The Drell-Yan q_T Spectrum and Its Uncertainty at N^{3}LL$'$

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TMD Collaboration Meeting
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The Drell-Yan q_T Spectrum and Its Uncertainty at $\mathrm{N^3LL'}$

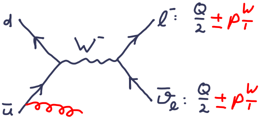
[to appear soon]

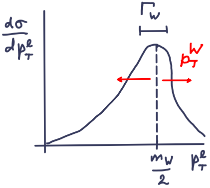
in collaboration with

G. Billis, M. Ebert, F. Tackmann



So you've lost a neutrino ... $\Rightarrow$ Best option is to extract m_W from $\mathrm{d}\sigma/\mathrm{d}p_T^{\ell}$!





$\Rightarrow$ Need precise theory predictions for $(\mathrm{d}\sigma/\mathrm{d}p_T^W)/(\mathrm{d}\sigma/\mathrm{d}p_T^Z)$ to model the p_T^W spectrum using precisely measured p_T^Z as input

$$\begin{aligned} m_W^{\mathrm{ATLAS}} &= 80370 \pm 7_{\text{stat.}} \\ &\pm 11_{\text{exp. syst.}} \\ &\pm 14_{\text{theory}}\ \mathrm{MeV} \\ &= 80370 \pm 19\ \mathrm{MeV} \end{aligned}$$

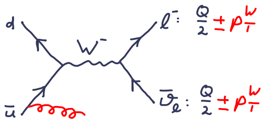
[ATLAS, 1701.07240]

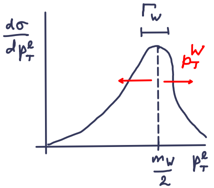
$$\begin{aligned} m_W^{\mathrm{LHCb}} &= 80354 \pm 23_{\text{stat.}} \\ &\pm 10_{\text{exp. syst.}} \\ &\pm 17_{\text{theory}} \\ &\pm 9_{\text{PDF}}\ \mathrm{MeV} \\ &= 80354 \pm 32\ \mathrm{MeV} \end{aligned}$$

[LHCb, 2109.01113]

Motivation: Measuring m_W at the LHC

So you've lost a neutrino … $\Rightarrow$ Best option is to extract m_W from $\mathrm{d}\sigma/\mathrm{d}p_T^\ell$!



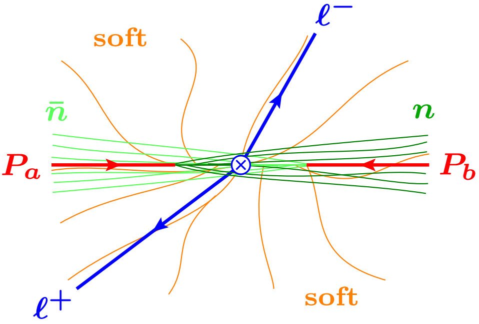


~~Challenges~~ Opportunities for theory

- Need sub-percent precision on $\mathrm{d}\sigma/\mathrm{d}p_T^Z$ and $\mathrm{d}\sigma/\mathrm{d}p_T^W$
  - Leave no stone unturned: QCD three-loop corrections, QED radiative corrections, quark mass effects, PDF & α_s parametric uncertainties
- Resum singular terms & large logarithms $\dfrac{\alpha_s^n}{q_T}\left(\ln\dfrac{q_T}{Q}\right)^{2n-1}$ to all orders in α_s
- Nonperturbative TMD dynamics of the proton at $q_T \sim 1/b_T \sim \Lambda_{\mathrm{QCD}}$ crucial

$$\frac{d\sigma}{dq_T} = \frac{d\sigma^{\rm res}_{\rm fact}}{dq_T} + \left[\frac{d\sigma^{\rm FO}_{\rm full}}{dq_T} - \frac{d\sigma^{\rm FO}_{\rm fact}}{dq_T}\right] \equiv \frac{d\sigma^{\rm res}_{\rm fact}}{dq_T} + \frac{d\sigma_{\rm nons}}{dq_T}$$

$$\frac{d\sigma_{\rm fact}}{dQ\,dY\,dq_T} = \sum_q H_{q\bar{q}}(Q,\mu)\;\; q_T \int_0^\infty db_T\, b_T\, J_0(q_T b_T) \times f_q^{\rm TMD}(x_a, b_T, \mu, \zeta)\, f_{\bar{q}}^{\rm TMD}(x_b, b_T, \mu, \zeta) + (q \leftrightarrow \bar{q})$$



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Implemented in SCETlib C++ numerical library [Ebert, JKLM, Tackmann]:

- Three-loop hard function [Baikov et al. '09; Lee et al. '10; Gehrmann et al. '10, '20; Czakon et al. '21]
- Three-loop matching of TMD PDFs onto collinear PDFs
  [Li, Zhu, '16; Luo, Yang, Zhu, Zhu '19; Ebert, Mistlberger, Vita '20]

$$f_i^{\rm TMD}(x,b_T,\mu,\zeta) = \sum_j \int \frac{dz}{z}\,C_{ij}(z,b_T,\mu,\zeta)\,f_j^{\rm coll}\Big(\frac{x}{z},\mu\Big)\Big[1+\mathcal{O}\Big(\Lambda^2_{\rm QCD} b_T^2\Big)\Big]$$

  - Prediction includes complete three-loop RG boundary conditions (N^{3}LL$'$)
  - Integral of spectrum is N^{3}LO-accurate after including $d\sigma^{\rm nons} = Y$ at α_s^3
- Four-loop cusp, three-loop noncusp anomalous dimension and CS kernel
  [Brüser, et al. '19; Henn et al. '20; v. Manteuffel et al. '20; Li, Zhu, '16; Vladimirov '16]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T} = \frac{\mathrm{d}\sigma_{\mathrm{fact}}^{\mathrm{res}}}{\mathrm{d}q_T} + \left[\frac{\mathrm{d}\sigma_{\mathrm{full}}^{\mathrm{FO}}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma_{\mathrm{fact}}^{\mathrm{FO}}}{\mathrm{d}q_T}\right] \equiv \frac{\mathrm{d}\sigma_{\mathrm{fact}}^{\mathrm{res}}}{\mathrm{d}q_T} + \frac{\mathrm{d}\sigma_{\mathrm{nons}}}{\mathrm{d}q_T}$$

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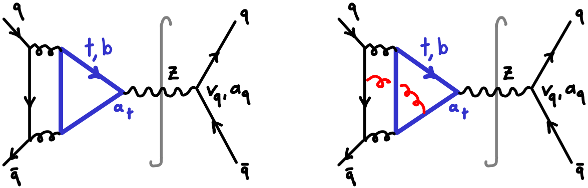
- Three-l…

[Li, Zhu, '1…



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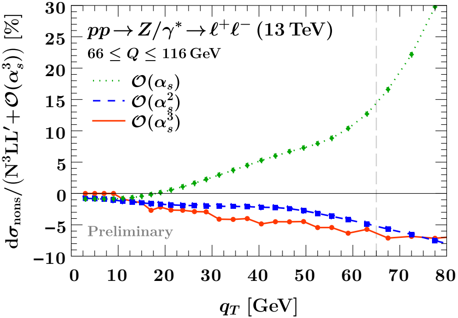
$$\frac{d\sigma_{\rm nons}}{dq_T} = \frac{d\sigma^{\rm FO}_{\rm full}}{dq_T} - \frac{d\sigma^{\rm FO}_{\rm fact}}{dq_T} = \frac{1}{q_T}\mathcal{O}\Big(\frac{q_T^2}{Q^2}\Big)$$



- Fiducial power corrections $\mathcal{O}(q_T/Q)$ captured through exact acceptance
  [TMD factorization, resummation & use in q_T FO subtractions: Ebert, JKLM, Stewart, Tackmann '20]
  - $d\sigma_{\rm nons} = Y$ pushed down to $\mathcal{O}(q_T^2)$ even in presence of fiducial cuts
- In-house analytic FO implementation of all helicity structure functions at $\mathcal{O}(\alpha_s)$
- Fiducial Z+jet MC data at $\mathcal{O}(\alpha_s^2)$ from MCFM
  [Campbell, Ellis, et al. '99, '15]
- Recently: Precise fiducial Z+jet MC data at $\mathcal{O}(\alpha_s^3)$ from NNLOjet
  [Chen et al., 2203.01565 – many thanks to the NNLOjet collaboration for providing the raw data.]

- Large Sudakov logs of $q_T/Q \sim 1/(b_T Q)$ are resummed by μ and CS evolution:

$$\mu_i, \sqrt{\zeta_i} = \frac{b_0}{b_T} \approx \frac{1.123}{b_T} \quad \longrightarrow \quad \mu_f, \sqrt{\zeta_f} = Q$$

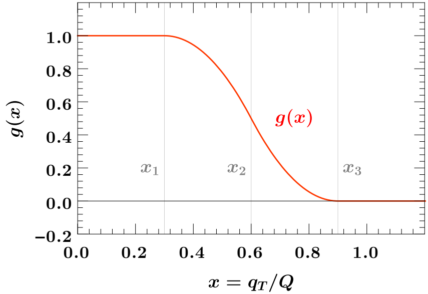
- Use exact analytic solutions of μ RGE and CS equation, combined with fast numerically exact solution of β function [Ebert '21]
  - Exact path independence in (μ, ζ) plane
  - Eliminates a source of truncation error at fraction of cost of full Runge-Kutta

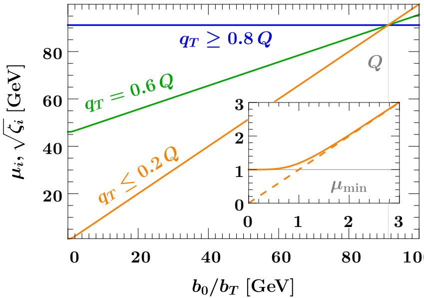
- Matching $\mathrm{d}\sigma = \mathrm{d}\sigma^{\mathrm{res}}_{\mathrm{fact}} + \left[\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{full}} - \mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{fact}}\right]$ governed by *(hybrid) profile scales*:
  [Lustermans, JKLM, Tackmann, Waalewijn '19]

$$\mu_i = \sqrt{\zeta_i} = Q\, f\left(\frac{q_T}{Q}, \frac{b_0}{b_T Q}\right)$$

$$f(x, y) = 1 + g(x)(y - 1)$$

  - $\mu_i = \sqrt{\zeta_i} = b_0/b_T$ and $[\ldots] = \mathcal{O}(q_T^2/Q^2)$ for $q_T \le x_1 Q$
  - $\mu_i = \sqrt{\zeta_i} = \mu_f = \sqrt{\zeta_f} = Q$ (pure FO) and $\mathrm{d}\sigma = \mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{full}}$ for $q_T \ge x_3 Q$



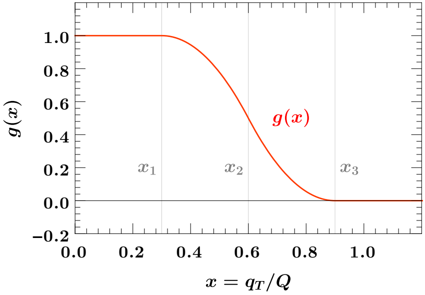


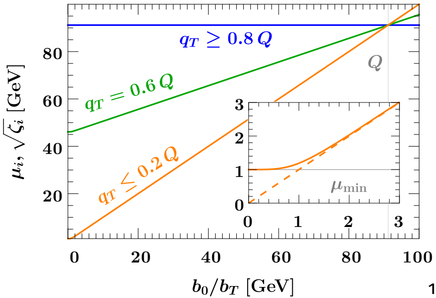
- Apply a b^* prescription starting at $\mathcal{O}(b_T^4)$:

$$\mu_i \to \mu_i^* = \left[(\mu_i^{\min})^4 + \left(\frac{b_0}{b_T}\right)^4\right]^{1/4} = \frac{b_0}{b_T}\left\{1 + \mathcal{O}\left[(\mu_i^{\min} b_T)^4\right]\right\}$$

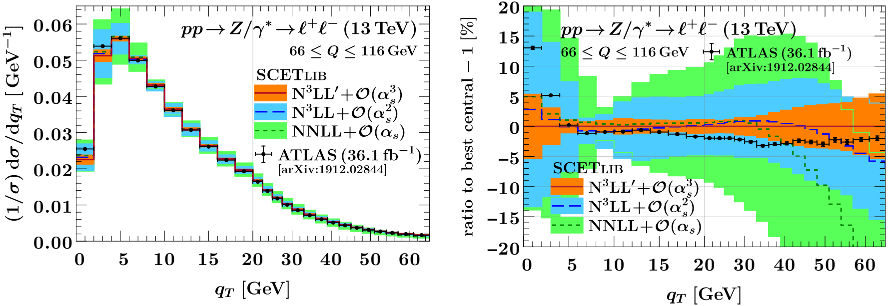
  - Avoids contaminating nonperturbative corrections at quadratic order
    [Conflict with b_T-space renormalon structure: Scimemi, Vladimirov '18]
    [Translation back to momentum space: Ebert, JKLM, Stewart, Sun '22]

- Our prescription is "local", i.e., only applied to μ_i (not overall b_T)





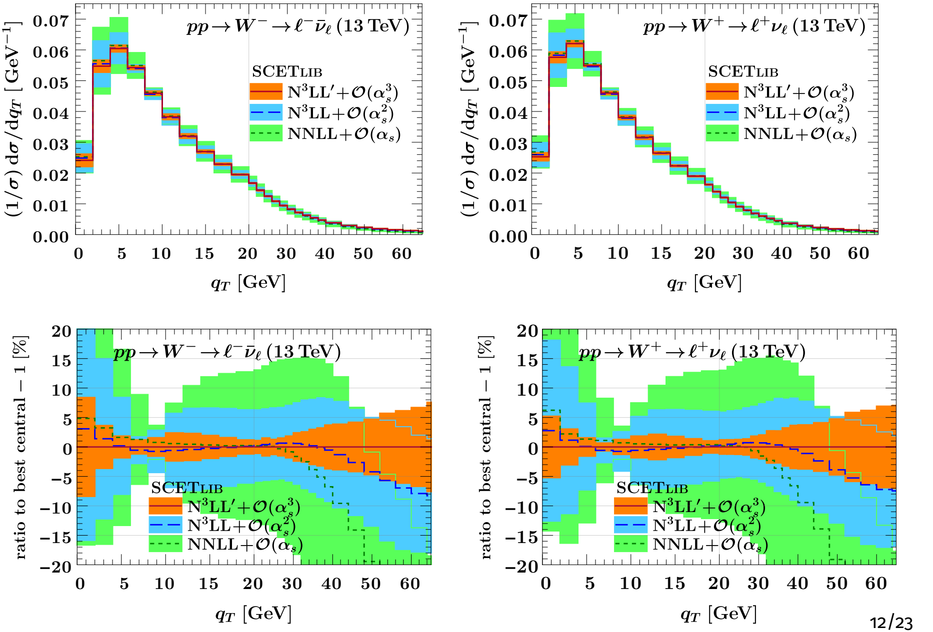
- Central results use MSHT20nnlo with $\alpha_s(m_Z) = 0.118$, $n_f = 5$
- NNLO (= three-loop!) collinear PDF evolution still formally sufficient at N^{3}LL′
  - Three-loop DGLAP formally cancels within three-loop TMD PDF
  - Separate question whether PDFs should have been extracted using three-loop $\hat{\sigma}_{ij}$ ...



- Excellent perturbative convergence towards three-loop result
- Higher orders are covered by lower-order uncertainty estimates

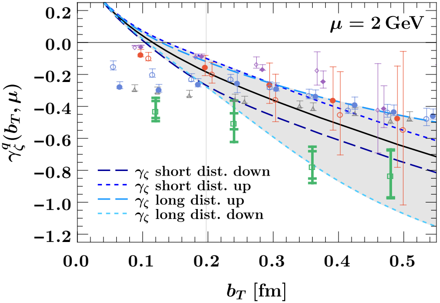
  [See backup for details of how they are estimated – (mostly) based on scale variations]

Results: Predictions for $W^{\pm} \to \ell\nu$



Nonperturbative model for the Collins-Soper kernel

$$\gamma^{q}_{\zeta\,\mathrm{NP}}(b_T) = c^i_\zeta \tanh\Big(\frac{\omega^2_{\zeta,i}}{|c_\zeta|} b_T^2\Big) = \mathrm{sgn}(c^i_\zeta)\,\omega^2_{\zeta,i} b_T^2 + \mathcal{O}(b_T^4)$$



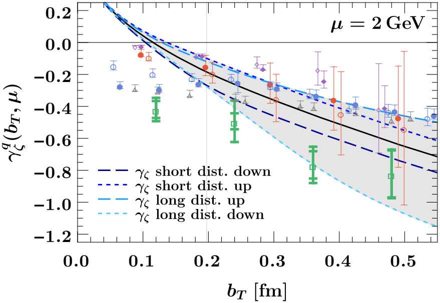
- Vary either ω_ζ ("short distance") or c_ζ ("long distance") to cover spread of lattice data at large b_T

  [Collection of lattice data reproduced from Shanahan, Wagman, Zhao, 2107.11930 – thanks to Y. Zhao]

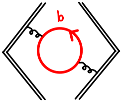
  [See talk by A. Avkhadiev for prospects for future improvements!]

NOTE Compatible with fits in [Scimemi, Vladimirov '19; Bacchetta et al. '19], but aim for a-priori prediction

$$\gamma^q_{\zeta\,\mathrm{NP}}(b_T) = c^i_\zeta \tanh\left(\frac{\omega^2_{\zeta,i}}{|c_\zeta|} b_T^2\right) = \mathrm{sgn}(c^i_\zeta)\,\omega^2_{\zeta,i} b_T^2 + \mathcal{O}(b_T^4)$$



- Pick central value of $\mathrm{sgn}(c^i_\nu)\,\omega^2_{\zeta,i}(1 \pm 2)$ to serve as bias correction for known leading (NNLL) bottom quark mass effect in γ^q_ζ:



$$\Delta\gamma^q_\zeta(b_T, m_b, \mu) = \frac{\alpha_s^2}{\pi^2} C_F T_F (m_b b_T)^2 \left(\ln \frac{b_T^2 m_b^2}{4e^{-2\gamma_E}} - 1\right) \approx -(0.25\,\mathrm{GeV})^2 b_T^2$$

- Leading term $f_i^{\mathrm{NP}}(x, b_T) = b_T^2 \Lambda_i^{(2)}(x) \sim \frac{\Lambda_{\mathrm{QCD}}^2}{q_T^2}$ in general complicated
- However, can show that for a given boson V and fiducial volume, only *a single average coefficient* $\overline{\Lambda}_V$ remains after integral over hard PS Φ_B: [Ebert, JKLM, Stewart, Sun '22]

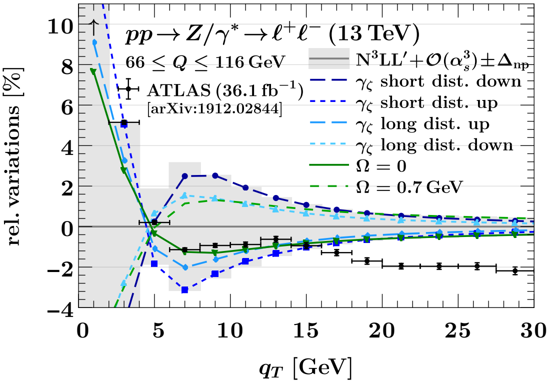
$$\tilde{\sigma}(b_T) = \tilde{\sigma}^{(0)}(b_T)\Big\{1 + b_T^2\Big(2\overline{\Lambda}_V^{(2)} + \gamma_{\zeta,q}^{(2)} L_{Q^2}\Big)\Big\} + \mathcal{O}\big[(\Lambda_{\mathrm{QCD}} b_T)^4\big]$$

$$\overline{\Lambda}_V^{(2)} = \frac{\int \mathrm{d}\Phi_B\, A(\Phi_B)\, \sum_{i,j} \sigma_{ij}^B(Q)\, f_i^{(0)}(x_a, \mu_0)\, f_j^{(0)}(x_b, \mu_0)\big[\Lambda_i^{(2)}(x_a) + \Lambda_j^{(2)}(x_b)\big]}{2\int \mathrm{d}\Phi_B\, A(\Phi_B)\, \sum_{i,j} \sigma_{ij}^B(Q)\, f_i^{(0)}(x_a, \mu_0)\, f_j^{(0)}(x_b, \mu_0)}$$

- Here: Promote $\overline{\Lambda}_V^{(2)}$ back to a single-parameter Gaussian model

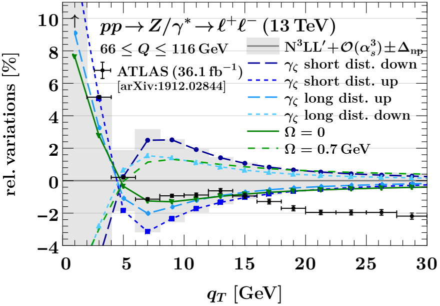
$$f_i^{\mathrm{NP}}(x, b_T) = \exp(-\Omega_V^2 b_T^2) \quad \text{with} \quad \overline{\Lambda}_V^{(2)} = -\Omega_V^2$$

  - ▶ Captures the most general form of the leading NP correction to the rapidity-integrated q_T spectrum up to $\mathcal{O}(\Lambda_{\mathrm{QCD}}^4/q_T^4)$
- In following: take central $\Omega_V = 0.5\,\mathrm{GeV}$ and vary it as $\Omega_V = \{0, 0.7\}\,\mathrm{GeV}$



- Taken at face value, the lowest bins seem to prefer *weaker* NP effects
- N^{3}LL closer to data for $q_T \leq 15\,\mathrm{GeV}$ with our default NP parameters, suggesting that three-loop and NP corrections can be traded off for one another
- Overshoot data at $q_T = 20 - 30\,\mathrm{GeV}$, way outside NP effect strength (and narrowly outside perturbative uncertainty)

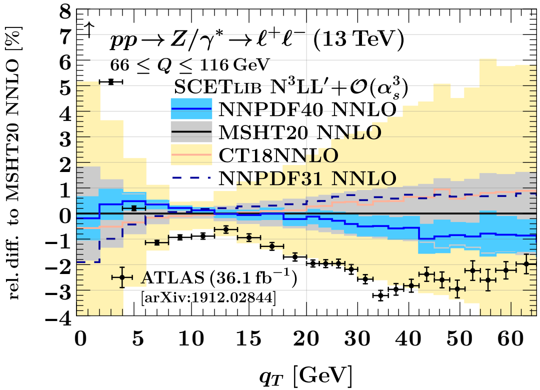
A (likely unnecessary) reminder about TMD PDF flavor dependence



- Important: $\Omega_Z \neq \Omega_W$ in general because $f_u^{\rm NP} \neq f_d^{\rm NP} \neq f_{\bar u}^{\rm NP} \neq f_{\bar d}^{\rm NP}$!
- Shifts in m_W from flavor dependent $f_i^{\rm NP}$ may be as large as $10\,{\rm MeV}$
  [Bacchetta, Bozzi, Radici, Ritzmann, Signori '18]

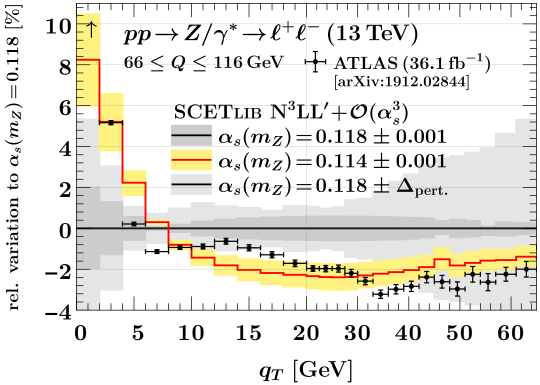
N.B. CDF 2022 measurement used flavor-independent BLNY ansatz

- $x \sim 10^{-2}$ at the LHC $\Leftrightarrow$ weakly constrained by lowest Mellin moments
  - Full x dependence of flavor ratios would be ideal
    [See talk by S. Schindler for a lattice-to-continuum factorization enabling this on formal footing!]

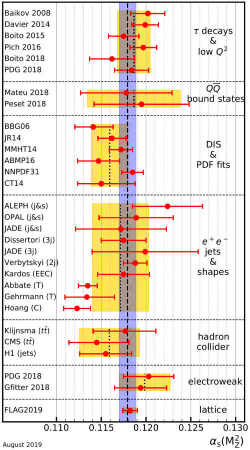


- Resummed 3-loop cross section is *analytic*, hold small $\alpha_s^{2,3}$ nonsingular fixed
  - Whole figure with complete PDF uncertainties at few 100 CPUh!
- PDF uncertainty largely cancels in normalized spectrum
- Cannot explain overshoot at $q_T = 20 - 30\,\mathrm{GeV}$

Results: Impact of α_s on normalized Z spectrum



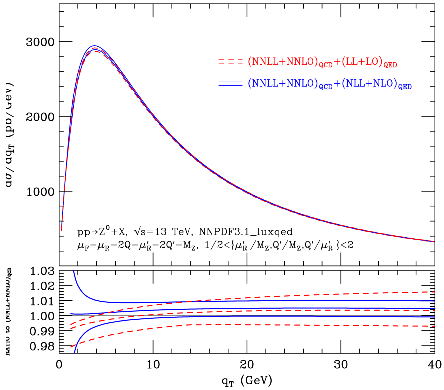
- Parametric uncertainty due to $\alpha_s(m_Z)$ on par with perturbative uncertainty
- Overshoot at $q_T = 20 - 30\,\mathrm{GeV}$ is naturally explained by lower $\alpha_s(m_Z)$



- Lower values of $\alpha_s(m_Z)$ have previously been reported in high-order resummed fits to e^+e^- event shapes (thrust and C parameter)
  - Like TMD observables these are driven by all-order resummation ...

Systematics at the theory frontier:

- QED TMD effects for on-shell Z well understood
  [Bacchetta, Echevarria '18; Cieri, Ferrera, Sborlini '18; Billis, Tackmann, Talbert '19]
  - Expected to be $\sim 1\%$, but would bring the tail up *more* …



[Cieri, Ferrera, Sborlini 1805.11948]

…but many caveats remain

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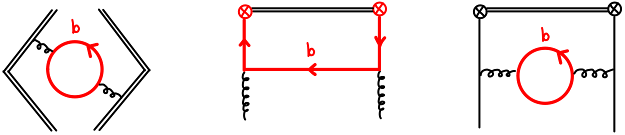
- Next challenge: Interface pp TMD factorization with QED+weak corrections to full process, including realistic lepton definitions
  [Connects to QED radiative corrections in SIDIS, cf. Liu Melnitchouk, Qiu, Sato '21]
- Subleading power resummation & factorization for *nonsingular* cross section
  [See talks by A. Gao, L. Gamberg, J. Terry in first Friday session!]
- Full resummed treatment of quark mass effects
  $\Leftrightarrow$ matched VFNS for every piece in factorization $\Rightarrow \# \, m_b^2/q_T^2$?



Side note: At least $g \to b\bar{b}$ matters less for Tevatron, cf. α_s fit in [Camarda, Ferrera, Schott '22]…

The Drell-Yan q_T Spectrum at N^{3}LL$'$ and Its Uncertainty:

- Percent-level perturbative predictions at the LHC are possible, and are required to fully leverage LHC data for TMD determinations
- Intriguing hints that the data may prefer a lower value of α_s ...
- Even small changes in α_s strongly impact the peak shape, in ways similar to nonperturbative effects
- Important to consistently match to fixed-order perturbation theory beyond $q_T \sim 20\,\mathrm{GeV}$ to isolate the two effects

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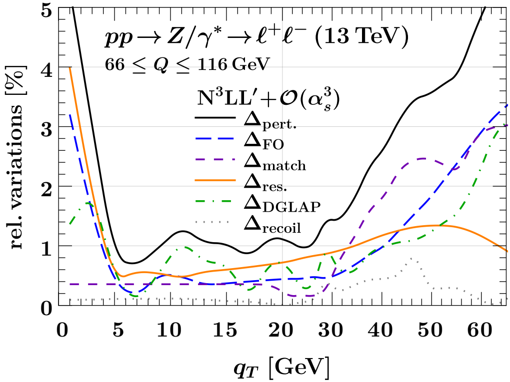
Thank you for your attention!

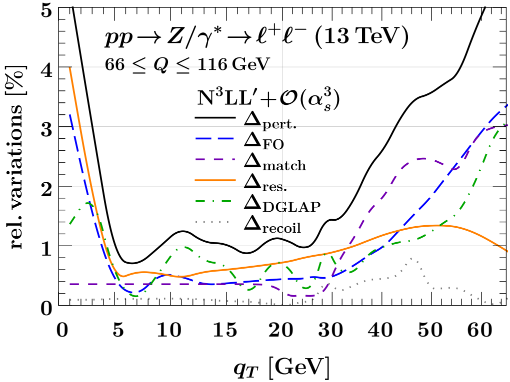
Backup

Breakdown of perturbative uncertainties



$$\Delta_{\mathrm{pert.}} = \Delta_{\mathrm{FO}} \oplus \Delta_{\mathrm{match}} \oplus \Delta_{\mathrm{res}} \oplus \Delta_{\mathrm{DGLAP}} \oplus \Delta_{\mathrm{recoil}}$$

- Fixed-order uncertainty, keeps resummed logarithms unchanged
- Estimated by standard variations of overall $\mu_R = \mu_{\mathrm{FO}}$
- All scales (except μ_f) are chosen $\propto \mu_{\mathrm{FO}}$, so e.g. μ_H/μ_S unchanged
- Frozen out at $b_T \lesssim 1/\Lambda_{\mathrm{QCD}}$ by μ_X^* prescription $\Rightarrow$ disentangled from NP



$$\Delta_{\rm pert.} = \Delta_{\rm FO} \oplus \Delta_{\rm match} \oplus \Delta_{\rm res} \oplus \Delta_{\rm DGLAP} \oplus \Delta_{\rm recoil}$$

- Uncertainty from matching scheme between resummed peak and fixed-order tail
- Estimated by varying the $x = q_T/Q$ transition points in hybrid profile as

$$\{x_1, x_2, x_3\} = \{0.3, 0.6, 0.9\} \pm \{0.1, 0.15, 0.2\}$$

- Checked that *inclusive* integrated cross section is recovered within $\Delta_{\rm match}$

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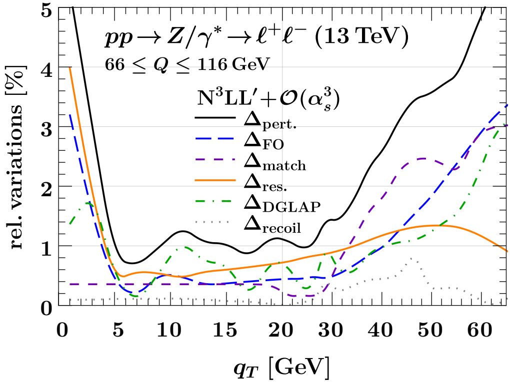
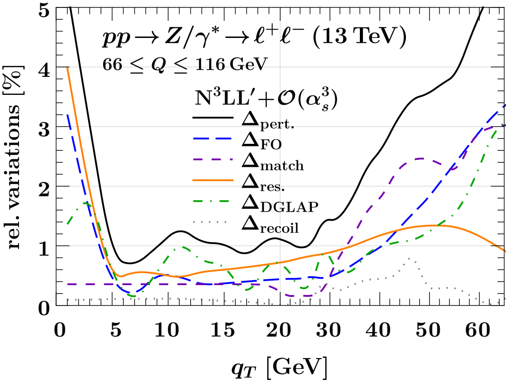


$$\Delta_{\rm pert.} = \Delta_{\rm FO} \oplus \Delta_{\rm match} \oplus \Delta_{\rm res} \oplus \Delta_{\rm DGLAP} \oplus \Delta_{\rm recoil}$$

- Probes higher-order resummed logarithms
- Estimated by envelope of 36 different combinations of independently varying $\{\mu_B, \mu_S, \nu_B, \ldots\}$ in $\sigma^{(0)} = H\, B \otimes B \otimes S$
- Also frozen out at $b_T \lesssim 1/\Lambda_{\rm QCD}$ by μ_X^* prescription $\Rightarrow$ disentangled from NP



$$\Delta_{\rm pert.} = \Delta_{\rm FO} \oplus \Delta_{\rm match} \oplus \Delta_{\rm res} \oplus \Delta_{\rm DGLAP} \oplus \Delta_{\rm recoil}$$

- Estimate of missing higher orders (four loops) in DGLAP running
- Estimated both in peak and tail by joint variations of $\mu_f(b_T, q_T, Q)$ and $\mu_F(Q)$
- Oscillatory due to b_T-space features at uncancelled m_b threshold

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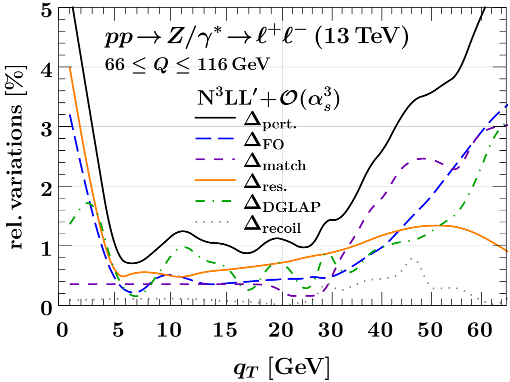
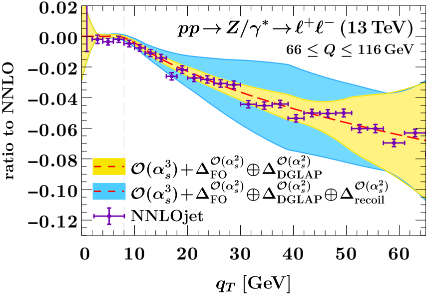
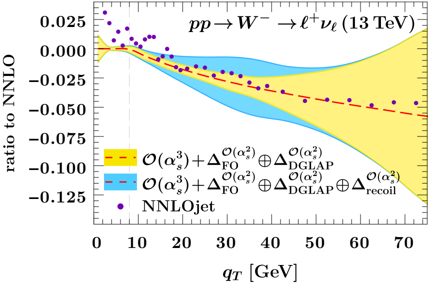


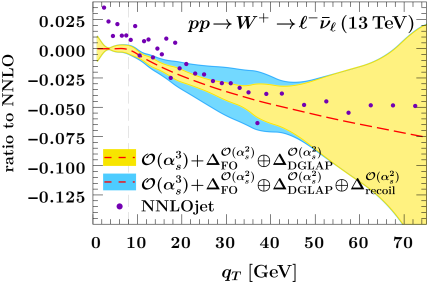
$$\Delta_{\mathrm{pert.}} = \Delta_{\mathrm{FO}} \oplus \Delta_{\mathrm{match}} \oplus \Delta_{\mathrm{res}} \oplus \Delta_{\mathrm{DGLAP}} \oplus \Delta_{\mathrm{recoil}}$$

- RPI-I transformation of n_a^μ, n_b^μ in $W_{\mathrm{LP}}^{\mu\nu} \sim g_\perp^{\mu\nu}(n_a, n_b)$
- Induces $O(q_T^2/Q^2)$ change in spectrum due to fiducial cuts on $L_{\mu\nu}$
  [Ebert, JKLM, Stewart, Tackmann '20]
- Equivalent to changing "recoil prescription"/choice of Z rest frame by $\mathcal{O}(q_T/Q)$

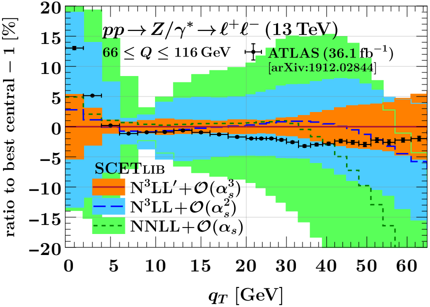
$\mathcal{O}(\alpha_s^3)$ nonsingular interpolations

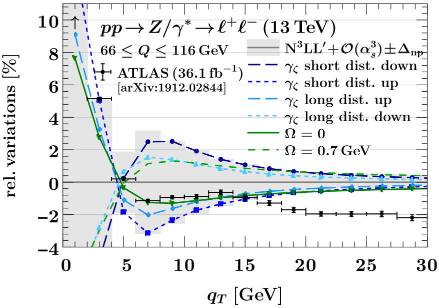


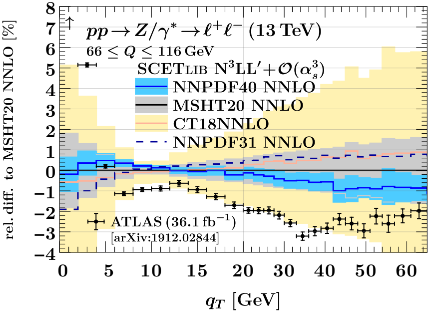


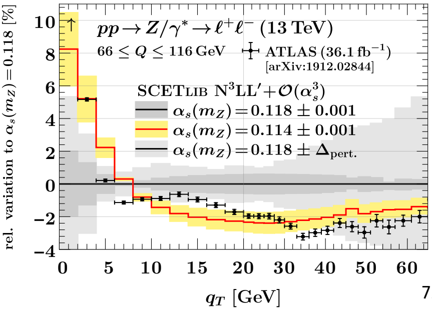


ATLAS normalized spectrum (Born leptons)









CMS normalized spectrum (dressed leptons)