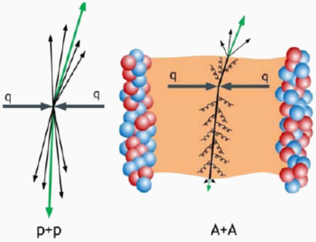
Medium modifications in the initial & final states to heavy-flavor in nuclear collisions

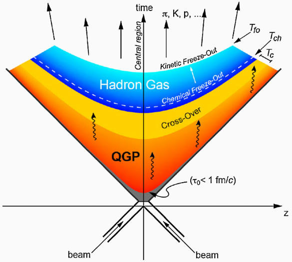
TMD Collaboration Meeting, Jun 15-17, 2022

Weiyao Ke, Los Alamos National Laboratory

June 16, 2022

Hard QCD processes created in a nuclear medium





$T_0 \sim 150...500$ MeV.

$R_T, \tau \sim$ a few ...10 fm.

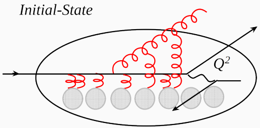
- Factorized formula in *p*-*p*

$$d\sigma_{pp\to h} = f_{i/p}(x_i,\mu) f_{j/p}(x_j,\mu) \otimes d\hat{\sigma}_{ij\to k} \otimes D_{h/k}(z,\mu)$$

- A colorful quark-gluon plasma is formed in nuclear collision. Cause parton energy loss & modify fragmentation.

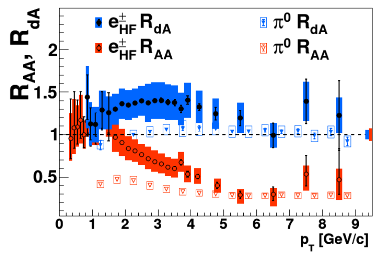
$$D_{h/k}(z,\mu) \to D_{h/k}(z,\mu; \underbrace{T(\tau,x), u^\mu(\tau,x)\cdots}_{\text{medium properties}})$$

- Modified initial-state: nuclear PDF $f_{i/p}(x_i,\mu_0) \to f_{i/A}(x_i,\mu_0)$ Evolution in cold nuclear matter (CNM).

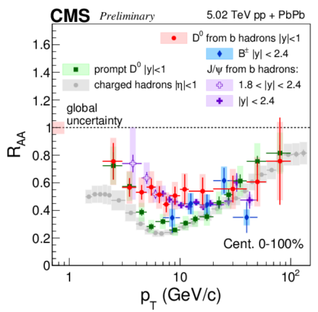


[I. Vitev PRC75(2007)064906]

Hadron & heavy-flavor as probes of CNM and QGP



[PHENIX: PRL109(2012)242301]



- Modified high-p_T hadron yield[1] $R_{AA} \equiv \frac{dN_{AA\to h}/dp_T}{\langle N_{\rm coll}\rangle dN_{pp\to h}/dp_T}$.
- Drastic suppression due to QGP effects in heavy-nucleus collisions $R_{AA} \ll 1$. A clear mass dependence (massive kinematics, dead-cone, mass v.s. in-medium scales).
- *p*-*A*, *d*-*A*: originally aims for CNM effects. But recent measurements also suggest important final-state effects.

[1] Low-p_T charged particle production in *A*-*A* mostly from medium freeze-out, does not follow $N_{\rm coll}$ scaling.

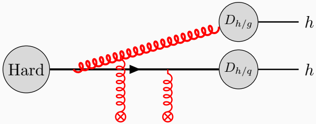
Outline

- Charged hadron & HF production in *A*-*A* and *p*-*A* [based on W. Ke, I. Vitev arXiv:2204.00634] .
    - The $\mathrm{SCET_G}$ in-medium QCD splitting functions.
    - Collinear evolution and momentum broadening in cold nuclear matter.
    - QGP effects (mostly discussing collinear physics) of inclusive hadron / HF production.
- How will TMD physics help future studies?
  Ongoing work: k_T-dependent medium modifications in cold nuclear matter.
  Largely encouraged by the TMD Winter School 2022.

The SCET$_G$ method of treating in-medium parton dynamics

- SCET$_G$: SCET Lagrangian coupled to background Glauber gluon of medium [G. Ovanesyan, I. Vitev, JHEP06(2011)080]

$$\mathcal{L}_{\rm SCET}(\xi_n, A_n) + \mathcal{L}_G(\xi_n, A_n, A_G)$$
$$\mathcal{L}_G = e^{-i(p-p')x}\left[\bar{\xi}_n \Gamma^{\mu,c}_{qqG}\xi_n - i\Gamma^{\mu\alpha\beta,cab}_{ggG} A^a_{n,\alpha} A^b_{n,\beta}\right] A^c_{G,\mu}(x)$$



- Background A_G is a superposition of the color field generate by medium sources,

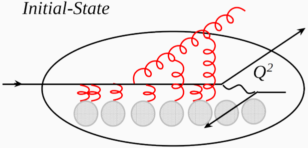
$$A^{\mu,a}(x) \quad = \quad \sum_i g_s \int e^{-iq(x-y)} \frac{g^{\mu\nu} + \cdots}{q^2 - \mu_D^2} J^a_{\nu,i}(y) dy^4, \quad q \sim (\lambda^2, \lambda^2, \vec{\lambda})$$

- In CNM: static sources with a constant non-perturbative screening mass μ_D.

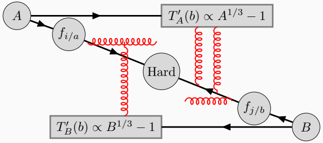
- In QGP: thermal sources $\langle J^0(x)\rangle = \frac{d_{q,g}}{e^{p\cdot u(x)/T(x)} \pm 1}$ for quarks and gluons with Debye screening mass $\mu_D^2(T) = \frac{g_s^2}{3}\left(N_c + \frac{N_f}{2}\right) T^2$. Neglect sources in the later hadronic stage.

Initial-state effects I: multiple collisions

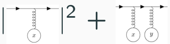
In p-A collisions ▽



Generalization to A-A collisions ▽



- $\mathrm{k}_\perp$ broadening from multiple collisions (elastic)



$$
\begin{aligned}
f(\mathrm{b}) &= f(\mathrm{b})\left\{1 + L/\lambda_{q,g}[\Sigma(b) - \Sigma(0)] + \cdots\right\} \\
&= f(\mathrm{b}) e^{\frac{L}{\lambda_{q,g}}[\Sigma(b)-\Sigma(0)]} \approx f(\mathrm{b}) e^{-\frac{L}{\lambda_{q,g}}\frac{\mu_D^2 \xi}{4} b^2}. \\
\Sigma(b) &= \frac{1}{\sigma}\int \frac{d\sigma}{d^2\mathrm{q}_\perp} e^{i\mathrm{b}\cdot\mathrm{q}_\perp} d^2\mathrm{q}_\perp \quad \text{collision form factor}
\end{aligned}
$$

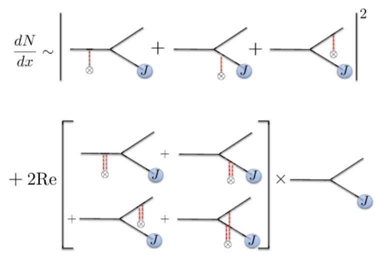
- Often approximated as a "Gaussian" ($\xi = \ln \frac{\mu_D^2 b^2 e^{2\gamma_E - 1}}{4}$). Phenomenology parameters $\mu_D^2 \sim 0.12\ \mathrm{GeV}^2,\ 1.0 < \lambda_g < 1.5\ \mathrm{fm}$.

- Coherent multiple collisions further leads to the dynamical shadowing [J.-W. Qiu, I. Vitev, PLB632 507-511] , effectively shift parton momentum fractions by

$$
\frac{\delta x_a}{x_a} \propto \frac{\langle \mathrm{k}_\perp^2 \rangle_B}{-u}, \qquad \frac{\delta x_b}{x_b} \propto \frac{\langle \mathrm{k}_\perp^2 \rangle_A}{-t}
$$

Initial-state effects II: medium-induced radiations

Medium-induced contributions to opacity order $\left(\frac{L}{\lambda}\right)^1$



- Full splitting functions $P_{qq}, P_{qg}, P_{gq}, P_{gg}$ in CNM available in [G. Ovanesyan, F. Ringer, I. Vitev PLB07(2016)054] .
- Currently applied in the soft-gluon limit [I. Vitev, PRC75(2007)064906] : further shift by $f(x)$ by the x-loss.

$$\frac{\Delta x_a}{x_a} \approx \frac{L_B}{\lambda_g} \int_{m_N/p^+}^{1} dx \int_{(x m_N)^2}^{(xp^+)^2} dk_\perp^2 \frac{dN}{dx dk_\perp^2}$$

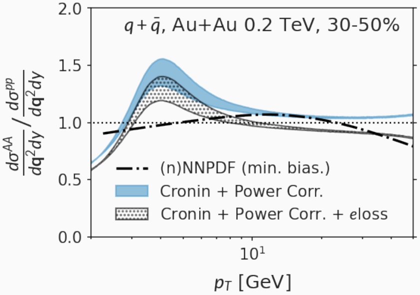
- Radiative momentum broadening not included!

$$\frac{dN(x_g \to 1)}{dx dk_\perp^2} = \frac{\alpha_s C_R}{\pi} \int_0^{\frac{\mu p^+}{4}} d^2 q_\perp \frac{\mu^2}{\pi (q_\perp^2 + \mu^2)^2} \left[\frac{q_\perp^2}{k_\perp^2 (k_\perp - q_\perp)^2} - \frac{2(q_\perp^2 - q_\perp \cdot k_\perp)}{k_\perp^2 (k_\perp - q_\perp)^2} \frac{\sin \frac{k_\perp^2 L_B}{x p^+}}{\frac{k_\perp^2 L_B}{x p^+}} \right]$$

Dynamical CNM effects: shadowing + elastic broadening + energy loss

Dynamical CNM effects[Z.-B. Kang et al. PLB718(2012)482-487] :

$$f(x,\mu) \to f(x + \delta x_{\text{dyn. shadowing}} + \Delta x_{\text{eloss}}, \mu)\frac{1}{\pi}e^{-\mathbf{k}_\perp^2/\langle \mathbf{k}_\perp^2\rangle_{\text{elastic}}}$$



Dynamical CNM calcualtion vs collinear nuclear PDF [R. A. Khalek et al. (nNNPDF3.0) 2201.12363.]

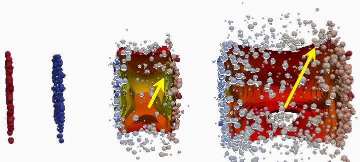
- Dynamical shadowing & elastic broadening: low-p_T depletion & a Cronin broadening at a few GeV.
- High-p_T (from large x partons): isospin effects and energy loss.
- Collinear nuclear PDF (nNNPDF): parametrized & fitted shadowing and anti-shadowing region.

Medium-modified splitting functions for heavy quark [Kang, Ringer, Vitev, JHEP03(2017)146] :

▽ ΔP_{QQ} ΔP_{Qg} ▷

$$\left(\frac{dN^{\rm med}}{dx d^2\boldsymbol{k}_\perp}\right)_{Q\to Qg} = \frac{\alpha_s}{2\pi^2} C_F \int \frac{d\Delta z}{\lambda_g(z)} \int d^2\boldsymbol{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\rm med}}{d^2\boldsymbol{q}_\perp} \left\{ \left(\frac{1+(1-x)^2}{x}\right) \left[\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2} \right.\right.$$
$$\times \left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2} - \frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z]) + \frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2} \cdot \left(2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2} - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2}\right.$$
$$\left. - \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_3)\Delta z]) + \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2} \cdot \frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2}(1-\cos[(\Omega_2-\Omega_3)\Delta z])$$
$$+ \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2} \cdot \left(\frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2+\nu^2} - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2}\right)(1-\cos[\Omega_4\Delta z]) - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2} \cdot \frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2+\nu^2}(1-\cos[\Omega_5\Delta z])$$
$$\left. + \frac{1}{N_c^2} \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2} \cdot \left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2} - \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z]) \right]$$
$$\left. + x^3 m^2 \left[\frac{1}{\boldsymbol{B}_\perp^2+\nu^2} \cdot \left(\frac{1}{\boldsymbol{B}_\perp^2+\nu^2} - \frac{1}{\boldsymbol{C}_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z]) + \ldots \right] \right\} \qquad (2.51)$$

$$\left(\frac{dN^{\rm med}}{dx d^2\boldsymbol{k}_\perp}\right)_{g\to Q\bar{Q}} = \frac{\alpha_s}{2\pi^2} T_R \int d\Delta z \frac{1}{\lambda_q(z)} \int d^2\boldsymbol{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\rm med}}{d^2\boldsymbol{q}_\perp} \left\{ (x^2+(1-x)^2) \right.$$
$$\times \left[2\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2} \cdot \left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2} - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z]) \right.$$
$$+ 2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2} \cdot \left(\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2} - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_3)\Delta z]) + \frac{1}{N_c^2-1}\left(2\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2}\right.$$
$$\times \left(\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2} - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z]) + 2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2} \cdot \left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2} - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2}\right)$$
$$\times (1-\cos[(\Omega_1-\Omega_3)\Delta z]) - 2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2} \cdot \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2} \cdot (1-\cos[(\Omega_2-\Omega_3)\Delta z])$$
$$+ 2\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2} \cdot \left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2} - \frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2+\nu^2}\right)(1-\cos[\Omega_4\Delta z])$$
$$\left.\left. + 2\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2} \cdot \frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2+\nu^2}(1-\cos[\Omega_5\Delta z])\right)\right]$$
$$\left. + m^2 \left[2\frac{1}{\boldsymbol{B}_\perp^2+\nu^2} \cdot \left(\frac{1}{\boldsymbol{B}_\perp^2+\nu^2} - \frac{1}{\boldsymbol{A}_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z]) + \ldots \right] \right\}. \qquad (2.52)$$

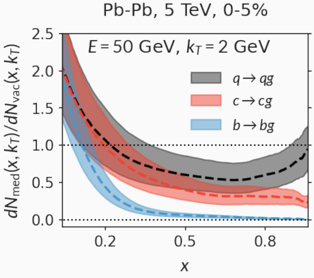


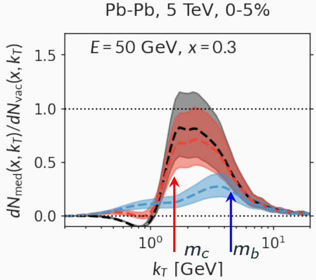
Hydrodynamic-based simulation of QGP

provides temperature profiles $T(\tau, x, y)$

[H. Song, U. Heinz, PRC77(2008)064901;

J. E. Bernhard, 1804.06469;]





QGP effects II: medium-modified splitting & colinear FF

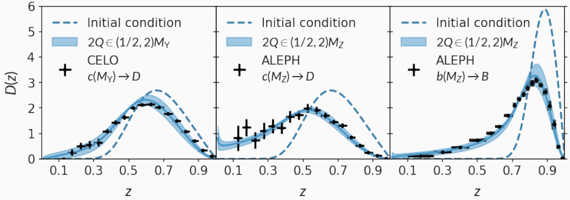
Modified DGLAP: $$\frac{\partial D_{h/i}(z,Q^2)}{\partial \ln Q^2} = [P_{ji}^{\rm vac} + \Delta P_{ji}^{\rm med}] \otimes D_{h/j}(z,Q^2),$$

$$Q^2 = \frac{k_\perp^2 + x m_3^2 + (1-x) m_2^2 - x(1-x) m_1^2}{x(1-x)}$$

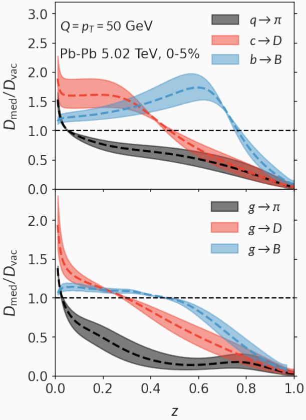
Lund-Bowler initial condition[2]($Q_0 = 0.4$ GeV) [Bowler ZPC11(1981)169] :

$$D(z) = \frac{(1-z)^a}{z^{1+bm_T^2}} e^{-bm_T^2/z}, \ \ a = 0.89, \ \ b = 3.3 \ {\rm GeV}^2.$$

Evolution in the vacuum ▽

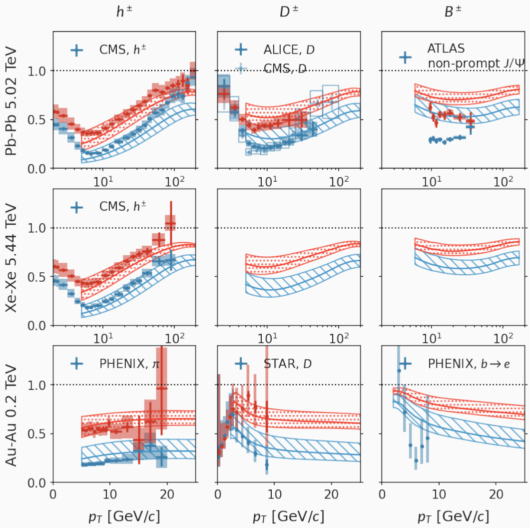


Medium modified FF ▽



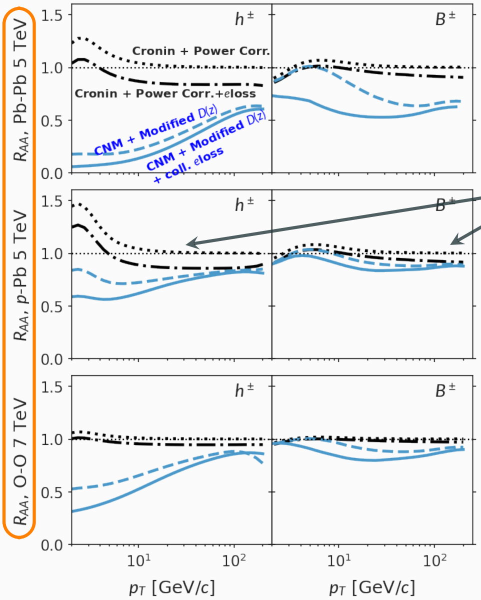
[2] $D_{D/g} = D_{B/g} = 0$ at $Q = Q_0$; *non-zero but small* at $Q > Q_0$ due to evolution. But non-perturbative input can be important for inclusive spectra [D. Anderle et al. PRD96(2017)034028] .

Nuclear modification factors in nuclear collisions

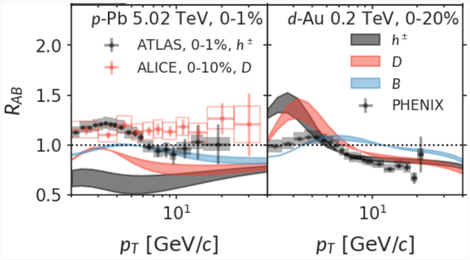


- Jet-medium coupling $g_s = 1.8 \pm 0.2$ $(0.20 < \alpha_s < 0.32)$.
- Reasonable description of R_{AA} for light and D.
- Underestimate suppression of B (may need to include NP $g \to B$ FF).

CNM and QGP effects in small colliding systems



- Heavy *A*-*A* collisions: CNM effects overwhelmed by QGP effects.
- Asymmetric *p*/*d*-*A* collisions: need better control of both CNM and QGP effects to understand the data. ▽



In progress: improve initial state calculations w/ TMD

Improve initial state calculations

- Understand $k_\perp$-dependent in-medium radiative contributions.
- Directly probe momentum broadening (not deducing it from energy loss).
  - Determine the jet transport parameter in CNM from, e.g., modified k_T distribution of HF pair and Drell-Yan [PHENIX prelim: PoS HardProbes2018 (2018) 160, NPA982(2019)695-698]
  - Improve the CNM baseline in p-A collisions to search for QGP in p-A.

How important is medium-induced radiative broadening?

Momentum broadening of an on-shell quark in a "brick" medium [B. Wu, JHEP1110(2011)029 , T. Liou, A.H. Mueller, B. Wu NPA916(2013)102-125, J.-P. Blaizot, Y. Mehtar-Tani, NPA929(2014)202-229] .

$$\langle \mathrm{k}_\perp^2 \rangle = \langle \mathrm{k}_\perp^2 \rangle_0 + \int d\omega \int d\mathrm{k}_\perp^2 \, \mathrm{k}_\perp^2 \frac{dN_g}{d\omega d\mathrm{k}_\perp^2}$$

Within BDMPS-Z framework [emissions induced by small-angle collisions with medium, but resumed to all-orders of opacity] , a double-log enhanced region is identified:

- $\hat{q}L \ll \mathrm{k}_\perp^2$: $1/\mathrm{k}_\perp^4$ power law region.
- $l_0 < \tau_f < L$, formation time $\tau_f = \frac{2xk^+}{\mathbf{k}_\perp^2}$, otherwise suppressed by Landau-Pomeranchuk-Migdal(LPM) effect.

$$\mathrm{k}_\perp^2 \frac{dN}{d\omega d\mathrm{k}_\perp^2} \propto \frac{\alpha_s C_A}{\pi} \frac{\hat{q}L}{\mathrm{k}_\perp^2} \frac{1}{\omega},$$

A two-dimensional evolution of the momentum broadening

[T. Liou, A.H. Mueller, B. Wu NPA916(2013)102-125] Define $\hat{q}(\tau,\mu^2) = \langle k_\perp^2 \rangle(\tau,\mu^2)/L$

- Time τ: $\tau_0 \to L$. $\tau_0^{-1} \sim$ in-medium screening mass / temperature.
- Renormalization μ^2: $\hat{q}\tau_0 \to \hat{q}L$, the typical momentum broadening scale.

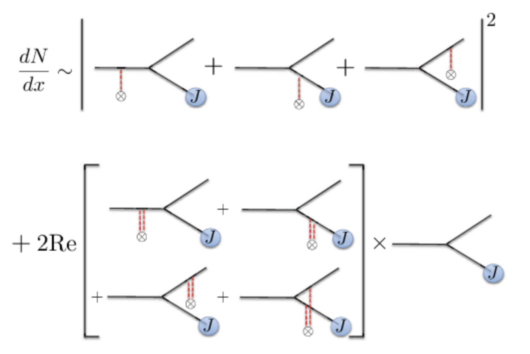
$$\frac{\partial \hat{q}(\tau,\mu^2)}{\partial \ln \tau} = \int_{\hat{q}\tau}^{\mu^2} \bar{\alpha} \frac{dk_\perp^2}{k_\perp^2} \hat{q}(\tau, k_\perp^2), \text{ notice the BC: } \left.\frac{\partial \hat{q}(\tau,\mu^2)}{\partial \ln \tau}\right|_{\mu^2=\hat{q}\tau} = 0.$$

- A solution with $\bar{\alpha} = \text{const}$ & $\hat{q}\tau \approx \hat{q}_0\tau$: $\hat{q}(L,\mu^2 = \hat{q}_0 L) \approx \hat{q}_0(\tau_0,\mu_0^2)\frac{1}{\sqrt{\bar{\alpha}}} I_1\left(2\sqrt{\bar{\alpha}}\ln\frac{L}{\tau_0}\right)$
  100% correction with $\alpha_s = 0.3$ and $\langle L_{\mathrm{Au}}\rangle\Lambda = \Lambda\frac{3}{4}r_0 197^{1/3} \approx 4.3$.

Definitely a large effect. *Can we directly compute the modified k_T distribution & perform vacuum and medium-induced emissions together?*

Let's try $k_\perp$-dependent modifications in CNM using TMD

Collinear sector at opacity $n = 1$



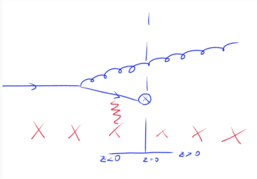
Single-born scattering $e^{i(k_\perp - q_\perp)\cdot b}$

Double-born unitary correction $e^{ik_\perp \cdot b}$

$$\frac{\alpha_s}{2\pi^2} P_{qq}(x) \int d^2 q_\perp \frac{1}{\sigma} \frac{d\sigma}{d^2 q_\perp} \left\{ |\text{Single Born}|^2 \times e^{-ik_\perp - q_\perp \cdot b} + [\text{Double Bornn}] \times e^{-ik_\perp \cdot b} \right\}$$

Integrate this path-length renormalization in the initial-state TMD

Hard process happens at $z^+ = 0$.
A train of target nucleons, constant density.
Path-length before and after hard collisions: L_I, L_F.



$|\text{Single born}|^2$

$$\int_0^\infty \frac{dz}{\lambda_g}\frac{1}{\mathbf{C}_\perp^2} + \int_{-\infty}^0 \frac{dz}{\lambda_g}\left\{\frac{1}{\mathbf{C}_\perp^2}+\frac{C_F/C_A}{\mathbf{A}_\perp^2}-\frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2}\cdot\frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2}+\left[\frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2}\left(\frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2}-\frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2}\right)-\frac{1}{N_c^2}\frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2}\left(\frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2}-\frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2}\right)\right](1-\cos(\omega_{12}z^+))\right\}$$

$2\text{Re}\{\text{Vac}^*\text{Double-Born}\}$

$$\int_0^\infty \frac{dz}{\lambda_g}\frac{1}{\mathbf{A}_\perp^2} + \int_{-\infty}^0 \frac{dz}{\lambda_g}\left[\frac{1}{\mathbf{A}_\perp^2}+\frac{C_F/C_A}{\mathbf{A}_\perp^2}-\frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2}\cdot\frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2}-\frac{1}{\mathbf{A}_\perp^2}(1-\cos(\omega_3 z))+\frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2}\cdot\frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2}(1-\cos(\omega_3 z))\right]$$

In the $x \to 1$ limit:

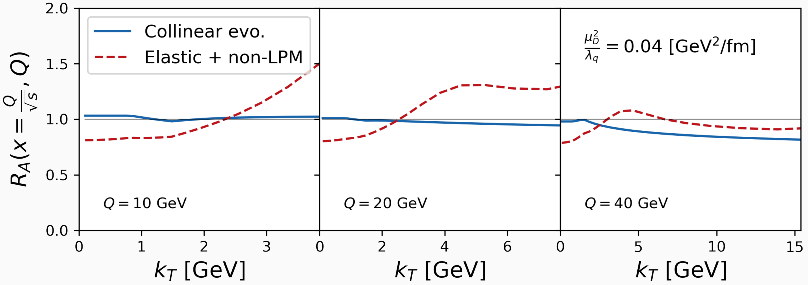
$$\underbrace{\int d^2\mathbf{q}_\perp \frac{1}{\sigma}\frac{d\sigma}{d^2\mathbf{q}_\perp}\left\{\frac{L_I}{\lambda_q}\left[\frac{e^{-i(\mathbf{k}_\perp-\mathbf{q}_\perp)\cdot b}}{\mathbf{k}_\perp^2}-\frac{e^{-i\mathbf{k}_\perp\cdot b}}{\mathbf{k}_\perp^2}\right]\right.}_{\text{Non-LPM term}} + \underbrace{\left.\frac{L_I}{\lambda_g}\frac{\mathbf{q}_\perp^2-\mathbf{k}_\perp\cdot\mathbf{q}_\perp}{\mathbf{k}_\perp^2(\mathbf{k}_\perp-\mathbf{q}_\perp)^2}\left[1-\frac{\sin\frac{\mathbf{k}_\perp^2 L_I}{2k^+}}{\frac{\mathbf{k}_\perp^2 L_I}{2k^+}}\right][e^{-i(\mathbf{k}_\perp-\mathbf{q}_\perp)\cdot b}+e^{-i\mathbf{k}_\perp\cdot b}]\right\}}_{\text{LPM-type contribution}}$$

Non-LPM term

$$\int d^2 \mathrm{q}_\perp \frac{1}{\sigma}\frac{d\sigma}{d^2\mathrm{q}_\perp}\frac{L_I^+}{\lambda_q}\left[\frac{e^{-i(\mathrm{k}_\perp-\mathrm{q}_\perp)\cdot b}}{\mathrm{k}_\perp^2}-\frac{e^{-i\mathrm{k}_\perp\cdot b}}{\mathrm{k}_\perp^2}\right]\to\frac{L_I^+}{\lambda_q}[\Sigma(b)-\Sigma(0)]\times\ \text{Vacuum NLO TMD}$$

Similar structure as elastic broadening: $\frac{L_I^+}{\lambda_q}[\Sigma(b)-\Sigma(0)]\delta(1-x).$

A simple convolution with $\frac{1}{\sigma}\frac{d\sigma}{d^2\mathrm{q}_\perp}$



The LPM contribution: multiscale problem b, L, λ, μ_D

Consider η-regulator [J.-Y. Chiu, A. Jain, D. Neil, I. Z. Rothstein JHEP1205(2012)084]
Combined with the LPM phase factor that suppress the spectra when $\frac{\mathrm{k}_\perp^2 L}{2k^+} \ll 1$.

$$\frac{\alpha_s(\mu^2)C_A}{2\pi}\left[\frac{k^+}{\nu}\right]^\alpha \left[1 - \frac{\sin\frac{\mathrm{k}_\perp^2 L}{2k^+}}{\frac{\mathrm{k}_\perp^2 L}{2k^+}}\right]$$

Region of relevance (when $\alpha > 0$) $\nu < k^+ < \mathrm{k}_\perp^2 L$, then:

$$\int_0^1 du \frac{1}{(1-u)^2}\int d^{2-2\epsilon}\mathrm{q}_\perp \frac{\mathrm{q}_\perp^2}{\sigma}\frac{d\sigma}{d^2\mathrm{q}_\perp} e^{-i(1-u)b\cdot\mathrm{q}_\perp}\int \frac{e^{ib\cdot\mathrm{k}_\perp}}{[\mathrm{k}_\perp^2 + u(1-u)\mathrm{q}_\perp^2]^2}\frac{d^{2-2\epsilon}\mathrm{k}_\perp}{(2\pi)^{2-2\epsilon}}$$

Focus on the region $\mathrm{k}_\perp \gg \mathrm{q}_\perp^2 \sim \mu_D^2$.

$$\frac{\alpha_s(\mu^2)C_A}{2\pi}\frac{L_I/\lambda_q}{4} b^2 \frac{1+x^2}{(1-x)^{1+\alpha}}\left(\frac{p^+}{\nu}\right)^\alpha \int_0^1 du \frac{-\nabla_b^2\Sigma_\epsilon((1-u)b)}{(1-u)^2}\left[\frac{1}{\epsilon} + L_\perp - 1\right]$$

Soft sector

η-regulator and the phase factor

$$\left[\frac{|k^+ - k^-|}{\nu/\sqrt{2}}\right]^\alpha \left[1 - \frac{\sin\frac{k_\perp^2 L}{2k^+}}{\frac{k_\perp^2 L}{2k^+}}\right]$$

The phase factor requires $k^- > 1/L^+ \sim \gamma/(r_0 A^{1/3})$, currently only consider region $k^+ < k^-$.

Combines contribution of the medium-induced soft & collinear sector for the projectile (ongoing...)

$$\frac{\alpha_s(\mu^2) C_A}{2\pi} \frac{L_l/\lambda_q}{4} b^2 \int_0^1 du \frac{-\nabla_b^2 \Sigma((1-u)b)}{(1-u)^2} \Bigg\{ [P_{qq}]_+ \left[\frac{1}{\epsilon} + L_\perp - 1\right] + (1-x)$$
$$+\delta(1-x)\left[-\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left[\ln\frac{\zeta}{\mu^2} - \frac{5}{2}\right] + (L_\perp - 1)\ln\frac{\zeta}{\mu^2} + \frac{1}{2}L_\perp^2 - \frac{7}{2}L_\perp + \frac{3}{2}\right]$$
$$+\frac{d\Sigma}{d\epsilon}(...) + (...)\Bigg\}$$

Towards medium-modified evolution at opacity $n = 1$

- Elastic collisions: $\delta(1-x)[1 + \frac{L}{\lambda_q}(\Sigma(b) - \Sigma(0))]$
- Non-LPM radiation: $[\text{Vacuum calc.}] \times [1 + \frac{L}{\lambda_q}(\Sigma(b) - \Sigma(0))]$
- LPM radiation:

$$-\frac{L_I}{\lambda_q}\overbrace{\int_0^1 du \frac{1}{4} b^2 \nabla^2_{b(1-u)} \Sigma((1-u)b)}^{\sim \Sigma(b) - \Sigma(0) + \mathcal{O}(b^4)} \frac{\alpha_s(\mu^2) C_A}{2\pi} \Bigg\{ [P_{qq}]_+ \left[\frac{1}{\epsilon} + L_\perp - 1\right]$$
$$+\delta(1-x)\left[-\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left[\ln\frac{\zeta}{\mu^2} - \frac{5}{2}\right] + (L_\perp - 1)\ln\frac{\zeta}{\mu^2} + \frac{1}{2}L_\perp^2 - \frac{7}{2}L_\perp + \cdots\right]\Bigg\}$$

Suppose to account for additional broadening.

- μ^2: $\mu_b^2 \to \mu_D^2$. Generates $\ln\frac{\mu_b^2}{\mu_D^2}$.
- $\sqrt{\zeta}$: $\mu_b \to \mu_b^2 L$? This clearly depends on the frame where L is measured. May need to include target evolution or go to medium rest frame.
- For almost on-shell parton: $\mu_b^2 \sim \hat{q}L$: $\ln\frac{\mu_b^2}{\mu_D^2} \to \ln\frac{\hat{q}L}{\mu_D^2}$, $\ln(\mu_b L) \to \frac{2}{3}\ln(\hat{q}^{1/3}L)$.
- For highly off-shell parton, typical $\ln\mu_b^2$ will not be $\ln\hat{q}L$.

Conclusion

- Hadron modifications in nuclear collisions probes both initial-state and final-state medium.
- In-medium splitting functions can be obtained within $\mathrm{SCET_G}$, has been used to study modified collinear evolution.
  - Inclusive hadron quenching well understood in *AA*.
  - In-medium emissions strongly modified by mass (v.s. $k_\perp^2$).
- Precision *p*-*A* & CNM studies requires improved calculation of $k_\perp$-dependent observables.
- Attempt to include medium modifications to the TMD evolution.

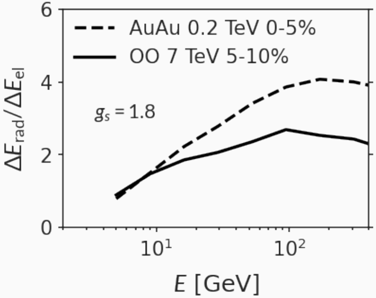
Questions?

Collisional energy loss in small systems

- Collisional and radiative energy loss scales different with medium geometry. For example $T^3 \propto \tau^{-\alpha}$, neglecting logs in $\alpha_s, \ln(E/T), \cdots$

$$\Delta E_{\rm rad} \propto L^{2-\alpha} \ \text{ v.s. } \ \Delta E_{\rm el} \propto L^{1-\frac{2}{3}\alpha}$$

- From realistic hydro simulations of Au-Au 0.2 TeV 0-5% and O-O 7 TeV 5-10%
  - Similar initial temperature $T \approx 0.32$ GeV.
  - QGP size differ by a factor of 2.3.

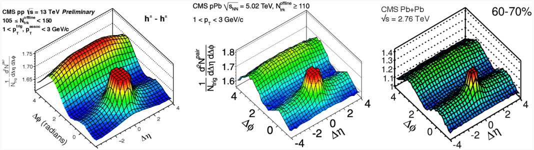


- Define a "radiative energy loss"

$$\Delta E_{\rm rad} = \int dk_\perp^2 \int_{1/2}^{1} \frac{d\Delta P_{qq}^{\rm med}}{dx dk_\perp^2}(1-x)dx$$

- Left: the relative importance of $\Delta E_{\rm rad}$ (charm) is reduced relative to $\Delta E_{\rm el}$ in the smaller system.

Identify QGP signals in small colliding system



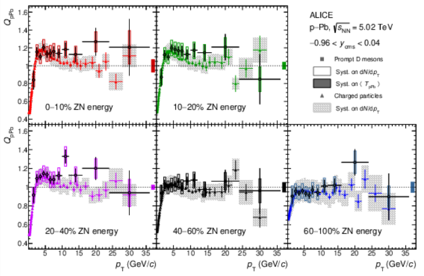
[CMS measured 2-particle correlations in *p*-*p*, *p*-Pb, Pb-Pb]

- Similarity between *p*-Pb and Pb-Pb can suggest final-state interactions in small systems.

T_{max} [GeV] achieved in hydro simulation

p-Pb 5 TeV		O-O 7 TeV	
0-1%	60-90%	0-10%	30-50%
0.315	0.174	0.325	0.263

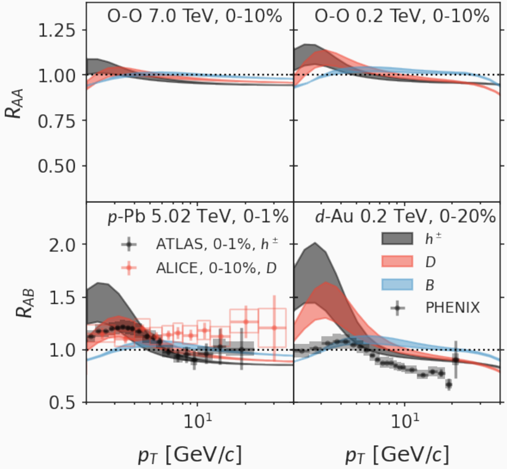
- *But quenching of high-p_T hadrons and heavy flavors is not yet unambiguously observed.*



△ *D*-meson $Q_{p\text{-Pb}}$, ALICE JHEP12(2019)092. Use neutrons in ZDC for centrality selection.

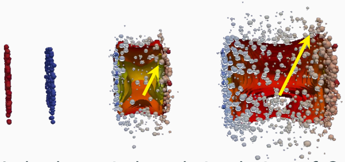
Need a better understanding of the baseline (no-QGP).

Scenario I: no QGP formation, only cold nuclear matter effects



- d-Au data: [PHENIX, PRC96(2017)064905] ; p-Pb data: [ATLAS, PLB763(2016)313-336 (with $\langle T_{pA} \rangle$ calculated from the Glauber-Gribov model)] .
- CNM effects alone qualitatively describes $h^\pm$ modifications in p-Pb, d-Au for $p_T > 5$ GeV.
- Cannot explain $R^D_{pA} > 1$ at high p_T.

QGP effects I: collisional effects



Hydrodynamic-based simulation of QGP provides temperature profiles $T(\tau, x, y)$ [H. Song, U. Heinz, PRC77(2008)064901; J. E. Bernhard, 1804.06469;]

- Hard thermal loop calculation of energy loss in a thermal QCD plasma [E. Braaten, M. H. Thoma PRD44(1991)R2625(R).] :

$$\Delta E_{\rm el} = \int d\tau \frac{C_R}{4} m_D^2(T)\alpha_s(ET) \ln\left(\frac{ET}{\mu_D^2}\right)\left(\frac{1}{v} - \frac{1-v^2}{2v^2}\ln\frac{1+v}{1-v}\right)$$

- $k_\perp$-broadening is complicated in a time-dependent QGP.

Often treated in simulations with Fokker-Planck/Boltzmann equation. $\langle \Delta k_T^2 \rangle \sim 1$ GeV^2 in high energy A-A collision.