Searching for the Stochastic Gravitational Wave Background with LISA

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August 2, 2022

Phase Transitions and Topological Defects in the Early Universe

Overview

- Gravitational waves and LISA
- Stochastic Gravitational Wave Background (SGWB)
- Challenges and complexities in searching for SGWB
- BLIP: a possible framework to handle these challenges
- Conclusions

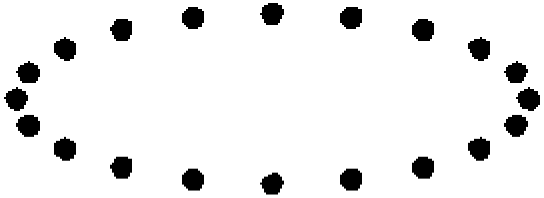
Gravitational Waves

- 1916: Linearized Einstein's Equation (weak field regime) becomes a wave equation in the metric.
  - Gravitational waves stretch and shrink space.
- Two polarizations:

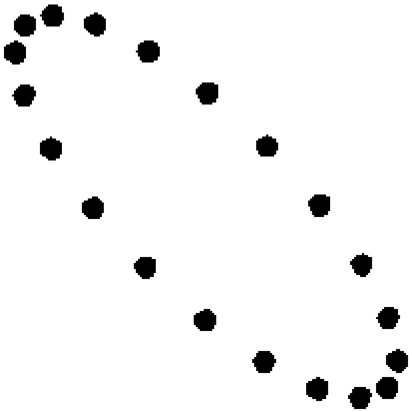
$$G_{\mu\nu} = K T_{\mu\nu}$$

$$\uparrow$$

$K \sim G / c^4$



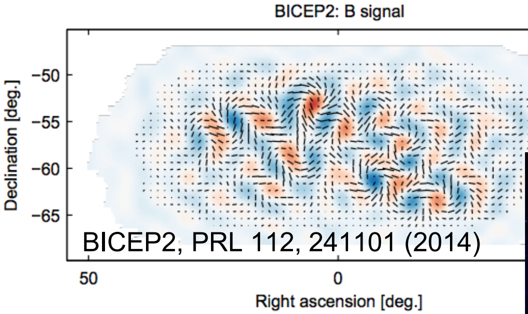
"+" Polarization



"x" Polarization

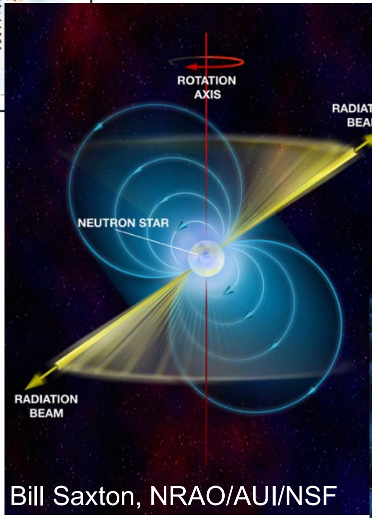
Gravitational-Wave Spectrum

Frequency



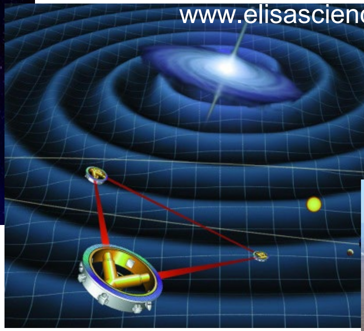
BICEP2, PRL 112, 241101 (2014)

CMB



Bill Saxton, NRAO/AUI/NSF

Pulsar timing



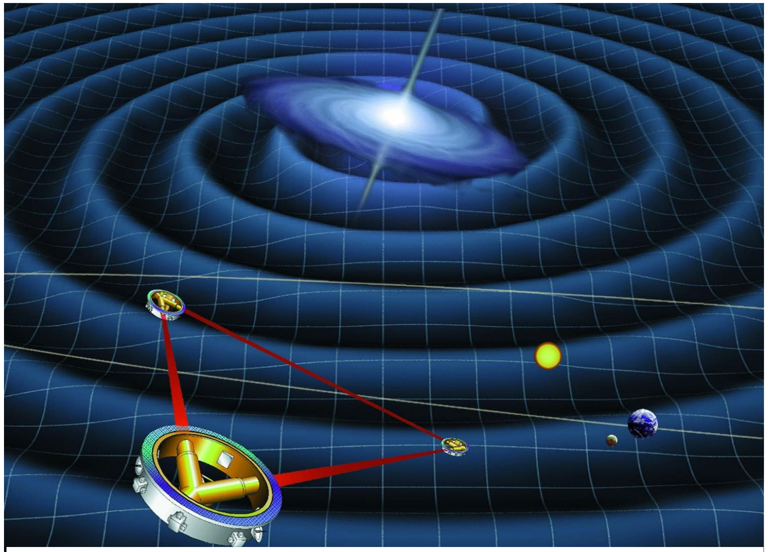
www.elisascience.org

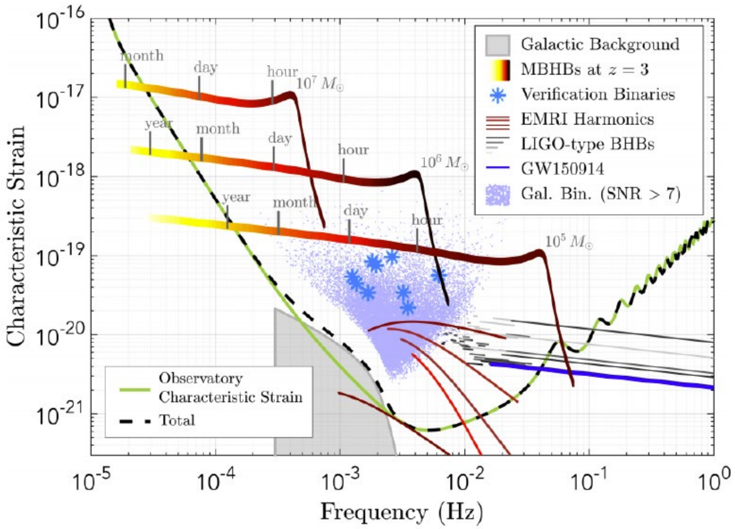
Satellite Detectors

Terrestrial Detectors

LISA

- Laser Interferometer Space Antenna (LISA)
- Joint ESA/NASA project, scheduled to launch in 2035.
  - » Three satellites in a triangular configuration, 2.5 Mkm sides.
  - » Operating at the Lagrange point, trailing the Earth.
  - » Pairs of laser beams in each arm, phases compared in each vertex.
  - » 5+ years of observation
- Rich science potential, many GW sources in the LISA band.

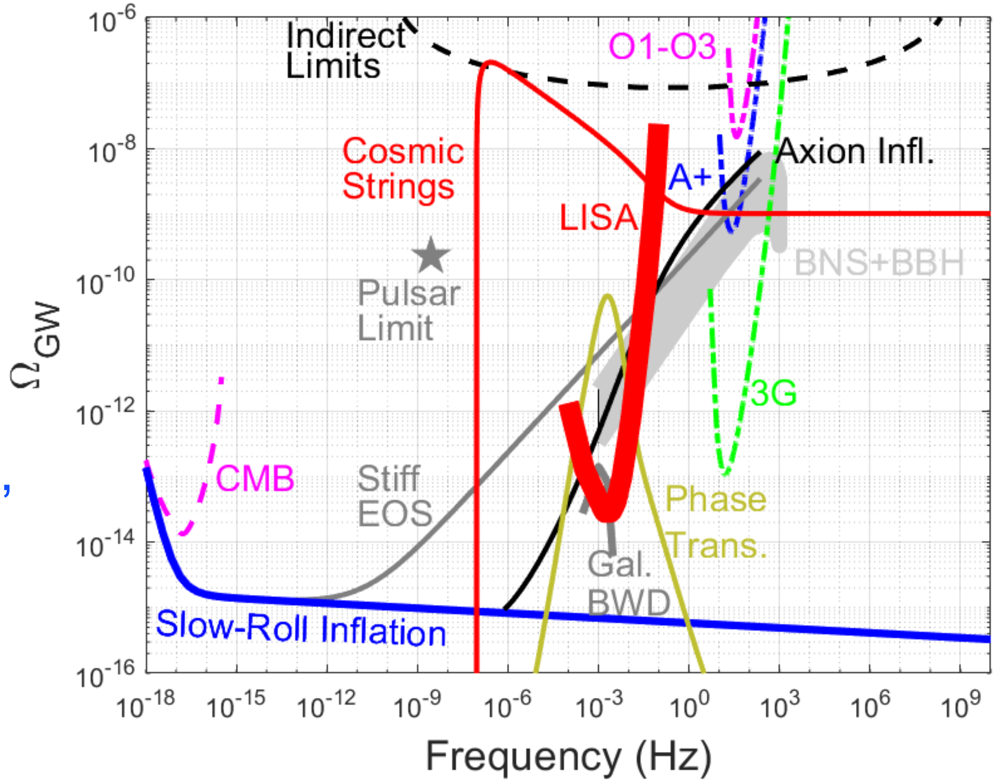




Stochastic GW Background

- Superposition of contributions from many uncorrelated GW sources.
- Cosmological:
  - » Inflationary models
  - » Phase transitions
  - » Cosmic (super)strings
  - » Alternative cosmologies
- Astrophysical:
  - » Compact binaries: DWD, BBH, BNS, BHNS…
  - » Supermassive BH, Rotating NS, planets…

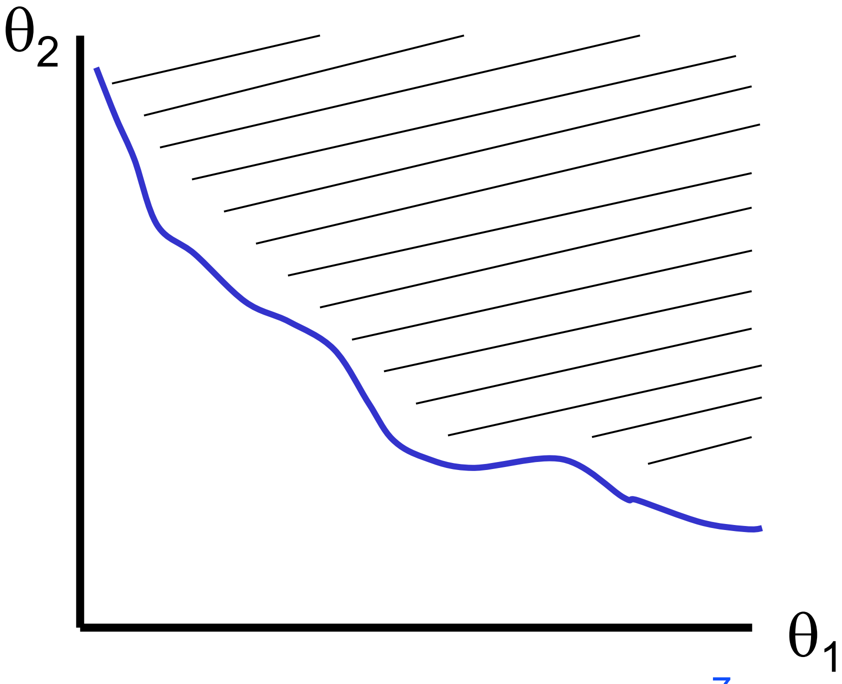
$$\Omega_{GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{GW}(f)}{d\ln f}$$



Parameter Estimation

- Define a likelihood comparing the measured spectrum with a model.
- If measurements are upper limits (no SGWB detected), this is easy:
    - » Upper limits on model parameters.
- In case of detection, much more complex:
    - » Which models contribute? Model selection…
    - » One vs more models? How do you set up the likelihood?
    - » Bias if your model is incorrect?
- Need a statistical framework to deal with this complexity.

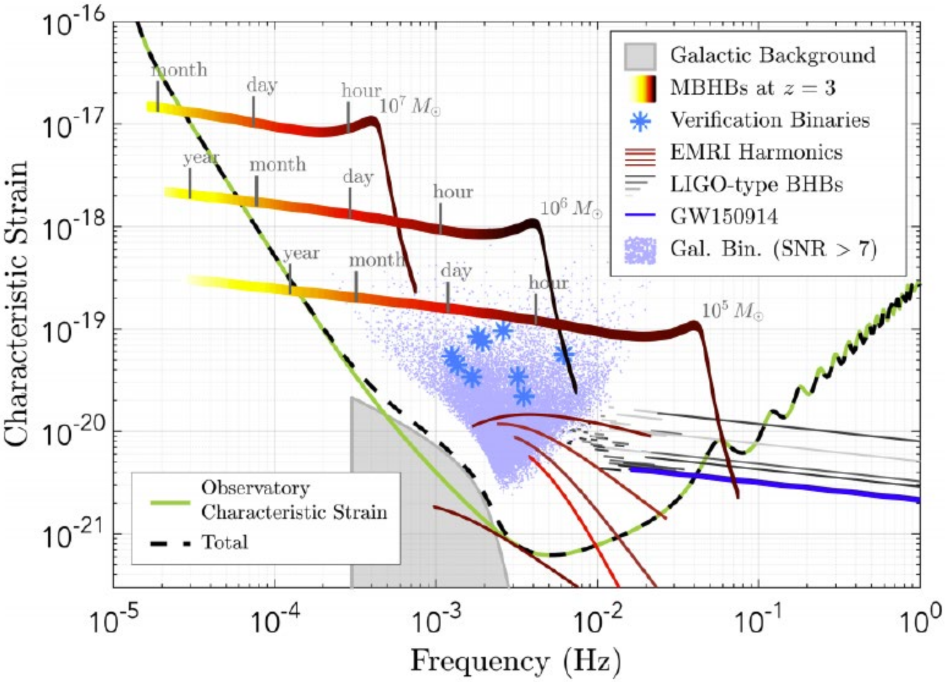
$$\ln L \propto \sum_f \frac{\left(\Omega_d(f) - \Omega_m\left(f;\vec{\theta}\right)\right)^2}{\sigma_f^2}$$



Challenges

- LISA band is very crowded.
- Many residual sources:
  - » Massive and solar BBH, DWD.
  - » Will these be subtracted before SGWB search is done? How? Residuals?
  - » If not, how does the SGWB search plug in with other searches?
- Galactic DWD foreground:
  - » SGWB, but directional.
  - » Must be included in the SGWB search – otherwise, bias…
  - » Impacts resolvable searches too.
  - » Other directional (e.g. LMC)?

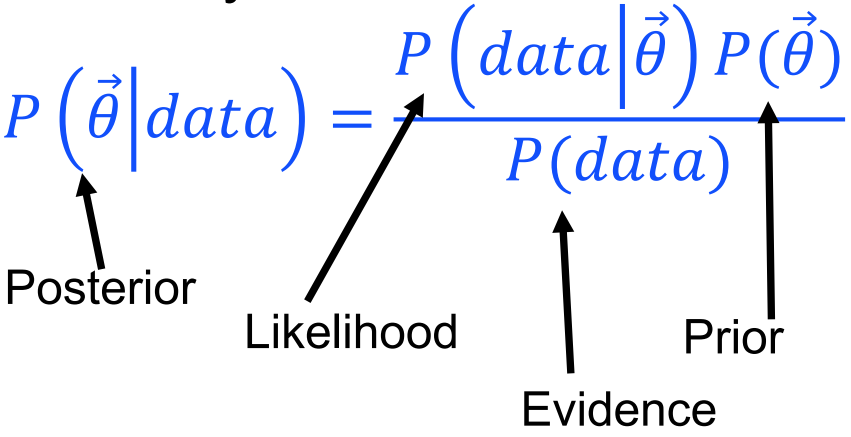
LISA proposal: 1702.00786 (2017)



Bayesian Formalism

- Directly estimate model parameters from data.
- Naturally combine multiple models.
    - » Including noise model.
    - » Existing infrastructure to handle many-dimensional parameter spaces.
- Naturally combine multiple experiments (eg LISA + 3G).
- Hierarchical formalisms: more complex parameter dependencies.
    - » Physical constraints.
- Model selection: Bayes factors
- Other searches (eg CBC) also use Bayesian formalisms.

Bayes Theorem:



$$P\left(\vec{\theta}\middle|data\right) = \frac{P\left(data\middle|\vec{\theta}\right)P(\vec{\theta})}{P(data)}$$

Bayes Factor:

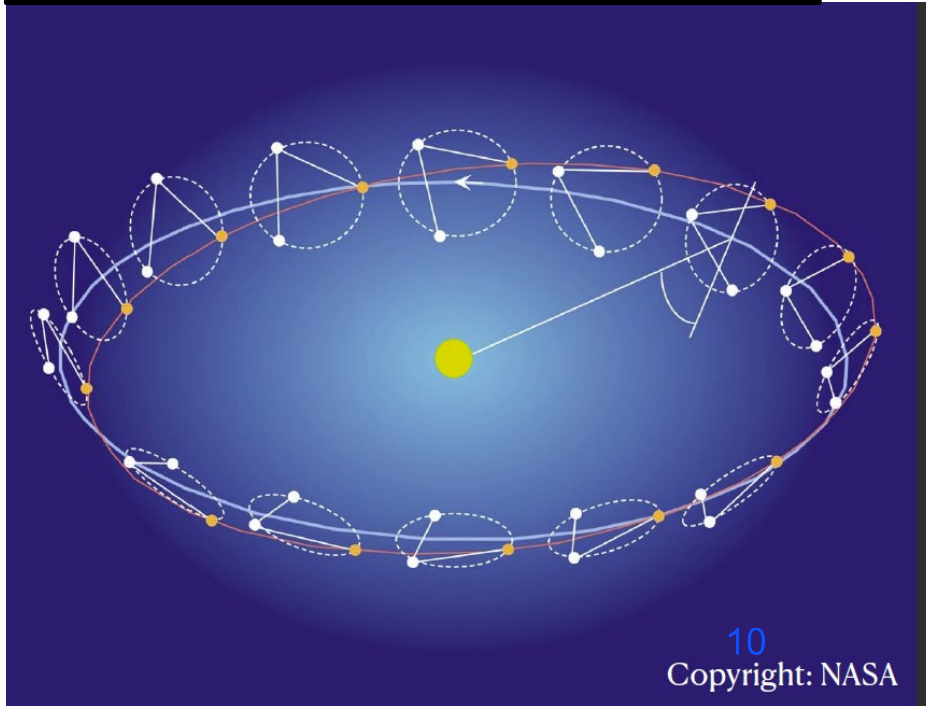
$$K = \frac{P(M_1|data)}{P(M_2|data)}$$

Bayesian LISA Pipeline (BLIP)

- Independent, python-based LISA SGWB search.
- Features:
  - » Instrumental Gaussian noise.
  - » Stationary and non-stationary LISA detector.
  - » TDI: XYZ and AET
    - – Simplified heliocentric rigid body orbits.
  - » Isotropic background, point sources, and spatially extended anisotropy.
  - » Bayesian inference with dynesty and emcee samplers.
  - » Physical constraint: non-negative SGWB across the sky.

Sharan Banagiri, Alexander Criswell,
T. Kuan, V. Mandic, J. Romano, S. Taylor
MNRAS 09, 0035 (2021); 2103.00826

Download:
https://github.com/sharanbngr/blip
pip install blip-gw



Copyright: NASA

BLIP (2)

- Assume frequency-direction factorization.
  - » Directional dependence expanded in spherical harmonics.

$$\Omega(f,\hat{n}) = \frac{\Omega(f)}{\sqrt{4\pi a_{00}}} \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

- Reality: $a_{l,-m} = (-1)^m a^*_{l,m}$
- Physical: $\Omega(f,\hat{n}) \geq 0$
  - » Essential for Bayesian mapping, especially for highly anisotropic distributions.

- Define another field on 2-sphere:

$$S(\hat{n}) = \sum_{lm} b_{lm} Y_{lm}(\hat{n}) = \left[\sum_{lm} a_{lm} Y_{lm}(\hat{n})\right]^{1/2}$$

- Non-negative background:
  $b_{l,-m} = (-1)^m b^*_{l,m}$

- a_{lm}'s and b_{lm}'s are related through Clebsch-Gordan coefficients.

BLIP (3)

$$\mathscr{L}(\tilde{d}|N_p, N_a, \Omega_{\text{ref}}, \alpha, \{b_{\ell,m}\}) = \prod_{t,f} \frac{1}{2\pi T_{\text{seg}}|C(t,f)|} \times \exp\left(-\frac{2\,\tilde{d}^*_{t,f}\,C(t,f)^{-1}\,\tilde{d}_{t,f}}{T_{\text{seg}}}\right)$$

- Assume $d_I(t) = n_I(t) + h_I(t)$
  - » Three channels, split into time-segments, then FFT-ed.
- Compute auto- and cross-correlation terms for all channels.
  - » Populate the covariance matrix *C.*
- Covariance matrix is a function of noise and signal parameters.

$$S_p(f) = N_p\left[1 + \left(\frac{2\,\text{mHz}}{f}\right)^4\right]\text{Hz}^{-1},$$

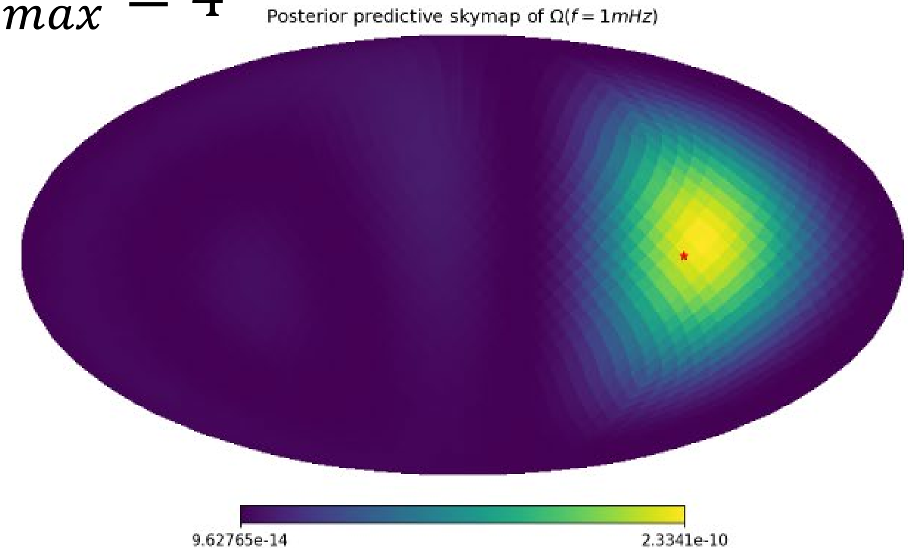
$$S_a(f) = \left[1 + \left(\frac{0.4\,\text{mHz}}{f}\right)^2\right]\left[1 + \left(\frac{f}{8\,\text{mHz}}\right)^4\right] \times \frac{N_a}{(2\pi f)^4}\text{Hz}^{-1}$$

$$\Omega(f,\hat{n}) = \Omega_{ref}\left(\frac{f}{f_0}\right)^{\alpha}\sum_{LM} a_{LM}Y_{LM}(\hat{n})$$

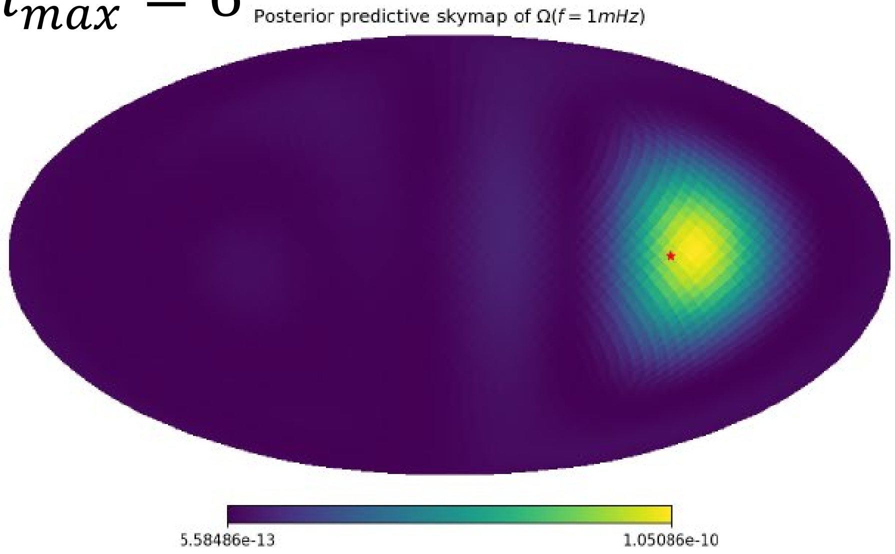
$$a_{L,M} = \sum_{\ell,m}\sum_{\ell',m'} b_{\ell,m}b_{\ell',m'}\beta_{L,M}^{\ell m,\ell' m'}$$

Recovery of Point Sources

$l_{max} = 4$



$l_{max} = 6$

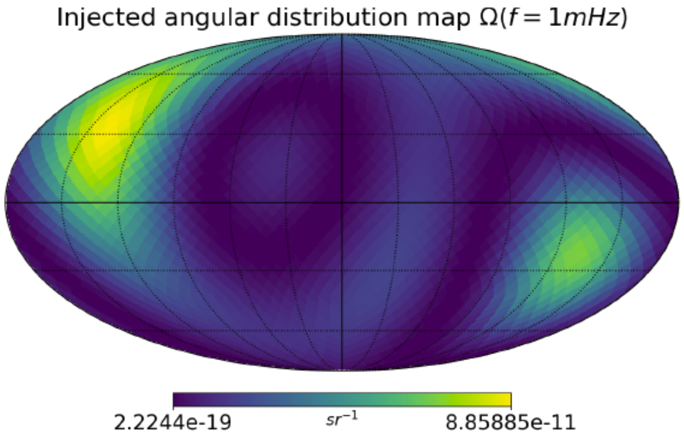


- Recovered sky-maps for simulated point sources.
- 3 months of simulated data.
- Red star indicates the true position of the source.
- Note improved angular resolution when larger l_{max} is used.

Banagiri et al, MNRAS 09, 0035 (2021); 2103.00826

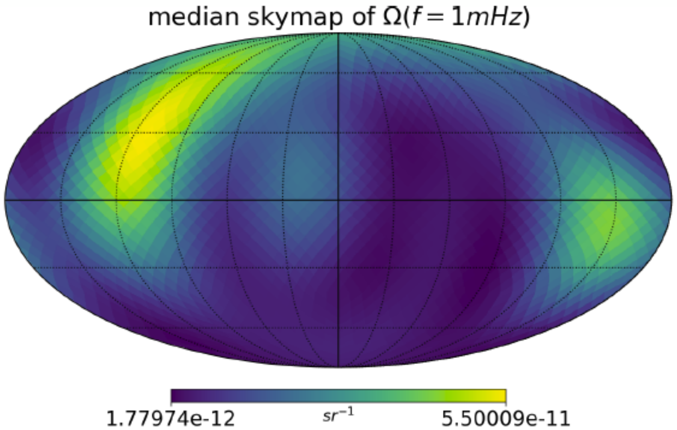
Recovery of Extended Anisotropy

S. Banagiri

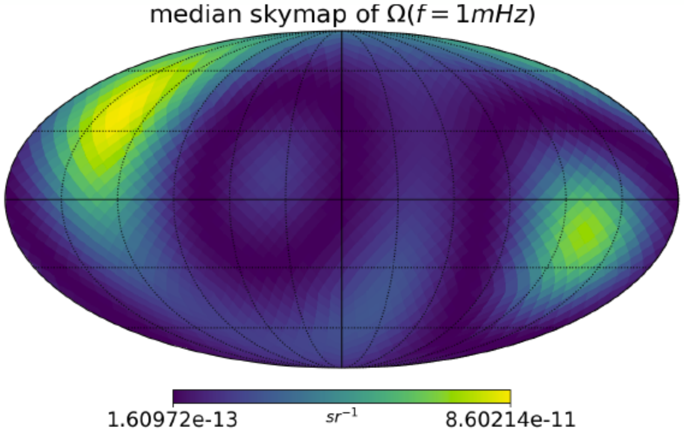

Injected angular distribution map $\Omega(f=1mHz)$

Adhoc power distribution, $\ell_{\max} = 4$

Two-month analysis


median skymap of $\Omega(f=1mHz)$

One-year analysis


median skymap of $\Omega(f=1mHz)$

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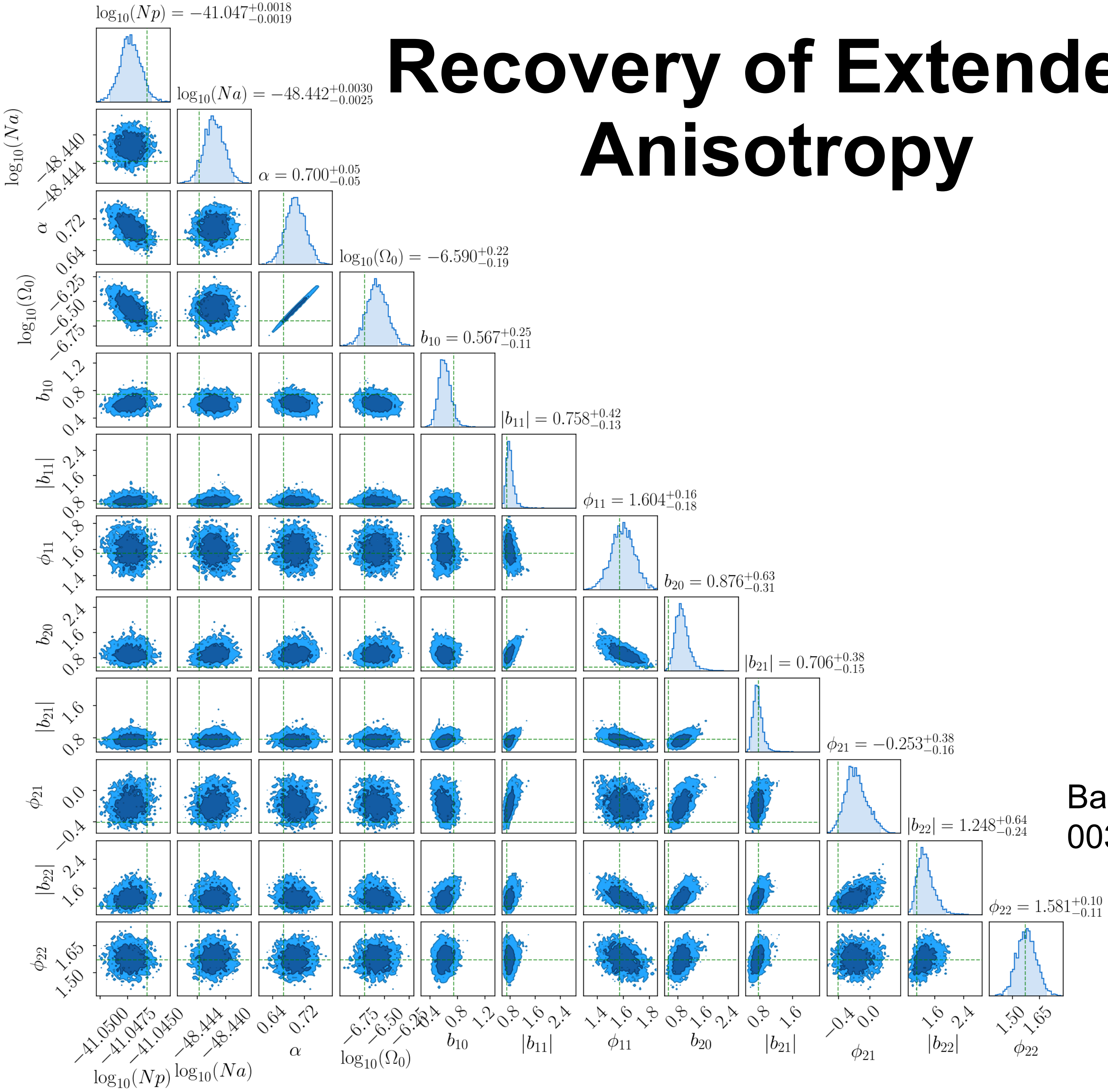
Recovery of Extended Anisotropy



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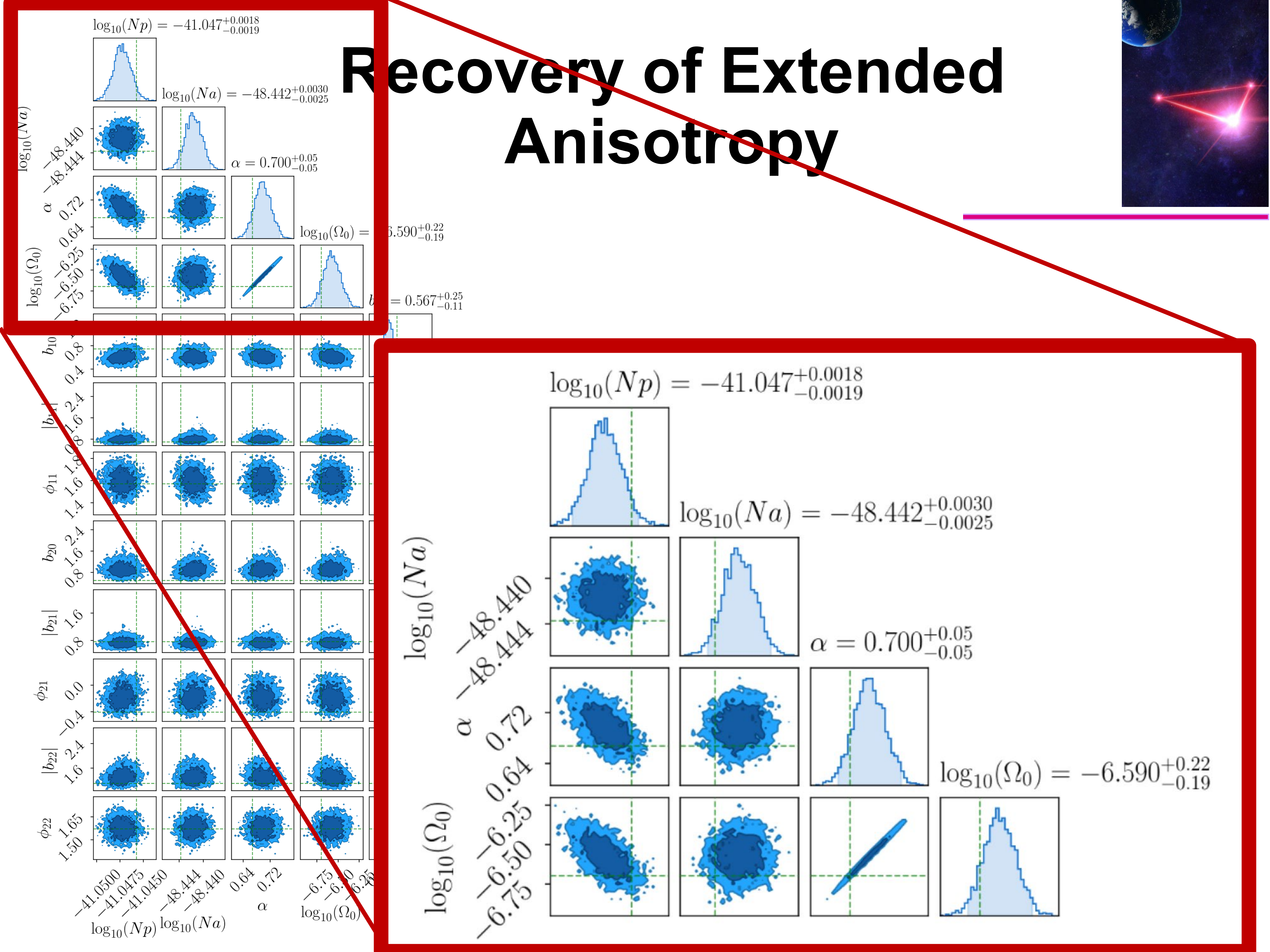
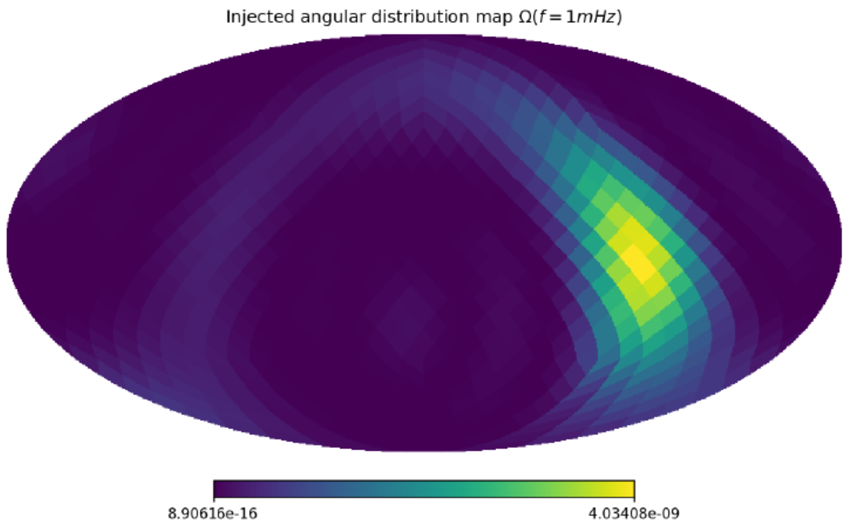
Recovery of Extended Anisotropy

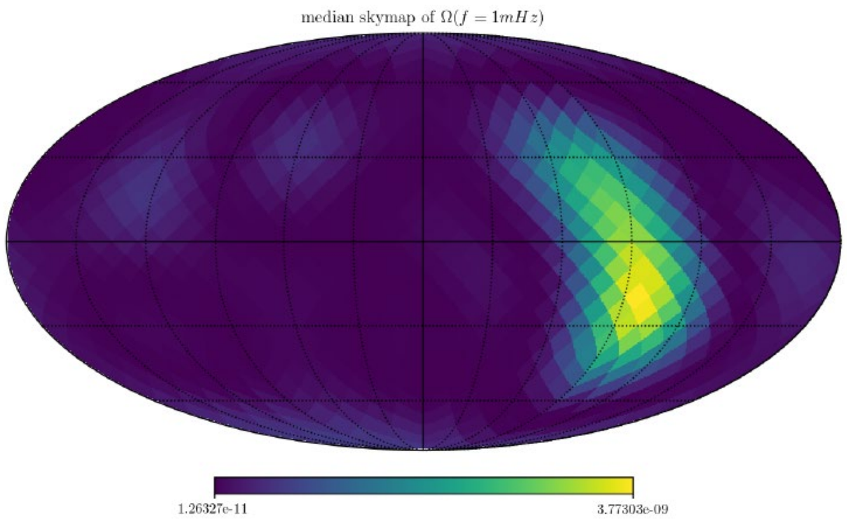


Recovery of Galactic Foreground

Injected



Recovered



A. Criswell

- Toy model of galactic foreground due to DWD inspirals.
- Power law in frequency, $\alpha = 2/3$.
- Disk and Bulge contributions, distributions following Breivik et al (2020) and McMillan (2011).
- $l_{max} = 4$.

Banagiri et al, MNRAS 09, 0035 (2021); 2103.00826

Conclusions

- Studying accessibility of a model cannot be done independently from other models (astrophysical, cosmological, detector-based).
- Need a unifying statistical framework that can allow:
    - Searching for (and distinguishing between) multiple models
    - Isotropic and anisotropic backgrounds
    - Polarized and unpolarized backgrounds
    - Instrumental noise and its uncertainty
    - Perhaps also handle resolved signals
    - Perhaps also combine with the 3G or pulsar data
- Complex problem, many challenges and biases.
    - Develop a simulations program to study each of them.
- **Need to start now!**