Quark spin-orbit correlations in the proton from lattice QCD

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Direct evaluation of quark spin-orbit correlations

$$2L_3S_3 = \int dx \int d^2k_T \int d^2r_T\, (r_T \times k_T)_3\, \Sigma\; \mathcal{W}^{\mathcal{U}}_{\Sigma}(x, k_T, r_T) \qquad \text{Wigner distribution}$$

$$\frac{2L_3S_3}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}}\, \langle P+\Delta_T/2 \mid \overline{\psi}(-z/2)\gamma^+\gamma^5\, \mathcal{U}[-z/2, z/2]\psi(z/2) \mid P-\Delta_T/2\, \rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle P+\Delta_T/2 \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid P-\Delta_T/2\, \rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

P in 3-direction, $P \to \infty$

Renormalization: Form ratio with number of valence quarks n – note: are using Domain Wall Fermions!

Connection to GTMDs: $$2L_3S_3 = \int dx \int d^2k_T\, \frac{k_T^2}{M^2}\, G_{11}\Big|_{\Delta_T=0}$$

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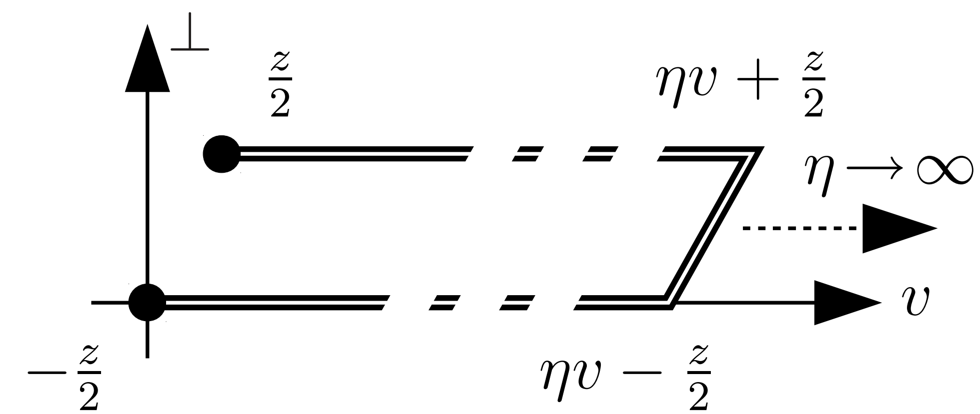
Role of the gauge link $\mathcal{U}$:

- Straight $\mathcal{U}[-z/2, z/2] \longrightarrow$ Ji OAM
- Staple-shaped $\mathcal{U}[-z/2, z/2] \longrightarrow$ Jaffe-Manohar OAM



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Wigner distribution

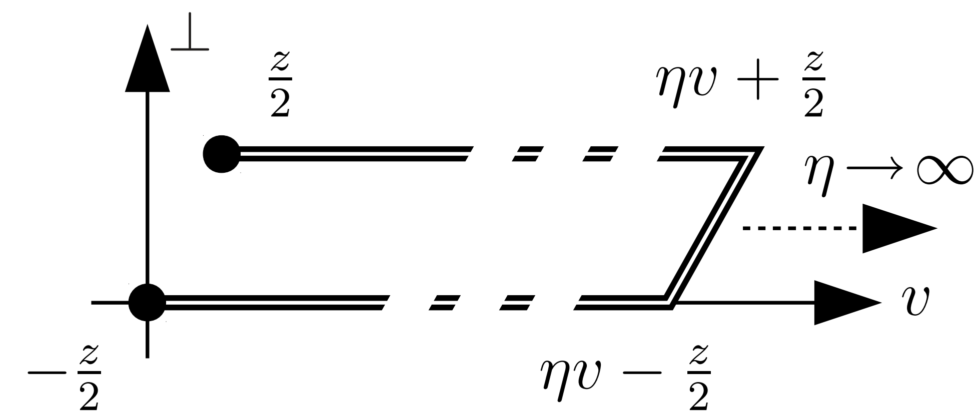
$$\frac{2L_3 S_3}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle P + \Delta_T/2 \mid \overline{\psi}(-z/2)\gamma^+ \gamma^5 \, \mathcal{U}[-z/2, z/2] \psi(z/2) \mid P - \Delta_T/2 \, \rangle \big|_{z^+ = z^- = 0 \, , \; \Delta_T = 0 \, , \; z_T \to 0}}{\langle P + \Delta_T/2 \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) \mid P - \Delta_T/2 \, \rangle \big|_{z^+ = z^- = 0 \, , \; \Delta_T = 0 \, , \; z_T \to 0}}$$

Role of the gauge link $\mathcal{U}$:

Direction of staple taken off light cone (rapidity divergences)

Characterized by Collins-Soper parameter $\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$

Are interested in $\hat{\zeta} \longrightarrow \infty$; synonymous with $P \longrightarrow \infty$ in the frame of the lattice calculation $(v = e_3)$



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- Derivative w.r.t. Δ_T taken using direct derivative method (Rome method)
- Derivative w.r.t. z_T taken as finite difference over lattice spacing a

$$\left. \frac{\partial f}{\partial z_{T,i}} \right|_{z_{T,i}=0} = \frac{1}{2a} \left(f(ae_i) - f(-ae_i) \right)$$

$\longrightarrow$ Corresponds to cutting off momentum integrations at the resolution scale of the calculation

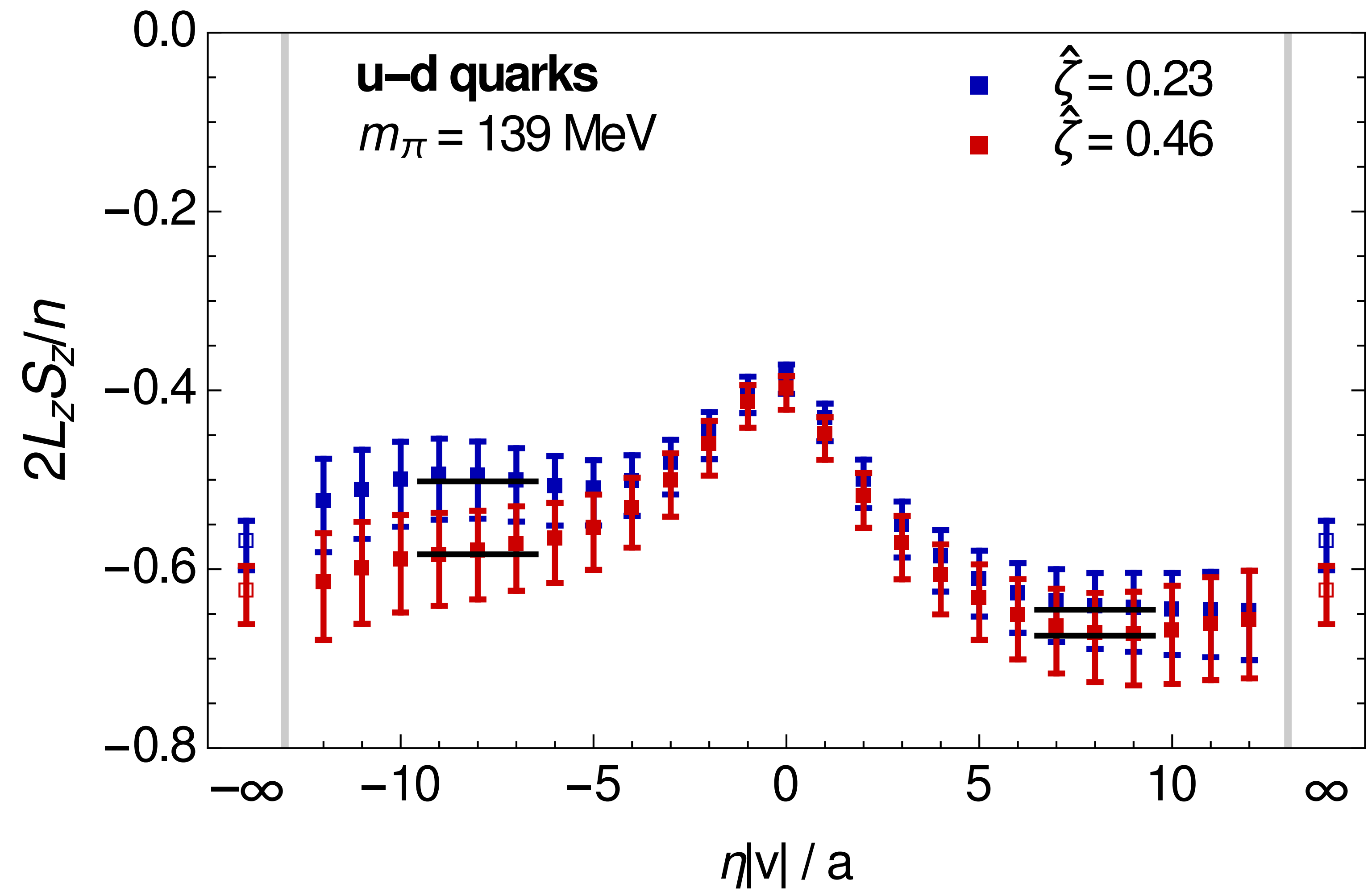
$\longrightarrow$ This is not identical to $\overline{MS}$ – matching factor needed to convert

Ensemble details

DWF ensemble provided by the RBC/UKQCD collaboration

$L^3 \times T$	a(fm)	m_π (MeV)	#conf.	#meas.
$48^3 \times 96$	0.114	139	130	(33280 sloppy + 520 exact) AMA

Result



For comparison, in a polarized proton (from C. Alexandrou et al., PRD 101 (2020) 094513; 2003.08486):

$$\langle L^u \rangle = -0.22(3)\ ,\ \langle 2S^u \rangle = 0.86(2) \Rightarrow \langle L^u \rangle \langle 2S^u \rangle = -0.2$$

$$\langle L^d \rangle = 0.26(2)\ ,\ \langle 2S^d \rangle = -0.42(2) \Rightarrow \langle L^d \rangle \langle 2S^d \rangle = -0.1$$

$$\Rightarrow \langle L^u \rangle \langle 2S^u \rangle - \langle L^d \rangle \langle 2S^d \rangle = -0.1$$

Conclusion

- Strong spin-orbit coupling of quarks in the proton observed.
- Effect of final-state interactions even more pronounced than for orbital angular momentum alone.
- To date, results only for one source-sink separation, one lattice spacing - further systematics to be explored.

Ji quark orbital angular momentum: $\eta = 0$