Lattice QCD Determination of the *x*-dependence of PDFs at NNLO

TMD Collaboration Meeting
Hilton Santa Fe Historic Plaza
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YONG ZHAO

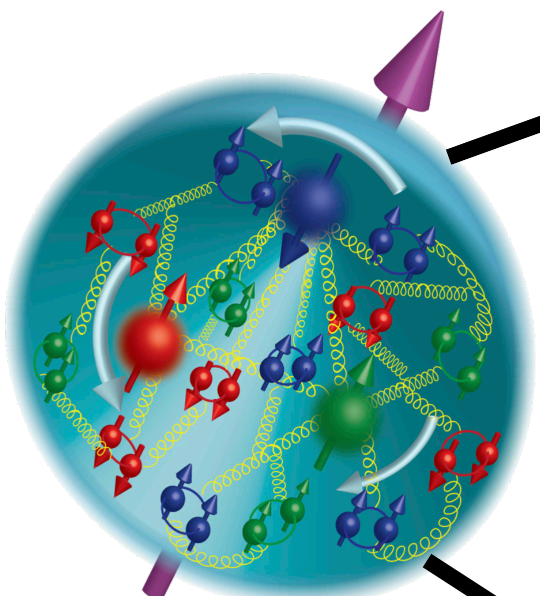
JUN 16, 2022

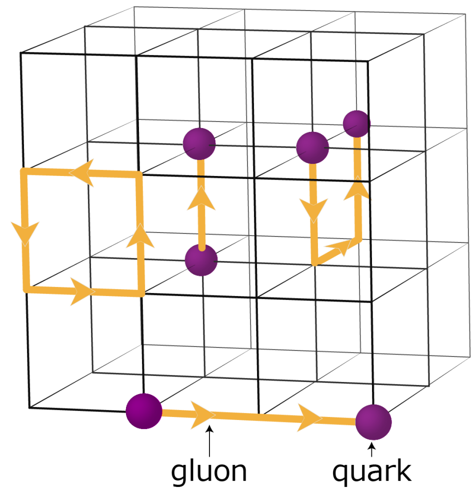
Argonne NATIONAL LABORATORY

Based on work with Xiang Gao, Andrew Hanlon, Swagato Mukherjee, Peter Petreczky, Philipp Scior, Sergey Syritsyn, PRL 128 (2022), 142003.

3D Tomography of the Proton (Hadrons)

Lattice QCD





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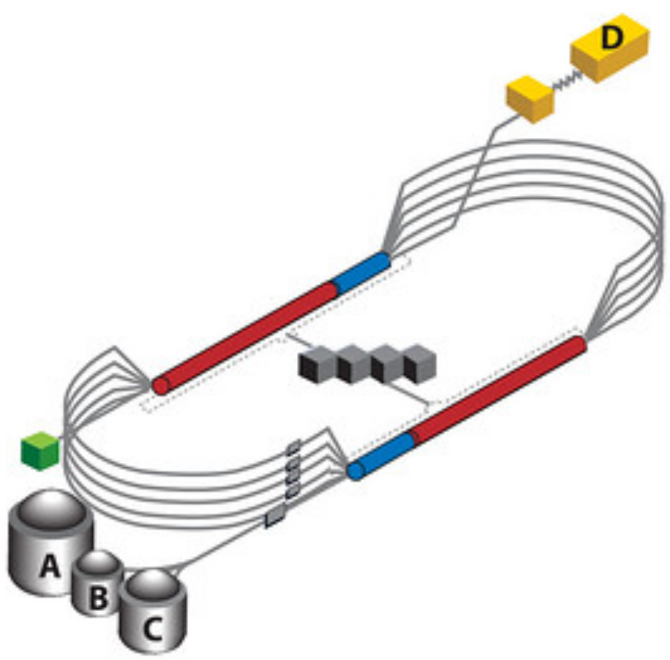
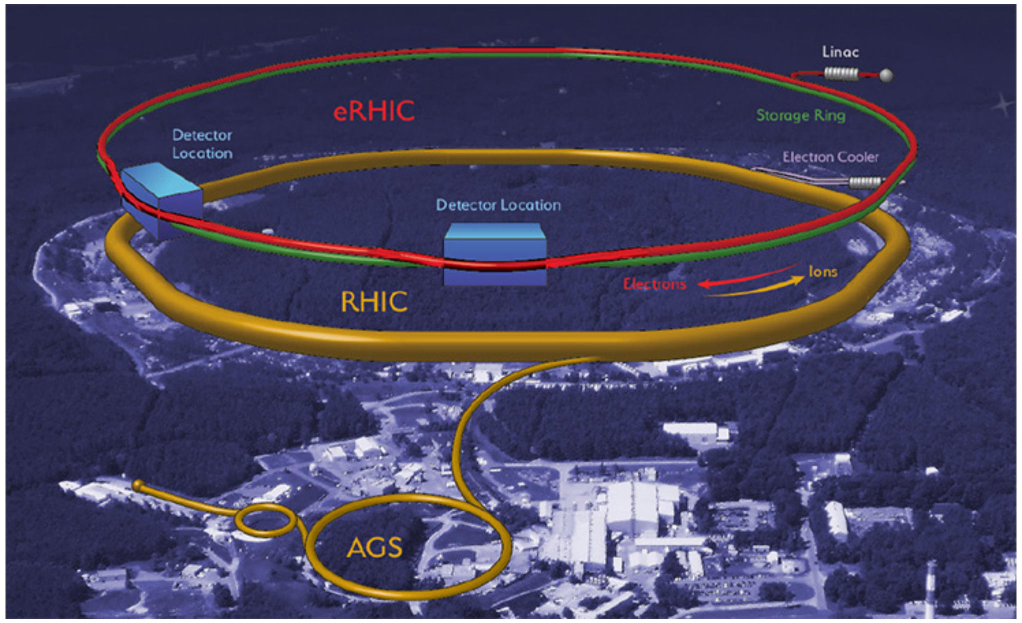
Precision is the key!

Hard Scattering



Jefferson Lab 12 GeV



The Electron-Ion Collider

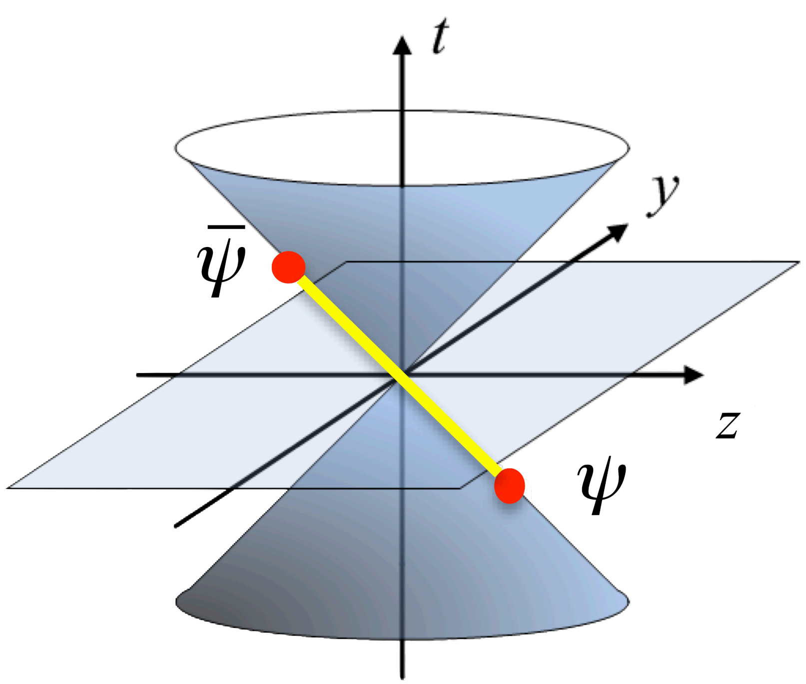
Large-Momentum Effective Theory (LaMET)

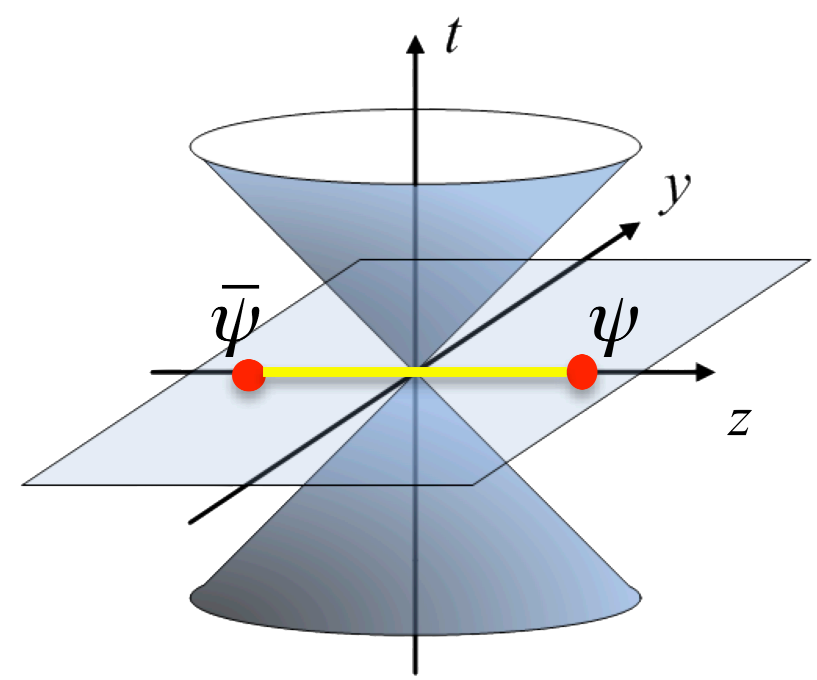
Infinite momentum frame ⇔ light front

X. Ji, PRL 110 (2013)

$z + ct = 0, \quad z - ct \neq 0$

$t = 0, \quad z \neq 0$





PDF $f(x)$:
Cannot be calculated on the lattice

$$f(x) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \langle P | \bar\psi(b^-) \times \frac{\gamma^+}{2} W[b^-, 0] \psi(0) | P \rangle$$

Quasi-PDF $\tilde f(x, P^z)$:
Directly calculable on the lattice

$$\tilde f(x, P^z) = \int \frac{dz}{2\pi} e^{ib^z(xP^z)} \langle P | \bar\psi(b^z) \times \frac{\gamma^z}{2} W[b^z, 0] \psi(0) | P \rangle$$

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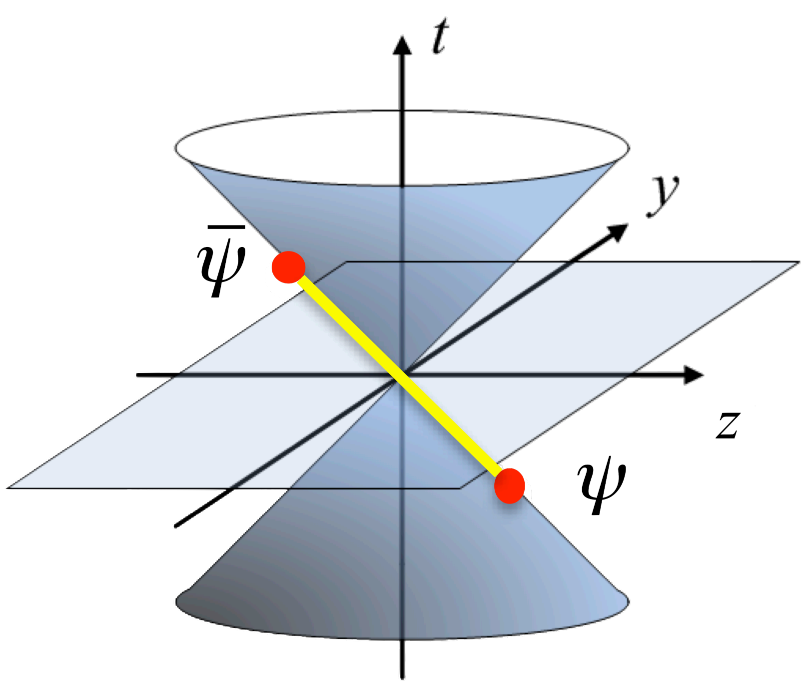

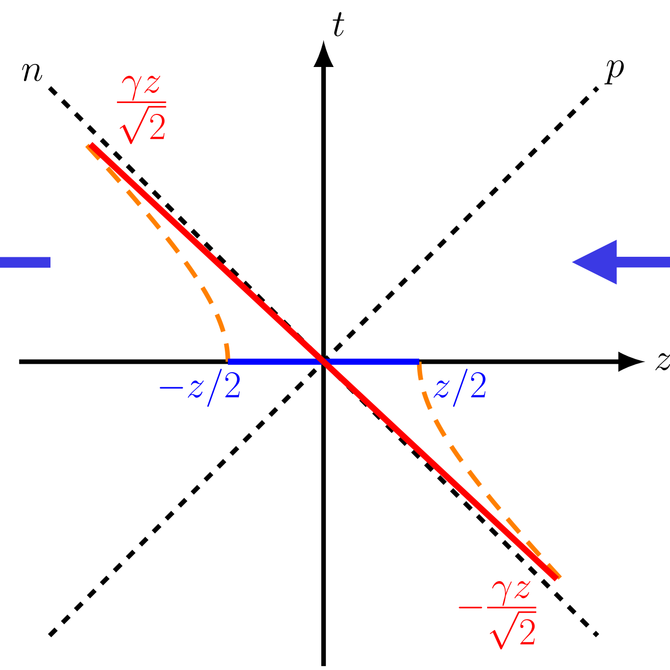

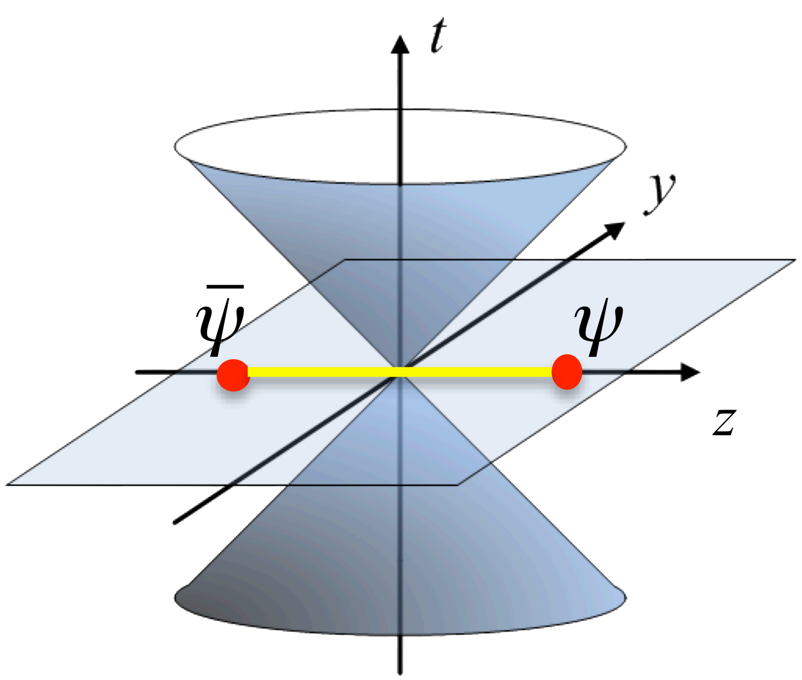

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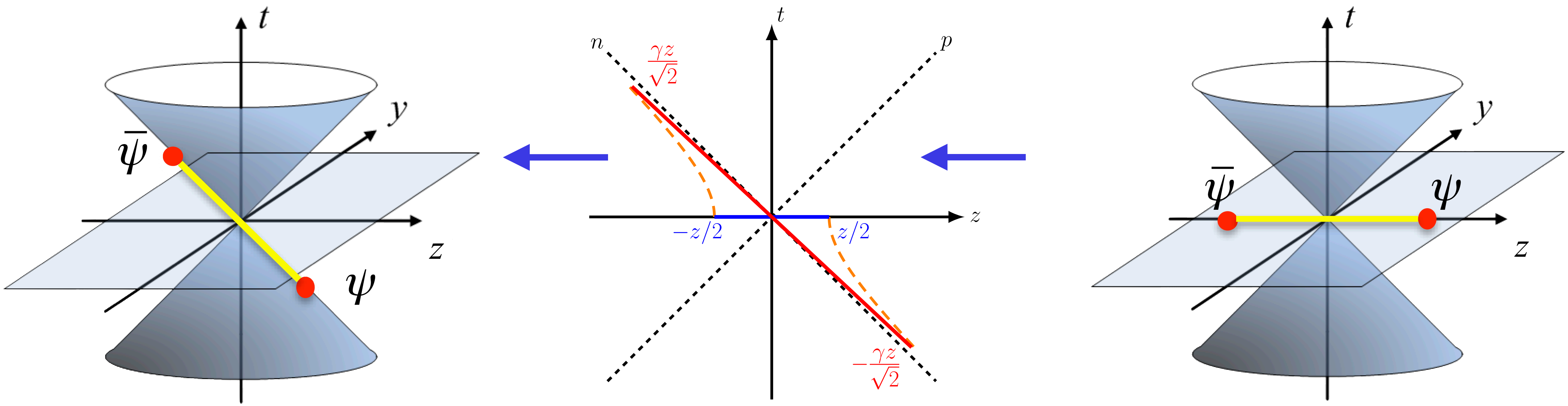

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$$\lim_{P^z \to \infty} \tilde{f}(x, P^z) \stackrel{?}{=} f(x)$$

✘

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Large-Momentum Effective Theory (LaMET)

- Quasi-PDF: $P^z \ll \Lambda$;

Λ: the ultraviolet cutoff, $\sim 1/a$

- PDF: $P^z = \infty$, implying $P^z \gg \Lambda$.
  - The limits $P^z \ll \Lambda$ and $P^z \gg \Lambda$ are not usually exchangeable;
  - For $P^z \gg \Lambda_{\rm QCD}$, the infrared (nonperturbative) physics is not affected, which allows for an EFT matching.

Large-Momentum Effective Theory (LaMET)

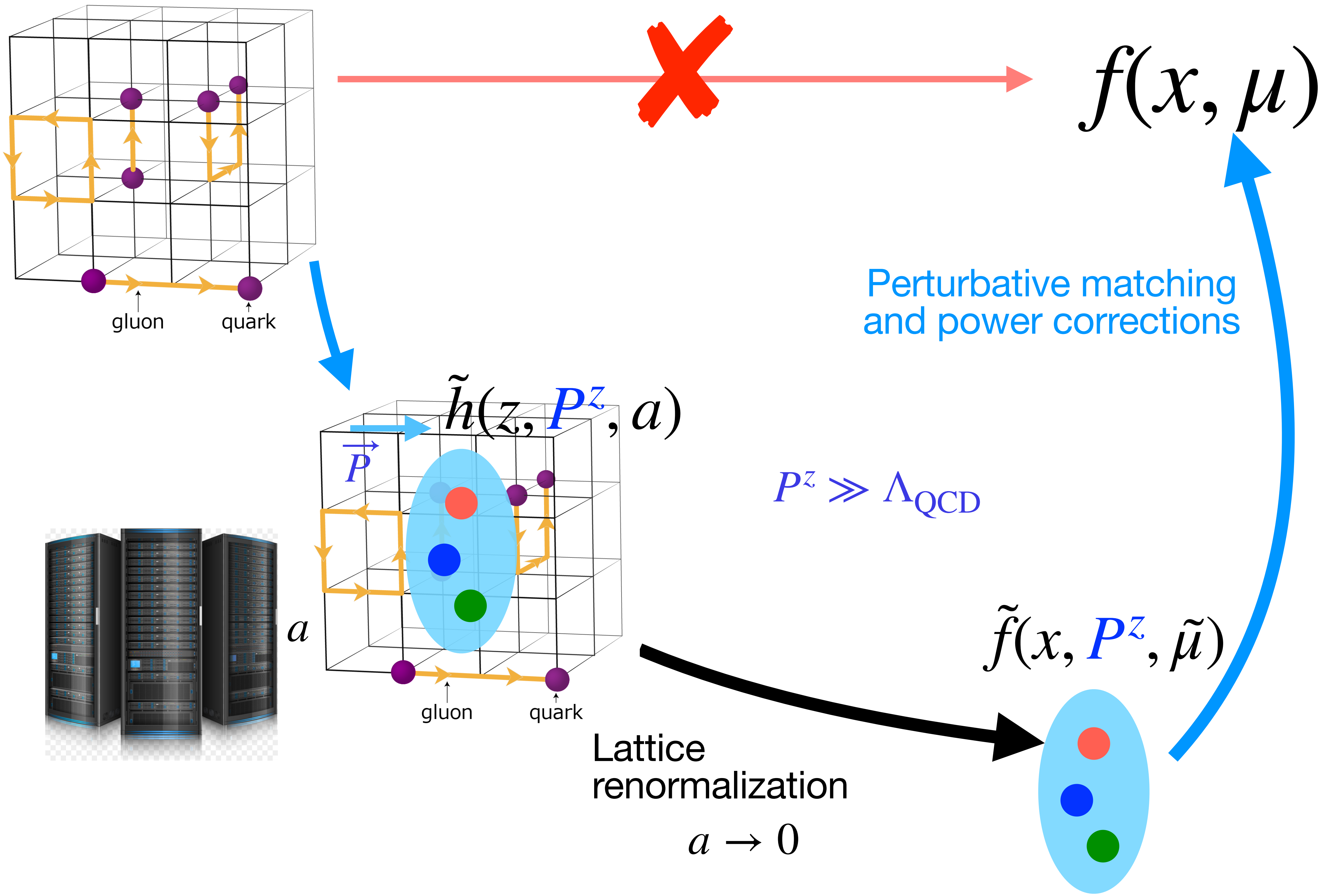
- Large-momentum expansion and perturbative matching:

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \bar{C}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

Systematics:

- Lattice: excited states, spacing $a \to 0$ (renormalization), physical m_π, lattice size $L \to \infty$, etc.;
- Perturbative matching: currently available at NNLO; resummation at small and large x. Only NLO has been used in calculations so far;
  - L.-B. Chen, R.-L. Zhu and W. Wang, PRL126 (2021);
  - Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, PRL126 (2021);
  - X. Gao, K. Lee, S. Mukherjee, C. Shugert and YZ, PRD103 (2021).
- Power corrections, controllable within [$x_{\rm min}$, $x_{\rm max}$] at a given finite P^z.

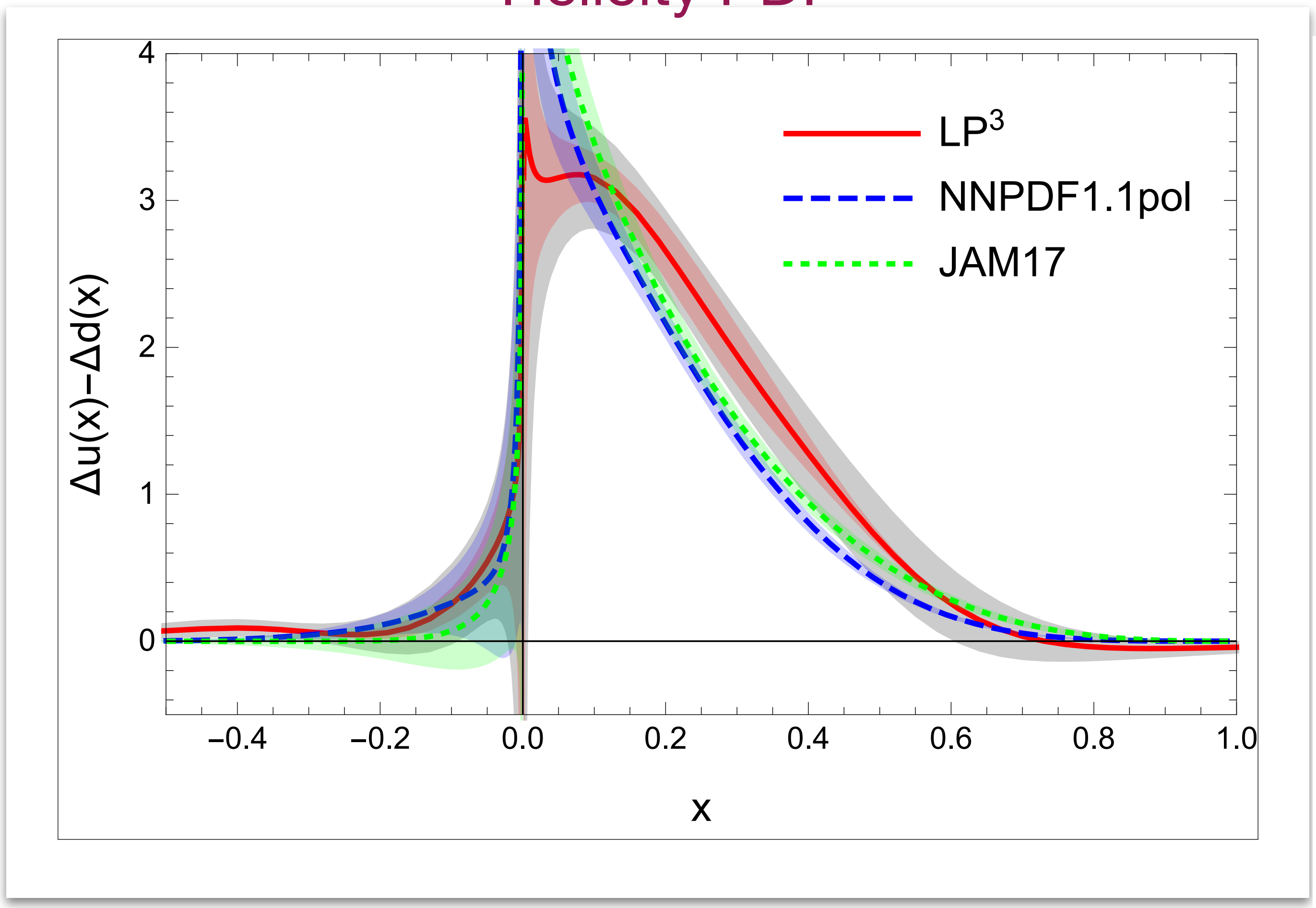
Systematic procedure in lattice calculation



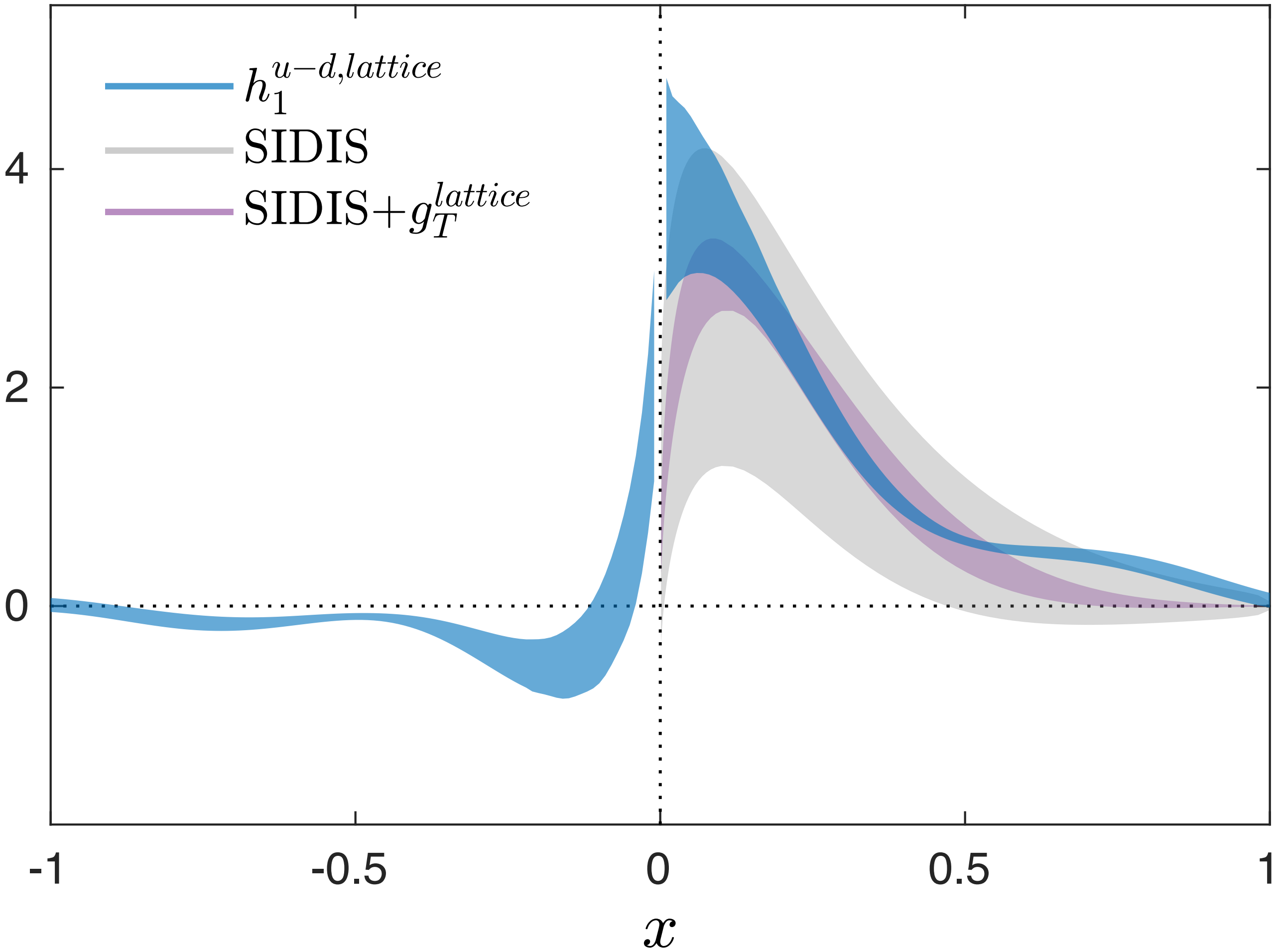
Encouraging results have been obtained:

For example, the isovector (u-d) PDFs of the proton, with RI/MOM lattice renormalization and NLO matching:

Helicity PDF



Transversity PDF



H.W. Lin, YZ, et al. (LP3 Collaboration), PRL 121 (2018)

C. Alexandrou, et al. (ETM), PRD 98 (2018).

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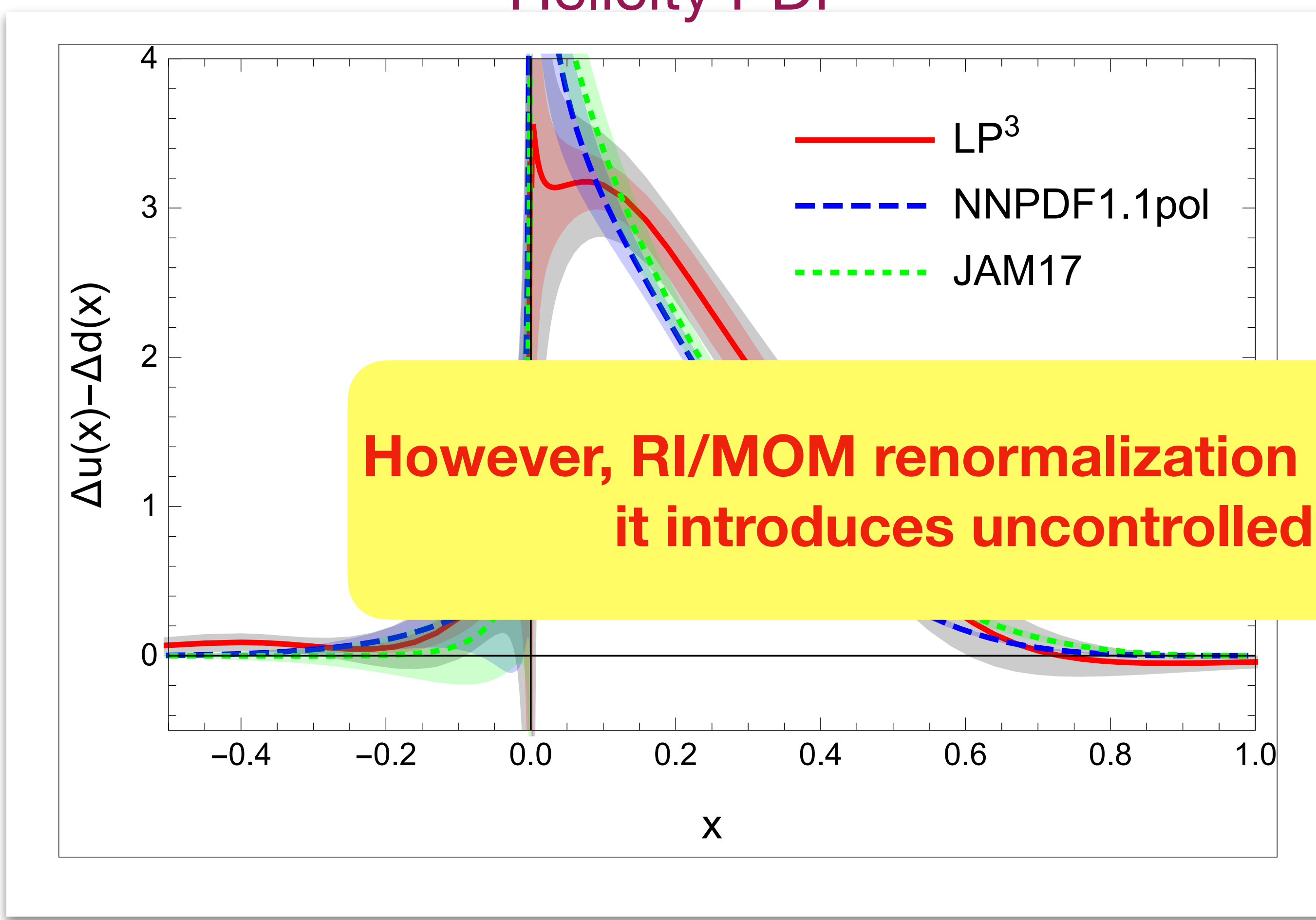

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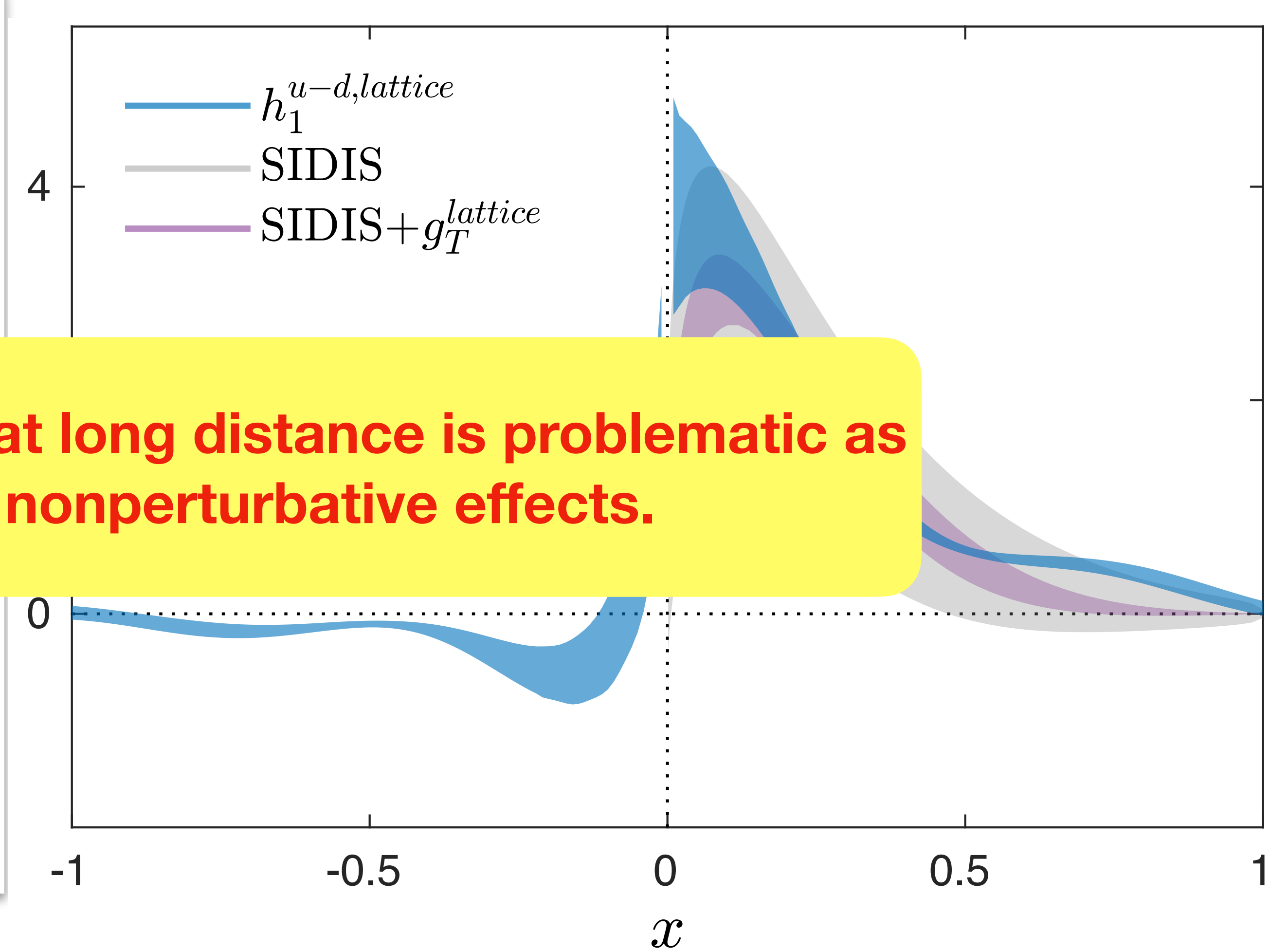


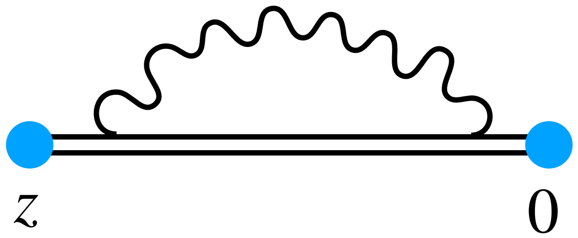
However, RI/MOM renormalization at long distance is problematic as it introduces uncontrolled nonperturbative effects.

H.W. Lin, YZ, et al. (LP3 Collaboration), PRL 121 (2018)

C. Alexandrou, et al. (ETM), PRD 98 (2018).

Lattice renormalization

$$O_B^\Gamma(z,a) = \bar{\psi}_0(z)\Gamma W_0[z,0]\psi_0(0) = e^{-\delta m(a)|z|}\, Z_O(a) O_R^\Gamma(z)$$



$$= \delta m(a)|z| \propto \frac{|z|}{a}$$

- Ji, Zhang and YZ, PRL 120 (2018);
- Ishikawa, Ma, Qiu and Yoshida, PRD 96 (2017);
- Green, Jansen and Steffens, PRL 121 (2018).

$$\tilde{f}_X(x, P^z, \tilde{\mu}) = \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{iz(xP^z)}\, \tilde{h}_X(z, P^z, \tilde{\mu})$$

$$= \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{iz(xP^z)} \lim_{a\to 0} \frac{\tilde{h}(z, P^z, a)}{Z_X(z, \tilde{\mu}, a)}$$

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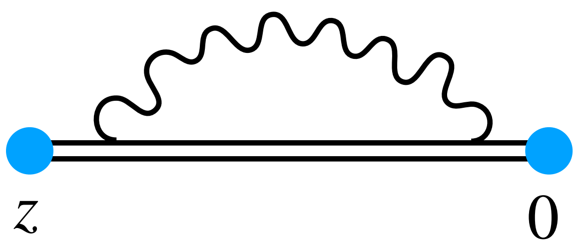

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Ratio-type schemes:

- RIMOM

  $$Z_X = \langle q | O^\Gamma(z) | q \rangle$$

- Hadron matrix elements

  $$Z_X = \langle P_0^z | O^\Gamma(z) | P_0^z \rangle$$

- Vacuum expectation value

  $$Z_X = \langle \Omega | O^\Gamma(z) | \Omega \rangle$$

See X. Ji, YZ, et al., NPB 964 (2021) and references therein.

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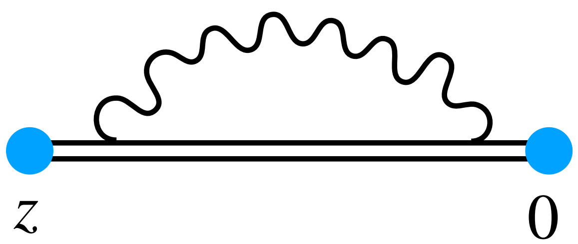

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- Vacuum expectation value

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- **For $z \sim a$, ratio-type schemes cancel cutoff effects;** 🙂
- **But for $z \sim \Lambda_{\rm QCD}^{-1}$, ratio-type schemes introduce uncontrolled nonperturbative effects.** 🙁

See X. Ji, YZ, et al., NPB 964 (2021) and references therein.

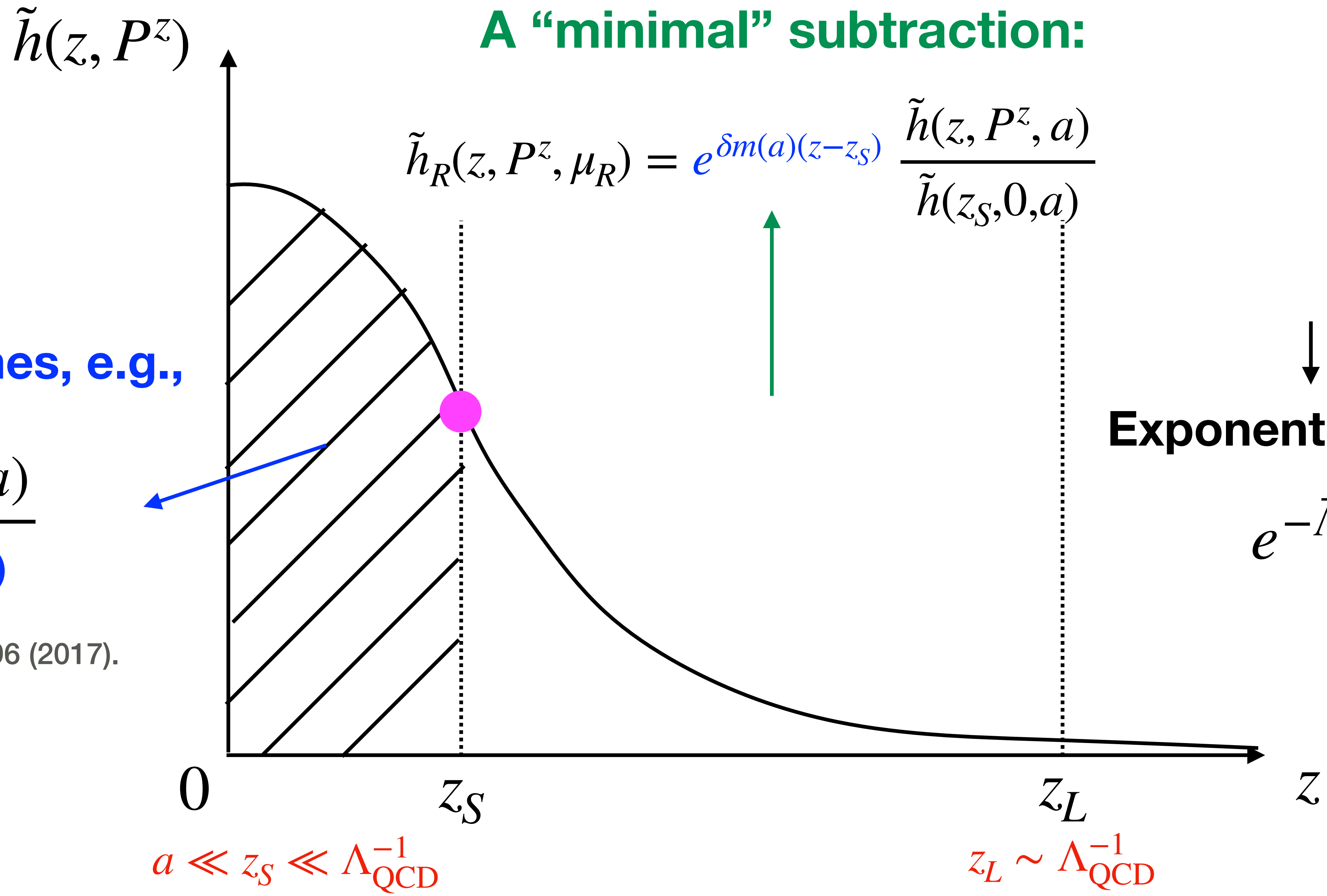
Hybrid renormalization scheme

$$O_B^\Gamma(z,a) = \bar{\psi}_0(z)\Gamma W_0[z,0]\psi_0(0) = e^{-\delta m(a)|z|}\, Z_O(a) O_R^\Gamma(z)$$

X. Ji, YZ, et al., NPB 964 (2021).

A “minimal” subtraction:

$$\tilde{h}_R(z,P^z,\mu_R) = e^{\delta m(a)(z-z_S)} \frac{\tilde{h}(z,P^z,a)}{\tilde{h}(z_S,0,a)}$$



Ratio schemes, e.g.,

$$\frac{\tilde{h}(z,P^z,a)}{\tilde{h}(z,0,a)}$$

Orginos et al., PRD 96 (2017).

Exponential decay

$$e^{-\bar{\Lambda}|z|}$$

Lattice data for the pion valence PDF

- Wilson-clover fermion on 2+1 flavor HISQ configurations.

n_z	P_z (GeV)		ζ
	$a = 0.06$ fm	$a = 0.04$ fm	
0	0	0	0
1	0.43	0.48	0
2	0.86	0.97	1
3	1.29	1.45	2/3
4	1.72	1.93	3/4
5	2.15	2.42	3/5
	$48^3 \times 64$	$64^3 \times 64$	

$m_\pi = 300$ MeV

- X. Gao, YZ, et al., PRD102 (2020).
- X. Gao, YZ, et al., PRD103 (2021).

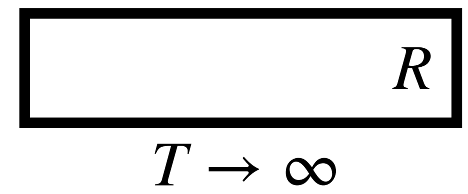
Why studying the pion?

- **Pseudo Nambu-Goldstone boson of QCD**
- **First excited state π(1300) much higher than ground state π(~140), good for control of excited-state contamination**

Hybrid scheme renormalization

- Wilson-line mass renormalization

Normalization scheme for the static quark-antiquark potential $V^{\rm lat}(r, a)$:

$\langle \Omega |$ 

$| \Omega \rangle \propto \exp[-V(R)T]$

$$V^{\rm lat}(r,a)\Big|_{r=r_0} + 2\delta m(a) = 0.95/r_0$$

$$r_0 = 0.469 \text{ fm}$$

$$a\delta m(a = 0.04 \text{ fm}) = 0.1508(12)$$

$$a\delta m(a = 0.06 \text{ fm}) = 0.1586(8)$$

A. Bazavov et al., TUMQCD, PRD98 (2018).

Renormalon ambiguity:

$$\delta m(a) = \frac{1}{a}\sum_n c_n \alpha_s^n(1/a) + m_0^{\rm lat}$$

$$m_0^{\rm lat} \sim \frac{1}{a}(a\Lambda_{\rm QCD}) + \text{ scheme dependent constant}$$

C. Bauer, G. Bali and A. Pineda, PRL108 (2012).

Wilson-line mass renormalization

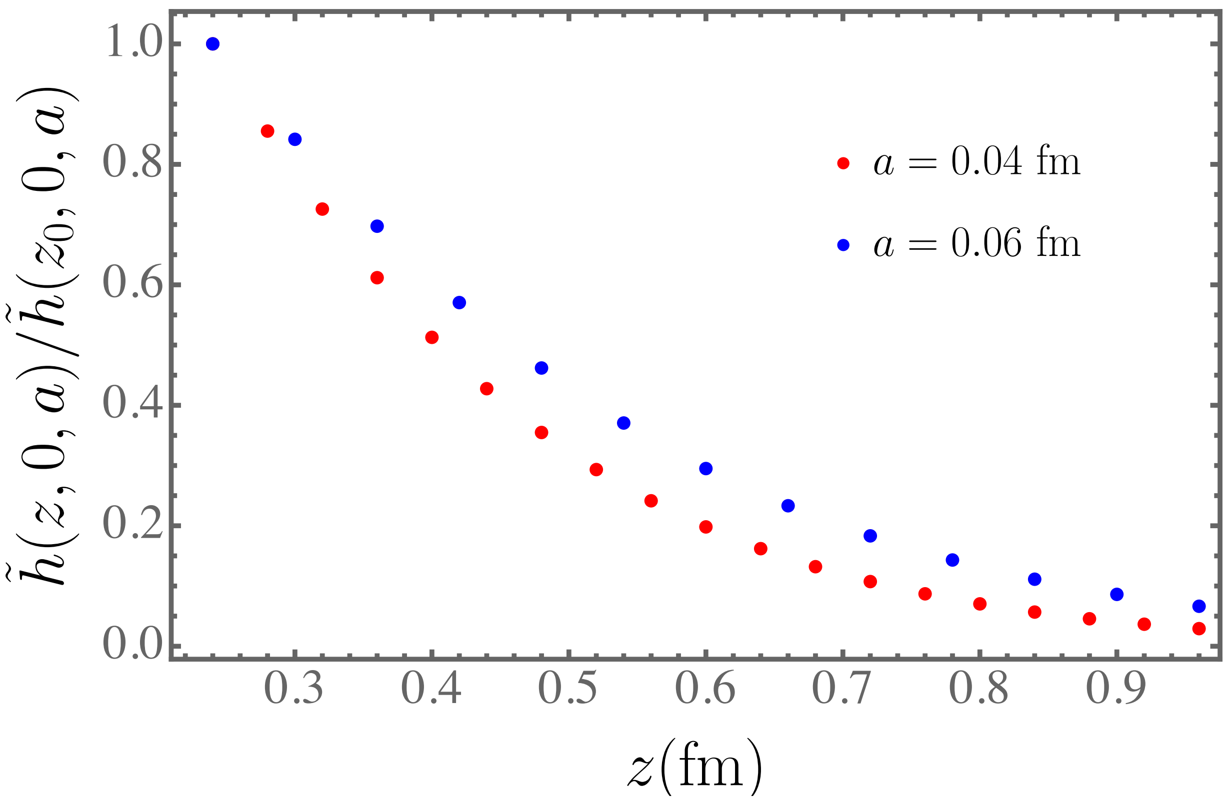
- Check of continuum limit

$$O_B^{\Gamma}(z,a) = e^{-\delta m|z|}\, Z_O(a) O_R^{\Gamma}(z)$$

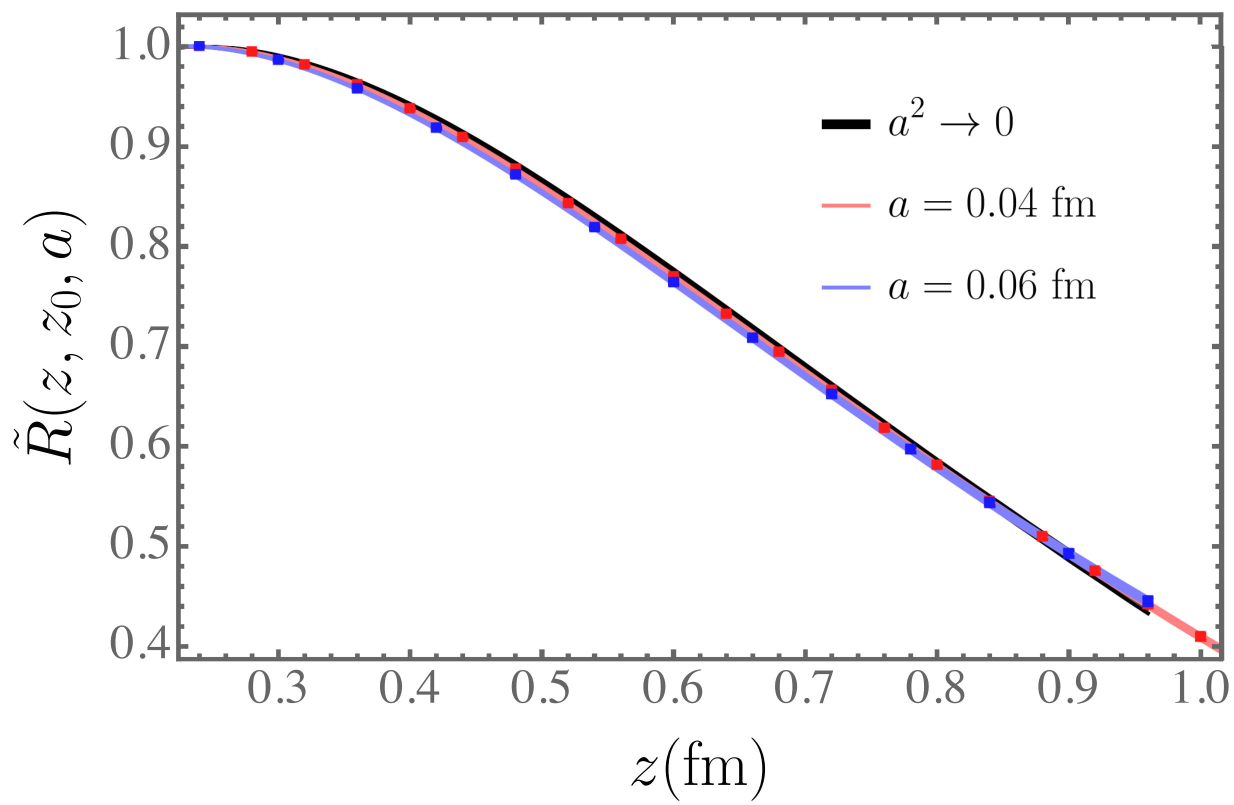
Renormalization-group invariant

$$\lim_{a\to 0} e^{\delta m(z-z_0)} \frac{\tilde{h}(z,a,P^z=0)}{\tilde{h}(z_0,a,P^z=0)} = \text{finite} \qquad z, z_0 \gg a$$

Before mass subtraction



After mass subtraction



Sub-precent level agreement!

Matching the Wilson-line mass to MSbar

- OPE of MSbar matrix element

$$\tilde{h}^{\overline{\rm MS}}(z, P^z = 0, \mu) = e^{-m_0^{\overline{\rm MS}}|z|}\ \tilde{h}_0^{\overline{\rm MS}}(z,0,\mu)$$

$$\stackrel{z\ll 1/\Lambda_{\rm QCD}}{=} e^{-m_0^{\overline{\rm MS}}(z-z_0)}\left[C_0(\alpha_s(\mu), z^2\mu^2) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)\right]$$

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Wilson coefficient:

Known to NNLO with 3-loop anomalous dimension

- L.-B. Chen, R.-L. Zhu and W. Wang, PRL126 (2021);
- Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, PRL126 (2021);
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IR renormalon

V. Braun, A. Vladimirov and J.-H. Zhang, PRD99 (2019).

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$m_0^{\overline{\rm MS}} \sim \Lambda_{\rm QCD}$

UV renormalon, similar to HQET

M. Beneke and V. Braun, NPB 426 (1994).

Wilson coefficient:

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- Matching to the MSbar OPE ratio

$$\bar{m}_0 = -\, m_0^{\rm lat} + m_0^{\overline{\rm MS}}$$

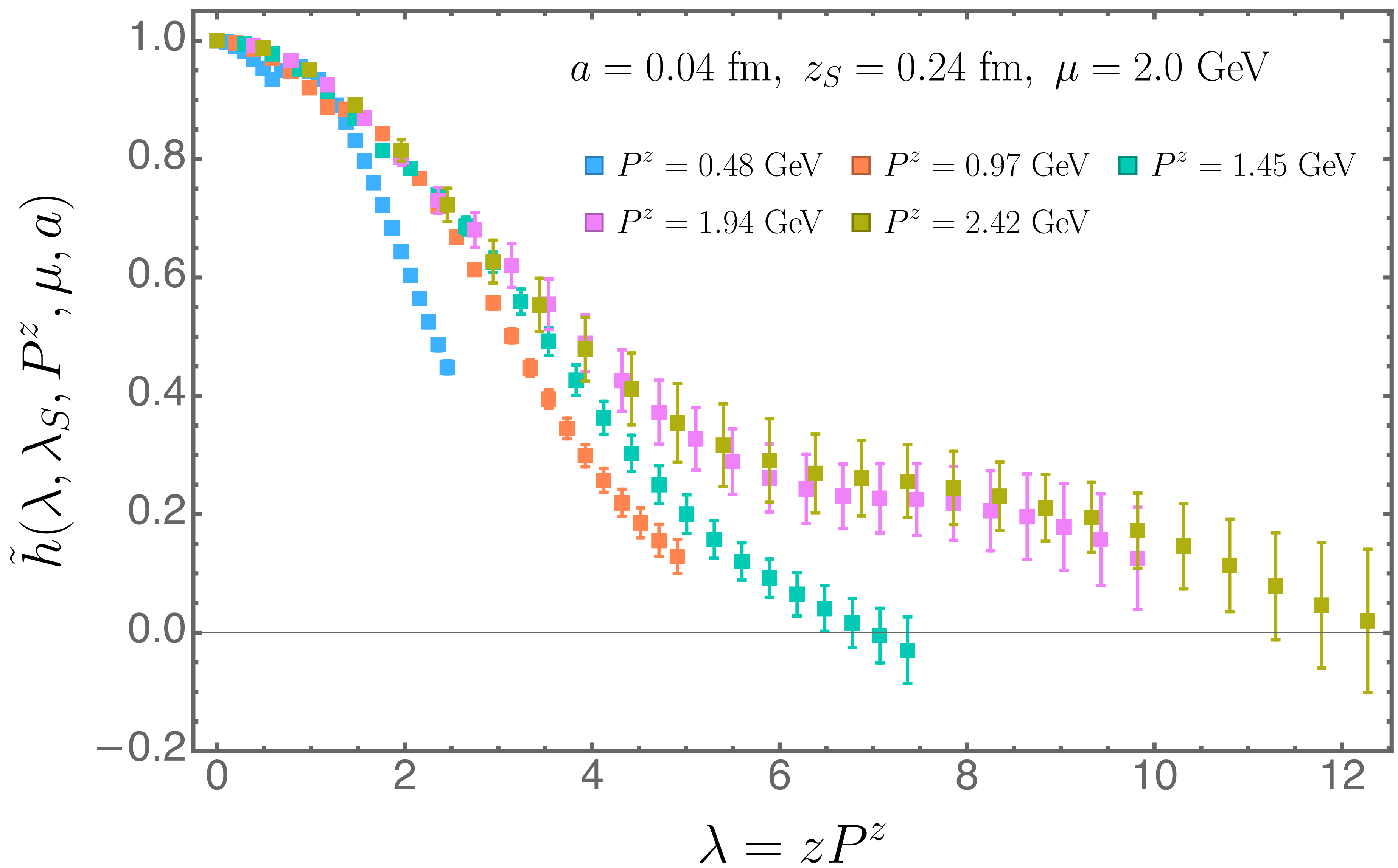
$$\lim_{a\to 0} e^{\delta m(z-z_0)}\frac{\tilde{h}(z, a, P^z = 0)}{\tilde{h}(z_0, a, P^z = 0)} = e^{-\bar{m}_0(z-z_0)}\frac{C_0(\alpha_s(\mu), z^2\mu^2) + \Lambda z^2}{C_0(\alpha_s(\mu), z_0^2\mu^2) + \Lambda z_0^2}$$

$$a \ll z, z_0 \ll 1/\Lambda_{\rm QCD}^{-1}$$

For related methods, see

- J. Green, K. Jansen, and F. Steffans, PRD 101 (2020);
- Y. Huo et al. (LPC), NPB 969 (2021).

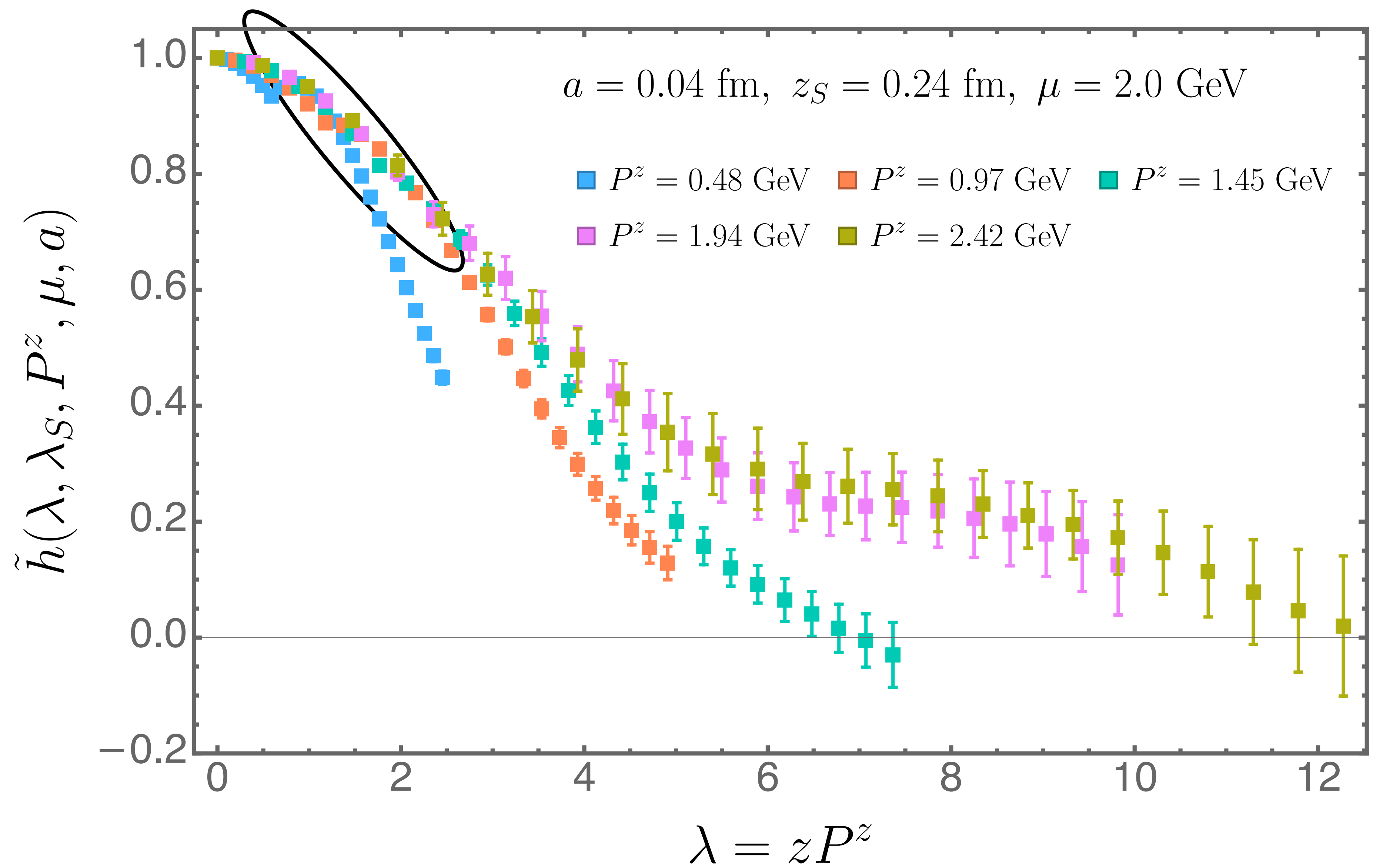
Renormalized and matched matrix elements



$z_{\max} = 1.0$ fm

Renormalized and matched matrix elements

At small λ (small *z*), the perturbative region, the matrix elements have mild P^z dependence due to slow QCD evolution.



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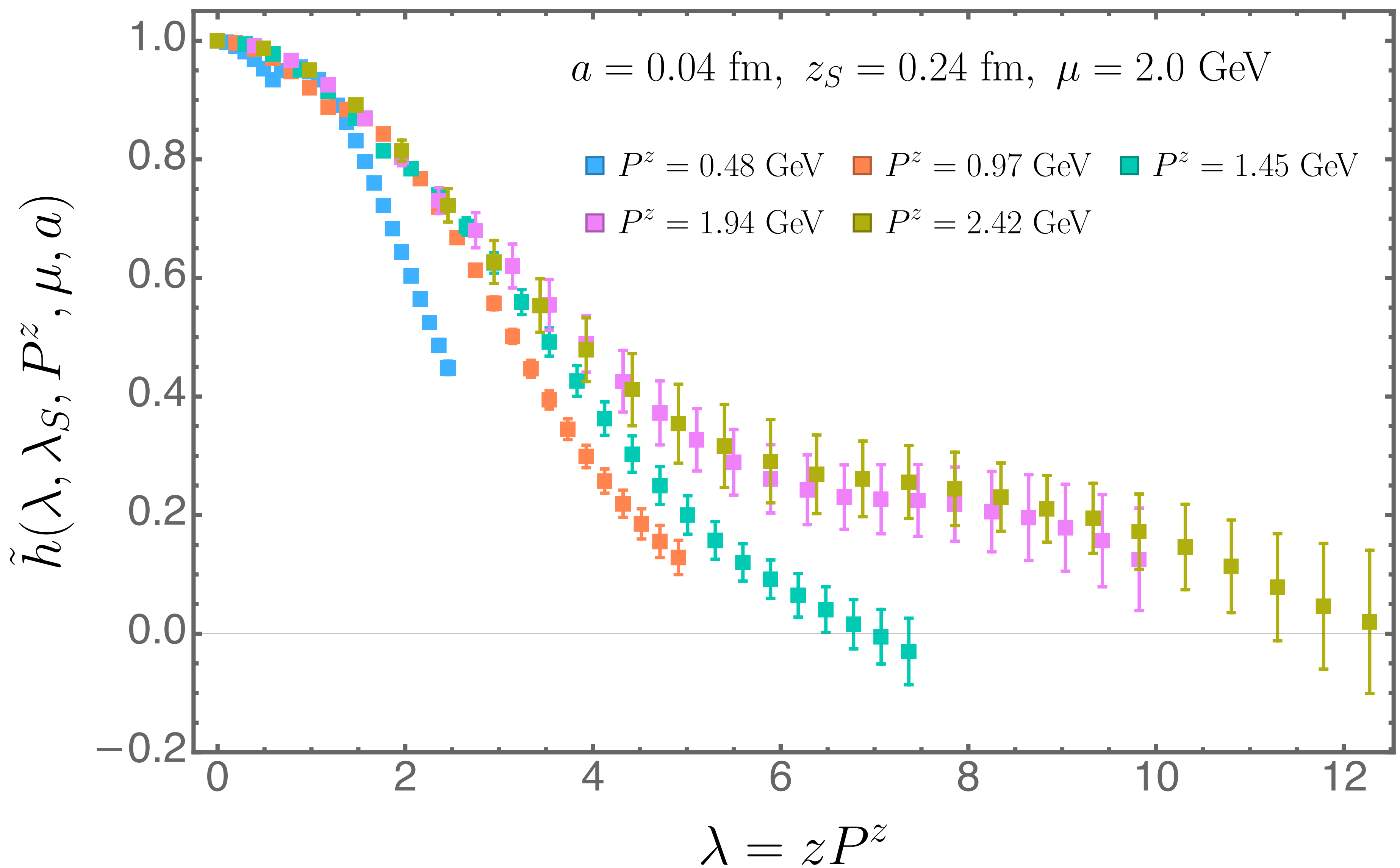

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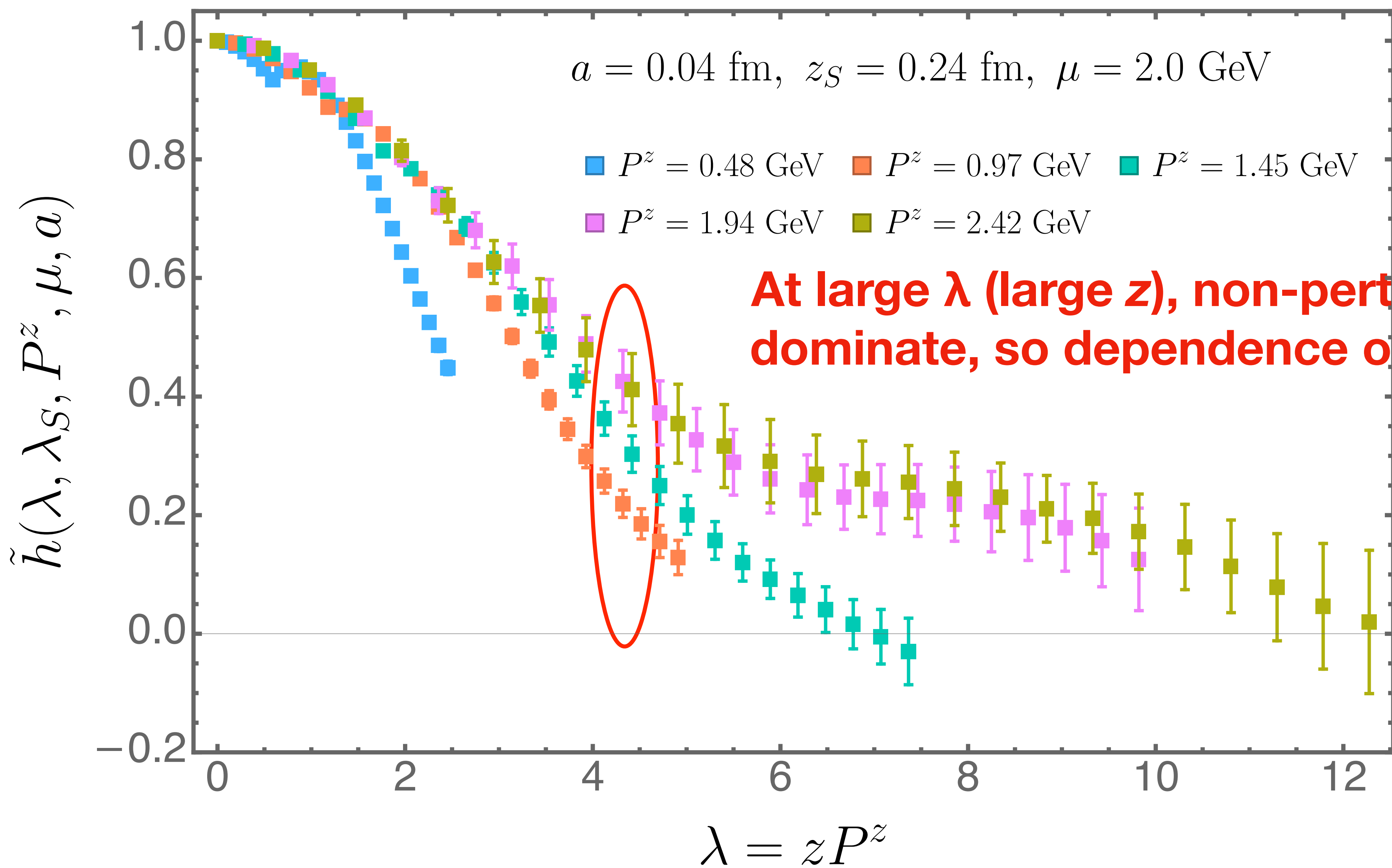

At large λ (large *z*), non-perturbative effects dominate, so dependence on P^z is stronger.

$z_{\text{max}} = 1.0$ fm

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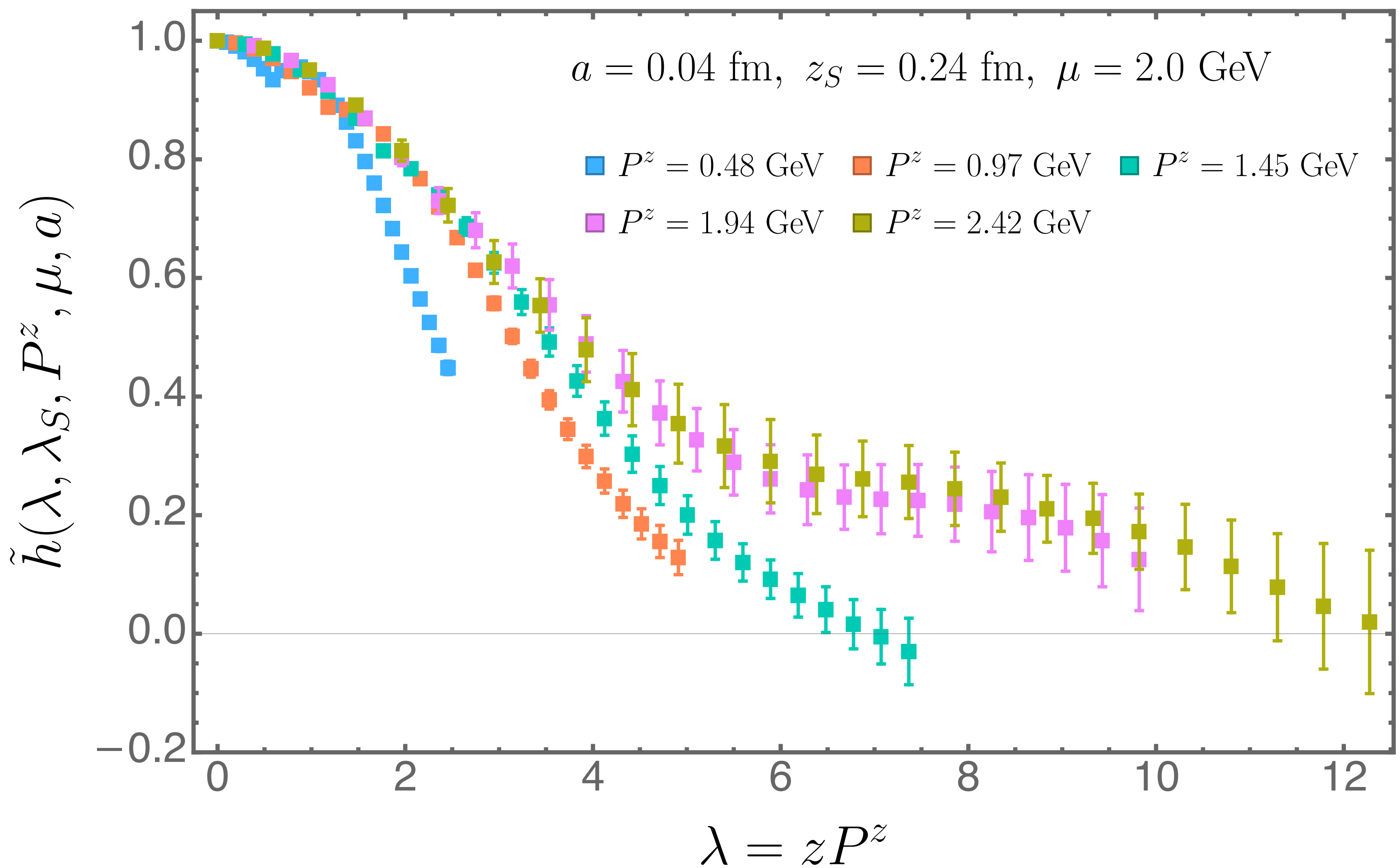
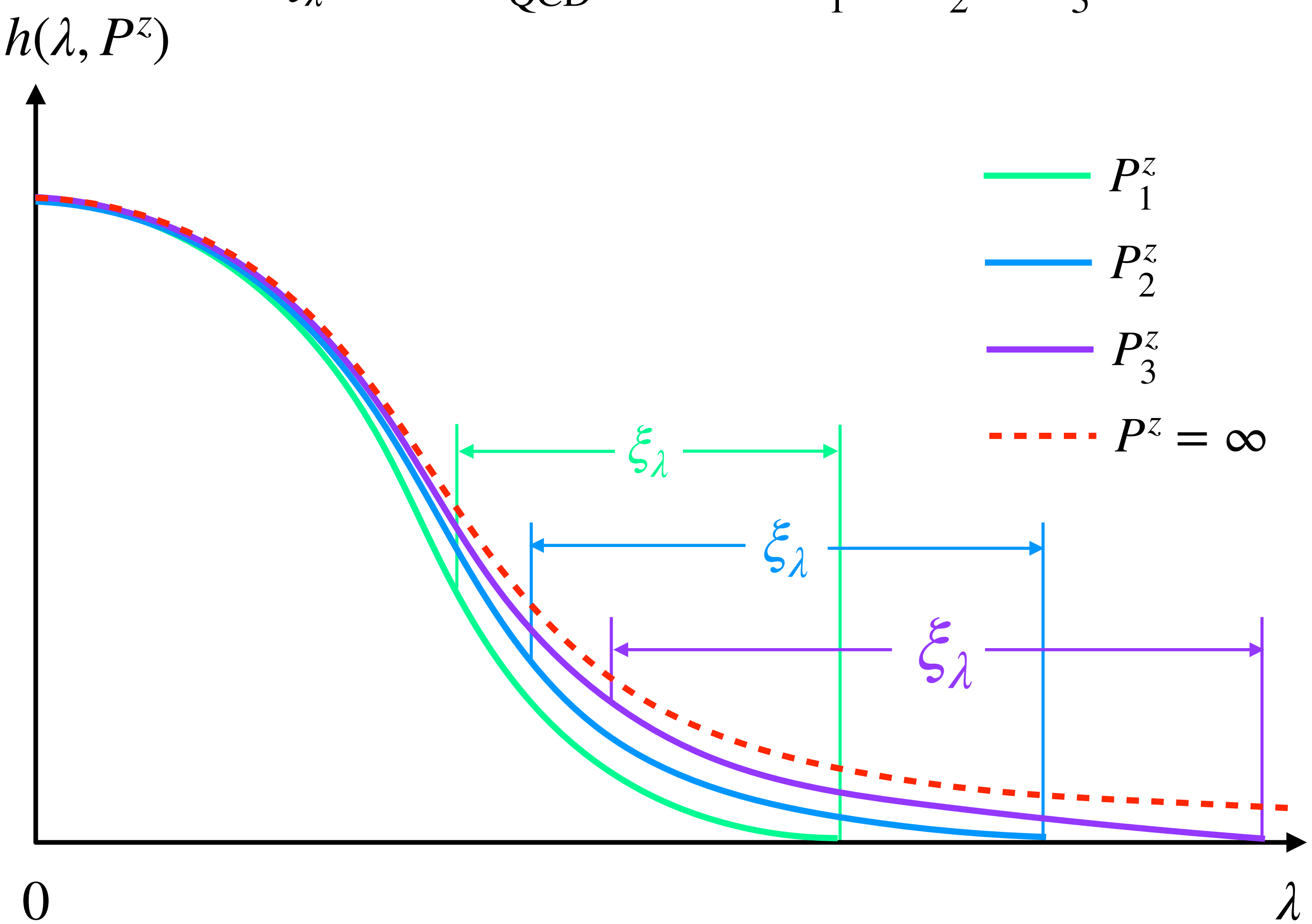


$z_{\max} = 1.0$ fm

Asymptotic behavior at large z

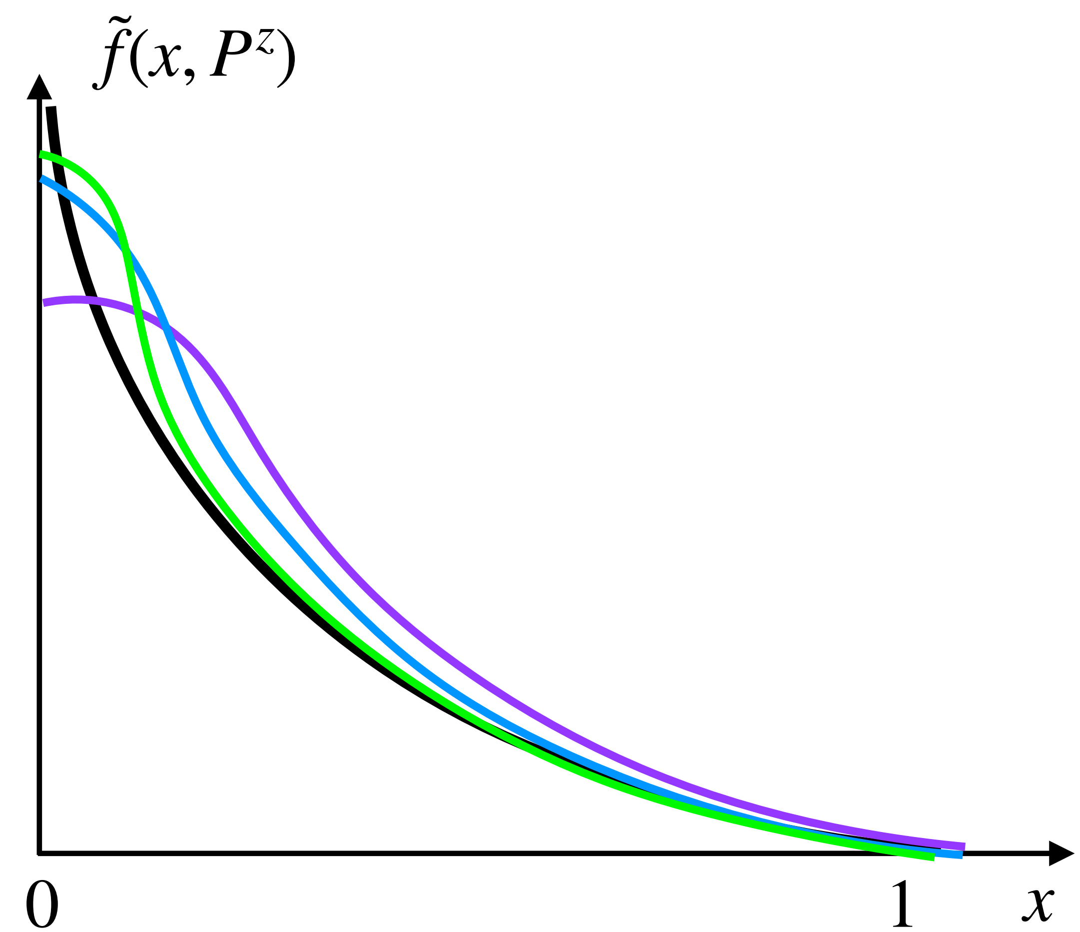
$$\tilde{h}(\lambda = zP^z, P^z) \xrightarrow{|z|\to\infty} \propto g(p\cdot z)\ e^{-\frac{\bar{\Lambda}}{P^z}|\lambda|}$$

$\xi_\lambda \sim P^z/\Lambda_{\rm QCD}\,, \qquad 0 < P_1^z < P_2^z < P_3^z$



Correlation length $\xi_\lambda \equiv P^z/\bar{\Lambda}$

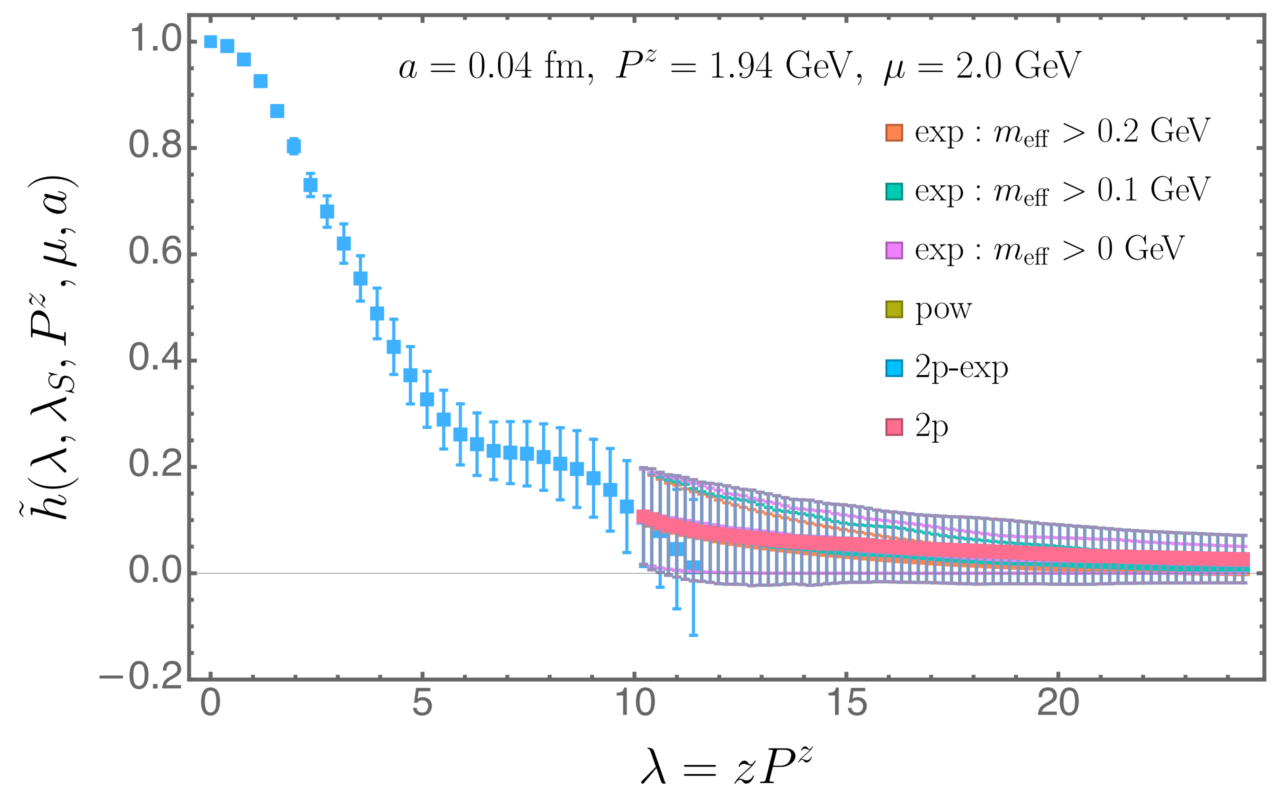
Fourier transform converges fast in *z* (or *λ*)



Physical extrapolation and Fourier transform (FT)

Extrapolation

- Removes unphysical oscillation;
- Moderate to large *x* regions are insensitive to the extrapolation form.



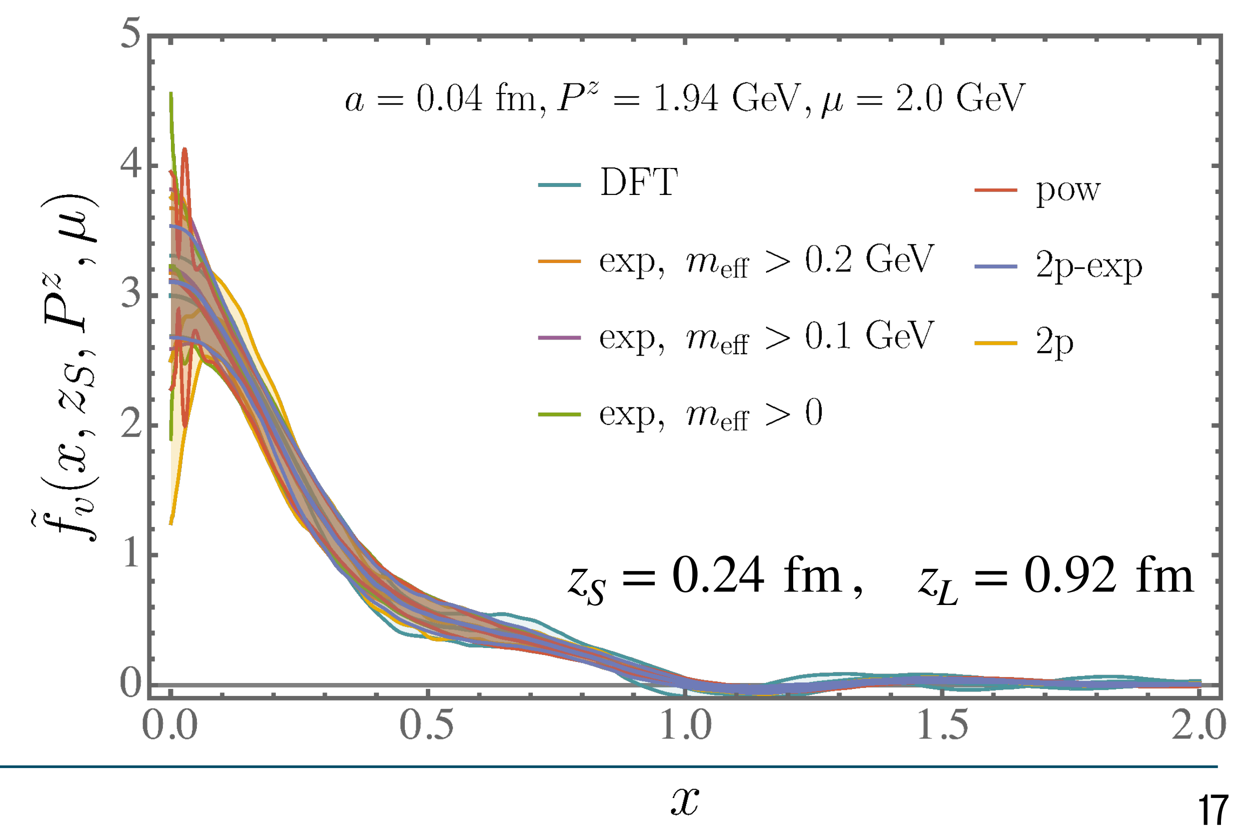
Extrapolation forms :

Discrete FT (DFT)

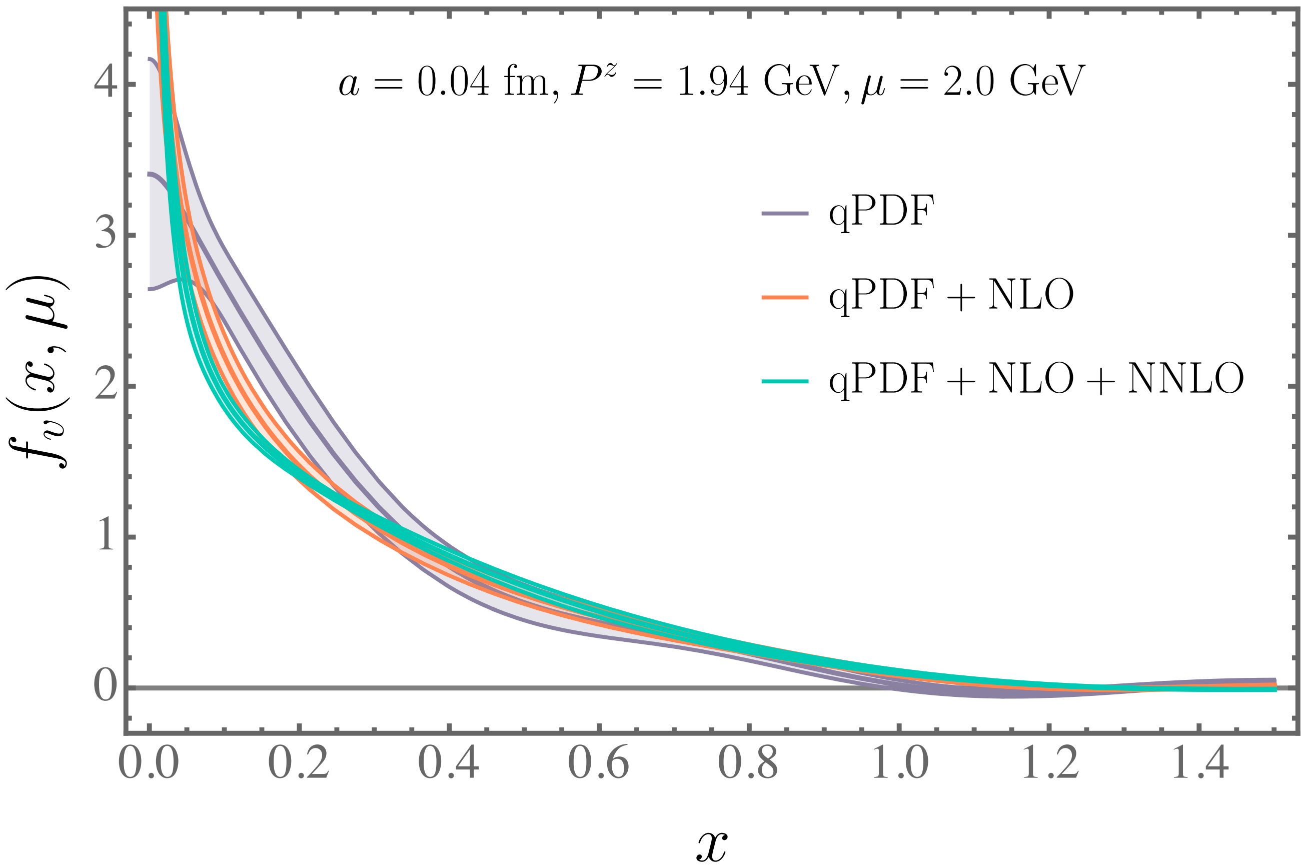
$$\text{exp}: \quad \frac{Ae^{-m_{\rm eff}|z|}}{\lambda^d} \qquad\qquad \text{pow}: \quad \frac{A}{\lambda^d}$$

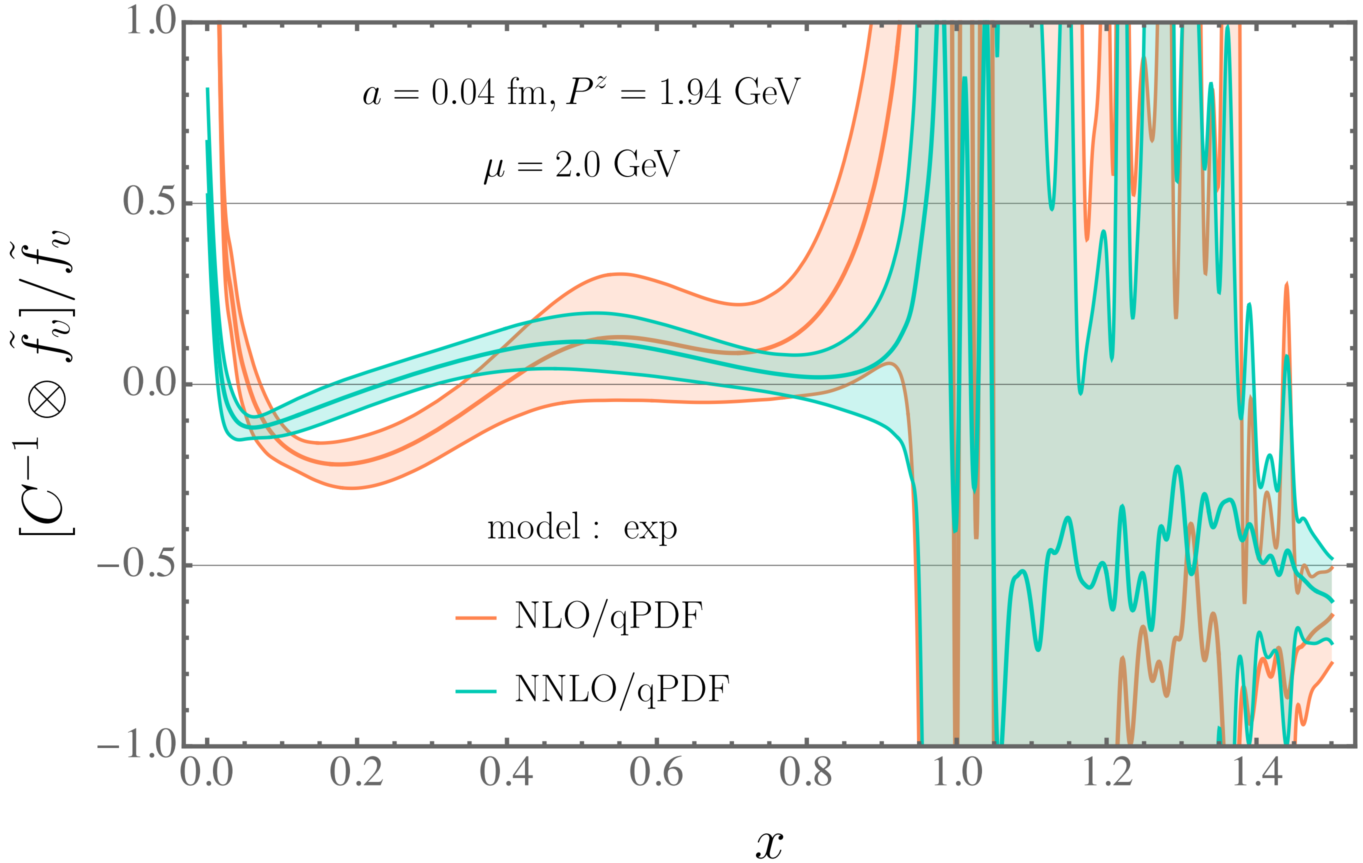
$$\text{2p-exp}: \quad A \ \text{Re}\left[\frac{\Gamma(1+a)}{(-i|\lambda|)^{a+1}} + e^{i\lambda}\frac{\Gamma(1+b)}{(i|\lambda|)^{b+1}}\right] e^{-m_{\rm eff}|z|}$$

$$\text{2p}: \quad A \ \text{Re}\left[\frac{\Gamma(1+a)}{(-i|\lambda|)^{a+1}} + e^{i\lambda}\frac{\Gamma(1+b)}{(i|\lambda|)^{b+1}}\right]$$



Perturbative matching at NNLO



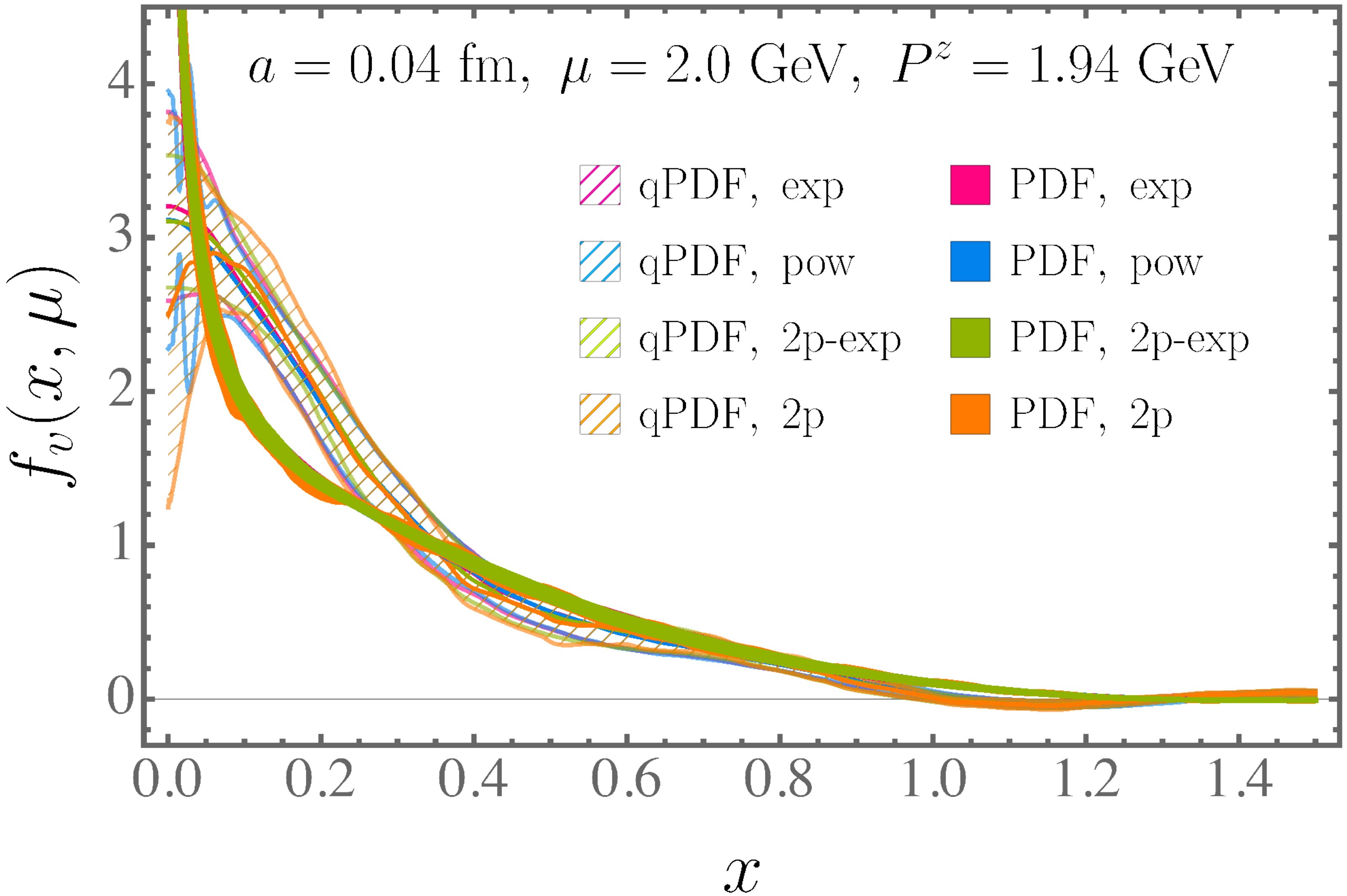


- Matching drives the quasi-PDF to smaller x;
- Good convergence at moderate x;
- Large corrections in end-point regions, need resummation;
- Surprisingly small corrections at x as small as 0.05.

$$\int_x^1 \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{f}(y, P^z, \mu) \xrightarrow{x \to 0} \alpha_s \int_x^1 \frac{dy}{|y|} \left[P_{qq}\left(\frac{x}{y}\right) \ln \frac{\mu^2}{4x^2 P_z^2} \right]_+ \tilde{f}(y, P^z, \mu)$$

Perturbative matching at NNLO

Extrapolation-model dependence further reduced:



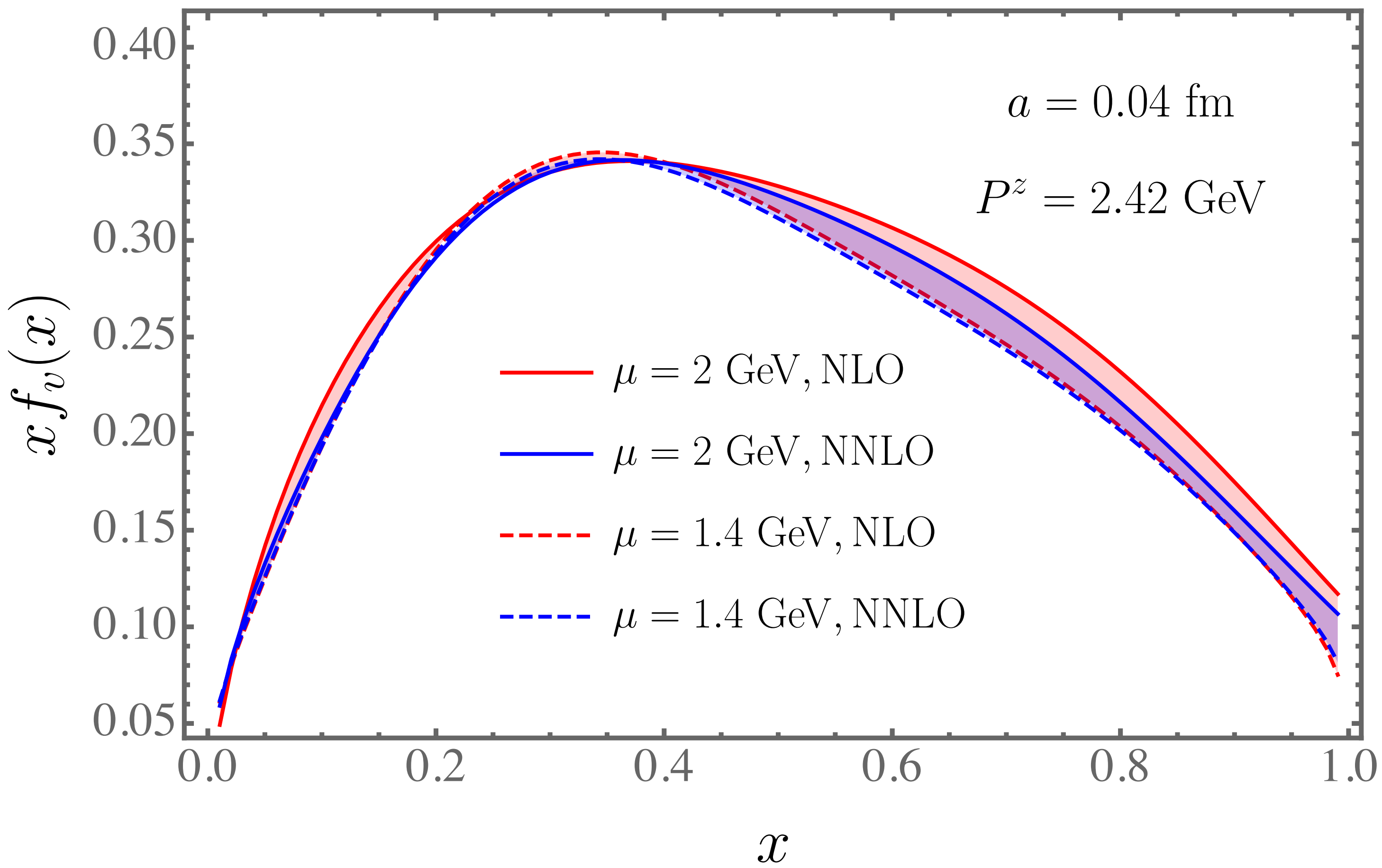
Agreement among different extrapolation models extends to smaller *x* region.

Perturbative matching at NNLO

Factorization scale variation uncertainty:

- Calculate the PDF at different $\mu = 1.4,\ 2.0$ GeV;
- Evolve the results to $\mu = 2.0\ \mathrm{GeV}$ with NLO DGLAP kernel.

Scale uncertainty reduced at NNLO 🙂

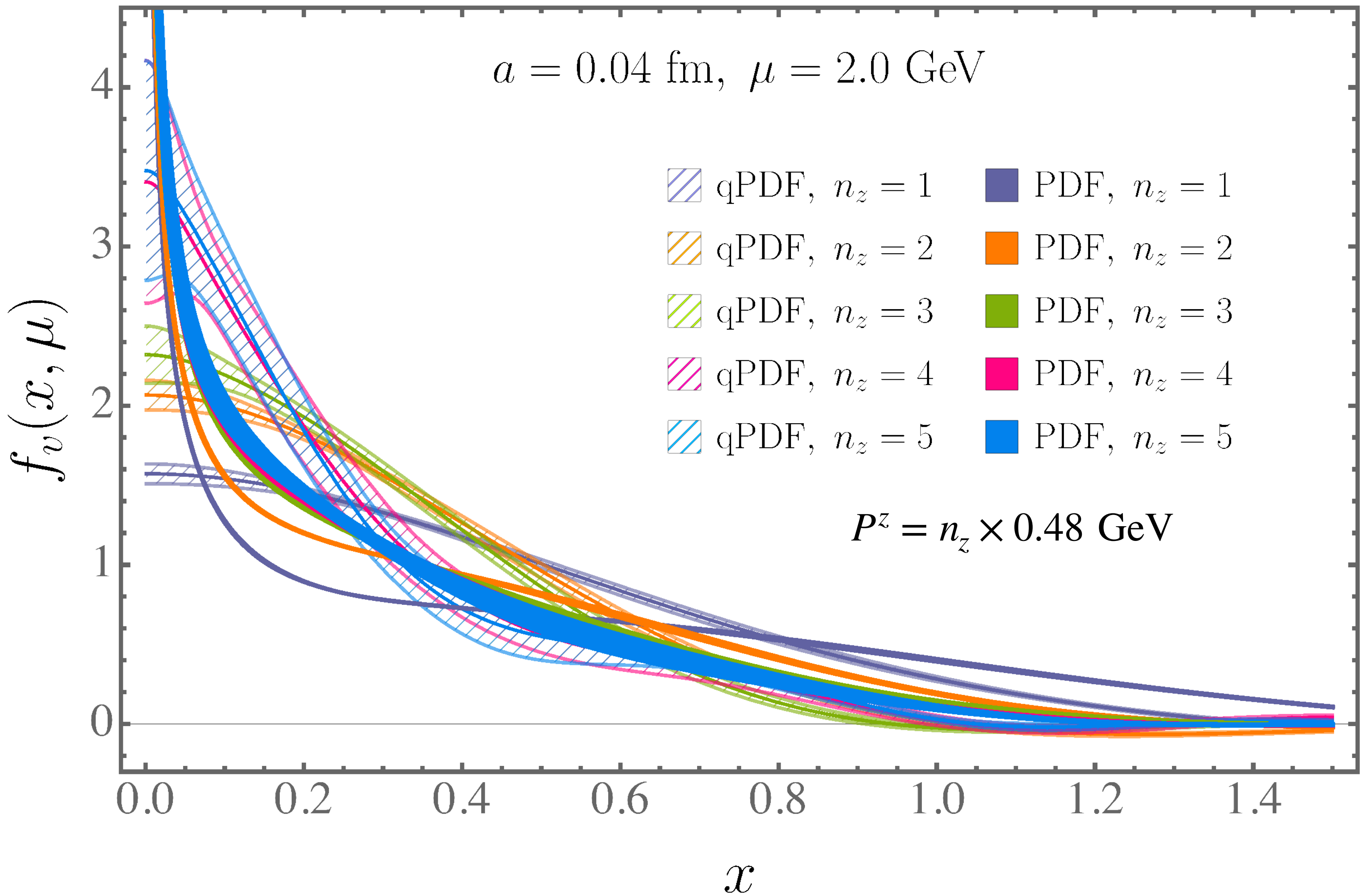


(Only the mean values are shown.)

- **Improved perturbation theory uncertainty;**
- **Uncertainty from matching $\delta m(a)$ to MSbar under control.**

Perturbative matching at NNLO

Momentum-dependence significantly reduced:



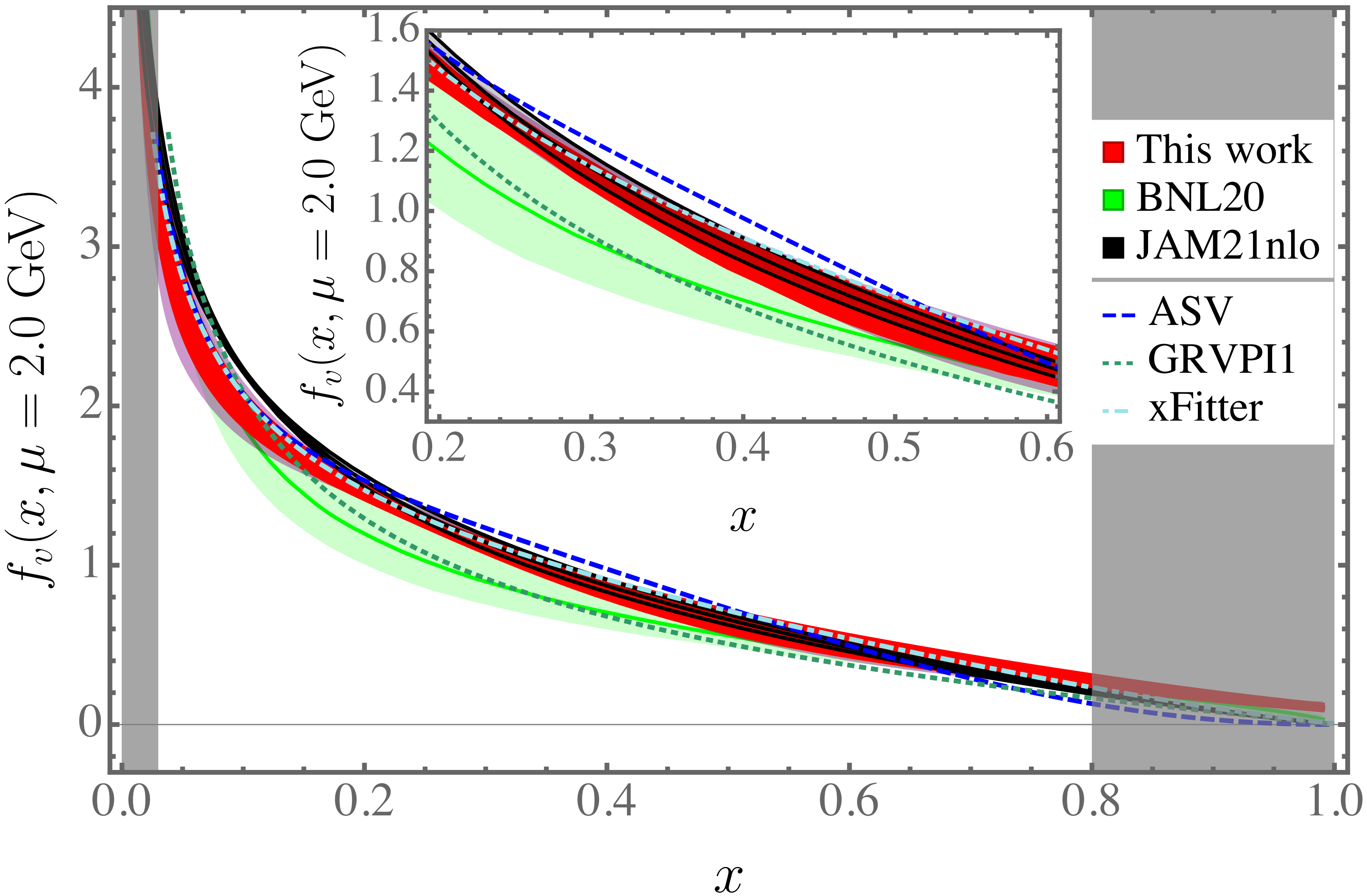
Convergence at P^z > 1.45 GeV (Lorentz boost factor ~ 5.0) and at moderate *x*.

Systematic uncertainties

- Statistical uncertainty: bootstrap resampling.
- Scale variation: error band covers results from $\mu = 1.4,\ 2.0,\ 2.8\ \mathrm{GeV}$, which are all evolved to $\mu = 2.0\ \mathrm{GeV}$ with NLO DGLAP kernel.
- Truncation point z_L: extremely small.
- Extrapolation model dependence: extremely small for the *x* of interest.
- Higher-order perturbative corrections:
  - Requiring N^{3}LO/LO $\leq 5\,\%$ $\Rightarrow$ NLO/LO $\leq 37\,\%$ and NNLO/LO $\leq 14\,\%$;
  - $0.03 \leq x \leq 0.88$.
- Power corrections:
  - Use P^z=2.42 GeV result as final prediction;
  - Fit P^z≥1.45 GeV results with $f_v(x) + \alpha(x)/P_z^2$ at each *x*;
  - $\left|\alpha(x)/[P_z^2 f_v(x)]\right| \leq 10\,\%$, $\Rightarrow$ $0.01 \leq x \leq 0.80$.

$0.03 \leq x \leq 0.80$

Final prediction



x	Statistical	Scale	$\mathcal{O}(\alpha_s^3)$	Power corrections	$\mathcal{O}(a^2 P_z^2)$
0.03	0.10	0.04	< 0.05	< 0.01	< 0.01
0.40	0.07	< 0.01	< 0.05	0.04	< 0.01
0.80	0.15	0.03	< 0.05	0.10	< 0.01

Global fits at NLO

- **JAM21nlo**, P. C. Barry, C.-R. Ji, N. Sato, and W. Melnitchouk, PRL 127 (2021);
- **xFitter**, I. Novikov et al., PRD 102 (2020);
- **ASV**, Aicher, A. Schafer, and W. Vogelsang, PRL 105 (2010);
- **GRVPI1,** M. Gluck, E. Reya, and A. Vogt, Z. Phys. C 53 (1992).

Short-distance factorization at NLO, with same lattice data:

BNL20, X. Gao, YZ, et al., PRD102 (2020).

The uncertainties are 5-20% for 0.03<*x*<0.80, not including pion mass dependence and finite volume effects.

Comparison to short-distance factorization

Operator product expansion:

T. Izubuchi, X. Ji, L. Jin, I. Stewart, and YZ, PRD98 (2018)

$$\tilde{h}(\lambda, z^2\mu^2) = \langle P | O_{\gamma^0}(z,\mu) | P \rangle / (2P^0)$$

$$\lambda = zP^z \qquad = \sum_{n=0}^{\infty} C_n(z^2\mu^2) \frac{(-i\lambda)^n}{n!} a_n(\mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD}),$$

Can calculate the lowest moments within finite λ_{max}=z_{max} p^z_{max}.

z_{max} must be small (0.2–0.3 fm?)

Reduce Ioffe-time pseudo distribution:

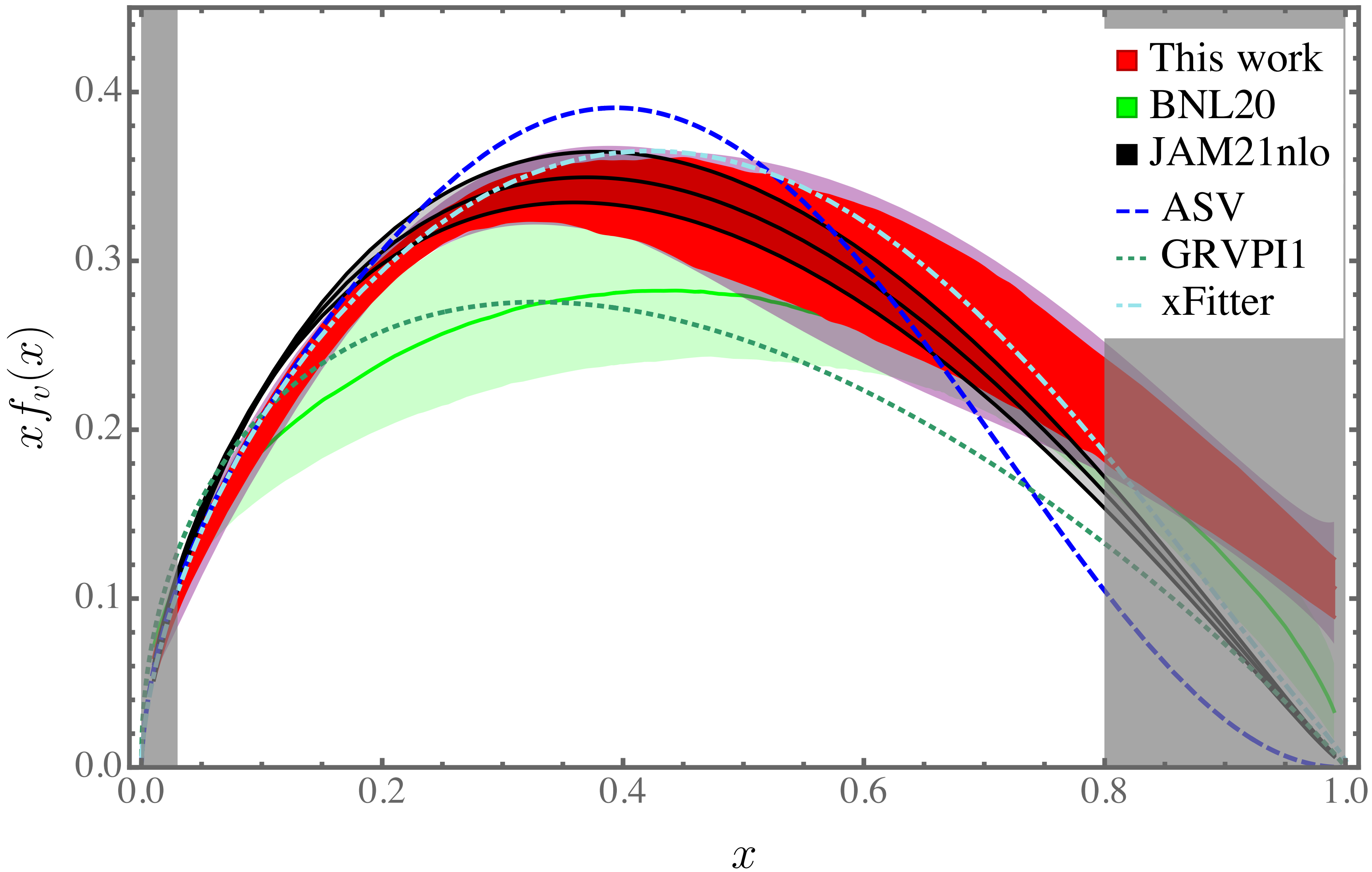
- A. Radyushkin, PRD 96 (2017);
- K. Orginos et al., PRD 96 (2017),

$$\tilde{h}(\lambda, z^2\mu^2) = \int_0^1 d\alpha\, \mathscr{C}(\alpha, z^2\mu^2)\, h(\alpha\lambda, \mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD}),$$

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-ix\lambda} h(\lambda,\mu)$$

- Power-law decrease at large λ: needs very large λ for controlled FT;
- With not very large λ_{max}, needs assumptions, e.g., $f(x) \propto x^a(1-x)^b(1+c\sqrt{x}+\ldots)$.

Comparison with short-distance factorization approach



Comparison to BNL20:

- **Better agreement with xFitter (2020) and JAM21nlo (2021);**
- **Reduced uncertainties.**

Short-distance factorization at NLO, with same lattice data:

BNL20, X. Gao, YZ, et al., PRD102 (2020).

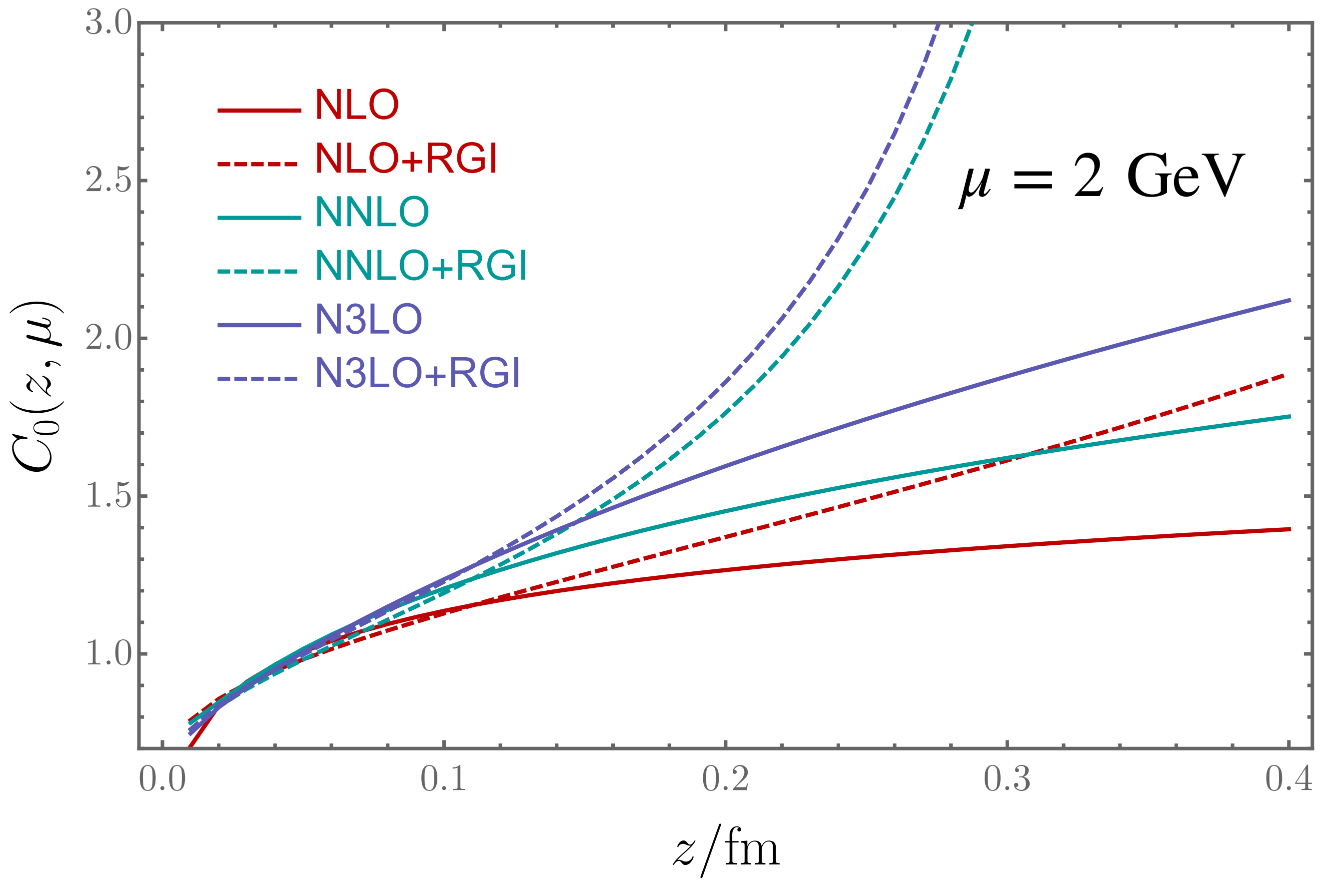
- With finite P^z and statistics, lattice QCD can only reliably predict $x \in [x_{\min}, x_{\max}]$;
- In short-distance factorization analysis, modeling the PDF correlates all $x \in [0,1]$;
- There can be bias from the model choice.

Conclusions

- We have carried out a lattice calculation of the x-dependence of pion valence PDF with an adapted hybrid renormalization scheme;
- The Wilson-line mass correction can be well determined from lattice and matched to the MSbar scheme, with the uncertainty under control;
- NNLO matching shows good perturbative convergence and reduced scale-variation uncertainty;
- We demonstrate that we can predict the x-dependence with controlled systematic uncertainties within $[x_{min}, x_{max}]$;
- Systematics to be analyzed: physical pion mass, lattice spacing dependence, finite volume effect, etc.
- Same renormalization method can also be used to calculate gluon PDFs, GPDs and TMDs.

Matching the Wilson-line mass to MSbar

How small should *z* be?



$$z, z_0 \gg a$$

Resummed coefficient:

$$C_0^{\mathrm{RGI}}\left(\mu^2, z^2\right) = C_0\left(\alpha_s(2e^{-\gamma_E}/z), 1\right) \times \exp\left[\int_{2e^{-\gamma_E}/z}^{\mu} d\alpha_s(\mu') \frac{\gamma_{\mathcal{O}}(\alpha(\mu'))}{\beta(\alpha_s(\mu'))}\right]$$

- For z ~ 0.2 fm, perturbation theory uncertainty is still under control;
- To suppress finite *a* effects, we choose

$$z_0 = 0.24 \text{ fm}, \; z_0 \le z \le 0.4 \text{ fm}$$

- and use NNLO C_0,
- and vary *μ* by a factor of 1/1.4 and 1.4 to estimate the uncertainty in this matching.

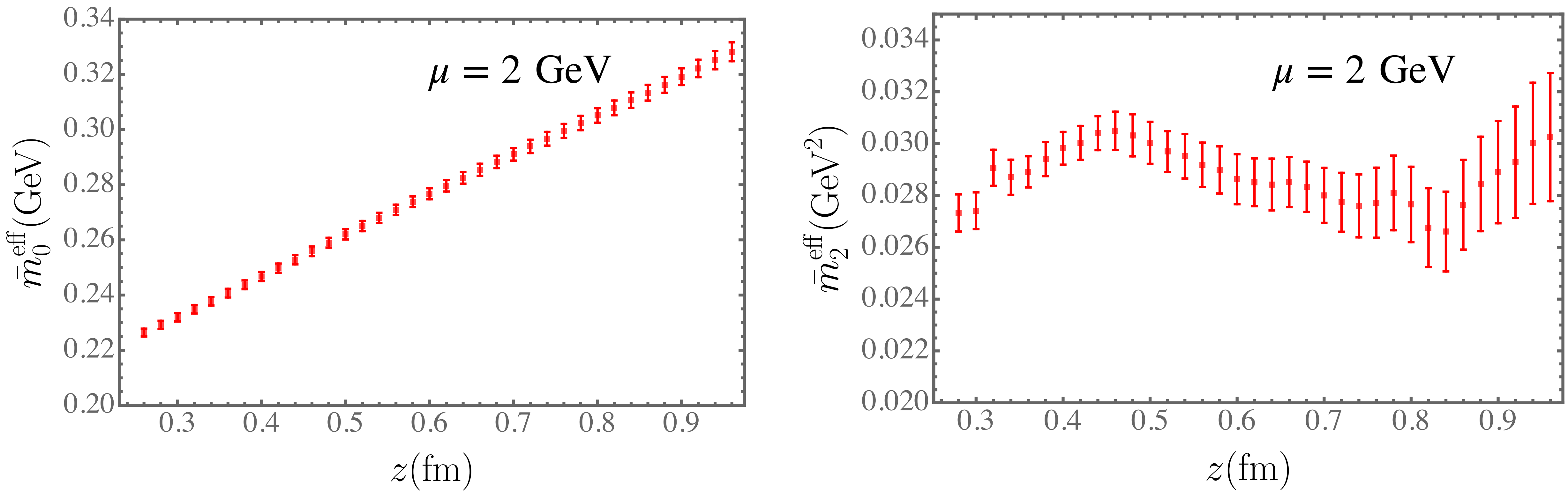
$$\lim_{a\to 0} e^{\delta m(a)(z-z_0)} \frac{\tilde{h}(z, P^z=0, a)}{\tilde{h}(z_0, P^z=0, a)} = e^{-\bar{m}_0(\mu)(z-z_0)} \frac{C_0(\alpha_s(\mu), z^2\mu^2) + \Lambda(\mu) z^2}{C_0(\alpha_s(\mu), z_0^2\mu^2) + \Lambda(\mu) z_0^2}$$

Matching the Wilson-line mass to MSbar

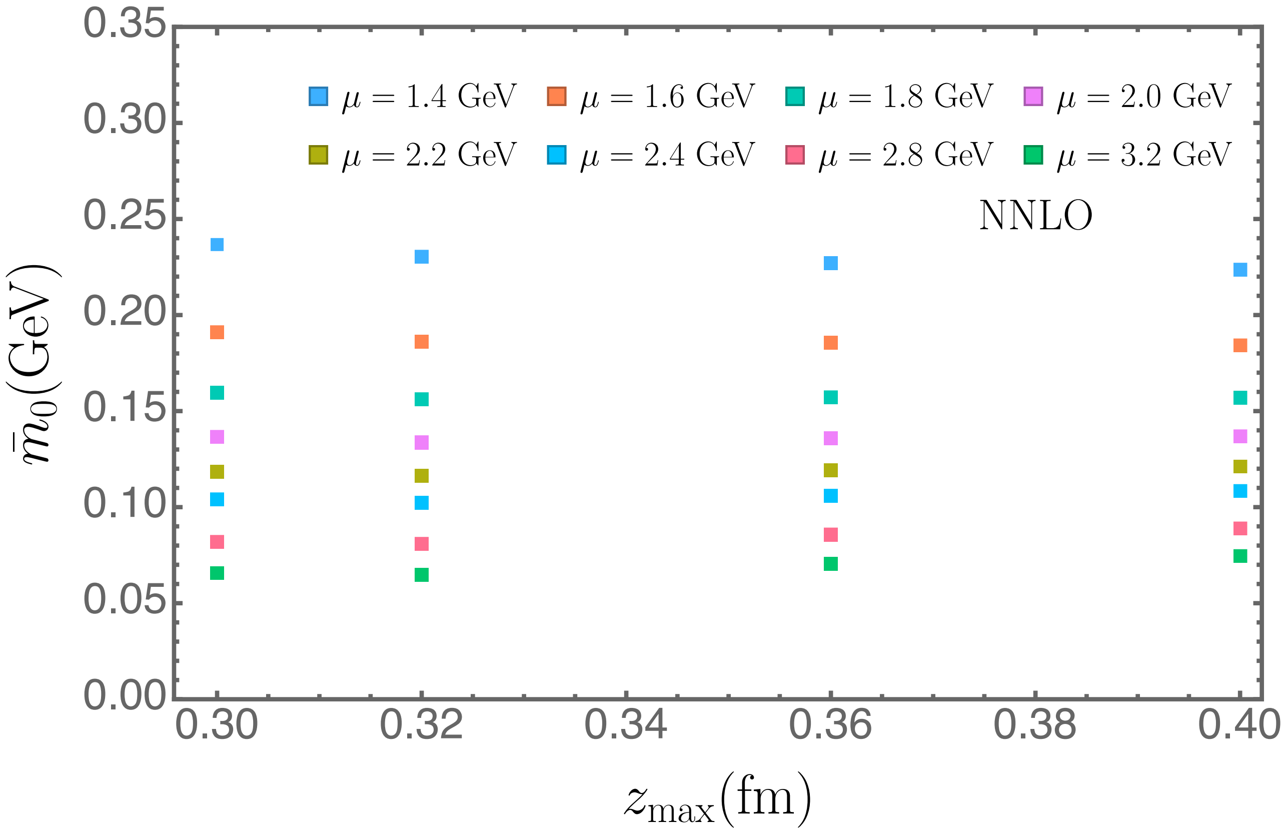
Define effective mass and its slope in *z*:

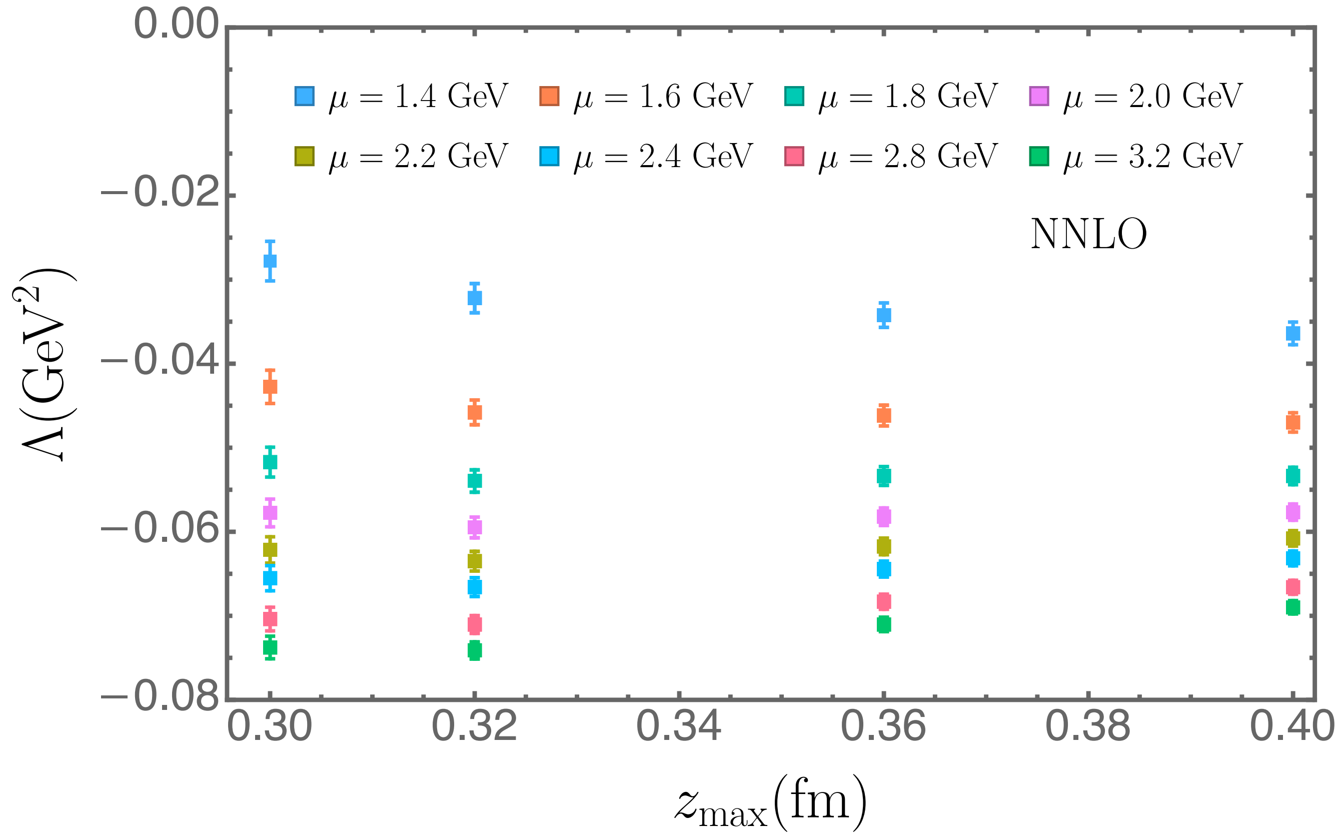
$$\bar{m}_0^{\rm eff}(z) \equiv \left[-\ln \frac{\tilde{h}(z,0,a)}{\tilde{h}(z_0,0,a)} + \ln \frac{C_0^{\rm NNLO}(z^2\mu^2)}{C_0^{\rm NNLO}(z_0^2\mu^2)} \right] \Big/ (z - z_0)$$

$$\bar{m}_2^{\rm eff}(z) = \frac{\bar{m}_0^{\rm eff}(z) - \bar{m}_0^{\rm eff}(z-a)}{a}$$



Matching the Wilson-line mass to MSbar





- Both $\bar{m}_0$ and Λ are sensitive to μ because we used fixed-order C_0 at NNLO;
- We vary μ by a factor of 1.4 to estimate the corresponding uncertainties in the final result.

Matched hybrid-scheme matrix element

Continuum limit:

$$\tilde{h}(z, z_S, P^z, \mu) = \frac{\tilde{h}_0^{\overline{\rm MS}}(z, P^z, \mu)}{C_0(z^2\mu^2)}\ \theta(z_S - |z|) + \frac{\tilde{h}_0^{\overline{\rm MS}}(z, P^z, \mu)}{C_0(z_S^2\mu^2)}\ \theta(|z| - z_S)$$

$$\tilde{h}^{\overline{\rm MS}}(z, P^z,\ \mu) = e^{-m_0^{\overline{\rm MS}}|z|}\ \tilde{h}_0^{\overline{\rm MS}}(z, P^z, \mu)$$

- Perturbatively matchable to $\tilde{h}_0^{\overline{\rm MS}}(z, P^z, \mu)$ as long as $z_S \ll 1/\Lambda_{\rm QCD}$!
- After Fourier transform, it preserves the perturbative matching in *x*-space.

Asymptotic behavior at large z

- Current-current correlator in a static hadron:

$$\langle \pi | j^0(z) j^0(0) | \pi \rangle = \sum_n \int \frac{d^3 p_n}{(2\pi)^3 2E_{p_n}} \langle \pi | j^0(z) | n \rangle \langle n | j^0(0) | \pi \rangle$$

$$\xrightarrow{|z|\to\infty} \propto e^{-M_0 |z|}$$

Burkardt, Grandy and Negele, Annals of Physics 238 (1995).

- Nonlocal quark bilinear ⇒ "heavy-to-light" current-current correlator, in a boosted hadron:

H. Dorn, Fortsch. Phys. 34 (1986).

$$\langle \pi(p) | \bar{\psi}(z) \Gamma W[z,0] \psi(0) | \pi(p) \rangle = \langle \pi(p) | \bar{\psi}(z) \Gamma Q(z) \bar{Q}(0) \psi(0) | \pi(p) \rangle_Q$$

$$\xrightarrow{|z|\to\infty} \propto e^{-\bar{\Lambda}|z|} g(p \cdot z)$$

$$\bar{\Lambda} = m_H - m_Q \sim 0.4 - 0.6 \text{ GeV}$$

Beneke and Braun, Nucl. Phys. B 426(1994).

Physical extrapolation and Fourier transform (FT)

z_L dependence: