Improving Lattice QCD Calculation of the Collins-Soper Kernel

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in collaboration with
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TMD Collaboration Meeting
June 15–17, 2022

TMD Quantities Probe Non-Perturbative Physics

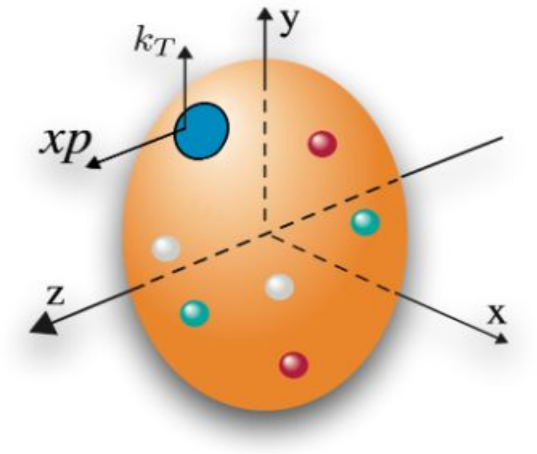
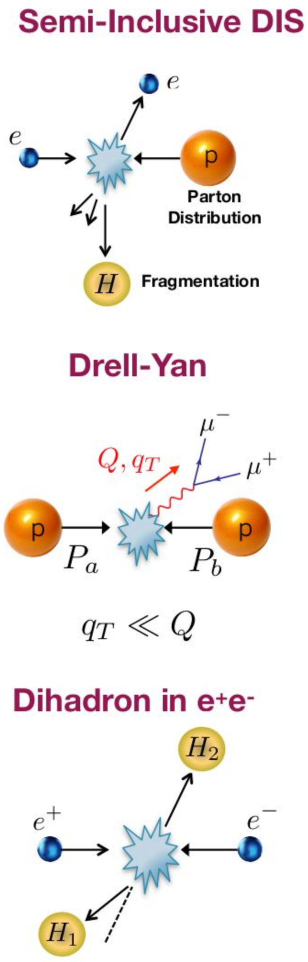


Fig. from TMD Handbook (modified).

- Transverse motion of partons in hadrons gives rise to TMD functions in factorized cross-sections.
- **Together with evolution equations**, TMDs let us study hadronic structure and test SM physics.



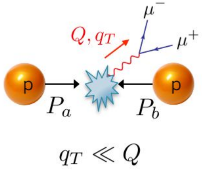
- For example, TMDs **and their evolution at non-perturbative scales** may impact QCD theory uncertainties of the W mass measurement — talk by Johannes Michel yesterday.

Fig. from TMD Handbook.

Collins-Soper (CS) Kernel is Important to the TMD Program

- Renormalization of rapidity divergences in TMDs leads to the Collins-Soper scale ζ and the evolution equation

e.g. TMD PDFs in Drell-Yan:



Fourier conjugate to parton's transverse momentum.

$$\zeta \frac{\mathrm{d}}{\mathrm{d}\zeta} \ln f_{i/h}(x, \mathbf{b}_\mathrm{T}; \mu, \zeta) = \frac{1}{2} \underbrace{\gamma_\zeta^i(\mu, b_\mathrm{T})}.$$

Fourier-transformed TMD for parton *i* in hadron *h*.

Collins-Soper Kernel (independent of external state *h*).

- The CS kernel is thus required to
  - Sum large logarithms in ζ and
  - Relate TMDs at different Collins-Soper scales.

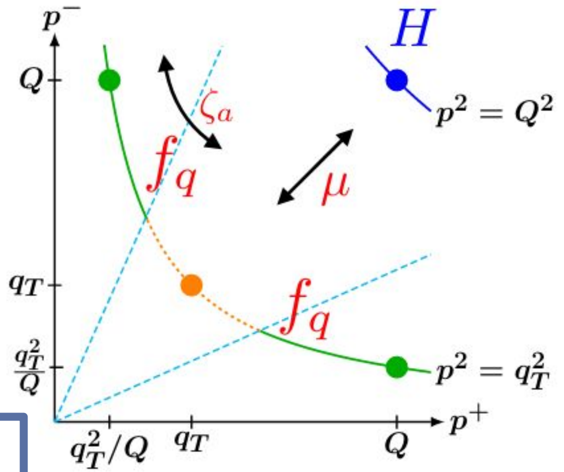
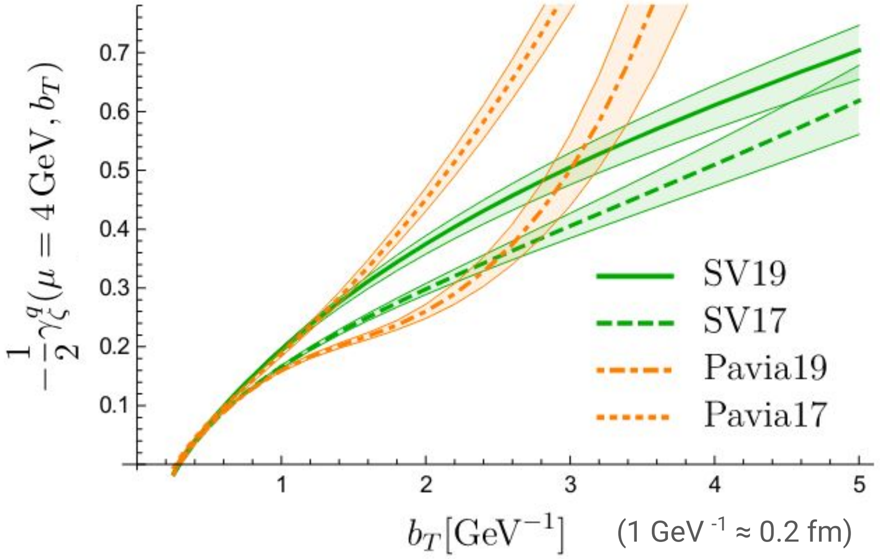


Fig. from Iain Stewart's talk at Lattice 2021.

$$\zeta_a \zeta_b = Q^4$$

The Collins-Soper (CS) Kernel is Non-Perturbative

- Even if μ is perturbative, $\gamma_\zeta^i(\mu, b_\mathrm{T})$ is non-perturbative at large b_T.
- The CS Kernel can be extracted from global fits to DY and SIDIS data.
- Non-perturbative modeling is significant for $b_\mathrm{T} \gtrsim 0.2\ \mathrm{fm}$.
- Non-perturbative CS kernel is a possible target for lattice QCD.

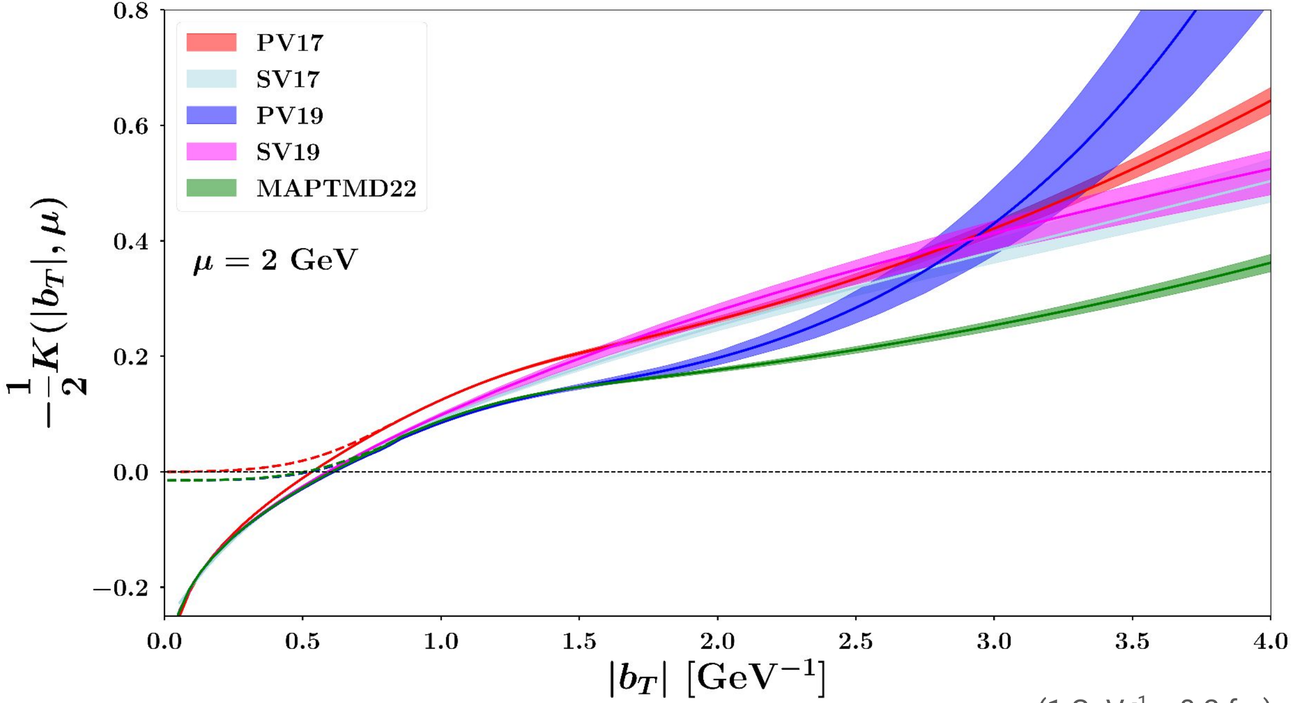


(1 GeV $^{-1}$ ≈ 0.2 fm)

Fig. from Vladimirov, Phys. Rev. Lett. 125 (2020), 2003.02288 [notation modified]

SV19 - Scimemi and Vladimirov, JEHP 06 (2020), 1912.06532;
SV17 - Scimemi and Vladimirov, Eur. Phys. J. C78 (2018), 1706.01473;
Pavia19 - Bacchetta et. al, JEHP 07 (2020), 1912.07550;
Pavia17 - JHEP 06, (2017), 1703.10157.

And another result from yesterday:



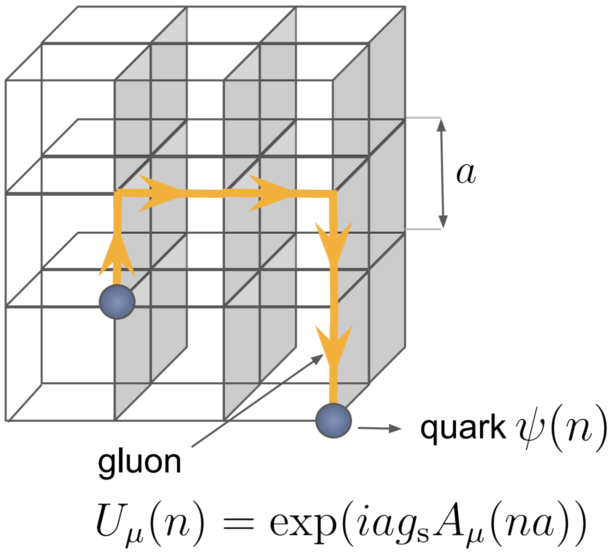
(1 GeV $^{-1}$ ≈ 0.2 fm)

Fig. from Bacchetta et. al [MAP], 2206.07598

LQCD Can Provide Non-Perturbative Input for the CS Kernel

- Since $\zeta \sim Q^2 \propto \mathbf{P}^2$, γ_ζ may be defined through ratios of certain matrix elements with different external-state momenta P.

- These matrix elements are lightlike correlations of **staple-shaped operators** coming from factorization formulas — but in LQCD, only spacelike correlations of related operators can be computed. **Calculations must relate space-like and light-like correlations.**



$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi \exp(-S[A,\bar{\psi},\psi]) \longrightarrow \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\text{conf}}} \sum_k \mathcal{O}(U^{(k)}),$$

with field configurations $U^{(k)}$ distributed according to $\exp(-S[U])$.

Extracting CS Kernel from LQCD Input at Large Momentum

Large Momentum Effective Theory (LaMET) provides a framework to match lightlike and boosted spacelike matrix elements, up to power corrections.

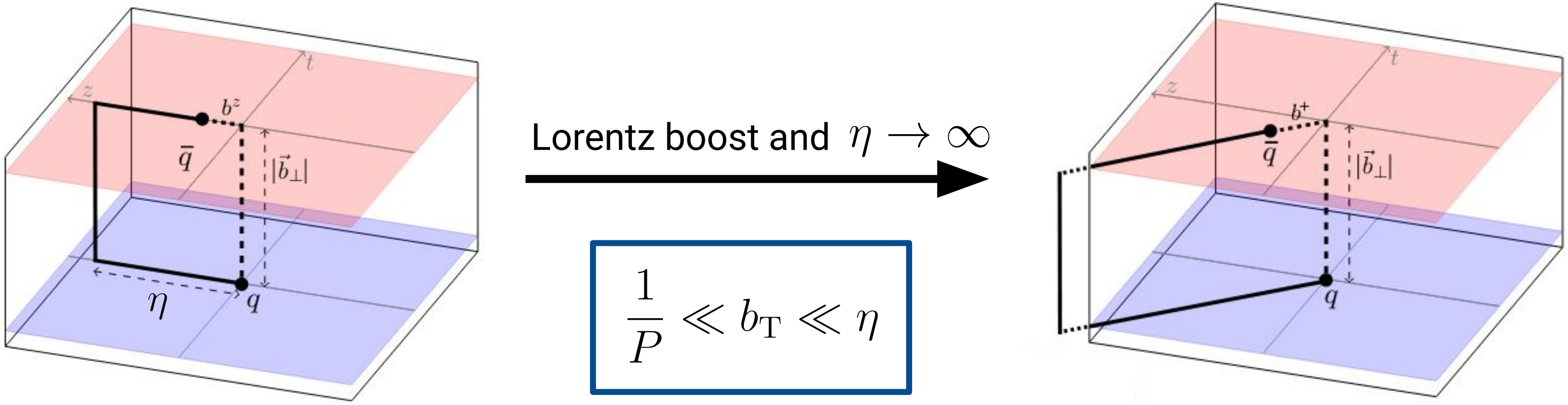


Fig. by Ebert, Stewart, Zhao, JHEP 1909 (2019) (notation changed).

Ji, Sun, Xiong and Yuan, PRD91 (2015);
Ji, Jin, Yuan, Zhang and Zhao, PRD99 (2019);
Ebert, Stewart, Zhao, PRD99 (2019), JHEP09 (2019) 037;
Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
Vladimirov and Schäfer, PRD 101 (2020);
Ebert, Schindler, Stewart and Zhao, JHEP04 (2022) 178.

Two Observables to Compute the CS Kernel with LQCD + LaMET

- Using **beam functions** from factorization formulas for cross-sections for processes such as Drell-Yan.
- The CS kernel is computed from ratios of beam functions.
- Beam functions are matched onto quasi-beam functions in LaMET.

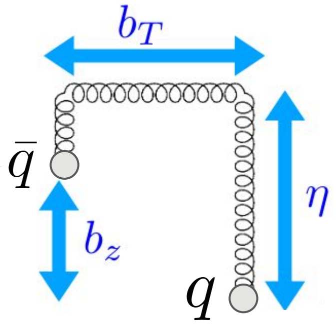
$$\gamma_\zeta^q \propto \ln \frac{\tilde{B}(x, \mathbf{b}_\mathrm{T}, P_1)}{\tilde{B}(x, \mathbf{b}_\mathrm{T}, P_2)}$$

- Using **quasi-TMD wavefunctions** from the factorization formula for a form-factor of a large-momentum pseudoscalar meson.
- The CS kernel is computed from ratios of TMD wavefunctions (WFs).
- TMD WFs are matched onto quasi-TMD WFs in LaMET.

$$\gamma_\zeta^q \propto \ln \frac{\tilde{\psi}(x, \mathbf{b}_\mathrm{T}, P_1)}{\tilde{\psi}(x, \mathbf{b}_\mathrm{T}, P_2)}$$

space-like matrix elements computed in LQCD

Two Observables to Compute the CS Kernel with LQCD + LaMET



Both approaches involve staple-shaped operators,

$$\mathcal{O}_\Gamma(b^\mu, z^\mu, \eta) = \bar{q}(z^\mu + b^\mu)\frac{\Gamma}{2}\widetilde{W}(\eta; b^\mu; z^\mu)q(z^\mu)$$

in respective matrix elements computed in LQCD:

1. Quasi-beam functions with $\Gamma \in \{\gamma^z, \gamma^t\}$ from two- and three-point functions,

$$\tilde{B}_\Gamma(b^z, \mathbf{b}_\mathrm{T}, \eta, P^z) = \langle \pi(P^z)|\mathcal{O}_\Gamma(b^\mu, 0, \eta)|\pi(P^z)\rangle;$$

2. Unsubtracted quasi-TMD WFs with $\Gamma \in \{\gamma^z\gamma^5, \gamma^t\gamma^5\}$ from two-point functions:

$$\tilde{\psi}(b^z, \mathbf{b}_\mathrm{T}, \eta, P^z) \propto \langle 0|\mathcal{O}_\Gamma(b^\mu, -P^z, \eta)|\pi(P^z)\rangle.$$

Fig. from Yong Zhao's Talk at CPHI-2022 (modified).

Lower computational cost, especially at large momenta and physical masses.

Two Observables to Compute the CS Kernel with LQCD + LaMET

With bare matrix elements computed, their ratio yields the CS kernel after:

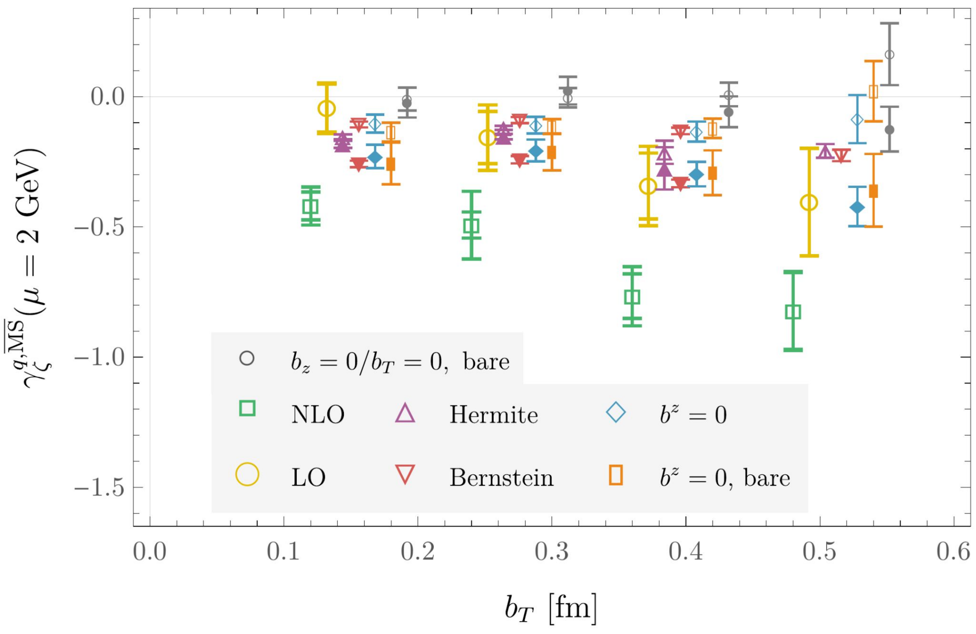
1. Renormalization; * Complicated by operator mixing
2. $\eta \to \infty$ extrapolation; * Needs careful treatment of divergences before taking the ratio.
3. Fourier Transform; * FT after analytic fits or a discrete FT.
4. Perturbative Matching. * NLO effects significant.

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{C_q^{\text{TMD PDF}}(\mu, xP_2^z)}{C_q^{\text{TMD PDF}}(\mu, xP_1^z)} \times \frac{\int db^z e^{ib^z xP_1^z} P_2^z \lim_{\eta\to\infty} \tilde{B}_q^{\text{ren.}}(\mu, b^z, \mathbf{b}_T, \eta, P_1^z)}{\int db^z e^{ib^z xP_1^z} P_1^z \lim_{\eta\to\infty} \tilde{B}_q^{\text{ren.}}(\mu, b^z, \mathbf{b}_T, \eta, P_2^z)}\right] + \mathcal{O}\left(\frac{1}{(xP^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(xP^z)^2}\right)$$

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{C_q^{\text{TMD WF}}(\mu, x, P_2^z)}{C_q^{\text{TMD WF}}(\mu, x, P_1^z)} \times \frac{\int db^z e^{ib^z xP_1^z} \lim_{\eta\to\infty} \tilde{\psi}_\pi^{\text{ren.}}(\mu, b^z, \mathbf{b}_T, \eta, P_1^z)}{\int db^z e^{ib^z xP_1^z} \lim_{\eta\to\infty} \tilde{\psi}_\pi^{\text{ren.}}(\mu, b^z, \mathbf{b}_T, \eta, P_2^z)}\right] + \mathcal{O}\left(\frac{1}{(xP^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(xP^z)^2}\right)$$

power corrections sensitive to the numerical setup in LQCD.

Systematic Uncertainties in Analyses are Significant



Same LQCD data with different treatments of matching, Fourier transform and renormalization leads to significant systematic effects and changes in uncertainty estimates.

$$\gamma_\zeta^q(\mu, b_{\rm T}) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{C_q^{\rm TMD\ PDF}(\mu, xP_2^z)}{C_q^{\rm TMD\ PDF}(\mu, xP_1^z)} \times \frac{\int db^z e^{ib^z xP_1^z} P_2^z \lim_{\eta\to\infty} \tilde{B}_q^{\rm ren.}(\mu, b^z, \mathbf{b}_{\rm T}, \eta, P_1^z)}{\int db^z e^{ib^z xP_1^z} P_1^z \lim_{\eta\to\infty} \tilde{B}_q^{\rm ren.}(\mu, b^z, \mathbf{b}_{\rm T}, \eta, P_2^z)} \right] + \mathcal{O}\left(\frac{1}{(xP^z b_{\rm T})^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z)^2} \right)$$

Fig. from Shanahan, Wagman, Zhao, Phys.Rev.D 104 (2021), 2107.11930

LQCD Calculations are Broadly Consistent; Actions and Systematics Differ

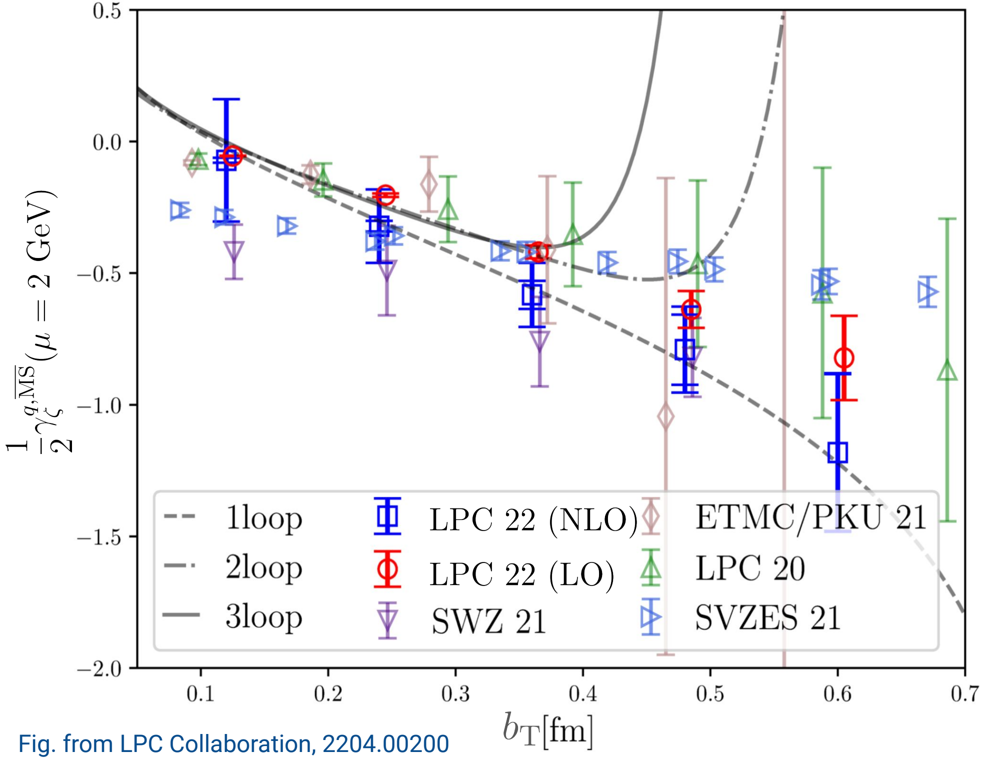


Fig. from LPC Collaboration, 2204.00200
[axes and legend labels modified]

Approach	Collaboration
Quasi-Beam Functions	**SWZ 21** PRD 104 (2021)
Quasi-TMD Wavefunctions	**LPC 20** PRL 125 (2020)
	ETMC/PKU 21 PRL 128 (2022)
	LPC 22 2204.00200
Mellin Moments of Quasi-TMDs	**SVZES 21** JHEP 08 (2021)

Comparisons with Phenomenology are Preliminary

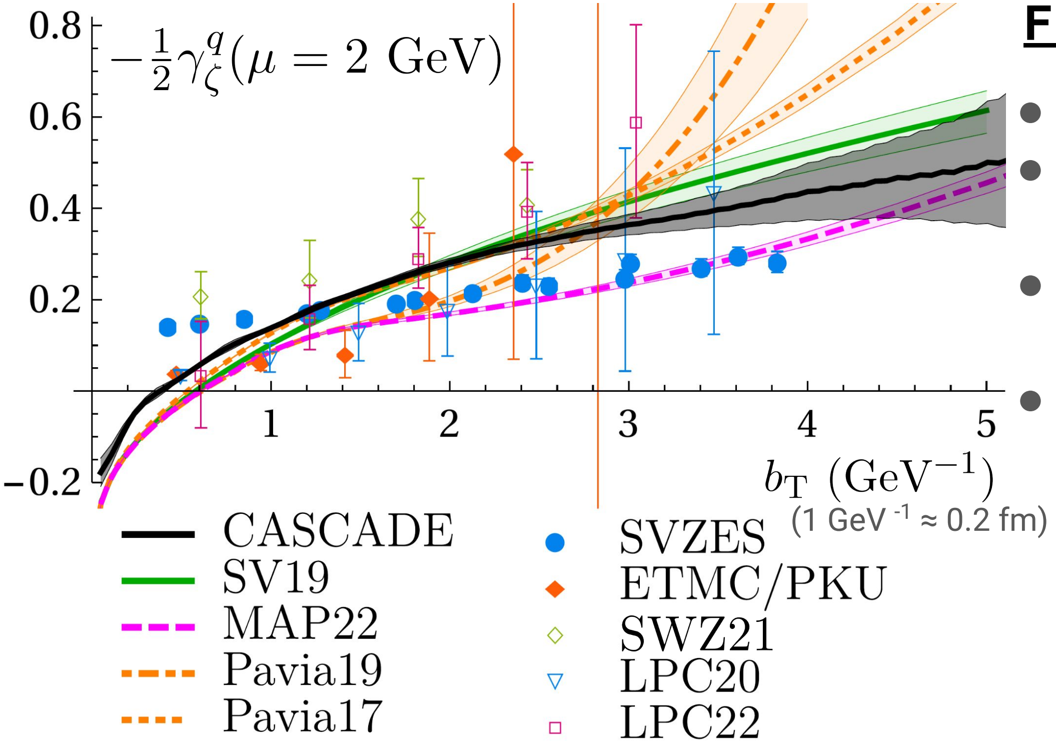


Fig. from Martinez and Vladimirov, 2206.01105
[axes and legend labels modified]

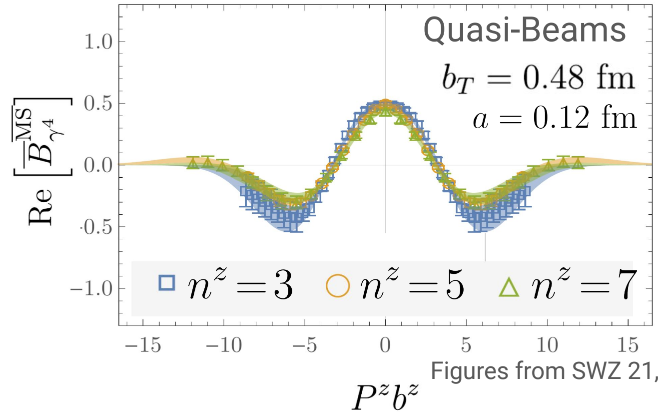
Further Improvements Needed:

- Lattice renormalization required.
- Fourier transform systematics need further study.
- Power corrections can be sensitive to valence quark masses.
- Discretization effects at small b_T unquantified.

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{C_q^{\text{TMD PDF}}(\mu, xP_2^z)}{C_q^{\text{TMD PDF}}(\mu, xP_1^z)} \times \frac{\int db^z e^{ib^z xP_1^z} P_2^z \lim_{\eta\to\infty} \tilde{B}_q^{\text{ren.}}(\mu, b^z, \mathbf{b}_T, \eta, P_1^z)}{\int db^z e^{ib^z xP_1^z} P_1^z \lim_{\eta\to\infty} \tilde{B}_q^{\text{ren.}}(\mu, b^z, \mathbf{b}_T, \eta, P_2^z)} \right] + \mathcal{O}\left(\frac{1}{(xP^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(xP^z)^2} \right)$$

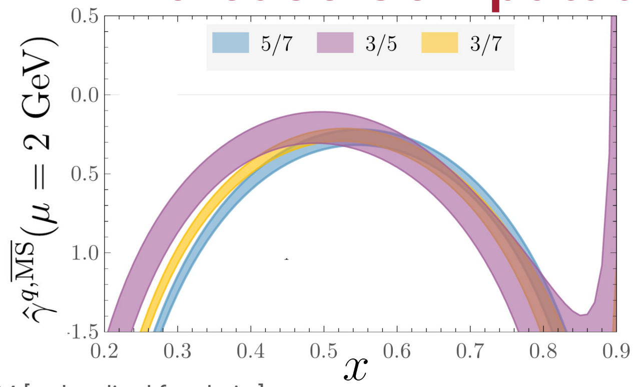
Observable	Collaboration	LQCD Setup	Matching	Fourier Transform	Operator Mixing
Quasi-Beam Functions	**SWZ 20** PRD 102 (2020)	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.219$ quenched	LO	**Yes**	✔
	SWZ 21 PRD 104 (2021)	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.129$	**NLO**	**Yes**	✔
Quasi-TMD Wavefunctions	**LPC 20** PRL 125 (2020)	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.067$	LO	N/A	✔
	PKU/ETMC 21 PRL 128 (2022)	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.063$	LO	N/A	✗
	LPC 22 2204.00200	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.067$	**NLO**	**Yes**	✗
	This work (in progress)	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.007$	**NLO**	**Yes**	✔
Mellin Moments of Quasi-TMDs	**SVZES 21** JHEP 08 (2021)	$\frac{(m_\pi^{\text{val}})^2}{(P^+_{\text{max}})^2} = 0.035$	**NLO**	N/A	✗

Target Improvements in the FT Increase Computational Costs

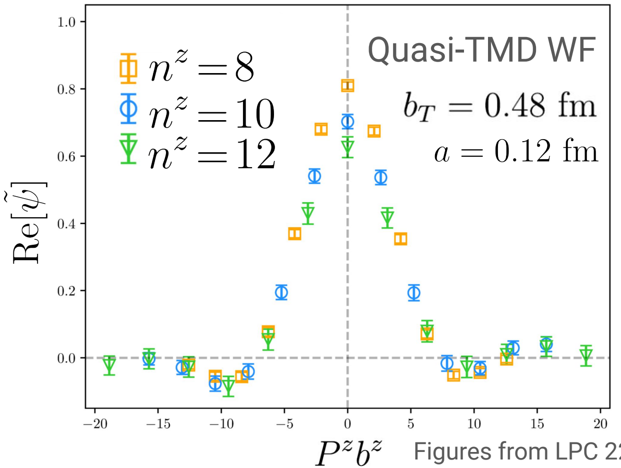


FT

& take ratios

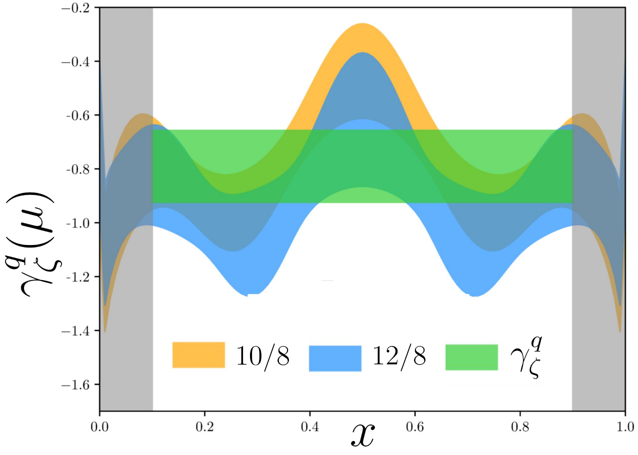


Figures from SWZ 21, Phys.Rev.D 104 [style edited for clarity]



FT

& take ratios

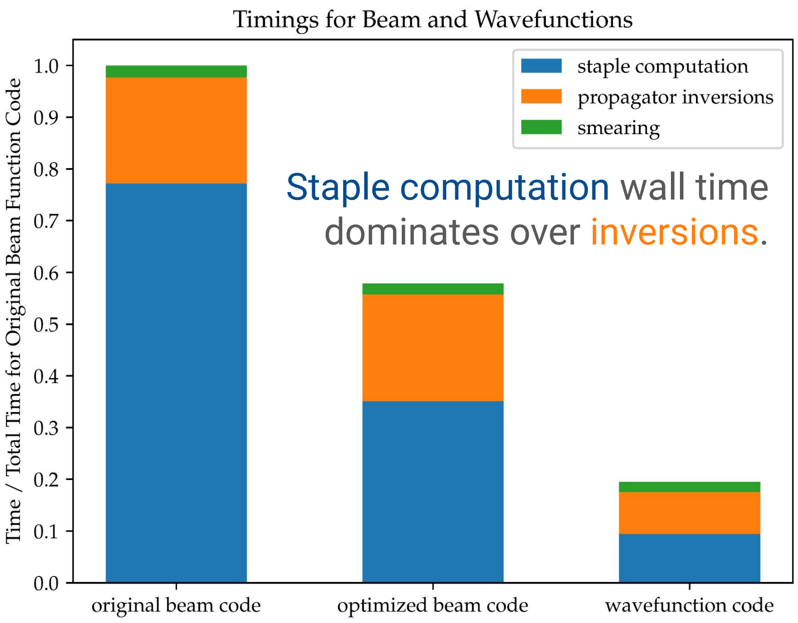


Figures from LPC 22, 2204.00200 [style edited for clarity]

- Aim to reach larger $|P^z b^z|$ to reduce systematic uncertainties in the Fourier Transform.
- Use ~physical m_π^{val} to remove partial quenching and suppress power corrections.

These lead to greater computational costs.

CS Kernel from Quasi-TMD Wavefunctions:
Recent Developments Increase Computational Efficiency



Estimate of fractional truncation effects
from Shanahan, Wagman, Zhao, Phys.Rev.D 102 (2020), 2003.06063

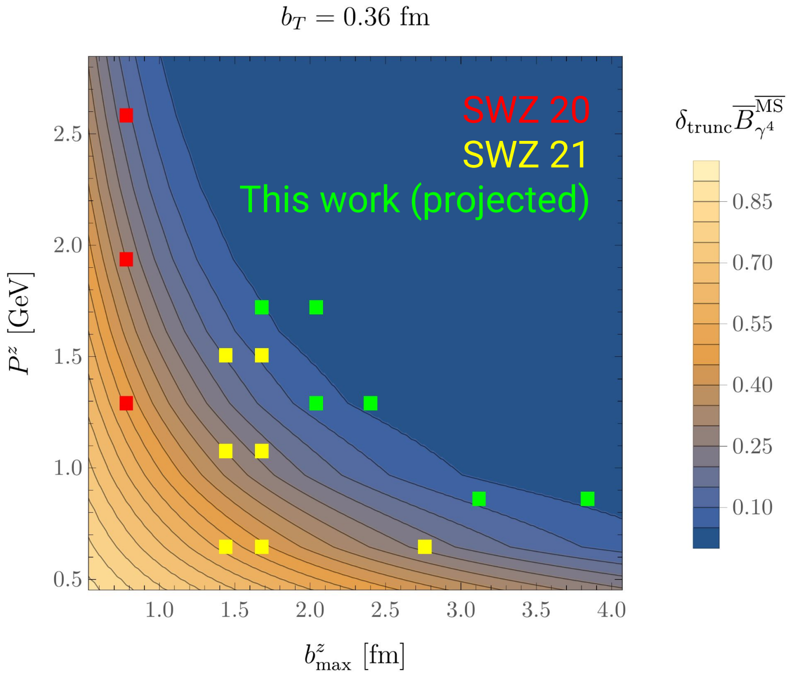
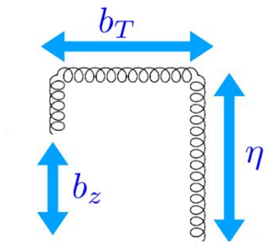


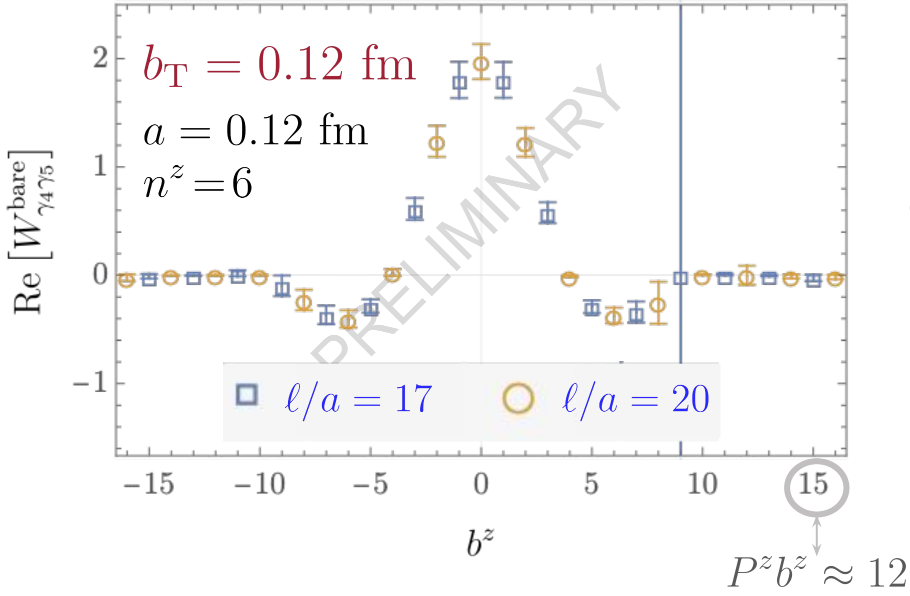
Fig. from Mike Wagman's Talk at USQCD 2022 Meeting

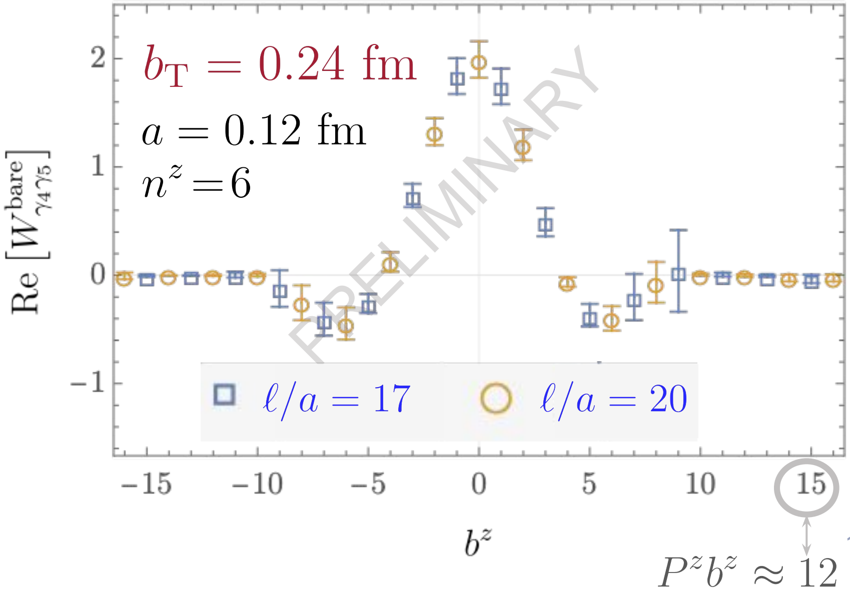
Preliminary Data Looks Promising

- Take ratio with $\tilde{\psi}(P^z = 0, \mathrm{b_T}, b^z = 0)$ to take care of ℓ dependence.
- Have bare data for $n^z \in \{0, 4, 6\}$ and $n^z = 8$ is underway.
- No $\eta \to \infty$ extrapolation yet.



$\ell = 2\eta - b_z$





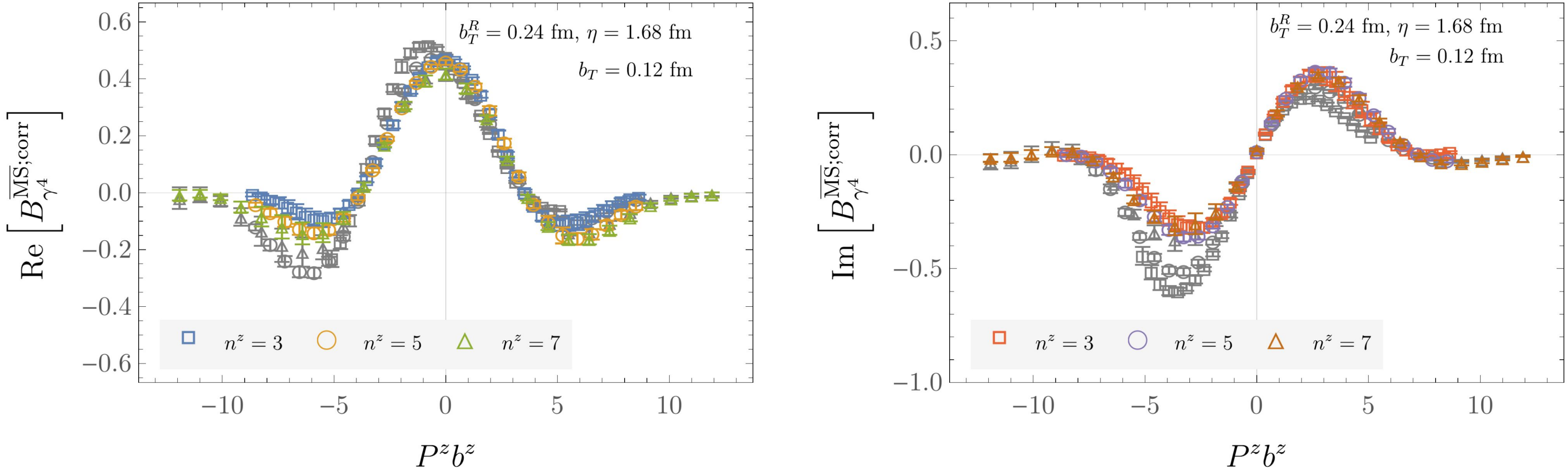
Summary and Outlook

- Determination of the Collins-Soper Kernel is critical to the TMD program.
- Non-Perturbative CS kernel can be determined with LQCD+LaMET.
- Further improvements are needed in LQCD calculations, which requires larger computational costs.
- Preliminary studies suggest quasi-TMD WFs will enable significantly more efficient CS kernel calculations.
- An improved quasi-TMD WF calculation is underway with NLO matching, robust non-local operator renormalization, and improved systematics in the Fourier transform at ~physical pion mass.

$$\gamma_\zeta^q(\mu, b_\mathrm{T}) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{C_q^{\mathrm{TMD\ WF}}(\mu, x, P_2^z)}{C_q^{\mathrm{TMD\ WF}}(\mu, x, P_1^z)} \times \frac{\int db^z e^{ib^z x P_1^z} \lim_{\eta\to\infty} \tilde{\psi}_\pi^{\mathrm{ren.}}(\mu, b^z, \mathbf{b}_\mathrm{T}, \eta, P_1^z)}{\int db^z e^{ib^z x P_1^z} \lim_{\eta\to\infty} \tilde{\psi}_\pi^{\mathrm{ren.}}(\mu, b^z, \mathbf{b}_\mathrm{T}, \eta, P_2^z)} \right] + \mathcal{O}\left(\frac{1}{(xP^z b_\mathrm{T})^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(xP^z)^2} \right)$$

Backup

Asymmetry Corrections to Beam Functions in RI'/MOM



The observed asymmetries of beam functions in RI'/MOM could arise from an incomplete cancellation of linear divergence, show also in previous calculations:

Zhang et al [QCD], PRD 104 (2021), 2012.05448
Huo et al [LPC], Nucl. Phys. B 969 (2021), 2103.02965