Effective field theory for cosmological phase transitions

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Cosmological first-order phase transitions

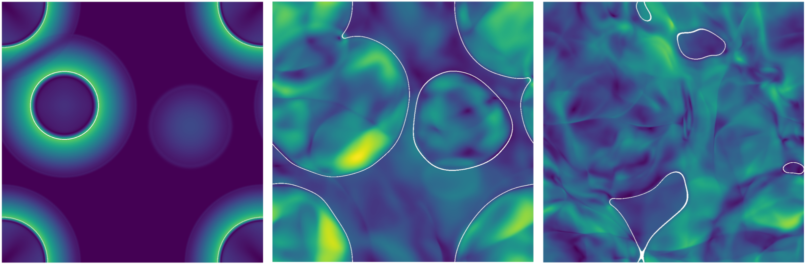


Figure: Cutting et al. 1906.00480.

- Transition dynamics
  - Bubbles nucleate, expand and collide
  - This creates fluid flows, and gravitational waves
- Observable remnants $\Rightarrow$ new probe of particle physics
  Such as $(n_B - n_{\bar{B}})/s$, stochastic gravitational wave backgrounds, topological defects, magnetic fields, . . .

Gravitational waves

- Gravitational waves directly observed by LIGO/Virgo →
- Future experiments will significantly improve sensitivity ↓

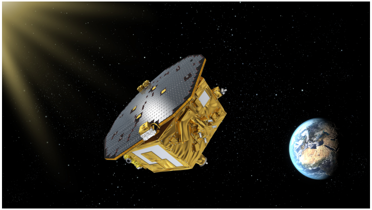


Figure: LISA Pathfinder

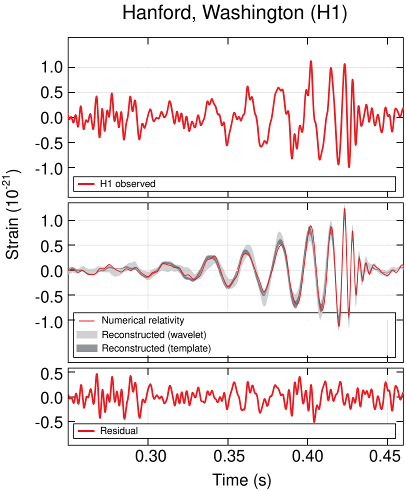


Figure: GW150914 1602.03837

Gravitational waves from phase transitions: the pipeline

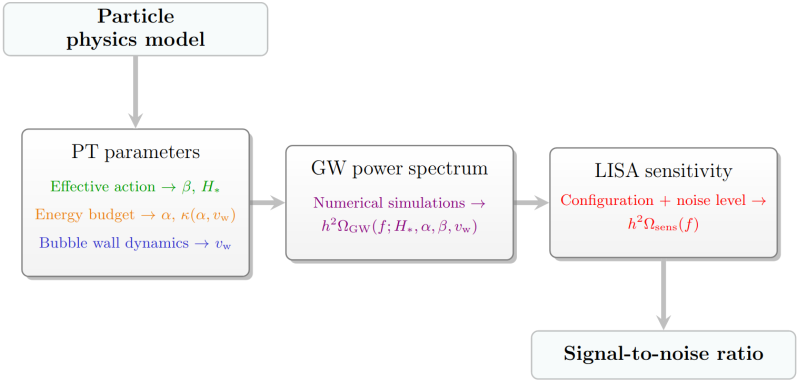


Figure: The Light Interferometer Space Antenna (LISA) pipeline $\mathscr{L} \to \text{SNR}(f)$, Caprini et al. 1910.13125.

Gravitational waves from phase transitions: the pipeline

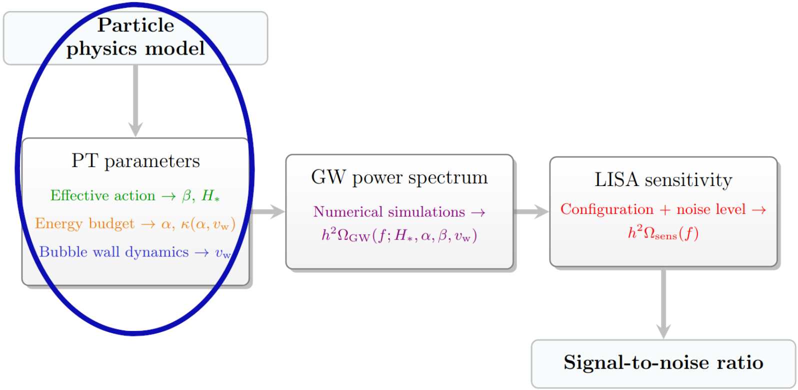
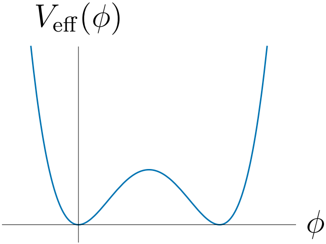
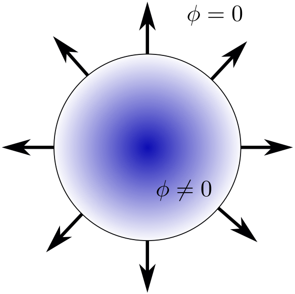


Figure: The Light Interferometer Space Antenna (LISA) pipeline $\mathscr{L} \to \text{SNR}(f)$, Caprini et al. 1910.13125.

Phase transition parameters





Equilibrium (hom.)

- order of transition
- T_c, critical temperature
- $\Delta\theta_c$, latent heat
- c_s^2, sound speed

Near-equilibrium

- Γ, bubble nucleation rate $\Rightarrow T_*$, $\Delta\theta_*$, α_*, β/H_*

Nonequilibrium

- v_w, bubble wall speed

Standard approach to computing parameters

One-loop resummed approximation is based on

$$V_{\text{eff}}(\phi, T) = V_{\text{tree}}(\phi) + \underbrace{\frac{1}{2}\int_{P\in\mathbb{R}^4} \log(P^2 + V''_{\text{tree}})}_{\text{Coleman-Weinberg}}$$

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$$\underbrace{-T\int_{p\in\mathbb{R}^3} \log\left[1 + n_{\text{B}}\left(\frac{\sqrt{p^2 + V''_{\text{tree}}}}{T}\right)\right]}_{\text{thermal}}$$

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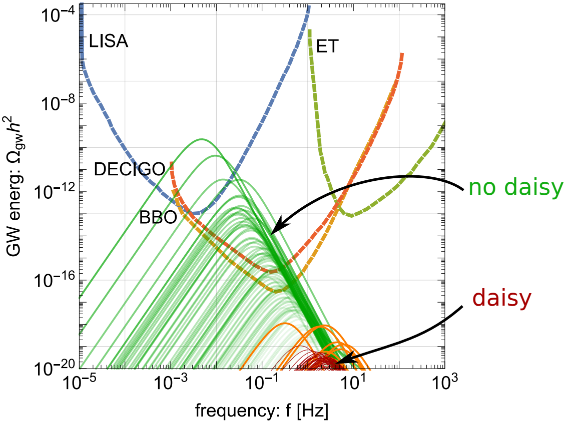
$$\underbrace{-\frac{T}{12\pi}\left((V''_{\text{tree}} + \Pi_T)^{3/2} - (V''_{\text{tree}})^{3/2}\right)}_{\text{daisy}}.$$

One then takes $\Re V_{\text{eff}}$ to determine the phases and critical bubbles (discarding $\Im V_{\text{eff}}$, fixing gauge parameter, fixing renormalization scale, and ignoring derivative corrections).

Theoretical uncertainties

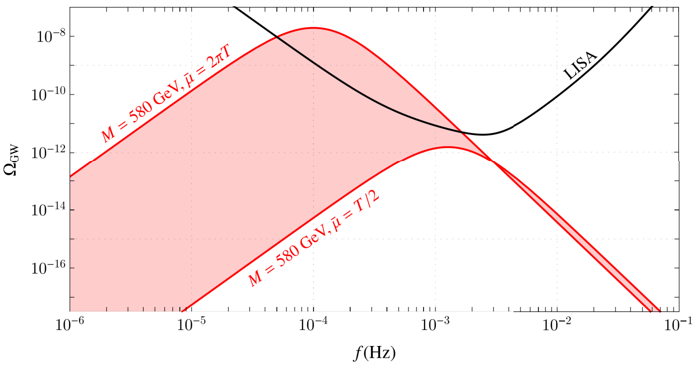


GW signals in two different 1-loop approximations for

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \frac{a_2}{2}(\Phi^\dagger\Phi)\sigma^2 + \frac{1}{2}(\partial\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{b_4}{4}\sigma^4$$

Carena, Liu & Wang 1911.10206

Theoretical uncertainties

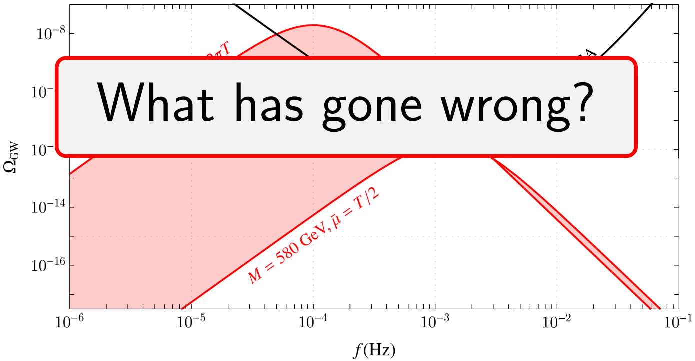


Renormalisation scale dependence of GW spectrum at one physical parameter point for

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \frac{1}{M^2}(\Phi^\dagger \Phi)^3.$$

Croon, OG, Schicho, Tenkanen & White 2009.10080

Theoretical uncertainties



What has gone wrong?

Renormalisation scale dependence of GW spectrum at one physical parameter point for

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Croon, OG, Schicho, Tenkanen & White 2009.10080

What has gone wrong?

Possible sources of theoretical uncertainties:

- nonperturbativity? — Linde '80
- inconsistencies? — E. Weinberg & Wu '87, E. Weinberg '92
- higher order perturbative corrections? — Arnold & Espinosa '92
- gauge dependence or infrared divergences? — Laine '94
- renormalisation scale dependence? — Farakos et al. '94
- ...

Overview

1. Motivation and theoretical uncertainties
2. Scale hierarchies in phase transitions
3. Problems and effective field theory solutions
4. Conclusions

Scale hierarchies in phase transitions

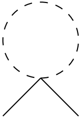
A hierarchy problem

Let's assume there is some very massive particle χ, $M_\chi \gg m_H$, coupled to the Standard Model Higgs Φ like

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + g^2 \Phi^\dagger \Phi \chi^\dagger \chi + \mathscr{L}_\chi .$$

If we integrate out χ, we find that the Higgs mass parameter gets a correction of the form

$$(\Delta m_H^2)\Phi^\dagger\Phi \ = $$



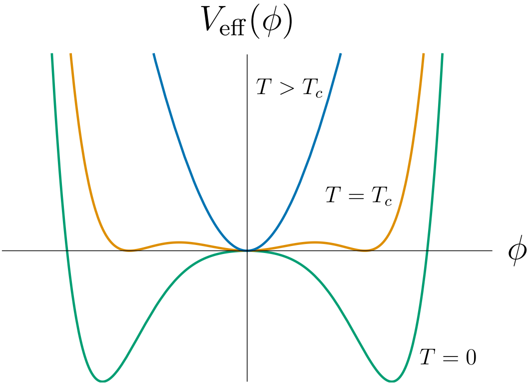
$$,$$

$$\sim g^2 M_\chi^2 \Phi^\dagger \Phi \ .$$

Relevant operators in the IR get large contributions from the UV,

$$\frac{\Delta m_H^2}{m_H^2} \sim g^2 \left(\frac{M_\chi}{m_H} \right)^2 .$$

Phase transitions



For there to be a phase transition, thermal/quantum fluctuations should modify the potential at leading order,

$$V_{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}}.$$

Hierarchies in phase transitions

So, for there to be a phase transition, we need

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 N \left(\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}}\right)^{\sigma} \overset{!}{\sim} 1,$$

where $\sigma > 0$ for relevant operators.

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$\Rightarrow$ either:

(i) $g^2 N \gtrsim 1$, i.e. strong coupling

(ii) $\Lambda_{\mathsf{fluct}} \gg \Lambda_{\mathsf{tree}}$, i.e. scale hierarchy

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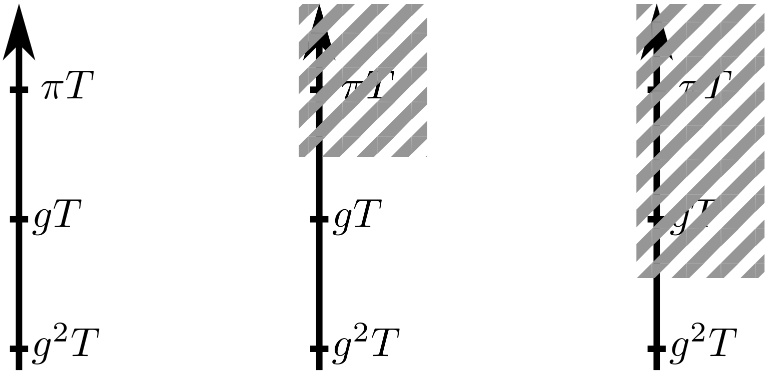
Perturbative phase transitions require scale hierarchies!

Problems and effective field theory solutions

High temperature effective field theory



$$\mathscr{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi} \not{D} \psi$$

$$\mathscr{L}_{\text{eff}} = \frac{1}{4} F_{ij} F_{ij} + (D_i A_0)^2 + m_D^2 A_0^2 + \lambda A_0^4$$

$$\mathscr{L}_{\text{eff}} = \frac{1}{4} F_{ij} F_{ij}$$

Farakos et al. '94, Braaten & Nieto '95, Kajantie et al. '95

Problem: renormalisation scale dependence

At zero temperature, the one-loop effective potential is renormalisation group invariant

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}(V_{\text{tree}} + V_{\text{1-loop}}) = 0.$$

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$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}(V_{\text{tree}} + V_{\text{1-loop}}) = 0.$$

But, at high temperatures this fails, even at leading order

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}(V_{\text{tree}} + V_{\text{1-loop}}^{\text{thermal}}) \neq 0.$$

The problem can be traced to the scale hierarchy $\pi T \gg m$, and to

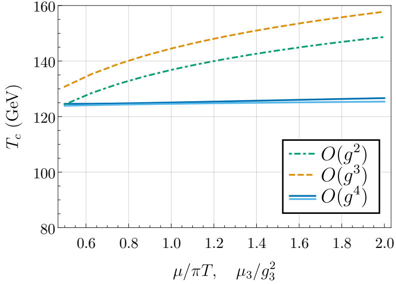
$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}\left(\frac{1}{2}\Pi_T\phi^2\right).$$

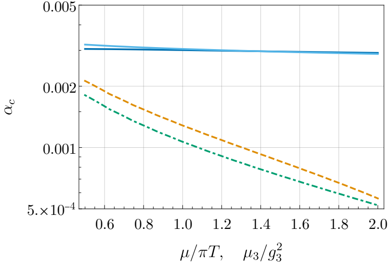
OG & Tenkanen, 2104.04399

EFT solution: renormalisation scale independence

Solution is clear in EFT approach, and requires all terms in V_{eff} up to $O(g^4 T^4)$, including (resummed) two-loop diagrams.

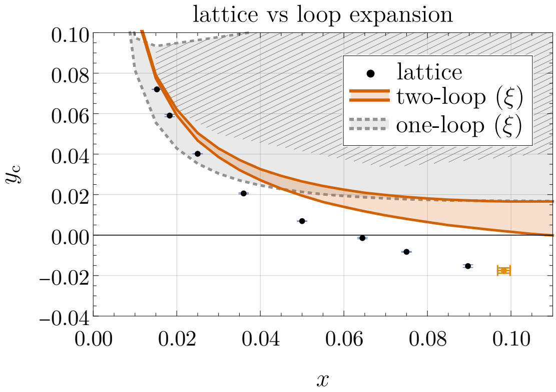
OG & Tenkanen, 2104.04399





$$\mathscr{L} = \mathscr{L}_{\text{SM}} + \frac{a_2}{2}(\Phi^\dagger\Phi)\sigma^2 + \frac{1}{2}(\partial\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{b_4}{4}\sigma^4$$

Problem: gauge dependence



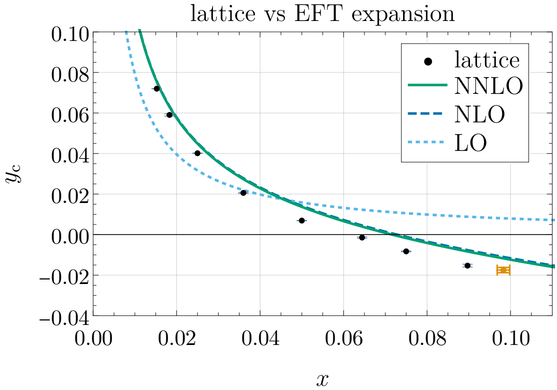
- One-loop V_{eff} is strongly gauge dependent at high T, here demonstrated for

$$\mathscr{L} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + (D_\mu\Phi)^\dagger D_\mu\Phi + m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

- Naive solutions lead to infrared divergences or inconsistencies.

Laine hep-ph/9411252, Patel & Ramsey-Musolf 1101.4665

EFT solution: gauge independence



EFT approach provides exact order-by-order gauge invariance.

Ekstedt, OG & Löfgren 2205.07241

(see also Löfgren et al. 2112.05472, Hirvonen et al. 2112.08912)

Problem: what is the thermal nucleation rate?

Linde '81

$$\Gamma_{\text{Linde}} \equiv T \left(\frac{S_3[\phi_{\mathrm{B}}]}{2\pi T} \right)^{3/2} \left| \frac{\det{}' S_3''[\phi_{\mathrm{B}}]}{\det S_3''[\phi_{\mathrm{F}}]} \right|^{-1/2} e^{-\frac{1}{T}\int_{\mathbb{R}^3}\left[\frac{1}{2}(\nabla\phi_{\mathrm{B}})^2 + V_T(\phi_{\mathrm{B}})\right]}$$

where V_T is a thermal effective potential.

Affleck '81

$$\Gamma_{\text{Affleck}} \equiv \frac{1}{2\pi} \left(\frac{S_3[\phi_{\mathrm{B}}]}{2\pi T} \right)^{3/2} \left(\frac{\det^{+} S_3''[\phi_{\mathrm{B}}]}{\det S_3''[\phi_{\mathrm{F}}]} \right)^{-1/2} e^{-\frac{1}{T}\int_{\mathbb{R}^3}\left[\frac{1}{2}(\nabla\phi_{\mathrm{B}})^2 + V_{\text{tree}}(\phi_{\mathrm{B}})\right]}$$

where V_{tree} is the tree-level potential at $T = 0$.

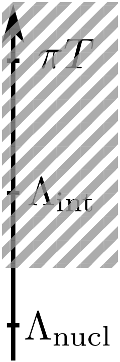
EFT solution: match to classical nucleation theory

First-principles definition of thermal nucleation rate:

1. Integrate out all energy scales $\Lambda \gg \Lambda_{\text{nucl}} \sim m_{\text{nucl}}$.
2. This yields a classical, statistical EFT,

$$S_{\text{nucl}}[\phi] = \int d^3x \left[\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m_{\text{nucl}}^2\phi^2 + \dots\right].$$

3. Classical nucleation theory then gives rate unambiguously.



OG & Hirvonen 2108.04377

EFT solution: what about the previous inconsistencies?

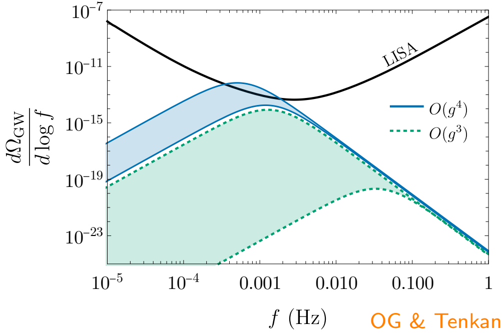
Resolutions in EFT approach:

- $S_{\rm nucl}[\phi]$ is real for all ϕ
- Derivative expansion justified by $\Lambda_{\rm nucl} \ll \Lambda$
- Modes counted only once in path integral,

$$\Gamma_{\mathsf{EFT}} = \underbrace{\frac{\kappa}{2\pi}\left(\frac{S_{\mathsf{nucl}}[\phi_{\rm B}]}{2\pi T}\right)^{3/2}\left|\frac{\det' S''_{\mathsf{nucl}}[\phi_{\rm B}]}{\det S''_{\mathsf{nucl}}[\phi_{\rm F}]}\right|^{-1/2}}_{\text{modes } E<\Lambda} \underbrace{e^{-S_{\rm nucl}[\phi_{\rm cb}]}}_{\text{modes } E>\Lambda}.$$

EFT methods ensure result is independent of Λ.

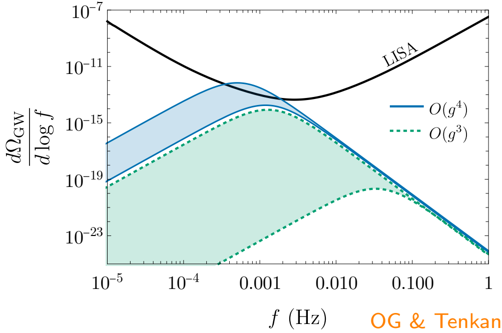
Conclusions



OG & Tenkanen 2104.04399

- Phase transitions may produce observable gravitational waves
- Large theoretical uncertainties in one-loop approximation
- EFT solves many problems of standard approach
- More work needed for non-equilibrium quantities

Conclusions



OG & Tenkanen 2104.04399

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Thanks for listening!

Backup slides

High temperature effective field theory

Equilibrium thermodynamics

- Can be formulated in $\mathbb{R}^3 \times S^1$.

- Fields are expanded into Fourier (Matsubara) modes:

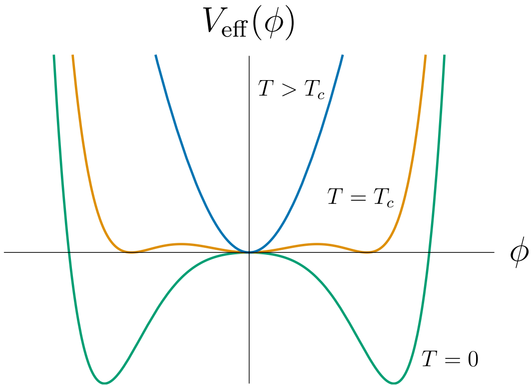
$$\Phi(x, \tau) = \sum_{n \text{ even}} \phi_n(x) e^{i(n\pi T)\tau} \leftarrow \text{boson}$$

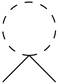
$$\Psi(x, \tau) = \sum_{n \text{ odd}} \psi_n(x) e^{i(n\pi T)\tau} \leftarrow \text{fermion}$$

- Masses of Matsubara modes are

$$m_n^2 = m^2 + (n\pi T)^2$$

Thermal mass hierarchies

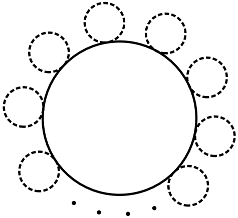


$m^2_{0,\text{eff}} = m^2 +$  $= m^2 + \#g^2 T^2$

- At $T \gg T_c$, thermal corrections dominate, so $m_{0,\text{eff}} \sim gT$ which is much less than πT.
- Near $T = T_c$, cancellations typically give $m_{0,\text{eff}} \ll gT$.

Resumming UV problems

$$\mathscr{L}_0 = \underbrace{\frac{1}{2}(\nabla\phi_0)^2 + \frac{1}{2}m^2\phi_0^2}_{\mathscr{L}_{\text{free}}} + \underbrace{\frac{1}{4!}g^2\phi_0^4}_{\mathscr{L}_{\text{int}}}.$$



Resummation by changing split between $\mathscr{L}_{\text{free}}$ and $\mathscr{L}_{\text{int}}$,

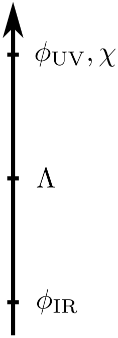
$$\mathscr{L}_{\text{free}} \to \mathscr{L}_{\text{free}} + \frac{1}{2}(m_{0,\text{eff}}^2 - m^2)\phi_0^2,$$
$$\mathscr{L}_{\text{int}} \to \mathscr{L}_{\text{int}} + \frac{1}{2}(m^2 - m_{0,\text{eff}}^2)\phi_0^2.$$

Top-down EFT

- Split degrees of freedom $\{\phi, \chi\}$ based on energy $\rightarrow$

- Integrate out the UV modes:

$$\int \mathcal{D}\phi \int \mathcal{D}\chi \ e^{-S[\phi,\chi]} = \int \mathcal{D}\phi_{\mathsf{IR}} \left(\int \mathcal{D}\phi_{\mathsf{UV}} \mathcal{D}\chi \ e^{-S[\phi,\chi]} \right)$$
$$= \int \mathcal{D}\phi_{\mathsf{IR}} \ e^{-S_{\mathsf{eff}}[\phi_{\mathsf{IR}}]}$$



- Careful power-counting cancels dependence on Λ.

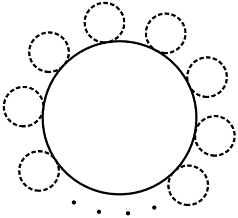
Burgess '21, Hirvonen '22

Resummations with EFT

By first integrating out the UV modes

$$S_{\text{eff}}[\phi_{\text{IR}}] = S_\phi[\phi_{\text{IR}}] - \log \int \mathcal{D}\phi_{\text{UV}}\mathcal{D}\chi \; e^{-S_\chi[\chi]-S_{\chi\phi}[\phi,\chi]},$$
$$\approx S_\phi[\phi_{\text{IR}}] + \int_x \frac{1}{2}(m_{\text{eff}}^2 - m^2)\phi_{\text{IR}}^2,$$

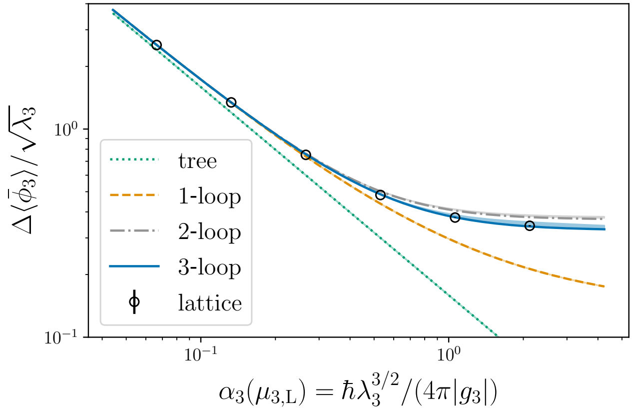
the daisy resummations arise naturally.



So do all other resummations necessary to resolve UV problems (i.e. large contributions to IR quantities from UV physics).

Lattice versus perturbation theory

Real scalar model

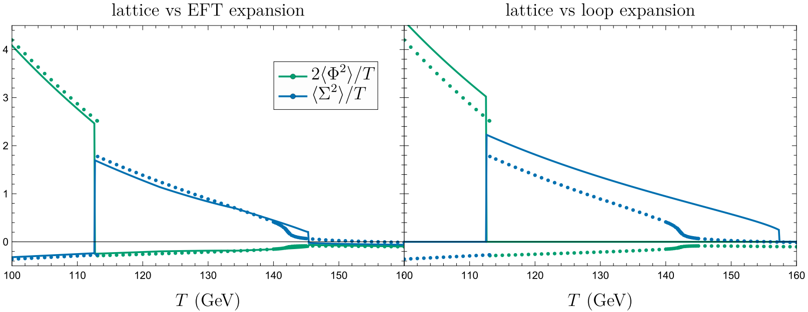


Perturbation theory converges towards the lattice for the jump in the scalar condensate in

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 + \frac{m_3^2}{2}\phi^2 + \frac{g_3}{3!}\phi^2 + \frac{\lambda_3}{4!}\phi^4$$

OG 2101.05528

Triplet extension of the Standard Model

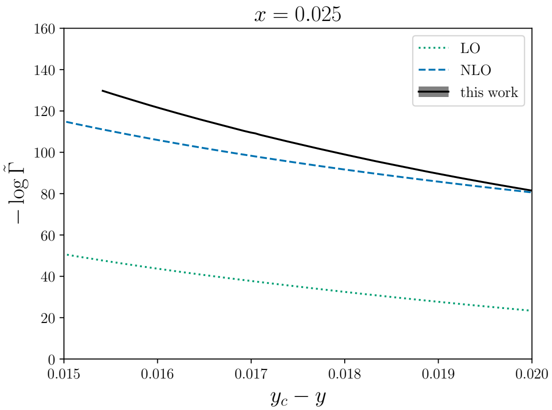


Perturbative EFT approach agrees well with lattice in triplet extension of Standard Model,

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \frac{a_2}{2}\Phi^\dagger\Phi\Sigma^a\Sigma^a + \frac{1}{2}D_\mu\Sigma^a D_\mu\Sigma^a + \frac{m_\Sigma^2}{2}\Sigma^a\Sigma^a + \frac{b_4}{4}(\Sigma^a\Sigma^a)^2$$

OG & Tenkanen forthcoming

SU(2) Higgs model - bubble nucleation



Lattice versus perturbative approaches for the nucleation rate in the SU(2) Higgs model

$$\mathscr{L} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + (D_\mu\Phi)^\dagger D_\mu\Phi + m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

OG, Güyer & Rummukainen 2205.07238