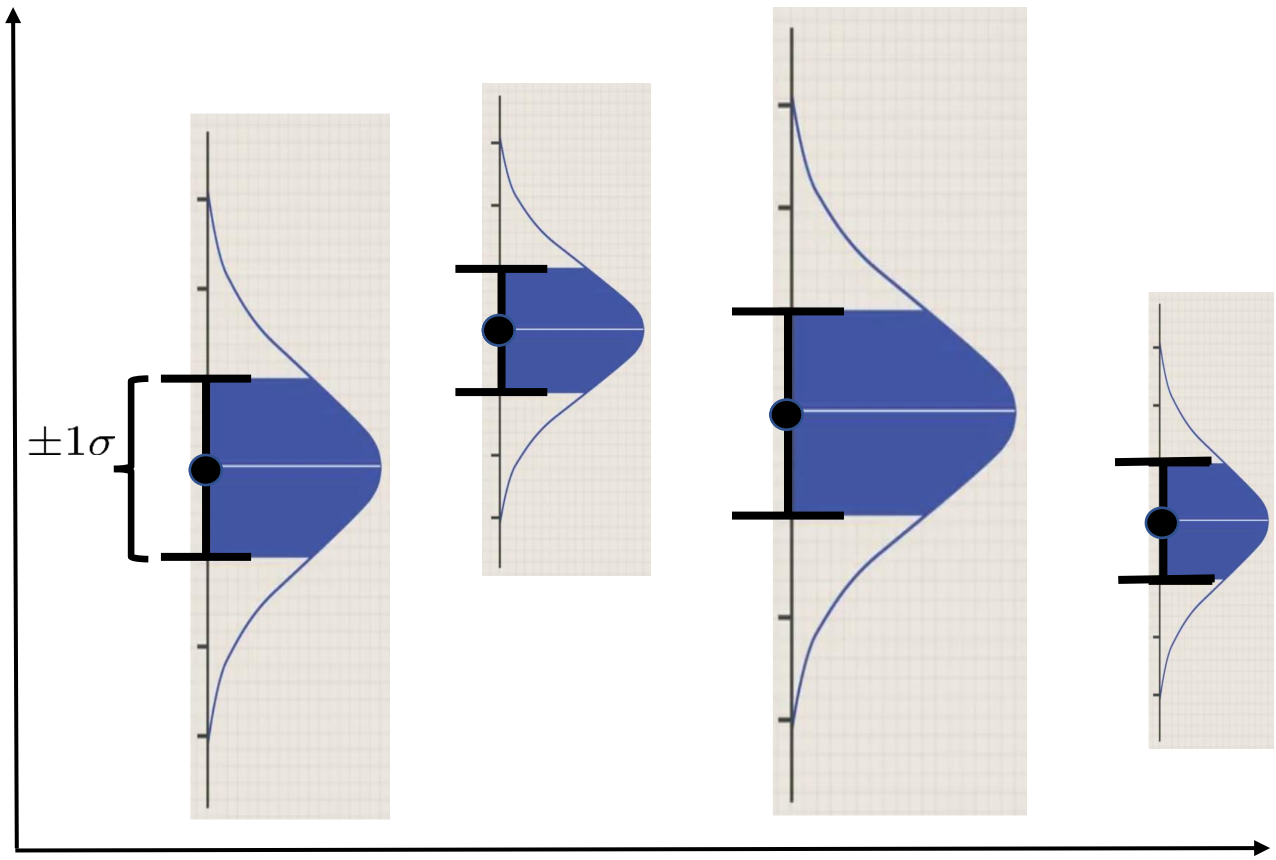


Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

- Theory
- Fit to exp. data
- Generate pseudo-data



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Fitting procedure: Monte-Carlo technique

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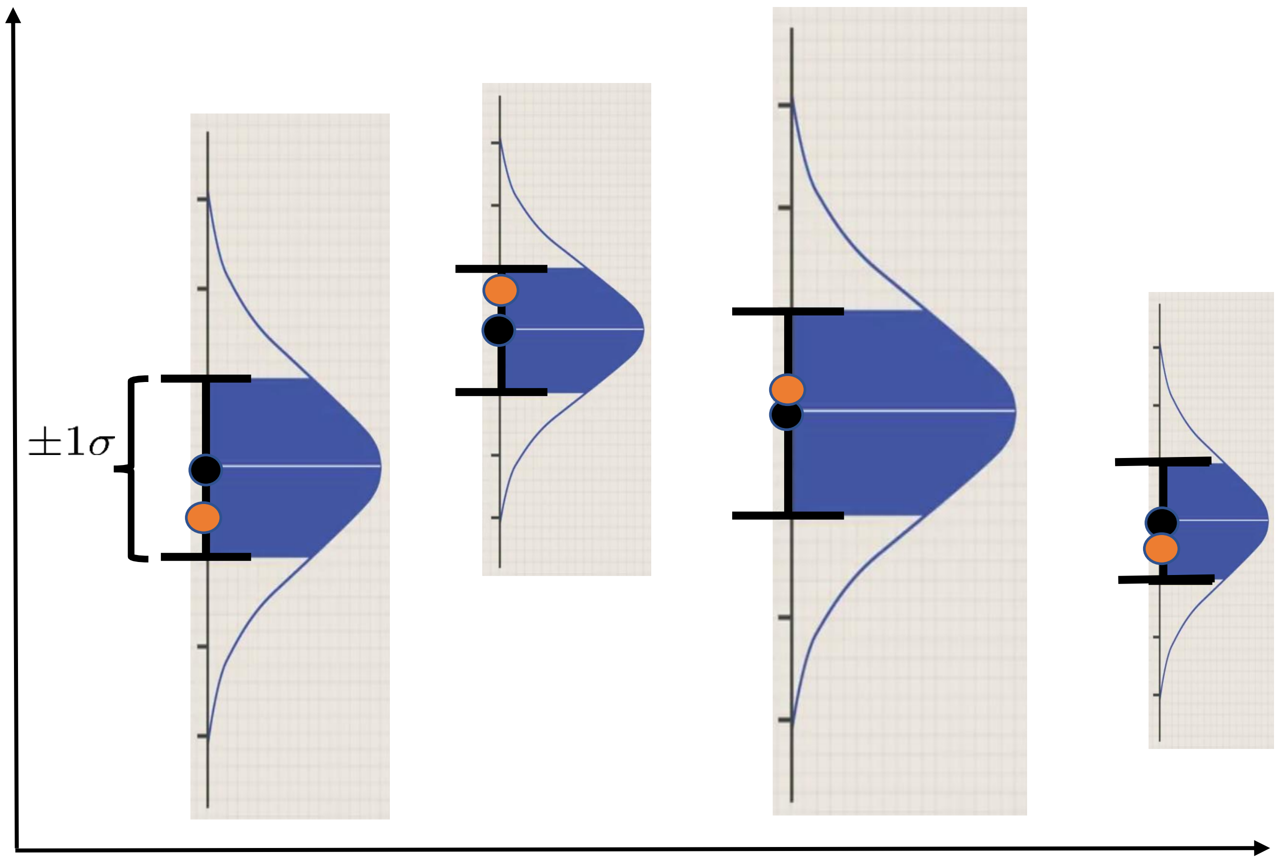
Fit to exp. data

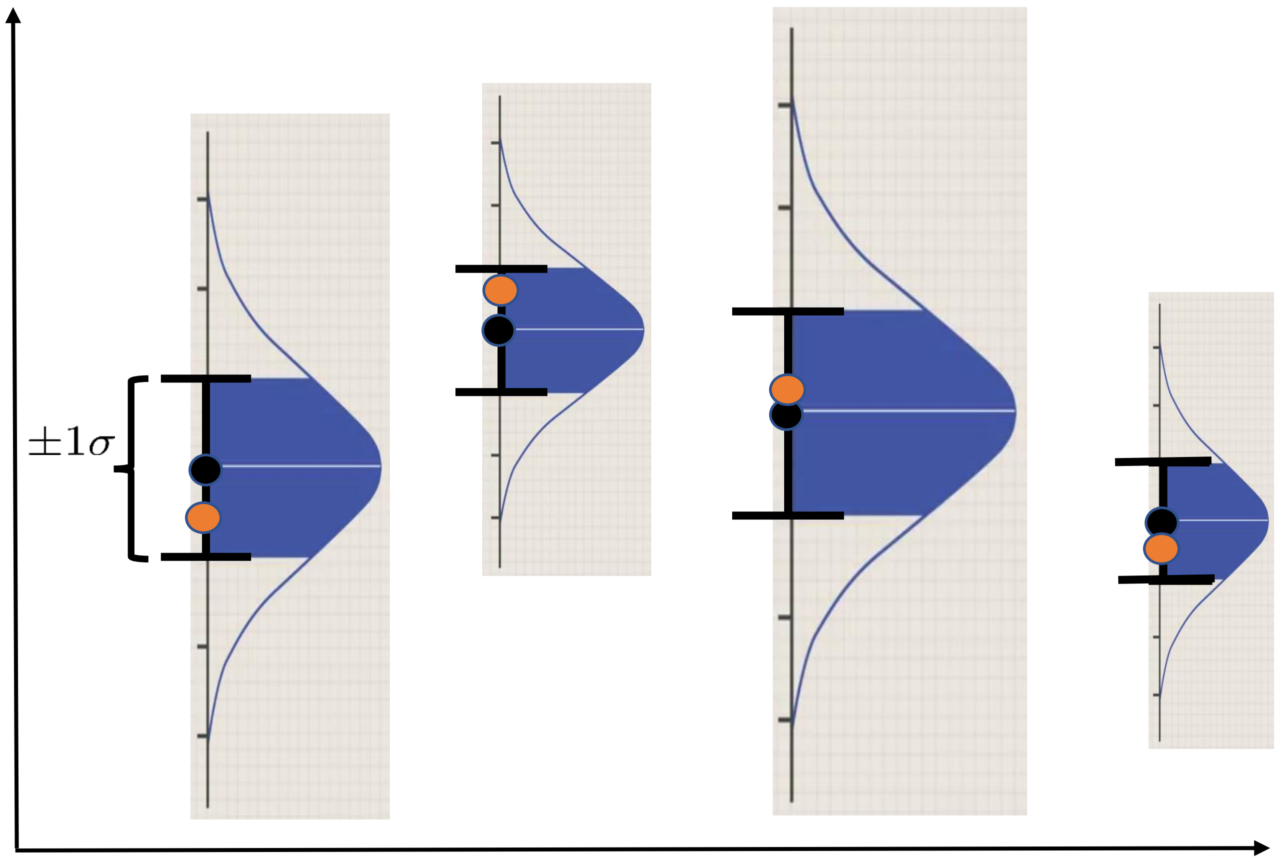
Generate pseudo-data



Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

- Theory
- Fit to exp. data
- Generate pseudo-data
- Fit to pseudo-data



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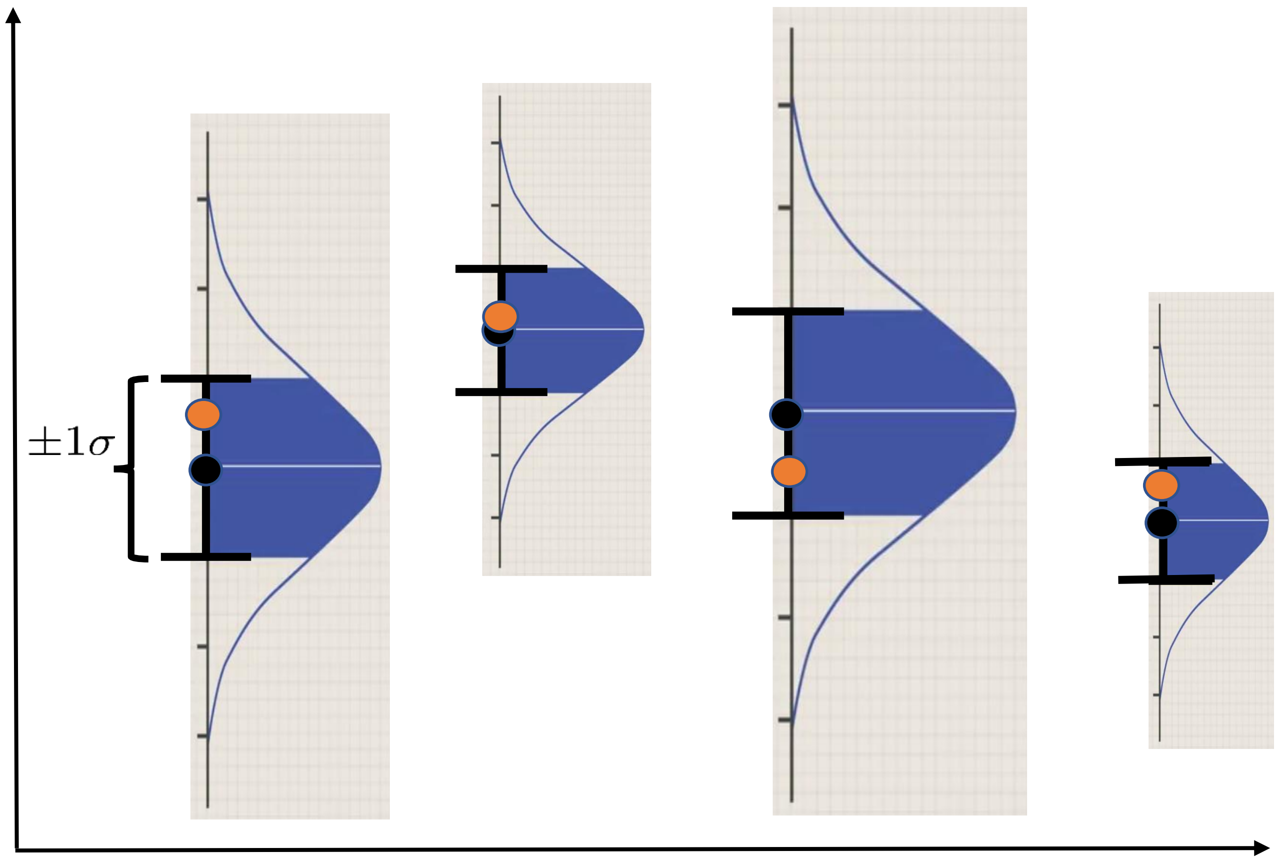
Fitting procedure: Monte-Carlo technique

Theory

Fit to exp. data

Generate pseudo-data

Fit to pseudo-data



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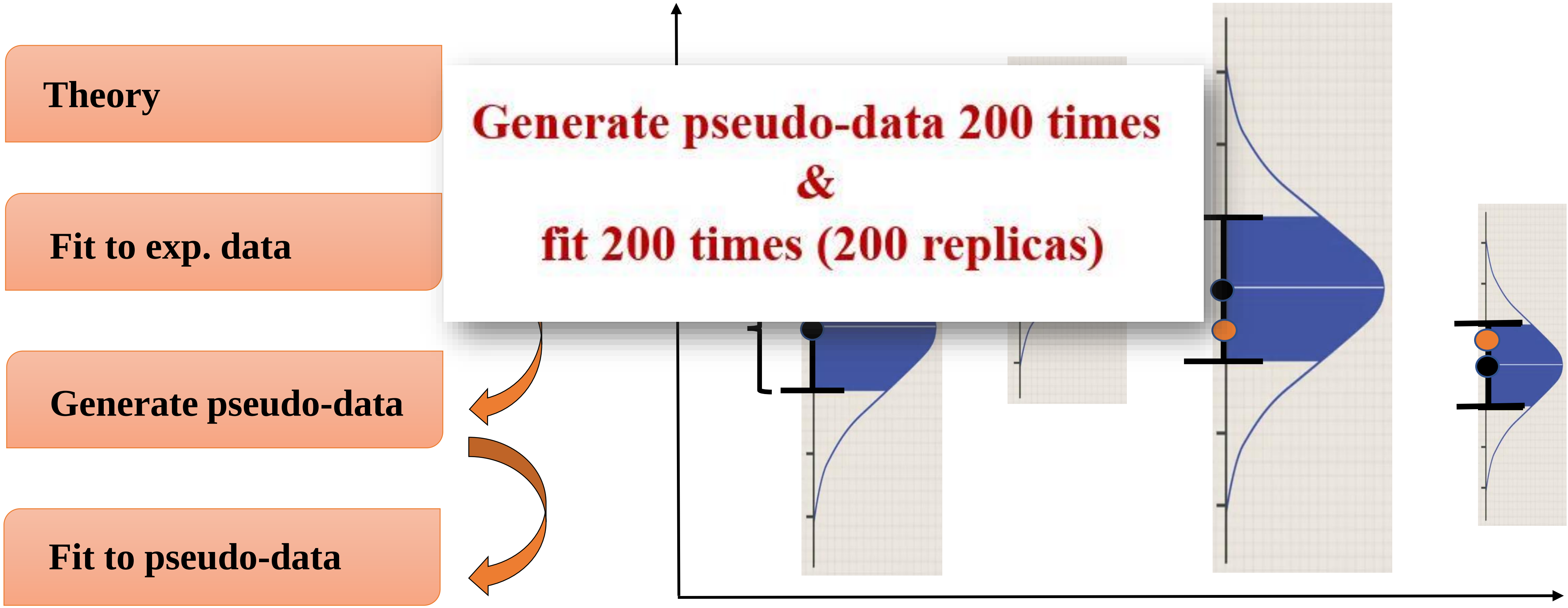


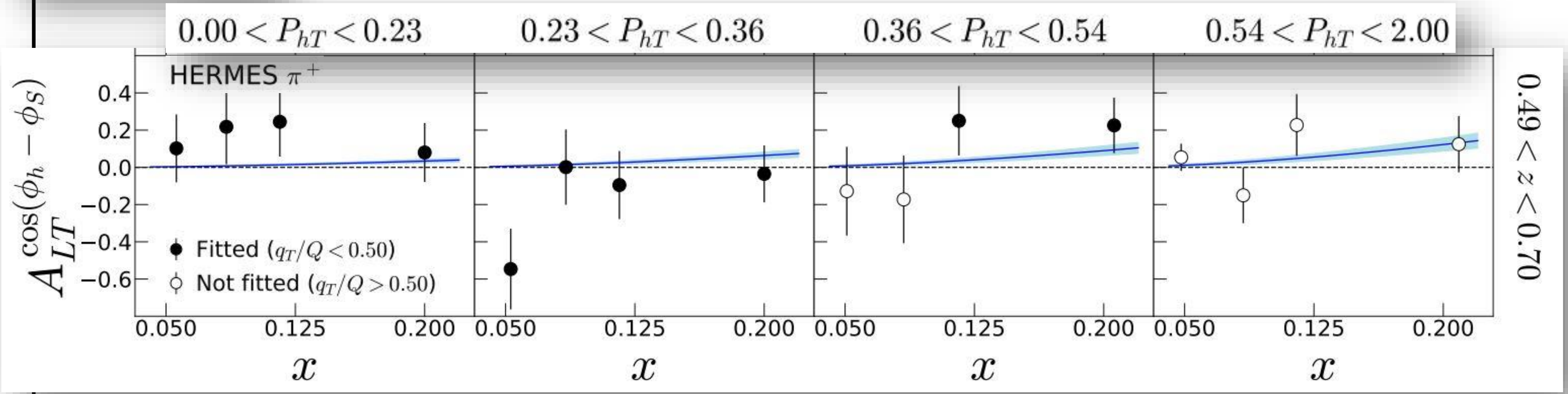
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data

Sample results

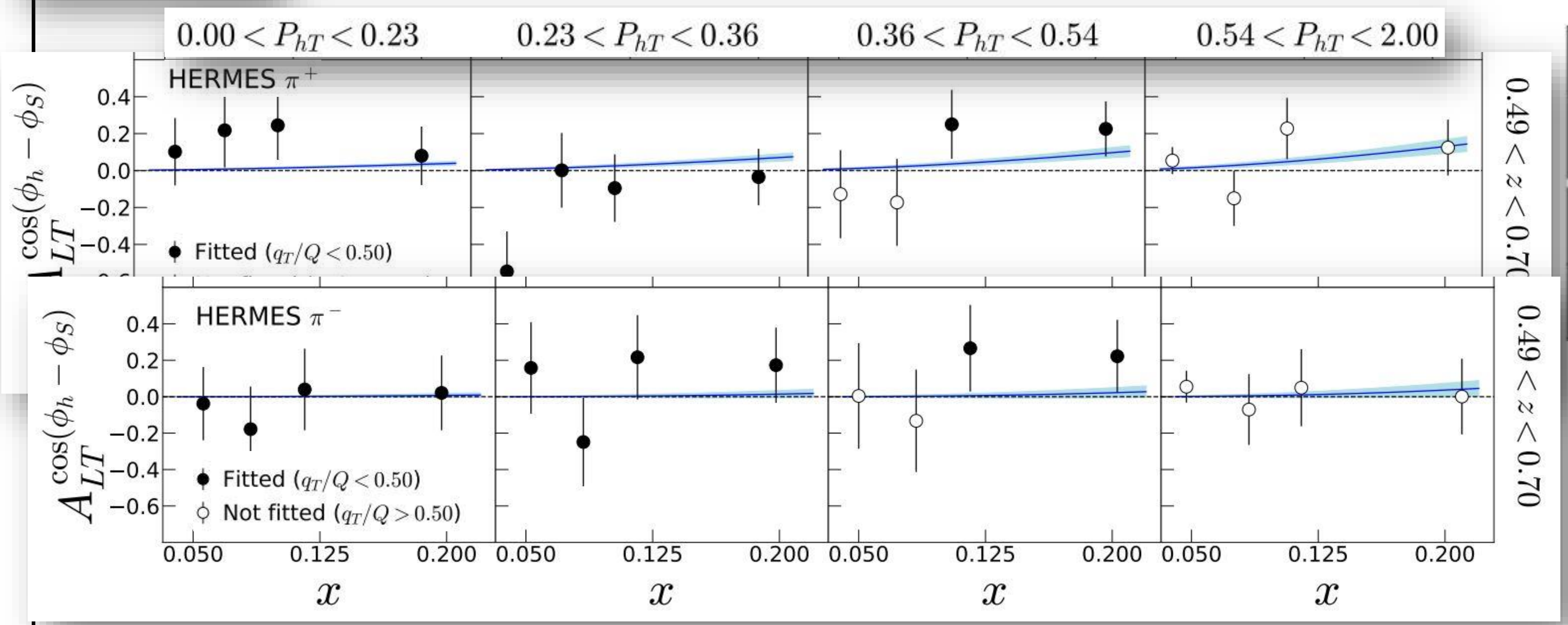


Data set	$\chi^2_w/N_{\text{pts.}}\|_{\text{Main}}$
HERMES π^+	1.20

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data

Sample results

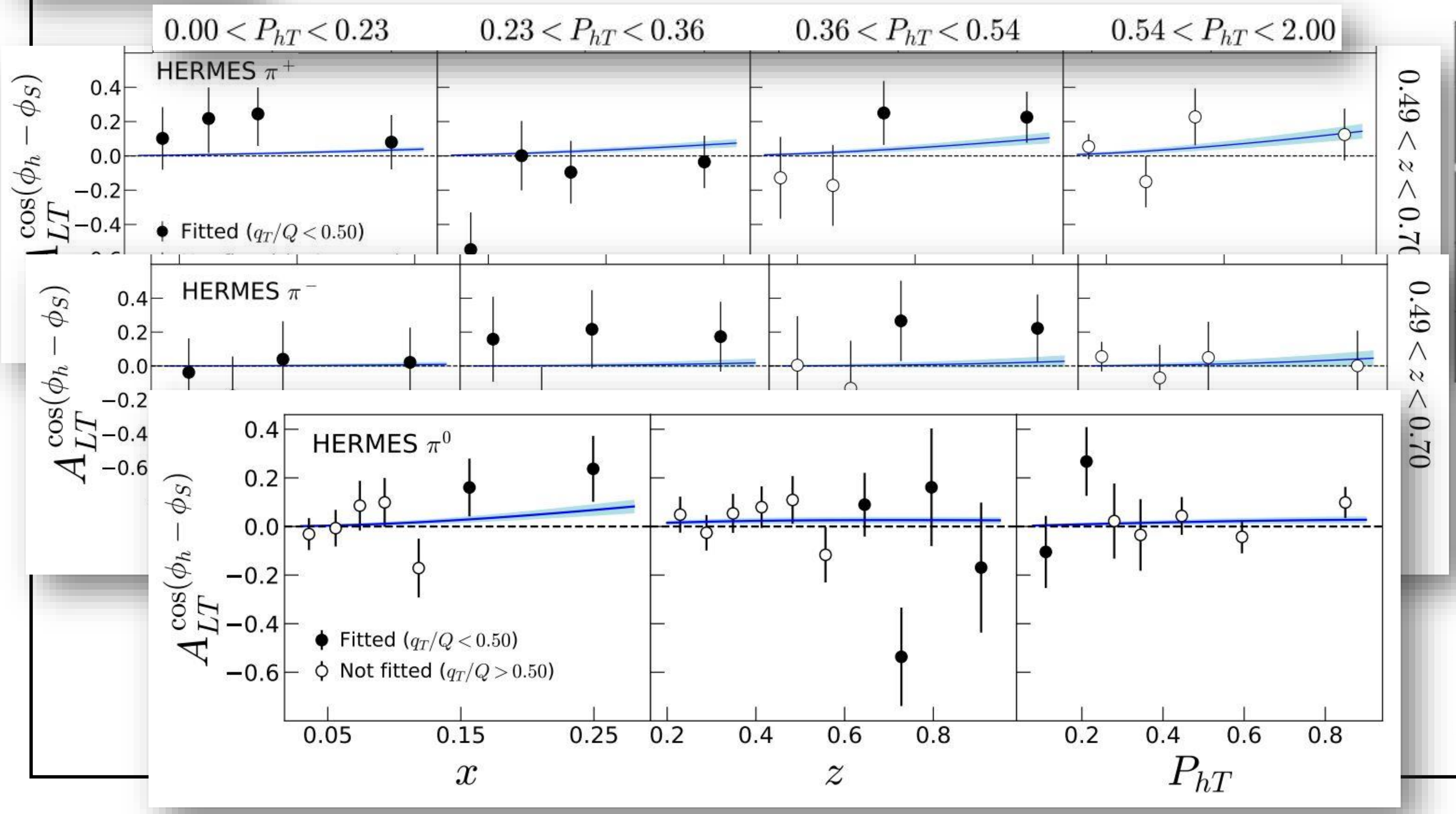


Data set	$\chi^2_w/N_{\text{pts.}}\big\vert_{\text{Main}}$
HERMES π^+	1.20
HERMES π^-	0.88

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data

Sample results

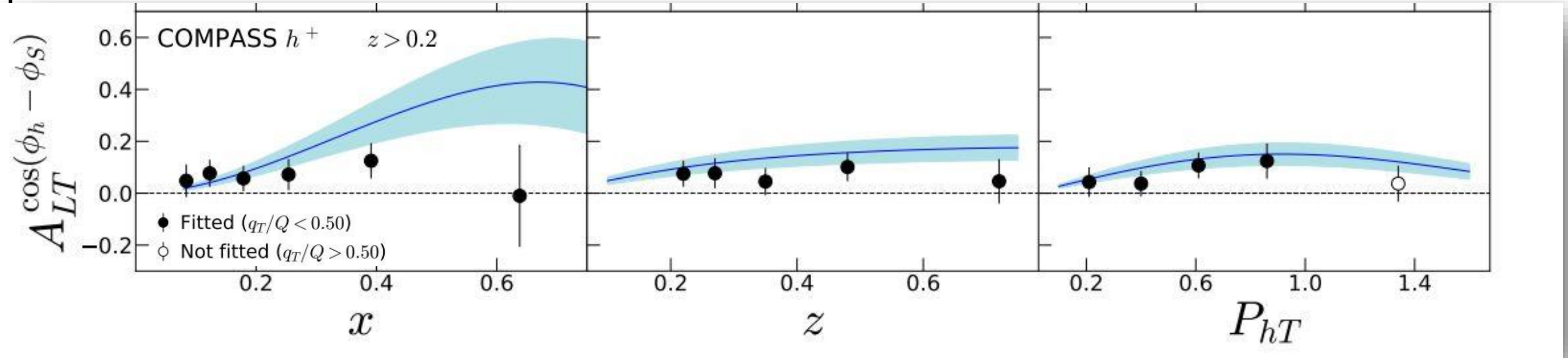


Data set	$\chi^2_w/N_{\text{pts.}}\|_{\text{Main}}$
HERMES π^+	1.20
HERMES π^-	0.88
HERMES π^0	1.94

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data

Sample results

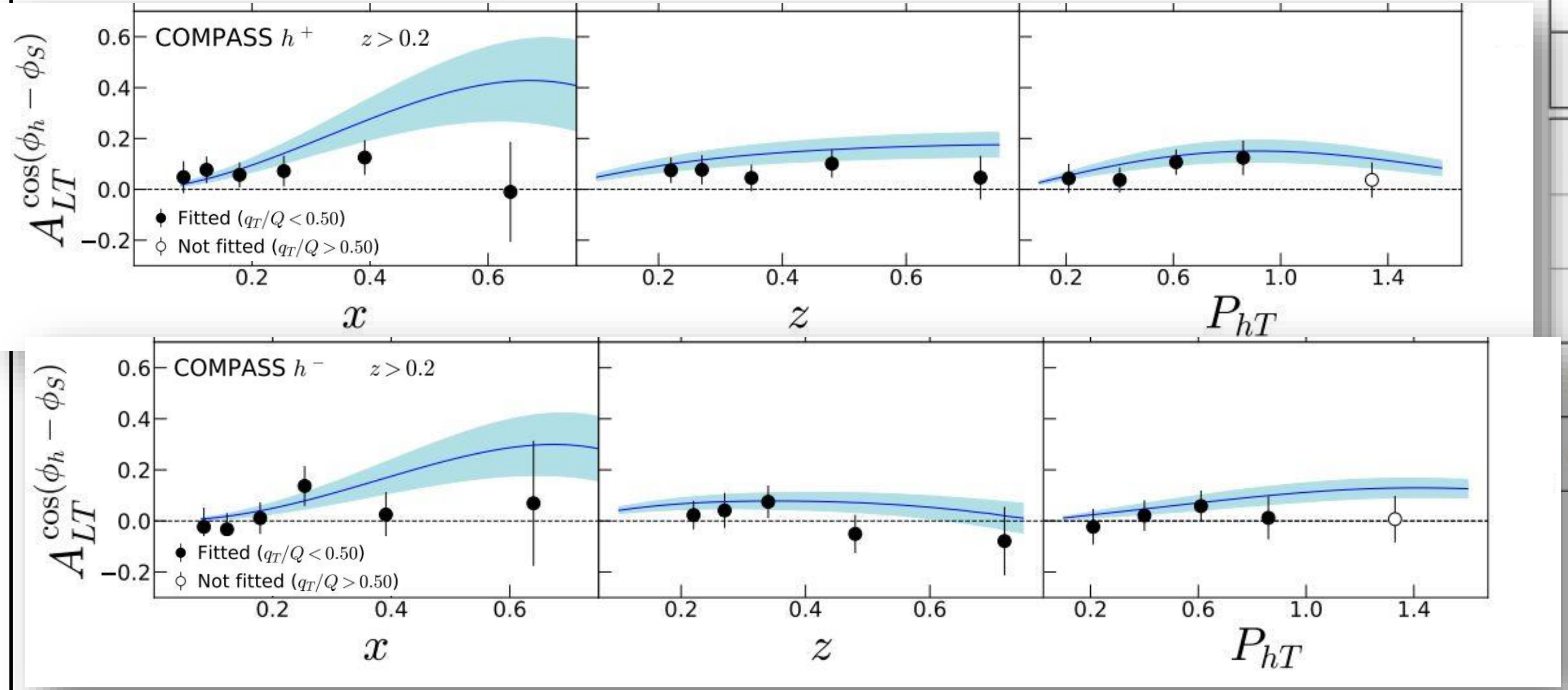


Data set	$\chi^2_w/N_{pts.}\|_{Main}$
HERMES π^+	1.20
HERMES π^-	0.88
HERMES π^0	1.94
COMPASS h^+	0.97

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data

Sample results



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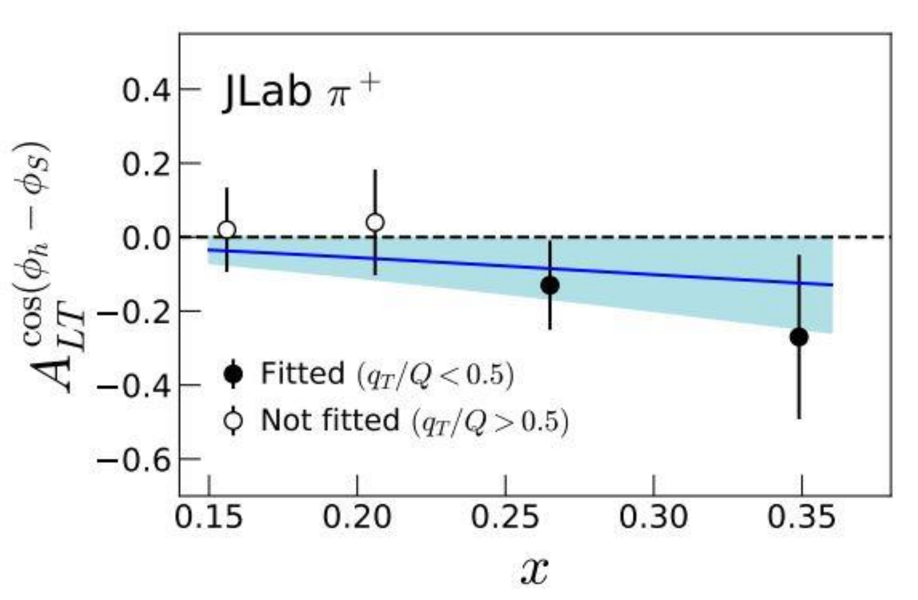
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data



Data set	$\chi^2_w/N_{pts.}\|_{Main}$
HERMES π^+	1.20
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Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

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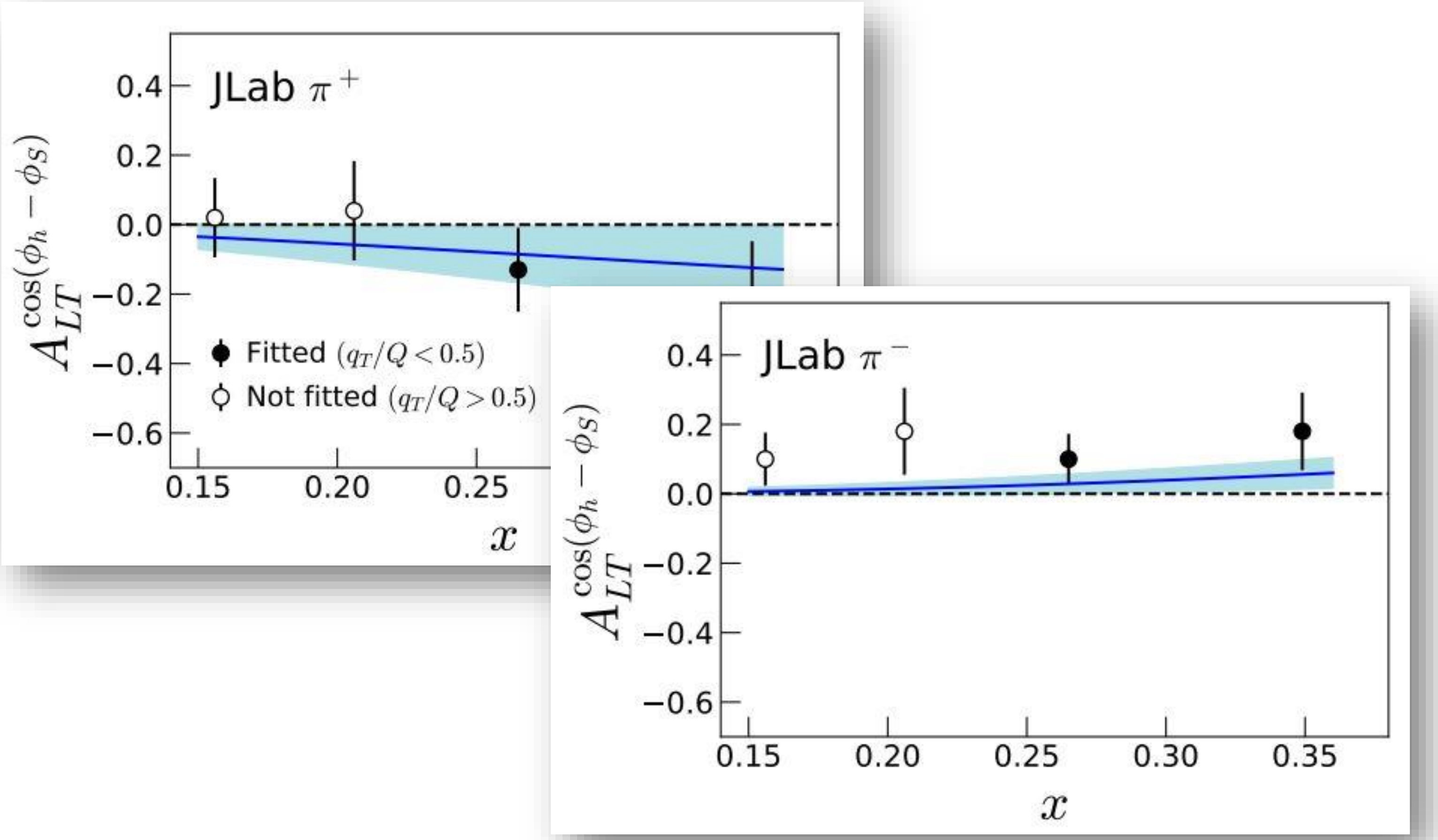

Data set	$\chi^2_w/N_{pts.}\vert_{Main}$
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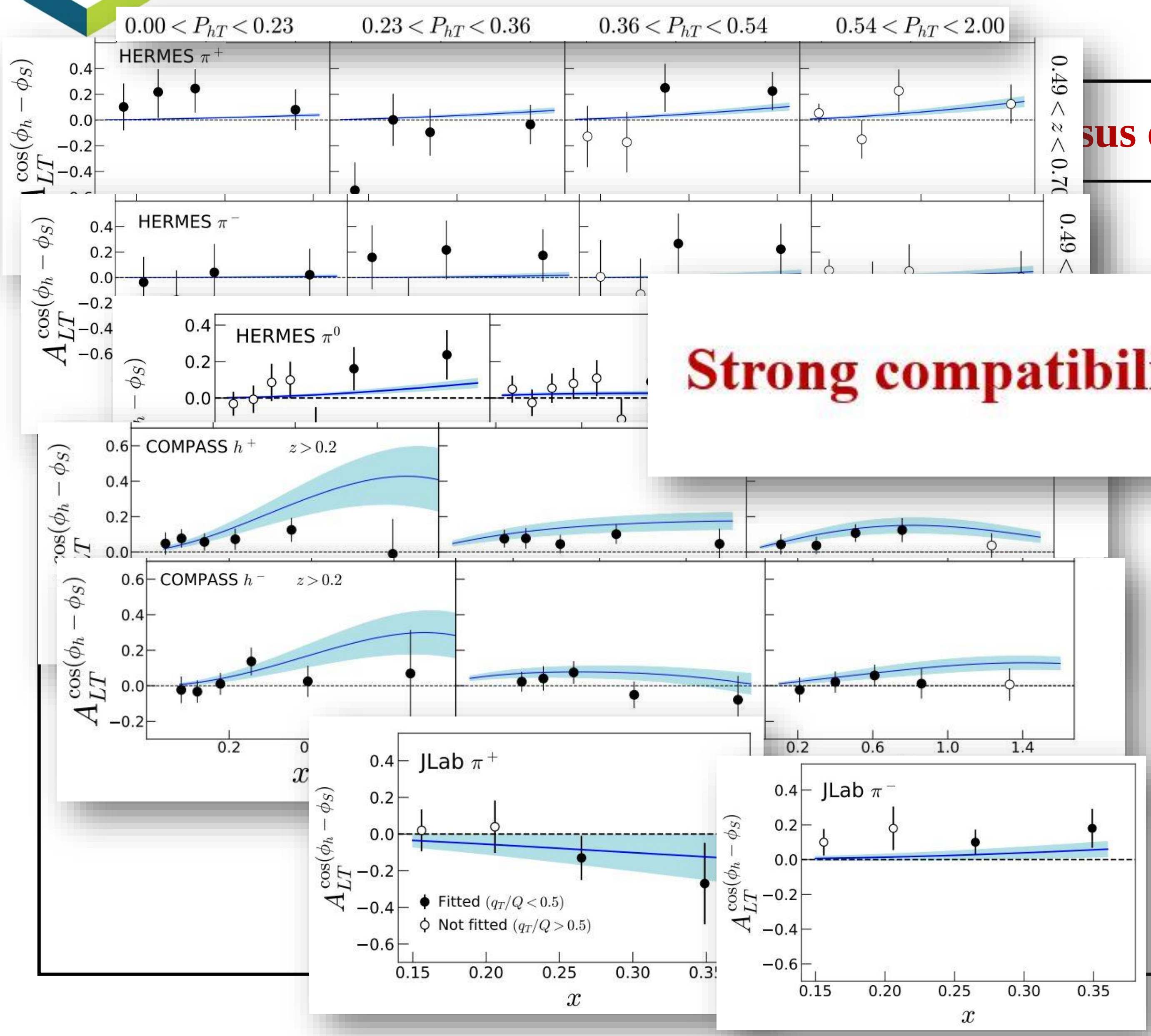
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data



sus data

Strong compatibility between our theory and data

HERMES π^-	0.88
HERMES π^0	1.94
COMPASS h^+	0.97
COMPASS h^-	0.71
JLab π^+	0.31
JLab π^-	1.13
Global	**0.86**

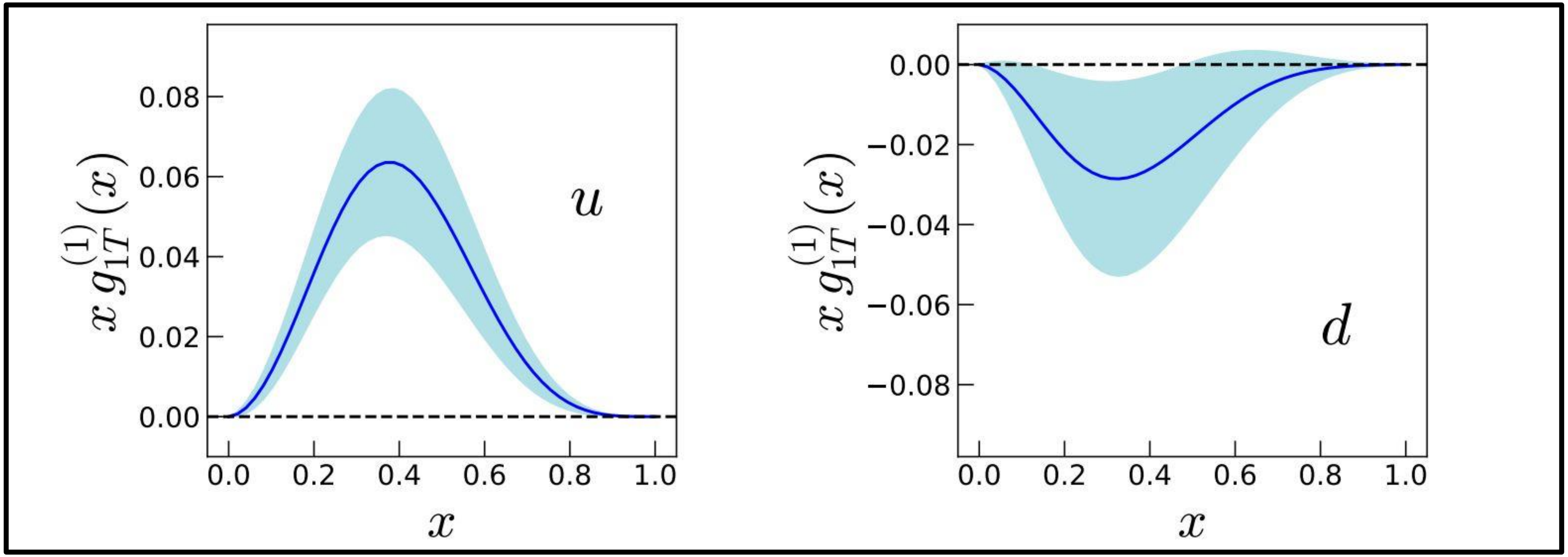
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Results for the x-dependence

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Average value of all replicas at a given x + 1-sigma error band

Results for the x-dependence

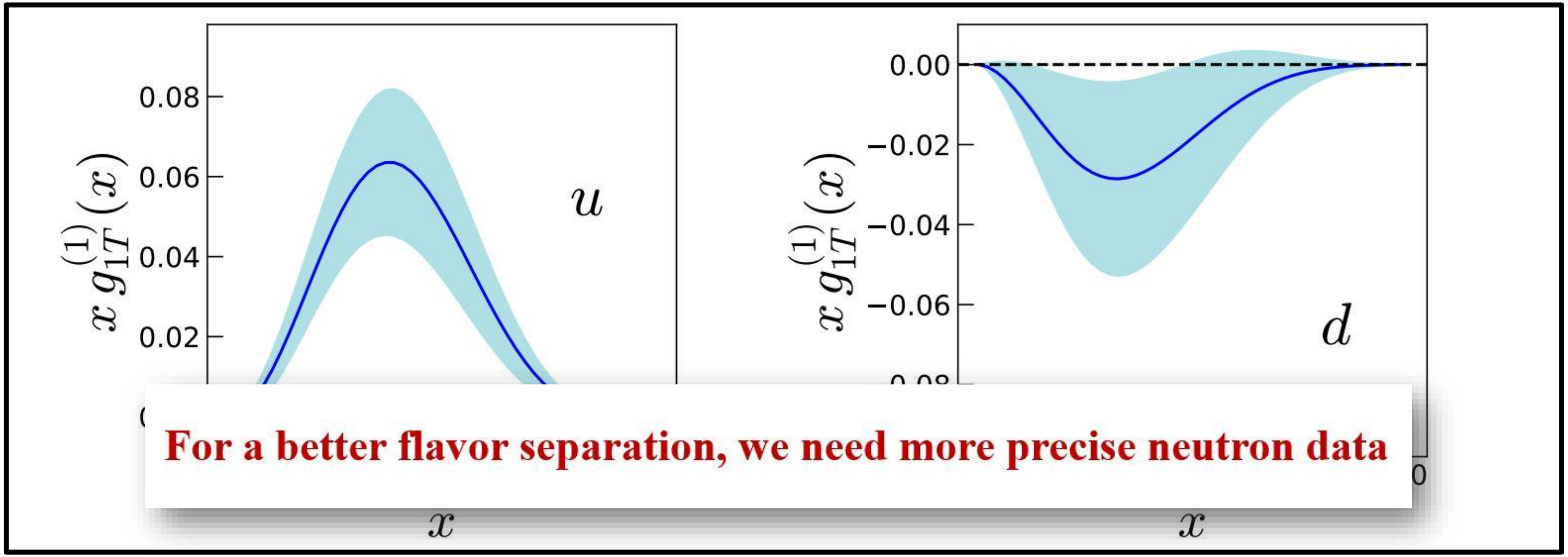


- **Up quark distribution is positive**
- **Down quark distribution is mostly negative**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Average value of all replicas at a given x + 1-sigma error band

Results for the x-dependence



For a better flavor separation, we need more precise neutron data

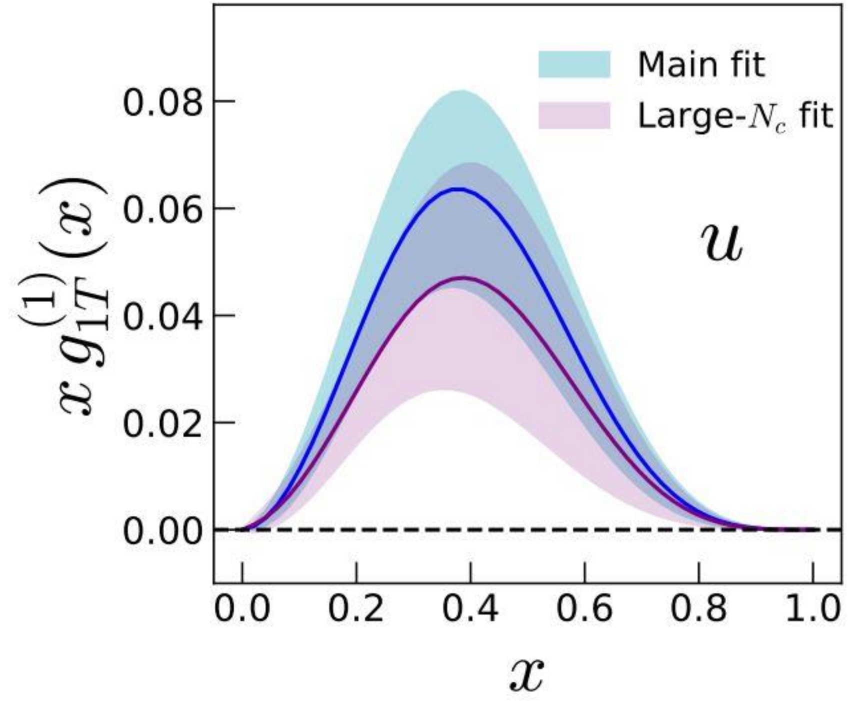
- **Up quark distribution is positive**
- **Down quark distribution is mostly negative**

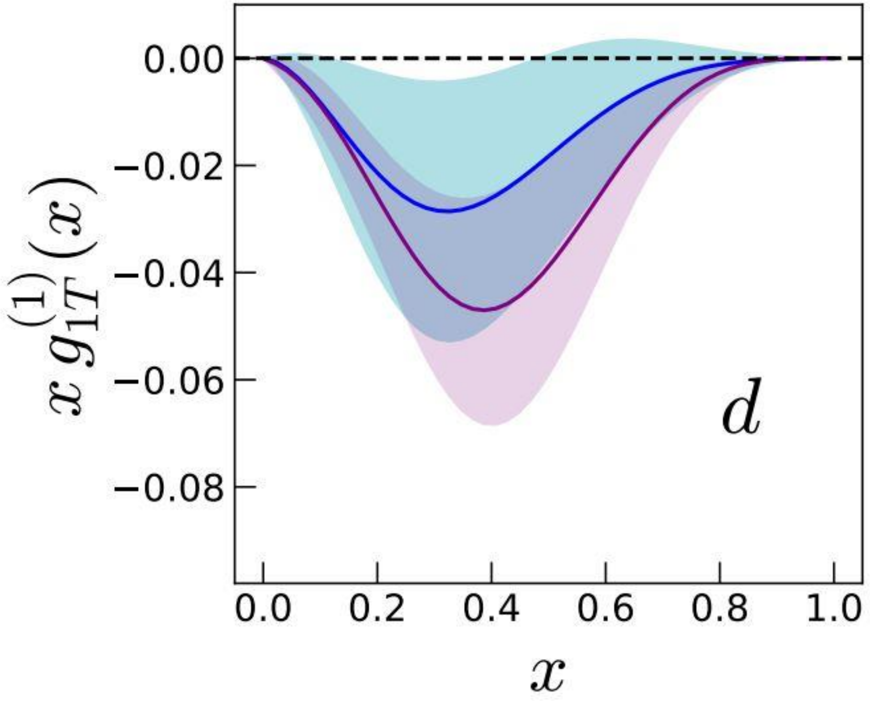
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Test of theoretical predictions

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Test of theoretical predictions



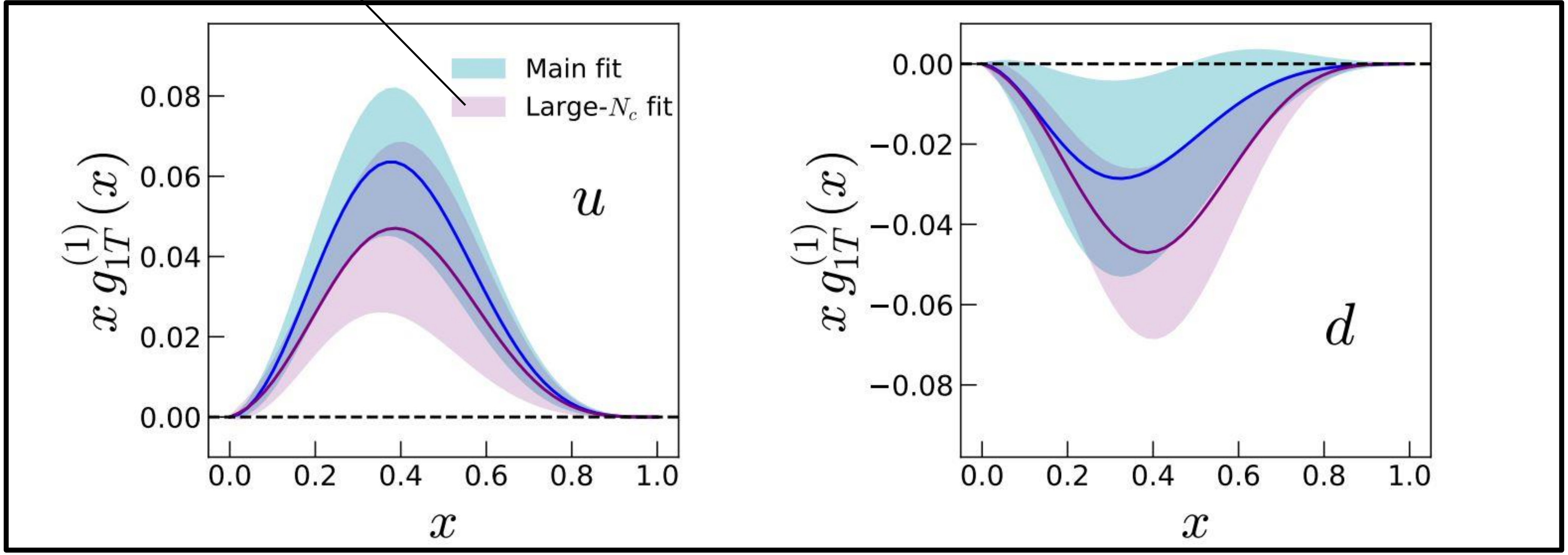


- **Qualitative agreement with large-$\mathbf{N_c}$ fit**
- **Slight preference to violate large-$\mathbf{N_c}$ approx.**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Set: $g_{1T}^{u}(x, \vec{k}_{\perp}^{2}) = -g_{1T}^{d}(x, \vec{k}_{\perp}^{2})$

Test of theoretical predictions



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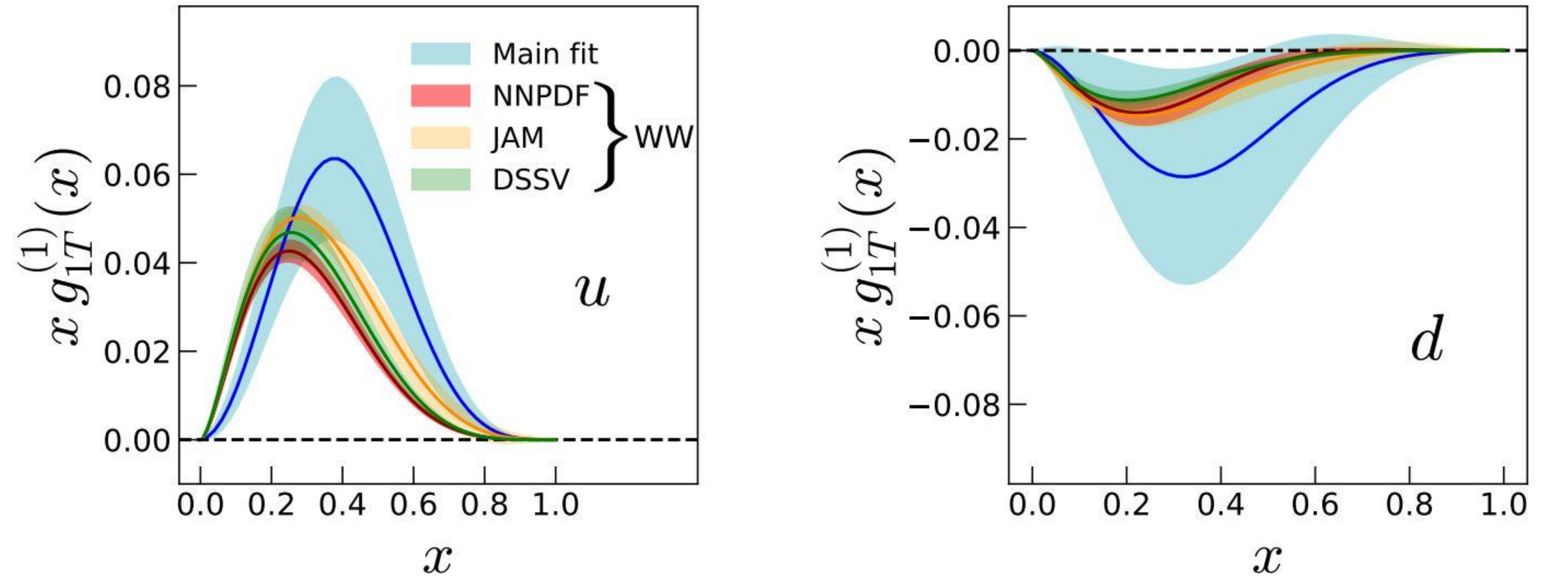
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Test of theoretical predictions

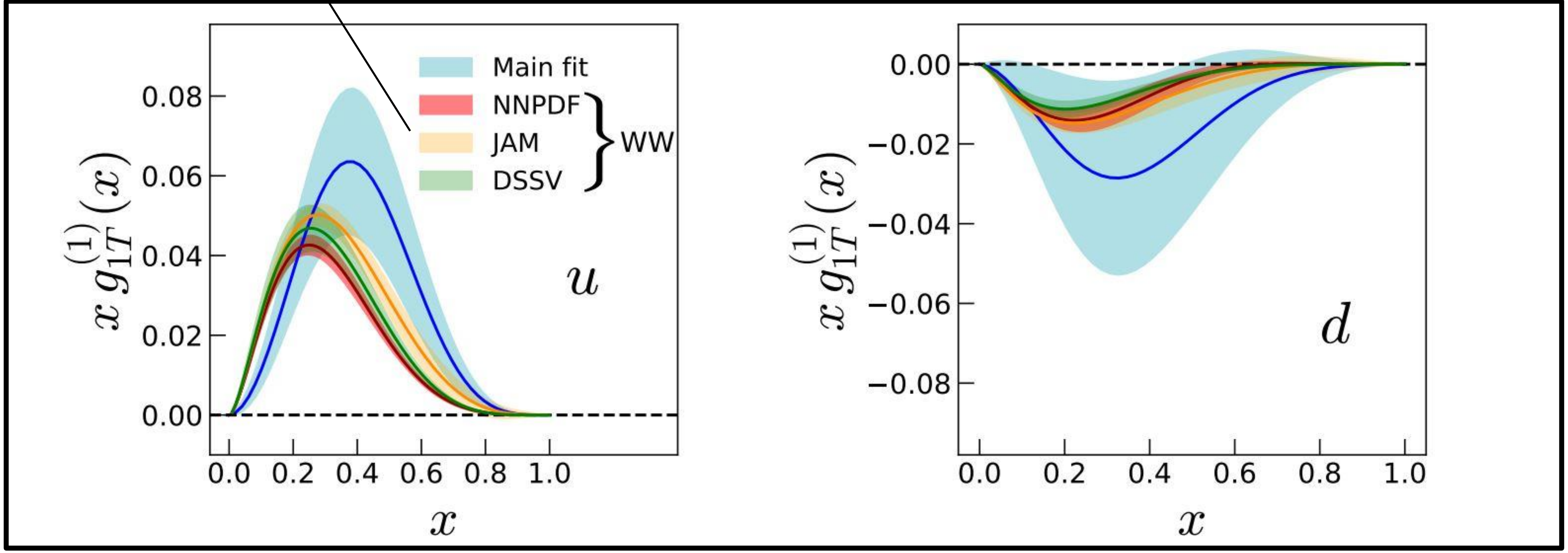


- **Qualitative agreement with WW-type approx.**
- **Hints of slight violation of WW-type approx.**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Set: $g_{1T}^{(1),q}(x) = x \int_x^1 dy \, \frac{g_1^q(y)}{y}$

Test of theoretical predictions



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Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Violation of existing theoretical predictions?

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Violation of existing theoretical predictions?

Summary of $\chi^2_w/N_{pts.}$					
Data set	$\chi^2_w/N_{pts.}\|_{Main}$	$\chi^2_w/N_{pts.}\|_{Large\text{-}N_c}$	$\chi^2_w/N_{pts.}\|_{NNPDF}$	$\chi^2_w/N_{pts.}\|_{JAM}$	$\chi^2_w/N_{pts.}\|_{DSSV}$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

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HERMES π^+	1.20	1.23			
HERMES π^-	0.88	0.88			
HERMES π^0	1.94	2.01			
COMPASS h^+	0.97	0.51			
COMPASS h^-	0.71	0.53			
JLab π^+	0.31	0.06			
JLab π^-	1.13	2.23			

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

cal predictions?

Similar or better chi-squared for some data sets

Summary of $\chi^2_w/N_{pts.}$					
Data set	$\chi^2_w/N_{pts.}\vert_{Main}$	$\chi^2_w/N_{pts.}\vert_{Large\text{-}N_c}$	$\chi^2_w/N_{pts.}\vert_{NNPDF}$	$\chi^2_w/N_{pts.}\vert_{JAM}$	$\chi^2_w/N_{pts.}\vert_{DSSV}$
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Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Violation of existing theoretical predictions?

Summary of $\chi^2_w/N_{\text{pts.}}$

Data set	$\chi^2_w/N_{\text{pts.}}\|_{\text{Main}}$	$\chi^2_w/N_{\text{pts.}}\|_{\text{Large-}N_c}$	$\chi^2_w/N_{\text{pts.}}\|_{\text{NNPDF}}$	$\chi^2_w/N_{\text{pts.}}\|_{\text{JAM}}$	$\chi^2_w/N_{\text{pts.}}\|_{\text{DSSV}}$
HERMES π^+	1.20		1.19	1.19	1.19
HERMES π^-	0.88		0.85	0.85	0.85
HERMES π^0	1.94		1.98	1.95	1.96
COMPASS h^+	0.97		0.71	1.02	0.89
COMPASS h^-	0.71		0.71	0.81	0.80
JLab π^+	0.31		0.81	0.78	0.96
JLab π^-	1.13		1.15	0.93	0.93

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

cal predictions?

Similar or better chi-squared for some data sets

Summary of $\chi^2_w/N_{\text{pts.}}$					
Data set	$\chi^2_w/N_{\text{pts.}}\vert_{\text{Main}}$	$\chi^2_w/N_{\text{pts.}}\vert_{\text{Large-}N_c}$	$\chi^2_w/N_{\text{pts.}}\vert_{\text{NNPDF}}$	$\chi^2_w/N_{\text{pts.}}\vert_{\text{JAM}}$	$\chi^2_w/N_{\text{pts.}}\vert_{\text{DSSV}}$
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Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Violation of existing theoretical predictions?

Summary of $\chi^2_w/N_{pts.}$					
Data set	$\chi^2_w/N_{pts.}\|_{Main}$	$\chi^2_w/N_{pts.}\|_{Large\text{-}N_c}$	$\chi^2_w/N_{pts.}\|_{NNPDF}$	$\chi^2_w/N_{pts.}\|_{JAM}$	$\chi^2_w/N_{pts.}\|_{DSSV}$
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HERMES π^-	0.88	0.88	0.85	0.85	0.85
HERMES π^0	1.94	2.01	1.98	1.95	1.96
COMPASS h^+	0.97	0.51	0.71	1.02	0.89
COMPASS h^-	0.71	0.53	0.71	0.81	0.80
JLab π^+	0.31	0.06	0.81	0.78	0.96
JLab π^-	1.13	2.23	1.15	0.93	0.93

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

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Global	**0.86**	**0.99**	**0.95**	**0.94**	**0.97**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Violation of existing theoretical predictions?

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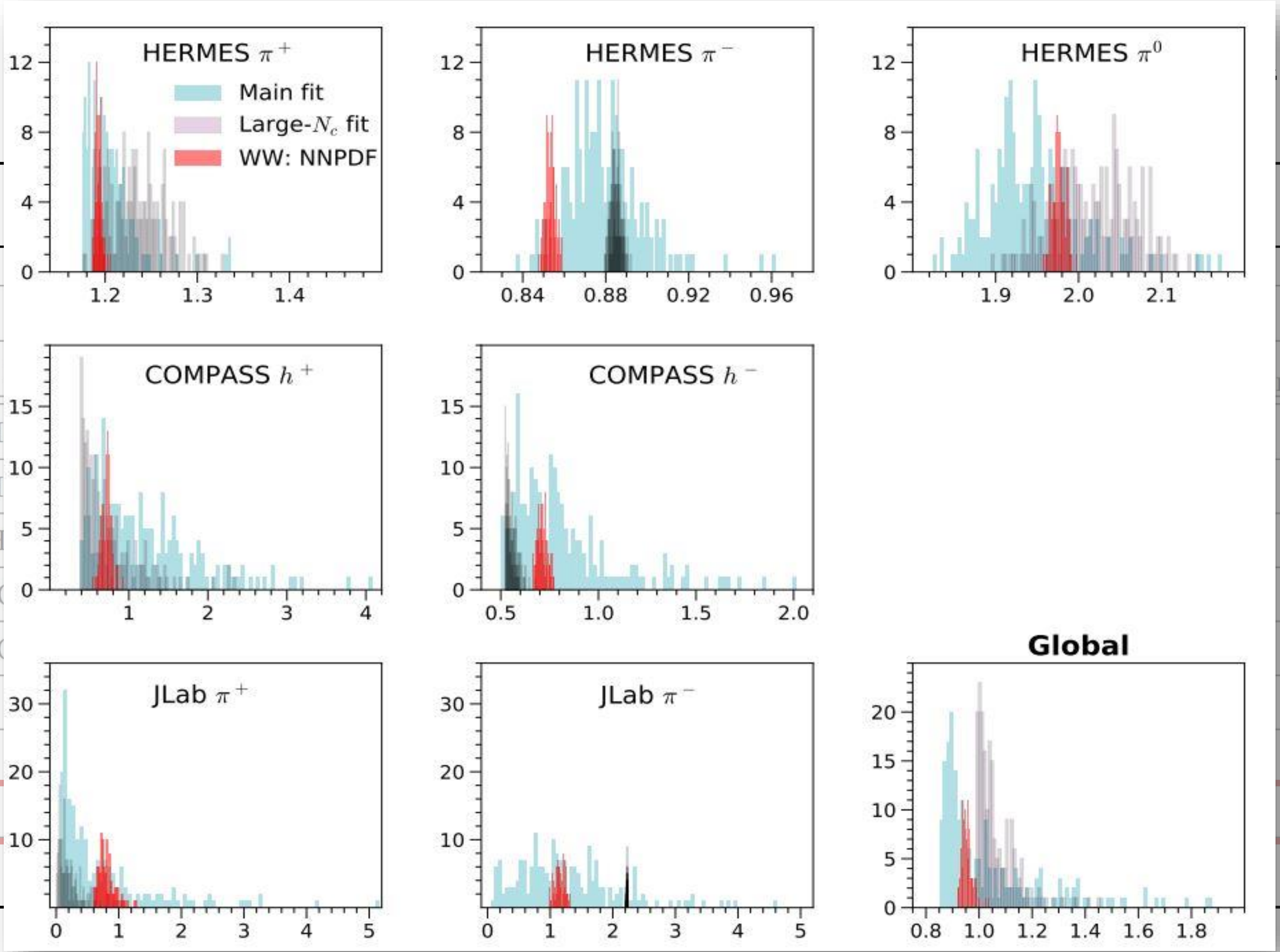
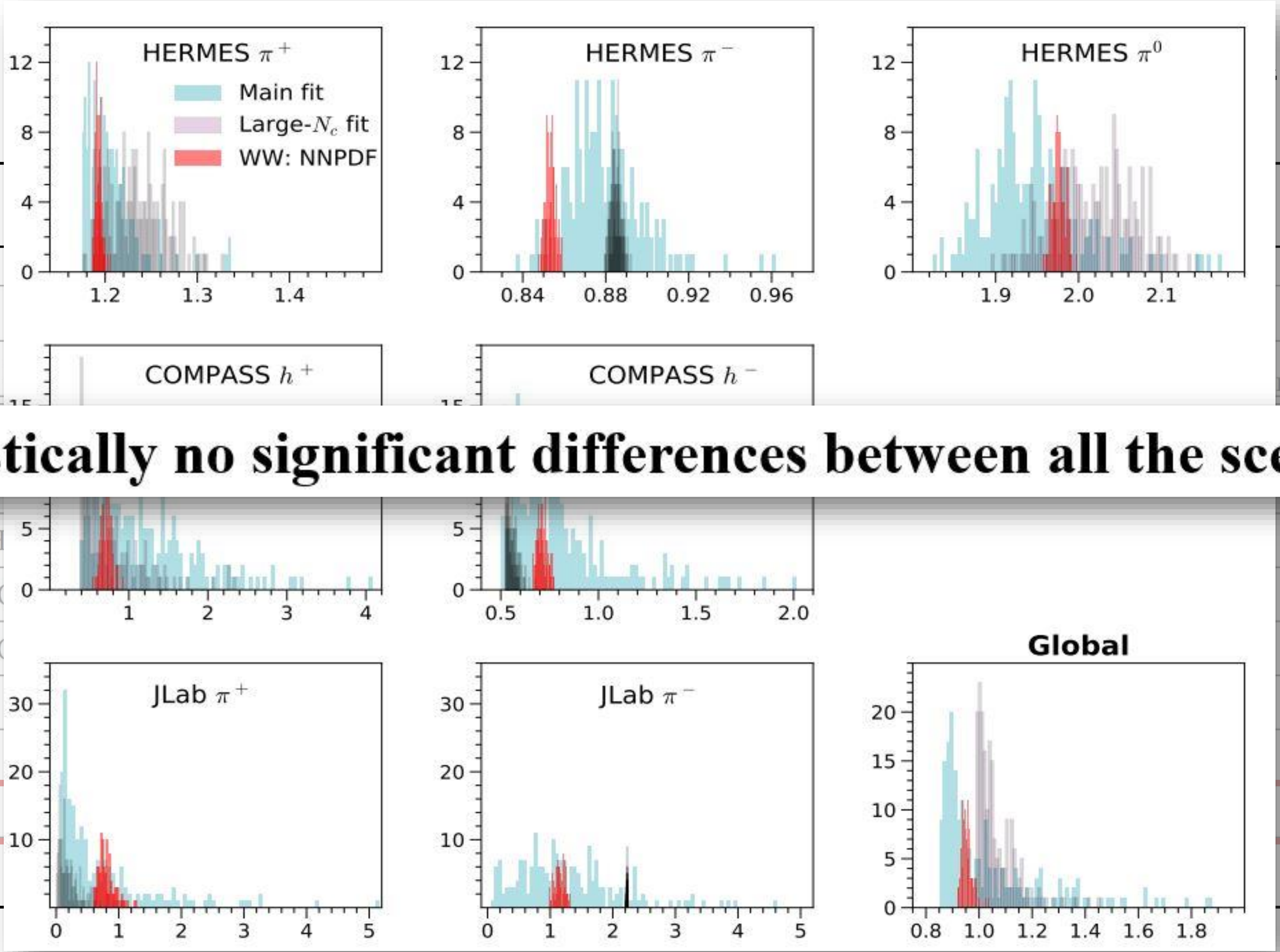
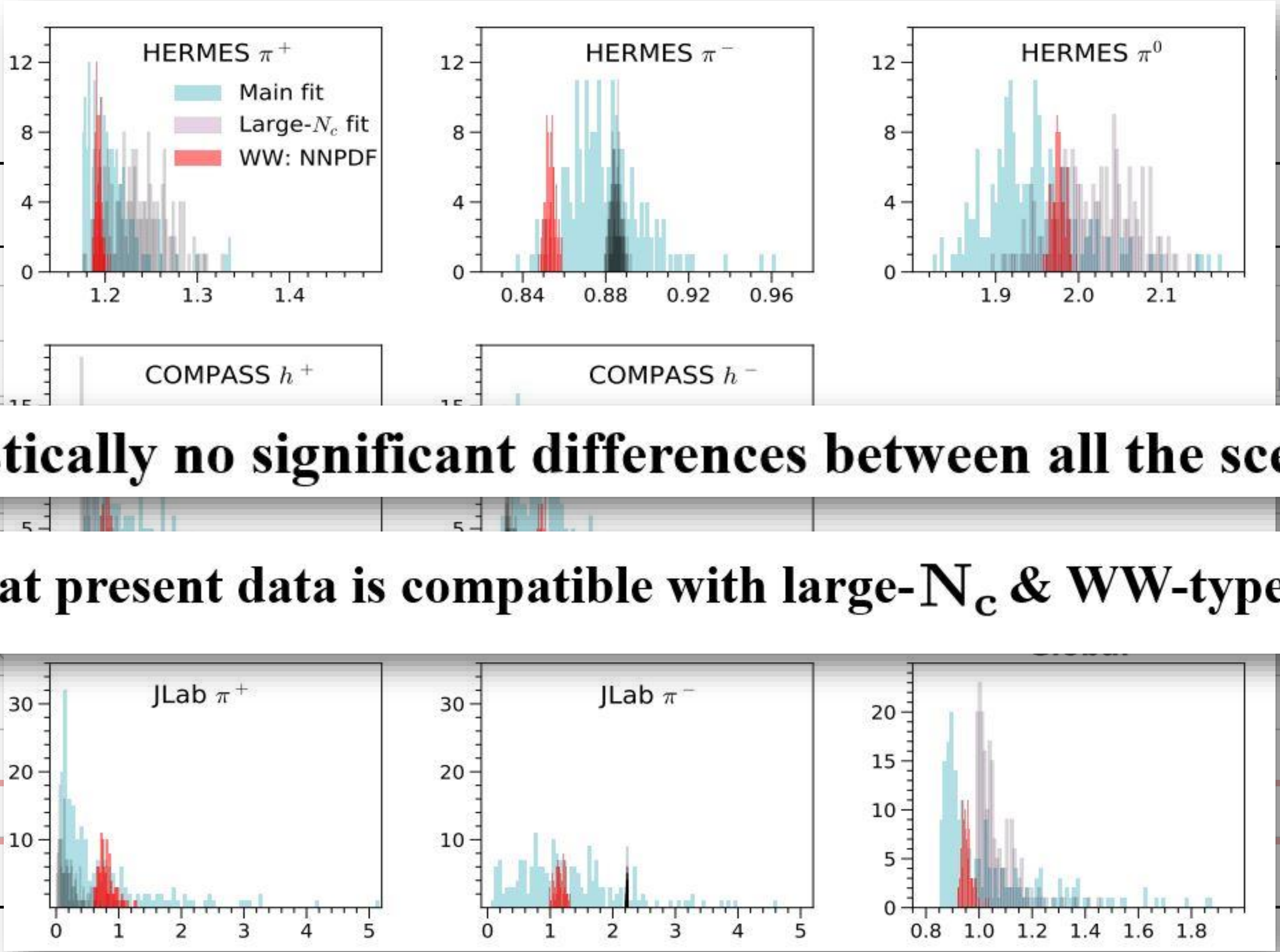
Global	**0.86**	**0.99**	**0.95**	**0.94**	**0.97**

Our global chi-squared is consistently better


HERMES π^+
Main fit
Large-N_c fit
WW: NNPDF
HERMES π^-
HERMES π^0
COMPASS h^+
COMPASS h^-
Global
JLab π^+
JLab π^-

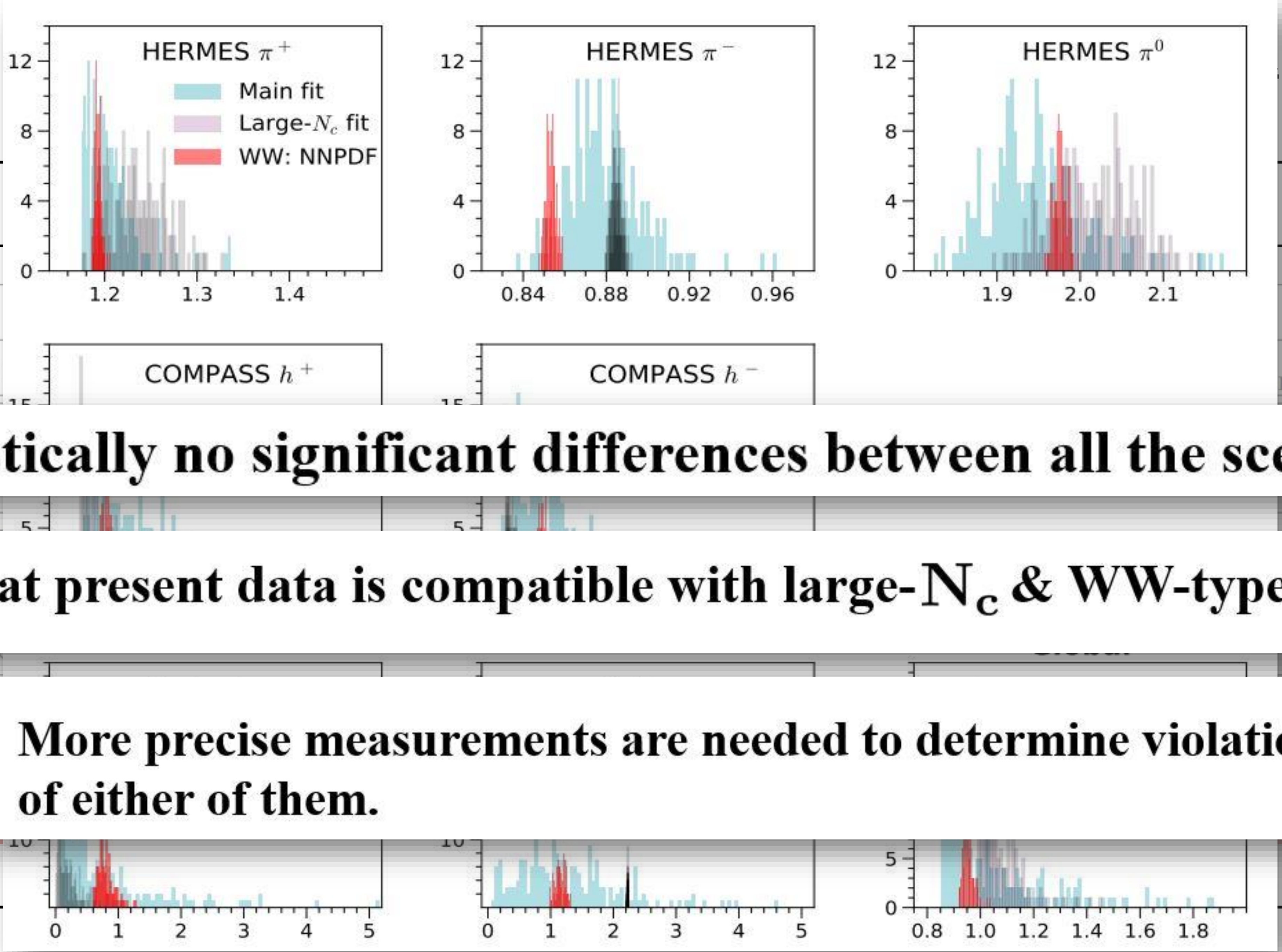


Statistically no significant differences between all the scenarios



Statistically no significant differences between all the scenarios

Hence, at present data is compatible with large-N_c & WW-type approx.



Statistically no significant differences between all the scenarios

Hence, at present data is compatible with large-N_c & WW-type approx.

More precise measurements are needed to determine violation of either of them.

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Comparison with lattice QCD results

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

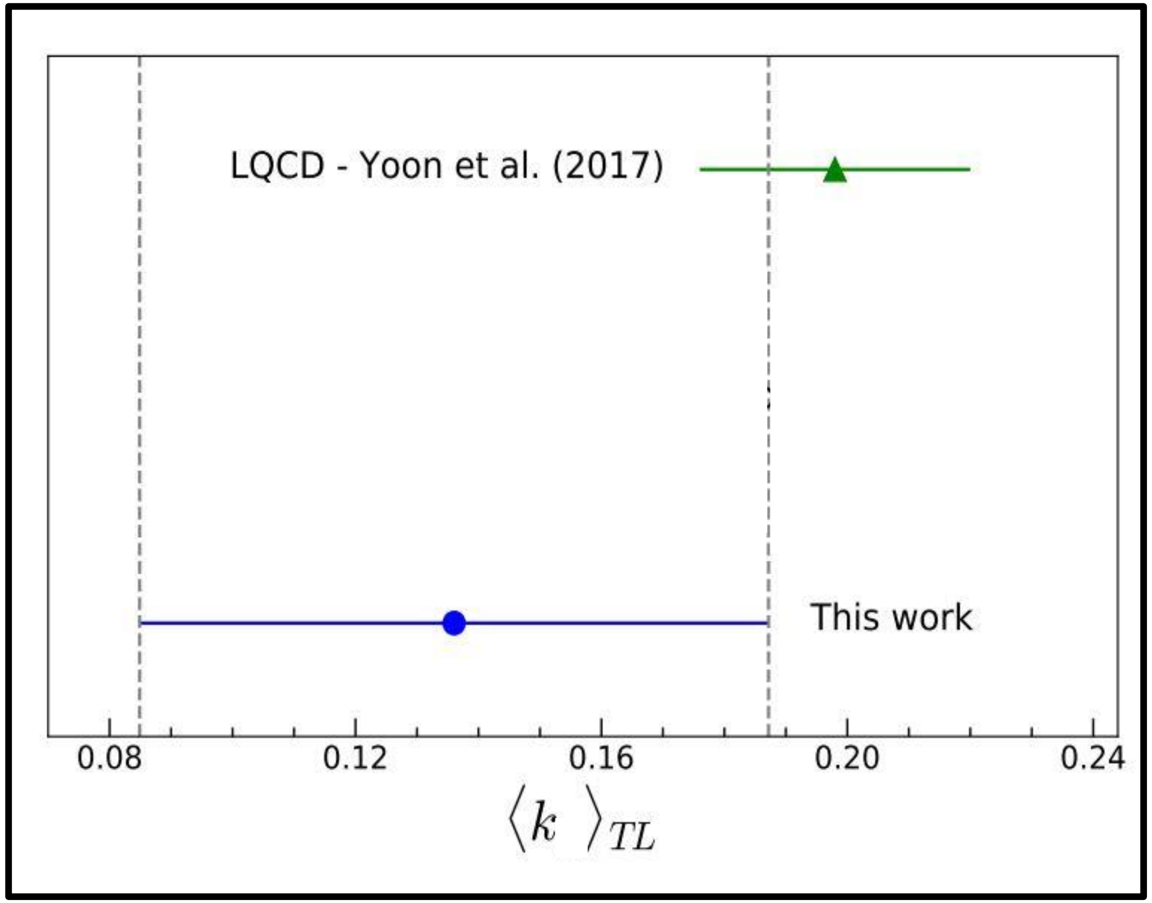
Comparison with lattice QCD results

Calculation of worm-gear shift:

$$[\langle k \quad \rangle_{TL}](Q^2) \equiv M \frac{\int_0^1 dx \left[g_{1T}^{(1)u}(x, Q^2) - g_{1T}^{(1)d}(x, Q^2) \right]}{\int_0^1 dx \left[f_1^u(x, Q^2) - f_1^d(x, Q^2) \right]}$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Comparison with lattice QCD results



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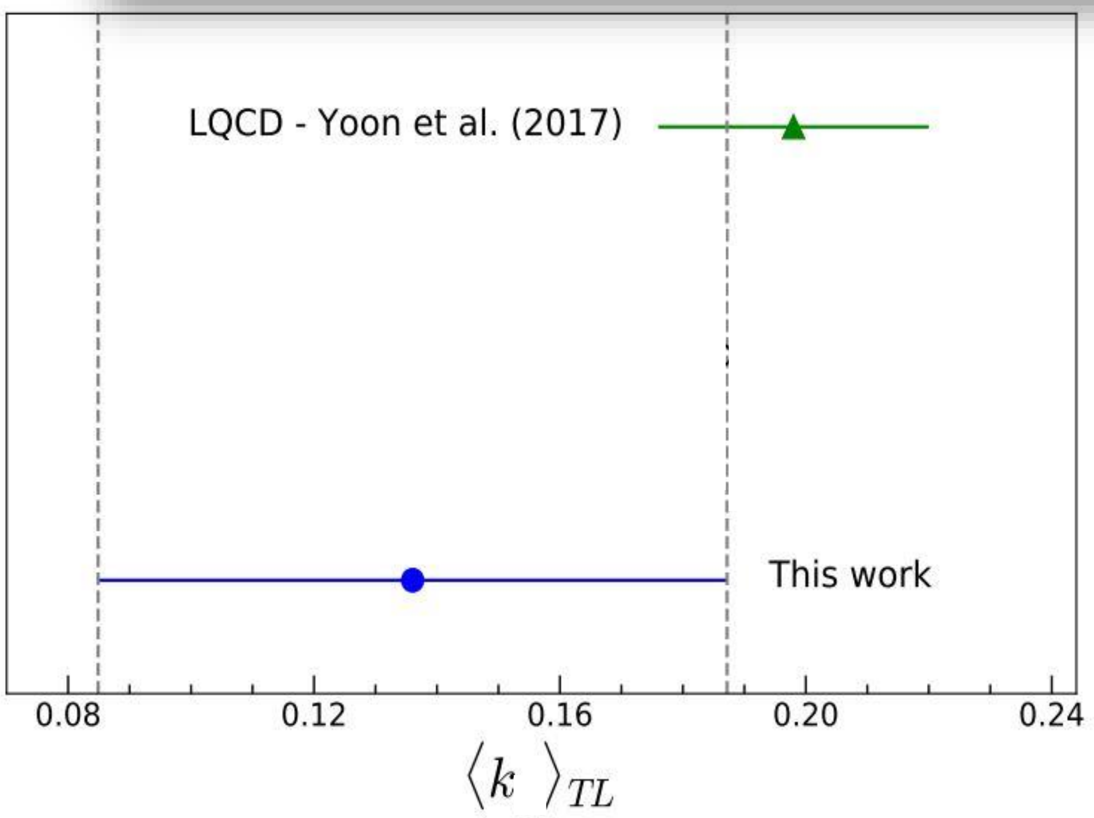
- **Consistency between lattice results & our main fit result**

MES, COMPASS & JLab data

Caveats in LQCD calculations:

- **No definite scale:** $Q \approx \frac{1}{a}$
- **Limits** $\hat{\zeta} \to \infty$ **&** $b_T \to 0$ **cannot be taken**

ce QCD results



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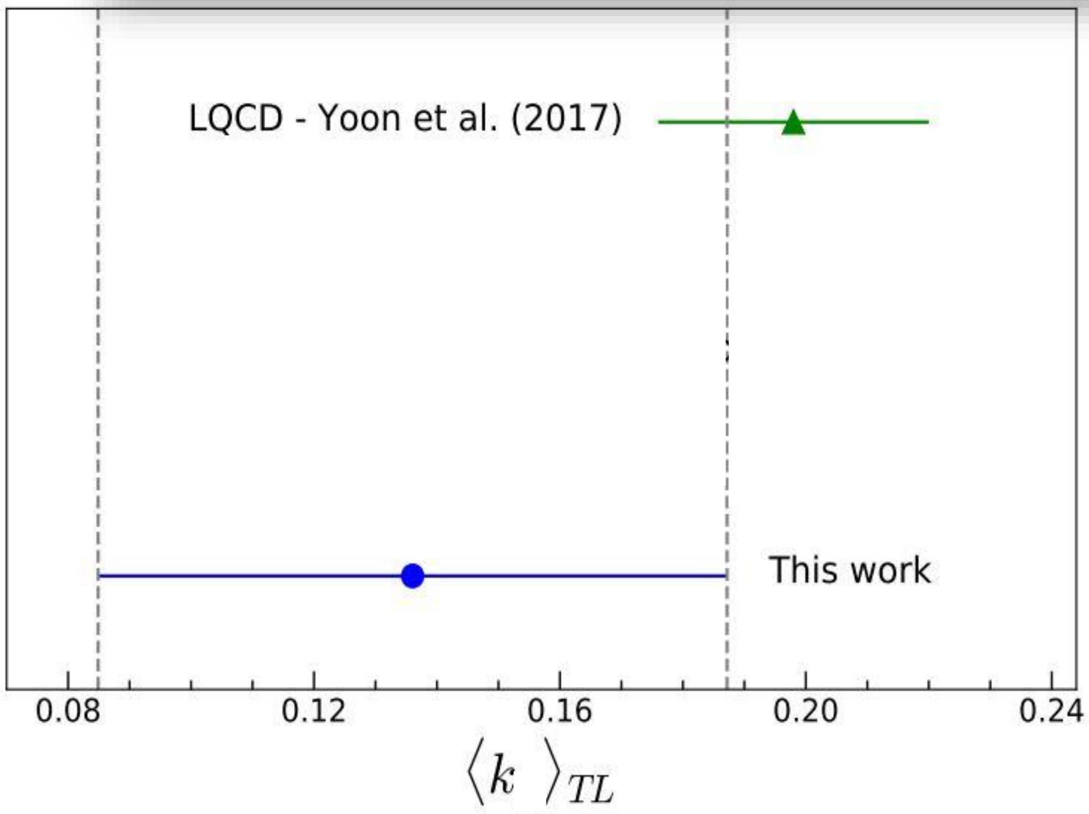

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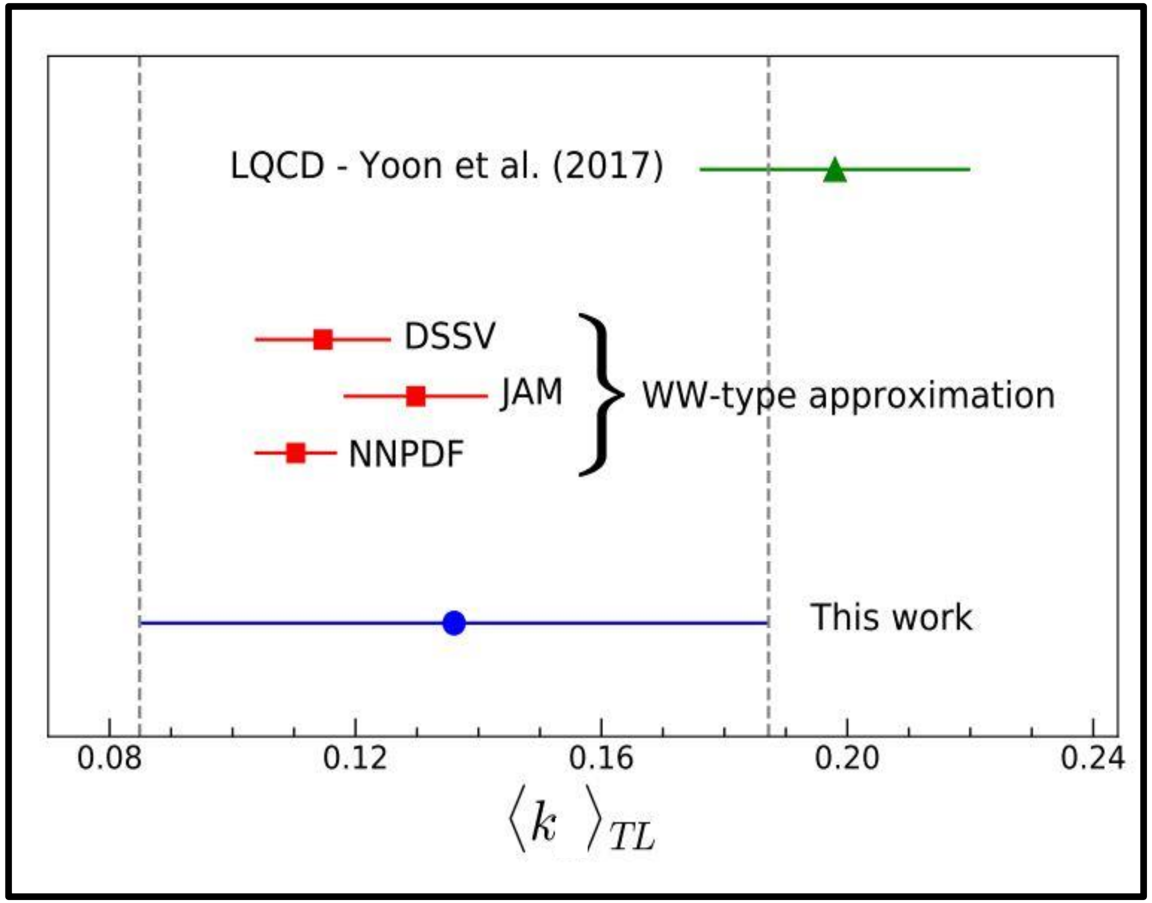
- **It is encouraging that lattice QCD & exp. data are in reasonable agreement**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Comparison with lattice QCD results



Calculation of worm-gear shift:

$$[\langle k \rangle_{TL}](Q^2) \equiv M \frac{\int_0^1 dx \left[g_{1T}^{(1)u}(x,Q^2) - g_{1T}^{(1)d}(x,Q^2)\right]}{\int_0^1 dx \left[f_1^u(x,Q^2) - f_1^d(x,Q^2)\right]}$$

- **Consistency between lattice results & our main fit result**
- **It is encouraging that lattice QCD & exp. data are in reasonable agreement**
- **Consistency between results from WW-type approx. & our main fit result**

Summary/Outlook

Summary

- **We have performed the first extraction of g_{1T} from experimental data, obtaining a very good fit simultaneously of HERMES, COMPASS, JLab data on SIDIS**
- **Additional deuteron and/or neutron measurements are needed for a cleaner flavor separation**
- **Qualitative agreements with large-N_c & WW-type approximation**
- **Although there is an indication of a slight violation of both the theoretical predictions, the data is not precise enough to affirm the degree of violation (if any)**
- **Any clear signal of violation of WW-type approximation would be a probe of quark-gluon-quark correlations**
- **Encouraging agreement in the worm-gear shift with lattice QCD results**

Summary/Outlook

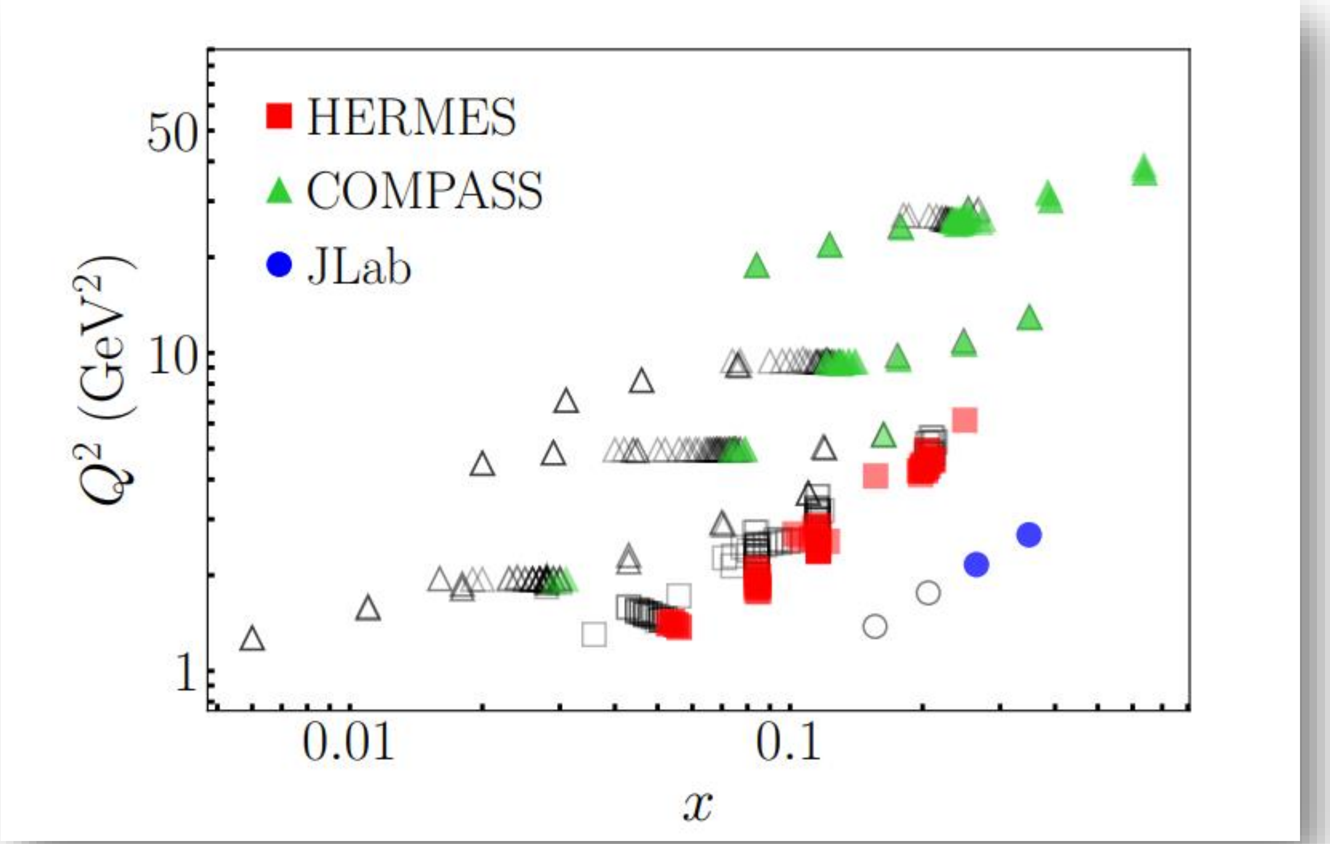
Outlook

Extend analysis to extract $h_{1L}^{\perp}$

…

Backup slides

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data



Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Why "weighted" technique after-all?

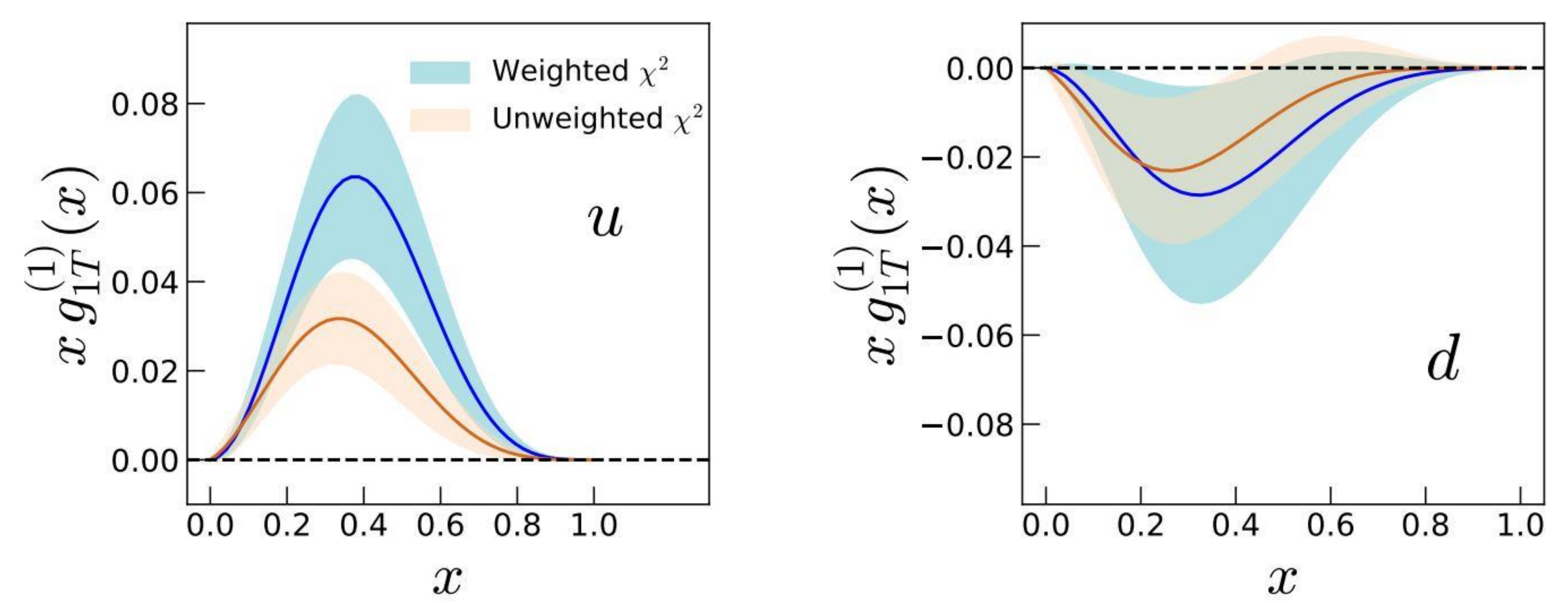
$$\chi^2 = \sum_{\text{H+C+J}} \frac{(\text{exp. data} - \text{theory})^2}{(\text{exp. error})^2}$$

Without weighting the chi-squared, we overfit COMPASS data, and we don't fit JLab π^- data particularly well.

HERMES π^-	0.89	0.88	0.85	0.85	0.85
HERMES π^0	2.03	2.03	1.98	1.95	1.96
COMPASS h^+	0.39	0.40	0.71	1.02	0.89
COMPASS h^-	0.54	0.53	0.71	0.81	0.80
JLab π^+	0.42	0.15	0.81	0.78	0.96
JLab π^-	1.88	2.23	1.15	0.93	0.93
Global	**0.83**	**0.83**	**0.93**	**1.02**	**0.99**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

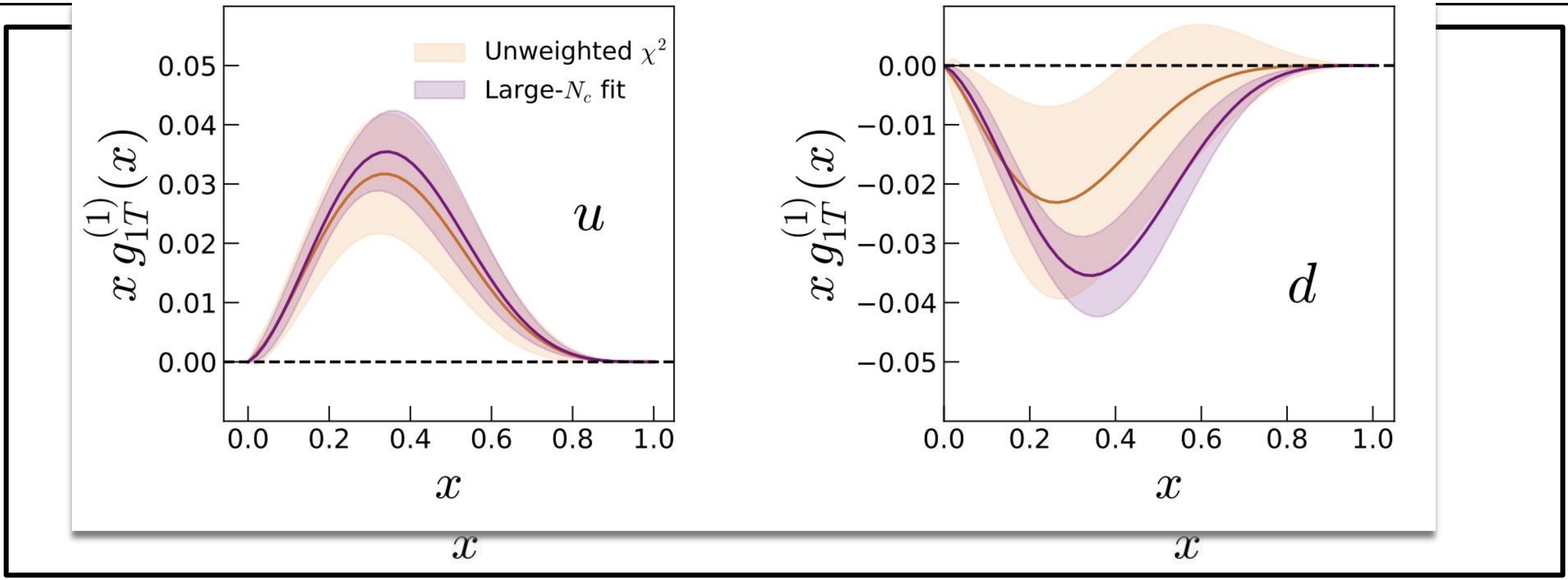
Weighted versus unweighted methods



- **Larger up quark distribution needed to describe JLab π^- data**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Weighted versus unweighted methods



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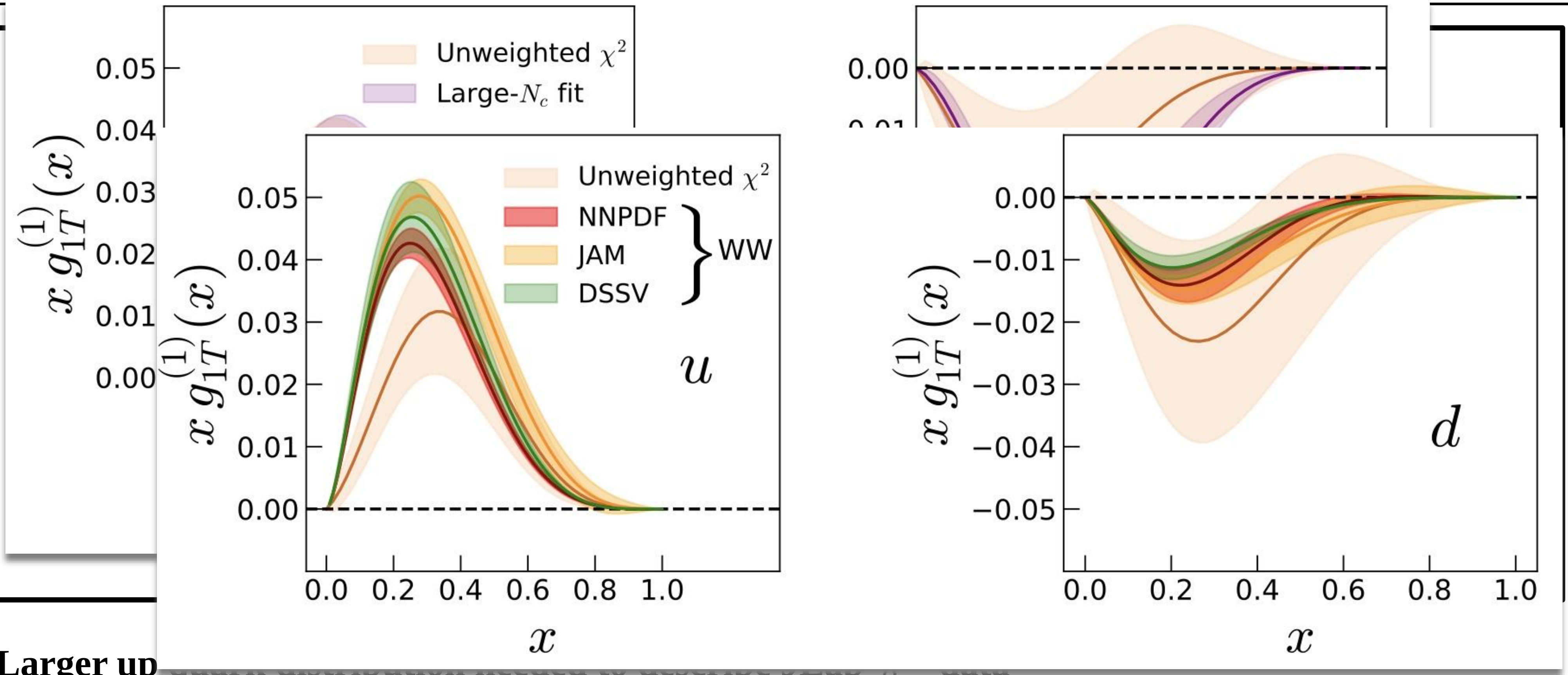
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Weighted versus unweighted methods



- **Larger up**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Remarks about evolution

- **Given the precision and range in Q of the data, a rigorous implementation of TMD evolution not needed**
- **The quantity** $g_{1T}^{(1)}(x)$ **evolves according to** $f_1(x)$
- **Actual evolution of** $g_{1T}^{(1)}(x)$ **follows a more complicated pattern: (Zhou, Yuan, Liang (2008))**

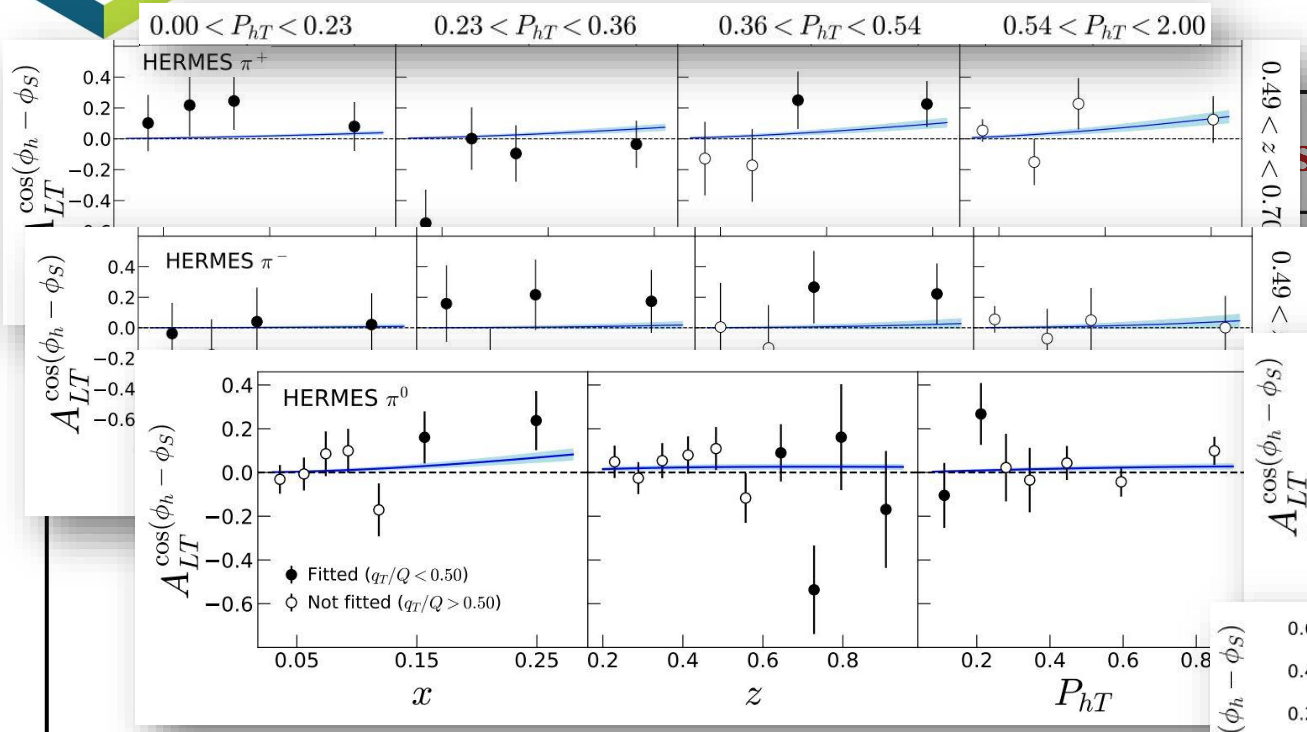
DGLAP kernel

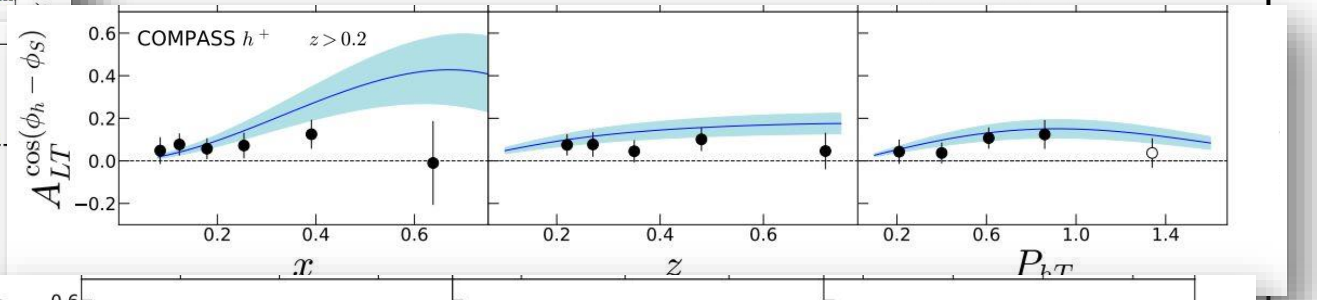
$$\frac{\partial \tilde{g}(x_B,\mu^2)}{\partial \ln\mu^2} = \frac{\alpha_s}{2\pi}\int\frac{dxdy}{x}\left\{\tilde{g}(x)\delta(y-x)\left[C_F\left(\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right)-\frac{C_A}{2}\frac{1+z^2}{1-z}\right]\right.$$
$$+\tilde{G}_D(x,y)\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{y}-\frac{2x_B^2}{xy}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\frac{(x_B^2+xy)(2x_B-x-y)}{(x_B-y)(x-y)y}\right]$$
$$\left.+G_D(x,y)\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{y}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\frac{x_B^2-xy}{(y-x_B)y}\right]\right\}$$

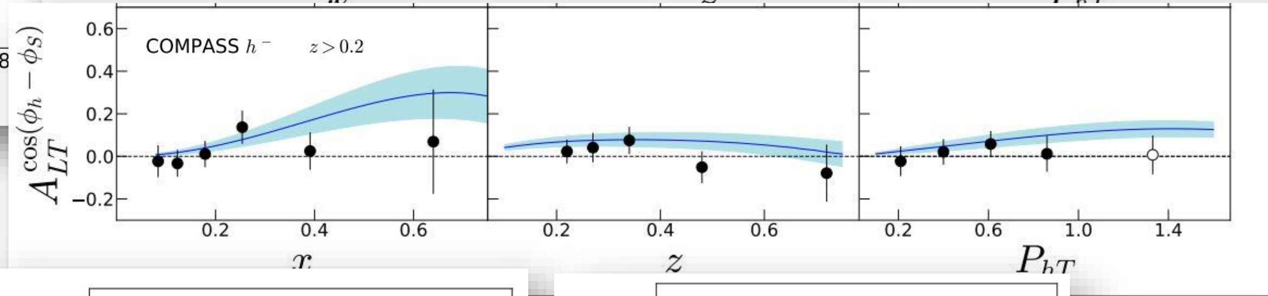
$$g_{1T}^{(1)}(x) \equiv \tilde{g}(x)$$

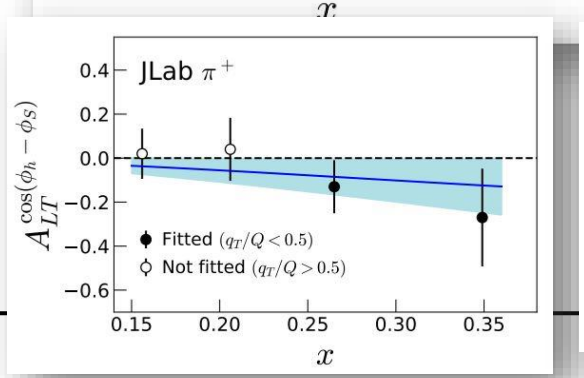
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

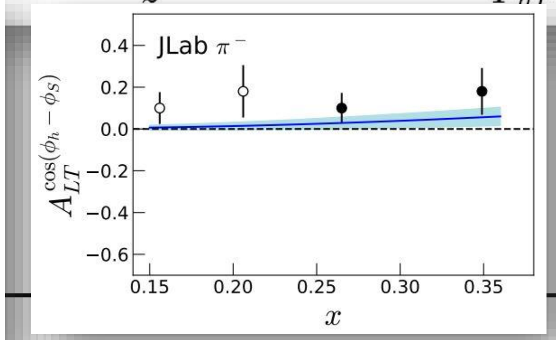
sus data



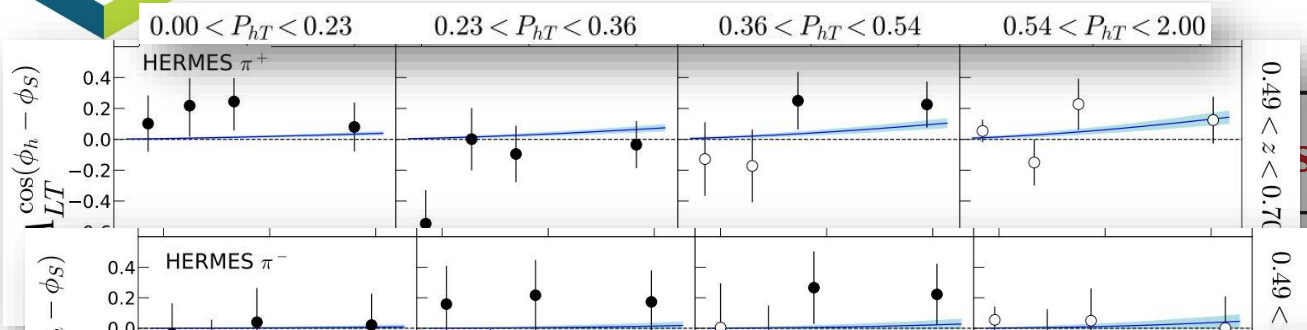


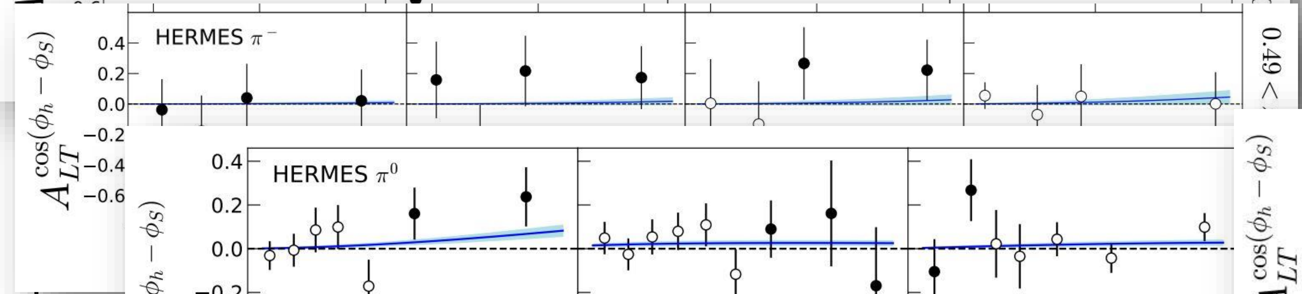


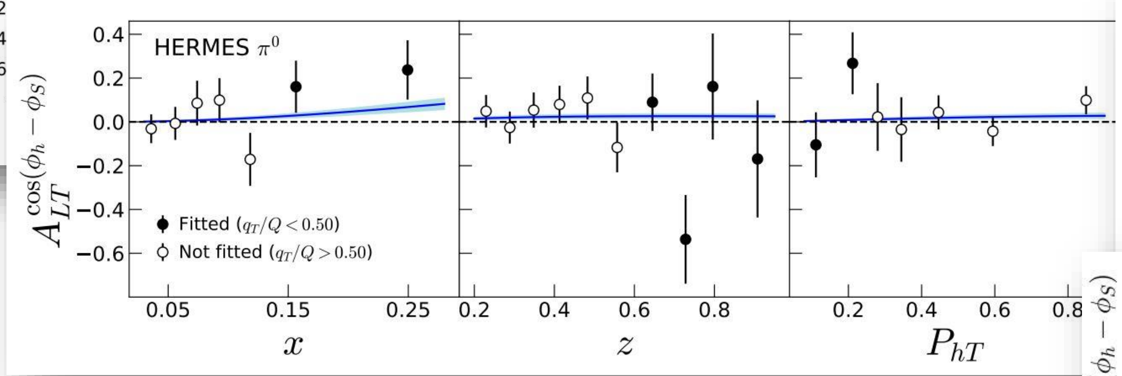




Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data







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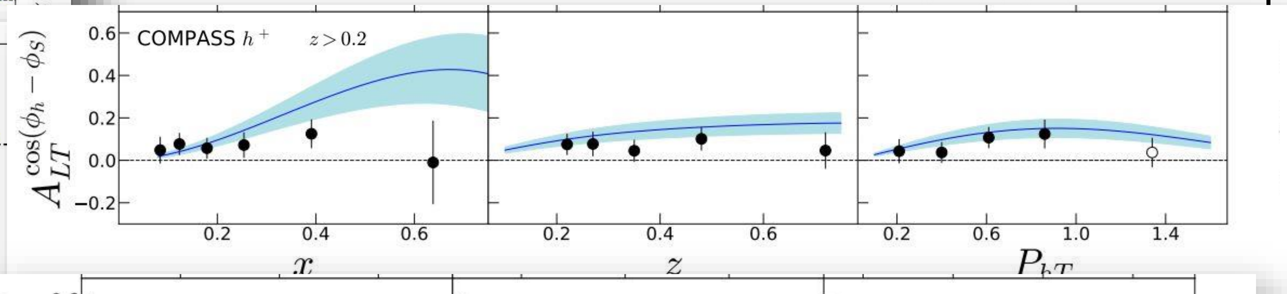
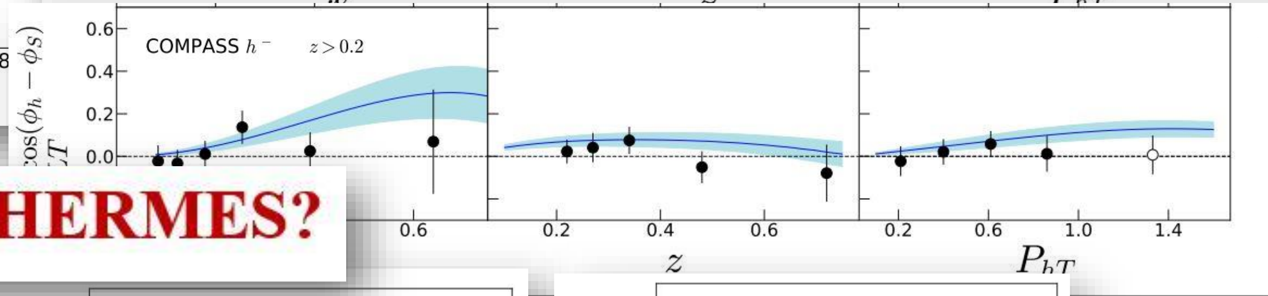
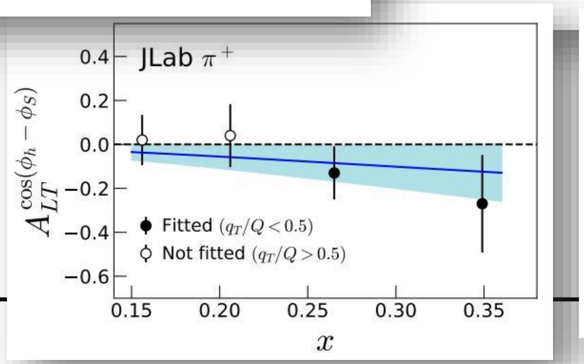
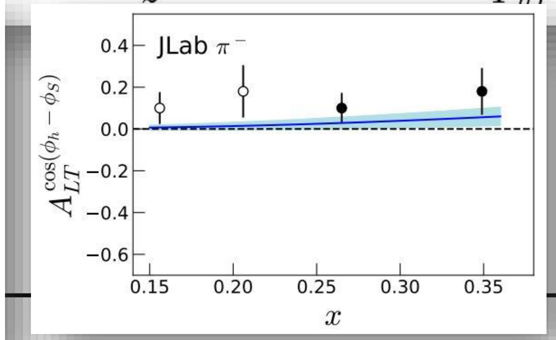




Smallness of the uncertainty bands for HERMES?





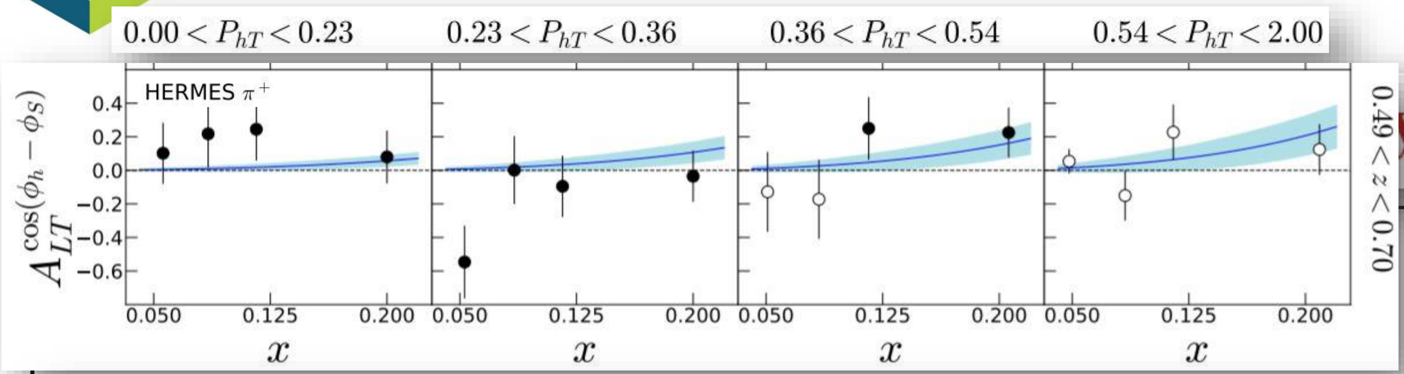
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

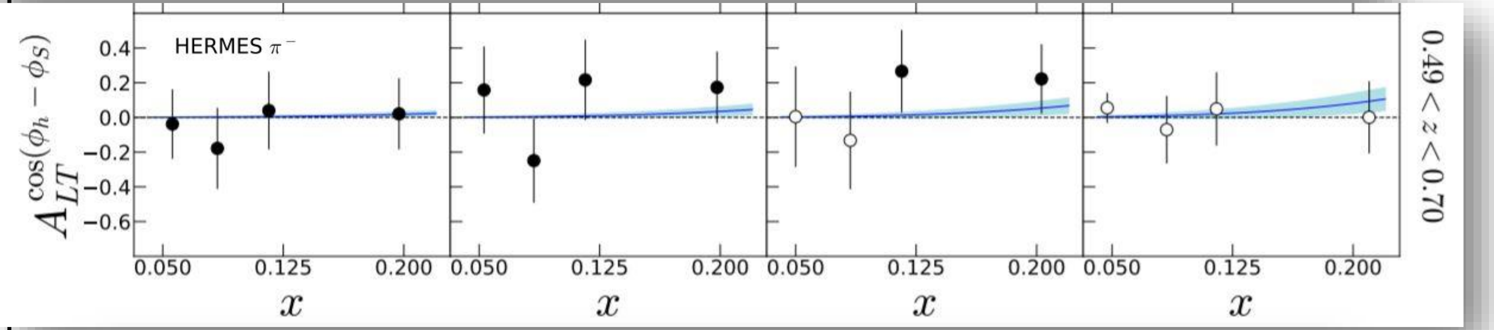
Smallness of the uncertainty bands for HERMES?

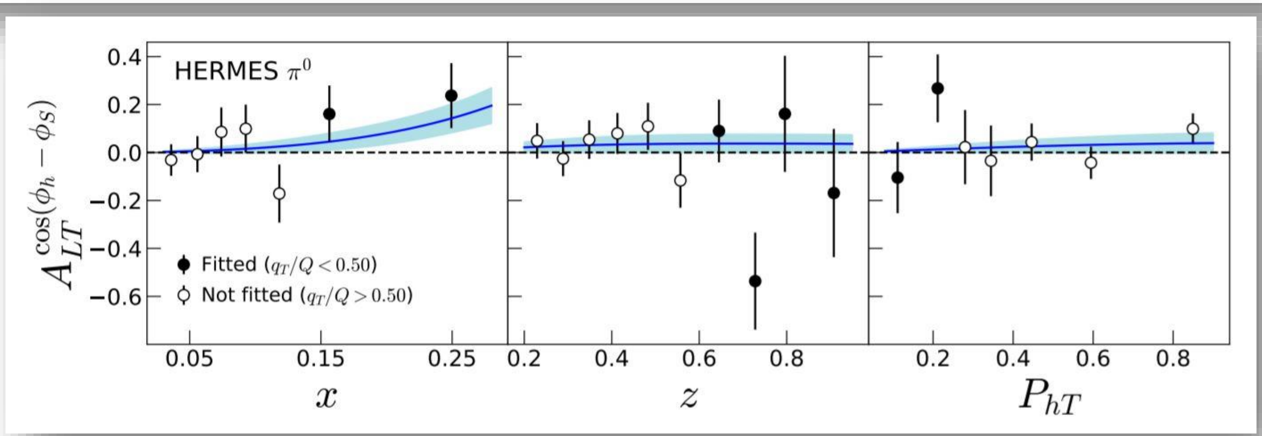
- **Performed a fit of HERMES + JLab**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

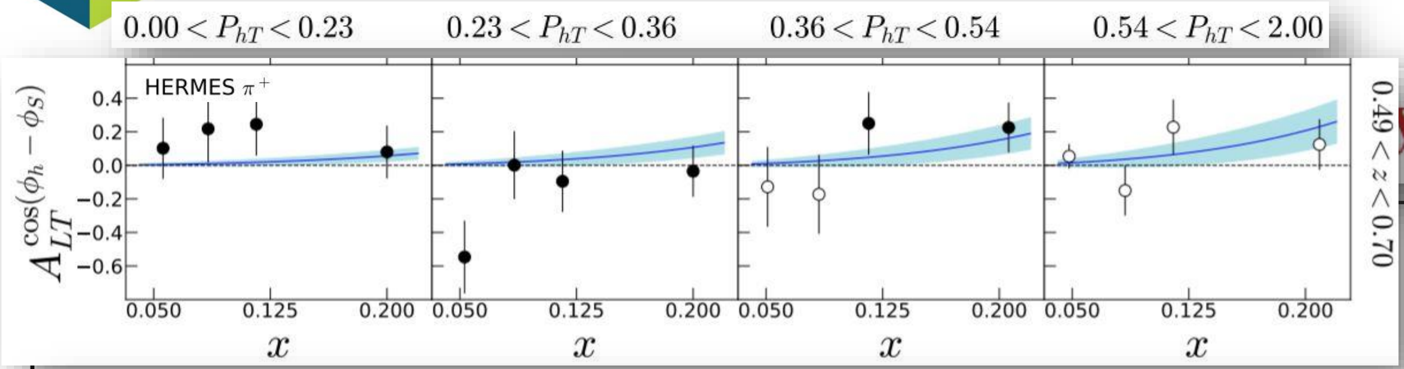
y bands for HERMES?







Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data



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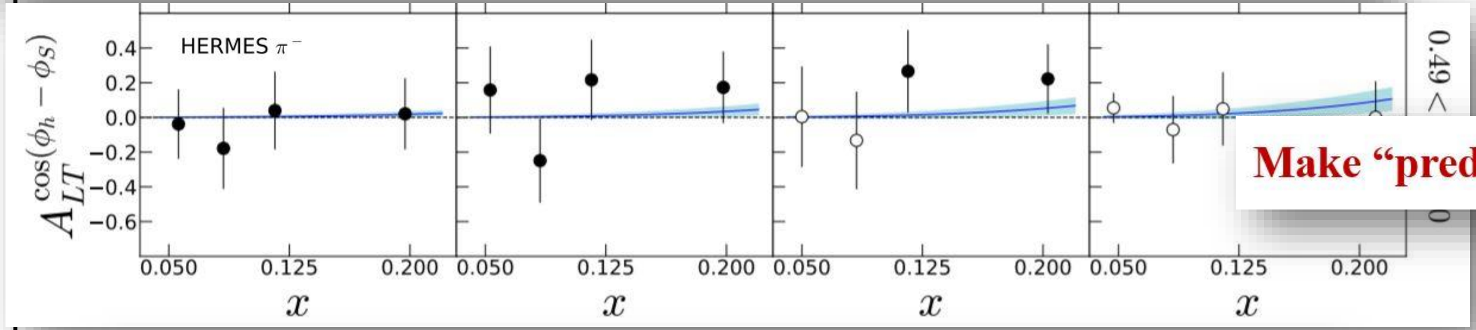
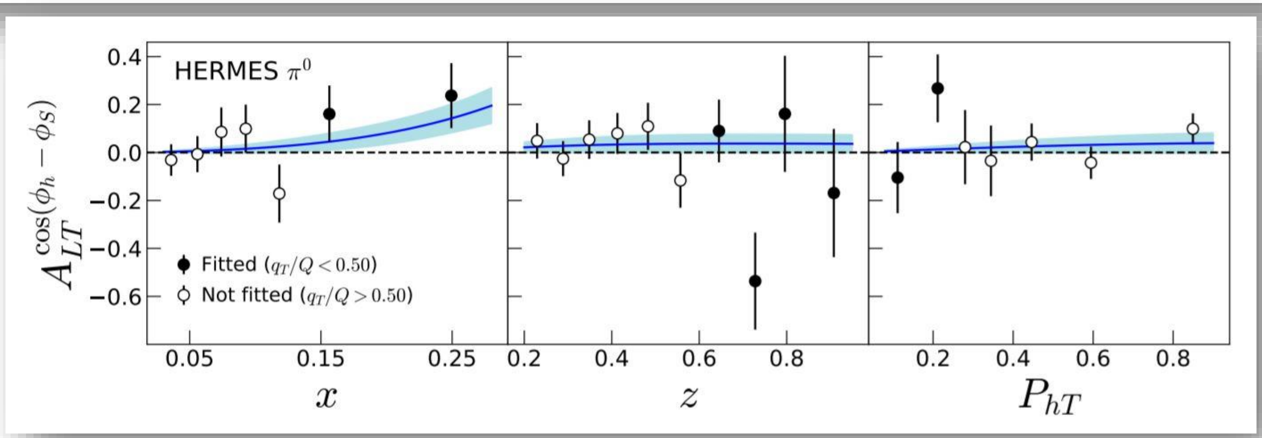
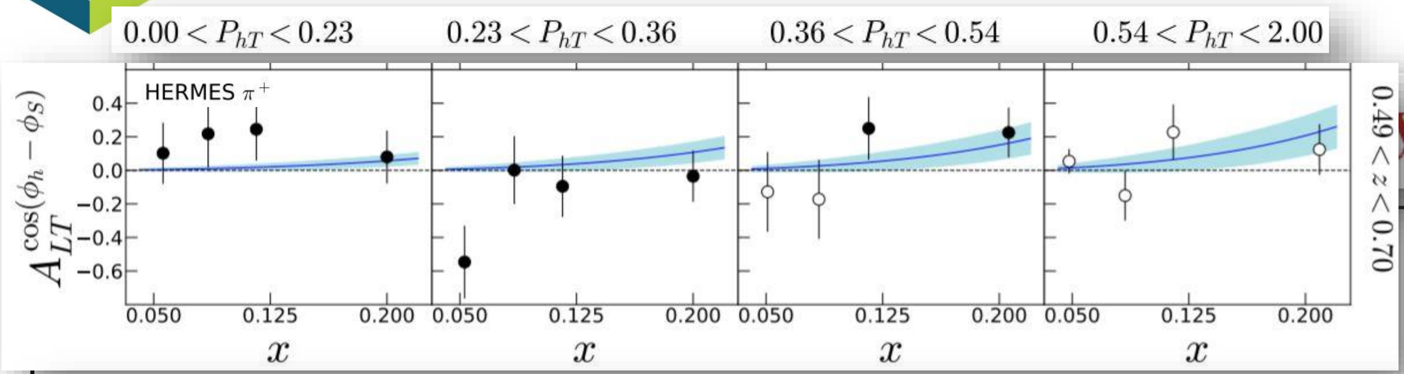


Make "predictions" for COMPASS using HERMES + JLab



Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data



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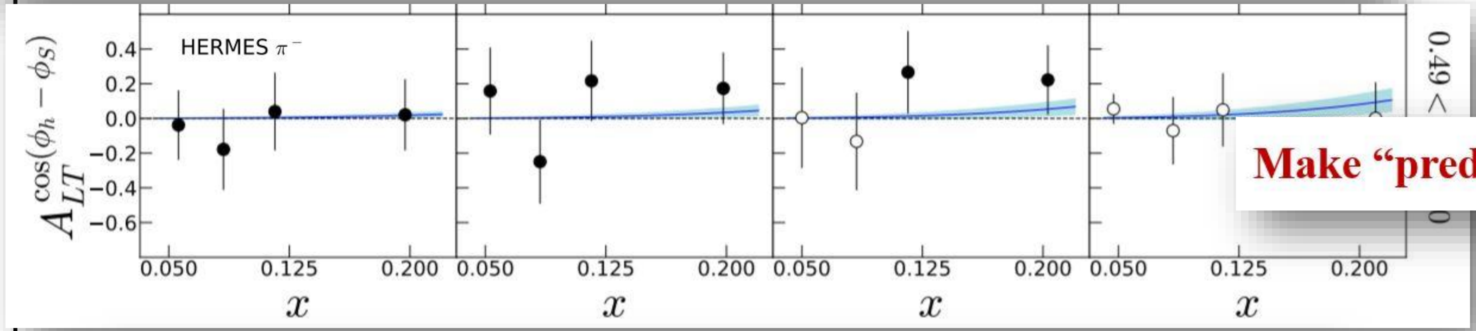

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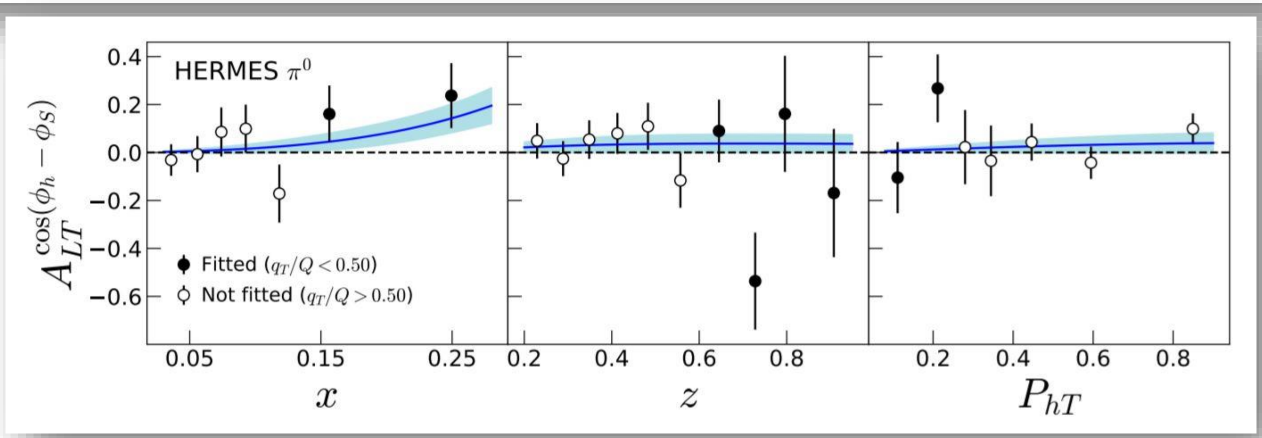
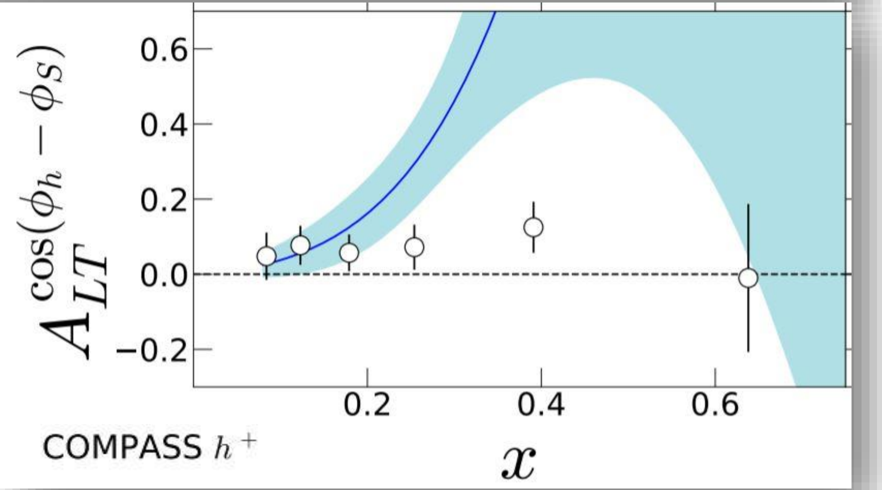
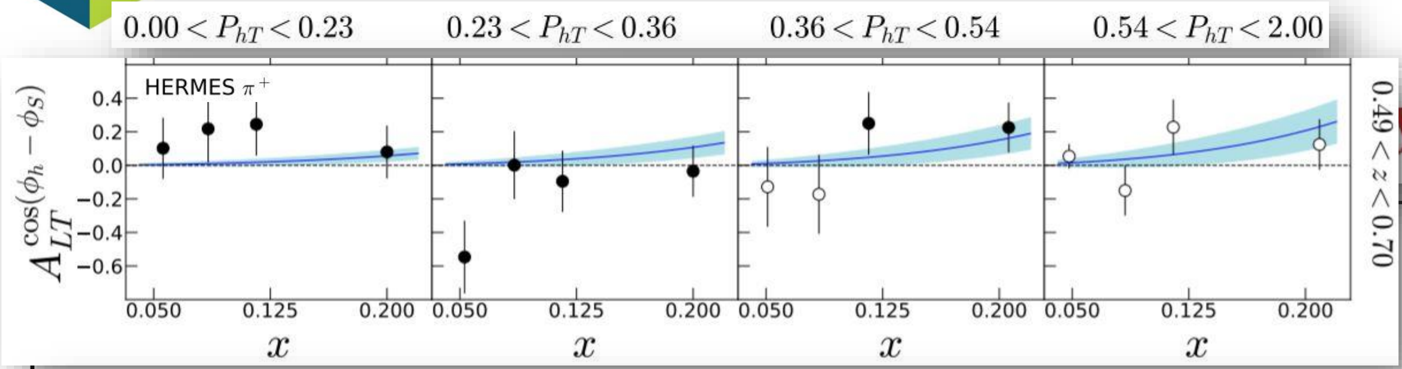
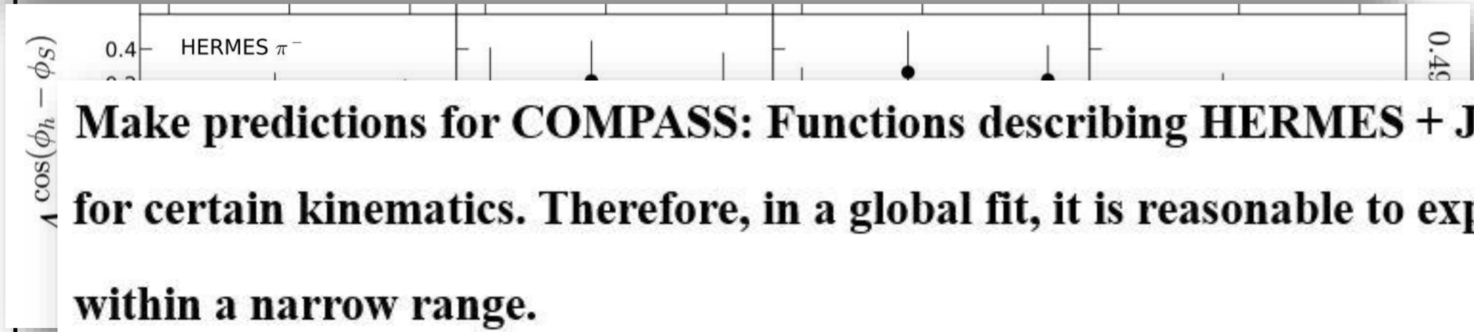




Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

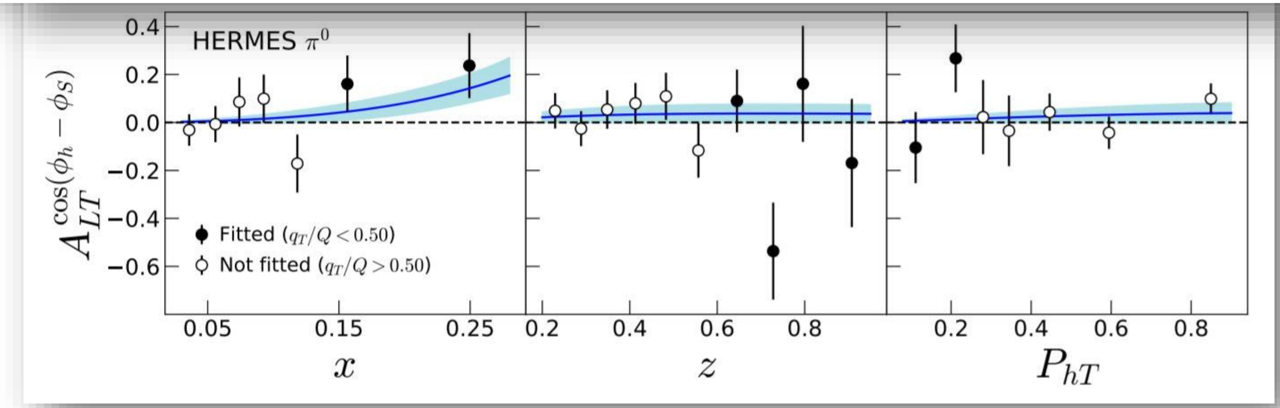


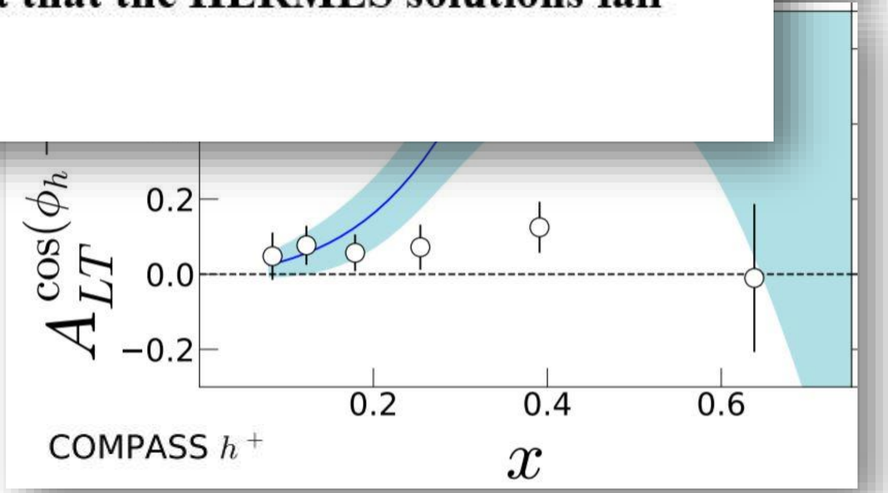
y bands for HERMES?



Make predictions for COMPASS: Functions describing HERMES + JLab are too large to describe COMPASS for certain kinematics. Therefore, in a global fit, it is reasonable to expect that the HERMES solutions fall within a narrow range.

Lab





Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Tests with other PDF sets (not shown in paper)

- **Parameterization with different unpolarized PDF sets?**

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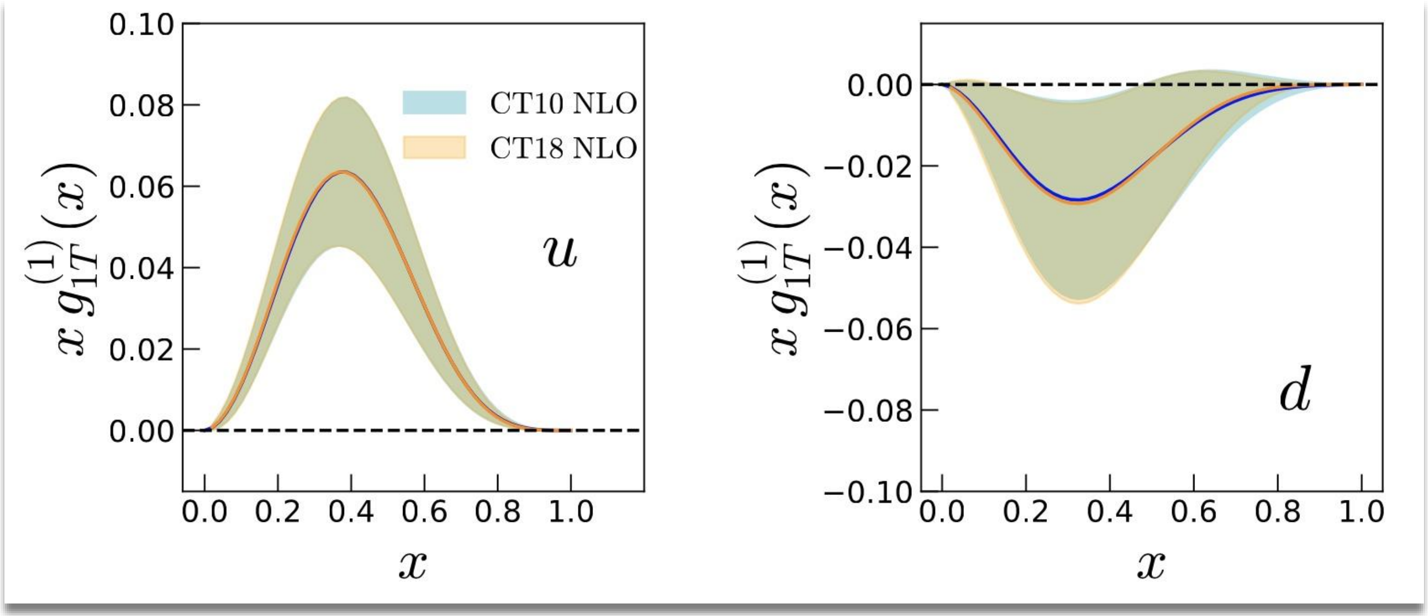

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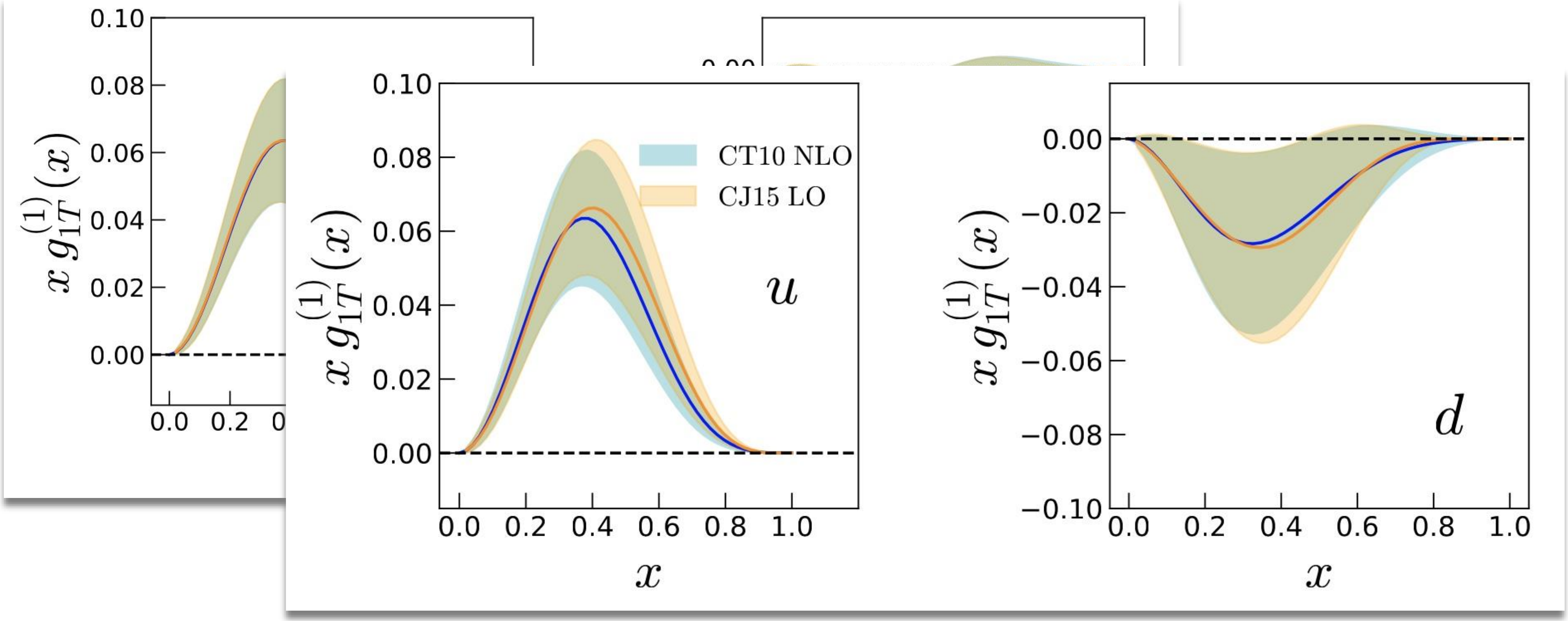

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Tests with other PDF sets (not shown in paper)

- **Different unpolarized PDF sets do not change our results**

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- **Parameterization with helicity PDF?**

    - **We were agnostic about any connection to helicity PDF through the WW-type relation**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Tests with other PDF sets (not shown in paper)

- **Different unpolarized PDF sets do not change our results**

- **Parameterization with helicity PDF?**
  - **We were agnostic about any connection to helicity PDF through the WW-type relation**
  - **Try:**

$$g_{1T}^{(1)}(x,Q^2) = \frac{n}{\int_0^1 dy\, y^{\alpha+1}(1-y)^{\beta} f_1(y,Q_0^2)} x^{\alpha}(1-x)^{\beta} f_1(x,Q^2)$$

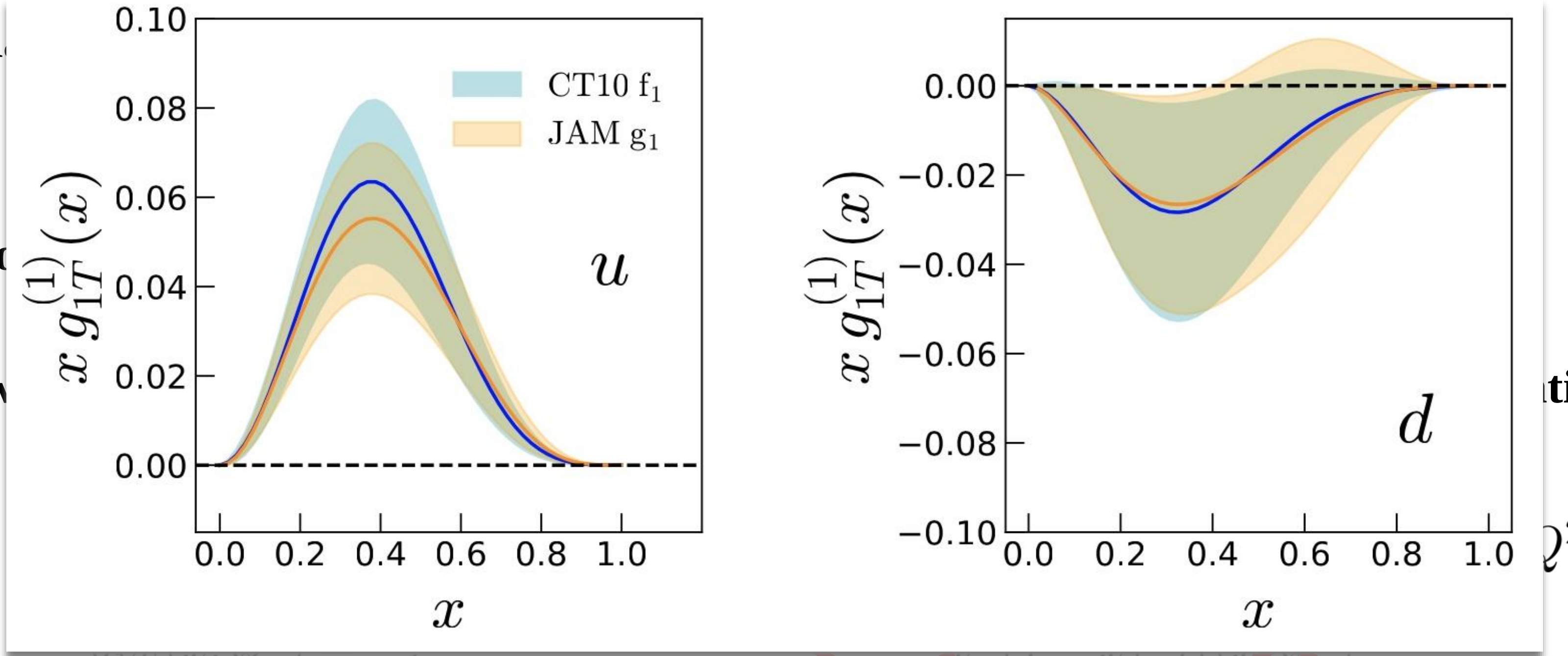
$g_1(x,Q^2)$

$g_1(y,Q_0^2)$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Tests with other PDF sets (not shown in paper)

- **Different unpol**
- **Parameterizatio**
  - **We w** ... **tion**
  - **Try:**



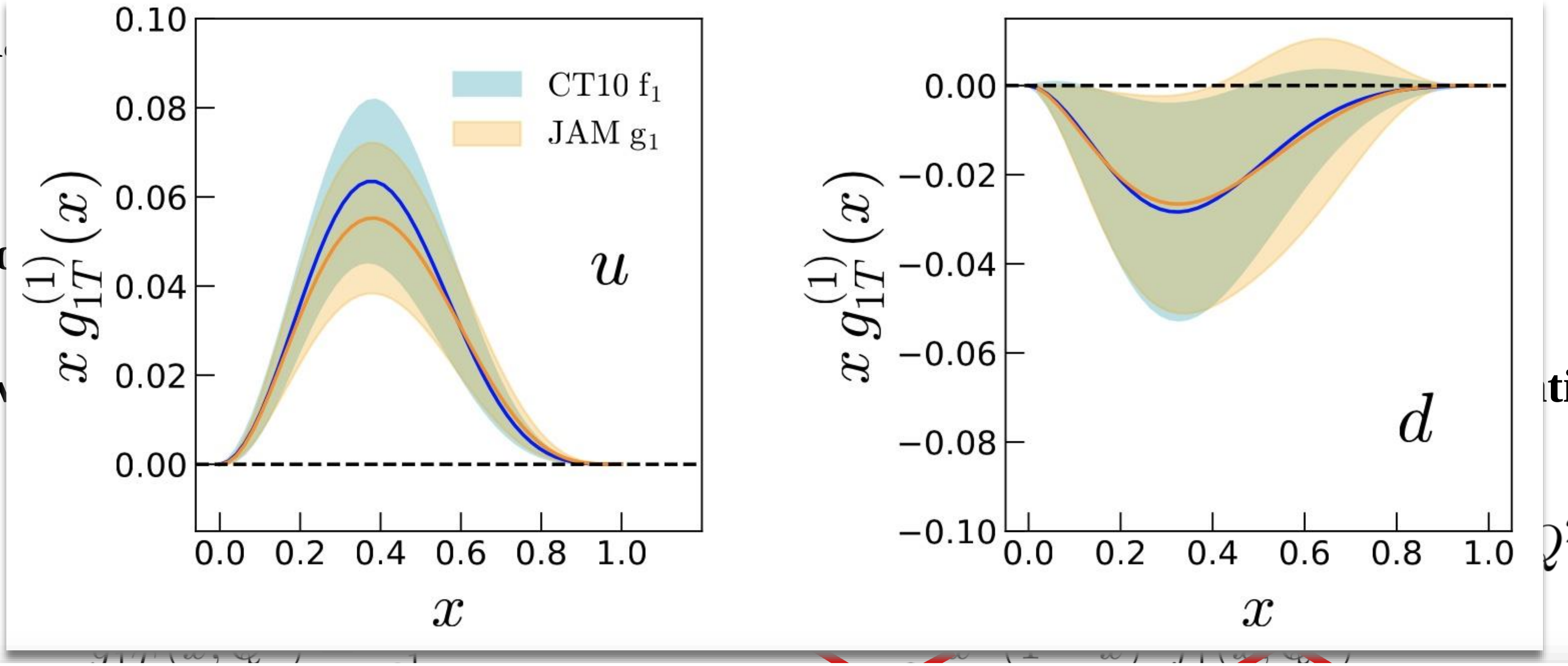
$$\int_0^1 dy\, y^{\alpha+1}(1-y)^{\beta} f_1(y, Q_0^2)$$

$g_1(y, Q_0^2)$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Tests with other PDF sets (not shown in paper)

- **Different unpol**
- **Parameterizatio**
  - **We w** ... **tion**
  - **Try:** ... $Q^2)$



- **Parameterizing in terms of helicity PDF does not change our results**