Factorization for Subleading Power TMD Observables

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arXiv: 2112.07680 with Markus Ebert, Iain Stewart

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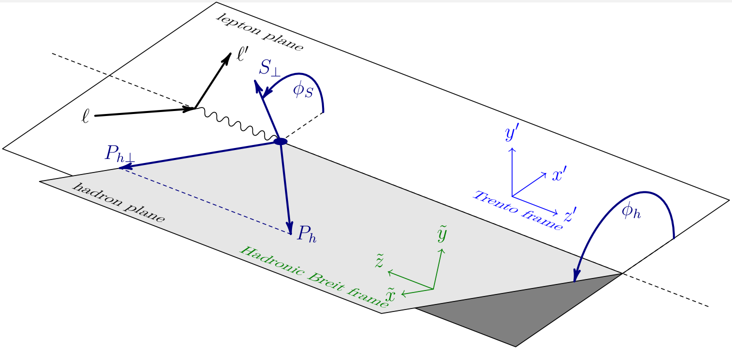
Outline

- Introduction to the Problem and Motivation
  - ▷ Goal: factorize NLP structure functions for SIDIS using SCET
- Review of (Ingredients from) Soft-Collinear Effective Theory (SCET)
- Deriving Factorization + Results
- Comparison with Literature at Tree Level

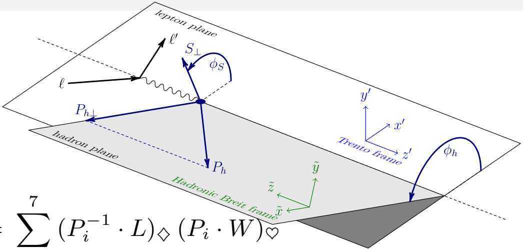
Tensor Decomposition and 18 Structure Functions in SIDIS



- $L^{\mu\nu} = \langle \ell | J^{\dagger\mu}_{\bar{\ell}\ell} | \ell' \rangle \langle \ell' | J^{\nu}_{\bar{\ell}\ell} | \ell \rangle = 2\Big[\Big(p^\mu_\ell p^\nu_{\ell'} + p^\nu_\ell p^\mu_{\ell'} - p_\ell \cdot p_{\ell'}\, g^{\mu\nu}\Big) + \mathrm{i}\lambda_\ell\, \epsilon^{\mu\nu\rho\sigma} p_{\ell\rho} p_{\ell'\sigma}\Big]$
- $W^{\mu\nu} = W^{\mu\nu}_U + S_L W^{\mu\nu}_L + S_T \cos(\phi_h - \phi_S) W^{\mu\nu}_{T\tilde{x}} + S_T \sin(\phi_h - \phi_S) W^{\mu\nu}_{T\tilde{y}}$
- Different polarization contributions of lepton/hadron $\left(\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}\right)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{Q^2}\frac{y}{z}\frac{\kappa_\gamma}{1-\epsilon}\Big[(L\cdot W)_{UU} + \lambda_\ell (L\cdot W)_{LU}$$
$$+ S_L (L\cdot W)_{UL} + \lambda_\ell S_L (L\cdot W)_{LL} + S_T (L\cdot W)_{UT} + \lambda_\ell S_T (L\cdot W)_{LT}\Big]$$

Tensor Decomposition and 18 Structure Functions in SIDIS



- Projection

$$(L \cdot W)_{\diamondsuit\heartsuit} = \sum_{i=-1}^{7} (P_i^{-1} \cdot L)_{\diamondsuit} \, (P_i \cdot W)_{\heartsuit}$$

- Projectors defined in the hadronic Breit frame

$P_{-1}^{\mu\nu} = (\tilde{x}^\mu \tilde{x}^\nu + \tilde{y}^\mu \tilde{y}^\nu)\,,\ P_0^{\mu\nu} = \tilde{t}^\mu \tilde{t}^\nu\,,\ P_1^{\mu\nu} = -(\tilde{t}^\mu \tilde{x}^\nu + \tilde{x}^\mu \tilde{t}^\nu)\,, \dots\,,\ P_7^{\mu\nu}$

- $q \cdot L = q \cdot W = 0 \Rightarrow$ no $\tilde{z}\,(\propto q) \Rightarrow 3 \times 3 = 9$ projectors
- Parity and hermiticity constraints reduce # of structure functions

$\Rightarrow$ In total 18 structure functions [Bacchetta et al '06]

$$(L{\cdot}W)_{UU} = W_{UU,T} + \epsilon W_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi_h) W_{UU}^{\cos(\phi_h)} + \epsilon\cos(2\phi_h) W_{UU}^{\cos(2\phi_h)}\,,$$

$$(L{\cdot}W)_{LU} = \sqrt{2\epsilon(1-\epsilon)}\sin(\phi_h) W_{LU}^{\sin(\phi_h)}\,,$$

$$(L{\cdot}W)_{LT} = \sqrt{1-\epsilon^2}\cos(\phi_h - \phi_S) W_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\left[\cos(\phi_S) W_{LT}^{\cos(\phi_S)} + \cos(2\phi_h - \phi_S) W_{LT}^{\cos(2\phi_h-\phi_S)}\right],$$

......

Power Expansion in $\lambda = P_{hT}/Q \ll 1$ and Motivation

- Focus on the unporlarized hadron (different notation for labeling)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{Q^2}\frac{y}{z}\frac{\delta_{\lambda_\ell\lambda_{\ell'}}}{1-\epsilon}\Big[\big(W_{-1}+\epsilon W_0\big)+\epsilon\cos(2\phi_h)W_3 + \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h W_1 + \lambda_\ell\sqrt{2\epsilon(1-\epsilon)}\sin\phi_h W_2\Big].$$

- $\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}$
- $W_i = P_i^{\mu\nu}W_{\mu\nu}$ with projectors $P_i^{\mu\nu}$ (defined in the hadronic Breit frame)
- $P_{-1}^{\mu\nu} = (\tilde{x}^\mu\tilde{x}^\nu + \tilde{y}^\mu\tilde{y}^\nu)\,, \quad P_3^{\mu\nu} = \tilde{x}^\mu\tilde{x}^\nu - \tilde{y}^\mu\tilde{y}^\nu\,,$

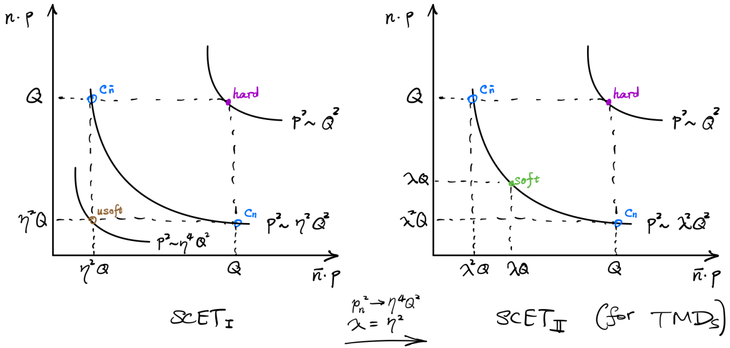
  $P_1^{\mu\nu} = -(\tilde{t}^\mu\tilde{x}^\nu + \tilde{x}^\mu\tilde{t}^\nu)\,, \quad P_2^{\mu\nu} = \mathrm{i}\big(\tilde{t}^\mu\tilde{x}^\nu - \tilde{x}^\mu\tilde{t}^\nu\big)\,, \quad P_0^{\mu\nu} = \tilde{t}^\mu\tilde{t}^\nu\,,$
- $W_{-1}\,,\ W_3 \sim \mathcal{O}\big(\lambda^0\big)$, standard factorization theorems (CSS, SCET)
- $W_1\,,\ W_2 \sim \mathcal{O}\big(\lambda\big)$
  - ▷ First treated in parton model (tree level matching) [Mulders, Tangerman '95]
  - ▷ Mismatch with perturbative results at tree level [Bacchetta et al '08]
  - ▷ Conjecture: Resolved by adding a LP soft function [Bacchetta et al '19]
- $W_0 \sim \mathcal{O}\big(\lambda^2\big)$, not considered in this work

⇒ *Use SCET to derive all-order factorization at subleading power*

Review of (Intro to) SCET

[Bauer, Fleming, Luke, Pirjol, Stewart '00, '01, '02]

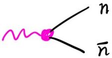
- EFT for collinear/soft d.o.f.s with power counting parameter $\lambda \ll 1$
- Lightcone coordinate $p^\mu = \frac{n^\mu}{2}\bar{n}\cdot p + \frac{\bar{n}^\mu}{2}n\cdot p + p_\perp$
- n_i-collinear particles: $(n_i \cdot p, \bar{n}_i \cdot p, p_{n_i\perp}) \sim Q(\lambda^2, 1, \lambda)$
- Ultrasoft $k^\mu \sim Q\lambda^2$ in SCET$_\text{I}$; Soft $k^\mu \sim Q\lambda$ in SCET$_\text{II}$ (for TMD)
- For SIDIS, take $n_1 = n /\!/ P_N$ and $n_2 = \bar{n} /\!/ P_h$



- SCET Lagrangian

$$\mathcal{L}_{\mathsf{SCET}_{\text{II}}} = \mathcal{L}_{\text{hard}} + \mathcal{L}_{\text{dyn}} = \Big(\sum_{i\geq 0} \mathcal{L}^{(i)}_{\text{hard}}\Big) + \Big(\sum_{i\geq 0} \mathcal{L}^{(i)}_{\text{dyn}} + \mathcal{L}^{(0)}_{G}\Big)\,,$$

  ▷ $\mathcal{L}^{(i)}_{\text{hard}} = \sum_k C^{(i)}_k \mathcal{O}^{(i)}_k = \frac{\mathrm{i}e^2}{Q^2} J_{\ell\ell' \mu} \sum J^{(i)\mu}_k\,,$ Hard scattering operators



  ▷ $\mathcal{L}^{(0)}_{\text{dyn}} = \mathcal{L}^{(0)}_{n} + \mathcal{L}^{(0)}_{\bar{n}} + \mathcal{L}^{(0)}_{s}\,,$ Collinear and soft dynamics factorize

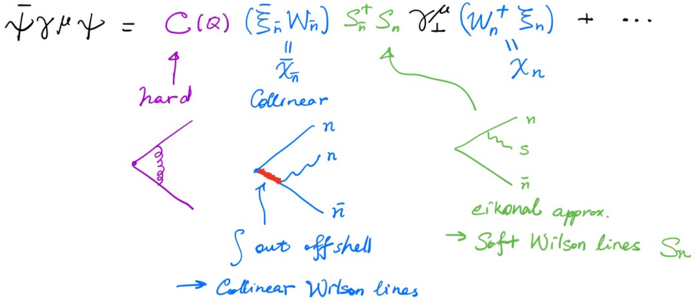
  ▷ $\mathcal{L}^{(0)}_{\text{G}}\,,$ Glauber: connect different sectors

Factorization = the Glauber contribution vanishes

- Here we assume $\mathcal{L}^{(0)}_{G}$ doesn't spoil factorization (left for future work)

Hard Operators for SIDIS

- Match QCD onto SCET $\Rightarrow$ Leading power current (operator)



- $\xi_n(x)$: n-collinear quark field, which obeys $\frac{1}{4}\not{n}\not{\bar n}\xi_n = \xi_n$ and $\frac{1}{4}\not{\bar n}\not{n}\xi_n = 0$, "good components" of the quark field.
- LP current $J^{(0)\mu} \sim \int \mathrm{d}\omega_a \mathrm{d}\omega_b \sum_f (\gamma_\perp^\mu)^{\alpha\beta}\, C_f^{(0)}(Q)\, \bar\chi^\alpha_{\bar n,\omega_b} [S_{\bar n}^\dagger S_n]\, \chi^\beta_{n,\omega_a}$
- Useful notation: $\chi_{n,\omega} = \delta(\omega - \bar n \cdot \mathcal{P})\chi_n$, $\mathcal{B}_{n\perp,\omega} = \delta(\omega + \bar n \cdot \mathcal{P})\mathcal{B}_{n\perp}$,

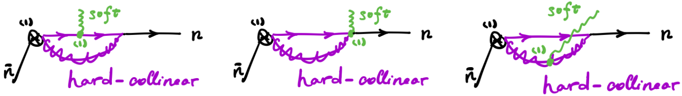
Hard Operators for SIDIS

- In general, operators are constructed using "building blocks" (No subleading collinear quark contribution (redundant in SCET)!)
  - ▷ Collinear quark and gluon χ_n, $\mathcal{B}^{\mu}_{n\perp} = \frac{1}{g}\Big[W_n^{\dagger}(x)\,\mathrm{i}D^{\mu}_{n\perp}W_n(x)\Big] \sim \lambda$
  - ▷ Soft quark and gluon $\psi_{s(n)} \sim \lambda^{3/2}$, $\mathcal{B}^{\mu}_{s(n)} \sim \lambda$
  - ▷ Momentum operators $\mathcal{P}_{\perp}$, $n \cdot \partial_s$, $\bar{n} \cdot \partial_s \sim \lambda$
- Operators get generated from two offshell scales
  - ▷ Hard $\mathcal{O}(\lambda)$ (tree-level and beyond) $\mathcal{L}_{\mathrm{h}}$

  - ▷ Hard-collinear $\mathcal{O}(\sqrt{\lambda}Q)$ (one-loop and beyond for NLP)

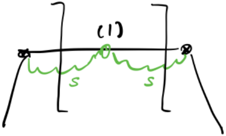
$T\big[J_{\mathrm{I}}^{(0)\mu}\mathcal{L}_{\mathrm{I}}^{(1)}\big]$, $T\big[J_{\mathrm{I}}^{(0)\mu}\mathcal{L}_{\mathrm{I}}^{(2)}\big]$, $T\big[J_{\mathrm{I}}^{(0)\mu}\mathcal{L}_{\mathrm{I}}^{(1)}\mathcal{L}_{\mathrm{I}}^{(1)}\big]$, $T\big[J_{\mathrm{I}}^{(1)\mu}\mathcal{L}_{\mathrm{I}}^{(1)}\big]$ in $\mathrm{SCET_I}$

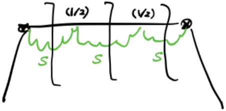
$\rightarrow$ hard scattering operators in $\mathrm{SCET_{II}}$: $\mathcal{L}_{\mathrm{hc}}$



Category of NLP for $W_i = P_i^{\mu\nu} W_{\mu\nu}$

- Kinematic power corrections: expanding $P^{\mu\nu}$ from the hadronic Breit frame ($z^\mu \propto q^\mu$) to the factorization frame ($n \propto P_N$, $\bar{n} \propto P_h$)
  $P_i^{\mu\nu} = P_i^{(0)\mu\nu} + P_i^{(1)\mu\nu} + \ldots,$
  e.g. $P_1^{\mu\nu} = \frac{1}{2}(\tilde{t}^\mu \tilde{x}^\nu + \tilde{x}^\mu \tilde{t}^\nu) = \frac{1}{2}(t^\mu x^\nu + x^\mu t^\nu) - \frac{q_T}{Q} x^\mu x^\nu + \ldots$
- Hard scattering power corrections from the hard region through $\mathcal{L}_{\mathrm{h}}^{(1)}$
- Hard scattering power corrections from the hard-collinear region through $\mathcal{L}_{\mathrm{hc}}^{(1)}$ and $T\big[\mathcal{L}_{\mathrm{hc}}^{(1/2)} \mathcal{L}_{\mathrm{dyn}}^{(1/2)}\big]$
- Subleading dynamic Lagrangian insertions (rescattering):
  $T\big[\mathcal{L}_{\mathrm{hard}}^{(0)} \mathcal{L}_{\mathrm{dyn}}^{(1/2)} \mathcal{L}_{\mathrm{dyn}}^{(1/2)}\big]$, $T\big[\mathcal{L}_{\mathrm{hard}}^{(0)} \mathcal{L}_{\mathrm{dyn}}^{(1)}\big]$





Factorization for $W^{\mu\nu}$ at Leading Power

LP current $J^{(0)\mu} \sim \sum_f (\gamma_\perp^\mu)^{\alpha\beta}\, C_f^{(0)}(Q)\, \bar\chi^\alpha_{\bar n,\omega_b} [S^\dagger_{\bar n} S_n]\, \chi^\beta_{n,\omega_a}$ 

- Plug it into $W^{(0)\mu\nu} \sim \langle N | J^{(0)\dagger\,\mu} | h, X\rangle \langle h, X | J^{(0)\nu} | N\rangle$
- Collinear fields yield quark correlators

$$\hat B_f^{\beta'\beta}(x, \vec b_T) = \Big\langle N \Big| \bar\chi^\beta_n(b_\perp)\, \delta(\omega_a - \overline{\mathcal{P}}_n)\, \chi^{\beta'}_n(0) \Big| N \Big\rangle$$

$$\hat{\mathcal{G}}_f^{\alpha\alpha'}(z, \vec b_T) = \frac{1}{2z} \sum\!\!\!\!\!\!\!\int_X \Big\langle 0 \Big| \delta(\omega_b - \overline{\mathcal{P}}_{\bar n})\, \chi^\alpha_{\bar n}(b_\perp) \Big| h, X \Big\rangle \Big\langle h, X \Big|\, \bar\chi^{\alpha'}_{\bar n}(0) \Big| 0 \Big\rangle$$

- Soft Wilson lines yield the TMD soft function

$$\mathcal{S}(b_T) = \frac{1}{N_c}\, \mathrm{tr} \Big\langle 0 \Big| \big[S^\dagger_n(b_\perp)\, S_{\bar n}(b_\perp)\big]\, \big[S^\dagger_{\bar n}(0)\, S_n(0)\big] \Big| 0 \Big\rangle\,.$$

- Combine into the quark correctors

$$B_f^{\beta'\beta}(x, \vec b_T) = \hat B_f^{\beta'\beta}(x, \vec b_T) \sqrt{\mathcal{S}(b_T)}\,, \qquad \mathcal{G}_f^{\alpha'\alpha}(z, \vec b_T) = \hat{\mathcal{G}}_f^{\alpha'\alpha}(z, \vec b_T) \sqrt{\mathcal{S}(b_T)}$$

$\Rightarrow$ Factorized leading power hadronic tensor

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T\, e^{\mathrm{i}\vec q_T\cdot\vec b_T}\, \mathcal{H}_f^{(0)}(Q)\, \mathrm{Tr}\Big[B_f(x, \vec b_T)\, \gamma_\perp^\mu\, \mathcal{G}_f(z, \vec b_T)\, \gamma_\perp^\nu \Big]\,.$$

- Hard function: $\mathcal{H}_f^{(0)}(Q) = \big|C_f^{(0)}(Q)\big|^2$

Structure Functions at Leading Power

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T \, e^{\mathrm{i}\vec{q}_T \cdot \vec{b}_T} \, \mathcal{H}_f^{(0)}(Q) \, \mathrm{Tr}\left[B_f(x, \vec{b}_T) \, \gamma_\perp^\mu \, \mathcal{G}_f(z, \vec{b}_T) \, \gamma_\perp^\nu \right] .$$

- In the momentum space, decompose into different Dirac structures

[Goeke, Metz, Schlegel '05]

$$B_f^{\beta'\beta}(x, \vec{p}_T) = \frac{1}{4} \left\{ f_1 \not{n} + \mathrm{i} h_1^\perp \frac{[\not{p}_\perp, \not{n}]}{2M_N} \right\}^{\beta'\beta} + \ldots ,$$

$$\mathcal{G}_f^{\alpha'\alpha}(z, \vec{k}_T) = \frac{1}{4} \left\{ D_1 \not{\bar{n}} + \mathrm{i} H_1^\perp \frac{[\not{k}_\perp, \not{\bar{n}}]}{2M_h} \right\}^{\alpha'\alpha}$$

- $h_1^\perp$ Boer-Mulders function, $H_1^\perp$ Collins function
- Contract $W^{(0)\mu\nu}$ with $P_{-1}^{(0)\mu\nu} = x^\mu x^\nu + y^\mu y^\nu$, $P_3^{(0)\mu\nu} = x^\mu x^\nu - y^\mu y^\nu$,

$$W_{-1}^{(0)} = \mathcal{F}\left[\mathcal{H}^{(0)} \, f_1 D_1 \right] ,$$

$$W_3^{(0)} = \mathcal{F}\left[-\frac{2\, p_{Tx} \, k_{Tx} - \vec{p}_T \cdot \vec{k}_T}{M_N M_h} \mathcal{H}^{(0)} \, h_1^\perp H_1^\perp \right] ,$$

[Collins, SCET, ...]

$$\mathcal{F}[\omega \, \mathcal{H} \, g \, D] = 2z \sum_f \int \mathrm{d}^2 p_T \, \mathrm{d}^2 k_T \, \delta^2\Big(\vec{q}_T + \vec{p}_T - \vec{k}_T \Big) \, \omega(\vec{p}_T, \vec{k}_T) \, \mathcal{H}_f(Q) \, g_f(x, p_T) \, D_f(z, k_T)$$

Kinematic Correction for W_1

- Taking

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T \, e^{\mathrm{i}\vec{q}_T \cdot \vec{b}_T} \, \mathcal{H}_f^{(0)}(Q) \, \mathrm{Tr}\left[B_f(x, \vec{b}_T) \, \gamma_\perp^\mu \, \mathcal{G}_f(z, \vec{b}_T) \, \gamma_\perp^\nu \right] ,$$

contract with $P_1^{\mu\nu} = \frac{1}{2}(\tilde{t}^\mu \tilde{x}^\nu + \tilde{x}^\mu \tilde{t}^\nu) = \frac{1}{2}(t^\mu x^\nu + x^\mu t^\nu) - \frac{q_T}{Q} x^\mu x^\nu + \dots$

$\Rightarrow$ kinematic corrections for W_1

$$\mathcal{F}\left\{ -\frac{P_{hT}}{zQ} \, \mathcal{H}^{(0)} \left[f_1 D_1 - \frac{2\, p_{Tx}\, k_{Tx} - \vec{p}_T \cdot \vec{k}_T}{M_N M_h} \, h_1^\perp H_1^\perp \right] \right\} \in W_1$$

Unique hard operator to all orders [Feige et al '17]

$$J_{\mathcal{P}}^{(1)\mu} \sim \frac{C_f^{(0)}}{2\omega_a} \bar{\chi}_{\bar{n},\omega_b} [S_{\bar{n}}^\dagger S_n] \gamma^\mu \, \mathcal{P\!\!\!/}_\perp \, \bar{n\!\!\!/} \chi_{n,\omega_a} + \text{h.c.} \sim$$

- Reparameterization $(n^\mu \to n'^\mu = n^\mu + \Delta_\perp^\mu)$ relates it with the LP one

$\Rightarrow$ The Wilson coefficient is identical to the leading power one, $C_f^{(0)}(Q)$

- Plug these currents into $J_{\mathcal{P}}^{(1)\dagger\mu} J^{(0)\nu} + J^{(0)\dagger\mu} J_{\mathcal{P}}^{(1)\nu}$

$$\hat{W}_{\mathcal{P}}^{(1)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2\vec{b}_T \, \mathcal{H}_f^{(0)}(Q) \, \mathcal{S}(b_T)$$

$$\times \left\{ \mathrm{Tr}\left[\hat{B}_{\mathcal{P}f}(x,\vec{b}_T) \, \gamma^\mu \, \hat{\mathcal{G}}_f(z,\vec{b}_T) \, \gamma^\nu \right] + \mathrm{Tr}\left[\hat{B}_f(x,\vec{b}_T) \, \gamma^\mu \, \hat{\mathcal{G}}_{\mathcal{P}f}(z,\vec{b}_T) \, \gamma^\nu \right] \right\} .$$

where $\hat{B}_{\mathcal{P}f}$ and $\hat{\mathcal{G}}_{\mathcal{P}f}$ are related to the LP corrector as

$$\hat{B}_{\mathcal{P}f}^{\beta'\beta}(x,\vec{b}_T)$$

$$\equiv \frac{1}{2Q} \theta(\omega_a) \left\{ \left\langle N \middle| \bar{\chi}_n^\beta(b_\perp^\mu) \, \left[\mathcal{P\!\!\!/}_\perp \bar{n\!\!\!/} \, \chi_{n,\omega_a}(0) \right]^{\beta'} \middle| N \right\rangle + \left\langle N \middle| \left[\bar{\chi}_n(b_\perp^\mu) \, \bar{n\!\!\!/} \, \mathcal{P\!\!\!/}_\perp^\dagger \right]^\beta \chi_{n,\omega_a}^{\beta'}(0) \middle| N \right\rangle \right\}$$

$$= \mathrm{i} \frac{1}{2Q} \frac{\partial}{\partial b_\perp^\rho} \left[\gamma_\perp^\rho \, \bar{n\!\!\!/} \, , \, \hat{B}_f(x,\vec{b}_T) \right]^{\beta'\beta} ,$$

Subleading Current: $\mathcal{P}_\perp$ Acting on the Collinear Fields

- Define $B_{\mathcal{P}f}$, $\mathcal{G}_{\mathcal{P}f}$ and $W_{\mathcal{P}}^{(1)\mu\nu}$

$$B_{\mathcal{P}f}^{\beta'\beta}(x,\vec{b}_T) \equiv \mathrm{i}\frac{1}{2Q}\frac{\partial}{\partial b_\perp^\rho}\left[\gamma_\perp^\rho\,\bar{n}\!\!\!/\,,\,B_f(x,\vec{b}_T)\right]^{\beta'\beta}\,,\ \text{where } B_f^{\beta'\beta}(x,\vec{b}_T) = \hat{B}_f^{\beta'\beta}(x,\vec{b}_T)\sqrt{\mathcal{S}(b_T)}$$

$$W_{\mathcal{P}}^{(1)\mu\nu} \equiv \frac{2z}{N_c}\sum_f\int \mathrm{d}^2\vec{b}_T\,\mathcal{H}_f^{(0)}(Q) \times\left\{\mathrm{Tr}\left[B_{\mathcal{P}f}(x,\vec{b}_T)\,\gamma^\mu\,\mathcal{G}_f(z,\vec{b}_T)\,\gamma^\nu\right] + \mathrm{Tr}\left[B_f(x,\vec{b}_T)\,\gamma^\mu\,\mathcal{G}_{\mathcal{P}f}(z,\vec{b}_T)\,\gamma^\nu\right]\right\}.$$

- Equivalent to $\hat{W}_{\mathcal{P}}^{(1)\mu\nu}$ (noticing that $(n_\mu - \bar{n}_\mu)P_i^{\mu\nu} = \mathcal{O}(P_{hT}/Q)$)

$$W_{\mathcal{P}}^{(1)\mu\nu} - \hat{W}_{\mathcal{P}}^{(1)\mu\nu} = \frac{2z}{N_c}\sum_f\int \mathrm{d}^2\vec{b}_T\,\mathcal{H}_f^{(0)}(Q)\frac{\mathrm{i}}{Q}\left(\frac{\partial}{\partial b_\perp^\rho}\sqrt{\mathcal{S}(b_T)}\right)\sqrt{\mathcal{S}(b_T)} \times\left\{(\bar{n}^\nu - n^\nu)\,\mathrm{Tr}\left[\gamma_\perp^\rho\,\hat{B}_f(x,\vec{b}_T)\,\gamma^\mu\,\hat{\mathcal{G}}_f(z,\vec{b}_T)\right] + (n^\mu - \bar{n}^\mu)\,\mathrm{Tr}\left[\hat{B}_f(x,\vec{b}_T)\,\gamma_\perp^\rho\,\hat{\mathcal{G}}_f(z,\vec{b}_T)\,\gamma^\nu\right]\right\}$$

- Same leading power functions appear, in momentum space

$$B_{\mathcal{P}f}(x,\vec{p}_T) = \frac{1}{2Q}\left[p\!\!\!/_\perp\bar{n}\!\!\!/\,,\,B_f\right] = \frac{1}{2Q}\left\{f_1 p\!\!\!/_\perp - ih_1^\perp\frac{p_T^2\,[n\!\!\!/,\bar{n}\!\!\!/]}{2M_N}\right\} + \ldots$$

Subleading Current: with $\mathcal{B}_{n\perp}$ Insertion

- Unique current to all order in α_s (denoting $\xi = \omega_c/Q$),

$$J_{\mathcal{B}}^{(1)\mu} \sim (n^\mu + \bar{n}^\mu)\int \mathrm{d}\omega_a \mathrm{d}\omega_b \mathrm{d}\omega_c \, C_f^{(1)}(Q, \xi)$$

$$\times \Big[\delta(\omega_a + \omega_c - Q)\,\delta(\omega_b - Q)\,\bar{\chi}_{\bar{n},\omega_b}[S_{\bar{n}}^\dagger S_n]\not{\mathcal{B}}_{\perp n,-\omega_c}\chi_{n,\omega_a}$$

$$+ \delta(\omega_a - Q)\,\delta(\omega_b + \omega_c - Q)\,\bar{\chi}_{\bar{n},\omega_b}\not{\mathcal{B}}_{\perp \bar{n},\omega_c}[S_{\bar{n}}^\dagger S_n]\chi_{n,\omega_a}\Big]$$

Subleading Current: with $\mathcal{B}_{n\perp}$ Insertion

Denoting $\xi = \omega_c/Q$, define the q-g-q correlators as

$$\hat{\tilde{B}}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T) \equiv Q\;\langle N|\,\big[\bar{\chi}^{\beta}_{n,\omega_a}\,\mathcal{B}^{\rho}_{\perp n,-\omega_c}\big](b^\mu_\perp)\,\chi^{\beta'}_{n}(0)\,|N\rangle\;,$$

$$\hat{\tilde{\mathcal{G}}}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T) \equiv \frac{Q}{2z}\sum\!\!\!\!\!\!\int_X \langle 0|\,\big[\bar{\chi}^{\beta}_{\bar{n},\omega_b}\,\mathcal{B}^{\rho}_{\perp\bar{n},\omega_c}\big](b^\mu_\perp)\,|h,X\rangle\,\langle h,X|\,\chi^{\beta'}_{\bar{n}}(0)\,|0\rangle$$

$$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T) = \hat{\tilde{B}}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T)\sqrt{\mathcal{S}(b_T)},$$

$$\tilde{\mathcal{G}}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T) = \hat{\tilde{\mathcal{G}}}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T)\sqrt{\mathcal{S}(b_T)}$$

$$W^{(1)\mu\nu}_{\mathcal{B}} = \frac{2z}{Q}\sum_f \int \mathrm{d}^2 b_T\, e^{\mathrm{i}\vec{q}_T\cdot\vec{b}_T}\int \mathrm{d}\xi\,\mathcal{H}^{(1)}(Q,\xi)(n^\mu + \bar{n}^\mu)$$

$$\times\,\mathrm{Tr}\Big[\tilde{B}^{\rho}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T)\,\gamma_\rho\,\mathcal{G}_f(z,\vec{b}_T)\,\gamma^\nu_\perp + B_f(x,\vec{b}_T)\,\gamma^\nu_\perp\,\tilde{\mathcal{G}}^{\rho}_{\mathcal{B}\,f}(z,\xi,\vec{b}_T)\,\gamma_\rho\Big] + \text{h.c.}\,.$$

$$\mathcal{H}^{(1)}(Q,\xi) = C^{(1)}_f(Q,\xi)\,C^{(0)}_f(Q)$$

In momentum space, $\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}f}$ can be decomposed as [Boer, Mulders, Pijlman '03] [Bacchetta, Mulders, Pijlman '04]

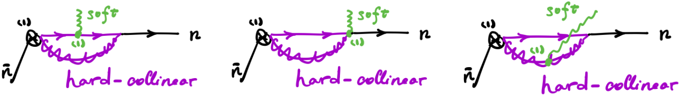
$$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}f}(x,\xi,\vec{p}_T) = \frac{xM_N}{2}\,\Big\{\Big[(\tilde{f}^\perp - \mathrm{i}\tilde{g}^\perp)\frac{p_{\perp\sigma}}{M_N}(g^{\rho\sigma}_\perp - \mathrm{i}\epsilon^{\rho\sigma}_\perp\gamma_5) + \mathrm{i}(\tilde{h} + \mathrm{i}\,\tilde{e})\gamma^\rho_\perp\Big]\frac{\not{n}}{2}\Big\}^{\beta'\beta} + \ldots$$

Vanishing Soft Contributions

- Subleading soft contributions exist in general, and are important in other processes
- In subleading SIDIS, we showed that they all vanish:

(1) Operators involving $\mathcal{B}_{s\perp}^{(n_i)\mu}$ get generated from Hard and hard-collinear

▷ $T\big[J_{\rm I}^{(1)\mu}\mathcal{L}_{\rm I}^{(1)}\big]$ in SCET$_{\rm I}$ $\rightarrow$ hard scattering operators in SCET$_{\rm II}$ in $\mathcal{L}_{\rm hc}$



Soft ME: $\hat{\mathcal{S}}_1^\rho(b_\perp) \sim \mathrm{tr}\left\langle 0 \middle| \left[S_n^\dagger(b_\perp)\, S_{\bar n}(b_\perp)\right]\left[S_{\bar n}^\dagger(0)\, S_n(0) g\mathcal{B}_{s\perp}^{(n)\rho}(0)\right] \middle| 0\right\rangle$

Vanish due to charge conjugation & parity invariance & Lorentz invariance & translation invariance of the vacuum

(2) Operators involving a $n\cdot\partial_s$, $n\cdot\mathcal{B}_s^{(\bar n)}, \ldots$ give $\frac{\partial}{\partial b_s^\mp}\mathcal{S}(b_T, b_s^+ b_s^-)\Big|_{b_s^\pm\to 0}$,

which scales linear in $\bar n$ or n under RPI-III ($n \to e^\alpha n$, $\bar n \to e^{-\alpha}\bar n$) of SCET , thus vanishes

Vanishing Soft Contributions

(3) SCET_{II} Subleading Lagrangian insertions

$$W_{\mathcal{L}}^{(1)\mu\nu} \sim \langle N|J^{(0)\dagger\,\mu}(b)|h,X\rangle\,\langle h,X|\int \mathrm{d}^4x\mathrm{d}^4y\,T\big[J^{(0)\nu}(0)\mathcal{L}^{(1/2)}(x)\mathcal{L}^{(1/2)}(y)\big]|N\rangle$$
$$+\,\langle N|J^{(0)\dagger\,\mu}(b)|h,X\rangle\,\langle h,X|\int \mathrm{d}^4x\,T\big[J^{(0)\nu}(0)\mathcal{L}^{(1)}(x)\big]|N\rangle + \ldots$$

Since μ, ν are transverse $\big(J^{(0)\mu} \sim (\gamma_\perp^\mu)^{\alpha\beta}\,C_f^{(0)}(Q)\,\bar{\chi}^\alpha_{\bar{n},\omega_b}[S^\dagger_{\bar{n}}S_n]\,\chi^\beta_{n,\omega_a}\big)$,
when contracting with $P_1^{\mu\nu} = -(\tilde{t}^\mu\tilde{x}^\nu + \tilde{x}^\mu\tilde{t}^\nu)$, $P_2^{\mu\nu} = \mathrm{i}\big(\tilde{t}^\mu\tilde{x}^\nu - \tilde{x}^\mu\tilde{t}^\nu\big)$, ...
such contributions vanish

(4) $T\big[J_{\mathrm{I}}^{(0)\mu}\mathcal{L}_{\mathrm{I}}^{(1)}\big]$, $T\big[J_{\mathrm{I}}^{(0)\mu}\mathcal{L}_{\mathrm{I}}^{(2)}\big]$, $T\big[J_{\mathrm{I}}^{(0)\mu}\mathcal{L}_{\mathrm{I}}^{(1)}\mathcal{L}_{\mathrm{I}}^{(1)}\big]$ in SCET_{I}

$\rightarrow$ hard scattering operators in $\text{SCET}_{\mathrm{II}}$
Vanish since μ, ν in $J^{(0)}$ are (again) transverse

Results

$$W_1 = \mathcal{F}\Big\{ -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\,h_1^\perp H_1^\perp\Big]$$ (Kinematic corrections)

$$- \mathcal{H}^{(0)}\Big[\frac{p_{Tx}+k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp\Big]$$ (From the $\mathcal{P}_\perp$ operators)

$$+ \mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(k_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} p_{Tx}\,\tilde{h}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{px}\,f_1\tilde{D}^\perp + \frac{M_h}{M_N} k_{Tx}\,h_1^\perp\tilde{H}\Big)\Big]\Big\}$$ (From the $\mathcal{B}_\perp$ operators)

$$\mathcal{F}[\omega\,\mathcal{H}\,g\,D] = 2z\sum_f \int \mathrm{d}^2p_T\,\mathrm{d}^2k_T\,\delta^2\big(\vec{q}_T + \vec{k}_T - \vec{p}_T\big)\,\omega(\vec{k}_T,\vec{p}_T) \times \int \mathrm{d}\xi\,\mathcal{H}_f(Q,(\xi))\,g_f(x,(\xi),k_T)\,D_f(z,(\xi),p_T)$$

- For example,

$$\mathcal{F}[k_{Tx}\mathcal{H}^{(1)}\tilde{f}^\perp D_1] = 2z\sum_f \int \mathrm{d}^2p_T\,\mathrm{d}^2k_T\,\delta^2\big(\vec{q}_T + \vec{k}_T - \vec{p}_T\big)\,k_{Tx} \times \int \mathrm{d}\xi\,\mathcal{H}_f^{(1)}(Q,\xi)\,\tilde{f}_f^\perp(x,\xi,k_T)\,D_{1f}(z,p_T)$$

Results

$$W_1 = \mathcal{F}\Big\{ -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\,h_1^\perp H_1^\perp\Big]$$ (Kinematic corrections)

$$- \mathcal{H}^{(0)}\Big[\frac{p_{Tx}+k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp\Big]$$ (From the $\mathcal{P}_\perp$ operators)

$$+ \mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(k_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} p_{Tx}\,\tilde{h}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{px}\,f_1\tilde{D}^\perp + \frac{M_h}{M_N} k_{Tx}\,h_1^\perp\,\tilde{H}\Big)\Big]\Big\}$$

(From the $\mathcal{B}_\perp$ operators)

$$W_2 = \mathcal{F}\Big\{\mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(p_{Tx}\,\tilde{g}^\perp D_1 + \frac{M_N}{M_h} k_{Tx}\,\tilde{e}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{Tx}\,f_1\tilde{G}^\perp + \frac{M_h}{M_N} p_{Tx}\,h_1^\perp\,\tilde{E}\Big)\Big]\Big\}$$

$$\mathcal{F}[\omega\,\mathcal{H}\,g\,D] = 2z\sum_f \int \mathrm{d}^2p_T\,\mathrm{d}^2k_T\,\delta^2\big(\vec{q}_T + \vec{k}_T - \vec{p}_T\big)\,\omega(\vec{k}_T,\vec{p}_T) \times \int \mathrm{d}\xi\,\mathcal{H}_f(Q,(\xi))\,g_f(x,(\xi),k_T)\,D_f(z,(\xi),p_T)$$

New in our results

- Vanishing of the subleading soft contributions
- Soft function, same as leading power (as conjectured in [Bacchetta et al '19])
- Two hard functions for all NLP structure functions, $\mathcal{H}^{(0)}(Q)$ and $\mathcal{H}^{(1)}(Q,\xi)$
- Dependence on ξ in $\mathcal{H}^{(1)}(Q,\xi)$ and the functions $\tilde{f}^\perp$, $\tilde{D}^\perp$, ...

Structure Functions with Full Spin Dependence

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{Q^2}\frac{y}{z}\frac{\kappa_\gamma}{1-\epsilon}\Big[(L{\cdot}W)_{UU} + S_L(L{\cdot}W)_{UL} + \lambda_\ell(L{\cdot}W)_{LU} + \lambda_\ell S_L(L{\cdot}W)_{LL} + S_T(L{\cdot}W)_{UT} + \lambda_\ell S_T(L{\cdot}W)_{LT}\Big],$$

$$(L{\cdot}W)_{UU} = W_{UU,T} + \epsilon W_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi_h)W_{UU}^{\cos(\phi_h)} + \epsilon\cos(2\phi_h)W_{UU}^{\cos(2\phi_h)},$$

$$(L{\cdot}W)_{UL} = \sqrt{2\epsilon(1+\epsilon)}\sin(\phi_h)W_{UL}^{\sin(\phi_h)} + \epsilon\sin(2\phi_h)W_{UL}^{\sin(2\phi_h)},$$

$$(L{\cdot}W)_{LU} = \sqrt{2\epsilon(1-\epsilon)}\sin(\phi_h)W_{LU}^{\sin(\phi_h)},$$

$$(L{\cdot}W)_{LL} = \sqrt{1-\epsilon^2}W_{LL} + \sqrt{2\epsilon(1-\epsilon)}\cos(\phi_h)W_{LL}^{\cos(\phi_h)},$$

$$(L{\cdot}W)_{UT} = \sin(\phi_h-\phi_S)\left[W_{UT,T}^{\sin(\phi_h-\phi_S)} + \epsilon W_{UT,L}^{\sin(\phi_h-\phi_S)}\right] + \epsilon\left[\sin(\phi_h+\phi_S)W_{UT}^{\sin(\phi_h+\phi_S)} + \sin(3\phi_h-\phi_S)W_{UT}^{\sin(3\phi_h-\phi_S)}\right] + \sqrt{2\epsilon(1+\epsilon)}\left[\sin(\phi_S)W_{UT}^{\sin(\phi_S)} + \sin(2\phi_h-\phi_S)W_{UT}^{\sin(2\phi_h-\phi_S)}\right],$$

$$(L{\cdot}W)_{LT} = \sqrt{1-\epsilon^2}\cos(\phi_h-\phi_S)W_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\left[\cos(\phi_S)W_{LT}^{\cos(\phi_S)} + \cos(2\phi_h-\phi_S)W_{LT}^{\cos(2\phi_h-\phi_S)}\right].$$

- We also have results for spin-dependent $\mathcal{O}(P_{hT}/Q)$ structure functions

Structure Function with Full Spin Dependence

- For example

$$W_{UT}^{\sin\phi_S} = \mathcal{F}\Bigg\{ -\frac{q_T}{2Q}\,\mathcal{H}^{(0)}\left(\frac{k_{Tx}}{M_N} f_{1T}^{\perp} D_1 - \frac{2p_{Tx}}{M_h} h_1 H_1^{\perp}\right) \text{(Kinematic corrections)}$$

$$+\,\mathcal{H}^{(0)}\left(-\frac{k_T^2 + \vec{k}_T\cdot\vec{p}_T}{2M_N Q} f_{1T}^{\perp} D_1 + \frac{p_T^2 + \vec{k}_T\cdot\vec{p}_T}{M_h Q} h_1 H_1^{\perp}\right) \text{(From the } \mathcal{P}_\perp \text{ operators)}$$

$$+\mathcal{H}^{(1)}\Bigg[\frac{xM_N}{Q}\left(2\tilde{f}_T D_1 - \frac{\vec{k}_T\cdot\vec{p}_T}{M_N M_h}\left(\tilde{h}_T - \tilde{h}_T^{\perp}\right) H_1^{\perp}\right)$$

$$-\frac{M_h}{zQ}\left(2h_1\tilde{H} + \frac{\vec{k}_T\cdot\vec{p}_T}{M_N M_h}\left(g_{1T}\tilde{G}^{\perp} + f_{1T}^{\perp}\tilde{D}^{\perp}\right)\right)\Bigg]\Bigg\} \text{(From the } \mathcal{B}_{n\perp} \text{ operators)}$$

Discussion

Anomalous dimensions

- Rapidity anomalous dimension is the same as at leading power
$\tilde{B}^{\rho\,\beta'\,\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T,\mu,\zeta) = \hat{\tilde{B}}^{\rho\,\beta'\,\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T,\mu,\nu^2/\zeta)\sqrt{\mathcal{S}(b_T,\mu,\nu)}$,

$$\Rightarrow \frac{\mathrm{d}\log \tilde{B}^{\rho\,\beta'\,\beta}_{\mathcal{B}\,f}}{\mathrm{d}\log\zeta} = \frac{1}{4}\frac{\mathrm{d}\log\mathcal{S}}{\mathrm{d}\log\nu} = \frac{1}{4}\gamma_\nu(\mu,b_T)$$

- Anomalous dimension of $C_f^{(1)}$ have been calculated to one loop, with single log dependence on ξ [Beneke et al, '17 '18]

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_f^{(1)}(Q,\xi,\mu) = \int \frac{\mathrm{d}\xi'}{\xi'}\gamma^{(1)}_{f\,f'}(\xi,\xi',Q,\mu)C^{(1)}_{f'}(\xi',Q,\mu)$$

- Complimentary results from [Vladimirov, Moos, Scimemi '21]:
  - ▷ also confirmed the rapidity divergence is the same as LP
  - ▷ calculated $C_f^{(1)}$ at $\mathcal{O}(\alpha_s)$, which is useful for improving precision

Comparison with Literature at Tree Level

- Literature [Mulders, Tangerman '95, Bacchetta et al '06]:
  uses different operator basis (subl. quark + qgq correlator)
  - ▷ $\Phi^{\beta\beta'}(x,\vec{b}_T) \sim \langle N|\bar{\psi}^{\beta'}(b)W_n(b)W_n^\dagger(0)\psi^\beta(0)|N\rangle$
    $$\sim \left(f_1 + \mathrm{i}h_1^\perp \frac{\not{k}_\perp}{M_N} + \cdots\right)\frac{\not{n}}{2} + \frac{xM_N}{2Q}\left\{f^\perp \frac{\not{k}_\perp}{M_N} + \frac{\mathrm{i}}{\sqrt{2}} h \frac{[\not{n},\bar{\not{n}}]}{2} + \cdots\right\} + \cdots$$
  - ▷ $$(\Phi_D^\rho)^{\beta\beta'}(x,\vec{k}_T) \sim \langle P|\bar{\psi}^{\beta'}(b)W_n(b)W_n^\dagger(0)\mathrm{i}D^\rho(0)\psi^\beta(0)|P\rangle\ .$$
    $$\tilde{\Phi}_A^\rho(x,\vec{k}_T) = \Phi_D^\rho(x,k_T) - k_T^\rho\Phi(x,k_T) \ni \tilde{f}^\perp,\ \tilde{h} \qquad \text{(No } \xi \text{ dependence)}\,,$$
  - ▷ Related by EOMs: $f_1 + x\tilde{f}^\perp = xf^\perp\,, \qquad x\tilde{h} - \frac{k_T^2}{M_N^2}h_1^\perp = xh$
- Our approach: purely LP qq correlator, NLP $\mathcal{P}_\perp$ and $\mathcal{B}_{n\perp}$ operators

$$\hat{B}^{\beta\beta'} \sim \langle N|\bar{\chi}_n^{\beta'}(b_\perp)\chi_{n,\omega_a}^\beta(0)|N\rangle \sim (\not{n}\bar{\not{n}})\,\Phi(x,\vec{b}_T)\,(\bar{\not{n}}\not{n}) \sim (f_1+\cdots)\frac{\not{n}}{2}$$

$$\hat{B}_{\mathcal{P}}^{\beta\beta'}(x,\vec{k}_T) = \frac{k_{\perp,\rho}}{2Q}\left[\gamma_\perp^\rho\,\bar{\not{n}}\,,\,\hat{B}_{f/N}(x,\vec{k}_T)\right]^{\beta\beta'} = \frac{1}{2Q}\left\{f_1\not{k}_\perp + \cdots - \mathrm{i}h_1^\perp\frac{k_T^2}{4M_N}[\not{n},\bar{\not{n}}]\right\}$$

$$\hat{B}_{\mathcal{B}}^{\rho\,\beta'\beta}(x,\xi\equiv\omega_c/Q,\vec{b}_T) \equiv Q\,\langle N|\left[\bar{\chi}_{n,\omega_a}^\beta\,\mathcal{B}_{\perp n,-\omega_c}^\rho\right](b_\perp^\mu)\,\chi_n^{\beta'}(0)\,|N\rangle \ni \tilde{f}^\perp,\ \tilde{h}\,,$$

- NLP quark can be expressed in terms of $\mathcal{P}_\perp$ and $\mathcal{B}_{n\perp}$ operators

$$\psi \sim \hat{\xi}_n + \frac{1}{\mathrm{i}\bar{n}\cdot D}\mathrm{i}\not{D}_\perp \frac{\not{\bar{n}}}{2}\hat{\xi}_n$$
$$= \hat{\xi}_n + W_n \frac{1}{\bar{n}\cdot\mathcal{P}}\left(\not{\mathcal{P}}_{n\perp} + g\not{\mathcal{B}}_{n\perp}\right)\frac{\not{\bar{n}}}{2} W_n^\dagger \hat{\xi}_n + \mathcal{O}(\lambda^2)$$

- At leading order, $C_f^{(1)}$ is independent on ξ from tree level matching. ξ can be integrated in qgq correlators

$$\int \mathrm{d}\xi\, \hat{B}^\rho_{\mathcal{B}}(x,\xi,\vec{b}_T) \to \tilde{\Phi}^\rho_A(x,\vec{b}_T)\,, \qquad \int \mathrm{d}\xi\, \tilde{f}^\perp(x,\xi,p_T) = \tilde{f}^\perp(x,p_T)$$

⇒ Reproduce literature using EOMs ($f_1 + x\tilde{f}^\perp = xf^\perp$, $x\tilde{h} - \frac{k_T^2}{M_N^2}h_1^\perp = xh$) (taking W_1 as example)

$$\frac{x}{2z}W_1 \to \frac{2M_N}{Q}\mathcal{C}\left\{\frac{-k_{Tx}}{M_N}\left[(f_1 + x\tilde{f}^\perp)D_1 + \frac{M_h}{M_N}xh_1^\perp\frac{\tilde{H}}{z}\right]\right.$$
$$\left. - \frac{p_{Tx}}{M_h}\left[\left(x\tilde{h} - \frac{k_T^2}{M_N^2}h_1^\perp\right)H_1^\perp + \frac{M_h}{M_N}f_1\frac{\tilde{D}^\perp}{z}\right]\right\}.$$

Summary & Outlook

$$W_1 = \mathcal{F}\Big\{ -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\,h_1^\perp H_1^\perp\Big]$$ (Kinematic corrections)

$$-\,\mathcal{H}^{(0)}\Big[\frac{p_{Tx}+k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp\Big]$$ (From the $\mathcal{P}_\perp$ operators)

$$+\,\mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(k_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} p_{Tx}\,\tilde{h}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{px}\,f_1\tilde{D}^\perp + \frac{M_h}{M_N} k_{Tx}\,h_1^\perp\tilde{H}\Big)\Big]\Big\}$$ (From the $\mathcal{B}_\perp$ operators)

- Demonstrated soft contribution at NLP is the same as LP
- Derived factorization of $W^{\mu\nu}$ at NLP, including contribution from subleading operators with insertion of $\mathcal{P}_\perp$ and $\mathcal{B}_\perp$
- Showed factorization formulae for NLP structure functions
- Future Directions: phenomenology including perturbative and resummation effects for NLP structure functions

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$$W_1 = \mathcal{F}\Big\{ -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\,h_1^\perp H_1^\perp\Big] \qquad \text{(Kinematic corrections)}$$

$$- \mathcal{H}^{(0)}\Big[\frac{p_{Tx}+k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp\Big] \qquad \text{(From the } \mathcal{P}_\perp \text{ operators)}$$

$$+ \mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(k_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} p_{Tx}\,\tilde{h}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{px}\,f_1\,\tilde{D}^\perp + \frac{M_h}{M_N} k_{Tx}\,h_1^\perp\,\tilde{H}\Big)\Big]\Big\}$$

(From the $\mathcal{B}_\perp$ operators)

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Thanks for your attention!