

# Early Universe Cosmology from Stochastic Gravitational Waves

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# LIGO

Exciting time for gravity waves



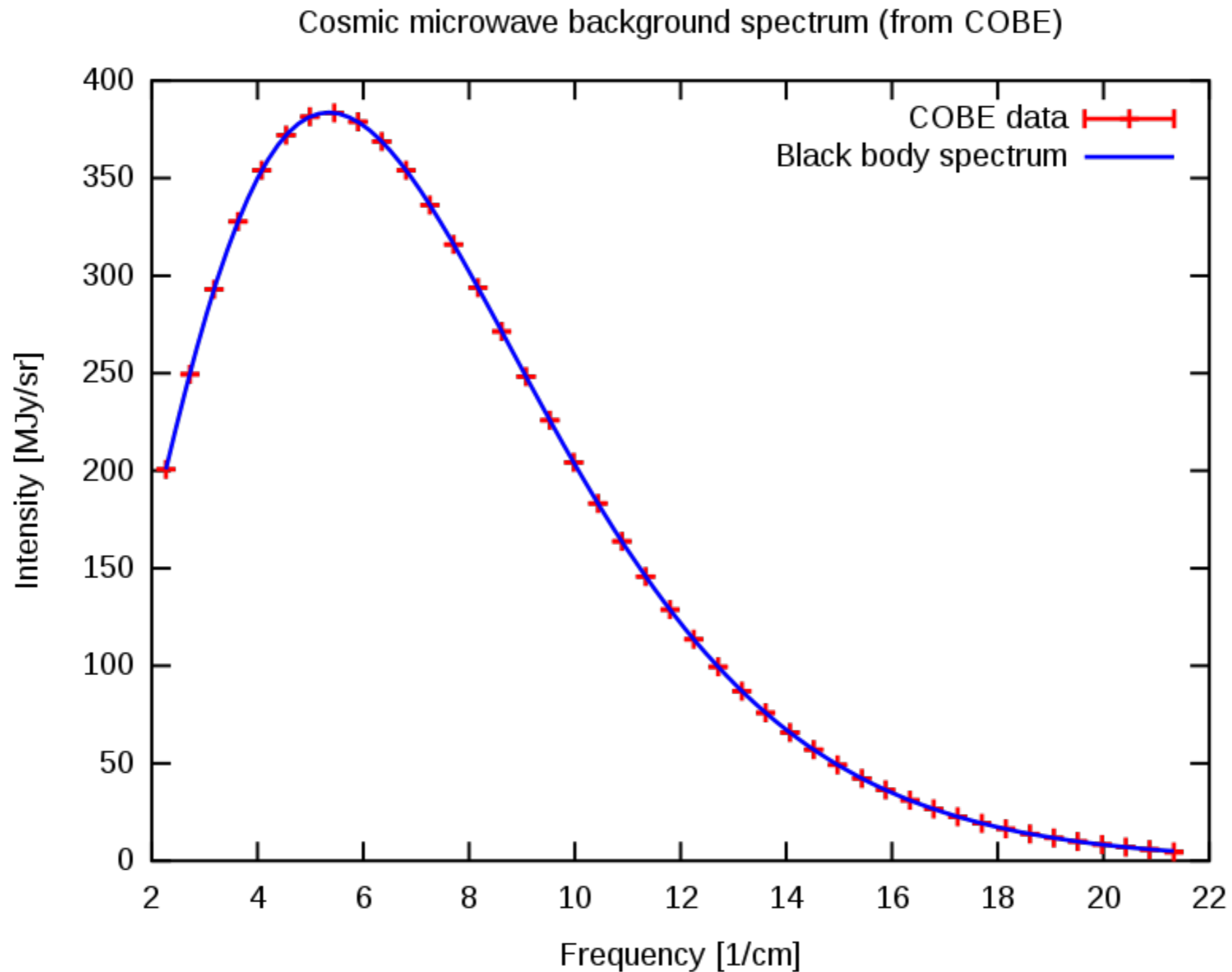
# Stochastic Gravitational Waves

CMB of Gravitational Waves

Produced in the early universe and free streamed to us

Anything the CMB taught us, GWs can teach us too

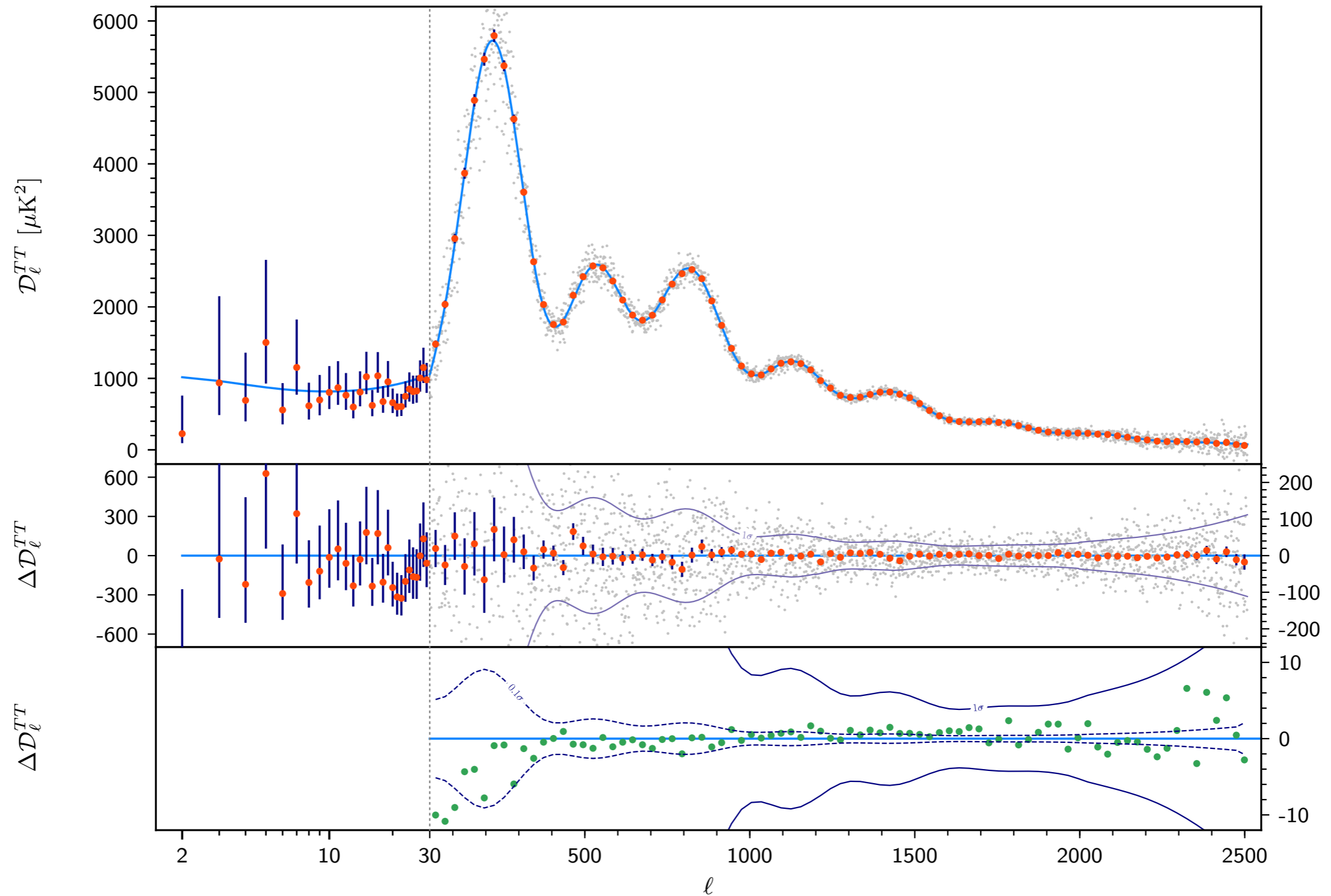
# CMB : Frequency spectrum



# CMB : Frequency spectrum

1. Black body radiation with a temperature of 2.7 K
  - Universe was in thermal equilibrium
  - CMB emitted as black body radiation
2. Small distortions, e.g. 21 cm
  - Propagation effects
  - Tells us about the universe between surface of last scattering and now

# CMB : Angular dependences



1907.12875

# CMB : Angular dependences

1. Uniform temperature
  - Acausal physics, e.g. Inflation
2. Scale invariance and small deviations
  - Inflationary parameters
  - Propagation effects

# CMB

## 1. Frequency

- Black Body - Production mechanism
- Deviation From Black Body - Propagation

## 2. Anisotropies

- Scale Invariance - Inflation
- Deviation From Scale Invariance - Propagation



# Stochastic Gravitational Waves

## 1. Frequency

- Spectrum - Production mechanism
- Deviation From Spectrum - Propagation

## 2. Anisotropies

- Scale Invariance - Inflation
- Deviation From Scale Invariance - Propagation

# Stochastic Gravitational Waves

## 1. Frequency

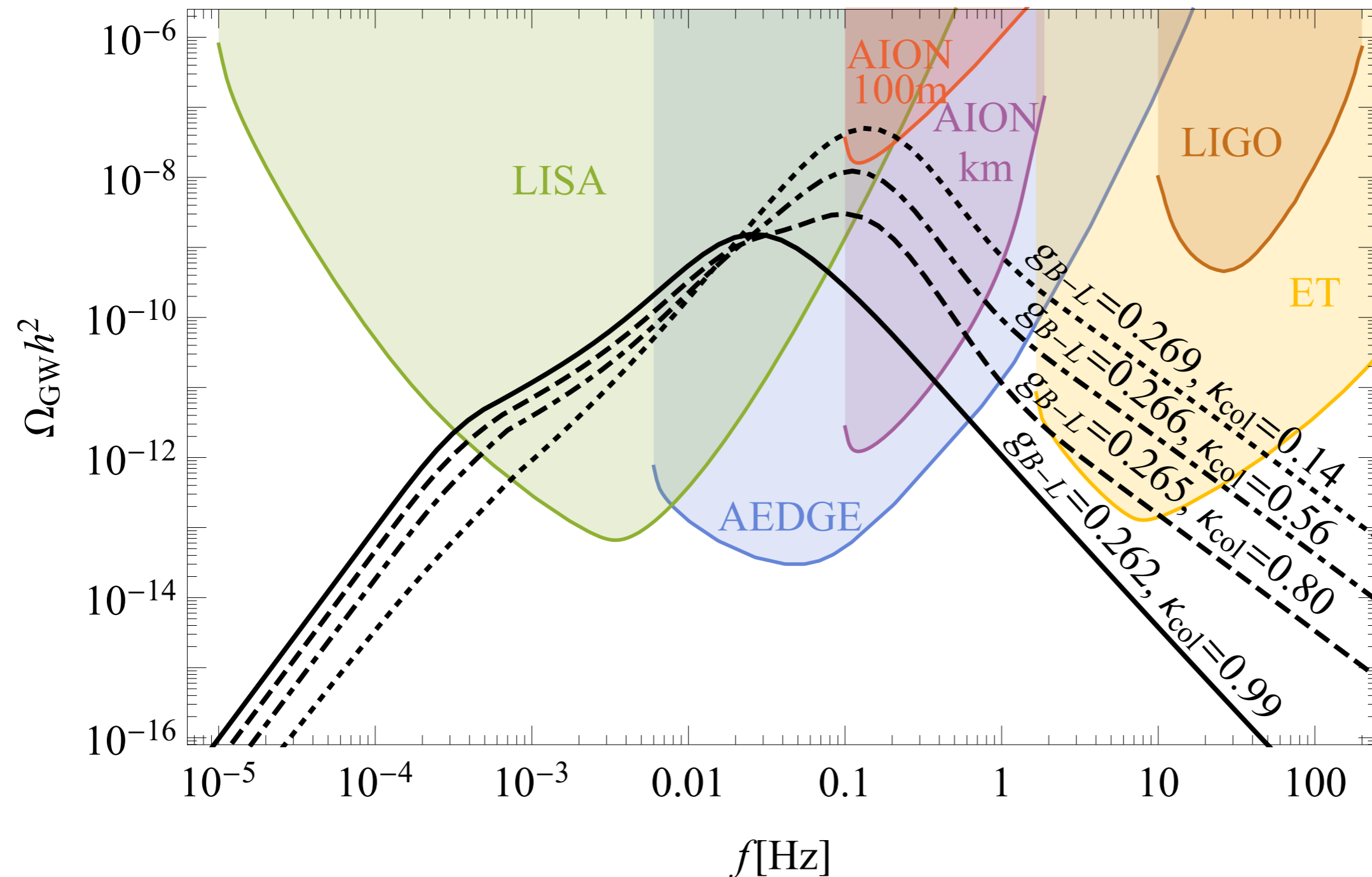
- Spectrum - Production mechanism
- Deviation From Spectrum - Propagation

## 2. Anisotropies

- Scale Invariance - Inflation
- Deviation From Scale Invariance - Propagation

# To Study Deviations...

We need to know the spectrum very very accurately



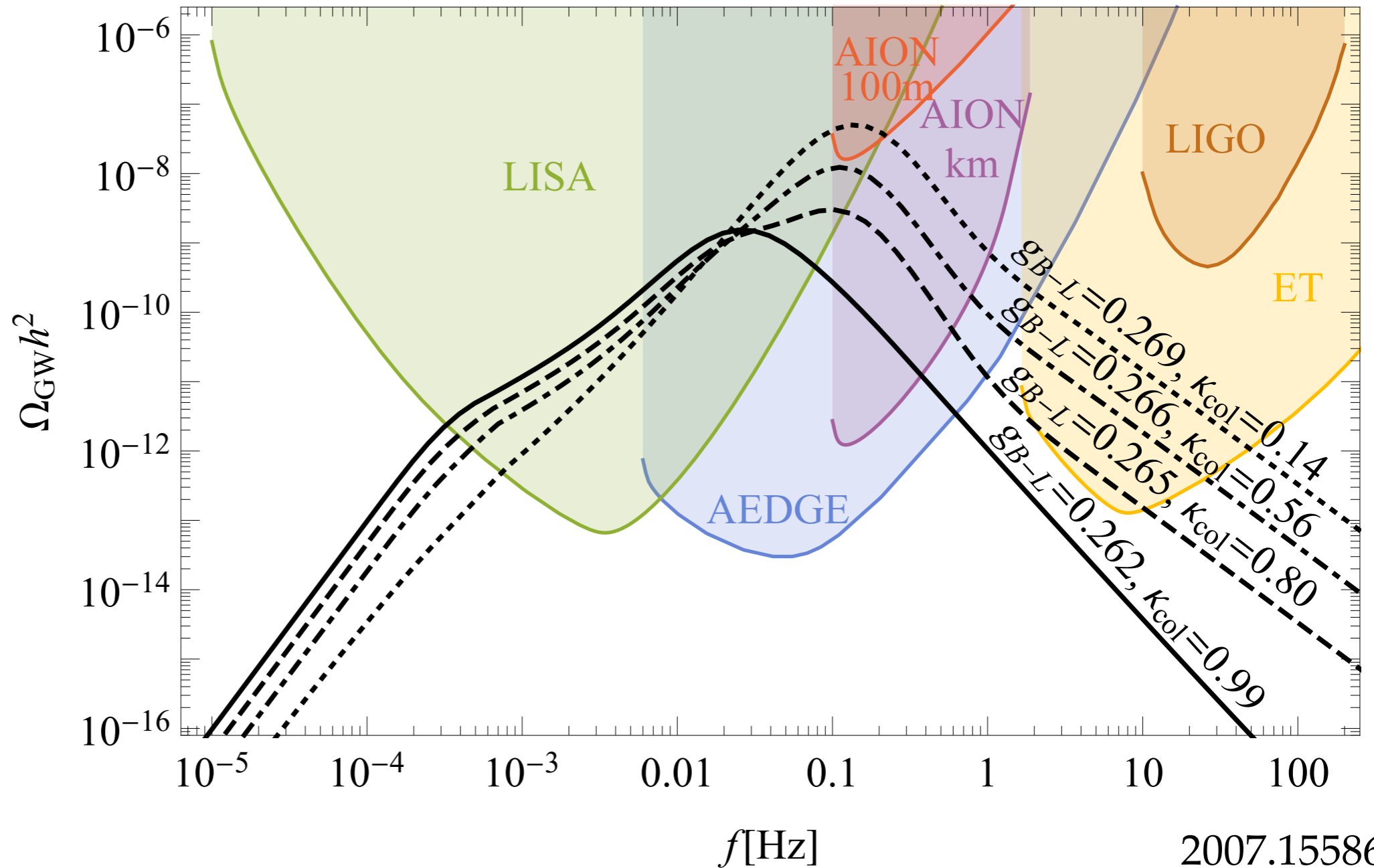
# GW Signal

Very model dependent shape

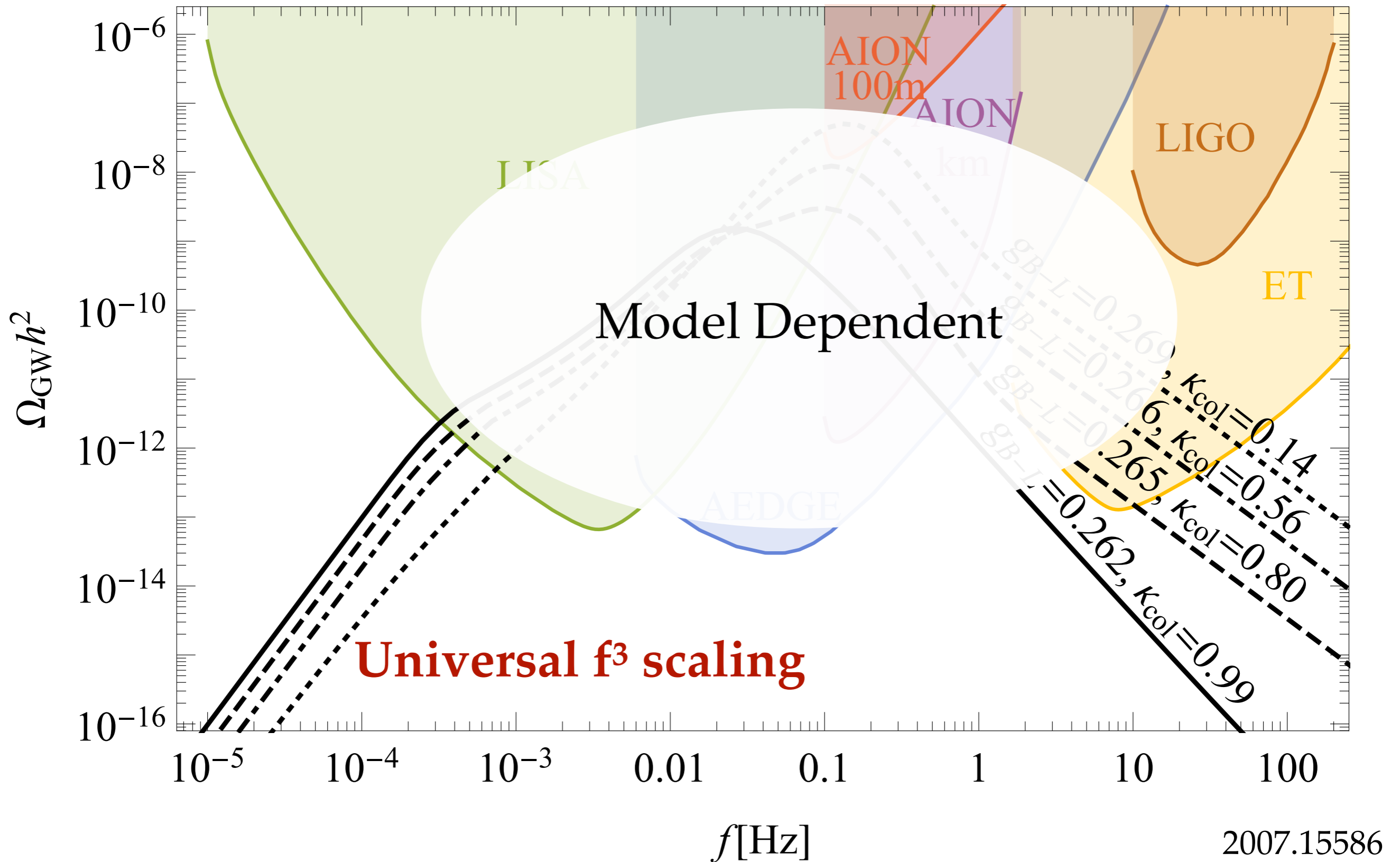
Turbulence

Collision of  
bubbles

Sound  
waves



# Signal at LISA



# Low frequency signatures

$f^3$  behavior is model independent so we know the “background”, which means we can look for a “signal”

Only things that change the propagation of gravity waves can create a signal

Equation of state of the universe

Free streaming particles

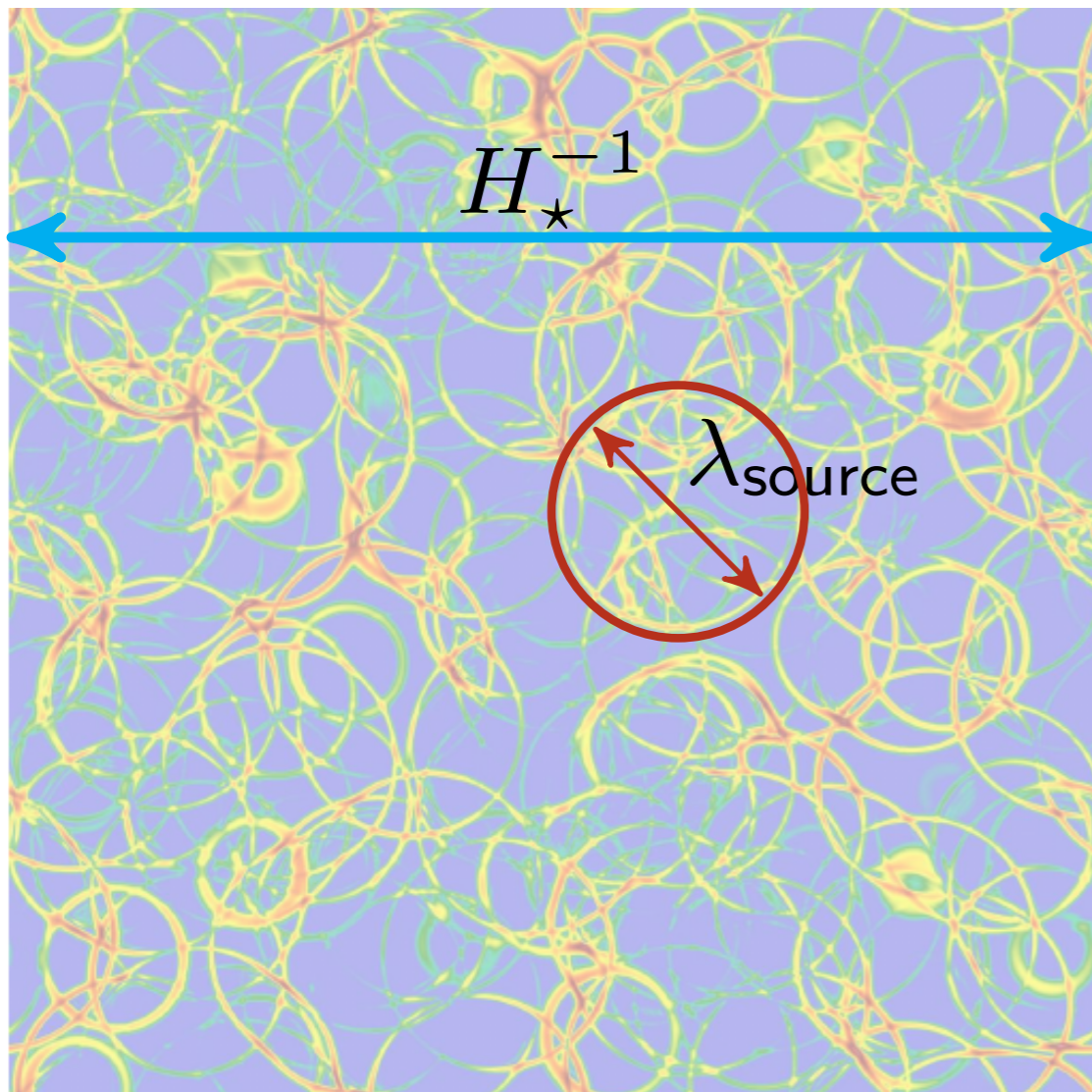
# Today

- What is the physical reason behind the low frequency behavior?
- Propagation effects
  - Equation of state of the universe
  - Free streaming particles
- LISA Sensitivity to Propagation effects
  - Equation of state of the universe
  - Free streaming particles



# Low frequency behavior

White noise behavior induced by causality



Wavelengths longer than the correlation length, source becomes independent of frequency



# Low frequency behavior

$$\Omega_{GW} = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d \log k} \propto \frac{k}{dk} \langle \dot{h}_{ij}(x) \dot{h}_{ij}(x) \rangle$$

$$\sim \frac{k}{dk} d^3 k k^2 \langle h_{ij}(k) h_{ij}^*(k) \rangle$$

**Phase space**

**Derivatives**

# Low frequency behavior

$$\Omega_{GW} \sim k^5 \langle h_{ij}(k) h_{ij}^*(k) \rangle$$

# Low frequency behavior

$$\Omega_{GW} \sim k^5 \langle h_{ij}(k) h_{ij}^*(k) \rangle$$

**Propagation**



$$h_{ij}(k) \sim G(t, t_0, k) \Pi(t_0, k)$$

**Source**



# Low frequency behavior

$$\Omega_{GW} \sim k^5 G^2(t, t_0, k) \langle \Pi(k) \Pi^*(k) \rangle$$

2 point function of the source in  
momentum space

How does the graviton propagate to us

# Low frequency behavior

$$P_{\Pi}(x) = \langle \Pi(x)\Pi(0) \rangle = 0 \quad x \gtrsim \lambda$$

Regions are uncorrelated beyond correlation length

$$\begin{aligned} P_{\Pi}(k) &= \int d^3\mathbf{x} e^{i\mathbf{k}\cdot\mathbf{x}} P_{\Pi}(x) = \int_0^{\lambda} dx 4\pi x \frac{\sin(kx)}{k} P_{\Pi}(x) \\ &\approx \int_0^{\lambda} dx 4\pi x^2 P_{\Pi}(x) \end{aligned}$$

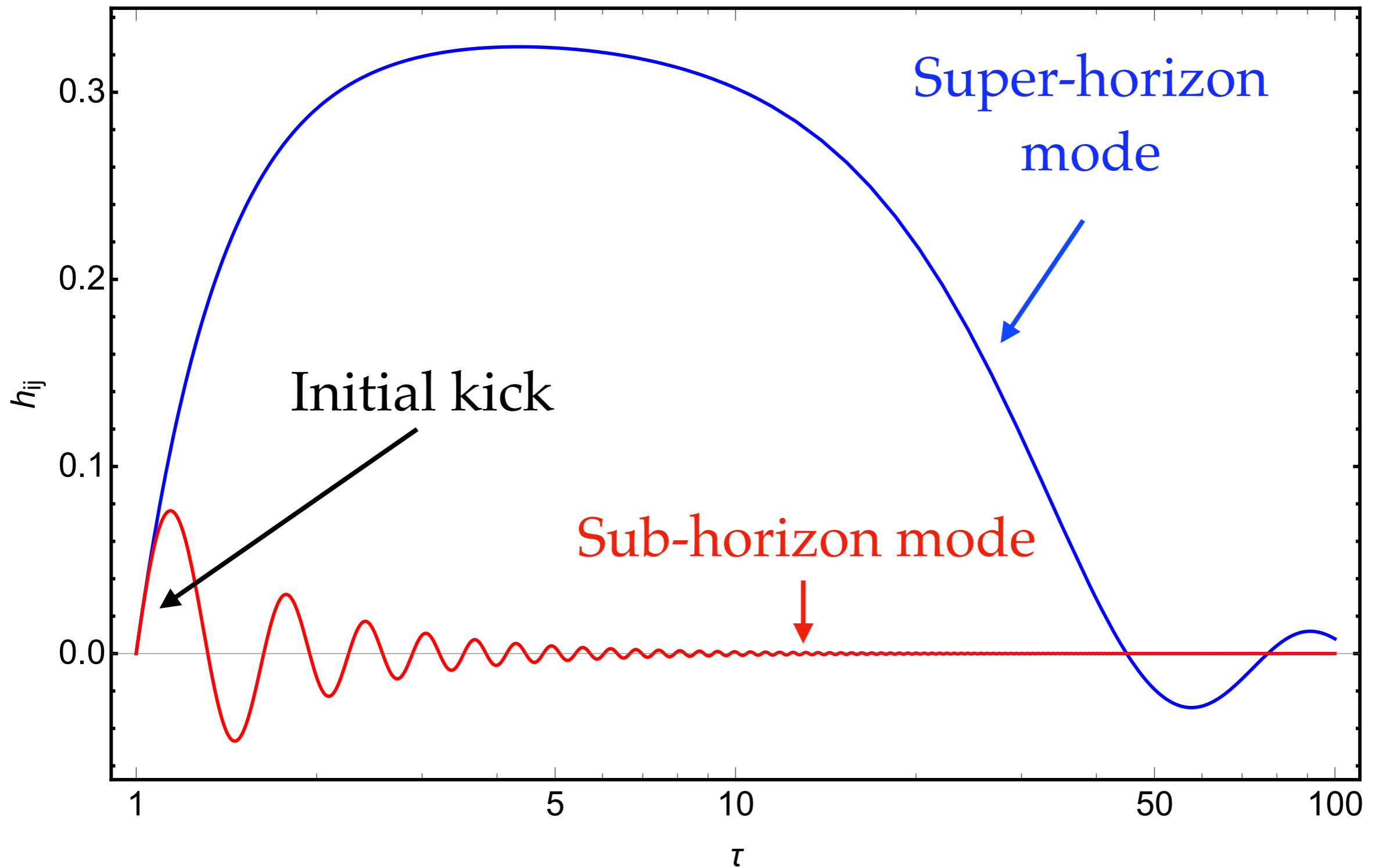
Source is frequency independent at low frequencies

# Low frequency behavior

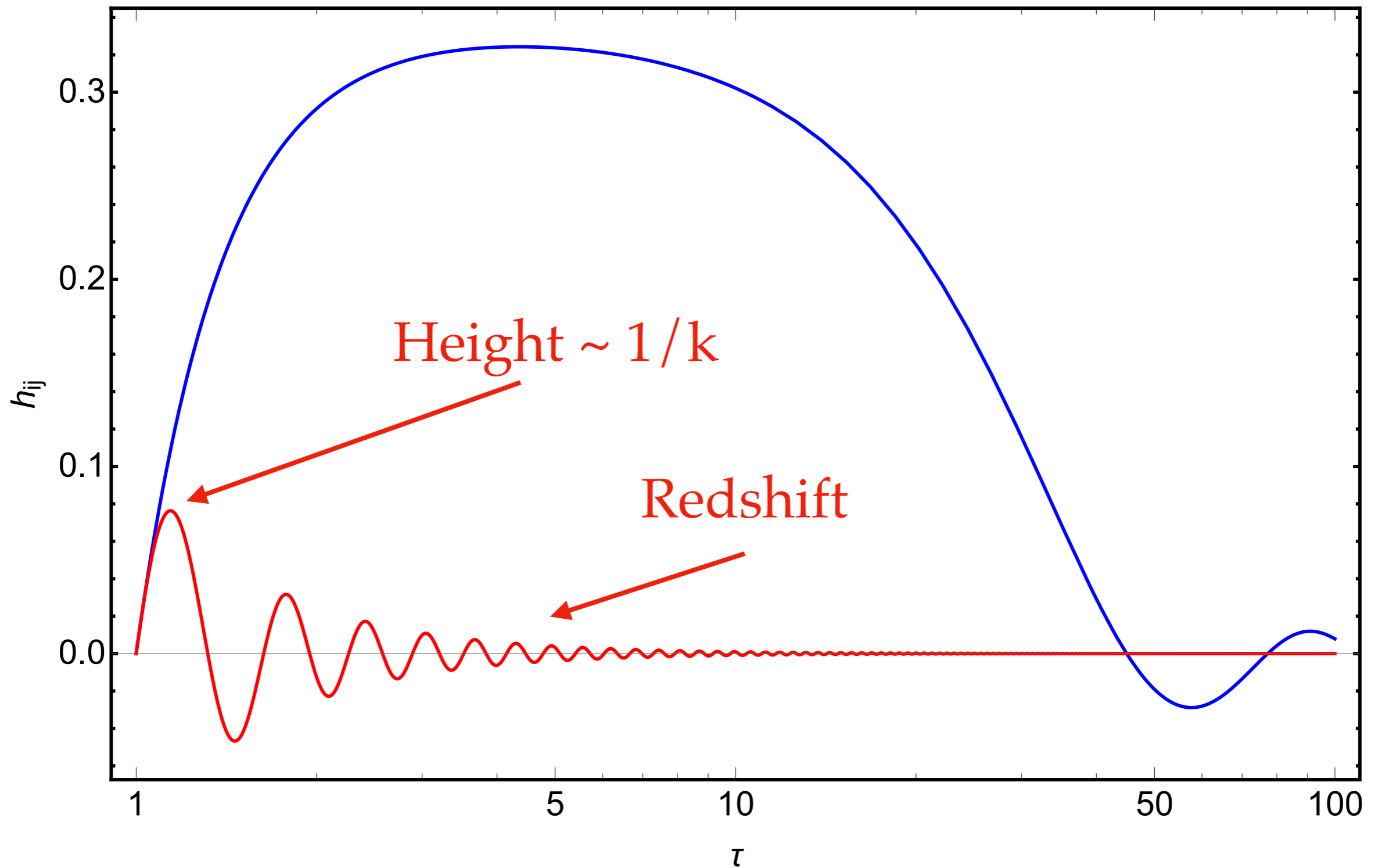
$$\Omega_{GW} \sim k^5 G^2(t, t_0, k)$$

Low frequency scaling determined entirely due to propagation from production to now

# Low frequency behavior

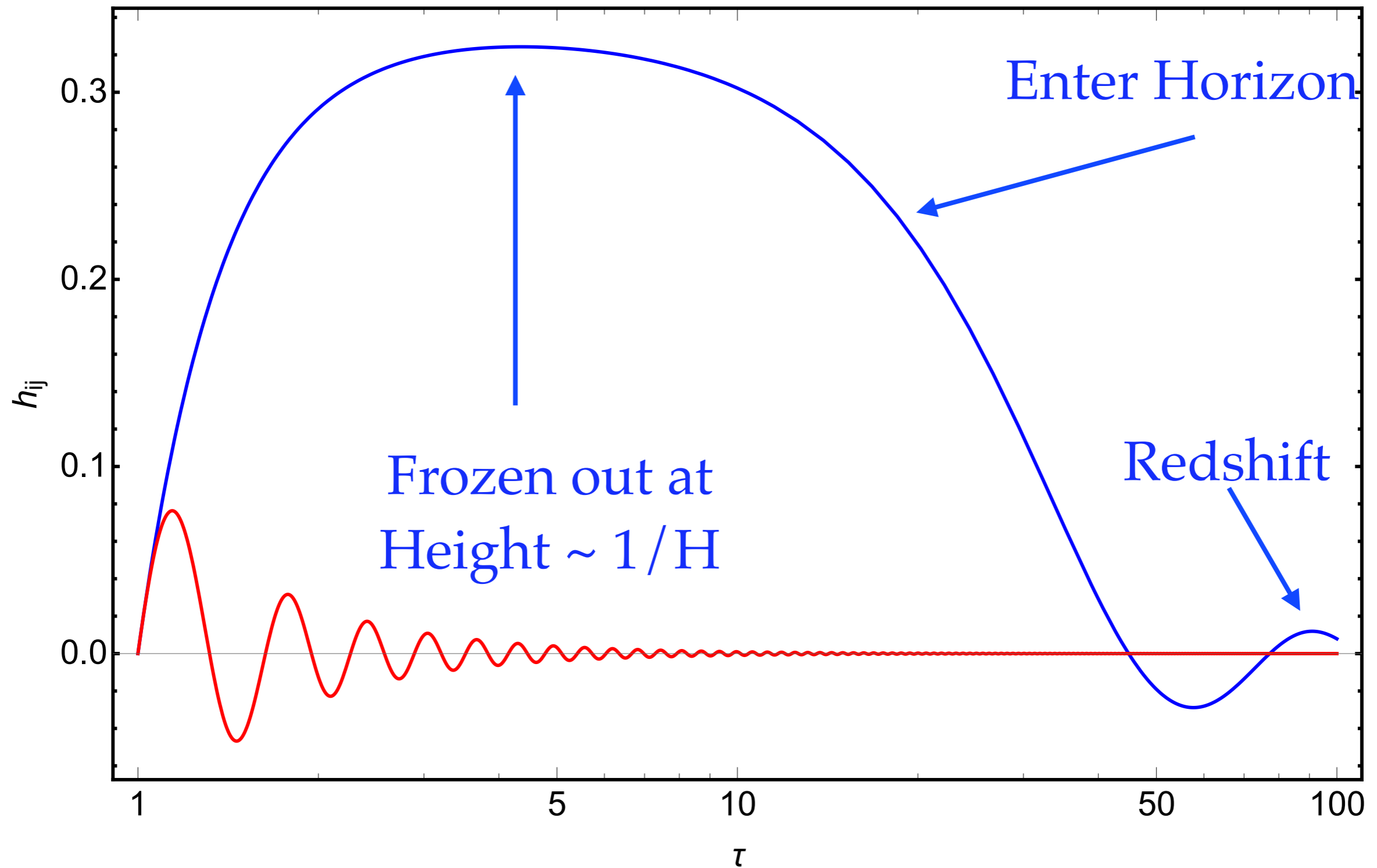


# Low frequency behavior

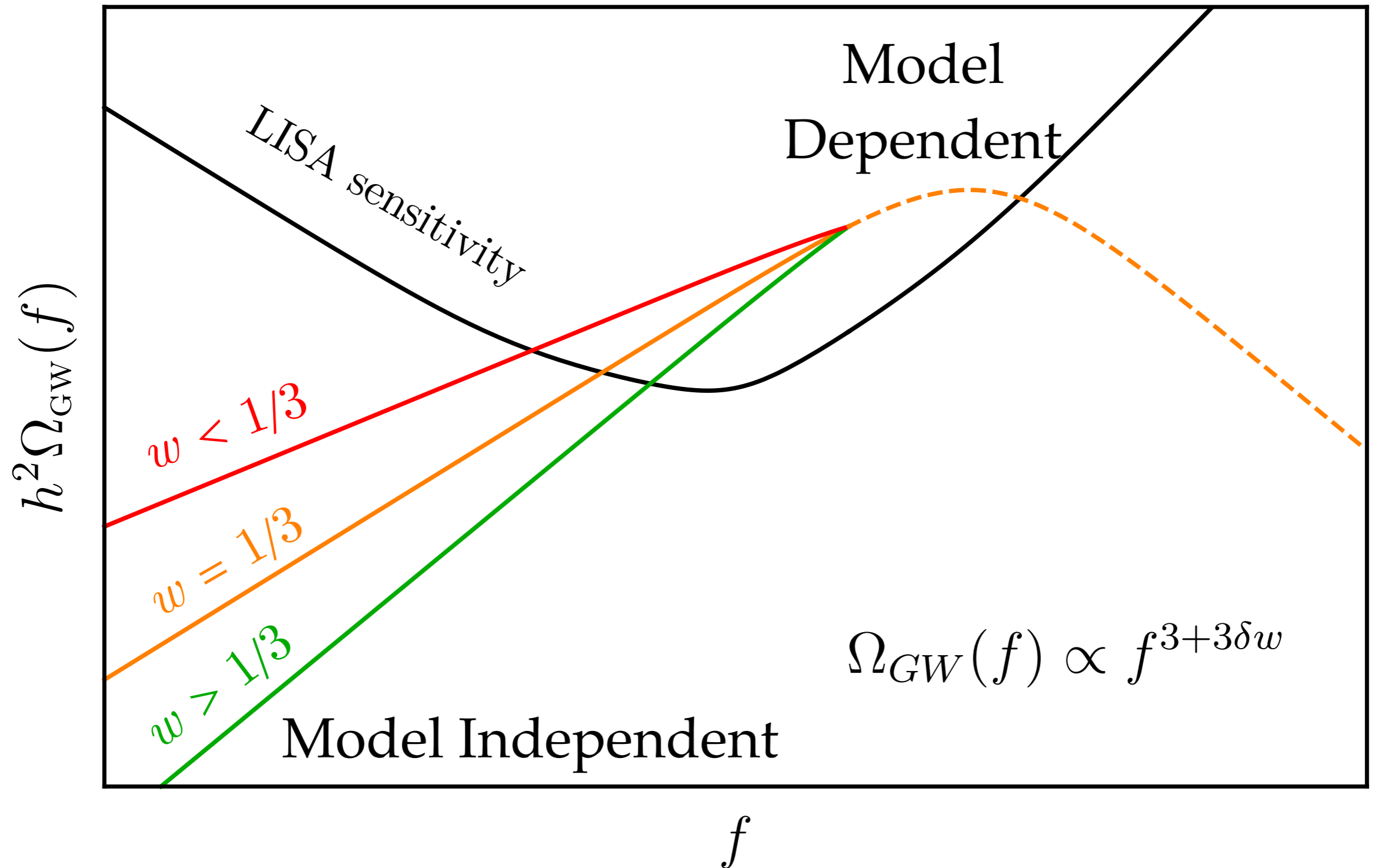




# Low frequency behavior



# Frequency spectrum



# Free-streaming particles

Thermal equilibrium

Free streaming

Constant scattering

No scattering

Random walk

Follow geodesics

# Free-streaming particles

Thermal equilibrium

Free streaming

Constant scattering

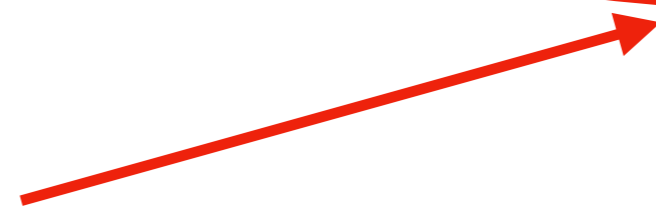
No scattering

Random walk

Follow geodesics

Cares about gravity waves!

Has the potential for back reaction



# Free-streaming particles

Effect on gravity waves in equation form

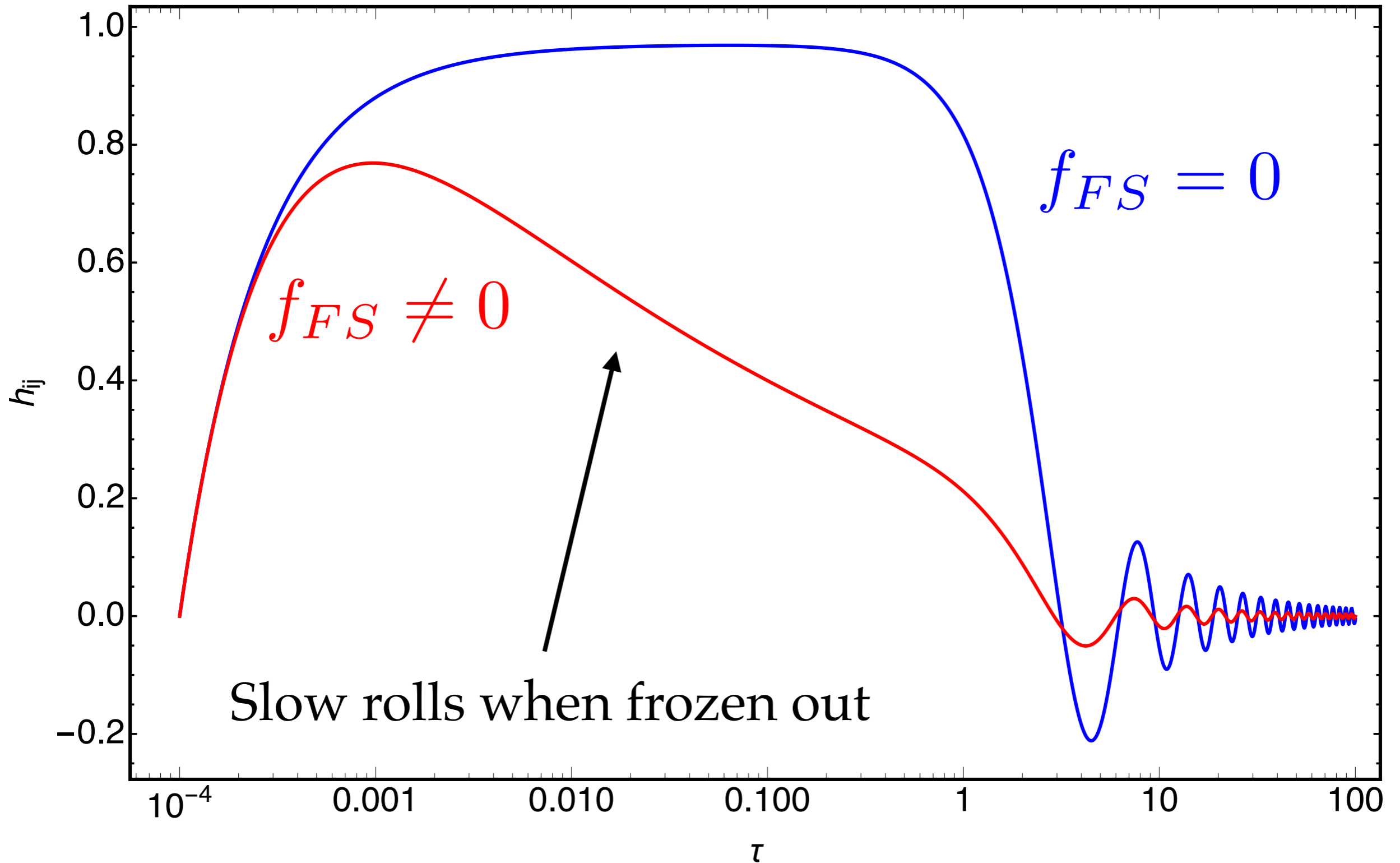
$$u^2 \partial_u^2 h(u) + 2u \partial_u h(u) + u^2 h(u) =$$
$$- 24 f_{\text{FS}} \int_{u_\star}^u dx \left( -\frac{\sin(u-x)}{(u-x)^3} - 3 \frac{\cos(u-x)}{(u-x)^4} + 3 \frac{\sin(u-x)}{(u-x)^5} \right) \partial_x h(x)$$

where

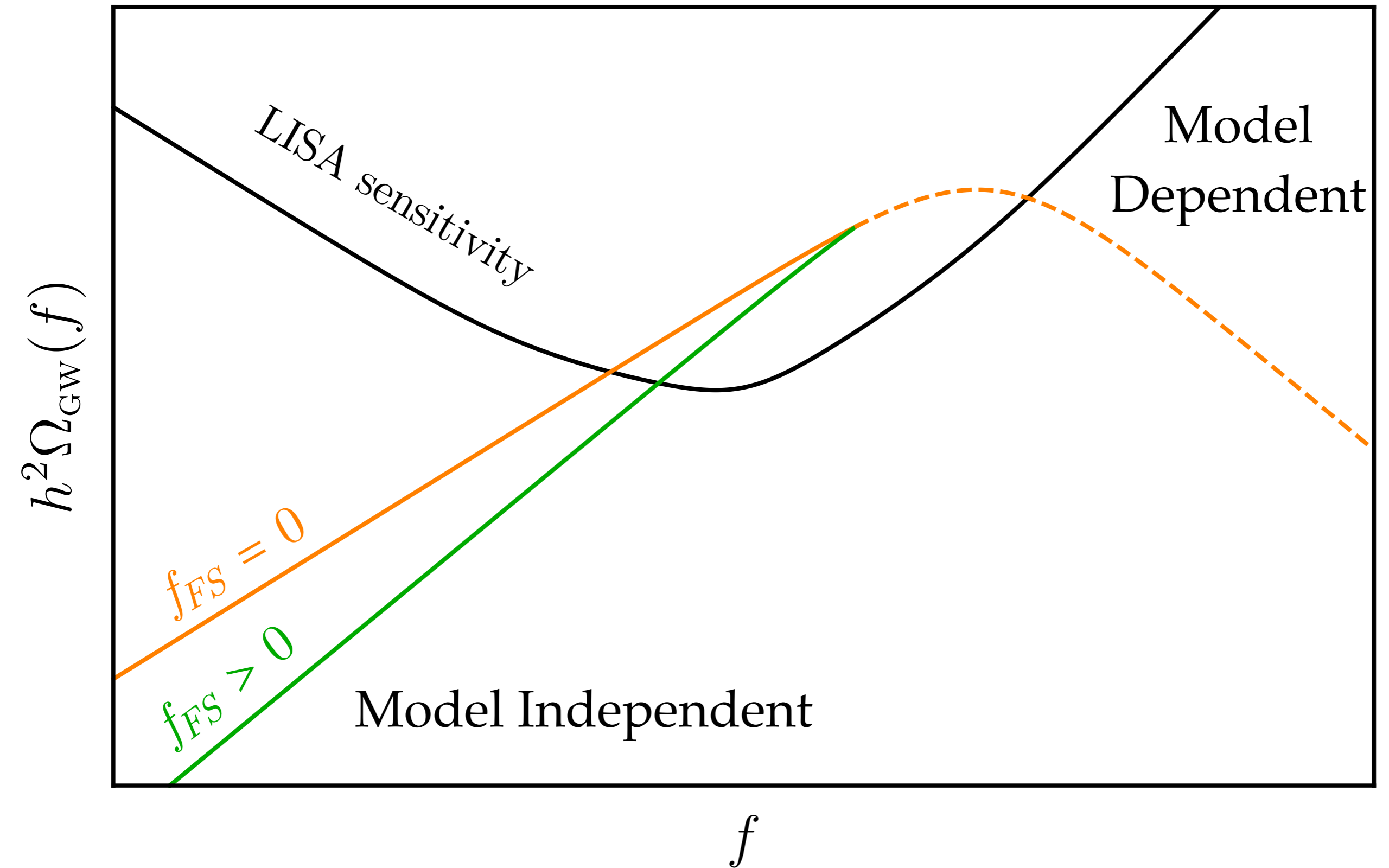
$$f_{\text{FS}} = \rho_{\text{FS}} / \rho_{\text{total}}$$

$$u = k\tau$$

# Free-streaming particles



# Frequency spectrum



# LISA Sensitivity

Estimate how accurately LISA can measure the free streaming fraction of the universe and its equation of state using Fisher Information Matrix

Assume GW source are the sound waves from a phase transition



# Intuition for Sensitivity

If you make  $N$  measurements, sensitive to  $1/\sqrt{N}$  effects

$$N \approx f T$$

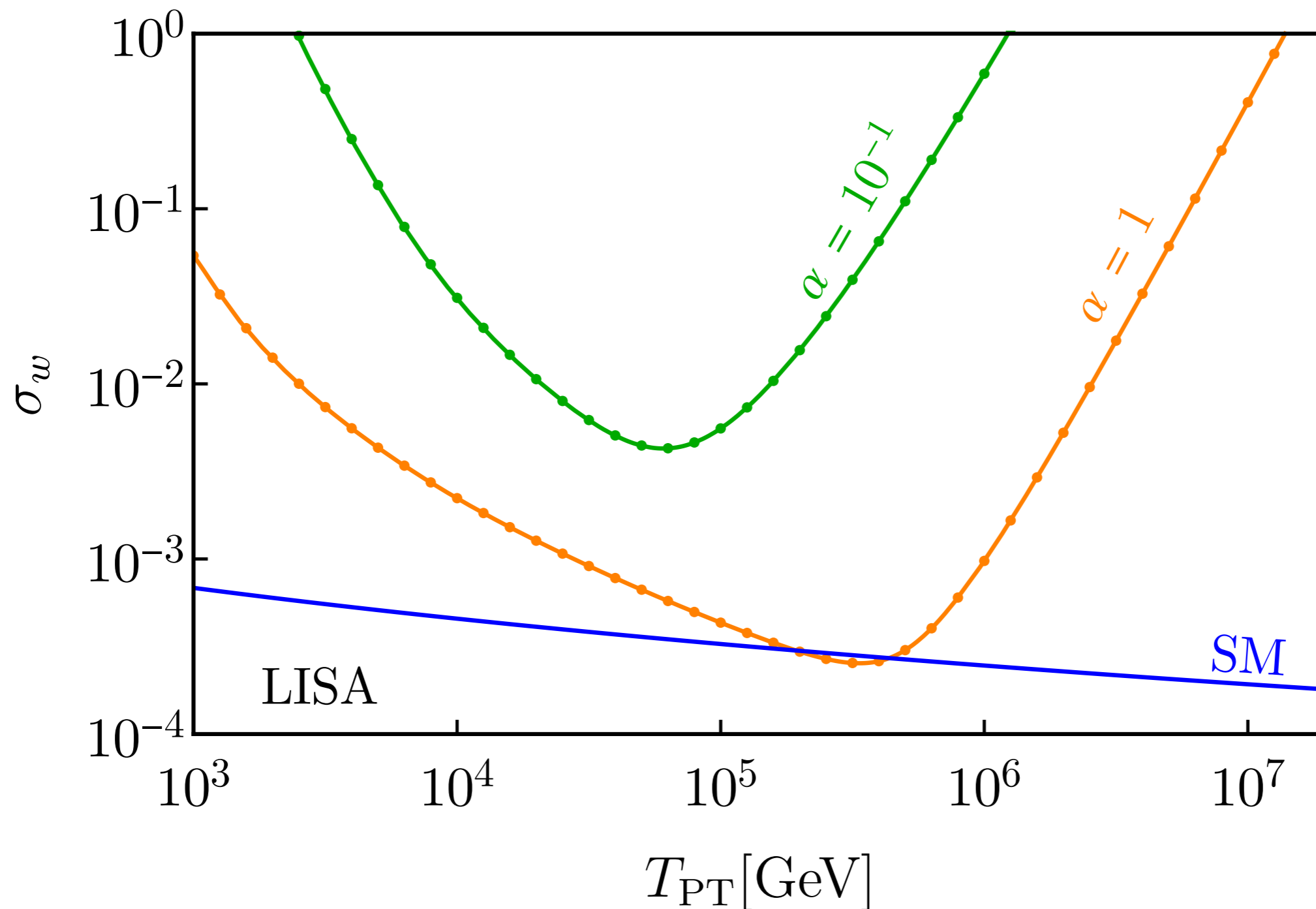
$N$  measurements given a frequency  $f$   
and time  $T$

$$1/\sqrt{\text{mHz year}} \sim 10^{-3}$$

Expect  $10^{-3}$  level sensitivity

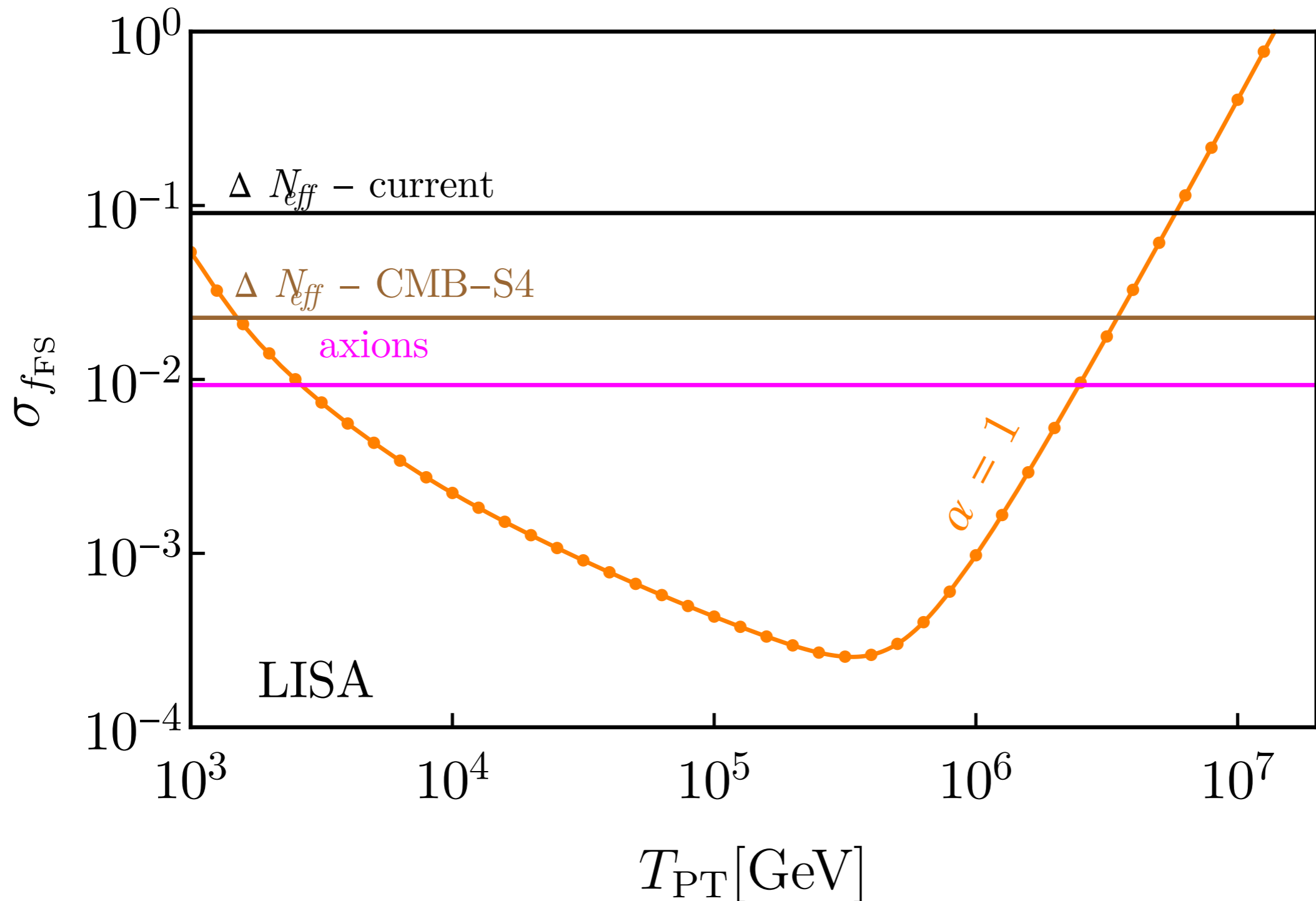
# LISA Sensitivity

Can measure equation of state to an accuracy of  $10^{-3}$



# LISA Sensitivity

Can measure free streaming fraction to  $10^{-3}$



# Implications

Can measure free streaming fraction  $10^2 - 10^3$  times better than current constraints at  $T \sim 10^3 - 10^6$  GeV

Reaches some interesting benchmarks such as free streaming axions

Can measure equation of state to an accuracy of  $10^{-3}$  at  $T \sim 10^5 - 10^6$  GeV

$$\delta w \sim 10^{-3} - 10^{-4}$$

**Is a Game Changer**

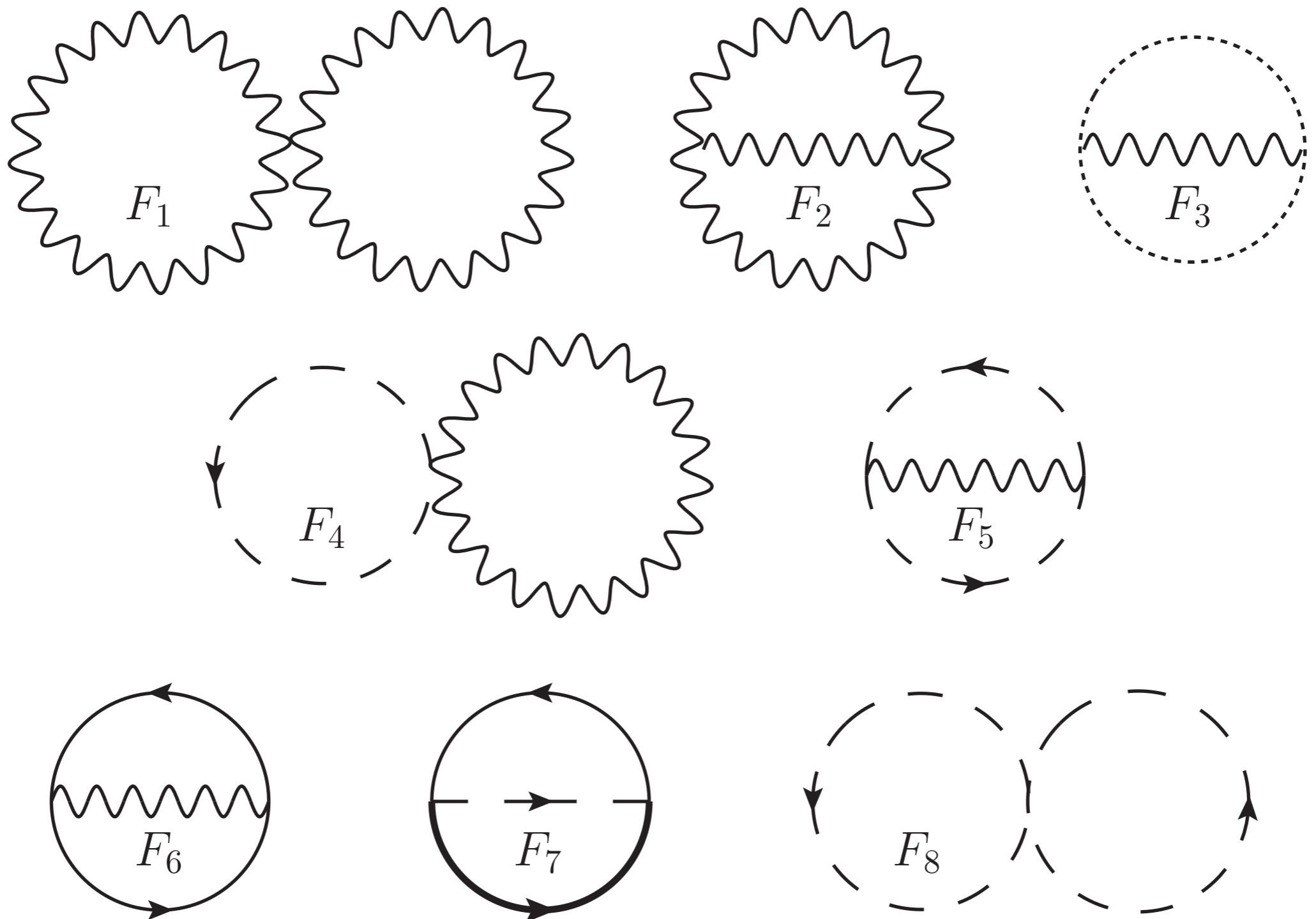
# Implications

What is the equation of state of the early universe?

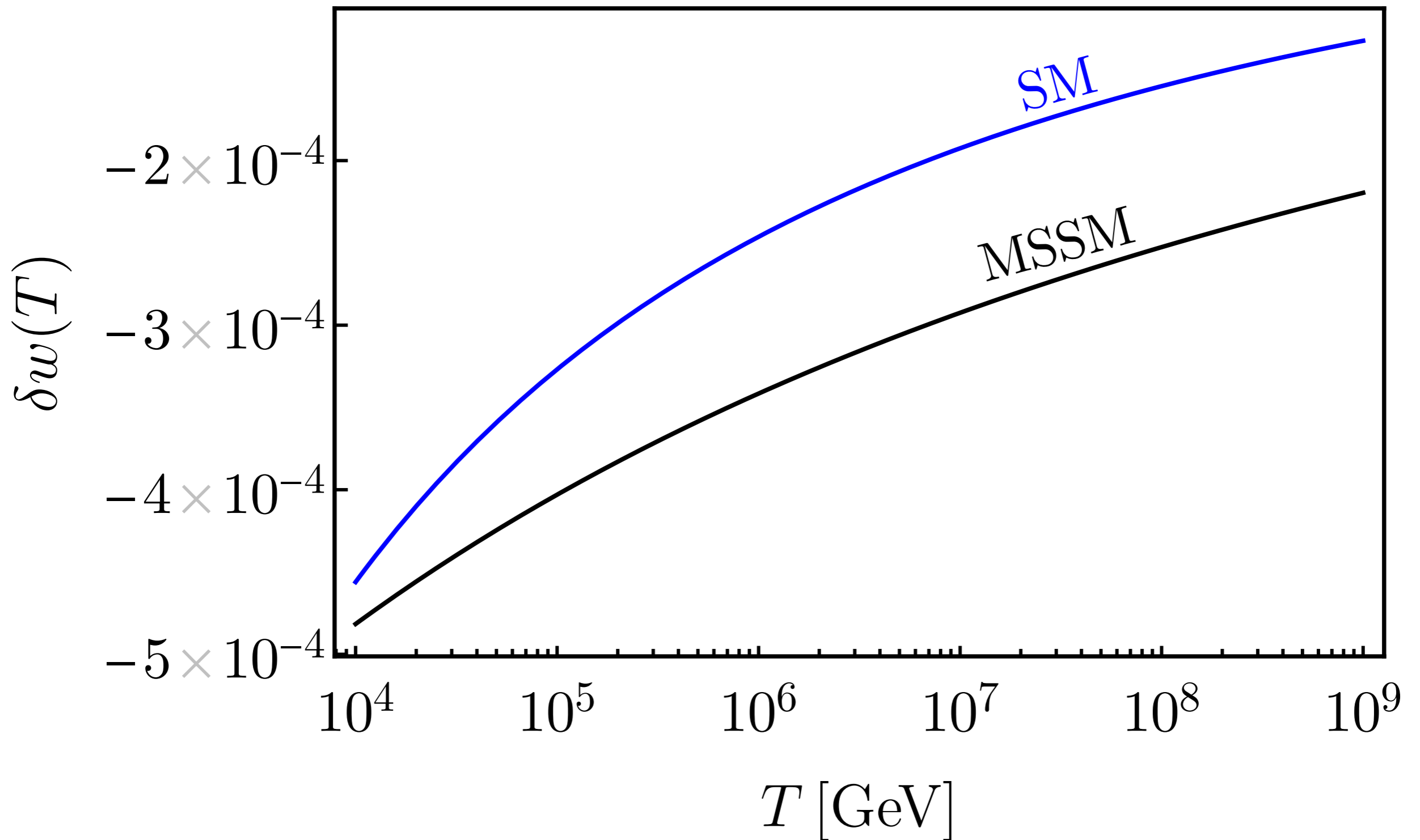
$$T_{\mu}^{\mu} \sim \rho - 3p \sim (1 - 3w)g_{\star} \sim \beta_{QCD}$$

Deviations from  $w = 1/3$  give us information about beta functions and number of degrees of freedom

# Implications



# Implications



# Implications

$$\delta w \sim 10^{-3} - 10^{-4}$$

You are  $O(1)$  sensitive to changes in the SM values of the  
QCD beta function and  $g_\star$

Many BSM models predict changes at this  
order

Any future detector only does better



# Conclusion

Low frequency part of a stochastic GW spectrum is model independent and sensitive to free streaming particles and EOS

$$\delta\omega, \delta f_{FS} \sim 10^{-3} - 10^{-4}$$

LISA can measure

$$T \sim 10^{5 \pm 1} \text{ GeV}$$

Most interesting case may be seeing a radiation dominated universe!